SHUBHAM GUPTA
180749
HW-5

1). $X_{n+1} = X_n - \min(X_n, C) + A_{n+1}$
$= \max(X_n - C, 0) + A_{n+1}$.

X_{n+1} only depends on X_n, hence it is a markov chain.

$$p_{i,j}^{n,n+1} = P(\max(X_n - c, 0) + A_{n+1} = j \mid X_n = i)$$

$$= \begin{cases} P(A_{n+1} = j \mid X_n = i) & i \le c \\ P(A_{n+1} = j + c - i \mid X_n = i) & i > c \end{cases}$$

$$= \begin{cases} P(A_{n+1} = j), & i \le c \\ P(A_{n+1} = j + c - i) & i > c \end{cases}$$

given, $P(A_1 = k) = 0.2 \quad \forall\ k = 0, 1$
$= 0.3 \quad \forall\ k = 2, 3$

a) $c = 2$

$$p_{i,j}^{n,n+1} = \begin{cases} P(A_{n+1} = j), & i \le c \\ P(A_{n+1} = j + c - i) & i > c \end{cases}$$

$p_{i,j}^{n,n+1}$	0	1	2	3	4	— —
0	0.2	0.2	0.3	0.3	0	
1	0.2	0.2	0.3	0.3	0	
2	0.2	0.2	0.3	0.3	0	
3	0	0.2	0.2	0.3	0.3	
4	0	0	0.2	0.2	0.3	
⋮						

b) $c = 3$

$$p_{i,j}^{m,m+1} = \begin{cases} P(A_{m+1} = j), & i \le c \\ P(A_{m+1} = j + c + i) & i > c \end{cases}$$

$p_{i,j}^{m,m+1}$	0	1	2	3	4
0	0·2	0·2	0·3	0·3	0
1	0·2	0·2	0·3	0·3	0
2	0·2	0·2	0·3	0·3	0
3	0·2	0·2	0·3	0·3	0
4	0	0·2	0·2	0·3	0·3