Lecture - 15

Motion of particles through fluids

$$\frac{du}{dt} = a_e \left(\frac{\rho_p - \rho}{\rho_p}\right) - \frac{C_D u^2 \rho A_p}{2m} \longrightarrow (*)$$

Case-1 $\quad a_e = g$

Spherical particle

$$u_t = \sqrt{\frac{4}{3} \frac{g(\rho_p - \rho) D_p}{\underbrace{C_D \cdot \rho}}} \longrightarrow (*) \quad \text{Generalized expression}$$

$\hookrightarrow f(u_t)$

→ For stoke's regime, $Re_p << 1$

$$\boxed{u_t = \frac{g D_p^2 (\rho_p - \rho)}{18 \mu}}$$

→ For $Re_p - 10^3 - 2 \times 10^5 \rightarrow C_D = 0.44$

$$* \quad u_t = 1.75 \sqrt{\frac{g D_p (\rho_p - \rho)}{\rho}} \longrightarrow \text{Newton's flow regime}$$

Criteria for settling regime of spherical particles:

For Stoke's law:

$$Re_p = \frac{D_p u_t \rho}{\mu} = D_p \frac{g D_p^2 (\rho_p - \rho)}{18\mu} \cdot \frac{\rho}{\mu}$$

$$Re_p = \frac{D_p^3 g \rho (\rho_p - \rho)}{18\mu^2}$$

$$18 Re_p = \Bigg[\underbrace{D_p \left(\frac{g \rho (\rho_p - \rho)}{\mu^2} \right)^{1/3}}_{K} \Bigg]^3$$

$$18 Re_p = K^3$$

$$\boxed{K = D_p \left(\frac{g \rho (\rho_p - \rho)}{\mu^2} \right)^{1/3}}$$

$$\Rightarrow \quad K = 18^{1/3} Re_p^{1/3}$$

upper limit

$$Re_p = 1$$

$$K = 18^{1/3} = 2.6$$

$$\boxed{K < 2.6}$$ Stoke's law applies

For Newton's flow regime,

$$Re_p - 10^3 - 2\times10^5$$

$$Re_p = \frac{D_p u_t \rho}{\mu} = 1.75 \left\{ D_p \left[\frac{g\rho(\rho_p-\rho)}{\mu^2}\right]^{1/3} \right\}^{1.5}$$

$\Rightarrow Re_p = 1.75\, K^{1.5}$

Lower limit, $Re_p = 1000 \Rightarrow K = 68.9$

upper limit, $Re_p = 2\times10^5 \Rightarrow K = 2360$

if $K = 68.9 - 2360$, then Newton's flow regime applies

For intermediate range, $\boxed{K = 2.6 - 68.9}$

Summary:

	Terminal velocity	k-criteria
* For all flow regimes	$u_t = \sqrt{\dfrac{4g(\rho_p-\rho)D_p}{3C_D\rho}}$	$k = D_p\left(\dfrac{g\rho(\rho_p-\rho)}{\mu^2}\right)^{1/3}$
* For Stoke's flow regime	$u_t = \dfrac{gD_p^2(\rho_p-\rho)}{18\mu}$	$k < 2.6$
* For Newton's flow regime	$u_t = 1.75\sqrt{\dfrac{gD_p(\rho_p-\rho)}{\rho}}$	$k = 68.9 - 2360$

Example-1 Calculate drag force on a submerged sphere (42 mm) in an air stream

$\rho_{air} = 1.137\ kg/m^3$

$\mu_{air} = 1.9 \times 10^{-5}\ Pa.s$

$u_t = 23\ m/s$

$Re_p - 5.7 \times 10^4 > 10^3$

$C_D = 0.44$

$F_D = \frac{1}{2} C_D \rho A_p u_t^2$

$\boxed{F_D = 0.18\ N}$

Example : 2

Oil droplet in air at 310 K & 101.3 kPa. Assume rigid spherical droplet of dia 20 μm & density 900 kg/m³. Calculate its terminal velocity if $\rho_{air} = 1.137$ kg/m³, $\mu_{air} = 1.9 \times 10^{-5}$ Pa.s.

$$K = D_p \left(\frac{g \rho (\rho_p - \rho)}{\mu^2} \right)^{1/3}$$

$$\boxed{K = 0.61 < 2.6} \Rightarrow \text{Stoke's law applicable}$$

$$u_t = \frac{g D_p^2 (\rho_p - \rho)}{18 \mu}$$

$$\Rightarrow \boxed{u_t = 0.0103 \text{ m/s}}$$

Example : 3

Estimate terminal velocity for −80+100 mesh particles of limestone ($\rho_p = 2800\ kg/m^3$) falling in water at 30°C.

$\rho = 995.7\ kg/m^3$, $\mu = 0.801\ cP$

$= 0.801 \times 10^{-3}\ Pa \cdot s$

Average size, $D_p = \dfrac{0.147 + 0.175}{2}$

$D_p = 0.161\ mm$

$K = 4.896$

$2.6 < K < 68.9 \Rightarrow$ Generalized expression

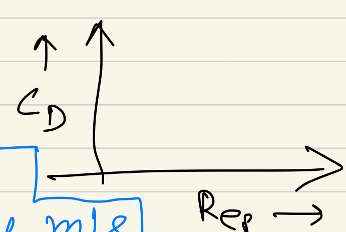
Assume Re_p,

$$u_t = \sqrt{\frac{4}{3}\frac{g(\rho_p - \rho) D_p}{C_D \rho}}$$

$$Re_p = \frac{D_p u_t \rho}{\mu}$$

$Re_p = 2.5$

$u_t = 0.014\ m/s$



Centrifugal settling:

$$F_e = m r \omega^2$$

* $\frac{du}{dt}$ is very small compared to R.H.S.

$$u_t = \omega \sqrt{\frac{2r(\rho_p - \rho) m}{A_p \rho_p C_D \rho}}$$

For spherical particle,

$$A_p = \frac{\pi}{4} D_p^2$$

$$m = \frac{\pi}{6} D_p^3 \rho_p$$

$$\Rightarrow \boxed{u_t = \sqrt{\frac{\frac{4}{3}(r\omega^2)(\rho_p - \rho) D_p}{g\, C_D}}}$$

$g \rightarrow r\omega^2$

* $$\frac{u_{t,c}}{u_{t,g}} = \frac{r\omega^2}{g}$$ (For Stoke's regime)

$$\frac{u_{t,c}}{u_{t,g}} = \sqrt{\frac{r\omega^2}{g}}$$ (For Newton's regime)