IME625: Introduction to Stochastic Processes Quiz-1 Part-1, January 31, 2022

1. Consider the transition probability matrix to the right. It depicts X_n – the wealth of a player after n rounds in a game of repeated gambling. Consider $X_0 = 2$ to be the starting

wealth. The game is deemed to have ended if $X_n = 0$ or 4 for some $n \ge 1$. Ending with 0 means loss and with 4 means win.

| p_{ij} | 0 | 1 | 2 | 3 | 4 |
|----------|-----|-----|-----|-----|-----|
| 0 | 1 | 0 | 0 | 0 | 0 |
| 1 | 0.4 | 0 | 0.6 | 0 | 0 |
| 2 | 0 | 0.4 | 0 | 0.6 | 0 |
| 3 | 0 | 0 | 0.4 | 0 | 0.6 |
| 4 | 0 | 0 | 0 | 0 | 1 |

a) Determine the probability of win by the 5th round.

[Marks: 1x5]

- b) Given the event in (a), determine the probability that the player did not lose a round.
- c) Given that the game ends by the 5th round, determine probability of the event in (b).
- d) Given that the game does not end by the 5^{th} round, determine mass function of X_4 .
- e) Determine the probability that the player is in a better position at the end of 3rd round.

Hint: Think in terms of path and remember that $X_0 = 2$ is known.