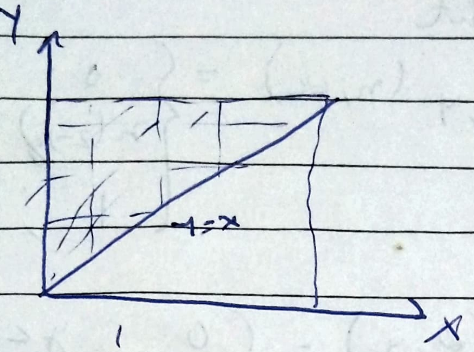
SHUBHAM GUPTA
180749

HW-3

1.)




$$\int_0^1 \int_x^1 f_{X,Y}(x,y)\, dy\, dx = 1$$

$$f_{X,Y}/2 = 1 \Rightarrow f_{X,Y} = 2$$
in shaded region

$$F_{X,Y}(x,y) = \begin{cases} 2\left\{\frac{1}{2}y^2 - \frac{1}{2}(y-x)^2\right\} = x(2y-x) & ; 0 \le x \le 1,\ 0 \le y \le 1,\ y > x \\ 0, & x<0, y<0 \\ 0, & x>0, y<0 \\ 0, & x<0, y>0 \\ y^2, & 0<y<1, x \ge y \\ x(2-x), & y>1, 0<x<1 \\ 1, & y>1, x>1 \end{cases}$$

$$f_{X,Y}(x,y) = \begin{cases} 2 ; & 0 \le x \le 1,\ 0 \le y \le 1,\ y > x \\ 0 ; & x<0, y<0 \\ 0 ; & x>0, y<0 \\ 0 ; & x<0, y>0 \\ 0 ; & 0 \le y < 1, x \ge y. \\ 0 ; & y>1, 0<x<1 \\ 0 ; & y>1, x>1 \end{cases}$$

Marginal distribut

$$F_X(x) = F_{X,Y}(x, \infty) = \begin{cases} 0 & ; x<0 \\ x(2-x) & ; 0<x<1 \\ 1 & ; x>1 \end{cases}$$

$$F_Y(y) = F_{X,Y}(\infty, y) = \begin{cases} 0 & ; y<0 \\ y^2 & ; 0<y<1 \\ 1 & ; y>1 \end{cases}$$

$$f_X(x) = \begin{cases} 0 & ; x<0 \\ 2-2x & ; 0\leq x<1 \\ 0 & ; x>1 \end{cases}$$

$$f_Y(y) = \begin{cases} 0 & ; y<0 \\ 2y & ; 0<y<1 \\ 0 & ; y>1 \end{cases}$$

$$F_{X|Y=y}(x) = P(X\leq x \mid Y=y) = \frac{P(X\leq x, Y=y)}{P(Y=y)}$$

$$= \frac{\int_{-\infty}^{x} f_{X,Y}(x,y)\,dx}{f_Y(y)}$$

$$F_{X|Y=y}(x) = \begin{cases} 0 & ; x<0 \\ \dfrac{\int_0^x f_{X,Y}(x,y)\,dx}{f_Y(y)} & ; 0<x<1 \\ \dfrac{\int_0^1 f_{X,Y}(x,y)\,dx}{f_Y(y)} & ; x>1 \end{cases}$$

$$= \begin{cases} 0 & x<0 \\ 2x/f_Y(y) & 0<x<1 \\ 2/f_Y(y) & x>1 \end{cases}$$

$$F_{X|Y=y}(x) = \begin{cases} 0 & x<0 \\ x/y & 0 \le x < 1,\ 0<y<1 \\ x/2y & x>1,\ 0<y<1 \end{cases}$$

$$f_{X|Y=y}(x) = \frac{f_{X,Y}(x,y)}{f_Y(y)}$$

$$= \begin{cases} \frac{1}{y}, & 0<x<1,\ 0<y<1,\ y>x \\ 0; & \text{elsewhere} \end{cases}$$

$$E_Y[E[X|Y]] = \int_{-\infty}^{\infty} E[X|Y]\, f_Y(y)\, dy$$

$$= \int_0^y x\left(\frac{1}{y}\right) dx = \frac{y^2}{2y} = y/2$$

$$\int_0^1 \frac{y}{2}(2y)\, dy = \boxed{\frac{1}{3}}$$

$$E[X] = \int_0^1 x f(x)\, dx$$

$$= \int_0^1 x(2-2x)\, dx$$

$$= 1 - \frac{2}{3} = \boxed{\frac{1}{3}}$$

verified

$$\mathrm{cov}(X,Y) = E[XY] - E[X]E[Y]$$

$$E[XY] = \int_0^1 \int_x^1 xy\, f_{X,Y}(x,y)\, dy\, dx$$

$$= \int_0^1 \int_x^1 2xy\, dy\, dx$$

$$= \int_0^1 \frac{2x}{2}(1-x^2)\,dx$$

$$= \frac{1}{2} - \frac{1}{4} = \frac{1}{4}$$

$$E[Y] = 2/3.$$

$$\mathrm{cov}(x,y) = \frac{1}{4} - \frac{1}{3} \times \frac{2}{3} = \frac{1}{4} - \frac{2}{9}$$

$$= \frac{9-8}{36} = \frac{1}{36}$$ Ans