SHUBHAM GUPTA
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Q4 P2

i) $\mathrm{cov}(N(s), N(t) - N(r))$.

from IIP $N(s)$, $N(t) - N(r)$ are independent.

$\therefore \boxed{\mathrm{cov}(N(s), N(t) - N(r)) = 0}$

ii) $P(N(r) = 1, N(s) = 2, N(t) = 3)$

$= P(N(r) = 1, N(s) - N_r = 1, N(t) - N(s) = 1)$

$= \underbrace{P(N(r) = 1)}_{e^{-\lambda r} \lambda r} \underbrace{P(N(s) - N(r) = 1)}_{P(N(s-r) = 1) \text{ by IIP}} \cdot P(N(t) - N(s) = 1)$

$P(N(t-s) = 1)$ by IIP.

$e^{-\lambda r} \lambda r \quad e^{-\lambda(s-r)} \lambda(s-r) \quad e^{-\lambda(t-s)} \lambda(t-s)$

$\boxed{\lambda^3 r (s-r)(t-s) e^{-\lambda t}}$

iii) $P(N(s) = 1 \mid N(r) = 1, N(t) = 3)$

$P(N(s) = 1 \mid N(r) = 1, N(t) - N(r) = 2)$ by IIP

$P(N(s) = 1 \mid N(r) = 1, N(t-r) = 2)$

$\dfrac{P(N(s), N(r) = 1, N(t-r) = 2)}{P(N(r) = 1, N(t-r) = 2)}$

$= \dfrac{e^{-\lambda s} \lambda s \cdot e^{-\lambda r} \lambda r \cdot e^{-\lambda(t-r)} \frac{\lambda(t-r)^2}{2}}{e^{-\lambda r} \lambda r \, e^{-\lambda(t-r)} \lambda(t-r)^2/2}$

$$= \boxed{\frac{e^{-\lambda s}}{\lambda s}}$$

2) rrlag $(\lambda, n-1)$

$$\lambda e^{-\lambda t} \frac{(\lambda t)^{n-1}}{(n-1)!}$$