

IME625: Introduction to Stochastic Processes
Quiz-1 Part-2, January 31, 2022

2. Let p denote the probability of getting head in a coin toss. We are tossing this coin until we get head. Then the probability of getting the desired outcome in the n -th toss is $q^{n-1}p$ where $q = 1 - p$. We are all too familiar with this. However, this simple question can become quite difficult if we toss the coin until we observe a generic pattern, e.g., HHH or HTTH, etc. Here, Markov chain can help. H denotes head and T denotes tail.

Construct a Markov chain model that tracks the pattern HHH in a repeated coin toss until the pattern is observed. Construct its transition probability matrix in terms of p and q . Note that the coin tossing stops once the pattern is observed, but the Markov chain, by definition, must continue forever. How will you answer the original question, i.e., finding the probability of observing the pattern HHH in the n -th toss, using this Markov chain? [Marks: 2+1]

We can be interested in any pattern. Think of a generic mechanism for constructing a Markov chain model to track the pattern in a repeated coin toss until the pattern is observed. Use it for the pattern HTTH and construct the transition probability matrix. [Marks: 2]