Mean Based on Total Number of Particles It is defined by

$$\bar{D}_A = \frac{\sum_{i=1}^{i=n}\left(\frac{N_i}{D_{pi\,avg}}\right)}{\sum_{i=1}^{i=n} N_i} = \frac{\sum_{i=1}^{i=n}\left(\frac{N_i}{D_{pi\,avg}}\right)}{N_T} \tag{2.16}$$

where, N_T = Total number of particles.

This is also known as the *arithmetic mean diameter*.

All these mean diameters are based on different factors like volume–surface, mass, volume, number of particles, etc. The values of the mean diameter differ and are suitable for specific applications. For example, the mean diameter based on surface area is useful in the study of mass transfer, catalytic reactions, etc.; the mean diameter based on volume or mass is useful in the study of spray drying, in the gravitational free settling velocity of a particle in a liquid, etc.

Example 2.2 *Finely divided clay is used as a catalyst in the petroleum industry. It has a density of 1.2 g/cc and a sphericity of 0.5. The size analysis is as follows.*

Average diameter, $D_{pi\,avg}$ (cm)	0.0252	0.0178	0.0126	0.0089	0.0038
Mass fraction, x_i (g/g)	0.088	0.178	0.293	0.194	0.247

Find the specific surface area and the Sauter mean diameter of the clay material

Solution The specific surface area and the Sauter mean diameters are given by the relations

$$A_{ss} = \frac{6}{\Phi\rho_p}\sum_{i=1}^{i=n}\frac{x_i}{D_{pi\,avg}} \text{ and } \bar{D}_{vs} = \frac{1}{\sum_{i=1}^{i=n}\frac{x_i}{D_{pi\,avg}}}.$$

For finding these, we have to proceed as follows:

Average diameter $_{pi\,avg}$ *cm*	*Mass fraction* x_i *g g*	$\frac{x_i}{D_{pi\,avg}}$
0.0252	0.088	3.492
0.0178	0.178	10.000
0.0126	0.293	23.254
0.0089	0.194	21.797
0.0038	0.247	65.000
	$\sum x_i = 1.000$	$\sum \frac{x_i}{D_{pi\,avg}} = 123.543$

Thus, the specific surface area, $A_{ss} = \dfrac{6}{\Phi \rho_p} \sum_{i=1}^{i=n} \dfrac{x_i}{D_{pi\ avg}} = \dfrac{6(123.543)}{0.5 \times 1.2} = 1235.43\ \text{cm}^2/\text{g}$

and the Sauter mean diameter,

$$\bar{D}_{vs} = \frac{1}{\sum_{i=1}^{i=n} \frac{x_i}{D_{pi\ avg}}} = \frac{1}{123.543} = 0.008094\ \text{cm} = 8.094 \times 10^{-3}\ \text{cm} \qquad \text{(Ans)}$$

Example 2.3 *Calculate the volume–surface mean diameter for the following particulate material.*

Size range, µm	* –710 + 300	–300 + 180	–180 + 90	–90 + 38	Pan
Mass of particles in the range, g	30	35	65	70	55

Solution The volume–surface mean diameter is given by the relation

$\bar{D}_{vs} = \dfrac{1}{\sum_{i=1}^{i=n} \frac{x_i}{D_{pi\ avg}}}$. For finding this, we have to proceed as follows:

Size Range, µm	$D_{pi\ avg}$	*Mass, g*	*Mass Fraction,* x_i	$\dfrac{x_i}{D_{pi\ avg}}$
–710 + 300	505	30	0.117	0.00023
–300 + 180	240	35	0.137	0.00057
–180 + 90	135	65	0.255	0.00189
–90 + 38	64	70	0.275	0.00429
Pan	38	55	0.215	0.00566
				$\sum \dfrac{x_i}{D_{pi\ avg}} = 0.01264$

Thus, $\bar{D}_{vs} = \dfrac{1}{0.01264} = 79.11\ \mu\text{m}$ (Ans)

The negative (–) sign indicates that the material passes through the screen and the positive (+) sign indicates that the material is retained on the screen. The details about screening are discussed in Chapter 5 .

Example 2.4 *The sieve analysis for a sample of feed is as follows:*

Mesh no.	*Screen opening, mm*	*Mass retained, g*
4	4.75	0
5	3.35	33.5
6	2.80	324
8	2.00	315.5

(*Continued*)

Mesh no.	Screen opening, mm	Mass retained, g
10	1.80	120
14	1.70	182
18	0.85	78
25	0.60	79
30	0.50	39
36	0.425	29
44	0.355	26
50	0.30	27
52	0.25	28
85	0.18	40
100	0.155	26
120	0.125	27
150	0.106	28
200	0.075	29
Pan	–	69
		Total = 1500 g

Find graphically, the average particle size for the above data.

Solution The average particle size (Eq. 2.13) is given by $\bar{D}_{vs} = \dfrac{1}{\sum_{i=1}^{i=n} \dfrac{x_i}{D_{pi\ avg}}}$.

Hence, by plotting a graph between 1/ $_{pi\ avg}$ and the cumulative mass fraction, , and then taking the inverse of the area under the curve, we can estimate the average particle size.

Mes no	*Mes opening mm*	*Avg particle si e $_{pi\ avg}$ mm*	*Mass retained g*	*Mass fraction overflo x_i*	*Cumulative mass fraction*	*$_{pi\ avg}$ mm*
4	4.75	> 4.75				
5	3.35	4.05	33.5	0.022	0.022	0.247
6	2.8	3.075	324	0.216	0.238	0.325
8	2	2.4	315.5	0.210	0.448	0.417
10	1.8	1.9	120	0.080	0.528	0.526
14	1.7	1.75	182	0.121	0.650	0.571
18	0.85	1.275	78	0.052	0.702	0.784
25	0.6	0.725	79	0.053	0.754	1.379
30	0.5	0.55	39	0.026	0.780	1.818
36	0.425	0.4625	29	0.019	0.800	2.162
44	0.355	0.39	26	0.017	0.817	2.564
50	0.3	0.3275	27	0.018	0.835	3.053
52	0.25	0.275	28	0.019	0.854	3.636
85	0.18	0.215	40	0.027	0.880	4.651
100	0.155	0.1675	26	0.017	0.898	5.970
120	0.125	0.14	27	0.018	0.916	7.143
150	0.106	0.1155	28	0.019	0.934	8.658
200	0.075	0.0905	29	0.019	0.954	11.050
Pan	–	0.0375	69	0.046	1.000	26.667

From the graph (Fig. 2.1) we have the area under the curve = 2.0995 mm^{-1}. *Area under t e curve as determined using rigin soft are*
Thus the average particle size = (area)$^{-1}$ = (2.0995)$^{-1}$ = 0.476 mm (Ans)

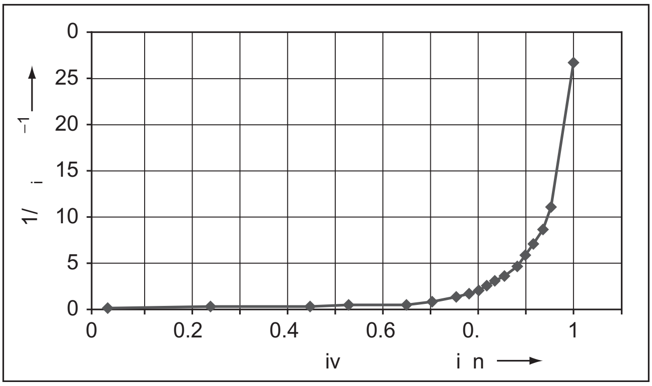


Fig. 2.1 *Plot of 1/$D_{pi\ avg}$ vs cumulative mass fraction, X*

Example 2.5

The size analysis of a powdered material on a weight basis is represented by a straight line from 0% weight at 1-micron particle size to 100% weight at 101-micron particle size. Calculate the Sauter mean diameter of the particles.

Solution

The Sauter mean diameter (Eq. 2.13) is given by

$$\bar{D}_{vs} = \frac{1}{\sum_{i=1}^{i=n} \frac{x_i}{D_{pi\ avg}}}.$$

Given that at $x = 0$, $_{pi\ avg} = 1\ \mu m$ and
$x = 1$, $_{pi\ avg} = 101\ \mu m$
The $_{pi\ avg}$ vs x_i line (Fig. 2.2) is given by $_{pi\ avg} = mx_i + c$ (A)

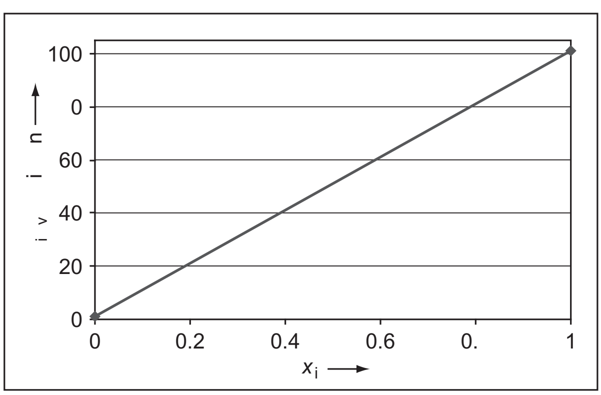


Fig. 2.2 *Plot of $D_{pi\ avg}$ vs x_i*

Here the slope, $m = \frac{101-1}{1-0} = \frac{100}{1} = 100.$

When $_{\text{pi avg}} = 1$, $x_i = 0$, then from Eq. (A), we have $c = 1$ micron.
Thus, Eq. (A) becomes:

$_{\text{pi avg}} = 100\, x_i + 1$ micron (B)

Now, for various values of x_i, $_{\text{pi avg}}$ is calculated and $x_i /$ $_{\text{pi avg}}$ for each fraction was found out as follows:

x_i	$_{pi\ avg}$	x_i $_{pi\ avg}$
0	1	0.0000
0.1	11	0.0091
0.2	21	0.0095
0.3	31	0.0097
0.4	41	0.0098
0.5	51	0.0098
0.6	61	0.0098
0.7	71	0.0099
0.8	81	0.0099
0.9	91	0.0099
1	101	0.0099
		Sum = 0.0972

Now, Sauter mean diameter, $\bar{D}_{\text{vs}} = \dfrac{1}{\sum_{i=1}^{i=n} \dfrac{x_i}{D_{\text{piavg}}}}$

$= 1/0.0972$

$= 10.29$ microns (Ans)

2.2 SOLIDS IN BULK

The properties of solids in bulk are dependent on the properties of the individual particles including their shape and size, and the way in which they interact with each other.

When solid particles are dry and non-cohesive, they behave like a fluid, for example, they flow through orifice or openings and exert pressure on the side walls of the container. But they differ in many ways, like they pose greater problems in storage, do not come out from the container like fluids, interlock under pressure, and do not slide over one another unless the force applied reaches a certain magnitude.

The pressure on them in one direction creates some pressure in another direction having lesser magnitude (than the applied pressure). For homogeneous solid particles, the ratio of the normal pressure, p_N to the applied pressure, p_A, is a constant. It is given by

$$K = \frac{p_N}{p_A} \tag{2.17}$$

which is the characteristic of the material and it is nearly independent of particle size. *K* is also known as the *coefficient of flowability*.

Example 3.1 *Particles of the average feed size of 50×10^{-4} m are crushed to an average product size of 10×10^{-4} m at the rate of 20 tonnes per hour. At this rate, the crusher consumes 40 kW of power of which 5 kW are required for running the mill empty. Calculate the power consumption if 12 tonnes/h of this product is further crushed to 5×10^{-4} m size in the same mill? Assume that Rittinger's law is applicable.*

Solution Rittinger s law (Eq. 3.13) is $\frac{}{\dot{m}} = K_R \left(\frac{1}{\bar{D}_{vsp}} - \frac{1}{\bar{D}_{vsf}} \right)$.

Given in this problem are $\bar{D}_{vsf} = 50 \times 10^{-4} m$, $\bar{D}_{vsp} = 10 \times 10^{-4} m$, and $\dot{m} = 20$ tonne / h.

As 5 kW of power is consumed for running the mill empty out of 40 kW of power fed to the mill, the actual power consumption is $P = 40 - 5 = 35$ kW.

Putting all these values in the above equation, we have

$$\frac{35}{20} = K_R \left(\frac{1}{10 \times 10^{-4}} - \frac{1}{50 \times 10^{-4}} \right)$$

$$\Rightarrow 1.75 = K_R \left(1000 - 200\right) = 800\ K_R$$

$$\Rightarrow K_R = 2.1875 \times 10^{-3} \frac{\text{kWh.m}}{\text{tonne}}$$

This value of K_R is constant for the machine. Now, for $\bar{D}_{vsf} = 10 \times 10^{-4}$ m, $\bar{D}_{vsp} = 5 \times 10^{-4}$ m, and $\dot{m} = 12$ tonne / h, we have

$$\frac{}{12} = 2.1875 \times 10^{-3} \left(\frac{1}{5 \times 10^{-4}} - \frac{1}{10 \times 10^{-4}} \right)$$

$$\Rightarrow \quad = 2.1875 \times 10^{-3} \times 12 \left(2000 - 1000\right)$$

$$\Rightarrow \quad = 26.25 \text{ kW}$$

(Ans)

Example 3.2 *Find the power required for crushing 5 tonne/h of limestone (Rittinger's number = 0.0765 m²/J) if the specific surface areas of the feed and the product are 100 and 200 m²/kg respectively. If the machine consumes a power of 4 hp, calculate its efficiency.*

Solution Rittinger s law (Eq. 3.12 and 3.13) is

$$\frac{}{\dot{m}} = K_R \left(A_{ssp} - A_{ssf}\right) = \frac{\left(A_{ssp} - A_{ssf}\right)}{\text{Rittinger's number}}.$$

Given in this problem are $\dot{m}$ = 5 tonne/h = 1.39 kg/s, A_{ssf} = 100 m^2/kg, A_{ssp} = 200 m^2/kg, and Rittinger s number = 0.0765 m^2/J.

Thus, the power required for crushing is

$$\frac{}{1.39} = \frac{200 - 100}{0.0765} = 1307.19$$

$$\Rightarrow \quad = 1816.99 \text{ J/s} = 1816.99 \text{ W}$$

$$\Rightarrow \quad = \frac{1816.99}{745.7} \text{hp} = 2.43 \text{ hp} \qquad \text{(Ans)}$$

The efficiency of the machine is

$$\frac{2.43}{4} \times 100 = 60.75\% \qquad \text{(Ans)}$$

Example 3.3 || *A sample of materials is crushed in a Blake jaw crusher such that the average size of the particles is reduced from 50 mm to 10 mm with the energy consumption of 13 kW/(kg/s). Determine the consumption of energy to crush the same material of 75-mm average size to an average size of 25 mm using Rittinger's and Kick's laws.*

Solution Given in this problem are

Volume surface mean diameter of feed materials, $\bar{D}_{\text{vsf}} = 50$ mm,

Volume surface mean diameter of crushed materials, $\bar{D}_{\text{vsp}} = 10$ mm, and

Energy consumption, $\frac{}{\dot{m}} = 13.0 \text{kW}/(\text{kg/s}) = 3.61 \frac{\text{kWh}}{\text{tonne}}$.

Case–I

Rittinger s law (Eq. 3.13) is $\frac{}{\dot{m}} = K_\text{R} \left(\frac{1}{\bar{D}_{\text{vsp}}} - \frac{1}{\bar{D}_{\text{vsf}}} \right)$.

Thus,

$$3.61 = K_\text{R} \left(\frac{1}{10} - \frac{1}{50} \right) = 0.08 K_\text{R}$$

$$K_\text{R} = 45.125 \frac{\text{kWh.mm}}{\text{tonne}}.$$

When the same machine is used to crush the same material from 75 mm $(= \bar{D}_{\text{vsf}})$ to 25 mm $(= \bar{D}_{\text{vsp}})$ size, then

$$\frac{}{\dot{m}} = 45.125 \left(\frac{1}{25} - \frac{1}{75} \right) = 45.125 \times 0.027$$

$$\Rightarrow 1.218 \frac{\text{kWh}}{\text{tonne}} \qquad \text{(Ans)}$$

Case–II

Kick s Law (Eq. 3.14) is $\frac{}{\dot{m}} = K_\text{K} \times \ln \left(\frac{\bar{D}_{\text{vsf}}}{\bar{D}_{\text{vsp}}} \right)$.

Thus,

$$3.61 = K_\text{K} \times \ln \left(\frac{50}{10} \right) = 1.609 K_\text{K}$$

$$\Rightarrow K_\text{K} = 2.24 \frac{\text{kWh}}{\text{tonne}}.$$

Now the energy consumption for crushing the same material from 75 mm $\left(=\bar{D}_{\text{vsf}}\right)$ to 25 mm $\left(=\bar{D}_{\text{vsp}}\right)$ size, is

$$\frac{\quad}{\dot{m}}=2.24\times\ln\left(\frac{75}{25}\right)=2.46\frac{\text{kWh}}{\text{tonne}} \qquad \text{(Ans)}$$

Example 3.4 || *A crusher and a grinder are connected to the same power drive. 2700 kg/h of limestone first passes through the crusher and then through the grinder in succession. Screen analysis of feed, product from the crusher, and product from the grinder indicated surface areas of 2.9, 103, and 865 m^2/kg respectively. Calculate the power required by the drive to run the crusher–grinder assembly, if the efficiency of the crusher is 20 %and that of the grinder is 25 % Rittinger's number for limestone is 77.4 m^2/kJ.*

Solution Rittinger s law (equations 3.12 and 3.13) is

$$\frac{\quad}{\dot{m}}=K_{\text{R}}\left(A_{\text{ssp}}-A_{\text{ssf}}\right)=\frac{\left(A_{\text{ssp}}-A_{\text{ssf}}\right)}{\text{Rittinger's number}}$$

Given in this problem are $\dot{m}$ = 2700 kg/h = 0.75 kg/s and Rittinger s number for limestone = 77.4 m^2/kJ = 0.0774 m^2/J.

For Crusher A_{ssf} = 2.9 m^2/kg, A_{ssp} = 103 m^2/kg

Thus, power required, $=\dot{m}\times K_{\text{R}}\left(A_{\text{ssp}}-A_{\text{ssf}}\right)$

$$\Rightarrow \quad =\frac{0.75\times(103-2.9)}{0.0774}=969.96\,\text{J/s}=969.96\,\text{W}$$

As the efficiency of the crusher is 20 %, the actual power requirement is

$$\frac{969.96}{0.20}=4849.8\,\text{W}\cong 4.85\,\text{kW}.$$

For Grinder A_{ssf} = 103 m^2/kg, A_{ssp} = 865 m^2/kg.

Thus, power required, $=\dot{m}\times K_{\text{R}}\left(A_{\text{ssp}}-A_{\text{ssf}}\right)$

$$\Rightarrow \quad =\frac{0.75\times(865-103)}{0.0774}=7383.72\,\text{J/s}=7383.72\,\text{W}$$

As the efficiency of the grinder is 25 %, the actual power requirement is

$$\frac{7383.72}{0.25}=29534.88\,\text{W}\cong 29.53\,\text{kW}.$$

Hence, the total power requirement for the drive to run both the crusher and grinder is

$$(\text{Actual Power})_{\text{Crusher}}+(\text{Actual Power})_{\text{Grinder}}=4.85+29.53=34.38\text{ kW} \qquad \text{(Ans)}$$

Example 3.5 || *270 kW of power is required to crush 150 tonnes/h of a material. If 80% of the feed passes through a 50-mm screen and 80% of the product passes through a 3-mm screen, calculate the work index of the material.*

And what will be the power required for the same feed at 150 tonnes/h to be crushed to a product such that 80 % is to pass through a 1.5-mm screen?

Solution Bond s law (Eq. 3.18) is $\frac{}{\dot{m}} = 0.3162 \quad _{i}\left(\frac{1}{\sqrt{D_{pp}}} - \frac{1}{\sqrt{D_{pf}}}\right)$.

(i) Given in this problem are $\dot{m}$ = 150 tonnes/h, P = 270 kW, $_{pf}$ = 50 mm, and $_{pp}$ = 3 mm.

Thus, work index, $_{i} = \dfrac{}{0.3162 \times \dot{m} \times \left(\frac{1}{\sqrt{D_{pp}}} - \frac{1}{\sqrt{D_{pf}}}\right)}$

$$\Rightarrow \quad _{i} = \frac{270}{0.3162 \times 150 \times \left(\frac{1}{\sqrt{3}} - \frac{1}{\sqrt{50}}\right)} = 13.06 \text{ kWh/tonne} \qquad \text{(Ans)}$$

(ii) For the same feed at same feed rate, if $_{pp}$ = 1.5 mm, the power required will be

$$= 0.3162 \times \dot{m} \times \quad _{i}\left(\frac{1}{\sqrt{D_{pp}}} - \frac{1}{\sqrt{D_{pf}}}\right)$$

$$= 0.3162 \times 150 \times 13.06\left(\frac{1}{\sqrt{1.5}} - \frac{1}{\sqrt{50}}\right) = 418.16\,\text{kW} \qquad \text{(Ans)}$$

Example 3.6 *The rate of grinding of uniform-sized particles is assumed to follow first-order breakage of particles. 60 grams of powder of 220-microns average diameter was ground in a laboratory grinding mill. The amount of unground materials (220 microns) was measured at various times of grinding and the results are given in the following table. Estimate the specific rate of grinding.*

Weight, g	60	20	15	10	8	4
Time, s	0	60	90	120	150	240

Solution The amount of unground materials (220 microns), the cumulative amount of ground materials, and the amount of materials ground with time are given in the table below.

Amount of unground materials (220 μm), g	60	20	15	10	8	4
Time, s	0	60	90	120	150	240
Cumulative amount of materials ground (W), g	0	40	45	50	52	56
Amount of materials ground (ΔW), g	0	40	5	5	2	4

Now, the percentage of materials ground can be calculated by the formula:

$$\left(\frac{\text{Weight in grams taken} - \text{Weight in grams remaining}}{\text{Weight in grams taken}}\right)\times 100 = \quad \text{(say)}$$

Now, the incremental % material ground and incremental time are presented in the following table.

M	Δt sec
$\frac{60-20}{60}\times 100 = 66\%$	$60 - 0 = 60$
$\frac{20-15}{20}\times 100 = 25\%$	$90 - 60 = 30$
$\frac{15-10}{15}\times 100 = 33.3\%$	$120 - 90 = 30$
$\frac{10-8}{10}\times 100 = 20\%$	$150 - 120 = 30$
$\frac{8-4}{8}\times 100 = 50\%$	$240 - 150 = 90$

The specific rate of grinding can be found using the following formula:

$$\frac{\sum(\quad\cdot\Delta\quad)}{\sum(\Delta\quad)}$$

$$= \frac{(66\times 40)+(25\times 5)+(33.3\times 5)+(20\times 2)+(50\times 4)}{56}$$

$= 56.63$ % per second (Ans)

3.4 SIZE-REDUCTION EQUIPMENTS

Size-reduction equipments are classified (Table 3.3) on the basis of:

(i) the mode of operation,
(ii) the method by which a force is applied, and
(iii) the size of feed and product.

Table 3.3 *Classification of size-reduction equipments*

(i)	Mode of operation → Batch operated → Continuous operated
(ii)	Method by which a force is applied → *Impact* Impact at one surface Impact between particles

(*Continued*)

(v) As grinding takes place inside the mill where an inert environment can be easily maintained and, therefore, a ball mill can be used for grinding of explosive materials, and

(vi) The mill can be used for open and closed-circuit grinding.

Example 3.9 || *What rotational speed in rpm would you recommend for a ball mill that is 1000 mm in diameter charged with 70 mm balls?*

Solution The critical speed from Eq. (3.31) is $N_C = \dfrac{1}{2\pi}\sqrt{\dfrac{g}{(\ \ -\ \ )}}$.

Given in this problem are = 1000 mm and d = 70 mm.

Thus, R = /2 = 500 mm = 0.5 m and $r = d/2$ = 35 mm = 0.035 m.

Now, the critical speed, $N_C = \dfrac{1}{2\pi}\sqrt{\dfrac{9.81}{(0.5-0.035)}} = 0.730\text{ rps} = 43.85\text{ rpm}$.

But, the operating speed of the ball mill is 50 to 75% of the critical speed. Hence, the operating speed is 21.90 to 32.88 rpm. (Ans)

Example 3.10 || *In a ball mill of 2000-mm diameter, 100-mm diameter steel balls are being used for grinding. Presently for the material being ground, the mill is running at 15 rpm. At what speed will the mill have to run if the 100-mm balls are replaced with 50-mm balls, all the other conditions remaining same?*

Solution We know for a ball mill from Eq. (3.30) $\cos\theta = \dfrac{4\pi^2 N^2(\ \ -\ \ )}{g}$.

Given in this problem are = 2000 mm, d = 100 mm, and N = 15 rpm.

Thus, R = /2 = 1000 mm = 1 m, $r = d/2$ = 50 mm = 0.05 m, and N = 0.25 rps.

Now, $$\cos\theta = \frac{4\times\pi^2\times(0.25)^2\times(1-0.05)}{9.81} = 0.238.$$

Now, the 100-mm steel balls are replaced with 50-mm balls. Hence, r = 50/2 = 25 mm = 0.025 m.

Thus, $$N = \sqrt{\frac{g\cos\theta}{4\pi^2(\ \ -\ \ )}}$$

$$\Rightarrow N = \sqrt{\frac{9.81\times0.238}{4\pi^2(1-0.025\ )}} = \sqrt{0.0606}$$

$\Rightarrow N = 0.2461\text{ rps} = 14.77\text{ rpm}$ is the speed of the ball mill when the balls are replaced with 50-mm balls. (Ans)

Rod Mills

Rod mills are almost similar to ball mills in appearance and working principle with the exceptions that the grinding medium here is the rods and the length of the mill is greater than its diameter. Rod mills can also be cylindro-conical with the cylindrical section being relatively long and smaller in diameter.

Capacity It is the mass of material that can be treated per unit time to a unit area of the screen to satisfactorily perform the desired size separation and this may be different from the actual feed rate.

Factors Affecting Effectiveness and Capacity The factors affecting the capacity as well as the effectiveness of screens are

(i) the feed rate to the screen,
(ii) the total area of the screening surface,
(iii) the size of the screen openings,
(iv) the number of contacts between the particle and the screen surface,
(v) the type of screening mechanism used,
(vi) the cohesion of particles to each other,
(vii) the blinding of screening surface,
(viii) the adhesion of particles to the screening surfaces,
(ix) the oblique direction of approach of particles, and
(x) the other particle characteristics such as density and moisture content.

The capacity and effectiveness are closely related. Large capacity can be obtained only at the expense of a reduction in effectiveness and for an improvement in efficiency, a reduction in capacity is required.

The capacity and effectiveness are used to measure the performance of industrial screens and a reasonable balance is made between them in actual industrial practices.

Example 5.1 || *A quartz mixture is screened through a 10-mesh screen. The cumulative screen analysis of the feed, overflow, and underflow are given in the following table.*

Mesh	*D_p, mm*	*Cumulative mass fraction greater than D_p*		
		Feed	*Overflow*	*Underflow*
4	4.699	0	0	0
8	2.362	0.15	0.43	0
10	1.651	0.47	0.85	0.195
28	0.589	0.94	1.00	0.91
65	0.208	0.98	–	0.975
Pan	–	1.00	–	1.00

Calculate the mass ratios of overflow to feed and underflow to feed. Also, calculate the overall effectiveness of screen.

Solution From the table, we have $x_F = 0.47$, $x_D = 0.85$, and $x_B = 0.195$.

(i) The mass ratio of overflow to feed (Eq. 5.8) is $\frac{D}{\ } = \frac{x\ \ - x_B}{x\ \ - x_B}$.

Thus, $$\frac{D}{\ } = \frac{0.47 - 0.195}{0.85 - 0.195} = 0.42 \qquad \text{(Ans)}$$

(ii) The mass ratio of underflow to feed (Eq. 5.9) is — $= \dfrac{x \quad - x}{x \quad - x_B}$.

Thus, $$— = \frac{0.85 - 0.47}{0.85 - 0.195} = 0.58 \qquad \text{(Ans)}$$

(iii) The overall effectiveness of screen (Eq. 5.10) is

$$= \frac{(x \quad)(1 - x_B)(x \quad - x_B)(x \quad - x \quad)}{(x \quad)(1 - x \quad)(x \quad - x_B)^2}.$$

Thus, $$= \frac{(0.85)(1 - 0.195)(0.47 - 0.195)(0.85 - 0.47)}{(0.47)(1 - 0.47)(0.85 - 0.195)^2} = 0.6692$$

The overall effectiveness of the screen is 66.92% (Ans)

Example 5.2 || *A sand mixture was screened through a standard 12 mesh screen. The mass fraction of the oversize material in feed, overflow, and underflow were found to be 0.4, 0.8, and 0.2 respectively. Calculate the screen effectiveness based on the oversize materials.*

Solution The screen effectiveness (Eq. 5.10) is

$$= \frac{(x \quad)(1 - x_B)(x \quad - x_B)(x \quad - x \quad)}{(x \quad)(1 - x \quad)(x \quad - x_B)^2}.$$

Given in this problem are $x_F = 0.4$, $x \quad = 0.8$, and $x_B = 0.2$.

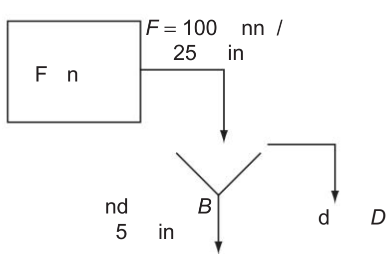
Thus, $$= \frac{(0.8)(1 - 0.2)(0.4 - 0.2)(0.8 - 0.4)}{(0.4)(1 - 0.4)(0.8 - 0.2)^2} = 0.5926$$

The overall effectiveness of screen is 59.26% (Ans)

Example 5.3 || *A sponge-iron industry uses a reciprocating screen of 5-mm aperture to separate oversize from undersize fines which is then recycled to the furnace. The screen analysis of the furnace output was found to contain 25% fines. The screen efficiency was known to be 50%. The underflow from the screen contains around 95% fines. If the furnace production rate is 100 tonne/h, find the product rate and the amount of fines present in it.*

Solution The pictorial representation of this problem is given below.

Given that the furnace production rate, $F = 100$ tonne/h, mass fraction of oversize in feed, $x_F = 1 - 0.25 = 0.75$, mass fraction of oversize in underflow, $x_B = 1 - 0.95 = 0.05$, and screen efficiency, $E = 50\%$.



The screen effectiveness (Eq. 5.10) is

$$= \frac{(x \quad)(1 - x_B)(x \quad - x_B)(x \quad - x \quad)}{(x \quad)(1 - x \quad)(x \quad - x_B)^2}.$$

Thus, $$0.5 = \frac{(x \quad)(1-0.05)(0.75-0.05)(x \quad -0.75)}{(0.75)(1-0.75)(x \quad -0.05)^2}$$

$$\Rightarrow x^2 \quad -0.855-0.00041=0$$

$$\Rightarrow x \quad = 0.8555 = \text{the amount of oversize in product.}$$

Thus, the amount of fines in the product = 1 – 0.8555

$$= 0.1445 = 14.45\% \qquad \text{(Ans)}$$

The mass ratio of overflow to feed (Eq. 5.8) is $\frac{D}{\quad} = \frac{x \quad - x_B}{x \quad - x_B}$.

Thus, the product rate, $D = \frac{\left(x \quad - x_B\right)}{\left(x \quad - x_B\right)}$

$$= \frac{100 \times \left(0.75 - 0.05\right)}{\left(0.8555 - 0.05\right)}$$

$$= 86.60 \text{ tonne/h} \qquad \text{(Ans)}$$

Example 5.4 || *A set of coarse-sized screens was constructed from steel rods and used to evaluate the efficiency of a 60 cm × 40 cm Blake jaw crusher. The standard Tyler relationship was maintained between the screen apertures in the above set of large screens. Calcite was fed to the crusher at the rate of 45 tonne/h driven by a 30-hp motor. The screen analysis of feed and product is given below. Calculate*

(a) the efficiency of the crusher assuming the motor to be operated at an average of 1/6th of the normal rating and

(b) the tonne/h of quartz that can be fed to the crusher and reduced on the same size range with the same power.

Data

Specific gravity of calcite = 2.71,
Specific gravity of quartz = 2.65, and
Rittinger's number of calcite = 76.05 cm^2/kg.cm, and
of quartz = 17.51 cm^2/kg.cm.

Assumption

The values of specific surface ratio may be taken same for both calcite and quartz.

Screen analysis of feed and product

Screen aperture (cm)	*Mass fraction feed (x_1)*	*Mass fraction product (x_2)*	*Specific surface ratio (N_{SSR})*
42.6	0.16	0.0	10.0
30.1	0.34	0.0	9.7
21.3	0.24	0.0	9.5
15.07	0.17	0.0	9.0
10.66	0.09	0.06	8.6
7.54	0.0	0.19	8.0
5.33	0.0	0.44	7.2
3.77	0.0	0.24	6.6
2.67	0.0	0.07	6.2

Solution The total surface area of mixture (Eq. 2.11) is

$$A_{ss} = \frac{6}{\rho_p} \sum_{i=1}^{i=n} \frac{N_{SSRi} x_i}{D_{pi\,avg}}.$$

Feed analysis

Screen aperture $_{pi\ avg}$ *cm*	*Specific surface ratio* N_{SSR}	*Mass fraction* x	$\frac{N_{SS\ i} x_i}{D_{pi\ avg}}$
42.6	10.0	0.16	0.038
30.1	9.7	0.34	0.110
21.3	9.5	0.24	0.107
15.07	9.0	0.17	0.102
10.66	8.6	0.09	0.073
		Total = 1.00	0.430

Product analysis

Screen aperture $_{pi\ avg}$ *cm*	*Specific surface ratio* N_{SSR}	*Mass fraction* x	$\frac{N_{SS\ i} x_i}{D_{pi\ avg}}$
10.66	8.6	0.06	0.048
7.54	8.0	0.19	0.202
5.33	7.2	0.44	0.594
3.77	6.6	0.24	0.420
2.67	6.2	0.07	0.163
		Total = 1.00	1.427

(a) The feed rate = 45 tonne/h = $\frac{45 \times 1000 \times 1000}{3600} = 12500$ g/s

The feed surface area = $\frac{6}{\rho_p} \sum_{i=1}^{i=n} \frac{N_{SSRi} x_{1i}}{D_{pi\,avg}} = \frac{6}{2.71} \times 0.430 = 0.952$ cm^2/g

The product surface area = $\frac{6}{2.71} \times 1.427 = 3.159$ cm^2/g.

Thus, the new surface created = 3.159 – 0.952 = 2.207 cm^2/g.

The new surface created/s = 2.207 × 12500 = 27587.5 cm^2/s.

The Rittinger s Number = 76.05 cm^2/kg.cm.

The, work done = $\frac{27587.5}{76.05 \times 76.2 \times 100} = 0.048$ hp.

The power supplied = 1/6 × 30 = 5 hp.

Thus, the efficiency of the crusher = $\frac{0.048}{5} \times 100 = 0.96\%$.

(b) The Rittinger s number for quartz = 17.51 cm^2/kg.cm.

The new surface created = $\frac{6}{\rho_p}\left[\left(\sum_{i=1}^{i=n} \frac{N_{SSRi}\, x_i}{D_{pi\,avg}}\right)_{product} - \left(\sum_{i=1}^{i=n} \frac{N_{SSRi}\, x_i}{D_{pi\,avg}}\right)_{feed}\right]$

$$= \frac{6}{2.65}(1.427 - 0.430) = 2.257 \text{ cm}^2/\text{g}.$$

Let the crushing capacity for quartz = W tonne/h.

Thus, the new surface created/s $= 2.257 \times \frac{\times 10^6}{3600} = 626.94 \quad \text{cm}^2$.

The work done $= \frac{626.94}{17.51 \times 76.2 \times 100} = 0.048$ hp.

So, $\quad = \frac{0.048 \times 17.51 \times 76.2 \times 100}{649.39} = 10.22$ tonne/h.

Thus, the amount of quartz that could be fed to the crusher and reduced on the same size range with the same power = 10.22 tonne/h (Ans)

Example 5.5 || *A catalyst used for gas oil cracking in a petroleum refinery industry has a density of 1.25 g/cm^3 and nearly the same specific surface ratio as quartz. A sample of the catalyst screened through the 200-mesh screen was further sized by air elutriation. From the resulting analysis given below, find the specific surface in cm^2/g and the mass mean diameter of the catalyst.*

Size analysis of the catalyst

Screened fraction		*Elutriated fraction (–200 mesh)*	
Mesh no.	*Mass fraction*	*Size limit (microns)*	*Mass fraction (of original sample)*
– 48 + 65	0.09	80 – 60	0.11
– 65 + 100	0.17	60 – 40	0.08
– 100 + 150	0.29	40 – 20	0.05
– 150 + 200	0.19	20 – 0	0.02
– 200	0.26		Total = 0.26
	Total = 1.000		

Solution The specific surface ratio for quartz is taken from Figure 17 (p. 22) [Brown, 1995].

Mes number si e limit	$_{pi\ avg}$ *cm*	N_{SSR}	x_i	$\frac{N_{SS\ i}x_i}{D_{pi\ avg}}$
– 48 + 65	0.0252	2.70	0.09	9.64
– 65 + 100	0.0178	2.50	0.17	23.88
– 100 + 150	0.0126	2.30	0.29	52.94
– 150 + 200	0.0089	2.15	0.19	45.90
80 – 60 µm	0.0070	2.07	0.11	32.53
60 – 40 µm	0.0050	2.00	0.08	32.00
40 – 20 µm	0.0030	1.85	0.05	30.83
20 – 0 µm	0.0010	1.55	0.02	31.00
			Total = 1.0	Total = 258.72

The specific surface (Eq. 2.11) is $A_{ss} = \dfrac{6}{\rho_p} \sum_{i=1}^{i=n} \dfrac{N_{SSRi}\, x_i}{D_{pi\ avg}}$

$$= \frac{6}{1.25} \times 258.72 = 1241.86 \frac{cm^2}{g} \qquad \text{(Ans)}$$

The mass mean diameter (Eq. 2.13) is $\bar{D}_m = \sum_{i=1}^{i=n} \left(x_i \times D_{pi\ avg} \right)$.

The value is obtained from the table below.

x_i	$_{pi\ avg}$ cm	x_i $_{pi\ avg}$
0.09	0.0252	0.00227
0.17	0.0178	0.00303
0.29	0.0126	0.00365
0.19	0.0089	0.00169
0.11	0.0070	0.00077
0.08	0.0050	0.00040
0.05	0.0030	0.00015
0.02	0.0010	0.00002

From the above table, $\sum (x_i D_{pi\,avg}) = 0.01198$ cm = 0.1198 mm.

Thus, the mass mean diameter = 0.1198 mm = 119.8 microns (Ans)

Example 5.6 || *A grinder is to be used to reduce an ore of the feed size shown below. Tests on similar equipment indicates that the product size given below will be satisfactory and that the grinder is approximately 10% efficient in converting input energy into size reduction as is evident by an increase in surface.*

It is estimated that a crusher to handle 9 tonne/h will cost about Rs 2,40,000. If the crusher operates on a 24-h basis for 300 days per year, it is estimated that maintenance, overhead, and ordinary replacement costs will be about 50% of power cost. Electric power costs Rs 3.0 per kWh. Considering linear depreciation for the grinder, which has an 8-year life period, calculate the processing cost per tonne of the ore.

Screen analysis

Mesh number (Tyler)	*Feed mass fraction*	*Product mass fraction*
– 6 + 8	0.14	–
– 8 + 10	0.21	–
– 10 + 14	0.23	–
– 14 + 20	0.19	0.10
– 20 + 28	0.12	0.23
– 28 + 35	0.07	0.28
– 35 + 48	0.04	0.15
– 48 + 65	–	0.10
– 65 + 100	–	0.07
– 100 + 150	–	0.04
– 150 + 200	–	0.03
	Total = 1.0	Total = 1.0

Physical property of the ore may be assumed to be same as that of quartz i.e., specific gravity = 2.65 and Rittinger's number = 17.51 cm^2/kg.cm.

Solution From the mesh number, the average diameter for each of the fractions is calculated with the help of Table 4 (p. 17) [Brown, 1995]. With the help of the average diameter, the specific surface ratio for each of the fractions is taken from Figure 17 (p. 22) [Brown, 1995]. These, along with other quantities, are tabulated below.

Mes number	*pi avg cm*	N_{SSR}	*Feed*		*Product*	
			x	$\frac{N_{SS\ i}\, x_i}{D_{pi\ avg}}$	x	$\frac{N_{SS\ i}\, x_i}{D_{pi\ avg}}$
– 6 + 8	0.2845	6.40	0.14	3.149	–	–
– 8 + 10	0.2007	5.10	0.21	5.336	–	–
– 10 + 14	0.1410	4.30	0.23	7.014	–	–
– 14 + 20	0.1001	3.90	0.19	7.403	0.10	3.896
– 20 + 28	0.0711	3.50	0.12	5.907	0.23	11.322
– 28 + 35	0.0503	3.10	0.07	4.314	0.28	17.256
– 35 + 48	0.0356	2.85	0.04	3.202	0.15	12.008
– 48 + 65	0.0252	2.70	–	–	0.10	10.714
– 65 + 100	0.0178	2.50	–	–	0.07	9.831
– 100 + 150	0.0126	2.30	–	–	0.04	7.302
– 150 + 200	0.0089	2.15	–	–	0.03	7.247
			Total = 1.0	Total = 36.325	Total = 1.0	Total = 79.576

The surface area of feed $= \dfrac{6}{\rho_p} \sum_{i=1}^{i=n} \dfrac{N_{SSRi}\, x_{1i}}{D_{pi\ avg}} = \dfrac{6}{2.65} \times 36.325 = 82.245\ \text{cm}^2/\text{g}$ and

the product surface area $= \dfrac{6}{\rho_p} \sum_{i=1}^{i=n} \dfrac{N_{SSRi}\, x_{2i}}{D_{pi\ avg}} = \dfrac{6}{2.65} \times 79.576 = 180.172\ \text{cm}^2/\text{g}.$

Thus, the new surface created = 180.172 – 82.245 = 97.927 cm^2/g.

$$\text{The new surface created/sec} = 97.927 \times \frac{9 \times 1000}{3600} \times 1000 = 244817\ 5\ \text{cm}^2.$$

$$\text{The work done} = \frac{244817.5}{17.51 \times 76.2 \times 100} = 1.835\ \text{hp}.$$

The efficiency of grinder = 10%.

$$\text{The power input} = \frac{1.835}{0.1} = 18.35\ \text{hp}.$$

Calculation of annual cost

$$\text{The annual power cost} = \frac{18.35 \times 24 \times 300}{1.341} \times \text{Rs } 3.0 = \text{Rs } 2,95,570.50$$

The maintenance, overhead, and replacement cost = 0.5 × Rs 295570.5 = Rs 147785.25.

The purchase cost of grinder = Rs 240000.0

The life of grinder = 8 years

$$\text{The annual depreciation} = \frac{\text{Rs } 240000}{8} = \text{Rs } 30,000.0$$

The total annual cost = Rs 295570.5 + Rs 147785.25 + Rs 30,000.0 = Rs 473355.75.

The ore processed per year = 9 × 24 × 300 = 64800 tonne

$$\text{Thus, the processing cost per tonne} = \frac{\text{Rs } 473355.75}{64800} = \text{Rs } 7.30 \qquad \text{(Ans)}$$

5.1.5 Ideal and Actual Screening

The objective of a single-step screening operation is to separate a mixture of particles into two parts: overflow containing mostly oversize and underflow containing mostly undersize particles.

The *ideal screening* process is the one which separates the feed mixture in such a way that all the oversize particles in feed go to the overflow and all the undersize particles go to the underflow streams. The ideal screening process does not exist in reality, it is a hypothetical one. This process makes the separation between oversize and undersize particles around a *cut diameter,* D_c, which is nearly equal to the size of the screen opening.

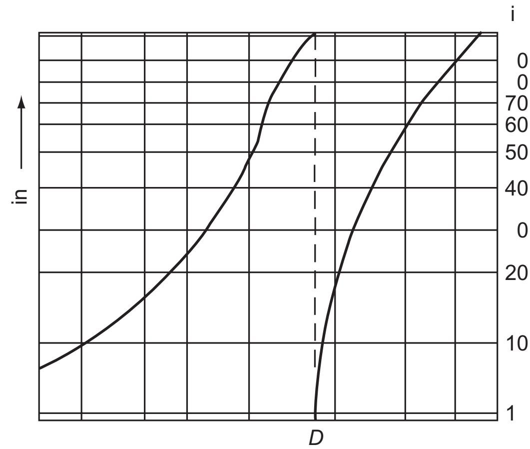


Fig. 5.18 *Plot of screen analysis of products from ideal screening (by permission, Metso Minerals Inc.)*

One typical screen analysis of products (overflow and underflow) from ideal screening is shown in Fig. 5.18. As there will be a sharp separation of overflow and underflow at $_c$, the first point on the curve for underflow and the last point on the curve for overflow will have the same abscissa with no overlapping.

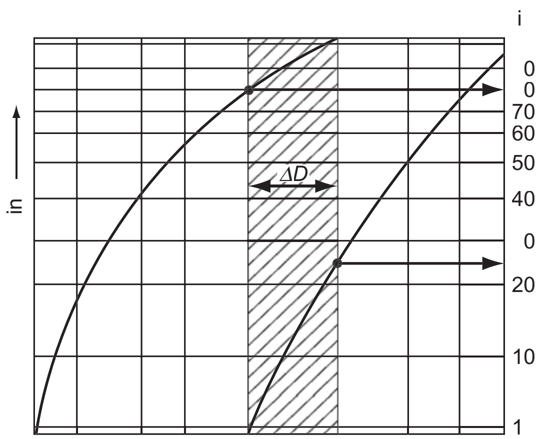


Fig. 5.19 *Plot of screen analysis of products from actual screening (by permission, Metso Minerals Inc.)*

However, in *actual screening*, the overflow is found to contain some particles smaller than the cut diameter and the underflow is found to contain some particles larger than the cut diameter, giving rise to an overlap of the two curves, as shown in Fig. 5.19. The