

## SECTION 2-1

# *Particulate Solids*

### PROBLEM 1.1

The size analysis of a powdered material on a mass basis is represented by a straight line from 0 per cent at 1  $\mu\text{m}$  particle size to 100 per cent by mass at 101  $\mu\text{m}$  particle size. Calculate the surface mean diameter of the particles constituting the system.

#### **Solution**

See Volume 2, Example 1.1.

### PROBLEM 1.2

The equations giving the number distribution curve for a powdered material are  $dn/dd = d$  for the size range 0–10  $\mu\text{m}$ , and  $dn/dd = 100,000/d^4$  for the size range 10–100  $\mu\text{m}$  where  $d$  is in  $\mu\text{m}$ . Sketch the number, surface and mass distribution curves and calculate the surface mean diameter for the powder. Explain briefly how the data for the construction of these curves may be obtained experimentally.

#### **Solution**

See Volume 2, Example 1.2.

### PROBLEM 1.3

The fineness characteristic of a powder on a cumulative basis is represented by a straight line from the origin to 100 per cent undersize at a particle size of 50  $\mu\text{m}$ . If the powder is initially dispersed uniformly in a column of liquid, calculate the proportion by mass which remains in suspension in the time from commencement of settling to that at which a 40  $\mu\text{m}$  particle falls the total height of the column. It may be assumed that Stokes' law is applicable to the settling of the particles over the whole size range.

## Solution

For settling in the Stokes' law region, the velocity is proportional to the diameter squared and hence the time taken for a  $40\text{ }\mu\text{m}$  particle to fall a height  $h\text{ m}$  is:

$$t = h/40^2k$$

where  $k$  a constant.

During this time, a particle of diameter  $d\text{ }\mu\text{m}$  has fallen a distance equal to:

$$kd^2h/40^2k = hd^2/40^2$$

The proportion of particles of size  $d$  which are still in suspension is:

$$= 1 - (d^2/40^2)$$

and the fraction by mass of particles which are still in suspension is:

$$= \int_0^{40} [1 - (d^2/40^2)]dw$$

Since  $dw/dd = 1/50$ , the mass fraction is:

$$\begin{aligned} &= (1/50) \int_0^{40} [1 - (d^2/40^2)]dd \\ &= (1/50)[d - (d^3/4800)]_0^{40} \\ &= 0.533 \text{ or } \underline{\underline{53.3 \text{ per cent}}} \text{ of the particles remain in suspension.} \end{aligned}$$

## PROBLEM 1.4

In a mixture of quartz of density  $2650\text{ kg/m}^3$  and galena of density  $7500\text{ kg/m}^3$ , the sizes of the particles range from  $0.0052$  to  $0.025\text{ mm}$ .

On separation in a hydraulic classifier under free settling conditions, three fractions are obtained, one consisting of quartz only, one a mixture of quartz and galena, and one of galena only. What are the ranges of sizes of particles of the two substances in the original mixture?

## Solution

Use is made of equation 3.24, Stokes' law, which may be written as:

$$u_0 = kd^2(\rho_s - \rho),$$

where  $k (= g/18\mu)$  is a constant.

For large galena:  $u_0 = k(25 \times 10^{-6})^2(7500 - 1000) = 4.06 \times 10^{-6}k\text{ m/s}$

For small galena:  $u_0 = k(5.2 \times 10^{-6})^2(7500 - 1000) = 0.176 \times 10^{-6}k\text{ m/s}$

For large quartz:  $u_0 = k(25 \times 10^{-6})^2(2650 - 1000) = 1.03 \times 10^{-6}k\text{ m/s}$

For small quartz:  $u_0 = k(5.2 \times 10^{-6})^2(2650 - 1000) = 0.046 \times 10^{-6}k\text{ m/s}$

If the time of settling was such that particles with a velocity equal to  $1.03 \times 10^{-6}k$  m/s settled, then the bottom product would contain quartz. This is not so and hence the maximum size of galena particles still in suspension is given by:

$$1.03 \times 10^{-6}k = kd^2(7500 - 1000) \quad \text{or} \quad d = 0.0000126 \text{ m} \quad \text{or} \quad 0.0126 \text{ mm.}$$

Similarly if the time of settling was such that particles with a velocity equal to  $0.176 \times 10^{-6}k$  m/s did not start to settle, then the top product would contain galena. This is not the case and hence the minimum size of quartz in suspension is given by:

$$0.176 \times 10^{-6}k = kd^2(2650 - 1000) \quad \text{or} \quad d = 0.0000103 \text{ m} \quad \text{or} \quad 0.0103 \text{ mm.}$$

It may therefore be concluded that, assuming streamline conditions, the fraction of material in suspension, that is containing quartz *and* galena, is made up of particles of sizes in the range 0.0103–0.0126 mm

## PROBLEM 1.5

A mixture of quartz and galena of a size range from 0.015 mm to 0.065 mm is to be separated into two pure fractions using a hindered settling process. What is the minimum apparent density of the fluid that will give this separation? How will the viscosity of the bed affect the minimum required density?

The density of galena is  $7500 \text{ kg/m}^3$  and the density of quartz is  $2650 \text{ kg/m}^3$ .

### Solution

See Volume 2, Example 1.4.

## PROBLEM 1.6

The size distribution of a dust as measured by a microscope is as follows. Convert these data to obtain the distribution on a mass basis, and calculate the specific surface, assuming spherical particles of density  $2650 \text{ kg/m}^3$ .

Size range ( $\mu\text{m}$ )	Number of particles in range (—)
0–2	2000
2–4	600
4–8	140
8–12	40
12–16	15
16–20	5
20–24	2

## Solution

From equation 1.4, the mass fraction of particles of size  $d_1$  is given by:

$$x_1 = n_1 k_1 d_1^3 \rho_s,$$

where  $k_1$  is a constant,  $n_1$  is the number of particles of size  $d_1$ , and  $\rho_s$  is the density of the particles = 2650 kg/m<sup>3</sup>.

$\Sigma x_1 = 1$  and hence the mass fraction is:

$$x_1 = n_1 k_1 d_1^3 \rho_s / \Sigma n k d^3 \rho_s.$$

In this case:

$d$	$n$	$k d^3 n \rho_s$	$x$
1	200	5,300,000 $k$	0.011
3	600	42,930,000 $k$	0.090
6	140	80,136,000 $k$	0.168
10	40	106,000,000 $k$	0.222
14	15	109,074,000 $k$	0.229
18	5	77,274,000 $k$	0.162
22	2	56,434,400 $k$	0.118
$\Sigma = 477,148,400k$			$\Sigma = 1.0$

The surface mean diameter is given by equation 1.14:

$$d_s = \Sigma(n_1 d_1^3) / \Sigma(n_1 d_1^2)$$

and hence:

$d$	$n$	$n d^2$	$n d^3$
1	2000	2000	2000
3	600	5400	16,200
6	140	5040	30,240
10	40	4000	40,000
14	15	2940	41,160
18	5	1620	29,160
22	2	968	21,296
$\Sigma = 21,968$			$\Sigma = 180,056$

Thus:  $d_s = (180,056/21,968) = 8.20 \mu\text{m}$

This is the size of a particle with the same specific surface as the mixture.

The volume of a particle 8.20  $\mu\text{m}$  in diameter =  $(\pi/6)8.20^3 = 288.7 \mu\text{m}^3$ .

The surface area of a particle  $8.20\text{ }\mu\text{m}$  in diameter  $= (\pi \times 8.20^2) = 211.2\text{ }\mu\text{m}^2$

and hence: the specific surface  $= (211.2/288.7)$

$$= 0.731\text{ }\mu\text{m}^2/\mu\text{m}^3 \text{ or } \underline{\underline{0.731 \times 10^6\text{ m}^2/\text{m}^3}}$$

### PROBLEM 1.7

The performance of a solids mixer was assessed by calculating the variance occurring in the mass fraction of a component amongst a selection of samples withdrawn from the mixture. The quality was tested at intervals of 30 s and the data obtained are:

mixing time (s)	30	60	90	120	150
sample variance (—)	0.025	0.006	0.015	0.018	0.019

If the component analysed represents 20 per cent of the mixture by mass and each of the samples removed contains approximately 100 particles, comment on the quality of the mixture produced and present the data in graphical form showing the variation of mixing index with time.

### Solution

See Volume 2, Example 1.3.

### PROBLEM 1.8

The size distribution by mass of the dust carried in a gas, together with the efficiency of collection over each size range is as follows:

Size range, ( $\mu\text{m}$ )	0–5	5–10	10–20	20–40	40–80	80–160
Mass (per cent)	10	15	35	20	10	10
Efficiency (per cent)	20	40	80	90	95	100

Calculate the overall efficiency of the collector and the percentage by mass of the emitted dust that is smaller than  $20\text{ }\mu\text{m}$  in diameter. If the dust burden is  $18\text{ g/m}^3$  at entry and the gas flow is  $0.3\text{ m}^3/\text{s}$ , calculate the mass flow of dust emitted.

### Solution

See Volume 2, Example 1.6.

### PROBLEM 1.9

The collection efficiency of a cyclone is 45 per cent over the size range  $0\text{--}5\text{ }\mu\text{m}$ , 80 per cent over the size range  $5\text{--}10\text{ }\mu\text{m}$ , and 96 per cent for particles exceeding  $10\text{ }\mu\text{m}$ .

## SECTION 2-3

# *Motion of Particles in a Fluid*

### PROBLEM 3.1

A finely ground mixture of galena and limestone in the proportion of 1 to 4 by mass, is subjected to elutriation by a current of water flowing upwards at 5 mm/s. Assuming that the size distribution for each material is the same, and is as follows, estimate the percentage of galena in the material carried away and in the material left behind. The absolute viscosity of water is 1 mN s/m<sup>2</sup> and Stokes' equation should be used.

Diameter (μm)	20	30	40	50	60	70	80	100
Undersize (per cent mass)	15	28	48	54	64	72	78	88

The density of galena is 7500 kg/m<sup>3</sup> and the density of limestone is 2700 kg/m<sup>3</sup>.

### Solution

See Volume 2, Example 3.2.

### PROBLEM 3.2

Calculate the terminal velocity of a steel ball, 2 mm diameter and of density 7870 kg/m<sup>3</sup> in an oil of density 900 kg/m<sup>3</sup> and viscosity 50 mN s/m<sup>2</sup>.

### Solution

For a sphere:

$$\begin{aligned}
 (R'_0/\rho u_0^2) Re_0'^2 &= (2d^3/3\mu^2)\rho(\rho_s - \rho)g && \text{(equation 3.34)} \\
 &= (2 \times 0.002^3/3 \times 0.05^2)900(7870 - 900)9.81 \\
 &= 131.3
 \end{aligned}$$

$$\log_{10} 131.3 = 2.118$$

From Table 3.4:  $\log_{10} Re_0' = 0.833$

or:  $Re_0' = 6.80$

Thus:  $u_0 = (6.80 \times 0.05)/(900 \times 0.002) = \underline{\underline{0.189 \text{ m/s}}}$

### PROBLEM 3.3

What is the terminal velocity of a spherical steel particle of 0.40 mm diameter, settling in an oil of density 820 kg/m<sup>3</sup> and viscosity 10 mN s/m<sup>2</sup>? The density of steel is 7870 kg/m<sup>3</sup>.

#### Solution

See Volume 2, Example 3.1.

### PROBLEM 3.4

What are the settling velocities of mica plates, 1 mm thick and ranging in area from 6 to 600 mm<sup>2</sup>, in an oil of density 820 kg/m<sup>3</sup> and viscosity 10 mN s/m<sup>2</sup>? The density of mica is 3000 kg/m<sup>3</sup>.

#### Solution

	Smallest particles	Largest particles
$A'$	$6 \times 10^{-6} \text{ m}^2$	$6 \times 10^{-4} \text{ m}^2$
$d_p$	$\sqrt{[(4 \times 6 \times 10^{-6})/\pi]} = 2.76 \times 10^{-3} \text{ m}$	$\sqrt{[(4 \times 6 \times 10^{-4})/\pi]} = 2.76 \times 10^{-2} \text{ m}$
$d_p^3$	$2.103 \times 10^{-8} \text{ m}^3$	$2.103 \times 10^{-5} \text{ m}^3$
volume	$6 \times 10^{-9} \text{ m}^3$	$6 \times 10^{-7} \text{ m}^3$
$k'$	0.285	0.0285

$$\begin{aligned}
 (R'_0/\rho u^2)Re'_0{}^2 &= (4k'/\mu^2\pi)(\rho_s - \rho)\rho d_p^3 g && \text{(equation 3.52)} \\
 &= [(4 \times 0.285)/(\pi \times 0.01^2)](3000 - 820)(820 \times 2.103 \times 10^{-8} \times 9.81) \\
 &= 1340 \text{ for smallest particle and } 134,000 \text{ for largest particle}
 \end{aligned}$$

	Smallest particles	Largest particles
$\log_{10}(R'_0/\rho u^2)Re'_0{}^2$	3.127	5.127
$\log_{10} Re'_0$	1.581	2.857 (from Table 3.4)
Correction from Table 3.6	-0.038	-0.300 (estimated)
Corrected $\log_{10} Re'_0$	1.543	2.557
$Re'_0$	34.9	361
$u$	<u>0.154 m/s</u>	<u>0.159 m/s</u>

Thus it is seen that all the mica particles settle at approximately the same velocity.

### PROBLEM 3.5

A material of density 2500 kg/m<sup>3</sup> is fed to a size separation plant where the separating fluid is water which rises with a velocity of 1.2 m/s. The upward vertical component of

### Example 1.4

A mixture of quartz and galena of a size range from 0.015 mm to 0.065 mm is to be separated into two pure fractions using a hindered settling process. What is the minimum apparent density of the fluid that will give this separation? The density of galena is 7500 kg/m<sup>3</sup> and the density of quartz is 2650 kg/m<sup>3</sup>.

### Solution

Assuming the galena and quartz particles are of similar shapes, then from equation 1.42, the required density of fluid when Stokes' law applies is given by:

$$\frac{0.065}{0.015} = \left( \frac{7500 - \rho}{2650 - \rho} \right)^{0.5}$$

and:

$$\rho = 2377 \text{ kg/m}^3$$

The required density of fluid when Newton's law applies is given by:

$$\frac{0.065}{0.015} = \left( \frac{7500 - \rho}{2650 - \rho} \right)^{1.0}$$

and hence:

$$\rho = 1196 \text{ kg/m}^3$$

Thus, the required density of the fluid is between 1196 and 2377 kg/m<sup>3</sup>.



What will be the terminal velocities of mica plates, 1 mm thick and ranging in area from 6 to 600 mm<sup>2</sup>, settling in an oil of density 820 kg/m<sup>3</sup> and viscosity 10 mN s/m<sup>2</sup>? The density of mica is 3000 kg/m<sup>3</sup>.

168

CHEMICAL ENGINEERING

### Solution

	smallest particles	largest particles
$A'$	$6 \times 10^{-6} \text{ m}^2$	$6 \times 10^{-4} \text{ m}^2$
$d_p$	$\sqrt{(4 \times 6 \times 10^{-6} / \pi)} = 2.76 \times 10^{-3} \text{ m}$	$\sqrt{(4 \times 6 \times 10^{-4} / \pi)} = 2.76 \times 10^{-2} \text{ m}$
$d_p^3$	$2.103 \times 10^{-8} \text{ m}^3$	$2.103 \times 10^{-5} \text{ m}^3$
volume	$6 \times 10^{-9} \text{ m}^3$	$6 \times 10^{-7} \text{ m}^3$
$k'$	0.285	0.0285

$$\left( \frac{R'_0}{\rho u^2} \right) Re_0'^2 = \frac{4k'}{\mu^2 \pi} (\rho_s - \rho) \rho d_p^3 g \quad \text{(equation 3.52)}$$

$$= (4 \times 0.285 / \pi \times 0.01^2) (3000 - 820) (820 \times 2.103 \times 10^{-8} \times 9.81)$$

$$= 1340 \text{ for the smallest particles and, similarly, } 134,000 \text{ for the largest particles.}$$

Thus:

	smallest particles	largest particles
$\log \left( \frac{R'_0}{\rho u_0^2} Re_0'^2 \right)$	3.127	5.127
$\log Re_0'$	1.581	2.857 (from Table 3.4)
Correction from Table 3.6	-0.038	-0.300 (estimated)
Corrected $\log Re_0'$	1.543	2.557
$Re_0'$	34.9	361
$u_0$	<u>0.154 m/s</u>	<u>0.159 m/s</u>

### Example 2.3

A ball mill, 1.2 m in diameter, is run at 0.80 Hz and it is found that the mill is not working properly. Should any modification in the conditions of operation be suggested?

### Solution

The angular velocity is given by:

$$\omega_c = \sqrt{g/r} \quad \text{(equation 2.10)}$$

In this equation,  $r$  = (radius of the mill – radius of the particle). For small particles,  $r = 0.6$  m and hence:

$$\omega_c = \sqrt{9.81/0.6} = 4.04 \text{ rad/s}$$

The actual speed =  $(2\pi \times 0.80) = 5.02$  rad/s and hence it may be concluded that the speed of rotation is too high and that the balls are being carried round in contact with the sides of the mill with little relative movement or grinding taking place.

The optimum speed of rotation lies in the range  $(0.5-0.75)\omega_c$ , say  $0.6\omega_c$  or:

$$(0.6 \times 4.04) = 2.42 \text{ rad/s}$$

This is equivalent to:  $(2.42/2\pi) = 0.39$  Hz, or, in simple terms:

the speed of rotation should be halved.

A cyclone separator, 0.3 m in diameter and 1.2 m long, has a circular inlet 75 mm in diameter and an outlet of the same size. If the gas enters at a velocity of 1.5 m/s, at what particle size will the theoretical cut occur?

The viscosity of air is  $0.018 \text{ mN s/m}^2$ , the density of air is  $1.3 \text{ kg/m}^3$  and the density of the particles is  $2700 \text{ kg/m}^3$ .

### Solution

Using the data provided:

cross-sectional area at the gas inlet,  $A_i = (\pi/4)(0.075)^2 = 4.42 \times 10^{-3} \text{ m}^2$

gas outlet diameter,  $d_0 = 0.075 \text{ m}$

gas density,  $\rho = 1.30 \text{ kg/m}^3$

height of separator,  $Z = 1.2 \text{ m}$ , separator diameter,  $d_t = 0.3 \text{ m}$ .

Thus: mass flow of gas,  $G = (1.5 \times 4.42 \times 10^{-3} \times 1.30) = 8.62 \times 10^{-3} \text{ kg/s}$

The terminal velocity of the smallest particle retained by the separator,

$$u_0 = 0.2A_i^2 d_0 \rho g / (\pi Z d_t G) \quad (\text{equation 1.54})$$

$$\begin{aligned} \text{or: } u_0 &= [0.2 \times (4.42 \times 10^{-3})^2 \times 0.075 \times 1.3 \times 9.81] / [\pi \times 1.2 \times 0.3 \times 8.62 \times 10^{-3}] \\ &= 3.83 \times 10^{-4} \text{ m/s} \end{aligned}$$

Use is now made of Stokes' law (Chapter 3) to find the particle diameter, as follows:

$$u_0 = d^2 g (\rho_s - \rho) / 18\mu \quad (\text{equation 3.24})$$

$$\begin{aligned} \text{or: } d &= [u_0 \times 18\mu / g (\rho_s - \rho)]^{0.5} \\ &= [(3.83 \times 10^{-4} \times 18 \times 0.018 \times 10^{-3}) / (9.81(2700 - 1.30))]^{0.5} \\ &= 2.17 \times 10^{-6} \text{ m or } 2.17 \text{ } \mu\text{m} \end{aligned}$$

- (e) Electrostatic precipitation.
- (f) Washing with a liquid.
- (g) Agglomeration of solid particles and coalescence of liquid droplets.

### Example 1.5

The collection efficiency of a cyclone is 45 per cent over the size range 0–5  $\mu\text{m}$ , 80 per cent over the size range 5–10  $\mu\text{m}$ , and 96 per cent for particles exceeding 10  $\mu\text{m}$ . Calculate the efficiency of collection for a dust with a mass distribution of 50 per cent 0–5  $\mu\text{m}$ , 30 per cent 5–10  $\mu\text{m}$  and 20 per cent above 10  $\mu\text{m}$ .

### Solution

*For the collector:*

Size ( $\mu\text{m}$ )	0–5	5–10	>10
Efficiency (per cent)	45	80	96

*For the dust:*

Mass (per cent)	50	30	20
-----------------	----	----	----

On the basis of 100 kg dust:

Mass at inlet (kg)	50	30	20
Mass retained (kg)	22.5	24.0	19.2—a total of 65.7 kg

Thus: Overall efficiency =  $100(65.7/100) = \underline{\underline{65.7 \text{ per cent}}}$

### Example 1.6

The size distribution by mass of the dust carried in a gas, together with the efficiency of collection over each size range, is as follows:

Size range ( $\mu\text{m}$ )	0–5	5–10	10–20	20–40	40–80	80–160
Mass (per cent)	10	15	35	20	10	10
Efficiency (per cent)	20	40	80	90	95	100

Calculate the overall efficiency of the collector, and the percentage by mass of the emitted dust that is smaller than 20  $\mu\text{m}$  in diameter. If the dust burden is 18  $\text{g}/\text{m}^3$  at entry and the gas flow 0.3  $\text{m}^3/\text{s}$ , calculate the mass flow of dust emitted.

### Solution

Taking 1  $\text{m}^3$  of gas as the basis of calculation, the following table of data may be completed noting that the inlet dust concentration is 18  $\text{g}/\text{m}^3$ . Thus:

Size range ( $\mu\text{m}$ )	0–5	5–10	10–20	20–40	40–80	80–160	Total
Mass in size range (g)	1.8	2.7	6.3	3.6	1.8	1.8	18.0
Mass retained (g)	0.36	1.08	5.04	3.24	1.71	1.80	13.23
Mass emitted (g)	1.44	1.62	1.26	0.36	0.09	0	4.77

Hence: overall efficiency =  $100(13.23/18.0) = \underline{\underline{73.5 \text{ per cent}}}$

$$\text{dust} < 20 \mu\text{m emitted} = 100[(1.44 + 1.62 + 1.26)/4.77] = \underline{\underline{90.1 \text{ per cent}}}$$

The inlet gas flow =  $0.3 \text{ m}^3/\text{s}$  and hence:

$$\text{mass emitted} = (0.3 \times 4.77) = 1.43 \text{ g/s or } \underline{\underline{1.43 \times 10^{-3} \text{ kg/s}}} \text{ (0.12 tonne/day)}$$

### 1.6.2. Gas cleaning equipment

The classification given in Section 1.6.1 is now used to describe commercially available equipment.

#### a) Gravity separators

If the particles are large, they will settle out of the gas stream if the cross-sectional area is increased. The velocity will then fall so that the eddy currents which are maintaining the particles in suspension are suppressed. In most cases, however, it is necessary to introduce baffles or screens as shown in Figure 1.51, or to force the gas over a series of trays as shown in Figure 1.52 which depicts a separator used for removing dust from sulphur dioxide produced by the combustion of pyrites. This equipment is suitable when the concentration of particles is high, because it is easily cleaned by opening the doors at the side. Gravity separators are seldom used as they are very bulky and will not remove particles smaller than  $50\text{--}100 \mu\text{m}$ .

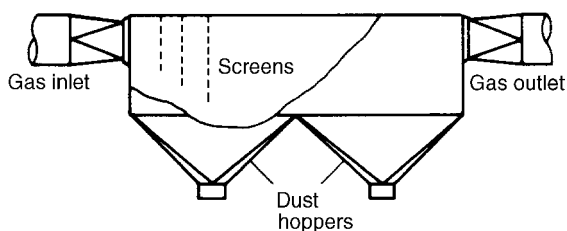


Figure 1.51. Settling chamber

Gravity separation is little used since the equipment must be very large in order to reduce the gas velocity to a reasonably low value which allows the finer particles to settle. For example, a settling chamber designed to remove particles with a diameter of  $20 \mu\text{m}$  and density  $2000 \text{ kg/m}^3$  from a gas stream flowing at  $10 \text{ m}^3/\text{s}$  would have a volume of about  $3000 \text{ m}^3$ . Clearly, this very large volume imposes a severe limitation and this type of equipment is normally restricted to small plants as a pre-separator which reduces the load on a more efficient secondary collector.

#### b) Centrifugal separators

The rate of settling of suspended particles in a gas stream may be greatly increased if centrifugal rather than gravitational forces are employed. In the cyclone separator shown

Equation 9.18 implies that the greater the depth  $h$  of liquid in the bowl, that is the lower the value of  $r_0$ , the smaller will be the value of  $\Sigma$ , but this is seldom borne out in practice in decanter centrifuges. This is probably due to high turbulence experienced and the effects of the scrolling mechanism.  $\Sigma$  is independent of the properties of the fluid and the particles and depends only on the dimensions of the centrifuge, the location of the overflow weir and the speed of rotation. It is equal to the cross-sectional area of a gravity settling tank with the same clarifying capacity as the centrifuge. Thus,  $\Sigma$  is a measure of the capacity of the centrifuge and gives a quantification of its performance for clarification. This treatment is attributable to the work of AMBLER<sup>(1)</sup>.

For cases where the thickness  $h$  of the liquid layer at the walls is comparable in order of magnitude with the radius  $R$  of the bowl, it is necessary to use equation 3.121 in place of equation 9.14 for the required residence time in the centrifuge or:

$$t_R = \frac{18\mu}{d^2(\rho_s - \rho)\omega^2} \ln \frac{R}{r_0} \quad (9.20)$$

(from equation 3.121)

Then:

$$Q = \frac{d^2(\rho_s - \rho)\omega^2 V'}{18\mu \ln(R/r_0)} \quad (9.21)$$

$$= \frac{d^2(\rho_s - \rho)g}{18\mu} \frac{\omega^2 V'}{g \ln(R/r_0)} \quad (9.22)$$

$$= u_0 \Sigma \quad (9.23)$$

In this case:

$$\begin{aligned} \Sigma &= \frac{\omega^2 V'}{g \ln(R/r_0)} \\ &= \frac{\pi(R^2 - r_i^2)H}{\ln(R/r_0)} \frac{\omega^2}{g} \end{aligned} \quad (9.24)$$

A similar analysis can be carried out with various geometrical arrangements of the bowl of the centrifuge. Thus, for example, for a disc machine (described later) the value of  $\Sigma$  is very much greater than for a cylindrical bowl of the same size. Values of  $\Sigma$  for different arrangements are quoted by HAYTER<sup>(2)</sup> and by TROWBRIDGE<sup>(3)</sup>.

This treatment leads to the calculation of the condition where all particles larger than a certain size are retained in the centrifuge. Other definitions are sometimes used, such as for example, the size of the particle which will just move half the radial distance from the surface of the liquid to the wall, or the condition when just half of the particles of the specified size will be removed from the suspension.

### Example 9.1

In a test on a centrifuge all particles of a mineral of density 2800 kg/m<sup>3</sup> and of size 5  $\mu$ m, equivalent spherical diameter, were separated from suspension in water fed at a volumetric throughput rate of 0.25 m<sup>3</sup>/s. Calculate the value of the capacity factor  $\Sigma$ .

What will be the corresponding size cut for a suspension of coal particles in oil fed at the rate of  $0.04 \text{ m}^3/\text{s}$ ? The density of coal is  $1300 \text{ kg/m}^3$  and the density of the oil is  $850 \text{ kg/m}^3$  and its viscosity is  $0.01 \text{ Ns/m}^2$ .

It may be assumed that Stokes' law is applicable.

## Solution

The terminal falling velocity of particles of diameter  $5 \text{ }\mu\text{m}$  in water, of density  $\rho = 1000 \text{ kg/m}^3$  and, of viscosity  $\mu = 10^{-3} \text{ Ns/m}^2$ , is given by:

$$u_0 = \frac{d^2(\rho_s - \rho)g}{18\mu} = \frac{25 \times 10^{-12} \times (2800 - 1000) \times 9.81}{18 \times 10^{-3}} \quad (\text{equation 3.24})$$

$$= 2.45 \times 10^{-5} \text{ m/s}$$

From the definition of  $\Sigma$ :

$$Q = u_0 \Sigma \quad (\text{equation 9.19})$$

and:

$$\Sigma = \frac{0.25}{(2.45 \times 10^{-5})} = 1.02 \times 10^4 \text{ m}^2.$$

For the coal-in-oil mixture:

$$u_0 = \frac{Q}{\Sigma} = \frac{0.04}{(1.02 \times 10^4)} = 3.92 \times 10^{-6} \text{ m/s}.$$

From equation 3.24:

$$d^2 = \frac{18\mu u_0}{(\rho_s - \rho)g}$$

$$= \frac{18 \times 10^{-2} \times 3.92 \times 10^{-6}}{(1300 - 850) \times 9.81}$$

and:

$$d = 4.0 \times 10^{-6} \text{ m or } \underline{\underline{4 \text{ }\mu\text{m}}}.$$

## Example 9.2

A centrifuge is fitted with a conical disc stack with an included angle of  $2\theta$ , and there are  $n$  flow passages between the discs. A suspension enters at radius  $r_1$  and leaves at radius  $r_2$ . Obtain an expression for the separating power  $\Sigma$  of the centrifuge. It may be assumed that the resistance force acting on the particles is given by Stokes' law.

## Solution

For two discs AA' and BB', as shown in Figure 9.4, the particle which is most unfavourably placed for collection will enter at point A at radius  $r_1$ , and be deposited on the upper plate at point B' at