SHUBHAM GUPTA
180749

HW-10

- $f_{1,1} \Rightarrow P(T_{1,1}=1) = P(\text{path } 1 \to 1) = 1$

  $P(T_{1,1}=2) = P(\text{path } 1 \to 1 \to 1)$ same as above.

  $\therefore$ state 1 is recurrent

- $f_{2,2} \Rightarrow P(T_{2,2}=1) = P(\text{path } 2 \to 2) = 0.5$

  $P(T_{2,2}=2) = P(\text{path } 2 \to 3 \to 2) = 0.4 \times 0.2$

  $P(T_{2,2}=3) = P(2 \to 3 \to 3 \to 2) = 0.4 \times 0.6 \times 0.2$

  $P(T_{2,2}=4) = P(2 \to 3 \to 3 \to 3 \to 2) = 0.4 \times (0.6)^2 \times 0.2$

  $$0.5 + 0.4 \times 0.2 \left(\frac{1}{1-0.6}\right) = 0.7$$

  state 2 is ~~transient~~ transient.

- $f_{3,3} \Rightarrow P(T_{3,3}=1) = P(3 \to 3) = 0.6$

  $P(T_{3,3}=2) = P(3 \to 2 \to 3) = 0.2 \times 0.4$

  $P(T_{3,3}=3) = P(3 \to 2 \to 2 \to 3) = 0.2 \times (0.5)(0.4)$

  $P(T_{3,3}=4) = P(3 \to 2 \to 2 \to 2 \to 3) = 0.2 \times (0.5)^2 \times 0.4$

  $$0.6 + 0.2 \times 0.4 \left(\frac{1}{1-0.5}\right) = 0.6 + 0.08 \times 2 = 0.76$$

  $\therefore$ state 3 is transient.

- $f_{4,4} \Rightarrow P(T_{4,4}=1) = P(4 \to 4) = 0.5$

  $P(T_{4,4}=2) = P(4 \to 5 \to 4) = 0.5^2 \times 0.3$

  $P(T_{4,4}=3) = P(4 \to 5 \to 5 \to 4) = 0.5^2 \times 0.7 \times 0.3$

  $P(T_{4,4}=4) = P(4 \to 5 \to 5 \to 5 \to 4) = 0.5^2 \times 0.7^2 \times 0.3$

  $$0.5 + (0.5)^2 \times 0.3 \left(\frac{1}{1-0.7}\right) = 1$$

∴ state 4 is recurrent

- $f_{s,s} = P(T_{s,s}=1) = P(s \to s) = 0.7$

$P(T_{s,s}=2) = P(s \to 4 \to s) = 0.3 \cdot 0.5$

$P(T_{s,s}=3) = P(s \to 4 \to 4 \to s) = 0.3 \times 0.5 \times$

$P(T_{s,s}=4) = P(s \to 4 \to 4 \to 4 \to s) = 0.3 \times (0.5)^2 \times 0.5$

$0.7 + 0.3 \times 0.5 \left(\frac{1}{1-0.5}\right) = 1$

∴ state S is recurrent.