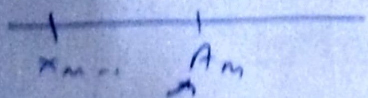
SHUBHAM GUPTA
180729
HW 14

$X_{m+1} = \max(x_m - c, 0) + A_{m+1}$ for $m \geq 0$.



$\sum_{i \in \Omega} \sigma_i P_{ij} = \sigma_j \quad \forall j \in \Omega$, $\sum_{i = \Omega} \sigma_i = 1$

must have unique sol^n

	0	1	2	3	4	5
0	a_0	a_1	a_2	0	0	0
1	a_0	a_1	a_2	0	0	0
2	0	a_0	a_1	a_2	0	0
3	0	0	a_0	a_1	a_2	0
4	0	0	0	a_0	a_1	a_2
5	0	0	0		a_0	a_0

$a_0\sigma_0 + a_0\sigma_1 = \sigma_0$

$a_1\sigma_0 + a_1\sigma_1 + a_0\sigma_2 = \sigma_1$

$a_2\sigma_0 + a_2\sigma_1 + a_1\sigma_2 + a_0\sigma_3 = \sigma_2$

$\boxed{\sigma_1 = \frac{\sigma_0(1-a_0)}{a_0}}$

$\sigma_2 = \frac{\sigma_0}{a_0} - \sigma_0 - \frac{\sigma_0}{a_0} a_1$

$\boxed{\sigma_2 = \frac{\sigma_0 a_2}{a_0}}$

$a_2(\sigma_0) + \frac{a_1 a_2 \sigma_0}{a_0} + a_0\sigma_3 = \frac{\sigma_0 a_2}{a_0}$

$\boxed{\sigma_3 = \frac{\sigma_0 a_2^2}{a_0^2}}$

$\sigma_2 a_2 + a_1\sigma_3 + a_0\sigma_4 = \sigma_3$ solving

$\boxed{\sigma_4 = \frac{a_2^3 \sigma_0}{a_0^3}}$

$\therefore \boxed{\sigma_m = \sigma_0\left(\frac{a_2}{a_0}\right)^{m-1} \quad \forall m \geq 2}$

Base case $m = 2, 3, 4$. holds true as seen above

$a_2\sigma_{N-2} + a_1\sigma_{N-1} + a_0\sigma_N = \sigma_{N-1}$

$\therefore \sigma_N = \sigma_0 \left(\frac{a_2}{a_0}\right)^{N-1}$

now $\sum_{i \in \Omega} \sigma_i = 1$

$\sigma_0 + \frac{\sigma_0}{a_0}(1-a_0) + \sigma_0 \left(\frac{a_2}{a_0}\right) - - - - = 1$

$\sigma_0 + \frac{\sigma_0}{a_0}(1-a_0) + \sigma_0 \left(\frac{a_2}{a_0}\right) \frac{1}{1-\left(\frac{a_2}{a_0}\right)} = 1$

$\Rightarrow a_2 < a_0 \quad \Rightarrow a_2 < 1 - a_1 - a_2 \Rightarrow a_2 < \frac{1-a_1}{2}$

solving $\sigma_0 = \frac{a_0(a_0 - a_2)}{a_0 - a_2 + a_0 a_2}$

$\sum_{k \geq 2} (k-1)\pi_k = \sum_{k \geq 2} (k-1)\sigma_k = \sum_{k \geq 2} (k-1)\sigma_0 \left(\frac{a_2}{a_0}\right)^{k-1}$

$= \sigma_0 \sum_{k \geq 2} (k-1) \left(\frac{a_2}{a_0}\right)^{k-1} = \sigma_0 \left(\frac{a_2}{a_0}\right) \Big/ \left(\frac{a_2}{a_0} - 1\right)^2$

$= \frac{a_2 a_0^2}{(a_0 - a_2 - a_0 a_2)(a_0 - a_2)}$

counter utilization $\sum_{k \leq C-1} \frac{k}{C} \pi_k + \sum_{k \geq C} \pi_k$.

$= \sum_{k \geq 1} \sigma_k = \frac{\sigma_0}{a_0}(1-a_0) + \frac{\sigma_0}{a_0} a_2 + \left(\frac{\sigma_0}{a_0^2} a_2^2\right)$

$= 1 - \sigma_0 = (a_0 - a_0^2 - a_2 - 2a_0 a_2) / (a_0 - a_2 + a_0 a_2)$