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HW-18

a) $\{N(t) : t \geq 0\}$ $\quad s < t$

$P(N(t) > N(s)) = P(N(t) > N(s) \mid s < t)$

$= 1 - P(N(t) = N(s) \mid s < t)$

$= 1 - e^{-\lambda(t-s)}$

b) $P(N(s) = 1, N(t) = 3)$

$P(N(s) = 1) = e^{-\lambda s} \lambda s$

$P(N(t) = 3) = \dfrac{e^{-\lambda t}(\lambda t)^3}{3!}$

$P(N(s) = 1) P(N(t) = 3) = \dfrac{e^{-\lambda s} e^{-\lambda t} \lambda^4 s t^3}{3!}$

c) $E[N(t) \mid N(s) = 2]$

as $s < t$.

$P(N(t) = 2) = e^{-\lambda(t-s)}$

$P(N(t) = 3) = e^{-\lambda(t-s)} \lambda(t-s)$

$P(N(t) = 4) = e^{-\lambda(t-s)} \dfrac{[\lambda(t-s)]^2}{2}$

$\downarrow$

in the $(t-s)$ expected no. of events.

ans $= 2 + [e^{-\lambda(t-s)}[\lambda(t-s) + \dfrac{4\lambda(t-s)^2}{2} + 3$

$= 2 + e^{-\lambda(t-s)} \lambda [\lambda^{(t-s)} e^{\lambda(t-s)}]$

$= 2 + \lambda(t-s) - e^{-\lambda(t-s)}$

d) $E[N(s) \mid N(t) = 4]$

$$P(N(s)=0) \Rightarrow P(N(t-s)=4)$$
$$= e^{-\lambda s}\, \frac{e^{-\lambda(t-s)}\,[\lambda(t-s)]^{4}}{4!}$$

$$P(N(s)=1) \Rightarrow P(N(t-s)=3)$$
$$= \frac{e^{-\lambda s}\,\lambda s\; e^{-\lambda(t-s)}\,[\lambda(t-s)]^{3}}{3!}$$

e) $\mathrm{cov}(N(s), N(t))$
$$= E[(N(s)-\mu_{N(s)})(N(t)-N(t))]$$
$$= E[N(s) \cdot N(t)] - \mu_{N(s)}\,\mu_{N(t)}$$