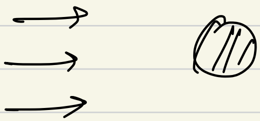
Lecture - 13

Particle mechanics :

- Flow through packed bed (drying, heat recovery) and fluidized bed (coal combustion)
- Catalytic reactors
- Gravity sedimentation
- Centrifugal separation
- Cyclone separators
- Floatation

Drag :



it is the force in the direction of flow exerted by fluid on the solid

* Drag has two components —

- Form drag or pressure drag
- Wall drag or friction drag or shear drag

Drag coefficient :

* For flow through pipes & channels,

friction factor, $f = \dfrac{\text{Wall shear stress}}{\text{velocity head} \times \text{density}}$

$$f = \frac{\tau_w}{\frac{1}{2}\rho V^2}$$

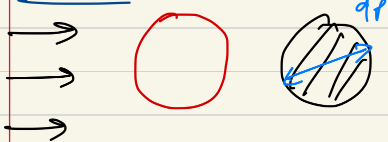
* For immersed solid objects, it is drag coeff,

$$C_d = \frac{\text{Total drag force / unit area}}{\text{velocity head} \times \text{density of fluid}}$$

$$C_d = \frac{F_D / A_P}{\frac{1}{2}\rho V^2}$$

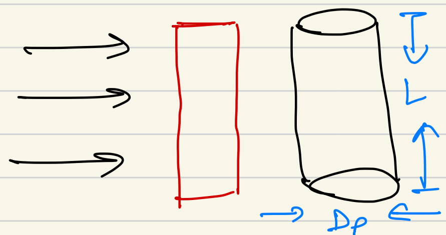
A_P - Projected area & is defined as area Obtained by projecting the object on a plane perpendicular to direction of flow

(i) Sphere



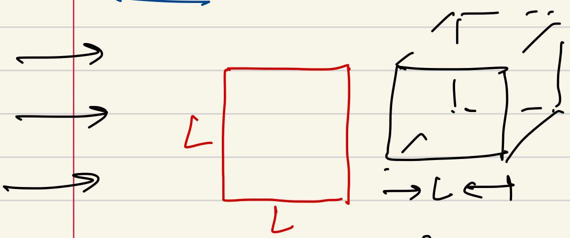
Projected area $= \dfrac{\pi D_p^2}{4}$

(ii) cylinder



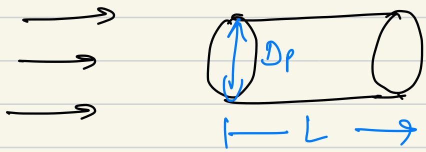
Projected area $= L \times D_p$

(iii) cube:



Projected area $= L^2$

(iv) cylinder:



Projected area $= \dfrac{\pi D_p^2}{4}$

Case: Spherical particle settling in a fluid

$D_p, \underbrace{\rho, \mu}_{\text{fluid}}, V, C_D$

From dimensional analysis, $C_D = f(Re)$

$$Re_p = \frac{D_p V \rho}{\mu} \left(\frac{\text{inertial force}}{\text{viscous force}} \right)$$

- C_D may depend on shape factor & orientation of the object

→ Different C_D & Re_p relations exist for each shape & orientation

* <u>For spherical particle settling in a Newtonian fluid</u>, $Re_p < 0.1$

$$C_D = \frac{F_D / A_p}{\frac{1}{2} \rho V^2}$$

$$F_D = 3\pi \mu V D_p \quad \text{(Stoke's law)}$$

$$C_D = \frac{3\pi \mu V D_p}{\frac{1}{2} \rho V^2 \left(\frac{\pi D_p^2}{4}\right)} = \frac{24 \mu}{\rho V D_p} = \frac{24}{Re_p}$$

$$C_D = \frac{24}{Re_p}$$ } valid for $Re_p \ll 1$ creeping flow

* At such low Re_p, terminal velocity of a sphere,

$$\boxed{v = \frac{g D_p^2 (\rho_p - \rho)}{18 \mu}}$$ — ☆

→ $v \propto D_p^2$

Terminal velocity,

$$v = \frac{g D_p^2 (\rho_p - \rho)}{18 \frac{D_p v \rho}{Re_p}} = \frac{g D_p^2 (\rho_p - \rho)}{18 \frac{D_p v \rho C_D}{24}}$$

$$\Rightarrow \boxed{v = \sqrt{\frac{4}{3} \frac{g (\rho_p - \rho) D_p}{C_D \cdot \rho}}} \quad (*)$$

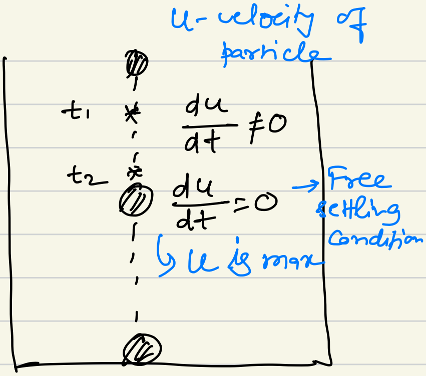
one dimensional motion of a particle through fluid :

Free settling condition:

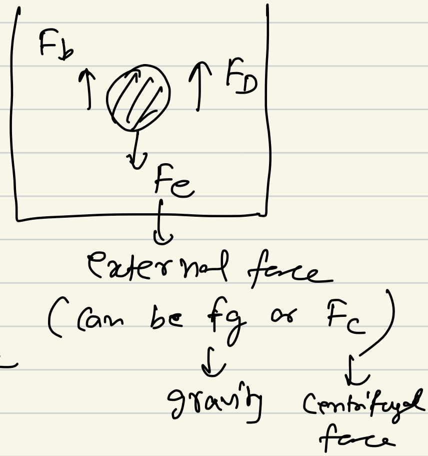
* Unbounded motion of particle in infinitely long cylindrical column

- No other particle nearby
- Container wall is also far away

u_t - Terminal velocity or free settling velocity



* Consider a particle of mass 'm' moving through a fluid under the action of external force Fe



m - mass of particle

ρ - density of fluid

ρ_p - density of particle

Ap - Projected area of particle

u - velocity of particle relative to the fluid

F_b - Buoyant force

F_D - Drag force

Net force,

$$F = \frac{d(mu)}{dt} = F_e - F_b - F_D$$

$$F_e = m a_e$$

$$F_D = \frac{C_D u^2 \rho A_P}{2} \qquad \left\{ C_D = \frac{F_D / A_P}{\frac{1}{2} \rho u^2} \right\}$$

Buoyancy force,

$$F_b = (\text{mass of fluid displaced}) \times \text{acceleration from external force}$$

$$F_b = \frac{m}{\rho_p} \cdot \rho \cdot a_e$$

$$m \frac{du}{dt} = m a_e - \frac{m \rho}{\rho_p} a_e - \frac{C_D u^2 \rho A_p}{2}$$

$$\Rightarrow \boxed{\frac{du}{dt} = a_e \left(\frac{\rho_p - \rho}{\rho_p} \right) - \frac{C_D u^2 \rho A_p}{2m}} \text{ — } (*)$$

<u>Case:</u> motion from gravitational force, $a_e = g$

$$\frac{du}{dt} = g \left(\frac{\rho_p - \rho}{\rho_p} \right) - \frac{C_D u^2 \rho A_p}{2m}$$

Terminal velocity (u_t) is the max. velocity attained by the particle under free settling conditions. i.e. $\frac{du}{dt} = 0$

$$\frac{g(\rho_p - \rho)}{\rho_p} = \frac{C_D u_t^2 \rho A_p}{2m}$$

$$\Rightarrow \boxed{u_t = \sqrt{\frac{2g(\rho_p - \rho) m}{A_p \rho_p C_D \rho}}} \text{ — } (*)$$

For spherical particle,

$$A_p = \frac{\pi}{4} D_p^2$$

$$m = \frac{\pi}{6} D_p^3 \rho_p$$

$$\Rightarrow \boxed{u_t = \sqrt{\frac{4}{3} \frac{g(\rho_p - \rho) D_p}{C_D \cdot \rho}}}$$

→ generalized expression valid for entire range of Reynold number.

$C_D \rightarrow f(u_t)$

Stoke's regime:

For $Re_p \ll 1$, $\quad C_D = \frac{24}{Re_p} = \frac{24 \mu}{D_p u_t \rho}$

$$u_t = \sqrt{\frac{g(\rho_p - \rho) D_p^2 u_t}{18 \mu}}$$

$$\Rightarrow \boxed{u_t = \frac{g D_p^2 (\rho_p - \rho)}{18 \mu}} \rightarrow (*)$$

→ Terminal velocity of a sphere in stoke's flow regime