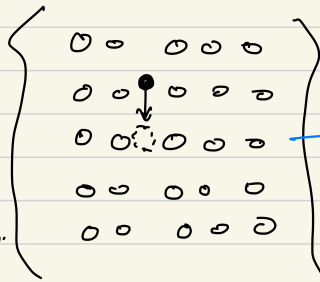
Lecture # 18

Hindered settling:

* Shape & size are same for all the particles.



Fluid is moving up due to downward motion of particle.

* For hindered flow, settling velocity of particles is less than that of free settling velocity.

viscosity of suspension or mixture,

$$\mu_m = \frac{\mu}{\psi_p} = \frac{\text{Actual viscosity of liquid}}{\text{Emperical correlation factor}}$$

ψ_p → depends on volume fraction of lq

$$\psi_p = \frac{1}{10^{1.82(1-\varepsilon)}} \quad \text{---} \; (*)$$

ε – volume fraction of lq in slurry

$$\varepsilon = \frac{\text{volume of lq}}{\text{Total volume}}$$

Bulk density of slurry,

$$\rho_m = \varepsilon \rho + (1-\varepsilon)\rho_p$$

$$\Rightarrow \rho_p - \rho_m = \rho_p - \{\rho_p + (1-\varepsilon)\rho_p\}$$

$$\boxed{\rho_p - \rho_m = \varepsilon(\rho_p - \rho)} \quad \text{———} \quad (*)$$

For stoke's flow of single particle,

$$u_t = \frac{g D_p^2 (\rho_p - \rho)}{18 \mu}$$

For Hindered settling,

$$\frac{u_s}{\varepsilon} = \frac{g D_p^2 (\rho_p - \rho_m)}{18 \frac{\mu}{\psi_p}}$$

$$\Rightarrow u_s = \frac{g D_p^2 (\rho_p - \rho_m)}{18 \mu} \varepsilon \cdot \psi_p$$

$$\Rightarrow u_s = \underbrace{\frac{g D_p^2 (\rho_p - \rho)\, \varepsilon \cdot \varepsilon \cdot \psi_p}{18 \mu}}$$

$$\Rightarrow \boxed{u_s = u_t \varepsilon^2 \psi_p}$$

$\{ \varepsilon^2 \psi_p$ is correction factor $\}$

$$Re_{p,s} = \frac{D_p u_s \rho_m}{\mu_m \varepsilon} = \frac{D_p^3 g(\rho_p - \rho)\rho_m \varepsilon^2 \psi_p}{\frac{\mu}{\psi_p} \cdot \varepsilon}$$

$$\Rightarrow Re_{p,s} = Re_p \underbrace{\frac{\rho_m}{\rho} \varepsilon \psi_p^2}$$

{much smaller than Reynold number of single particle}

$u_s = u_t (\varepsilon)^n$ { $n = f(Re_p)$ } For intermediate regime

For stoke's regime $(Re_p < 0.1) \rightarrow n = 4.6$

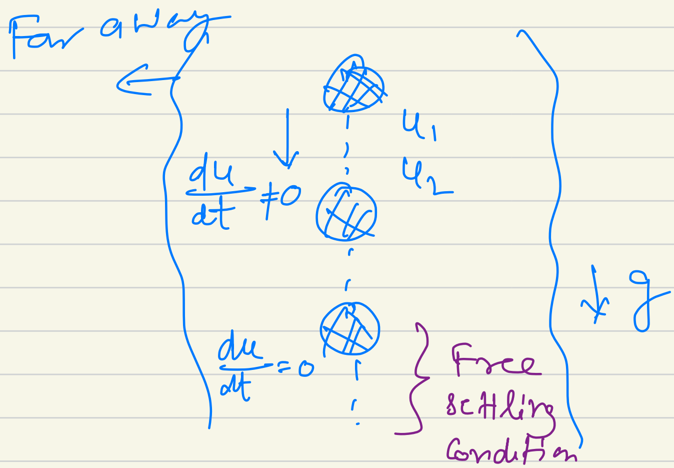
For Newton's regime $(Re_p > 10^3) \rightarrow n = 2.5$

Experimentally

(X) n vs Re_p is given graphically

Gravity sedimentation:

* Gravity sedimentation processes are impelled by force of gravity. In such processes, the fluid may be at rest or flowing at certain velocity.



* **Clarifier:** A settler that removes virtually all the particles from a liquid is known as clarifier (such as sedimentation tanks)

* **Classifier:** A device that separates the solids into two fractions is called classifier. These are the devices that separate particles of different densities.

Gravity classifiers:

1) Sink-and-float method:

A liquid of intermediate density is chosen such that lighter material will float and heavier settles in the liquid. (independent of size of particles & depends only on relative density of materials)

* make sure particles are crushed to have same size before settling process starts.

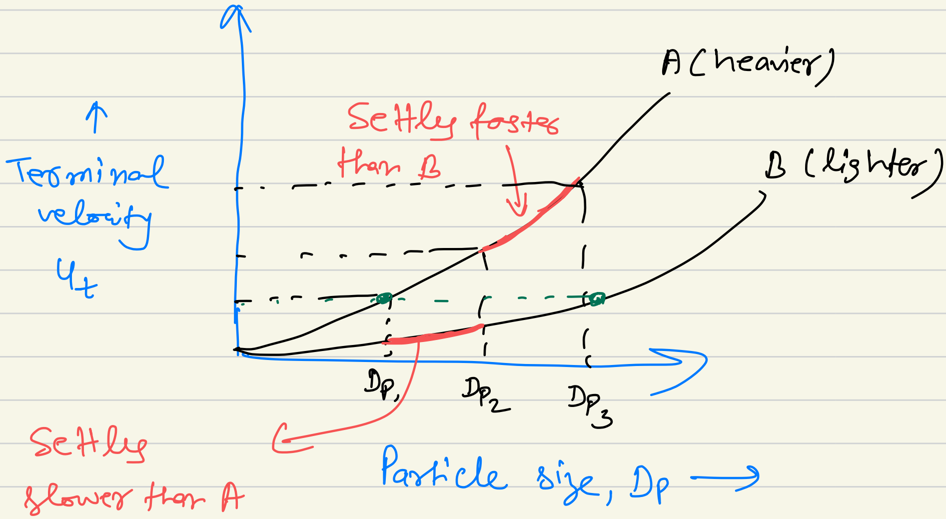
2) Differential settling method:

The density of the medium is less than that of either of two substances to be separated.

A- Heavy density material

B- Light " "

$$u_t = \sqrt{\frac{4}{3} \frac{g(\rho_p - \rho) D_p}{C_D \cdot \rho}}$$

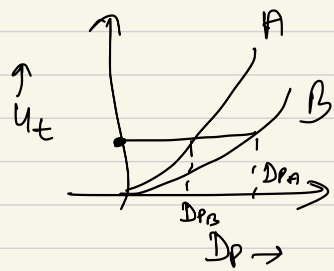


* Smaller heavy particles settle at approximately same terminal velocity as larger light particles.

<u>Terminal velocity expressions:</u>

$$u_{t,A} = \sqrt{\frac{\frac{4}{3} g (\rho_{p_A} - \rho) D_{p_A}}{C_{D_A} \cdot \rho}}$$

$$u_{t,B} = \sqrt{\frac{\frac{4}{3} g (\rho_{p_B} - \rho) D_{p_B}}{C_{D_B} \cdot \rho}}$$



$$u_{t,A} = u_{t,B}$$

$$\frac{(\rho_{p_A} - \rho) D_{p_A}}{C_{D_A} \cdot \rho} = \frac{(\rho_{p_B} - \rho) D_{p_B}}{C_{D_B} \cdot \rho}$$

$$\Rightarrow \frac{D_{pA}}{D_{pB}} = \frac{(\rho_{pB}-\rho)}{(\rho_{pA}-\rho)} \cdot \frac{C_{DA}}{C_{DB}}$$

* For spherical particles at very high Re (i.e. Newton's ~~flow~~ regime), $C_D = 0.44$

$$\Rightarrow \boxed{\frac{D_{pA}}{D_{pB}} = \left(\frac{\rho_{pB}-\rho}{\rho_{pA}-\rho}\right)} \longrightarrow (*)$$

* <u>For laminar Stoke's settling:</u>

$$C_{DA} = \frac{24\mu}{D_{pA}\, u_{tA} \cdot \rho} \quad \& \quad C_{DB} = \frac{24\mu}{D_{pB}\, u_{tB} \cdot \rho}$$

For $u_{t,A} = u_{t,B}$

$$\Rightarrow \frac{D_{pA}}{D_{pB}} = \left(\frac{\rho_{pB}-\rho}{\rho_{pA}-\rho}\right) \cdot \frac{D_{pB}}{D_{pA}}$$

$$\Rightarrow \boxed{\frac{D_{pA}}{D_{pB}} = \sqrt{\frac{\rho_{pB}-\rho}{\rho_{pA}-\rho}}} \longrightarrow (*)$$

For intermediate regime,

$$\frac{D_{pA}}{D_{pB}} = \left(\frac{\rho_{pB} - \rho}{\rho_{pA} - \rho}\right)^n \quad \left\{0.5 \leq n \leq 1\right\}$$

* For equal size particles in Newton's regime ($Re_p > 10^3$),

$$\frac{u_{t,A}}{u_{t,B}} = \sqrt{\frac{\rho_{pA} - \rho}{\rho_{pB} - \rho}} = \left(\frac{\rho_{pA} - \rho}{\rho_{pB} - \rho}\right)^{1/2}$$

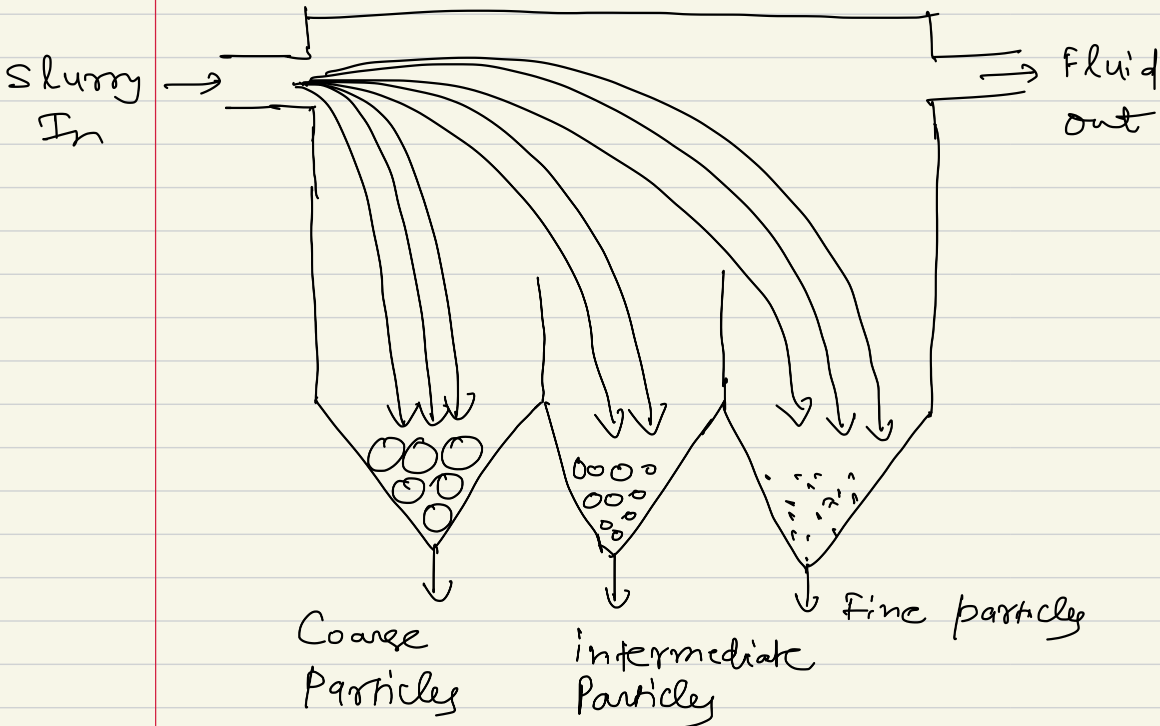
* For equal size particles in Stokes regime, i.e., $C_D = \frac{24}{Re_p}$

$$\frac{u_{t,A}}{u_{t,B}} = \left(\frac{\rho_{pA} - \rho}{\rho_{pB} - \rho}\right)^{1/3}$$

* For intermediate regime,

$$\frac{u_{t,A}}{u_{t,B}} = \left(\frac{\rho_{pA} - \rho}{\rho_{pB} - \rho}\right)^n \quad \left\{\frac{1}{3} \leq n \leq \frac{1}{2}\right\}$$

Simple gravity settling classifier:



* Large tank is subdivided into several sections.

* A Liquid slurry feed enters the tank containing a size range of solid particles.

* Larger, faster settling particles settle to bottom close to the entrance

* Slower settling particles settle to bottom close to the exit.