IME625: Stochastic Processes

2021-22 Sem-II

Homework-5

Let us consider a discrete-time model of traffic congestion in a highway toll plaza. There are c number of counters. Each counter takes 1 minute to serve a vehicle. For ease in modelling, let us assume that the vehicles arrive in a "discrete" manner – let A_1 vehicles arrive at the end of 1st minute, A_2 vehicles arrive at the end of 2nd minute, and so on, and no vehicle arrives during $(0,1)$ minute, $(1,2)$ minute, and so on. Consider $A_1, A_2, \ldots$ to be *iid* random variables. Let X_n, for $n = 0,1,2,\ldots$, denote the number of vehicles in the toll plaza at the end of n-th minute, after arrivals and departures of that minute. Note that n +1st minute begins with X_n vehicles. Argue that $\{X_n : n = 0,1,2,\ldots\}$ is a Markov chain.

Consider $P(A_1 = k)$ is 0.2 for $k = 0,1$ and 0.3 for $k = 2,3$. Determine transition probability matrix of the Markov chain for $c = 2$, and $c = 3$.