IME625: Stochastic Processes

2021-22 Sem-II

Homework-14

In the queueing model of Homework 5, number of vehicles in the toll plaza at the end of $n+1^{\text{st}}$ minute, $X_{n+1} = \max(X_n - c, 0) + A_{n+1}$ for $n \geq 0$, where c is the number of counters and A_{n+1} is the number of vehicles arriving during $n+1^{\text{st}}$ minute. Consider $c = 1$ and the arrivals to be independent and identically distributed with mass function: $P(A = 0) = a_0$, $P(A = 1) = a_1$, $P(A = 2) = a_2$, $P(A \geq 3) = 0$. Determine the condition involving a_1, a_2 for the Markov chain to be positive recurrent. Assuming the condition holding, calculate the quantities of interest mentioned earlier, i.e., the long-run traffic congestion $\sum_{k \geq c+1} (k - c)\pi_k$ and counter utilization $\sum_{k \leq c-1} (k/c)\pi_k + \sum_{k \geq c} \pi_k$. What happens to these quantities if the condition does not hold?