Wyner VAE
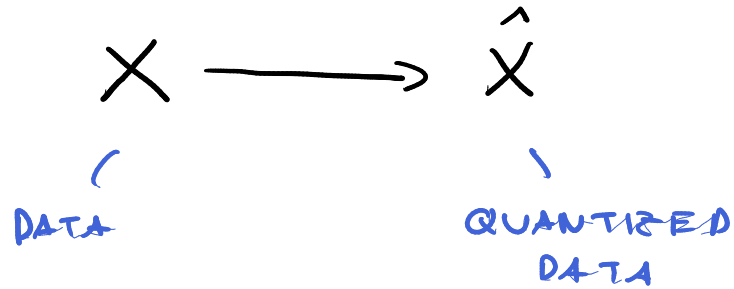
Joint & conditional generation
with succint common representation learning

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RELEVANT QUANTIZATION

$$X \longrightarrow \hat{X}$$

DATA   QUANTIZED DATA

- P_X FIXED & KNOWN

- FIND CODEBOOK $P_{\hat{X}|X}$

- JUST MINIMIZE $I(X;\hat{X})$?

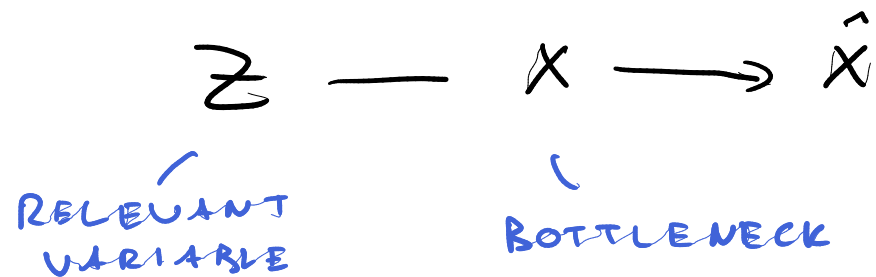# Rate Distortion Theory

$$X \longrightarrow \hat{X} \qquad\qquad d : \mathcal{X} \times \hat{\mathcal{X}} \longrightarrow \mathbb{R}^+$$

- WE MAKE THE PROBLEM WELL-DEFINED BY REQUIRING A CHOSEN DISTORTION TO BE BOUNDED:

$$\min_{P_{\hat{X}|X}} I(X ; \hat{X}) \qquad s.t. \qquad \mathbb{E}[d(X, \hat{X})] < D$$

$P_{\hat{X}|X}$ — CODEBOOK

$\mathbb{E}[d(X, \hat{X})]$ — EXPECTED DISTORTION

D — UPPER BOUND

- HOW TO CHOOSE d ?

Information Bottleneck
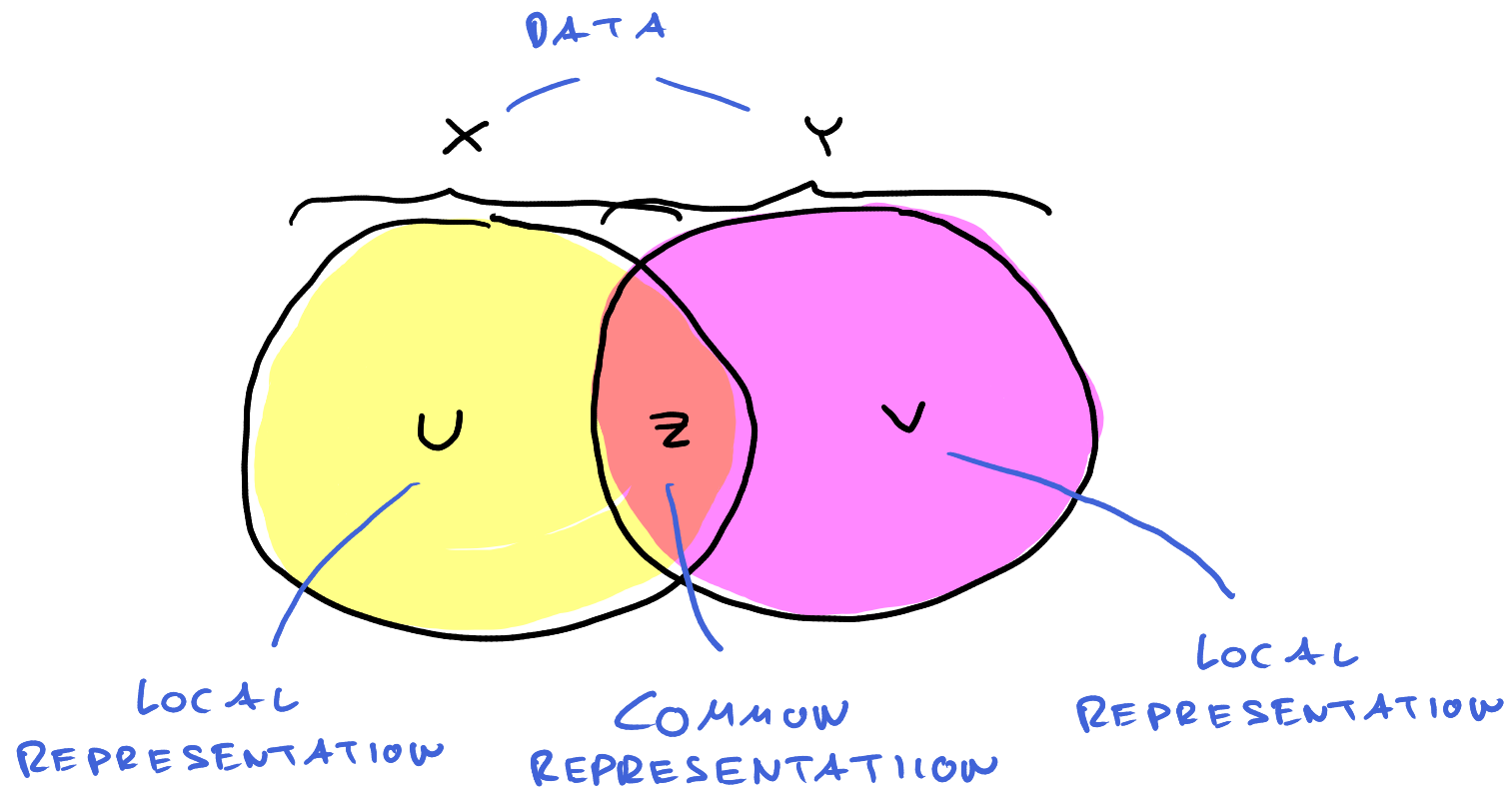
$$Z \text{ --- } X \longrightarrow \hat{X}$$

Z: RELEVANT VARIABLE

X: BOTTLENECK

- P_{zx} IS KNOWN & FIXED
- FIND $P_{\hat{x}|x}$, BUT PRESERVE KNOWLEDGE ABOUT Z!

$$\min_{P_{\hat{x}|x}} I(x;\hat{x}) \quad \text{s.t.} \quad I(\hat{x};z) \geq \alpha$$
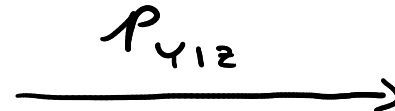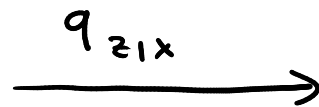
WYNER COMMON INFORMATION

DATA

X   Y

U

Z

V

LOCAL REPRESENTATION

COMMON REPRESENTATIION

LOCAL REPRESENTATION

OPERATIONAL PROBLEMS

$$X — Z — Y$$

- q_{xy} KNOWN FROM DATA

- FIND SUCCINT, NONTRIVIAL Z

- TWO EQUIVALENT OPERATIONAL PROBLEMS:

  > CHANNEL SYNTHESIS $\qquad X \longrightarrow Z \longrightarrow Y$

  > DISTRIBUTED SIMULATION $\quad X \longleftarrow Z \longrightarrow Y$

Channel Synthesis

$$X \longrightarrow Z \longrightarrow Y$$

$$\xrightarrow{q_{Z|X}} \quad \boxed{\equiv} \quad \xrightarrow{P_{Y|Z}}$$

Alice Describes
a Childhood Portrait

Bob Draws An
Adulthood Portrait

DISTRIBUTED SIMULATION

$$Z \xrightarrow{P_{X|Z}} X$$

$$Z \xrightarrow{P_{Y|Z}} Y$$
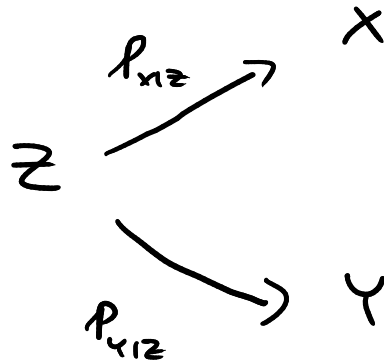
LOCAL NOISE
REPRESENTATION:

$$U \sim P_U \searrow$$
$$X = x(U, Z)$$
$$Z \sim P_Z$$
$$Y = y(V, Z)$$
$$V \sim P_V \nearrow$$
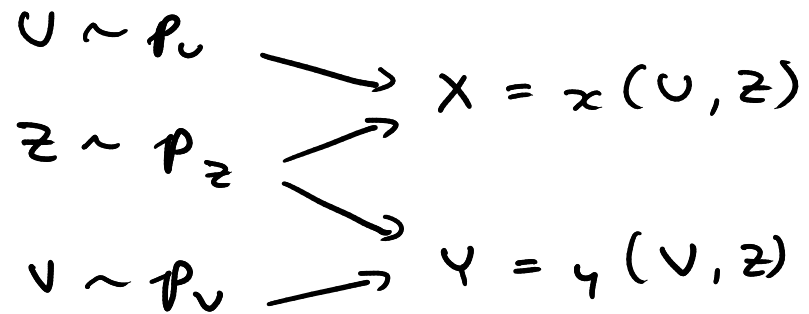
WYNER'S OPTIMIZATION PROBLEM

$$J(X;Y) = \min_{q_\varphi} I(X,Y;Z)$$

$$\text{s.t.} \quad X - Z - Y$$

$$(X,Y,Z) \sim q_{data}(x,y)\, q_\varphi(z|x,y)$$

$J(X;Y)$ IS THE FUNDAMENTAL LIMIT OF DISTRIBUTED SIMULATION

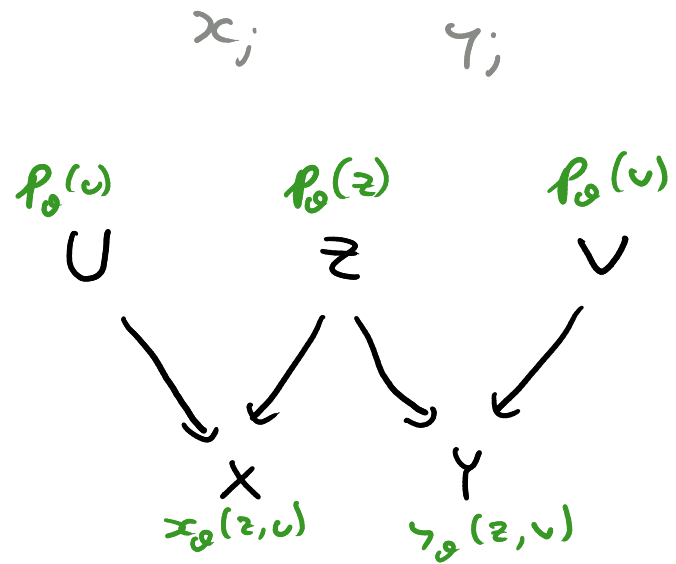# Recap: Variational Inference

- VI : Inference via optimization

$$\min_{\varphi} D_{KL}(q_{\varphi}(\cdot | x) \| p(\cdot | x))$$
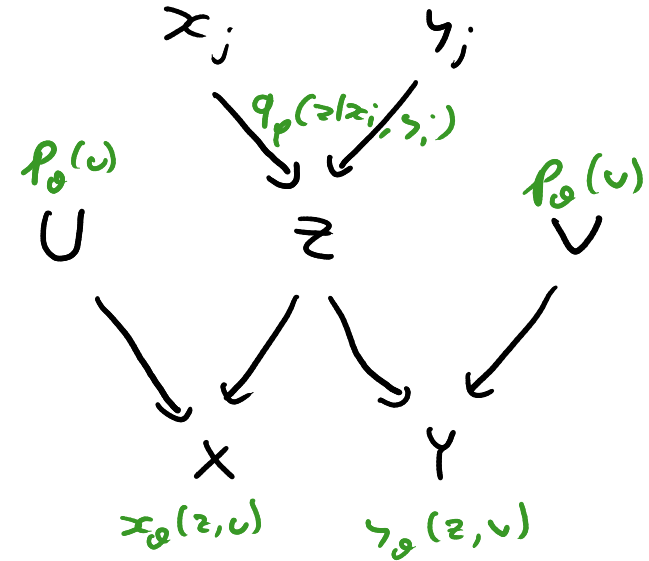
- VAE : Additionally, find generative model!

$$\min_{\vartheta, \varphi} D_{KL}(q_{\varphi}(\cdot | x) \| p_{\vartheta}(\cdot | x))$$

where $p_{\vartheta}(z | x)$ is the generative network

VARIATIONAL WYNER MODEL

x_i y_i

$P_\theta(u)$ $P_\theta(z)$ $P_\theta(v)$

U Z V

X Y

$x_\theta(z,u)$ $y_\theta(z,v)$

JOINT SAMPLING

x_j y_j

$q_\phi(z|x_i,y_i)$

$P_\theta(u)$ $P_\theta(v)$

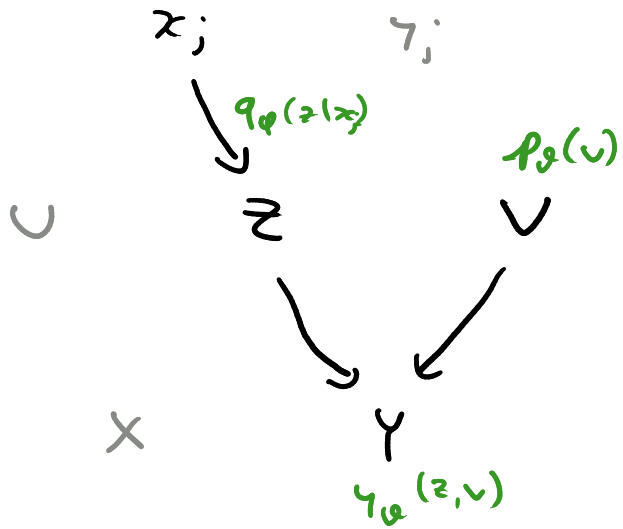U Z V

X Y

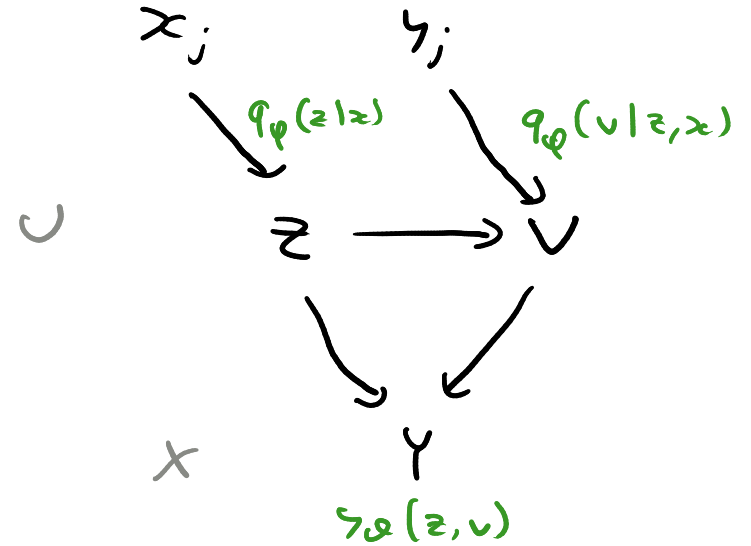$x_\theta(z,u)$ $y_\theta(z,v)$

JOINT STOCHASTIC
RECONSTRUCTION

Variational Wyner Model

Conditional Sampling

Conditional Sampling with Style Control

TRAINING OBJECTIVES

- MAIN OBJECTIVES

$$\min_{\varphi} \quad I_{xy\to}(X, Y \mid Z)$$

$$\text{s.t.} \quad X - Z - Y$$

$$\rightsquigarrow$$

$$\min_{\varphi} \quad I_m(X, Y \mid Z)$$

$$\text{s.t.} \quad D_m^{xyz} = \emptyset$$

- RECONSTRUCTION LOSSES

$$R_{xy\to x} = \mathbb{E}_{q_{xy} \to p_0}[dl_x(x, \tilde{x})]$$

- WEIGHTED COMBINATION, SELECTION PER MODEL

TRAINING PROCEDURE

- SELECT NECESSARY OBJECTIVES:

$$\mathcal{D}_{x \to y}^{xyzuv} + \mathcal{D}_{y \to x}^{xyzuv} + \mathcal{D}_{x \leftrightarrow y}^{xyzuv}$$
$$+ \lambda^{CI}(I_{x \to y} + I_{y \to x})$$
$$+ \lambda^{rec}(\mathcal{R}_{x \to y} + \mathcal{R}_{y \to x} + \mathcal{R}_{x \to x} + \mathcal{R}_{y \to y})$$

<div align="center">MODEL LOSS</div>

$$\tilde{\mathcal{D}}_{x \to y}^{xyzuv} + \tilde{\mathcal{D}}_{y \to x}^{xyzuv} + \tilde{I}_{x \to y} + \tilde{I}_{y \to x}$$

<div align="center">DISCRIMINATOR LOSS</div>

- ADVERSERIAL TRAINING WITH VARIATIONAL DENSITY RATIO ESTIMATION (GAN-LIKE)

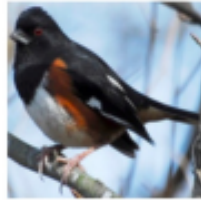# EXAMPLE: BIRD PICTURES & CAPTIONS
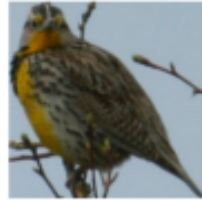
→ (IMAGE, CAPTION)

this small bird is black white white with a small bill bill and black feet

this bird is grey with grey and a black beak , pointy short pointy beak .

this is a black and white black bird and a short black beak .

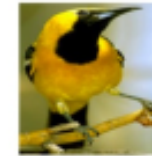this bird has a black and and white and white feathers and

IMAGE → CAPTION

this bird has a black crown and breast , with a crown , and and and black red its . .

this is a very , and white and and color with with a , and and a long blue patches . .

this bird has a very , thin beak with a breast and a brown beak , the body rimmed body .

CAPTION → IMAGE

This bird has yellow topped black and white striped wings and some red markings on its belly.

GROUND TRUTH

GENERATED