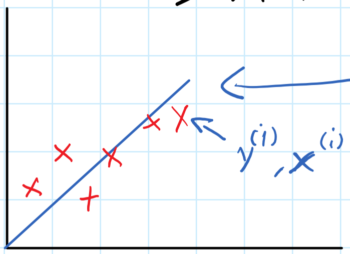
Linear Regression

Mittwoch, 29. Januar 2020 15:45

distance $\rightarrow$ less $\rightarrow$ the better

$\Rightarrow$ minimize distance btw line and point



$y_{guess}^{(i)} = m \cdot x^{(i)} + c$

$$\sum_i \left(y_{guess}^{(i)} - y^{(i)}\right)^2$$

$$\Rightarrow \operatorname{cost}(m,c) = \sum_i \left(m x^{(i)} + c - y^{(i)}\right)^2$$

$$\frac{\partial}{\partial m} \operatorname{cost} = 0$$

$$\frac{\partial}{\partial c} \operatorname{cost} = 0$$

$$0 = \frac{\partial}{\partial m} \operatorname{cost} = \frac{\partial}{\partial m} \sum_i \left(m x^{(i)} + c - y^{(i)}\right)^2$$

$$= \sum_i \frac{\partial}{\partial m} \left(m x^{(i)} + c - y^{(i)}\right)^2 = \sum_i 2\left(m x^{(i)} + c - y^{(i)}\right) \cdot x^{(i)}$$

$$= \sum_i 2\left(m x^{(i)} + c - y^{(i)}\right) \cdot x^{(i)}$$

$$= m \sum_i x^{(i)2} + c \sum_i x^{(i)} - \sum_i y^{(i)} x^{(i)}$$

$$0 = \frac{\partial}{\partial c} \operatorname{cost} = \dots = m \sum x^{i} + c n - \sum y^{(i)}$$

$$\begin{bmatrix} \sum y^{(i)} x^{(i)} \\ \sum y^{(i)} \end{bmatrix} = \begin{bmatrix} \sum x^{(i)2} & \sum x^{(i)} \\ \sum x^{(i)} & n \end{bmatrix} \begin{bmatrix} m \\ c \end{bmatrix}$$

$$y^{(i)} = a\, x^{(i)2} + b\, x^{(i)} + c$$

$$\sum \left(a x^{(i)2} + b\, x^{(i)} + c - y \right)^2 = \mathrm{cost}(a, b, c)$$

$$\frac{\partial}{\partial a} \mathrm{cost} = 0 = \sum 2 \left(a x^{(i)2} + b\, x^{(i)} + c - y \right) \cdot \; x^2 \qquad 1:2$$

$$\frac{\partial}{\partial b} \mathrm{cost} = 0 = \sum 2 \left(a x^{(i)2} + b x^{(i)} + c - y \right) \; x \qquad 1:2$$

$$\frac{\partial}{\partial c} \mathrm{cost} = 0 = \sum 2 \left(a x^{(i)2} + b\, x^{(i)} + c - y \right) \qquad 1:2$$

$$\Leftrightarrow \sum y^i x^{(i)2} = a \sum x^{(i)4} + b \sum x^{(i)3} + c \sum x^i$$

$$\sum y^i x^i = \ldots$$

$$\vdots$$

$$\begin{bmatrix} \sum y^i x^{i2} \\ \sum y^i x^i \\ \sum y^i \end{bmatrix} = \begin{bmatrix} \sum x^{i4} & \sum x^{i3} & \sum x^{i2} \\ \sum x^{i3} & \sum x^2 & \sum x^i \\ \sum x^{i2} & \sum x & n \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

$$f(x) = a \cdot e^{-x} + c$$

$$\sum \left(a \cdot e^{-x^{(i)}} + c - y \right)^2 \qquad = cost(a, c)$$

$$\frac{\partial}{\partial a} = 0 = \sum 2 \cdot (a \cdot e^{-x} + c - y) - e^{-x} \quad | : 2$$

$$\frac{\partial}{\partial c} = 0 = \sum 2(a \cdot e^{-x} + c - y) \qquad | : 2$$

$$\Leftrightarrow -a \sum e^{-2x} - c \sum e^{-x} + \sum y e^{-x}$$

$$\Leftrightarrow \; a \sum e^{-x} + c \sum 1 \quad - \sum y$$

$$\rightarrow \begin{bmatrix} \sum y e^{-x} \\ \sum y \end{bmatrix} = \begin{bmatrix} \sum e^{-2x} & \sum e^{-x} \\ \sum e^{-x} & n \end{bmatrix} \cdot \begin{bmatrix} a \\ c \end{bmatrix}$$

$$cost(a, b) = \sum \left(b \cdot \ln(v^i) + \ln(a) - \ln(c^i) \right)^2$$

$$\frac{\partial}{\partial a} = 0 = \sum 2 \cdot (b \cdot \ln(v^i) + \ln(a) - \ln(c^i)) \frac{1}{a}$$

$$\frac{\partial}{\partial b} = 0 = \sum 2 (b \cdot \ln(v^i) + \ln(a) - \ln(c^i)) \ln(v^i)$$

$$\frac{b}{a} \sum \ln(v^i) + \frac{1}{a} \ln(a) \sum 1 - \frac{1}{a} \sum \ln(c^i) \quad | \cdot a$$

$$b \sum \ln^2(v^i) + \ln(a) \sum \ln(v^i) - \sum \ln(c^i) \cdot \ln(v^i)$$

$$b \sum \ln(v^i) + \ln(a) \sum 1 = \sum \ln(c^i)$$

$$b\sum \ln^2(v^i) + \ln(a)\sum \ln(v^i) = \sum \ln(c^i)\cdot \ln(v^i)$$