Midterm 2019 – Philip Singer

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Problem 2

1) $f(x) = \sin^2(x)$

$$f(x) \approx \sum_{i=0}^{n} \frac{f^{(i)}(0)}{i!} \cdot x^i$$

$$
\begin{aligned}
f(x) \approx\ & \overset{=0}{\cancel{\sin^2(0)}} \\
& + \overset{=0}{\cancel{2\sin(0)\cdot\cos(0)}}\cdot x \\
& + \frac{2\,(\cos(0)\cdot\cos(0) + \sin(0)\cdot -\sin(0))\cdot x^2}{2} \\
& + \cancel{\frac{-8\cdot\cos(0)\cdot\sin(0)\cdot x^3}{3!}} = 0 \\
& + \frac{(8\cdot\sin(0) - 8\cdot\cos(0))\cdot x^4}{4!} \\
& + \cancel{\frac{32\sin(0)\cdot\cos(0)\cdot x^5}{5!}} = 0 \\
& + \frac{(32\cos^2(0) - 32\sin(0))\cdot x^6}{6!} \\
& + \cancel{\frac{-128\cos(0)\cdot\sin(0)\cdot x^7}{7!}} = 0
\end{aligned}
$$

$$+ \frac{-128(\cos^2(0)-\sin^2(0))\cdot x^8}{8!}$$

$$+ O(x^9)$$

$$f(x) \approx x^2 - \frac{x^4}{3} + \frac{2x^6}{45} - \frac{x^8}{315} + O(x^9)$$

$$f(0.3) = \sin^2(0.3) \approx 0.087332192$$

Real value : $\sin^2(0.3) = 0.087332192$

2.

error :

$$\frac{f^{(n+1)}(\xi)\cdot x^{n+1}}{(n+1)!}, \quad \xi \in [0;x]$$

$O(x^9) = 0$ so we use $O(x^{10})$:

$$\Rightarrow \frac{512(\cos^2(\xi)-\sin^2(\xi))\cdot x^{10}}{10!}$$

We choose $\xi = 0$ to get the uncertainty :

$$\frac{512(\cos^2(0)-\sin^2(0))\cdot 0.3^{10}}{10!} = 8.3314\cdot 10^{-10}$$

Problem 3

3.1
$$cost(A,B,C) = \sum_{i=1}^{n} (A \cdot e^{x_i} + B \cdot \ln(x_i) + C - y_i)^2$$

$$\frac{\partial}{\partial A} cost = 0 = \sum 2(A \cdot e^{x_i} + B \cdot \ln(x_i) + C - y_i) \cdot e^{x_i} \quad | : 2$$

$$\frac{\partial}{\partial B} cost = 0 = \sum 2(A \cdot e^{x} + B \cdot \ln(x_i) + C - y_i) \cdot \ln(x_i) \quad | : 2$$

$$\frac{\partial}{\partial C} cost = 0 = \sum 2(A \cdot e^{x} + B \cdot \ln(x_i) + C - y_i) \quad | : 2$$

$$\Rightarrow \sum y \cdot e^{x} = A \cdot \sum e^{x+2} + B \sum \ln(x) \cdot e^{x_i} + C \sum e^{x_i}$$

$$\sum y \cdot \ln(x) = \ldots$$

$$\sum y = \ldots$$

$\Rightarrow$ Resulting matrixes:

$$\begin{bmatrix} \sum y \cdot e^{x} \\ \sum y \cdot \ln(x) \\ \sum y \end{bmatrix} = \begin{bmatrix} \sum e^{x+2} & \sum \ln(x) \cdot e^{x} & \sum e^{x} \\ \sum e^{x} \cdot \ln(x) & \sum \ln^2(x) & \sum \ln(x) \\ \sum e^{x} & \sum \ln(x) & \sum 1 \end{bmatrix} \cdot \begin{bmatrix} A \\ B \\ C \end{bmatrix}$$