

## Hierarchical Clustering

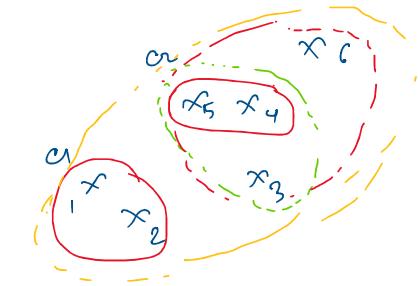


- problem
- 1) number of cluster specific
  - 2) Elbow method ✓
  - 3) Sensitive to outliers ✓
  - 4) non spherical shape

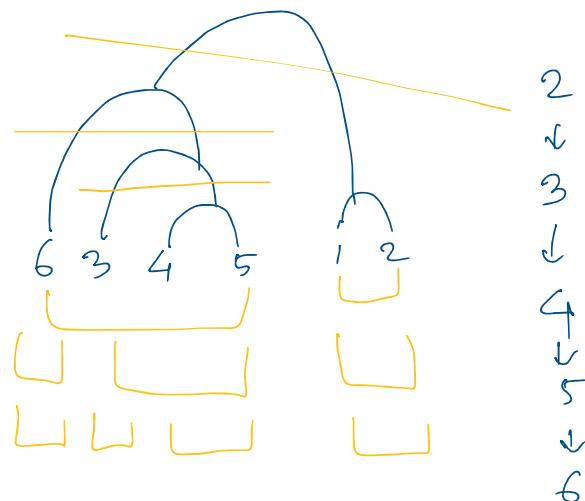


### Types

- 1) Agglomerative clustering → most used
- 2) Divisive clustering



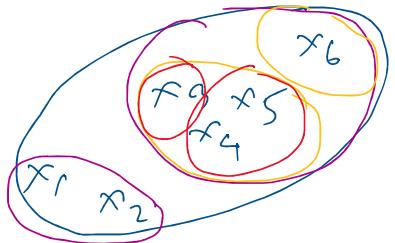
$$\begin{aligned} c_1 - c_3 & x \\ c_2 - c_3 & \\ c_1 - c_2 & \end{aligned}$$



2  
↓  
3  
↓  
4  
↓  
5  
↓  
6

### Divisive clustering

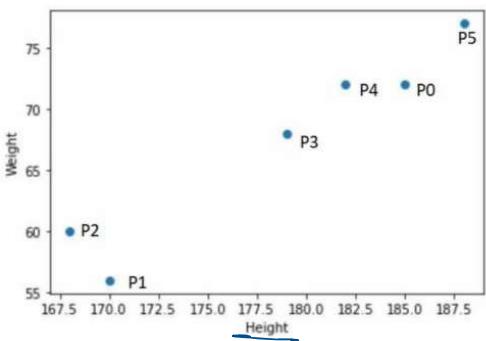




↓  
2  
↓  
3  
↓  
4  
↓  
5  
↓  
6

### Agglomerative clustering

	<u>Height</u>	<u>Weight</u>
P <sub>0</sub> →	185	72
P <sub>1</sub> →	170	56
P <sub>2</sub> →	168	60
P <sub>3</sub> →	179	68
P <sub>4</sub> →	182	72
P <sub>5</sub> →	188	77



$$A(x_1, y_1) \quad B(x_2, y_2)$$

$$D(A, B) = \text{Euclidean Formula} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$(P_0, P_1) \quad (P_0, P_2) \quad (P_0, P_3) \quad (P_0, P_4) \quad (P_0, P_5)$$

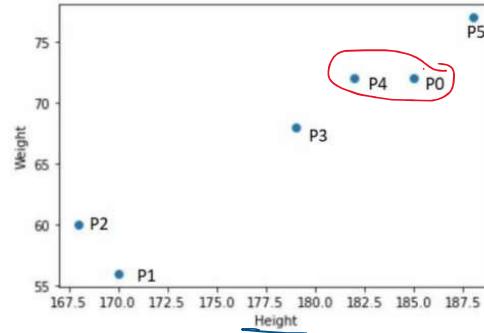
$$(P_1, P_2) \quad (P_1, P_3) \quad (P_1, P_4) \quad (P_1, P_5)$$

$$(P_2, P_3) \quad (P_2, P_4) \quad (P_2, P_5)$$

$$(P_3, P_4) \quad (P_3, P_5)$$

$(P_0, P_5)$

	P0	P1	P2	P3	P4	P5
P0	0					
P1	21.93	0				
P2	20.81	4.47	0			
P3	7.21	15	13.6	0		
P4	3	20	18.44	5	0	
P5	5.83	27.66	26.25	12.73	7.81	0



$P_0, P_4 \rightarrow 3$

①

	[P0,P4]	P1	P2	P3	P5
[P0,P4]	0				
P1	20	0			
P2	18.44	4.47	0		
P3	5	15	13.6	0	
P5	5.83	27.66	26.25	12.73	0

After merging P0 and P4

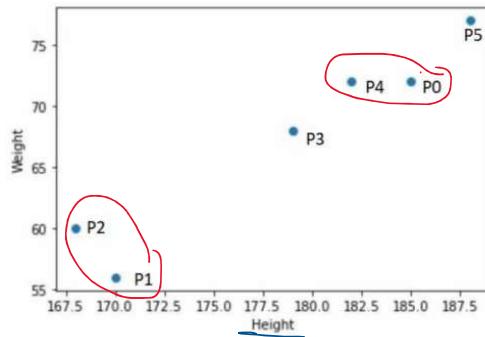
$$P_1, [P_0, P_4] \rightarrow \min(\delta(P_1, P_0), \delta(P_1, P_4)) \\ (21.93, 20) \\ = 20$$

$$P_2, [P_0, P_4] \rightarrow \min(\delta(P_2, P_0), \delta(P_2, P_4)) \\ \rightarrow (20.81, 18.44) \\ = 18.44$$

2

	[P0,P4]	[P1,P2]	P3	P5
[P0,P4]	0			
[P1,P2]	18.44	0		
P3	~5	13.6	0	
P5	5.83	26.25	12.73	0

Merging P1 and P2

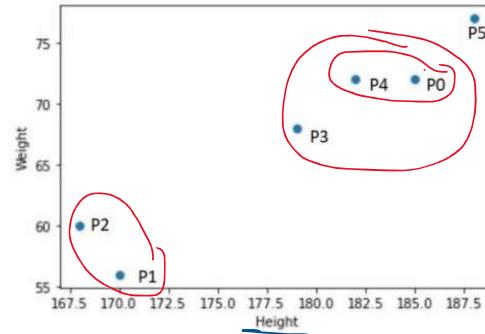


$$\begin{aligned}
 [P_0, P_4], [P_1, P_2] &\rightarrow \min(d(P_0, P_1), d(P_0, P_2), d(P_1, P_2), d(P_4, P_2)) \\
 &= \min(21.93, 20.81, 20, 18.44) \\
 &= 18.44
 \end{aligned}$$

	P0	P1	P2	P3	P4	P5
P0	0					
P1	21.93	0				
P2	20.81	4.47	0			
P3	7.21	15	13.6	0		
P4	3	20	18.44	5	0	
P5	5.83	27.66	26.25	12.73	7.81	0

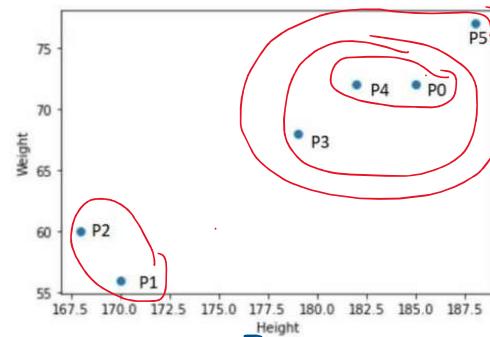
	[P3,[P0,P4]]	[P1,P2]	P5
[P3,[P0,P4]]	0		
[P1,P2]	13.6	0	
P5	5.83	26.25	0

Merging of P3 and [P0,P4]



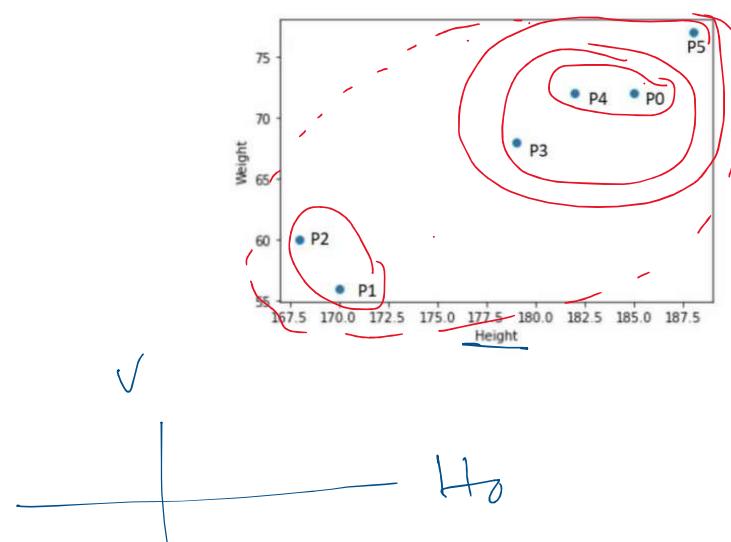
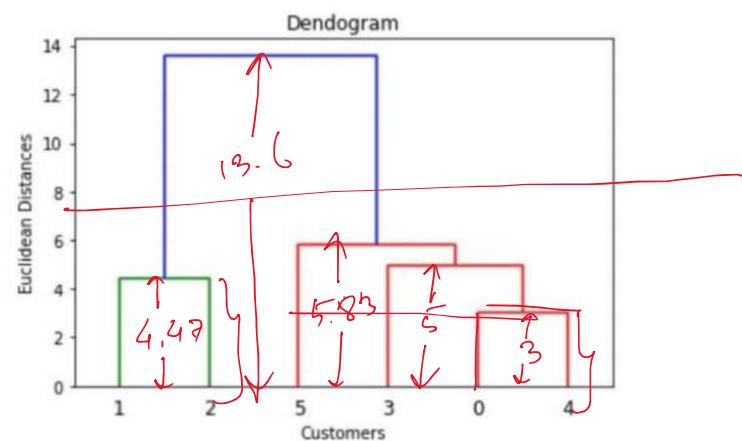
	[P5,[P3,[P0,P4]]]	[P1,P2]
[P5,[P3,[P0,P4]]]	0	
[P1,P2]	13.6	0

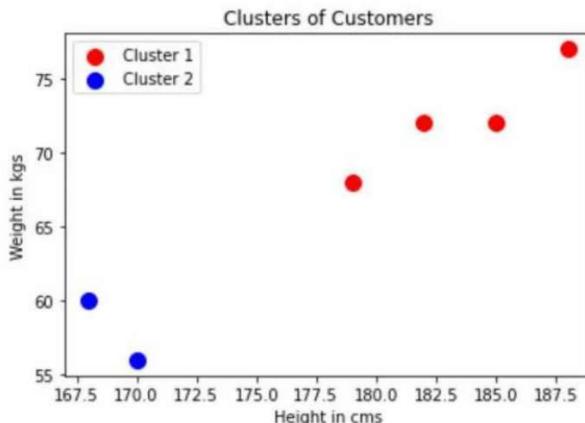
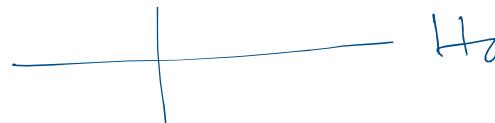
Merging of P5 and [P3,[P0,P4]]



	[[P1,P2],[P5,[P3,[P0,P4]]]]	0
[[P1,P2],[P5,[P3,[P0,P4]]]]		0

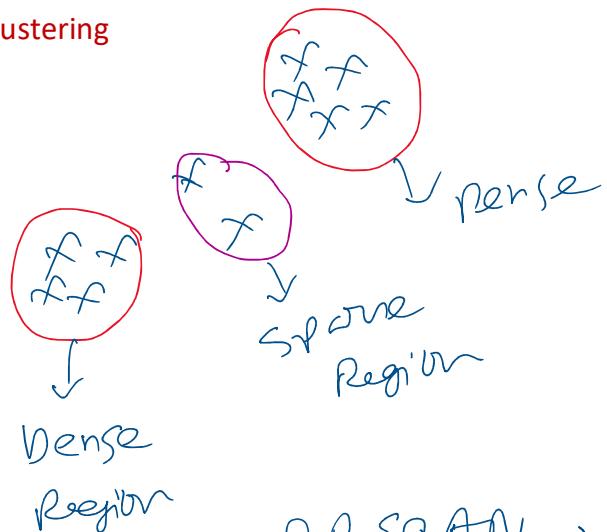
Merging of [P1,P2] and [P5,[P3,[P0,P4]]]





Final Clusters

Density Base Clustering



Dense →   
sparse →

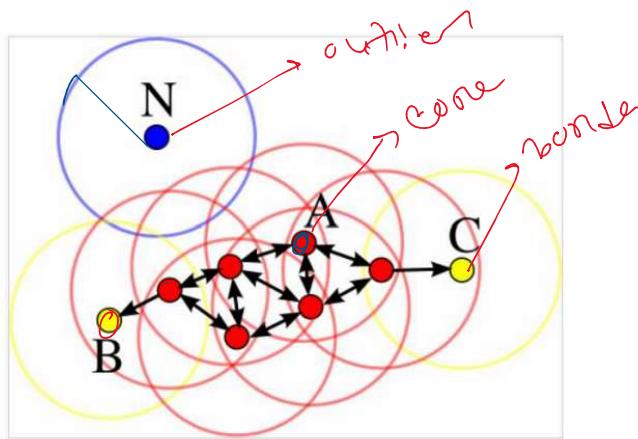
DBSCAN → Density-Based spatial clustering

Region

DBSCAN → Density-Based spatial clustering  
of Application with noise

OPTICS → Another Density Based Algorithm

DBSCAN



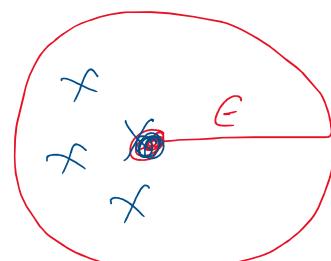
In this diagram, minPts = 4. Point A and the other red points are core points, because the area surrounding these points in an  $\epsilon$  radius contain at least 4 points (including the point itself). Because they are all reachable from one another, they form a single cluster. Points B and C are not core points, but are reachable from A (via other core points) and thus

- → core point
- → border point
- → Outlier

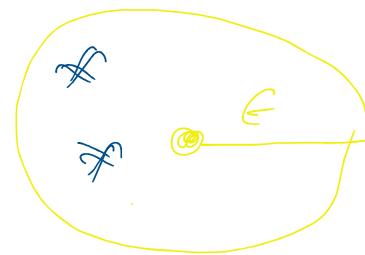
$$\text{minpts} = 4 \quad \epsilon = \text{radius}$$

\* core point

number of point  $\geq 4$



\* border point

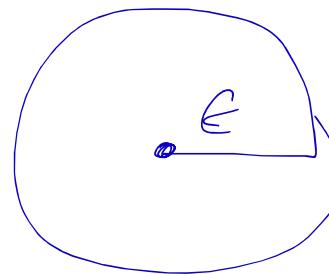


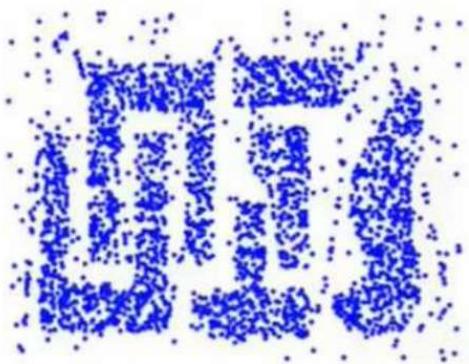
number of point  $3 < 4$   
and

1 at least = core

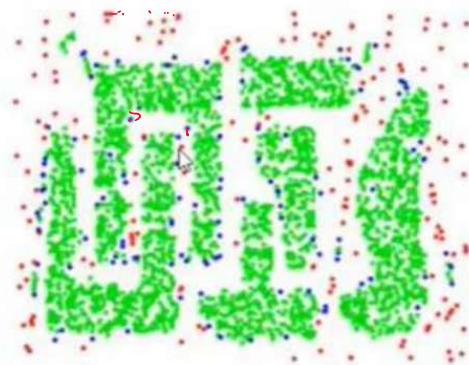
\* outlier :

No core point and No Border  
point



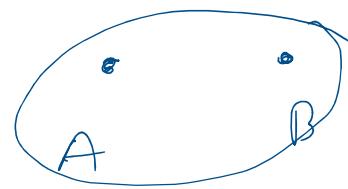


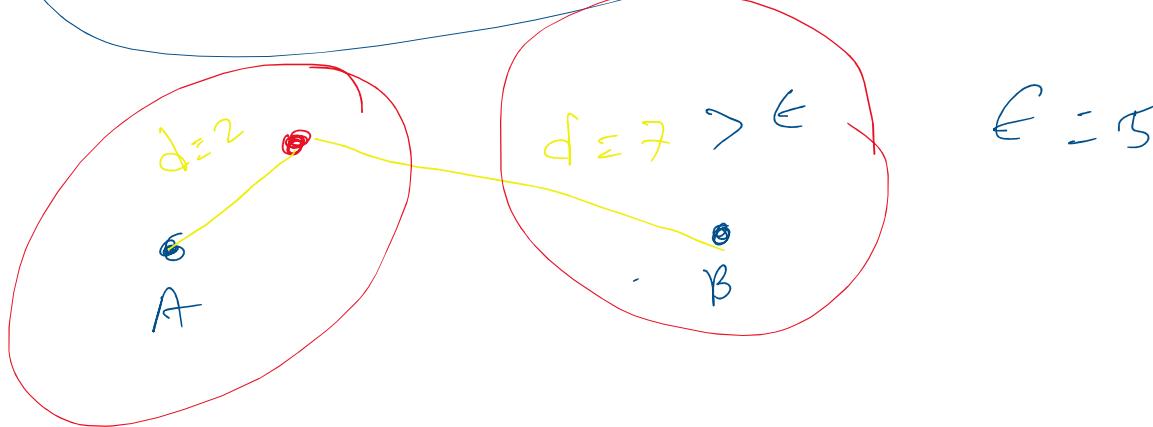
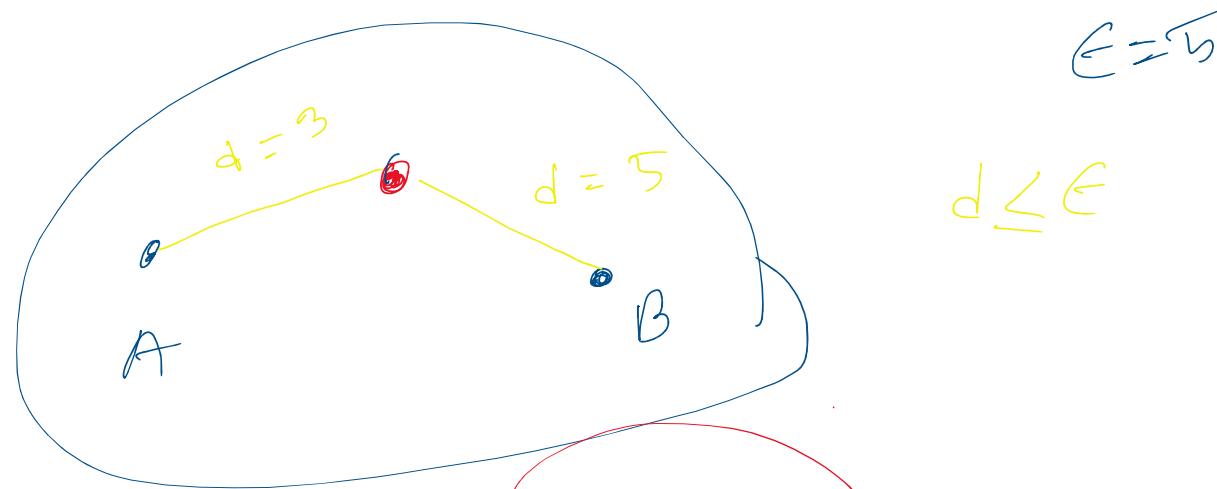
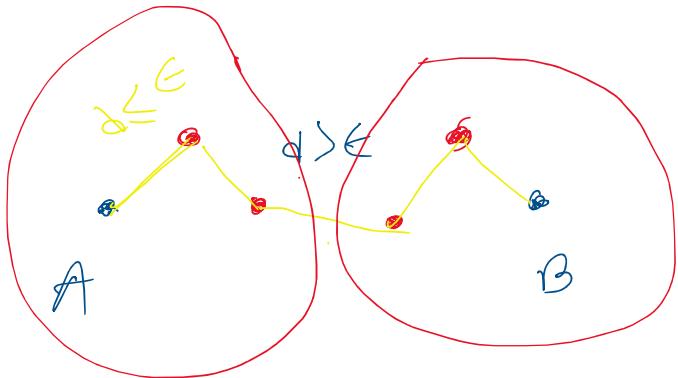
Original Points



Point types: core,  
border and noise

Density connected Point





DBSCAN Algorithm

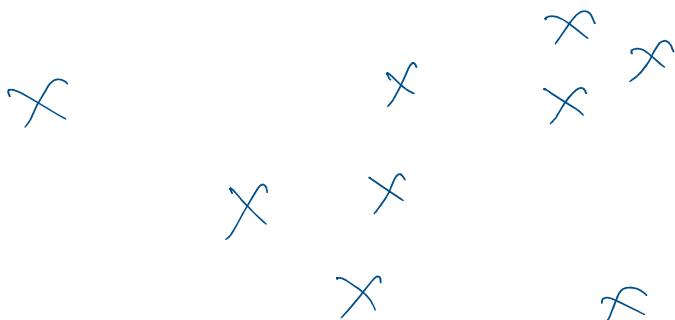
ε

## DBSCAN Algorithm

Step 0:

minpts = 3

$\epsilon = 5$



Step 1:

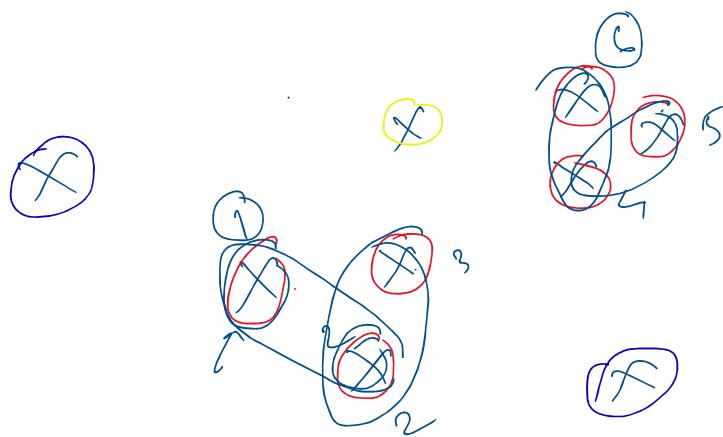
$$P_1 P_2 = d = 3 < \epsilon \checkmark$$

$$P_2 P_3 = d = 4 < \epsilon \checkmark$$

$$P_3 P_4 = d = 7 > \epsilon \times$$

$$P_6 P_4 = d = 3 < \epsilon \checkmark$$

$$P_1, P_5 = d = 2 < \epsilon \checkmark$$

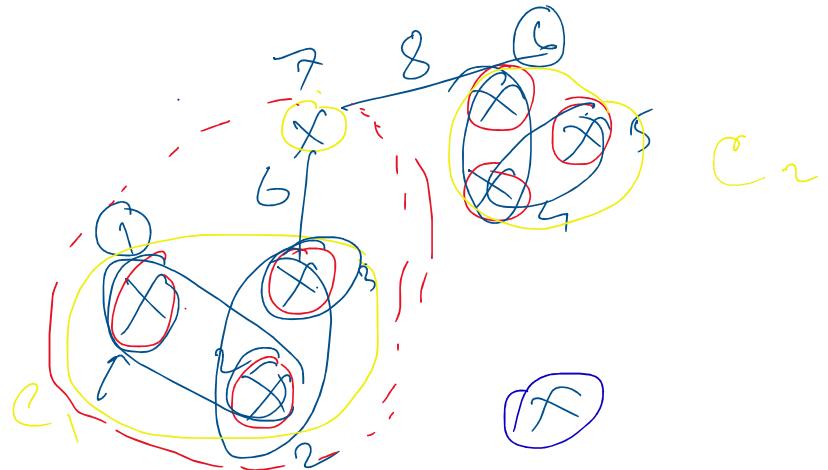


$P_4 P_5 = d = 2 \in \mathbb{Z}$

Step 2  $\Rightarrow$

Border point 7,  $c_1 \ni P_3 = 6$

Border point 7,  $c_2 \ni P_6 = 8$



Step 3  $\Rightarrow$  outliers  $\rightarrow$