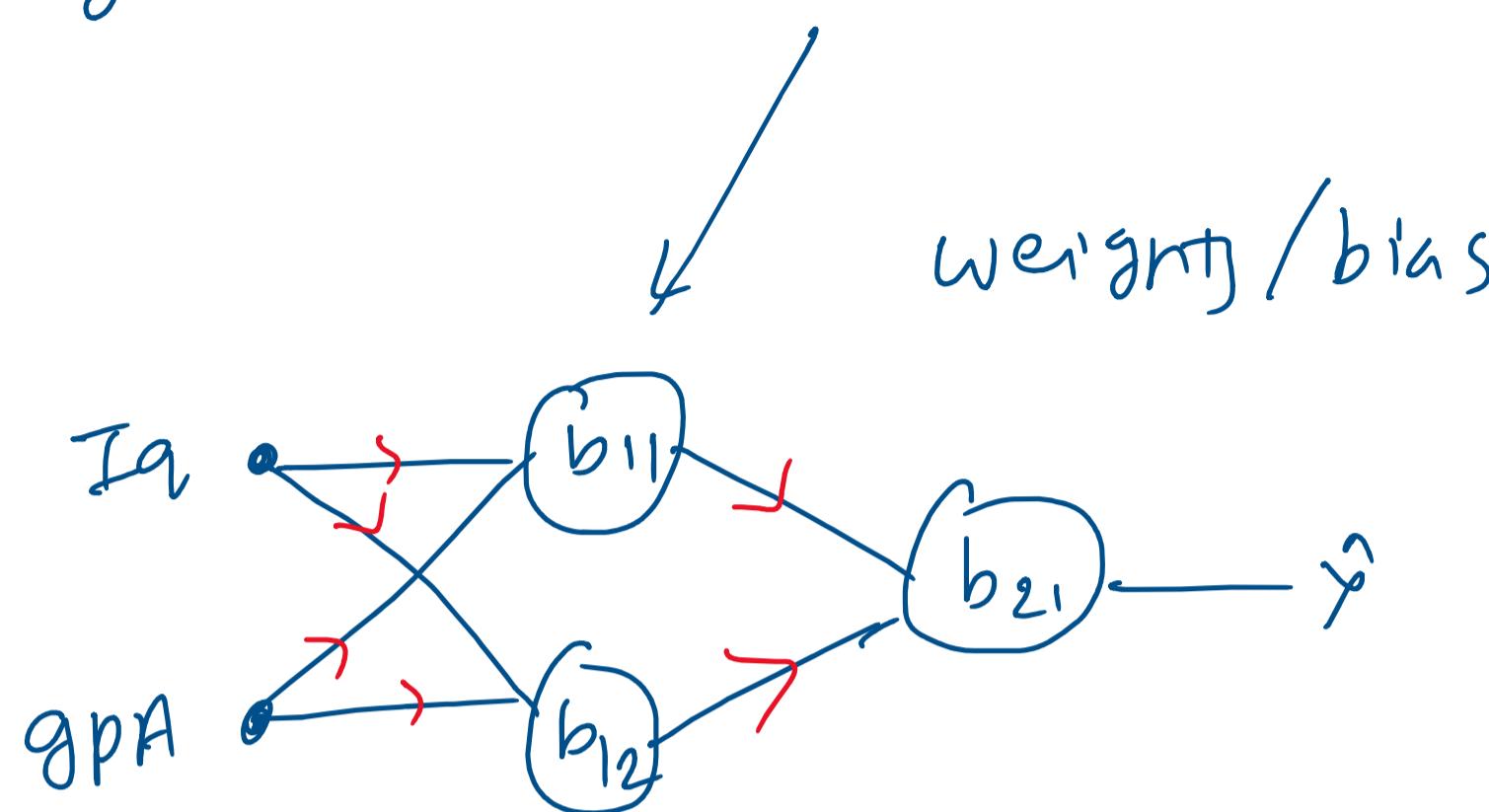


Back propagation → Algorithm → Train NN

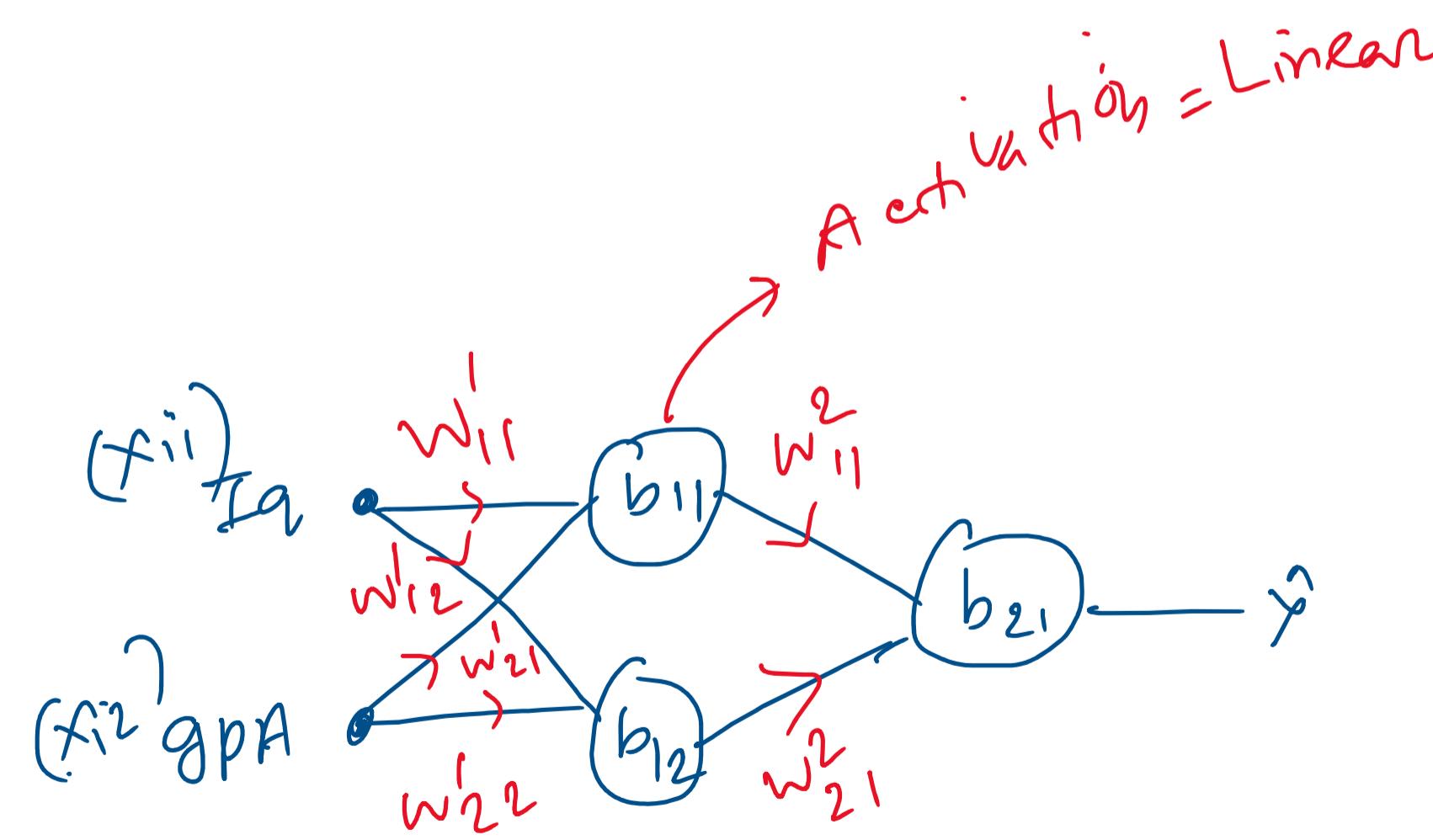
I_q	gpa	Salary
60	3.5	40
80	4.7	60
20	3	25
0) 0	3	50



- * Gradient Descent
- * Forward propagation

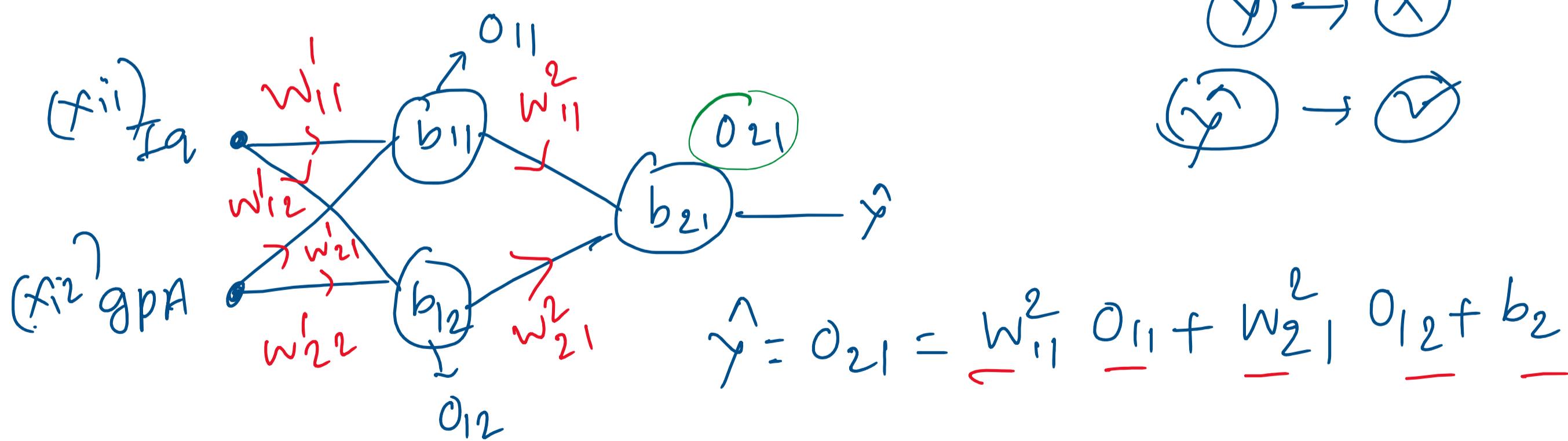
How BP works

I_q	gpa	Salary
60	3.5	40
80	4.7	60
20	3	25
0) 0	3	50



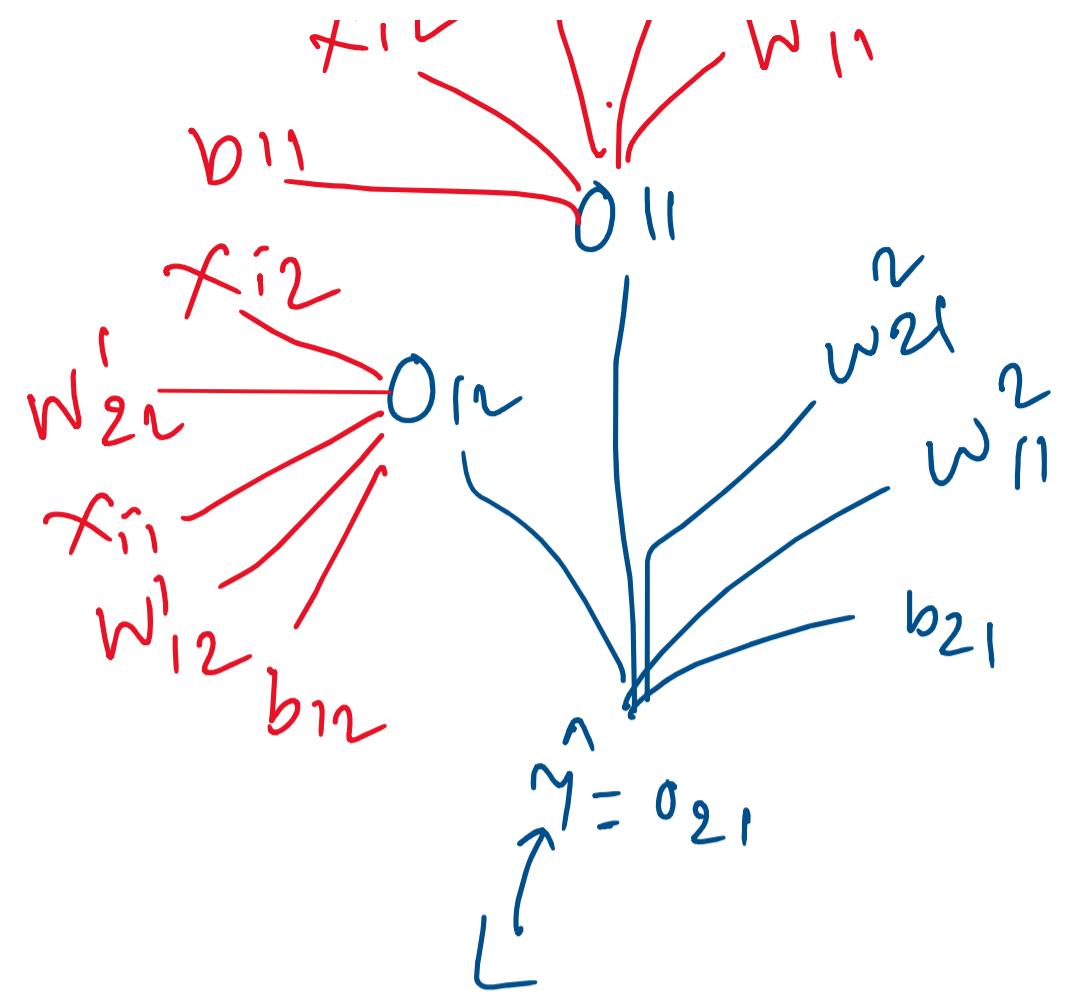
regression

- Step :
- 0) Initialization $w, b \rightarrow (\omega, b) \rightarrow$ Random Values
 $\omega = 1, b = 0$
Predict $\rightarrow 60$
 - 1) select a row
 - 2) Predict (Salary) \rightarrow Forward propagation
 - 3) choose a Loss Function (MSE) \rightarrow Regression Problem
 $L = (y - \hat{y})^2$
 $= (40 - 60)^2$
 $= 400 \leftarrow \text{error}$
 - 4) weights and bias update
Gradient descent

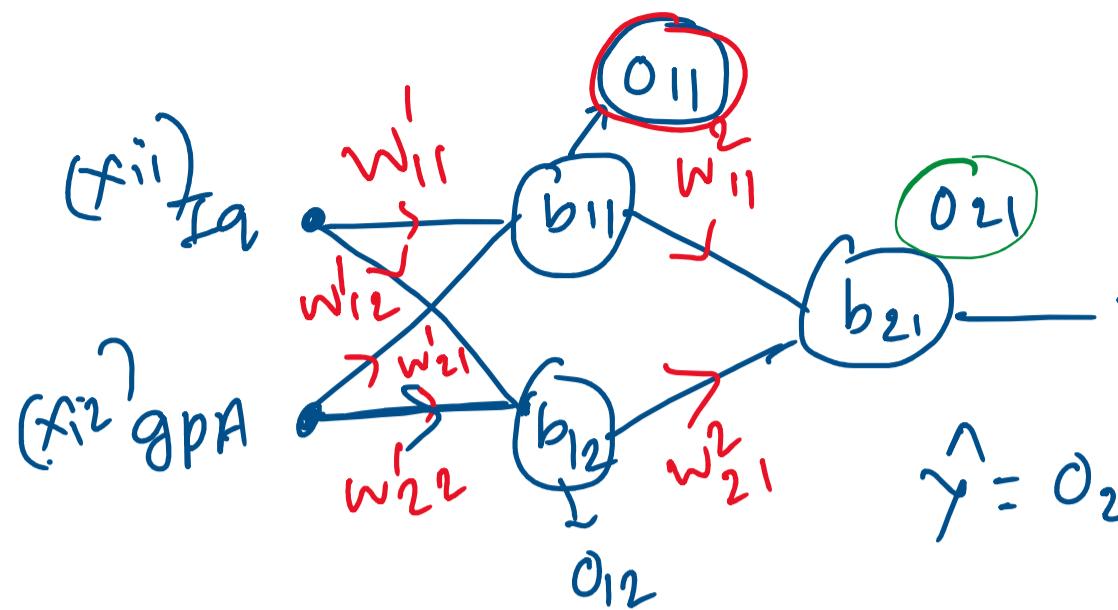


$$f_{12} \quad | \quad f_{11}$$
$$w_{11} \quad w_{12}$$

$$L \downarrow \min = \hat{y}$$
$$= -\frac{1}{m} \sum_{i=1}^m (y_i - \hat{y}_i)^2$$



$$o_{12} = w_{12}^1 x_{i1} + w_{22}^2 x_{i2} + b_{12}$$

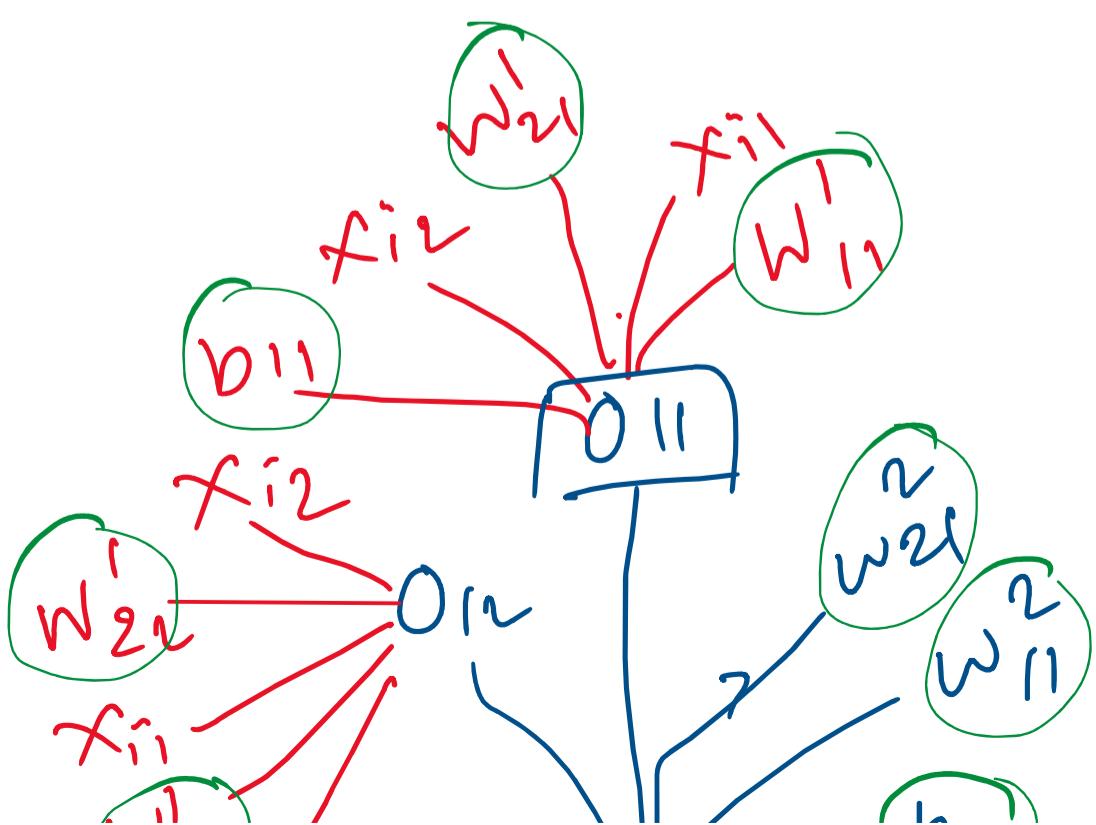


$$o_{11} = w_{11}^1 x_{i1} + w_{21}^1 x_{i2} + b_{11}$$

Gradient Descent

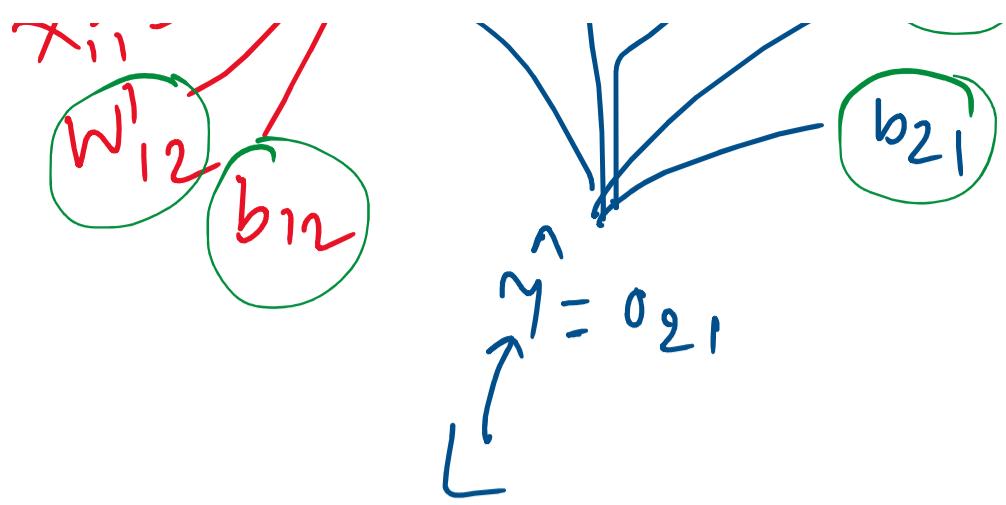
(*) $w_{\text{new}} = \boxed{w_{\text{old}}} - n \frac{\partial L}{\partial w_{o12}}$ ✓

(*) $b_{\text{new}} = b_{o12} - n \frac{\partial L}{\partial b_{o12}}$ ✓



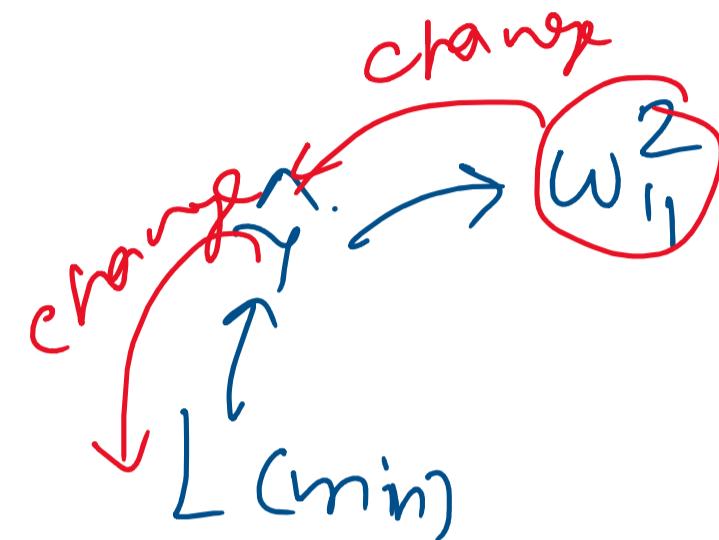
$$\frac{\partial L}{\partial w_{21}^2}, \frac{\partial L}{\partial w_{11}^2}, \frac{\partial L}{\partial b_{21}}$$

$$\frac{\partial L}{\partial w_{11}^1}, \frac{\partial L}{\partial w_{21}^1}, \frac{\partial L}{\partial b_{11}}$$



$$\frac{\partial L}{\partial w'_{11}}, \frac{\partial L}{\partial w'_{12}}, \frac{\partial L}{\partial b_{12}}$$

$\frac{\partial L}{\partial w^2_{21}}, \frac{\partial L}{\partial w^2_{11}}, \frac{\partial L}{\partial b_{21}}$ } *Finding*



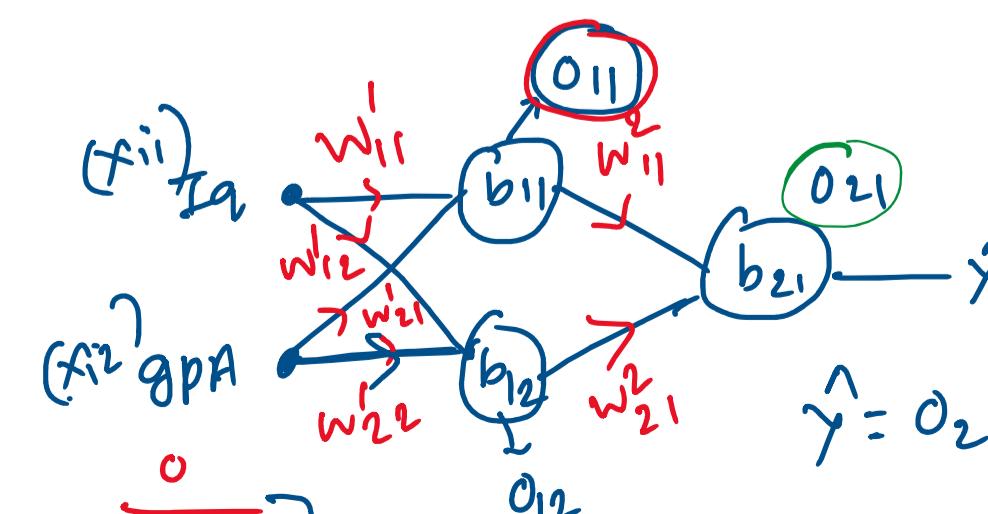
$$\frac{\partial L}{\partial w^2_{21}} = \boxed{\frac{\partial L}{\partial \hat{y}} \times \frac{\partial \hat{y}}{\partial w^2_{21}}} \rightarrow \text{chain rule of differentiation}$$

$$\frac{\partial L}{\partial w^2_{11}} = \frac{\partial L}{\partial \hat{y}} \frac{\partial \hat{y}}{\partial w^2_{11}}$$

$$\frac{\partial L}{\partial \hat{y}} = \frac{\partial}{\partial \hat{y}} (\gamma - \hat{y})^2$$

$$\hookrightarrow = -2(\gamma - \hat{y})$$

$$\boxed{\frac{\partial L}{\partial \hat{y}} = -2(\gamma - \hat{y})}$$



$$\frac{\partial \hat{y}}{\partial w_{21}^2} = \frac{\partial}{\partial w_{21}^2} \left[\underbrace{w_{11}^2}_{\text{WL}} o_{11} + \underbrace{w_{21}^2}_{\text{I}} o_{12} + \underbrace{b_{21}}_{\gamma - \hat{y}} \right]$$

$$\hookrightarrow = o_{12}$$

$$\frac{\partial L}{\partial w_{21}^2} = \frac{\partial L}{\partial \hat{y}} \times \frac{\partial \hat{y}}{\partial w_{21}^2} = -2(\hat{y} - \hat{y}) o_{12}$$

$$\frac{\partial L}{\partial w_{11}^2} = \frac{\partial L}{\partial \hat{y}} \frac{\partial \hat{y}}{\partial w_{11}^2} = -2(\hat{y} - \hat{y}) o_{11}$$

$$\frac{\partial \hat{y}}{\partial w_{11}^2} = \frac{\partial}{\partial w_{11}^2} \left[\underbrace{w_{11}^2}_{\text{o}} o_{11} + \underbrace{w_{21}^2}_{\text{o}} o_{12} + \underbrace{b_{21}}_{\gamma} \right]$$

$$\hookrightarrow = o_{11}$$

$$\frac{\partial L}{\partial b_{21}} = \frac{\partial L}{\partial \hat{y}} \frac{\partial \hat{y}}{\partial b_{21}} = -2(\hat{y} - \hat{y})$$

$$\frac{\partial \hat{y}}{\partial b_{21}} = \frac{\partial}{\partial b_{21}} \left[\underbrace{w_{11}^2}_{\text{o}} o_{11} + \underbrace{w_{21}^2}_{\text{o}} o_{12} + \underbrace{b_{21}}_{\gamma} \right]$$

- ↑

$$= 1$$

$$\frac{\partial L}{\partial w_{21}^2} = -2(y - \hat{y}) o_{12}$$

$$\frac{\partial L}{\partial w_{11}^2} = -2(y - \hat{y}) o_{11}$$

$$\frac{\partial L}{\partial b_{21}} = -2(y - \hat{y})$$

$\frac{\partial L}{\partial w_{11}^1}, \frac{\partial L}{\partial w_{21}^1}, \frac{\partial L}{\partial b_{11}}$

\Rightarrow Finding

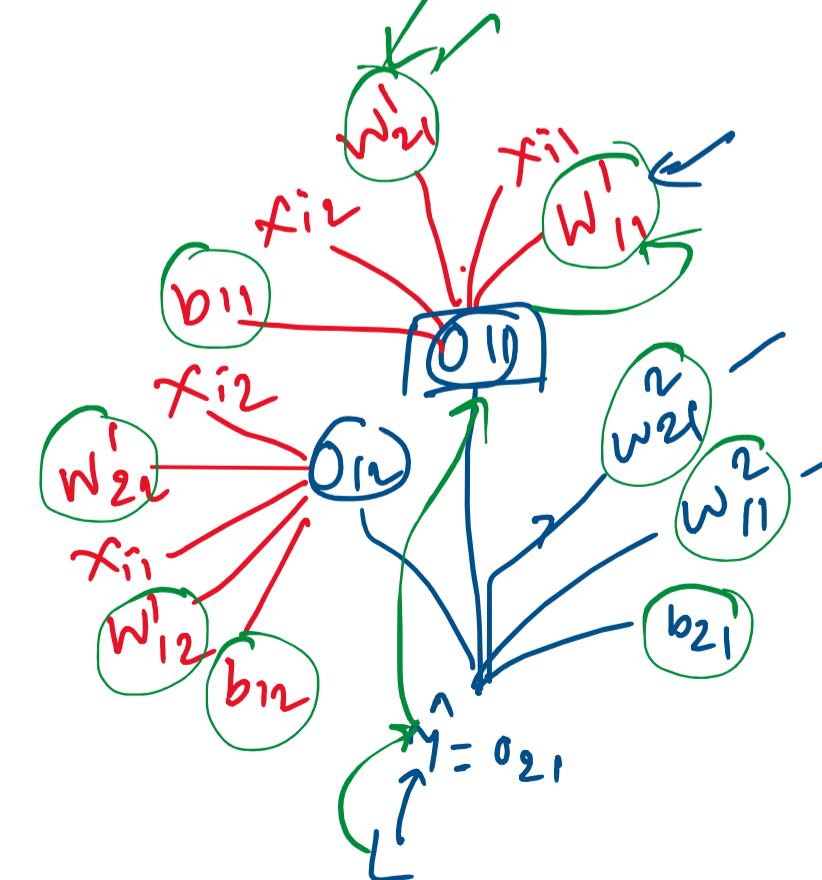
$$\frac{\partial L}{\partial \hat{y}} = -2(y - \hat{y})$$

$$\frac{\partial L}{\partial w_{11}^1} = \frac{\partial L}{\partial \hat{y}} \times \frac{\partial \hat{y}}{\partial o_{11}} \times \frac{\partial o_{11}}{\partial w_{11}^1}$$

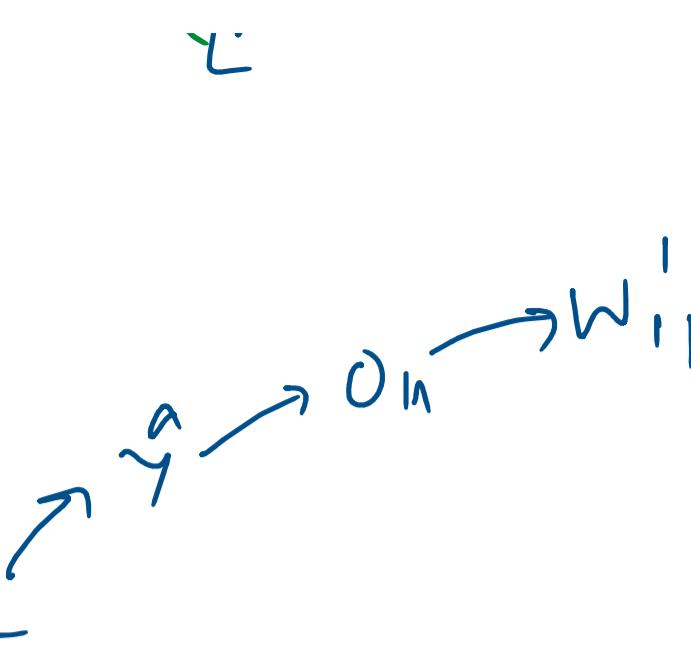
$$\frac{\partial L}{\partial w_{21}^1} = \frac{\partial L}{\partial \hat{y}} \frac{\partial \hat{y}}{\partial o_{11}} \frac{\partial o_{11}}{\partial w_{21}^1},$$

... \approx \approx \approx

$$\frac{\partial L}{\partial o_{11}}$$



$$\frac{\partial L}{\partial b_{11}} = \frac{\partial L}{\partial y} \cdot \frac{\partial y}{\partial o_{11}} \cdot \frac{\partial o_{11}}{\partial b_{11}}$$



$$\frac{\partial y}{\partial o_{11}} = \frac{\partial}{\partial o_{11}} \left[w_{11}^2 o_{11} + w_{21}^2 o_{12} + b_{11} \right]$$

$$\hookrightarrow = w_{11}^2$$

$$[w_{11}^1 x_{i1} + w_{21}^1 x_{i2} + b_{11}]$$

$$\frac{\partial o_{11}}{\partial w_{11}^1} = \frac{\partial}{\partial w_{11}^1} \left[\underline{w_{11}^1 x_{i1}} + \underline{w_{21}^1 x_{i2}} + \underline{b_{11}} \right].$$

$$\hookrightarrow = x_{i1}$$

$$\frac{\partial o_{11}}{\partial w_{21}^1} = \frac{\partial}{\partial w_{21}^1} \left[\underline{w_{11}^1 x_{i1}} + \underline{w_{21}^1 x_{i2}} + \underline{b_{11}} \right]$$

$$\hookrightarrow = x_{i2}$$

$$\begin{aligned} \frac{\partial o_{11}}{\partial b_{11}} &= \frac{\partial}{\partial b_{11}} \left[\underline{\frac{w_{11}^1 x_{i1}}{\sigma}} + \underline{\frac{w_{21}^1 x_{i2}}{\sigma}} + \underline{b_{11}} \right] \\ &= 1 \end{aligned}$$

$$\frac{\partial L}{\partial y} = -2(y - \hat{y})$$

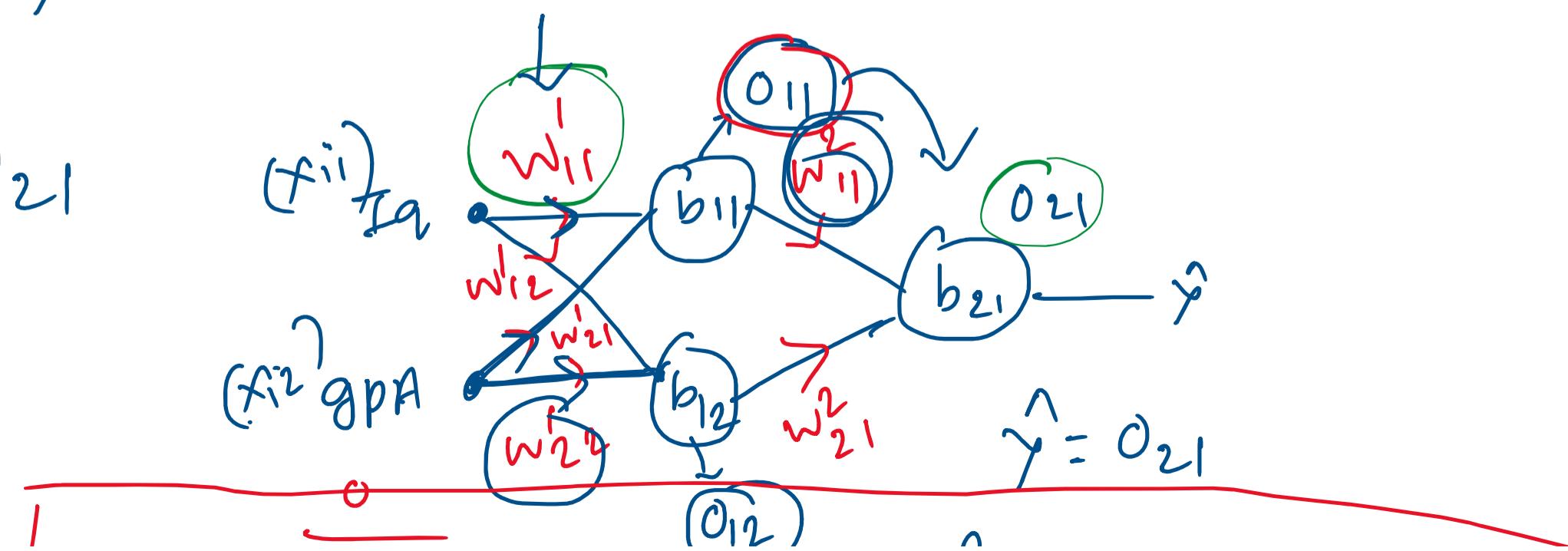
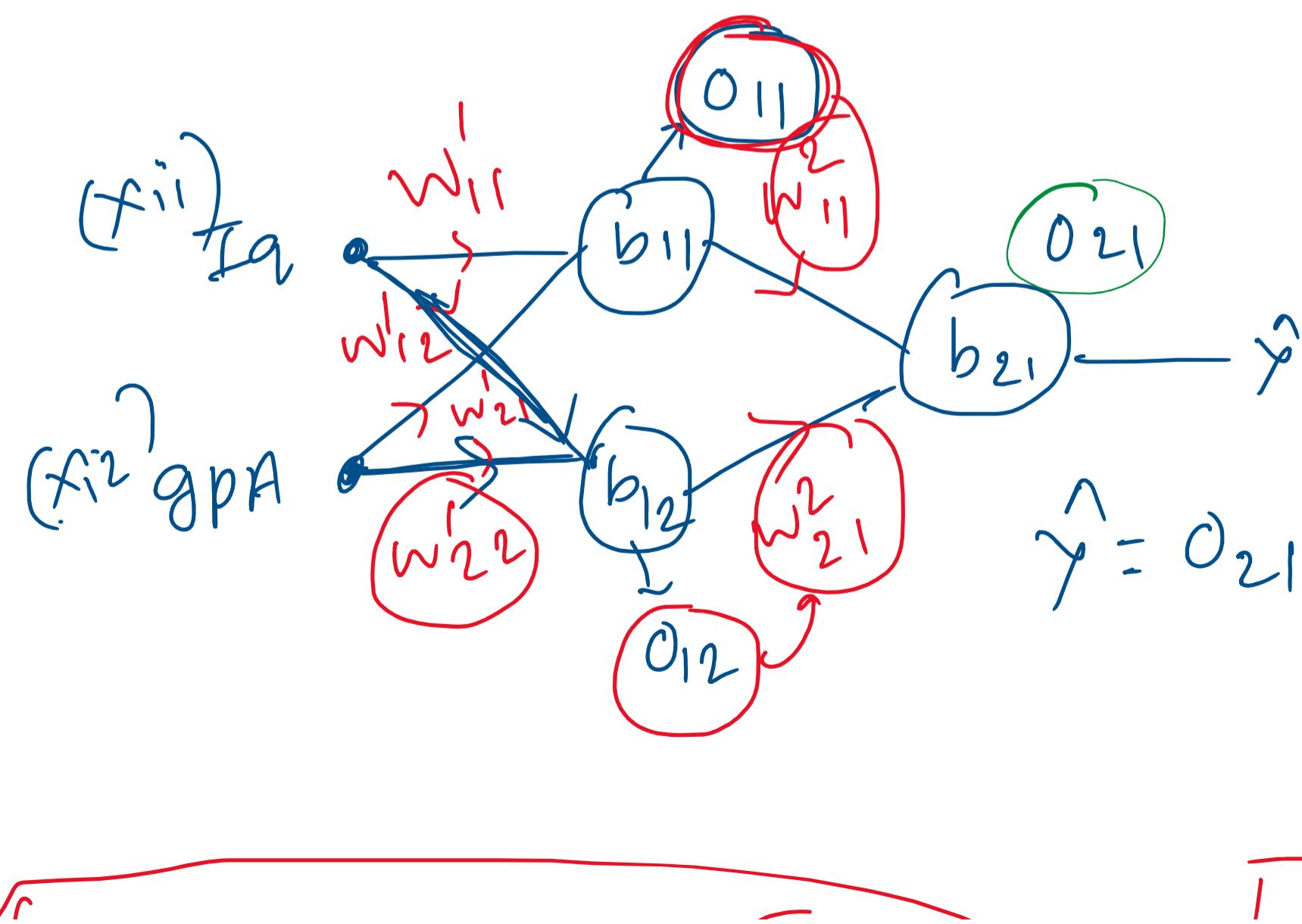
$$\frac{\partial y}{\partial o_{11}} = w_{11}^2$$

$$\frac{\partial L}{\partial w_{11}^1} = \frac{\partial L}{\partial y} \cdot \frac{\partial y}{\partial o_{11}} \cdot \frac{\partial o_{11}}{\partial w_{11}^1} = -2(y - \hat{y}) w_{11}^2 x_{i1}$$

$$\frac{\partial L}{\partial w_{11}^1} = \frac{\partial L}{\partial y} \cdot \frac{\partial y}{\partial o_{11}} \cdot \frac{\partial o_{11}}{\partial w_{11}^1} = -2(y - \hat{y}) \cdot 1 \cdot 1$$

$$\frac{\partial L}{\partial w_{21}^1} = \frac{\partial L}{\partial y} \cdot \frac{\partial y}{\partial o_{11}} \cdot \frac{\partial o_{11}}{\partial w_{21}^1} = -2(y - \hat{y}) \cdot w_{11}^2 \cdot x_{i2}$$

$$\frac{\partial L}{\partial b_{11}} = \frac{\partial L}{\partial y} \cdot \frac{\partial y}{\partial o_{11}} \cdot \frac{\partial o_{11}}{\partial b_{11}} = -2(y - \hat{y}) \cdot w_{11}^2$$



$$\frac{\partial L}{\partial w_{11}^1} = -2(y - \hat{y}) w_{11}^2 x_{i1}$$

$$\frac{\partial L}{\partial w_{21}^1} = -2(y - \hat{y}) w_{11}^2 x_{i2}$$

$$\frac{\partial L}{\partial b_{11}} = -2(y - \hat{y}) w_{11}^2$$

$$\frac{\partial L}{\partial w_{12}^1} = -2(y - \hat{y}) w_{21}^2 x_{i2}$$

$$\frac{\partial L}{\partial w_{12}^1} = -2(y - \hat{y}) w_{21}^2 x_{i1}$$

$$\frac{\partial L}{\partial b_{12}} = -2(y - \hat{y}) w_{21}^2$$

$$\frac{\partial L}{\partial w_{21}^2} = -2(y - \hat{y}) o_{12}$$

$$\frac{\partial L}{\partial w_{11}^2} = -2(y - \hat{y}) o_{11}$$

$$\frac{\partial L}{\partial b_{21}} = -2(y - \hat{y})$$

x_{i1}

x_{i2}

$o_{11} \quad o_{12}$

$o_{21} \quad o_{22}$

$$\frac{\partial \hat{y}}{\partial w_{11}^2} = 0_{11}$$

$$\frac{\partial L}{\partial w_{11}^1} = -2(\hat{y} - y) w_{11}^2 x_{i1}$$

$$\frac{\partial L}{\partial w_{12}^1} = -2(\hat{y} - y) w_{21}^2 x_{i1}'$$

$$\frac{\partial \hat{y}}{\partial w_{21}^2} = 0_{12}$$

$$\frac{\partial L}{\partial w_{21}^1} = -2(\hat{y} - y) w_{11}^2 x_{i2}$$

$$\frac{\partial L}{\partial w_{22}^1} = -2(\hat{y} - y) w_{21}^2 x_{i2}'$$

$$\frac{\partial \hat{y}}{\partial b_{21}} = 1$$

$$\frac{\partial L}{\partial b_{11}} = -2(\hat{y} - y) w_{11}^2$$

$$\frac{\partial L}{\partial b_{21}} = -2(\hat{y} - y) w_{21}^2$$

I _i	gpa	salary
60	3.5	40
80	4.7	60
20	3	25
90	3	50

Step

o) Initialization

$w/b \rightarrow w=1, b=0$

epoch
↑
loop()
↓

L (min)

i) for i in range(4) : <

* Take a row \rightarrow Forward Propagation

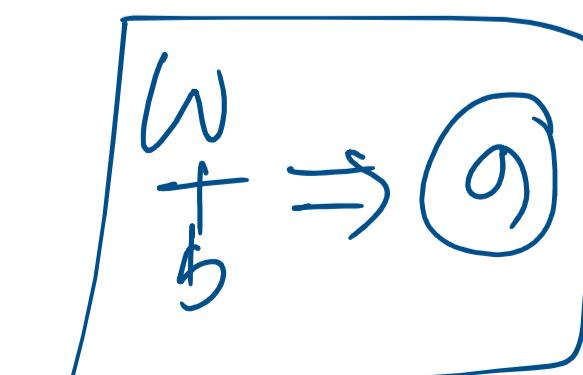
* Loss Function (MSE)

Back propagation

Predict (Salary)

* Adjust weights / bias

$\frac{\partial L}{\partial w}$



$$w_{\text{new}} = w_{\text{old}} - \eta \frac{\partial L}{\partial w_0}_{\text{old}}$$

$$b_{\text{new}} = b_{\text{old}} - \eta \frac{\partial L}{\partial b_0}_{\text{old}}$$

⑤ \Rightarrow