CONTINUOUS PROBABILITY DISTRIBUTIONS

https://phitter.io

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Abstract

This document provides an overview of the continuous probability distributions utilized in Phitter. It includes a detailed description for each distribution, covering aspects such as the definition, domain, parameter definitions and domains, probability density function, cumulative distribution function, percentile point function, raw moments, mean, variance, skewness, kurtosis, median, and mode in a concise and clear manner.

Contents

1 Alpha Distribution

1.1 Distribution definition

$$X \sim \text{Alpha}\left(\alpha, \text{Loc}, \text{Sc}\right)$$

1.2 Distribution domain

$$x \in \left(\text{Loc}, \infty\right)$$

1.3 Parameters domain and parameters constraints

$$\alpha \in \mathbb{R}^{+}, \text{Loc} \in \mathbb{R}, \text{Sc} \in \mathbb{R}^{+}$$

1.4 Cumulative distribution function

$$F_X\left(x\right) = \frac{\Phi\left(\alpha - \frac{1}{z(x)}\right)}{\Phi\left(\alpha\right)}$$

1.5 Probability density function

$$f_X\left(x\right) = \frac{1}{\text{Sc} \cdot z(x)^2 \cdot \Phi\left(\alpha\right) \cdot \sqrt{2\pi}} \exp\left(-\frac{1}{2}\left(\alpha - \frac{1}{z(x)}\right)^2\right)$$

1.6 Percent point function/Sample

$$F_X^{-1}\left(u\right) = \text{Loc} + \text{Sc} \times \frac{1}{\alpha - \Phi^{-1}\left(u\Phi\left(\alpha\right)\right)}$$

1.7 Parametric centered moments

$$\tilde{\mu}'_k = E[\tilde{X}^k] = \int_0^\infty x^k f_{\tilde{X}}\left(x\right) dx$$

1.8 Parametric mean

$$\text{Mean}(X) = \text{Loc} + \text{Sc} \cdot \tilde{\mu}'_1$$

1.9 Parametric variance

$$\text{Variance}(X) = \text{Sc}^2 \cdot (\tilde{\mu}'_2 - \tilde{\mu}'^2_1)$$

1.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\tilde{\mu}'_3 - 3\tilde{\mu}'_2\tilde{\mu}'_1 + 2\tilde{\mu}'^3_1}{(\tilde{\mu}'_2 - \tilde{\mu}'^2_1)^{1.5}}$$

1.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\tilde{\mu}'_4 - 4\tilde{\mu}'_1\tilde{\mu}'_3 + 6\tilde{\mu}'^2_1\tilde{\mu}'_2 - 3\tilde{\mu}'^4_1}{(\tilde{\mu}'_2 - \tilde{\mu}'^2_1)^2}$$

1.12 Parametric median

$$\text{Median}(X) = \text{Loc} + \frac{\text{Sc}}{\alpha - \Phi^{-1}\left(\frac{1}{2}\Phi\left(\alpha\right)\right)}$$

1.13 Parametric mode

$$\text{Mode}(X) = \text{Loc} + \text{Sc}\frac{(\sqrt{\alpha^2 + 8} - \alpha)}{4}$$

1.14 Additional information and definitions

- $\tilde{X} \sim \text{Alpha}\,(\alpha, 0, 1)$
- Loc : Location parameter
- Sc : Scale parameter
- $z(x) = (x - \text{Loc})\,/\text{Sc}$
- u : Uniform[0,1] random varible
- $\Phi\,(x)$: CDF normal standard distribution
- $\Phi^{-1}\,(x)$: PPF normal standard distribution

1.15 Spreadsheet documents

- Excel file from GitHub repository
- Google spreadsheet document

2 Arcsine Distribution

2.1 Distribution definition

$$X \sim \operatorname{Arcsine}(a, b)$$

2.2 Distribution domain

$$x \in (a, b)$$

2.3 Parameters domain and parameters constraints

$$a \in \mathbb{R}, b \in \mathbb{R}, a < b$$

2.4 Cumulative distribution function

$$F_X(x) = \frac{2}{\pi} \arcsin\left(\sqrt{\frac{x-a}{b-a}}\right)$$

2.5 Probability density function

$$f_X(x) = \frac{1}{\pi\sqrt{(x-a)(b-x)}}$$

2.6 Percent point function/Sample

$$F_X^{-1}(u) = a + (b-a) \times \sin^2\left(\frac{\pi}{2}u\right)$$

2.7 Parametric centered moments

$$\tilde{\mu}'_k = E[\tilde{X}^k] = \int_0^1 x^k f_{\tilde{X}}(x)\, dx = \frac{1}{\pi}\text{Beta}\left(\frac{1}{2}, k + \frac{1}{2}\right) = \frac{(2k-1)!!}{2^k k!}$$

2.8 Parametric mean

$$\text{Mean}(X) = a + \tilde{\mu}'_1 (b-a) = a + \frac{1}{2}(b-a)$$

2.9 Parametric variance

$$\text{Variance}(X) = (b-a)^2 \times (\tilde{\mu}'_2 - \tilde{\mu}'^2_1) = \frac{(b-a)^2}{8}$$

2.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\tilde{\mu}'_3 - 3\tilde{\mu}'_2\tilde{\mu}'_1 + 2\tilde{\mu}'^3_1}{(\tilde{\mu}'_2 - \tilde{\mu}'^2_1)^{1.5}} = 0$$

2.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\tilde{\mu}'_4 - 4\tilde{\mu}'_1\tilde{\mu}'_3 + 6\tilde{\mu}'^2_1\tilde{\mu}'_2 - 3\tilde{\mu}'^4_1}{(\tilde{\mu}'_2 - \tilde{\mu}'^2_1)^2} = 3 - \frac{3}{2}$$

2.12 Parametric median

$$\text{Median}(X) = a + (b-a) \times \sin^2\left(\frac{\pi}{4}\right)$$

2.13 Parametric mode

$$\text{Mode}(X) = \text{undefined}$$

2.14 Additional information and definitions

- $\tilde{X} \sim \text{Arcsine}\,(0,1)$
- u : Uniform[0,1] random varible
- $\text{Beta}\,(x,y)$: Beta function

2.15 Spreadsheet documents

- Excel file from GitHub repository
- Google spreadsheet document

3 Argus Distribution

3.1 Distribution definition

$$X \sim \text{Argus}\,(\chi, \text{Loc}, \text{Sc})$$

3.2 Distribution domain

$$x \in (\text{Loc}, \text{Loc} + \text{Sc})$$

3.3 Parameters domain and parameters constraints

$$\chi \in \mathbb{R}^{+}, \text{Loc} \in \mathbb{R}, \text{Sc} \in \mathbb{R}^{+}$$

3.4 Cumulative distribution function

$$F_X\,(x) = 1 - \frac{\Psi\left(\chi\sqrt{1 - z(x)^2}\right)}{\Psi(\chi)}$$

3.5 Probability density function

$$f_X\,(x) = \frac{1}{\text{Sc}} \cdot \frac{\chi^3}{\sqrt{2\pi}\,\Psi(\chi)} \cdot z(x)\sqrt{1 - z(x)^2}\exp\left(-\frac{1}{2}\chi^2\left(1 - z(x)^2\right)\right)$$

3.6 Percent point function/Sample

$$F_X^{-1}\,(u) = \text{Loc} + \text{Sc}\sqrt{1 - \frac{2\text{P}^{-1}(\frac{3}{2}, (1-u)\text{P}(\frac{3}{2}, \frac{\chi^2}{2}))}{\chi^2}}$$

3.7 Parametric centered moments

$$\mu'_k = E[X^k] = \int_{\text{Loc}}^{\text{Loc}+\text{Sc}} x^k f_X\,(x)\,dx$$

3.8 Parametric mean

$$\text{Mean}(X) = \mu'_1 = \text{Loc} + \text{Sc}\sqrt{\pi/8}\,\frac{\chi e^{-\frac{\chi^2}{4}} I_1(\frac{\chi^2}{4})}{\Psi(\chi)}$$

3.9 Parametric variance

$$\text{Variance}(X) = \mu'_2 - \mu'^2_1 = \text{Sc}^2 \cdot \left(1 - \frac{3}{\chi^2} + \frac{\chi\phi(\chi)}{\Psi(\chi)}\right) - (\mu - \text{Loc})^2$$

3.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\mu'_3 - 3\mu'_2\mu'_1 + 2\mu'^3_1}{(\mu'_2 - \mu'^2_1)^{1.5}}$$

3.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\mu'_4 - 4\mu'_1\mu'_3 + 6\mu'^2_1\mu'_2 - 3\mu'^4_1}{(\mu'_2 - \mu'^2_1)^2}$$

3.12 Parametric median

$$\text{Median}(X) = \text{Loc} + \text{Sc}\sqrt{1 - \frac{2\text{P}^{-1}(\frac{3}{2}, \frac{1}{2}\text{P}(\frac{3}{2}, \frac{\chi^2}{2}))}{\chi^2}}$$

3.13 Parametric mode

$$\mathrm{Mode}(X) = \mathrm{Loc} + \frac{\mathrm{Sc}}{\sqrt{2}\chi}\sqrt{(\chi^2 - 2) + \sqrt{\chi^4 + 4}}$$

3.14 Additional information and definitions

- Loc : Location parameter
- Sc : Scale parameter
- $z(x) = (x - \mathrm{Loc})\,/\mathrm{Sc}$
- u : Uniform[0,1] random varible
- $\Psi(\chi) = \Phi(\chi) - \chi\phi(\chi) - \frac{1}{2}$
- $\Phi(x)$: CDF normal standard distribution
- $\phi(x)$: PDF normal standard distribution
- $I_\alpha\left(x\right)$: Modified Bessel function of the first kind of order $\alpha \in \mathbb{N}$
- $\mathrm{P}(a, x) = \frac{\gamma(a,x)}{\Gamma(a)}$: Regularized lower incomplete gamma function
- $\mathrm{P}^{-1}(a, y)$: Inverse of regularized lower incomplete gamma function
- $\gamma(a, x)$: Lower incomplete gamma function
- $\Gamma(x)$: Gamma function

3.15 Spreadsheet documents

- Excel file from GitHub repository
- Google spreadsheet document

4 Beta Distribution

4.1 Distribution definition

$$X \sim \mathrm{Beta}\left(\alpha, \beta, A, B\right)$$

4.2 Distribution domain

$$x \in (A, B)$$

4.3 Parameters domain and parameters constraints

$$\alpha \in \mathbb{R}^{+}, \beta \in \mathbb{R}^{+}, A \in \mathbb{R}, B \in \mathbb{R}, A < B$$

4.4 Cumulative distribution function

$$F_X\left(x\right) = I\left(z(x), \alpha, \beta\right)$$

4.5 Probability density function

$$f_X\left(x\right) = \frac{z(x)^{\alpha-1}\left(1 - z(x)\right)^{\beta-1}}{\mathrm{Beta}(\alpha, \beta)(B - A)}$$

4.6 Percent point function/Sample

$$F_X^{-1}\left(u\right) = A + (B - A) \times I^{-1}\left(u, \alpha, \beta\right)$$

4.7 Parametric centered moments

$$\tilde{\mu}'_k = E[\tilde{X}^k] = \int_0^1 x^k f_{\tilde{X}}\left(x\right) dx$$

4.8 Parametric mean

$$\mathrm{Mean}(X) = A + (B - A) \cdot \tilde{\mu}'_1 = A + \frac{\alpha\left(B - A\right)}{\alpha + \beta}$$

4.9 Parametric variance

$$\mathrm{Variance}(X) = (B - A)^2 \cdot (\tilde{\mu}'_2 - \tilde{\mu}'^2_1) = \frac{\alpha\beta\left(B - A\right)^2}{(\alpha + \beta)^2(\alpha + \beta + 1)}$$

4.10 Parametric skewness

$$\mathrm{Skewness}(X) = \frac{\tilde{\mu}'_3 - 3\tilde{\mu}'_2\tilde{\mu}'_1 + 2\tilde{\mu}'^3_1}{(\tilde{\mu}'_2 - \tilde{\mu}'^2_1)^{1.5}} = \frac{2\left(\beta - \alpha\right)\sqrt{\alpha + \beta + 1}}{(\alpha + \beta + 2)\sqrt{\alpha\beta}}$$

4.11 Parametric kurtosis

$$\mathrm{Kurtosis}(X) = \frac{\tilde{\mu}'_4 - 4\tilde{\mu}'_1\tilde{\mu}'_3 + 6\tilde{\mu}'^2_1\tilde{\mu}'_2 - 3\tilde{\mu}'^4_1}{(\tilde{\mu}'_2 - \tilde{\mu}'^2_1)^2} = 3 + \frac{6[(\alpha - \beta)^2(\alpha + \beta + 1) - \alpha\beta(\alpha + \beta + 2)]}{\alpha\beta(\alpha + \beta + 2)(\alpha + \beta + 3)}$$

4.12 Parametric median

$$\mathrm{Median}(X) = A + (B - A) \times I^{-1}\left(\frac{1}{2}, \alpha, \beta\right) \quad \text{if } \alpha, \beta > 1$$

4.13 Parametric mode

$$\mathrm{Mode}(X) = A + (B - A)\frac{\alpha - 1}{\alpha + \beta - 2} \quad \text{if } \alpha, \beta > 1$$

4.14 Additional information and definitions

- $\tilde{X} \sim \text{Beta}\left(\alpha, \beta, 0, 1\right)$
- $z\left(x\right) = \left(x - A\right) / \left(B - A\right)$
- u : Uniform[0,1] random varible
- $I\left(x, a, b\right)$: Regularized incomplete beta function
- $I^{-1}\left(x, a, b\right)$: Inverse of regularized incomplete beta function
- $\text{Beta}\left(x, y\right)$: Beta function

4.15 Spreadsheet documents

- Excel file from GitHub repository
- Google spreadsheet document

5 Beta Prime Distribution

5.1 Distribution definition

$$X \sim \text{BetaPrime}\,(\alpha, \beta)$$

5.2 Distribution domain

$$x \in [0, \infty)$$

5.3 Parameters domain and parameters constraints

$$\alpha \in \mathbb{R}^+, \beta \in \mathbb{R}^+$$

5.4 Cumulative distribution function

$$F_X\,(x) = I\left(\frac{x}{1+x}, \alpha, \beta\right)$$

5.5 Probability density function

$$f_X\,(x) = \frac{x^{\alpha-1}(1+x)^{-\alpha-\beta}}{\text{Beta}(\alpha,\beta)}$$

5.6 Percent point function/Sample

$$F_X^{-1}\,(u) = \frac{I^{-1}\,(u,\alpha,\beta)}{1 - I^{-1}\,(u,\alpha,\beta)}$$

5.7 Parametric centered moments

$$\mu'_k = E[X^k] = \int_0^\infty x^k f_X\,(x)\,dx = \frac{\Gamma\,(k+\alpha)\,\Gamma\,(\beta-k)}{\Gamma\,(\alpha)\,\Gamma\,(\beta)} \quad \text{if } \beta > k$$

5.8 Parametric mean

$$\text{Mean}(X) = \mu'_1 = \frac{\alpha}{\beta-1} \quad \text{if } \beta > 1$$

5.9 Parametric variance

$$\text{Variance}(X) = \mu'_2 - \mu'^2_1 = \frac{\alpha(\alpha+\beta-1)}{(\beta-2)(\beta-1)^2} \quad \text{if } \beta > 2$$

5.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\mu'_3 - 3\mu'_2\mu'_1 + 2\mu'^3_1}{(\mu'_2 - \mu'^2_1)^{1.5}} = \frac{2(2\alpha+\beta-1)}{\beta-3}\sqrt{\frac{\beta-2}{\alpha(\alpha+\beta-1)}} \quad \text{if } \beta > 3$$

5.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\mu'_4 - 4\mu'_1\mu'_3 + 6\mu'^2_1\mu'_2 - 3\mu'^4_1}{(\mu'_2 - \mu'^2_1)^2} \quad \text{if } \beta > 4$$

5.12 Parametric median

$$\text{Median}(X) = \frac{I^{-1}\left(\frac{1}{2},\alpha,\beta\right)}{1 - I^{-1}\left(\frac{1}{2},\alpha,\beta\right)}$$

5.13 Parametric mode

$$\text{Mode}(X) = \frac{\alpha - 1}{\beta + 1}$$

5.14 Additional information and definitions

- u : Uniform[0,1] random varible
- $I\left(x, a, b\right)$: Regularized incomplete beta function
- $I^{-1}\left(x, a, b\right)$: Inverse of regularized incomplete beta function
- $\Gamma\left(x\right)$: Gamma function
- $\text{Beta}\left(x, y\right)$: Beta function

5.15 Spreadsheet documents

- Excel file from GitHub repository
- Google spreadsheet document

6 Beta Prime 4P Distribution

6.1 Distribution definition

$$X \sim \mathrm{BetaPrime}_{\mathrm{4P}}\left(\alpha, \beta, \mathrm{Loc}, \mathrm{Sc}\right)$$

6.2 Distribution domain

$$x \in [\mathrm{Loc}, \infty)$$

6.3 Parameters domain and parameters constraints

$$\alpha \in \mathbb{R}^{+}, \beta \in \mathbb{R}^{+}, \mathrm{Loc} \in \mathbb{R}, \mathrm{Sc} \in \mathbb{R}^{+}$$

6.4 Cumulative distribution function

$$F_X\left(x\right) = I\left(\frac{z(x)}{1+z(x)}, \alpha, \beta\right)$$

6.5 Probability density function

$$f_X\left(x\right) = \frac{z(x)^{\alpha-1}(1+z(x))^{-\alpha-\beta}}{\mathrm{Sc} \times \mathrm{Beta}(\alpha, \beta)}$$

6.6 Percent point function/Sample

$$F_X^{-1}\left(u\right) = \mathrm{Loc} + \mathrm{Sc}\frac{I^{-1}\left(u, \alpha, \beta\right)}{1 - I^{-1}\left(u, \alpha, \beta\right)}$$

6.7 Parametric centered moments

$$\tilde{\mu}'_k = E[\tilde{X}^k] = \int_0^{\infty} x^k f_{\tilde{X}}\left(x\right) dx = \frac{\Gamma\left(k+\alpha\right)\Gamma\left(\beta-k\right)}{\Gamma\left(\alpha\right)\Gamma\left(\beta\right)} \quad \text{if } \beta > k$$

6.8 Parametric mean

$$\mathrm{Mean}(X) = \mathrm{Loc} + \mathrm{Sc}\tilde{\mu}'_1 = \mathrm{Loc} + \mathrm{Sc}\frac{\alpha}{\beta-1} \quad \text{if } \beta > 1$$

6.9 Parametric variance

$$\mathrm{Variance}(X) = \mathrm{Sc}^2(\tilde{\mu}'_2 - \tilde{\mu}'^2_1) = \mathrm{Sc}^2\frac{\alpha(\alpha+\beta-1)}{(\beta-2)(\beta-1)^2} \quad \text{if } \beta > 2$$

6.10 Parametric skewness

$$\mathrm{Skewness}(X) = \frac{\tilde{\mu}'_3 - 3\tilde{\mu}'_2\tilde{\mu}'_1 + 2\tilde{\mu}'^3_1}{(\tilde{\mu}'_2 - \tilde{\mu}'^2_1)^{1.5}} = \frac{2(2\alpha+\beta-1)}{\beta-3}\sqrt{\frac{\beta-2}{\alpha(\alpha+\beta-1)}} \quad \text{if } \beta > 3$$

6.11 Parametric kurtosis

$$\mathrm{Kurtosis}(X) = \frac{\tilde{\mu}'_4 - 4\tilde{\mu}'_1\tilde{\mu}'_3 + 6\tilde{\mu}'^2_1\tilde{\mu}'_2 - 3\tilde{\mu}'^4_1}{(\tilde{\mu}'_2 - \tilde{\mu}'^2_1)^2} \quad \text{if } \beta > 4$$

6.12 Parametric median

$$\mathrm{Median}(X) = \mathrm{Loc} + \mathrm{Sc}\frac{I^{-1}\left(\frac{1}{2}, \alpha, \beta\right)}{1 - I^{-1}\left(\frac{1}{2}, \alpha, \beta\right)}$$

6.13 Parametric mode

$$\text{Mode}(X) = \text{Loc} + \text{Sc}\frac{\alpha - 1}{\beta + 1}$$

6.14 Additional information and definitions

- $\tilde{X} \sim \text{BetaPrime}\,(\alpha, \beta)$
- Loc : Location parameter
- Sc : Scale parameter
- $z\,(x) = (x - \text{Loc})\,/\text{Sc}$
- u : Uniform[0,1] random varible
- $I\,(x, a, b)$: Regularized incomplete beta function
- $I^{-1}\,(x, a, b)$: Inverse of regularized incomplete beta function
- $\Gamma\,(x)$: Gamma function
- $\text{Beta}\,(x, y)$: Beta function

6.15 Spreadsheet documents

- Excel file from GitHub repository
- Google spreadsheet document

7 Bradford Distribution

7.1 Distribution definition

$$X \sim \text{Bradford}\left(c, \min, \max\right)$$

7.2 Distribution domain

$$x \in \left(\min, \max\right)$$

7.3 Parameters domain and parameters constraints

$$c \in \mathbb{R}^{+}, \min \in \mathbb{R}, \max \in \mathbb{R}, \min < \max$$

7.4 Cumulative distribution function

$$F_X\left(x\right) = \frac{\ln\left(1 + c \cdot z(x)\right)}{k}$$

7.5 Probability density function

$$f_X\left(x\right) = \frac{c}{k\left(1 + c \cdot z(x)\right)\left(\max - \min\right)}$$

7.6 Percent point function/Sample

$$F_X^{-1}\left(u\right) = \min + \left(\max - \min\right) \times \frac{\left(1 + c\right)^{u} - 1}{c}$$

7.7 Parametric centered moments

$$\tilde{\mu}'_k = E[\tilde{X}^k] = \int_0^1 x^k f_{\tilde{X}}\left(x\right) dx$$

7.8 Parametric mean

$$\text{Mean}(X) = \min + \left(\max - \min\right) \cdot \tilde{\mu}'_1 = \min + \left(\max - \min\right) \cdot \frac{c - k}{ck}$$

7.9 Parametric variance

$$\text{Variance}(X) = \left(\max - \min\right)^2 \cdot \left(\tilde{\mu}'_2 - \tilde{\mu}'^2_1\right) = \left(\max - \min\right)^2 \cdot \frac{\left(c + 2\right)k - 2c}{2ck^2}$$

7.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\tilde{\mu}'_3 - 3\tilde{\mu}'_2\tilde{\mu}'_1 + 2\tilde{\mu}'^3_1}{\left(\tilde{\mu}'_2 - \tilde{\mu}'^2_1\right)^{1.5}} = \frac{\sqrt{2}\left(12c^2 - 9kc\left(c + 2\right) + 2k^2\left(c\left(c + 3\right) + 3\right)\right)}{\sqrt{c\left(c\left(k - 2\right) + 2k\right)}\left(3c\left(k - 2\right) + 6k\right)}$$

7.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\tilde{\mu}'_4 - 4\tilde{\mu}'_1\tilde{\mu}'_3 + 6\tilde{\mu}'^2_1\tilde{\mu}'_2 - 3\tilde{\mu}'^4_1}{\left(\tilde{\mu}'_2 - \tilde{\mu}'^2_1\right)^2} = 3 + \frac{c^3\left(k - 3\right)\left(k\left(3k - 16\right) + 24\right) + 12kc^2\left(k - 4\right)\left(k - 3\right) + 6ck^2\left(3k - 14\right) + 1}{3c\left(c\left(k - 2\right) + 2k\right)^2}$$

7.12 Parametric median

$$\text{Median}(X) = \min + \left(\max - \min\right) \cdot \frac{\left(1 + c\right)^{frac12} - 1}{c}$$

7.13 Parametric mode

$$\text{Mode}(X) = \min$$

7.14 Additional information and definitions

- $\tilde{X} \sim \text{Bradford}\,(c, 0, 1)$
- $k = \ln(1 + c)$
- $z\,(x) = (x - \min)\,/\,(\max - \min)$
- u : Uniform[0,1] random varible

7.15 Spreadsheet documents

- Excel file from GitHub repository
- Google spreadsheet document

8 Burr Distribution

8.1 Distribution definition

$$X \sim \mathrm{Burr}\left(A, B, C\right)$$

8.2 Distribution domain

$$x \in [0, \infty)$$

8.3 Parameters domain and parameters constraints

$$A \in \mathbb{R}^{+}, B \in \mathbb{R}, C \in \mathbb{R}^{+}$$

8.4 Cumulative distribution function

$$F_X\left(x\right) = 1 - \left[1 + \left(\frac{x}{A}\right)^{B}\right]^{-C}$$

8.5 Probability density function

$$f_X\left(x\right) = \frac{BC}{A}\left(\frac{x}{A}\right)^{B-1}\left[1 + \left(\frac{x}{A}\right)^{B}\right]^{-C-1}$$

8.6 Percent point function/Sample

$$F_X^{-1}\left(u\right) = A\left[\left(1-u\right)^{-\frac{1}{c}} - 1\right]^{\frac{1}{B}}$$

8.7 Parametric centered moments

$$\mu_k' = E[X^k] = \int_0^{\infty} x^k f_X\left(x\right) dx = A^k C \times \mathrm{Beta}\left(\frac{BC-k}{B}, \frac{B+K}{B}\right)$$

8.8 Parametric mean

$$\mathrm{Mean}(X) = \mu_1'$$

8.9 Parametric variance

$$\mathrm{Variance}(X) = \mu_2' - \mu_1'^2$$

8.10 Parametric skewness

$$\mathrm{Skewness}(X) = \frac{\mu_3' - 3\mu_2'\mu_1' + 2\mu_1'^3}{(\mu_2' - \mu_1'^2)^{1.5}}$$

8.11 Parametric kurtosis

$$\mathrm{Kurtosis}(X) = \frac{\mu_4' - 4\mu_1'\mu_3' + 6\mu_1'^2\mu_2' - 3\mu_1'^4}{(\mu_2' - \mu_1'^2)^2}$$

8.12 Parametric median

$$\mathrm{Median}(X) = A\left[\left(\frac{1}{2}\right)^{-\frac{1}{c}} - 1\right]^{\frac{1}{B}}$$

8.13 Parametric mode

$$\mathrm{Mode}(X) = A\left(\frac{B-1}{BC+1}\right)^{\frac{1}{B}}$$

8.14 Additional information and definitions

- u : Uniform[0,1] random varible
- Beta (x, y) : Beta function

8.15 Spreadsheet documents

- Excel file from GitHub repository
- Google spreadsheet document

9 Burr 4P Distribution

9.1 Distribution definition

$$X \sim \text{Burr}_{\text{4P}}\left(A, B, C, \text{Loc}\right)$$

9.2 Distribution domain

$$x \in [\text{Loc}, \infty)$$

9.3 Parameters domain and parameters constraints

$$A \in \mathbb{R}^{+}, B \in \mathbb{R}, C \in \mathbb{R}^{+}, \text{Loc} \in \mathbb{R}$$

9.4 Cumulative distribution function

$$F_X\left(x\right) = 1 - \left[1 + \left(\frac{x - \text{Loc}}{A}\right)^{B}\right]^{-C}$$

9.5 Probability density function

$$f_X\left(x\right) = \frac{BC}{A}\left(\frac{x - \text{Loc}}{A}\right)^{B-1}\left[1 + \left(\frac{x - \text{Loc}}{A}\right)^{B}\right]^{-C-1}$$

9.6 Percent point function/Sample

$$F_X^{-1}\left(u\right) = \text{Loc} + A\left[(1-u)^{-\frac{1}{c}} - 1\right]^{\frac{1}{B}}$$

9.7 Parametric centered moments

$$\tilde{\mu}'_k = E[\tilde{X}^k] = \int_0^{\infty} x^k f_{\tilde{X}} = A^k C \times \text{Beta}\left(\frac{BC-k}{B}, \frac{B+K}{B}\right)$$

9.8 Parametric mean

$$\text{Mean}(X) = \text{Loc} + \tilde{\mu}'_1$$

9.9 Parametric variance

$$\text{Variance}(X) = \tilde{\mu}'_2 - \tilde{\mu}'^2_1$$

9.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\tilde{\mu}'_3 - 3\tilde{\mu}'_2\tilde{\mu}'_1 + 2\tilde{\mu}'^3_1}{(\tilde{\mu}'_2 - \tilde{\mu}'^2_1)^{1.5}}$$

9.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\tilde{\mu}'_4 - 4\tilde{\mu}'_1\tilde{\mu}'_3 + 6\tilde{\mu}'^2_1\tilde{\mu}'_2 - 3\tilde{\mu}'^4_1}{(\tilde{\mu}'_2 - \tilde{\mu}'^2_1)^2}$$

9.12 Parametric median

$$\text{Median}(X) = \text{Loc} + A\left[\left(\frac{1}{2}\right)^{-\frac{1}{c}} - 1\right]^{\frac{1}{B}}$$

9.13 Parametric mode

$$\text{Mode}(X) = \text{Loc} + A\left(\frac{B-1}{BC+1}\right)^{\frac{1}{B}}$$

9.14 Additional information and definitions

- $\tilde{X} \sim \text{Burr}\,(A, B, C)$
- Loc : Location parameter
- u : Uniform[0,1] random varible
- $\text{Beta}\,(x, y)$: Beta function

9.15 Spreadsheet documents

- Excel file from GitHub repository
- Google spreadsheet document

10 Cauchy Distribution

10.1 Distribution definition

$$X \sim \text{Cauchy}\left(x0, \gamma\right)$$

10.2 Distribution domain

$$x \in (-\infty, +\infty)$$

10.3 Parameters domain and parameters constraints

$$x_0 \in \mathbb{R}, \gamma \in \mathbb{R}^+$$

10.4 Cumulative distribution function

$$F_X\left(x\right) = \frac{1}{\pi} \arctan\left(\frac{x - x_0}{\gamma}\right) + \frac{1}{2}$$

10.5 Probability density function

$$f_X\left(x\right) = \frac{1}{\pi\gamma\left[1 + \left(\frac{x-x_0}{\gamma}\right)^2\right]}$$

10.6 Percent point function/Sample

$$F_X^{-1}\left(u\right) = x_0 + \gamma\ \tan\left[\pi\left(u - \frac{1}{2}\right)\right]$$

10.7 Parametric centered moments

$$\mu'_k = E[X^k] = \int_{-\infty}^{\infty} x^k f_X\left(x\right) dx$$

10.8 Parametric mean

$$\text{Mean}(X) = \text{undefined}$$

10.9 Parametric variance

$$\text{Variance}(X) = \text{undefined}$$

10.10 Parametric skewness

$$\text{Skewness}(X) = \text{undefined}$$

10.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \text{undefined}$$

10.12 Parametric median

$$\text{Median}(X) = x_0$$

10.13 Parametric mode

$$\text{Mode}(X) = x_0$$

10.14 Additional information and definitions

- x_0 : Location parameter
- γ : Scale parameter
- u : Uniform[0,1] random varible

10.15 Spreadsheet documents

- Excel file from GitHub repository
- Google spreadsheet document

11 Chi Square Distribution

11.1 Distribution definition

$$X \sim \chi^2\left(\mathrm{df}\right)$$

11.2 Distribution domain

$$x \in (0, \infty)$$

11.3 Parameters domain and parameters constraints

$$\mathrm{df} \in \mathbb{N}^+$$

11.4 Cumulative distribution function

$$F_X\left(x\right) = \frac{\gamma(\frac{\mathrm{df}}{2}, \frac{x}{2})}{\Gamma(\frac{\mathrm{df}}{2})} = \mathrm{P}\left(\frac{\mathrm{df}}{2}, \frac{x}{2}\right)$$

11.5 Probability density function

$$f_X\left(x\right) = \frac{1}{2^{\mathrm{df}/2}\Gamma(\mathrm{df}/2)}\, x^{\mathrm{df}/2-1}e^{-x/2}$$

11.6 Percent point function/Sample

$$F_X^{-1}\left(u\right) = 2\mathrm{P}^{-1}\left(\frac{\mathrm{df}}{2}, u\right)$$

11.7 Parametric centered moments

$$\mu'_k = E[X^k] = \int_0^\infty x^k f_X\left(x\right) dx = \mathrm{df}(\mathrm{df}+2)\cdots(\mathrm{df}+2k-2) = 2^k \frac{\Gamma\left(k+\frac{\mathrm{df}}{2}\right)}{\Gamma\left(\frac{\mathrm{df}}{2}\right)}$$

11.8 Parametric mean

$$\mathrm{Mean}(X) = \mu'_1 = \mathrm{df}$$

11.9 Parametric variance

$$\mathrm{Variance}(X) = \mu'_2 - \mu'^2_1 = 2\mathrm{df}$$

11.10 Parametric skewness

$$\mathrm{Skewness}(X) = \frac{\mu'_3 - 3\mu'_2\mu'_1 + 2\mu'^3_1}{(\mu'_2 - \mu'^2_1)^{1.5}} = \sqrt{\frac{8}{\mathrm{df}}}$$

11.11 Parametric kurtosis

$$\mathrm{Kurtosis}(X) = \frac{\mu'_4 - 4\mu'_1\mu'_3 + 6\mu'^2_1\mu'_2 - 3\mu'^4_1}{(\mu'_2 - \mu'^2_1)^2} = 3 + \frac{12}{\mathrm{df}}$$

11.12 Parametric median

$$\mathrm{Median}(X) = 2\mathrm{P}^{-1}\left(\frac{\mathrm{df}}{2}, \frac{1}{2}\right)$$

11.13 Parametric mode

$$\mathrm{Mode}(X) = \max(\mathrm{df}-2, 0)$$

11.14 Additional information and definitions

- u : Uniform[0,1] random varible
- $\mathrm{P}\left(a,x\right)=\frac{\gamma(a,x)}{\Gamma(a)}$: Regularized lower incomplete gamma function
- $\mathrm{P}^{-1}\left(a,u\right)$: Inverse of regularized lower incomplete gamma function
- $\gamma\left(a,x\right)$: Lower incomplete gamma function
- $\Gamma\left(x\right)$: Gamma function

11.15 Spreadsheet documents

- Excel file from GitHub repository
- Google spreadsheet document

12 Chi Square 3P Distribution

12.1 Distribution definition

$$X \sim \chi^2_{3\mathrm{P}}\left(\mathrm{df}, \mathrm{Loc}, \mathrm{Sc}\right)$$

12.2 Distribution domain

$$x \in (\mathrm{Loc}, \infty)$$

12.3 Parameters domain and parameters constraints

$$\mathrm{df} \in \mathbb{N}^+, \mathrm{Loc} \in \mathbb{R}, \mathrm{Sc} \in \mathbb{R}^+$$

12.4 Cumulative distribution function

$$F_X\left(x\right) = \frac{\gamma(\frac{\mathrm{df}}{2}, \frac{z(x)}{2})}{\Gamma(\frac{\mathrm{df}}{2})} = \mathrm{P}\left(\frac{\mathrm{df}}{2}, \frac{z(x)}{2}\right)$$

12.5 Probability density function

$$f_X\left(x\right) = \frac{1}{\mathrm{Sc}}\frac{1}{2^{\mathrm{df}/2}\Gamma(\mathrm{df}/2)}\, x^{\mathrm{df}/2-1}e^{-z(x)/2}$$

12.6 Percent point function/Sample

$$F_X^{-1}\left(u\right) = 2\mathrm{P}^{-1}\left(\frac{\mathrm{df}}{2}, u\right)$$

12.7 Parametric centered moments

$$\tilde{\mu}'_k = E[\tilde{X}^k] = \int_0^\infty x^k f_{\tilde{X}}\left(x\right) dx = \mathrm{df}(\mathrm{df}+2)\cdots(\mathrm{df}+2k-2) = 2^k\frac{\Gamma\left(k+\frac{\mathrm{df}}{2}\right)}{\Gamma\left(\frac{\mathrm{df}}{2}\right)}$$

12.8 Parametric mean

$$\mathrm{Mean}(X) = \mathrm{Loc} + \mathrm{Sc}\cdot\tilde{\mu}'_1 = \mathrm{Loc} + \mathrm{Sc}\cdot\mathrm{df}$$

12.9 Parametric variance

$$\mathrm{Variance}(X) = \mathrm{Sc}^2\cdot(\tilde{\mu}'_2 - \tilde{\mu}'^2_1) = 2\cdot\mathrm{df}\cdot\mathrm{Sc}^2$$

12.10 Parametric skewness

$$\mathrm{Skewness}(X) = \frac{\tilde{\mu}'_3 - 3\tilde{\mu}'_2\tilde{\mu}'_1 + 2\tilde{\mu}'^3_1}{(\tilde{\mu}'_2 - \tilde{\mu}'^2_1)^{1.5}} = \sqrt{\frac{8}{\mathrm{df}}}$$

12.11 Parametric kurtosis

$$\mathrm{Kurtosis}(X) = \frac{\tilde{\mu}'_4 - 4\tilde{\mu}'_1\tilde{\mu}'_3 + 6\tilde{\mu}'^2_1\tilde{\mu}'_2 - 3\tilde{\mu}'^4_1}{(\tilde{\mu}'_2 - \tilde{\mu}'^2_1)^2} = 3 + \frac{12}{\mathrm{df}}$$

12.12 Parametric median

$$\mathrm{Median}(X) = \mathrm{Loc} + \mathrm{Sc}\times 2\mathrm{P}^{-1}\left(\frac{\mathrm{df}}{2}, \frac{1}{2}\right)$$

12.13 Parametric mode

$$\mathrm{Mode}(X) = \mathrm{Loc} + \mathrm{Sc}\times\max(\mathrm{df}-2, 0)$$

12.14 Additional information and definitions

- $\tilde{X} \sim \chi^2\left(\mathrm{df}\right)$
- Loc : Location parameter
- Sc : Scale parameter
- $z\left(x\right) = \left(x - \mathrm{Loc}\right)/\mathrm{Sc}$
- u : Uniform[0,1] random varible
- $\mathrm{P}\left(a, x\right) = \frac{\gamma(a,x)}{\Gamma(a)}$: Regularized lower incomplete gamma function
- $\mathrm{P}^{-1}\left(a, u\right)$: Inverse of regularized lower incomplete gamma function
- $\gamma\left(a, x\right)$: Lower incomplete gamma function
- $\Gamma\left(x\right)$: Gamma function

12.15 Spreadsheet documents

- Excel file from GitHub repository
- Google spreadsheet document

13 Dagum Distribution

13.1 Distribution definition

$$X \sim \text{Dagum}\,(a, b, p)$$

13.2 Distribution domain

$$x \in (0, \infty)$$

13.3 Parameters domain and parameters constraints

$$a \in \mathbb{R}^{+}, b \in \mathbb{R}^{+}, p \in \mathbb{R}^{+}$$

13.4 Cumulative distribution function

$$F_X\,(x) = \left(1 + \left(\frac{x}{b}\right)^{-a}\right)^{-p}$$

13.5 Probability density function

$$f_X\,(x) = \frac{ap}{x}\left(\frac{(\frac{x}{b})^{ap}}{\left((\frac{x}{b})^{a} + 1\right)^{p+1}}\right)$$

13.6 Percent point function/Sample

$$F_X^{-1}\,(u) = b(u^{-1/p} - 1)^{-1/a}$$

13.7 Parametric centered moments

$$\mu'_k = E[X^k] = \int_0^\infty x^k f_X\,(x)\,dx = pb^k \cdot \text{Beta}\left(\frac{ap+k}{a}, \frac{a-k}{a}\right)$$

13.8 Parametric mean

$$\text{Mean}(X) = \mu'_1$$

13.9 Parametric variance

$$\text{Variance}(X) = \mu'_2 - \mu'^2_1$$

13.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\mu'_3 - 3\mu'_2\mu'_1 + 2\mu'^3_1}{(\mu'_2 - \mu'^2_1)^{1.5}}$$

13.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\mu'_4 - 4\mu'_1\mu'_3 + 6\mu'^2_1\mu'_2 - 3\mu'^4_1}{(\mu'_2 - \mu'^2_1)^2}$$

13.12 Parametric median

$$\text{Median}(X) = b\left(-1 + 2^{\frac{1}{p}}\right)^{-\frac{1}{a}}$$

13.13 Parametric mode

$$\text{Mode}(X) = b\left(\frac{ap-1}{a+1}\right)^{\frac{1}{a}}$$

13.14 Additional information and definitions

- b : Scale parameter
- u : Uniform[0,1] random varible
- Beta (x, y) : Beta function

13.15 Spreadsheet documents

- Excel file from GitHub repository
- Google spreadsheet document

14 Dagum 4P Distribution

14.1 Distribution definition

$$X \sim \text{Dagum}_{\text{4P}}\left(a, b, p, \text{Loc}\right)$$

14.2 Distribution domain

$$x \in (\text{Loc}, \infty)$$

14.3 Parameters domain and parameters constraints

$$a \in \mathbb{R}^{+}, b \in \mathbb{R}^{+}, p \in \mathbb{R}^{+}, \text{Loc} \in \mathbb{R}$$

14.4 Cumulative distribution function

$$F_X\left(x\right) = \left(1 + \left(\frac{x - \text{Loc}}{b}\right)^{-a}\right)^{-p}$$

14.5 Probability density function

$$f_X\left(x\right) = \frac{ap}{x - \text{Loc}}\left(\frac{(\frac{x-\text{Loc}}{b})^{ap}}{\left((\frac{x-\text{Loc}}{b})^{a} + 1\right)^{p+1}}\right)$$

14.6 Percent point function/Sample

$$F_X^{-1}\left(u\right) = \text{Loc} + b(u^{-1/p} - 1)^{-1/a}$$

14.7 Parametric centered moments

$$\tilde{\mu}'_k = E[\tilde{X}^k] = \int_0^{\infty} x^k f_{\tilde{X}}\left(x\right) dx = pb^k \cdot \text{Beta}\left(\frac{ap+k}{a}, \frac{a-k}{a}\right)$$

14.8 Parametric mean

$$\text{Mean}(X) = \text{Loc} + \tilde{\mu}'_1$$

14.9 Parametric variance

$$\text{Variance}(X) = \tilde{\mu}'_2 - \tilde{\mu}'^2_1$$

14.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\tilde{\mu}'_3 - 3\tilde{\mu}'_2\tilde{\mu}'_1 + 2\tilde{\mu}'^3_1}{(\tilde{\mu}'_2 - \tilde{\mu}'^2_1)^{1.5}}$$

14.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\tilde{\mu}'_4 - 4\tilde{\mu}'_1\tilde{\mu}'_3 + 6\tilde{\mu}'^2_1\tilde{\mu}'_2 - 3\tilde{\mu}'^4_1}{(\tilde{\mu}'_2 - \tilde{\mu}'^2_1)^2}$$

14.12 Parametric median

$$\text{Median}(X) = \text{Loc} + b\left(-1 + 2^{\frac{1}{p}}\right)^{-\frac{1}{a}}$$

14.13 Parametric mode

$$\text{Mode}(X) = \text{Loc} + b\left(\frac{ap-1}{a+1}\right)^{\frac{1}{a}}$$

14.14 Additional information and definitions

- $\bar{X} \sim \text{Dagum}\,(a, b, p)$
- Loc : Location parameter
- b : Scale parameter
- u : Uniform[0,1] random varible
- $\text{Beta}\,(x, y)$: Beta function

14.15 Spreadsheet documents

- Excel file from GitHub repository
- Google spreadsheet document

15 Erlang Distribution

15.1 Distribution definition

$$X \sim \text{Erlang}\,(k, \beta)$$

15.2 Distribution domain

$$x \in [0, \infty)$$

15.3 Parameters domain and parameters constraints

$$k \in \mathbb{N}^{+}, \beta \in \mathbb{R}^{+}$$

15.4 Cumulative distribution function

$$F_X\,(x) = \mathrm{P}(k, \frac{x}{\beta}) = \frac{\gamma(k, \frac{x}{\beta})}{(k-1)!}$$

15.5 Probability density function

$$f_X\,(x) = \frac{x^{k-1} e^{-\frac{x}{\beta}}}{\beta^k (k-1)!}$$

15.6 Percent point function/Sample

$$F_X^{-1}\,(u) = \beta \mathrm{P}^{-1}\,(k, u)$$

15.7 Parametric centered moments

$$\mu'_n = E[X^n] = \int_0^\infty x^n f_X\,(x)\,dx = \beta^k \frac{\Gamma\,(n+k)}{\Gamma(k)}$$

15.8 Parametric mean

$$\text{Mean}(X) = \mu'_1$$

15.9 Parametric variance

$$\text{Variance}(X) = \mu'_2 - \mu'^2_1$$

15.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\mu'_3 - 3\mu'_2\mu'_1 + 2\mu'^3_1}{(\mu'_2 - \mu'^2_1)^{1.5}}$$

15.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\mu'_4 - 4\mu'_1\mu'_3 + 6\mu'^2_1\mu'_2 - 3\mu'^4_1}{(\mu'_2 - \mu'^2_1)^2}$$

15.12 Parametric median

$$\text{Median}(X) = \mathrm{P}(k, \frac{1}{2\beta})$$

15.13 Parametric mode

$$\text{Mode}(X) = \beta\,(k-1)$$

15.14 Additional information and definitions

- β : Scale parameter
- u : Uniform[0,1] random varible
- $\mathrm{P}\left(a, x\right) = \frac{\gamma(a,x)}{\Gamma(a)}$: Regularized lower incomplete gamma function
- $\mathrm{P}^{-1}\left(a, u\right)$: Inverse of regularized lower incomplete gamma function
- $\gamma\left(a, x\right)$: Lower incomplete gamma function
- $\Gamma\left(x\right)$: Gamma function

15.15 Spreadsheet documents

- Excel file from GitHub repository
- Google spreadsheet document

16 Erlang 3P Distribution

16.1 Distribution definition

$$X \sim \text{Erlang}_{\text{3P}}\left(k, \beta, \text{Loc}\right)$$

16.2 Distribution domain

$$x \in [\text{Loc}, \infty)$$

16.3 Parameters domain and parameters constraints

$$k \in \mathbb{N}^{+}, \beta \in \mathbb{R}^{+}, \text{Loc} \in \mathbb{R}$$

16.4 Cumulative distribution function

$$F_X\left(x\right) = \text{P}(k, \frac{x - \text{Loc}}{\beta}) = \frac{\gamma(k, \frac{x-\text{Loc}}{\beta})}{(k-1)!}$$

16.5 Probability density function

$$f_X\left(x\right) = \frac{(x - \text{Loc})^{k-1} e^{-\frac{x-\text{Loc}}{\beta}}}{\beta^k (k-1)!}$$

16.6 Percent point function/Sample

$$F_X^{-1}\left(u\right) = \text{Loc} + \beta \text{P}^{-1}\left(k, u\right)$$

16.7 Parametric centered moments

$$\tilde{\mu}'_n = E[\tilde{X}^n] = \int_0^\infty x^n f_{\tilde{X}}\left(x\right) dx = \beta^k \frac{\Gamma\left(n + k\right)}{\Gamma(k)}$$

16.8 Parametric mean

$$\text{Mean}(X) = \text{Loc} + \tilde{\mu}'_1$$

16.9 Parametric variance

$$\text{Variance}(X) = \tilde{\mu}'_2 - \tilde{\mu}'^2_1$$

16.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\tilde{\mu}'_3 - 3\tilde{\mu}'_2\tilde{\mu}'_1 + 2\tilde{\mu}'^3_1}{(\tilde{\mu}'_2 - \tilde{\mu}'^2_1)^{1.5}}$$

16.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\tilde{\mu}'_4 - 4\tilde{\mu}'_1\tilde{\mu}'_3 + 6\tilde{\mu}'^2_1\tilde{\mu}'_2 - 3\tilde{\mu}'^4_1}{(\tilde{\mu}'_2 - \tilde{\mu}'^2_1)^2}$$

16.12 Parametric median

$$\text{Median}(X) = \text{Loc} + \text{P}(k, \frac{1}{2\beta})$$

16.13 Parametric mode

$$\text{Mode}(X) = \text{Loc} + \beta \cdot (k - 1)$$

16.14 Additional information and definitions

- $\tilde{X} \sim \text{Erlang}\,(k, \beta)$
- Loc : Location parameter
- β : Scale parameter
- u : Uniform[0,1] random varible
- $\text{P}\,(a, x) = \frac{\gamma(a,x)}{\Gamma(a)}$: Regularized lower incomplete gamma function
- $\text{P}^{-1}\,(a, u)$: Inverse of regularized lower incomplete gamma function
- $\gamma\,(a, x)$: Lower incomplete gamma function
- $\Gamma\,(x)$: Gamma function

16.15 Spreadsheet documents

- Excel file from GitHub repository
- Google spreadsheet document

17 Error Function Distribution

17.1 Distribution definition

$$X \sim \text{ErrorFunction}\,(h)$$

17.2 Distribution domain

$$x \in (-\infty, \infty)$$

17.3 Parameters domain and parameters constraints

$$h \in \mathbb{R}^{+}$$

17.4 Cumulative distribution function

$$F_X\,(x) = \Phi(\sqrt{2}hx)$$

17.5 Probability density function

$$f_X\,(x) = \frac{h}{\sqrt{\pi}} e^{-h^2 x^2}$$

17.6 Percent point function/Sample

$$F_X^{-1}\,(u) = \frac{\Phi^{-1}(u)}{\sqrt{2}h}$$

17.7 Parametric centered moments

$$\mu'_k = E[X^k] = \int_{-\infty}^{\infty} x^k f_X\,(x)\,dx$$

17.8 Parametric mean

$$\text{Mean}(X) = \mu'_1 = 0$$

17.9 Parametric variance

$$\text{Variance}(X) = \mu'_2 - \mu'^2_1 = \frac{1}{2h^2}$$

17.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\mu'_3 - 3\mu'_2\mu'_1 + 2\mu'^3_1}{(\mu'_2 - \mu'^2_1)^{1.5}} = 0$$

17.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\mu'_4 - 4\mu'_1\mu'_3 + 6\mu'^2_1\mu'_2 - 3\mu'^4_1}{(\mu'_2 - \mu'^2_1)^2} = 3$$

17.12 Parametric median

$$\text{Median}(X) = 0$$

17.13 Parametric mode

$$\text{Mode}(X) = 0$$

17.14 Additional information and definitions

- h : Inverse of scale parameter
- u : Uniform[0,1] random varible
- $\Phi(x)$: CDF normal standard distribution

17.15 Spreadsheet documents

- Excel file from GitHub repository
- Google spreadsheet document

18 Exponential Distribution

18.1 Distribution definition

$$X \sim \text{Exponential}\,(\lambda)$$

18.2 Distribution domain

$$x \in [0, \infty)$$

18.3 Parameters domain and parameters constraints

$$\lambda \in \mathbb{R}^+$$

18.4 Cumulative distribution function

$$F_X\,(x) = 1 - e^{-\lambda x}$$

18.5 Probability density function

$$f_X\,(x) = \lambda e^{-\lambda x}$$

18.6 Percent point function/Sample

$$F_X^{-1}\,(u) = -\frac{\ln(1-u)}{\lambda}$$

18.7 Parametric centered moments

$$\mu'_k = E[X^k] = \int_0^\infty x^n f_X\,(x)\,dx = \frac{k!}{\lambda^k}$$

18.8 Parametric mean

$$\text{Mean}(X) = \mu'_1 = \frac{1}{\lambda}$$

18.9 Parametric variance

$$\text{Variance}(X) = \mu'_2 - \mu'^2_1 = \frac{1}{\lambda^2}$$

18.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\mu'_3 - 3\mu'_2\mu'_1 + 2\mu'^3_1}{(\mu'_2 - \mu'^2_1)^{1.5}} = 2$$

18.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\mu'_4 - 4\mu'_1\mu'_3 + 6\mu'^2_1\mu'_2 - 3\mu'^4_1}{(\mu'_2 - \mu'^2_1)^2} = 9$$

18.12 Parametric median

$$\text{Median}(X) = \frac{\ln 2}{\lambda}$$

18.13 Parametric mode

$$\text{Mode}(X) = 0$$

18.14 Additional information and definitions

- λ : Inverse of scale parameter
- u : Uniform[0,1] random varible

18.15 Spreadsheet documents

- Excel file from GitHub repository
- Google spreadsheet document

19 Exponential 2P Distribution

19.1 Distribution definition

$$X \sim \text{Exponential}_{\text{2P}}\left(\lambda, \text{Loc}\right)$$

19.2 Distribution domain

$$x \in [\text{Loc}, \infty)$$

19.3 Parameters domain and parameters constraints

$$\lambda \in \mathbb{R}^{+}, \text{Loc} \in \mathbb{R}$$

19.4 Cumulative distribution function

$$F_X\left(x\right) = 1 - e^{-\lambda(x-\text{Loc})}$$

19.5 Probability density function

$$f_X\left(x\right) = \lambda e^{-\lambda(x-\text{Loc})}$$

19.6 Percent point function/Sample

$$F_X^{-1}\left(u\right) = \text{Loc} - \frac{\ln(1-u)}{\lambda}$$

19.7 Parametric centered moments

$$\tilde{\mu}'_k = E[\tilde{X}^k] = \int_0^{\infty} x^k f_{\tilde{X}}\left(x\right) dx = \frac{k!}{\lambda^k}$$

19.8 Parametric mean

$$\text{Mean}(X) = \text{Loc} + \tilde{\mu}'_1 = \text{Loc} + \frac{1}{\lambda}$$

19.9 Parametric variance

$$\text{Variance}(X) = \tilde{\mu}'_2 - \tilde{\mu}'^2_1 = \frac{1}{\lambda^2}$$

19.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\tilde{\mu}'_3 - 3\tilde{\mu}'_2\tilde{\mu}'_1 + 2\tilde{\mu}'^3_1}{(\tilde{\mu}'_2 - \tilde{\mu}'^2_1)^{1.5}} = 2$$

19.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\tilde{\mu}'_4 - 4\tilde{\mu}'_1\tilde{\mu}'_3 + 6\tilde{\mu}'^2_1\tilde{\mu}'_2 - 3\tilde{\mu}'^4_1}{(\tilde{\mu}'_2 - \tilde{\mu}'^2_1)^2} = 9$$

19.12 Parametric median

$$\text{Median}(X) = \text{Loc} + \frac{\ln 2}{\lambda}$$

19.13 Parametric mode

$$\text{Mode}(X) = \text{Loc}$$

19.14 Additional information and definitions

- $\tilde{X} \sim \text{Exponential}\,(\lambda)$
- Loc : Location parameter
- λ : Inverse of scale parameter
- u : Uniform[0,1] random varible

19.15 Spreadsheet documents

- Excel file from GitHub repository
- Google spreadsheet document

20 F Distribution

20.1 Distribution definition

$$X \sim \mathrm{F}\left(\mathrm{df}_1, \mathrm{df}_2\right)$$

20.2 Distribution domain

$$x \in [0, \infty)$$

20.3 Parameters domain and parameters constraints

$$\mathrm{df}_1 \in \mathbb{R}^+, \mathrm{df}_2 \in \mathbb{R}^+$$

20.4 Cumulative distribution function

$$F_X\left(x\right) = I_{\mathrm{df}_1 x/(\mathrm{df}_1 x + \mathrm{df}_2)}\left(\tfrac{\mathrm{df}_1}{2}, \tfrac{\mathrm{df}_2}{2}\right)$$

20.5 Probability density function

$$f_X\left(x\right) = \frac{\sqrt{\frac{(\mathrm{df}_1 x)^{\mathrm{df}_1} \mathrm{df}_2^{\mathrm{df}_2}}{(\mathrm{df}_1 x + \mathrm{df}_2)^{\mathrm{df}_1 + \mathrm{df}_2}}}}{x \;\times \mathrm{Beta}\left(\tfrac{\mathrm{df}_1}{2}, \tfrac{\mathrm{df}_2}{2}\right)}$$

20.6 Percent point function/Sample

$$F_X^{-1}\left(u\right) = \frac{\mathrm{df}_2 \times I^{-1}\left(u, \tfrac{\mathrm{df}_1}{2}, \tfrac{\mathrm{df}_2}{2}\right)}{\mathrm{df}_1 \times \left(1 - I^{-1}\left(u, \tfrac{\mathrm{df}_1}{2}, \tfrac{\mathrm{df}_2}{2}\right)\right)}$$

20.7 Parametric centered moments

$$\mu'_k = E[X^k] = \int_0^\infty x^k f_X\left(x\right) dx = \left(\frac{\mathrm{df}_2}{\mathrm{df}_1}\right)^k \frac{\Gamma\left(\frac{\mathrm{df}_1}{2} + k\right)}{\Gamma\left(\frac{\mathrm{df}_1}{2}\right)} \frac{\Gamma\left(\frac{\mathrm{df}_2}{2} - k\right)}{\Gamma\left(\frac{\mathrm{df}_2}{2}\right)} \quad \text{if } \mathrm{df}_2 > 2k$$

20.8 Parametric mean

$$\mathrm{Mean}(X) = \mu'_1 = \frac{\mathrm{df}_2}{\mathrm{df}_2 - 2} \quad \text{if } \mathrm{df}_2 > 2$$

20.9 Parametric variance

$$\mathrm{Variance}(X) = \mu'_2 - \mu'^2_1 = \frac{2\,\mathrm{df}_2^2\left(\mathrm{df}_1 + \mathrm{df}_2 - 2\right)}{\mathrm{df}_1(\mathrm{df}_2 - 2)^2(\mathrm{df}_2 - 4)} \quad \text{if } \mathrm{df}_2 > 4$$

20.10 Parametric skewness

$$\mathrm{Skewness}(X) = \frac{\mu'_3 - 3\mu'_2\mu'_1 + 2\mu'^3_1}{(\mu'_2 - \mu'^2_1)^{1.5}} = \frac{(2\mathrm{df}_1 + \mathrm{df}_2 - 2)\sqrt{8(\mathrm{df}_2 - 4)}}{(\mathrm{df}_2 - 6)\sqrt{\mathrm{df}_1(\mathrm{df}_1 + \mathrm{df}_2 - 2)}} \quad \text{if } \mathrm{df}_2 > 6$$

20.11 Parametric kurtosis

$$\mathrm{Kurtosis}(X) = \frac{\mu'_4 - 4\mu'_1\mu'_3 + 6\mu'^2_1\mu'_2 - 3\mu'^4_1}{(\mu'_2 - \mu'^2_1)^2} = \frac{3\left(8 + (\mathrm{df}_2 - 6) \times \mathrm{Skewness}(X)^2\right)}{2\mathrm{df}_2 - 16} + 3 \quad \text{if } \mathrm{df}_2 > 8$$

20.12 Parametric median

$$\mathrm{Median}(X) = \frac{\mathrm{df}_2 \times I^{-1}\left(\tfrac{1}{2}, \tfrac{\mathrm{df}_1}{2}, \tfrac{\mathrm{df}_2}{2}\right)}{\mathrm{df}_1 \times \left(1 - I^{-1}\left(\tfrac{1}{2}, \tfrac{\mathrm{df}_1}{2}, \tfrac{\mathrm{df}_2}{2}\right)\right)}$$

20.13 Parametric mode

$$\mathrm{Mode}(X) = \frac{\mathrm{df}_2\left(\mathrm{df}_1 - 2\right)}{\mathrm{df}_1\left(\mathrm{df}_2 + 2\right)} \quad \text{if } \mathrm{df}_1 > 2$$

20.14 Additional information and definitions

- u : Uniform[0,1] random varible
- $I\left(x, a, b\right)$: Regularized incomplete beta function
- $I^{-1}\left(x, a, b\right)$: Inverse of regularized incomplete beta function
- $\mathrm{Beta}\left(x, y\right)$: Beta function

20.15 Spreadsheet documents

- Excel file from GitHub repository
- Google spreadsheet document

21 F 4P Distribution

21.1 Distribution definition

$$X \sim \mathrm{F}_{\mathrm{4P}}\left(\mathrm{df}_1, \mathrm{df}_2, \mathrm{Loc}, \mathrm{Sc}\right)$$

21.2 Distribution domain

$$x \in [\mathrm{Loc}, \infty)$$

21.3 Parameters domain and parameters constraints

$$\mathrm{df}_1 \in \mathbb{R}^+, \mathrm{df}_2 \in \mathbb{R}^+, \mathrm{Loc} \in \mathbb{R}, \mathrm{Sc} \in \mathbb{R}^+$$

21.4 Cumulative distribution function

$$F_X\left(x\right) = I_{\mathrm{df}_1 z(x)/(\mathrm{df}_1 z(x)+\mathrm{df}_2)}\left(\tfrac{\mathrm{df}_1}{2}, \tfrac{\mathrm{df}_2}{2}\right)$$

21.5 Probability density function

$$f_X\left(x\right) = \frac{1}{\mathrm{Sc}} \times \frac{\sqrt{\frac{(\mathrm{df}_1 z(x))^{\mathrm{df}_1}\mathrm{df}_2^{\mathrm{df}_2}}{(\mathrm{df}_1 z(x)+\mathrm{df}_2)^{\mathrm{df}_1+\mathrm{df}_2}}}}{z(x)\,\mathrm{Beta}\left(\frac{\mathrm{df}_1}{2}, \frac{\mathrm{df}_2}{2}\right)}$$

21.6 Percent point function/Sample

$$F_X^{-1}\left(u\right) = \mathrm{Loc} + \mathrm{Sc}\frac{\mathrm{df}_2 \times I^{-1}\left(u, \frac{\mathrm{df}_1}{2}, \frac{\mathrm{df}_2}{2}\right)}{\mathrm{df}_1 \times \left(1 - I^{-1}\left(u, \frac{\mathrm{df}_1}{2}, \frac{\mathrm{df}_2}{2}\right)\right)}$$

21.7 Parametric centered moments

$$\tilde{\mu}'_k = E[\tilde{X}^k] = \int_0^\infty x^k f_{\tilde{X}}\left(x\right)dx = \frac{\Gamma\left(\frac{\mathrm{df}_1}{2}+k\right)}{\Gamma\left(\frac{\mathrm{df}_1}{2}\right)}\frac{\Gamma\left(\frac{\mathrm{df}_2}{2}-k\right)}{\Gamma\left(\frac{\mathrm{df}_2}{2}\right)}\left(\frac{\mathrm{df}_2}{\mathrm{df}_1}\right)^k \quad \text{if } \mathrm{df}_2 > 2k$$

21.8 Parametric mean

$$\mathrm{Mean}(X) = \mathrm{Loc} + \mathrm{Sc}\tilde{\mu}'_1 = \mathrm{Loc} + \mathrm{Sc}\frac{\mathrm{df}_2}{\mathrm{df}_2 - 2} \quad \text{if } \mathrm{df}_2 > 2$$

21.9 Parametric variance

$$\mathrm{Variance}(X) = \mathrm{Sc}^2(\tilde{\mu}'_2 - \tilde{\mu}'^2_1) = \mathrm{Sc}^2\frac{2\,\mathrm{df}_2^2\,(\mathrm{df}_1 + \mathrm{df}_2 - 2)}{\mathrm{df}_1(\mathrm{df}_2-2)^2(\mathrm{df}_2-4)} \quad \text{if } \mathrm{df}_2 > 4$$

21.10 Parametric skewness

$$\mathrm{Skewness}(X) = \frac{\tilde{\mu}'_3 - 3\tilde{\mu}'_2\tilde{\mu}'_1 + 2\tilde{\mu}'^3_1}{(\tilde{\mu}'_2 - \tilde{\mu}'^2_1)^{1.5}} = \frac{(2\mathrm{df}_1 + \mathrm{df}_2 - 2)\sqrt{8(\mathrm{df}_2-4)}}{(\mathrm{df}_2-6)\sqrt{\mathrm{df}_1(\mathrm{df}_1+\mathrm{df}_2-2)}} \quad \text{if } \mathrm{df}_2 > 6$$

21.11 Parametric kurtosis

$$\mathrm{Kurtosis}(X) = \frac{\tilde{\mu}'_4 - 4\tilde{\mu}'_1\tilde{\mu}'_3 + 6\tilde{\mu}'^2_1\tilde{\mu}'_2 - 3\tilde{\mu}'^4_1}{(\tilde{\mu}'_2 - \tilde{\mu}'^2_1)^2} = \frac{3\left(8 + (\mathrm{df}_2 - 6)\times\mathrm{Skewness}(X)^2\right)}{2\mathrm{df}_2 - 16} + 3 \quad \text{if } \mathrm{df}_2 > 8$$

21.12 Parametric median

$$\mathrm{Median}(X) = \mathrm{Loc} + \mathrm{Sc}\frac{\mathrm{df}_2 \times I^{-1}\left(\frac{1}{2}, \frac{\mathrm{df}_1}{2}, \frac{\mathrm{df}_2}{2}\right)}{\mathrm{df}_1 \times \left(1 - I^{-1}\left(\frac{1}{2}, \frac{\mathrm{df}_1}{2}, \frac{\mathrm{df}_2}{2}\right)\right)}$$

21.13 Parametric mode

$$\text{Mode}(X) = \text{Loc} + \text{Sc}\frac{\text{df}_2\left(\text{df}_1 - 2\right)}{\text{df}_1\left(\text{df}_2 + 2\right)} \quad \text{if } \text{df}_1 > 2$$

21.14 Additional information and definitions

- $\tilde{X} \sim \text{F}\left(\text{df}_1, \text{df}_2\right)$
- Loc : Location parameter
- Sc : Scale parameter
- $z\left(x\right) = \left(x - \text{Loc}\right)/\text{Sc}$
- u : Uniform[0,1] random varible
- $I\left(x, a, b\right)$: Regularized incomplete beta function
- $I^{-1}\left(x, a, b\right)$: Inverse of regularized incomplete beta function
- $\text{Beta}\left(x, y\right)$: Beta function

21.15 Spreadsheet documents

- Excel file from GitHub repository
- Google spreadsheet document

22 Fatigue Life Distribution

22.1 Distribution definition

$$X \sim \text{FatigueLife}\left(\gamma, \text{Loc}, \text{Sc}\right)$$

22.2 Distribution domain

$$x \in \left(\text{Loc}, \infty\right)$$

22.3 Parameters domain and parameters constraints

$$\gamma \in \mathbb{R}^{+}, \text{Loc} \in \mathbb{R}, \text{Sc} \in \mathbb{R}^{+}$$

22.4 Cumulative distribution function

$$F_X\left(x\right) = \Phi\left(\frac{\sqrt{z(x)} - \sqrt{\frac{1}{z(x)}}}{\gamma}\right)$$

22.5 Probability density function

$$f_X\left(x\right) = \frac{\sqrt{z(x)} + \sqrt{\frac{1}{z(x)}}}{2\gamma z(x)}\phi\left(\frac{\sqrt{z(x)} - \sqrt{\frac{1}{z(x)}}}{\gamma}\right)$$

22.6 Percent point function/Sample

$$F_X^{-1}\left(u\right) = \text{Loc} + \text{Sc}\frac{1}{4}\left[\gamma\Phi^{-1}(u) + \sqrt{4 + \left(\gamma\Phi^{-1}(u)\right)^2}\right]^2$$

22.7 Parametric centered moments

$$\mu'_k = E[X^k] = \int_{-\infty}^{\infty} x^k f_X\left(x\right) dx$$

22.8 Parametric mean

$$\text{Mean}(X) = \text{Loc} + \text{Sc}\cdot\tilde{\mu}'_1 = \text{Loc} + \text{Sc}\left(1 + \frac{\gamma^2}{2}\right)$$

22.9 Parametric variance

$$\text{Variance}(X) = \text{Sc}^2\cdot(\tilde{\mu}'_2 - \tilde{\mu}'^2_1) = \text{Sc}^2\gamma^2\left(1 + \frac{5\gamma^2}{4}\right)$$

22.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\mu'_3 - 3\mu'_2\mu'_1 + 2\mu'^3_1}{(\mu'_2 - \mu'^2_1)^{1.5}} = \frac{4\gamma(6 + 11\gamma^2)}{(4 + 5\gamma^2)^{1.5}}$$

22.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\mu'_4 - 4\mu'_1\mu'_3 + 6\mu'^2_1\mu'_2 - 3\mu'^4_1}{(\mu'_2 - \mu'^2_1)^2} = 3 + \frac{6\gamma^2(93\gamma^2 + 40)}{(5\gamma^2 + 4)^2}$$

22.12 Parametric median

$$\text{Median}(X) = \text{Loc} + \text{Sc}\frac{1}{4}\left[\gamma\Phi^{-1}\left(1/2\right) + \sqrt{4 + \left(\gamma\Phi^{-1}\left(1/2\right)\right)^2}\right]^2$$

22.13 Parametric mode

$$\mathrm{Mode}(X) = \arg\max_x f_X(x)$$

22.14 Additional information and definitions

- Loc : Location parameter
- Sc : Scale parameter
- $z(x) = (x - \mathrm{Loc}) / \mathrm{Sc}$
- u : Uniform[0,1] random varible
- $\Phi(x)$: CDF normal standard distribution
- $\phi(x)$: PDF normal standard distribution

22.15 Spreadsheet documents

- Excel file from GitHub repository
- Google spreadsheet document

23 Folded Normal Distribution

23.1 Distribution definition

$$X \sim \text{FoldedNormal}\left(\mu, \sigma\right)$$

23.2 Distribution domain

$$x \in [0, \infty)$$

23.3 Parameters domain and parameters constraints

$$\mu \in \mathbb{R}, \sigma \in \mathbb{R}^{+}$$

23.4 Cumulative distribution function

$$F_X\left(x\right) = \frac{1}{2}\left[\text{erf}\left(\frac{x+\mu}{\sigma\sqrt{2}}\right) + \text{erf}\left(\frac{x-\mu}{\sigma\sqrt{2}}\right)\right]$$

23.5 Probability density function

$$f_X\left(x\right) = \frac{1}{\sigma\sqrt{2\pi}}\, e^{-\frac{(x-\mu)^2}{2\sigma^2}} + \frac{1}{\sigma\sqrt{2\pi}}\, e^{-\frac{(x+\mu)^2}{2\sigma^2}}$$

23.6 Percent point function/Sample

$$\text{Sample}_X\left(u\right) = \left|\mu + \sigma\Phi^{-1}(u)\right|$$

23.7 Parametric centered moments

$$\mu_k' = E[X^k] = \int_0^{\infty} x^k f_X\left(x\right) dx$$

23.8 Parametric mean

$$\text{Mean}(X) = \mu_1' = \sigma\sqrt{\tfrac{2}{\pi}}\, e^{(-\mu^2/2\sigma^2)} + \mu\left(1 - 2\,\Phi(-\tfrac{\mu}{\sigma})\right)$$

23.9 Parametric variance

$$\text{Variance}(X) = \mu_2' - \mu_1'^2 = \mu^2 + \sigma^2 - \text{Mean}(X)^2$$

23.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\mu_3' - 3\mu_2'\mu_1' + 2\mu_1'^3}{(\mu_2' - \mu_1'^2)^{1.5}}$$

23.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\mu_4' - 4\mu_1'\mu_3' + 6\mu_1'^2\mu_2' - 3\mu_1'^4}{(\mu_2' - \mu_1'^2)^2}$$

23.12 Parametric median

$$\text{Median}(X) = \left|\mu + \sigma\Phi^{-1}\left(1/2\right)\right|$$

23.13 Parametric mode

$$\text{Mode}(X) = \arg\max_x f_X\left(x\right)$$

23.14 Additional information and definitions

- Computing an analytic expression for the inverse of the cumulative distribution function is not feasible. Nonetheless, it is possible to generate a random sample from the distribution.
- μ : Location parameter
- σ : Scale parameter
- u : Uniform[0,1] random varible
- $\Phi(x)$: CDF normal standard distribution
- $\phi(x)$: PDF normal standard distribution
- $\Phi^{-1}(x)$: PPF normal standard distribution

23.15 Spreadsheet documents

- Excel file from GitHub repository
- Google spreadsheet document

24 Frechet Distribution

24.1 Distribution definition

$$X \sim \text{Frechet}\left(\alpha, \text{Loc}, \text{Sc}\right)$$

24.2 Distribution domain

$$x \in [\text{Loc}, \infty)$$

24.3 Parameters domain and parameters constraints

$$\alpha \in \mathbb{R}^{+}, \text{Loc} \in \mathbb{R}, \text{Sc} \in \mathbb{R}^{+}$$

24.4 Cumulative distribution function

$$F_X\left(x\right) = e^{\left(-z(x)\right)^{-\alpha}}$$

24.5 Probability density function

$$f_X\left(x\right) = \frac{\alpha}{\text{Sc}}\;\left(z(x)\right)^{-1-\alpha}\;e^{-\left(z(x)\right)^{-\alpha}}$$

24.6 Percent point function/Sample

$$F_X^{-1}\left(u\right) = \text{Loc} + \text{Sc}\left(-\ln(u)\right)^{-\frac{1}{\alpha}}$$

24.7 Parametric centered moments

$$\mu'_k = E[X^k] = \int_{\text{Loc}}^{\infty} x^k f_X\left(x\right)dx = \Gamma\left(1 - \frac{k}{\alpha}\right)$$

24.8 Parametric mean

$$\text{Mean}(X) = \text{Loc} + \text{Sc} \cdot \tilde{\mu}'_1 \quad \text{if } \alpha > 1$$

24.9 Parametric variance

$$\text{Variance}(X) = \text{Sc}^2 \cdot (\tilde{\mu}'_2 - \tilde{\mu}'^2_1) \quad \text{if } \alpha > 2$$

24.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\mu'_3 - 3\mu'_2\mu'_1 + 2\mu'^3_1}{(\mu'_2 - \mu'^2_1)^{1.5}} \quad \text{if } \alpha > 3$$

24.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\mu'_4 - 4\mu'_1\mu'_3 + 6\mu'^2_1\mu'_2 - 3\mu'^4_1}{(\mu'_2 - \mu'^2_1)^2} \quad \text{if } \alpha > 4$$

24.12 Parametric median

$$\text{Median}(X) = \text{Loc} + \frac{\text{Sc}}{\sqrt[\alpha]{\ln(2)}}$$

24.13 Parametric mode

$$\text{Mode}(X) = \text{Loc} + \text{Sc}\left(\frac{\alpha}{1+\alpha}\right)^{1/\alpha}$$

24.14 Additional information and definitions

- Loc : Location parameter
- Sc : Scale parameter
- $z(x) = (x - \text{Loc})/\text{Sc}$
- u : Uniform[0,1] random varible
- $\Gamma(x)$: Gamma function

24.15 Spreadsheet documents

- Excel file from GitHub repository
- Google spreadsheet document

25 Gamma Distribution

25.1 Distribution definition

$$X \sim \text{Gamma}\left(\alpha, \beta\right)$$

25.2 Distribution domain

$$x \in (0, \infty)$$

25.3 Parameters domain and parameters constraints

$$\alpha \in \mathbb{R}^{+}, \beta \in \mathbb{R}^{+}$$

25.4 Cumulative distribution function

$$F_X\left(x\right) = \mathrm{P}\left(\alpha, \frac{x}{\beta}\right) = \frac{1}{\Gamma(\alpha)}\gamma\left(\alpha, \frac{x}{\beta}\right)$$

25.5 Probability density function

$$f_X\left(x\right) = \frac{1}{\Gamma(\alpha)\beta^{\alpha}}x^{\alpha-1}e^{-\frac{x}{\beta}}$$

25.6 Percent point function/Sample

$$F_X^{-1}\left(u\right) = \beta \mathrm{P}^{-1}\left(\alpha, u\right)$$

25.7 Parametric centered moments

$$\mu'_k = E[X^k] = \int_0^{\infty} x^k f_X\left(x\right) dx = \beta^k \frac{\Gamma(k+\alpha)}{\Gamma(\alpha)}$$

25.8 Parametric mean

$$\text{Mean}(X) = \mu'_1 = \alpha\beta$$

25.9 Parametric variance

$$\text{Variance}(X) = \mu'_2 - \mu'^2_1 = \alpha\beta^2$$

25.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\mu'_3 - 3\mu'_2\mu'_1 + 2\mu'^3_1}{(\mu'_2 - \mu'^2_1)^{1.5}} = \frac{2}{\sqrt{\alpha}}$$

25.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\mu'_4 - 4\mu'_1\mu'_3 + 6\mu'^2_1\mu'_2 - 3\mu'^4_1}{(\mu'_2 - \mu'^2_1)^2} = 3 + \frac{6}{\alpha}$$

25.12 Parametric median

$$\text{Median}(X) = (\alpha - 1)\beta \quad \text{if } \alpha > 1$$

25.13 Parametric mode

$$\text{Mode}(X) = \beta \mathrm{P}^{-1}\left(\alpha, \frac{1}{2}\right)$$

25.14 Additional information and definitions

- β : Scale parameter
- u : Uniform[0,1] random varible
- $\mathrm{P}(a, x) = \frac{\gamma(a,x)}{\Gamma(a)}$: Regularized lower incomplete gamma function
- $\mathrm{P}^{-1}(a, u)$: Inverse of regularized lower incomplete gamma function
- $\gamma(a, x)$: Lower incomplete gamma function
- $\Gamma(x)$: Gamma function

25.15 Spreadsheet documents

- Excel file from GitHub repository
- Google spreadsheet document

26 Gamma 3P Distribution

26.1 Distribution definition

$$X \sim \mathrm{Gamma}_{3\mathrm{P}}\left(\alpha, \mathrm{Loc}, \beta\right)$$

26.2 Distribution domain

$$x \in (\mathrm{Loc}, \infty)$$

26.3 Parameters domain and parameters constraints

$$\alpha \in \mathbb{R}^{+}, \mathrm{Loc} \in \mathbb{R}, \beta \in \mathbb{R}^{+}$$

26.4 Cumulative distribution function

$$F_X\left(x\right) = \mathrm{P}\left(\alpha, \frac{x - \mathrm{Loc}}{\beta}\right) = \frac{1}{\Gamma(\alpha)}\gamma\left(\alpha, \frac{x - \mathrm{Loc}}{\beta}\right)$$

26.5 Probability density function

$$f_X\left(x\right) = \frac{1}{\Gamma(\alpha)\beta^{\alpha}}(x - \mathrm{Loc})^{\alpha-1}e^{-\frac{x-\mathrm{Loc}}{\beta}}$$

26.6 Percent point function/Sample

$$F_X^{-1}\left(u\right) = \mathrm{Loc} + \beta\mathrm{P}^{-1}\left(\alpha, u\right)$$

26.7 Parametric centered moments

$$\tilde{\mu}'_k = E[\tilde{X}^k] = \int_0^{\infty} x^k f_{\tilde{X}}\left(x\right) dx = \beta^k \frac{\Gamma(k+\alpha)}{\Gamma(\alpha)}$$

26.8 Parametric mean

$$\mathrm{Mean}(X) = \mathrm{Loc} + \tilde{\mu}'_1 = \mathrm{Loc} + \alpha\beta$$

26.9 Parametric variance

$$\mathrm{Variance}(X) = \tilde{\mu}'_2 - \tilde{\mu}'^2_1 = \alpha\beta^2$$

26.10 Parametric skewness

$$\mathrm{Skewness}(X) = \frac{\tilde{\mu}'_3 - 3\tilde{\mu}'_2\tilde{\mu}'_1 + 2\tilde{\mu}'^3_1}{(\tilde{\mu}'_2 - \tilde{\mu}'^2_1)^{1.5}} = \frac{2}{\sqrt{\alpha}}$$

26.11 Parametric kurtosis

$$\mathrm{Kurtosis}(X) = \frac{\tilde{\mu}'_4 - 4\tilde{\mu}'_1\tilde{\mu}'_3 + 6\tilde{\mu}'^2_1\tilde{\mu}'_2 - 3\tilde{\mu}'^4_1}{(\tilde{\mu}'_2 - \tilde{\mu}'^2_1)^2} = 3 + \frac{6}{\alpha}$$

26.12 Parametric median

$$\mathrm{Median}(X) = \mathrm{Loc} + (\alpha - 1)\beta \quad \text{if } \alpha > 1$$

26.13 Parametric mode

$$\mathrm{Mode}(X) = \mathrm{Loc} + \beta\mathrm{P}^{-1}\left(\alpha, \frac{1}{2}\right)$$

26.14 Additional information and definitions

- $\tilde{X} \sim \text{Gamma}\,(\alpha, \beta)$
- Loc : Location parameter
- β : Scale parameter
- u : Uniform[0,1] random varible
- $\mathrm{P}\,(a, x) = \frac{\gamma(a,x)}{\Gamma(a)}$: Regularized lower incomplete gamma function
- $\mathrm{P}^{-1}\,(a, u)$: Inverse of regularized lower incomplete gamma function
- $\gamma\,(a, x)$: Lower incomplete gamma function
- $\Gamma\,(x)$: Gamma function

26.15 Spreadsheet documents

- Excel file from GitHub repository
- Google spreadsheet document

27 Generalized Extreme Value Distribution

27.1 Distribution definition

$$X \sim \text{GeneralizedExtremeValue}\,(\xi, \mu, \sigma)$$

27.2 Distribution domain

$$\text{if } \xi > 0: \; x \in (z(x), \infty)\,, \quad \text{if } \xi = 0: \; x \in (-\infty, \infty)\,, \quad \text{if } \xi < 0: \; x \in (-\infty, z(x))$$

27.3 Parameters domain and parameters constraints

$$\xi \in \mathbb{R}, \mu \in \mathbb{R}, \sigma \in \mathbb{R}^+$$

27.4 Cumulative distribution function

$$F_X\,(x) = \begin{cases} \exp\Big(-\exp(-z(x))\Big) & \text{if } \; \xi = 0 \\ \exp\Big(-(1+\xi z(x))^{-1/\xi}\Big) & \text{if } \; \xi \neq 0 \end{cases}$$

27.5 Probability density function

$$f_X\,(x) = \begin{cases} \frac{1}{\sigma}\exp(-z(x))\exp\Big(-\exp(-z(x))\Big) & \text{if } \; \xi = 0 \\ \frac{1}{\sigma}\Big(1+\xi z(x)\Big)^{-(1+1/\xi)}\exp\Big(-(1+\xi z(x))^{-1/\xi}\Big) & \text{if } \; \xi \neq 0 \end{cases}$$

27.6 Percent point function/Sample

$$F_X^{-1}\,(u) = \begin{cases} \mu - \sigma \ln\left(-\ln(u)\right) & \text{if } \; \xi = 0 \\ \mu + \frac{\sigma}{\xi}\Big(\left(-\ln(u)\,\right)^{-\xi} - 1\Big) & \text{if } \; \xi \neq 0 \end{cases}$$

27.7 Parametric centered moments

$$\mu'_k = E[X^k] = \int_{-\infty}^{\infty} x^k f_X\,(x)\,dx = \Gamma(1 - k\xi)$$

27.8 Parametric mean

$$\text{Mean}(X) = \begin{cases} \mu + \sigma(\mu'_1 - 1)/\xi & \text{if } \xi \neq 0, \xi < 1 \\ \mu + \sigma\,\gamma & \text{if } \xi = 0 \end{cases}$$

27.9 Parametric variance

$$\text{Variance}(X) = \begin{cases} \sigma^2\,(\mu'_2 - \mu'^2_1)/\xi^2 & \text{if } \xi \neq 0, \xi < \frac{1}{2} \\ \sigma^2\,\frac{\pi^2}{6} & \text{if } \xi = 0 \end{cases}$$

27.10 Parametric skewness

$$\text{Skewness}(X) = \begin{cases} \text{sign}(\xi)\frac{\mu'_3 - 3\mu'_2\mu'_1 + 2\mu'^3_1}{(\mu'_2 - \mu'^2_1)^{1.5}} & \text{if } \xi \neq 0, \xi < \frac{1}{3} \\ \frac{12\sqrt{6}\,\zeta(3)}{\pi^3} & \text{if } \xi = 0 \end{cases}$$

27.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \begin{cases} 3 + \frac{\mu'_4 - 4\mu'_1\mu'_3 + 6\mu'^2_1\mu'_2 - 3\mu'^4_1}{(\mu'_2 - \mu'^2_1)^2} & \text{if } \xi \neq 0, \xi < \frac{1}{4} \\ 3 + \frac{12}{5} & \text{if } \xi = 0 \end{cases}$$

27.12 Parametric median

$$\text{Median}(X) = \begin{cases} \mu + \sigma \frac{(\ln 2)^{-\xi} - 1}{\xi} & \text{if } \xi \neq 0 \\ \mu - \sigma \ln \ln 2 & \text{if } \xi = 0 \end{cases}$$

27.13 Parametric mode

$$\text{Mode}(X) = \begin{cases} \mu + \sigma \frac{(1+\xi)^{-\xi} - 1}{\xi} & \text{if } \xi \neq 0 \\ \mu & \text{if } \xi = 0 \end{cases}$$

27.14 Additional information and definitions

- μ : Location parameter
- σ : Scale parameter
- $z(x) = (x - \mu)/\sigma$
- u : Uniform[0,1] random varible
- $\Gamma(x)$: Gamma function
- γ : Euler-Mascheroni constant = 0.5772156649

27.15 Spreadsheet documents

- Excel file from GitHub repository
- Google spreadsheet document

28 Generalized Gamma Distribution

28.1 Distribution definition

$$X \sim \text{GeneralizedGamma}\,(a, d, p)$$

28.2 Distribution domain

$$x \in (0, \infty)$$

28.3 Parameters domain and parameters constraints

$$a \in \mathbb{R}^+, d \in \mathbb{R}^+, p \in \mathbb{R}^+$$

28.4 Cumulative distribution function

$$F_X\,(x) = \mathrm{P}(d/p, (x/a)^p) = \frac{\gamma(d/p, (x/a)^p)}{\Gamma(d/p)}$$

28.5 Probability density function

$$f_X\,(x) = \frac{p/a^d}{\Gamma(d/p)} x^{d-1} e^{-(x/a)^p}$$

28.6 Percent point function/Sample

$$F_X^{-1}\,(u) = a\mathrm{P}^{-1}\left(\frac{d}{p}, u\right)^{\frac{1}{p}}$$

28.7 Parametric centered moments

$$\mu'_k = E[X^k] = \int_0^\infty x^k f_X\,(x)\,dx = a^k \frac{\Gamma(\frac{d+k}{p})}{\Gamma(\frac{d}{p})}$$

28.8 Parametric mean

$$\text{Mean}(X) = \mu'_1$$

28.9 Parametric variance

$$\text{Variance}(X) = \mu'_2 - \mu'^2_1$$

28.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\mu'_3 - 3\mu'_2\mu'_1 + 2\mu'^3_1}{(\mu'_2 - \mu'^2_1)^{1.5}}$$

28.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\mu'_4 - 4\mu'_1\mu'_3 + 6\mu'^2_1\mu'_2 - 3\mu'^4_1}{(\mu'_2 - \mu'^2_1)^2}$$

28.12 Parametric median

$$\text{Median}(X) = a\mathrm{P}^{-1}\left(\frac{d}{p}, \frac{1}{2}\right)^{\frac{1}{p}}$$

28.13 Parametric mode

$$\text{Mode}(X) = a\left(\frac{d-1}{p}\right)^{\frac{1}{p}} \quad \text{if } d > 1$$

28.14 Additional information and definitions

- a : Scale parameter
- u : Uniform[0,1] random varible
- $\mathrm{P}(a, x) = \frac{\gamma(a,x)}{\Gamma(a)}$: Regularized lower incomplete gamma function
- $\mathrm{P}^{-1}(a, u)$: Inverse of regularized lower incomplete gamma function
- $\gamma(a, x)$: Lower incomplete gamma function
- $\Gamma(x)$: Gamma function

28.15 Spreadsheet documents

- Excel file from GitHub repository
- Google spreadsheet document

29 Generalized Gamma 4P Distribution

29.1 Distribution definition

$$X \sim \text{GeneralizedGamma}_{4\text{P}}\left(a, d, p, \text{Loc}\right)$$

29.2 Distribution domain

$$x \in (\text{Loc}, \infty)$$

29.3 Parameters domain and parameters constraints

$$a \in \mathbb{R}^{+}, d \in \mathbb{R}^{+}, p \in \mathbb{R}^{+}, \text{Loc} \in \mathbb{R}$$

29.4 Cumulative distribution function

$$F_X\left(x\right) = \text{P}(d/p, ((x - \text{Loc})/a)^{p}) = \frac{\gamma(d/p, ((x - \text{Loc})/a)^{p})}{\Gamma(d/p)}$$

29.5 Probability density function

$$f_X\left(x\right) = \frac{p/a^{d}}{\Gamma(d/p)}(x - \text{Loc})^{d-1}e^{-((x-\text{Loc})/a)^{p}}$$

29.6 Percent point function/Sample

$$F_X^{-1}\left(u\right) = \text{Loc} + a\text{P}^{-1}\left(\frac{d}{p}, u\right)^{\frac{1}{p}}$$

29.7 Parametric centered moments

$$\tilde{\mu}'_k = E[\tilde{X}^k] = \int_0^{\infty} x^k f_{\tilde{X}}\left(x\right)dx = a^k\frac{\Gamma(\frac{d+k}{p})}{\Gamma(\frac{d}{p})}$$

29.8 Parametric mean

$$\text{Mean}(X) = \text{Loc} + \tilde{\mu}'_1$$

29.9 Parametric variance

$$\text{Variance}(X) = \tilde{\mu}'_2 - \tilde{\mu}'^2_1$$

29.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\tilde{\mu}'_3 - 3\tilde{\mu}'_2\tilde{\mu}'_1 + 2\tilde{\mu}'^3_1}{(\tilde{\mu}'_2 - \tilde{\mu}'^2_1)^{1.5}}$$

29.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\tilde{\mu}'_4 - 4\tilde{\mu}'_1\tilde{\mu}'_3 + 6\tilde{\mu}'^2_1\tilde{\mu}'_2 - 3\tilde{\mu}'^4_1}{(\tilde{\mu}'_2 - \tilde{\mu}'^2_1)^2}$$

29.12 Parametric median

$$\text{Median}(X) = \text{Loc} + a\text{P}^{-1}\left(\frac{d}{p}, \frac{1}{2}\right)^{\frac{1}{p}}$$

29.13 Parametric mode

$$\text{Mode}(X) = \text{Loc} + a\left(\frac{d-1}{p}\right)^{\frac{1}{p}} \quad \text{if } d > 1$$

29.14 Additional information and definitions

- $\tilde{X} \sim \text{GeneralizedGamma}\,(a, d, p)$
- Loc : Location parameter
- a : Scale parameter
- u : Uniform[0,1] random varible
- $\mathrm{P}\,(a, x) = \frac{\gamma(a,x)}{\Gamma(a)}$: Regularized lower incomplete gamma function
- $\mathrm{P}^{-1}\,(a, u)$: Inverse of regularized lower incomplete gamma function
- $\gamma\,(a, x)$: Lower incomplete gamma function
- $\Gamma\,(x)$: Gamma function

29.15 Spreadsheet documents

- Excel file from GitHub repository
- Google spreadsheet document

30 Generalized Logistic Distribution

30.1 Distribution definition

$$X \sim \mathrm{GeneralizedLogistic}\left(c, \mathrm{Loc}, \mathrm{Sc}\right)$$

30.2 Distribution domain

$$x \in (\mathrm{Loc}, \infty)$$

30.3 Parameters domain and parameters constraints

$$c \in \mathbb{R}^{+}, \mathrm{Loc} \in \mathbb{R}, \mathrm{Sc} \in \mathbb{R}^{+}$$

30.4 Cumulative distribution function

$$F_X\left(x\right) = \frac{1}{\left(1 + \exp\left(-z(x)\right)\right)^{c}}$$

30.5 Probability density function

$$f_X\left(x\right) = \frac{c \exp\left(-z(x)\right)}{\mathrm{Sc}\left(1 + \exp\left(-z(x)\right)\right)^{c+1}}$$

30.6 Percent point function/Sample

$$F_X^{-1}\left(u\right) = \mathrm{Loc} - \mathrm{Sc}\ln\left(u^{-1/c} - 1\right)$$

30.7 Parametric centered moments

$$\mu_k' = E[X^k] = \int_{-\infty}^{\infty} x^k f_X\left(x\right) dx$$

30.8 Parametric mean

$$\mathrm{Mean}(X) = \mathrm{Loc} + \mathrm{Sc} \cdot \tilde{\mu}_1' = \mathrm{Loc} + \mathrm{Sc}\left(\gamma + \psi_0\left(c\right)\right)$$

30.9 Parametric variance

$$\mathrm{Variance}(X) = \mathrm{Sc}^2 \cdot (\tilde{\mu}_2' - \tilde{\mu}_1'^2) = \mathrm{Sc}^2\left(\frac{\pi^2}{6} + \psi_1\left(c\right)\right)$$

30.10 Parametric skewness

$$\mathrm{Skewness}(X) = \frac{\mu_3' - 3\mu_2'\mu_1' + 2\mu_1'^3}{(\mu_2' - \mu_1'^2)^{1.5}} = \frac{\psi_2\left(c\right) + 2\zeta\left(3\right)}{\left(\frac{\pi^2}{6} + \psi_1\left(c\right)\right)^{3/2}}$$

30.11 Parametric kurtosis

$$\mathrm{Kurtosis}(X) = \frac{\mu_4' - 4\mu_1'\mu_3' + 6\mu_1'^2\mu_2' - 3\mu_1'^4}{(\mu_2' - \mu_1'^2)^2} = \frac{\left(\frac{\pi^4}{15} + \psi_3\left(c\right)\right)}{\left(\frac{\pi^2}{6} + \psi_1\left(c\right)\right)^2}$$

30.12 Parametric median

$$\mathrm{Median}(X) = \mathrm{Loc} - \mathrm{Sc}\ln\left(2^{1/c} - 1\right)$$

30.13 Parametric mode

$$\mathrm{Mode}(X) = \mathrm{Loc} + \mathrm{Sc}\ln\left(c\right)$$

30.14 Additional information and definitions

- Loc : Location parameter
- Sc : Scale parameter
- $z(x) = (x - \text{Loc})/\text{Sc}$
- u : Uniform[0,1] random varible
- γ : Euler-Mascheroni constant = 0.5772156649
- $\psi_0(x)$: Digamma function
- $\psi_n(x)$: Polygamma function of order $n \in \mathbb{N}$

30.15 Spreadsheet documents

- Excel file from GitHub repository
- Google spreadsheet document

31 Generalized Normal Distribution

31.1 Distribution definition

$$X \sim \text{GeneralizedNormal}\left(\beta, \mu, \alpha\right)$$

31.2 Distribution domain

$$x \in (-\infty, +\infty)$$

31.3 Parameters domain and parameters constraints

$$\beta \in \mathbb{R}^+, \mu \in \mathbb{R}, \alpha \in \mathbb{R}^+$$

31.4 Cumulative distribution function

$$F_X\left(x\right) = \frac{1}{2} + \frac{\text{sign}(x-\mu)}{2\Gamma(1/\beta)}\gamma\left(1/\beta, \left|\frac{x-\mu}{\alpha}\right|^{\beta}\right) = \frac{1}{2} + \frac{\text{sign}(x-\mu)}{2}\mathrm{P}\left(1/\beta, \left|\frac{x-\mu}{\alpha}\right|^{\beta}\right)$$

31.5 Probability density function

$$f_X\left(x\right) = \frac{\beta}{2\alpha\Gamma(1/\beta)}\exp\left(-\left(\frac{|x-\mu|}{\alpha}\right)^{\beta}\right)$$

31.6 Percent point function/Sample

$$F_X^{-1}\left(u\right) = \text{sign}(u-\frac{1}{2})\left[\alpha^{\beta}\mathrm{P}^{-1}\left(\frac{1}{\beta}, 2|u-\frac{1}{2}|\right)\right]^{1/\beta} + \mu$$

31.7 Parametric centered moments

$$\mu_k' = E[X^k] = \int_{-\infty}^{\infty} x^k f_X\left(x\right)dx = \begin{cases} 0 & \text{if } k \text{ is odd} \\ \alpha^k\Gamma\left(\frac{k+1}{\beta}\right)\Big/\Gamma\left(\frac{1}{\beta}\right) & \text{if } k \text{ is even} \end{cases}$$

31.8 Parametric mean

$$\text{Mean}(X) = \mu + \alpha\mu_1' = \mu$$

31.9 Parametric variance

$$\text{Variance}(X) = \alpha^2(\mu_2' - \mu_1'^2) = \frac{\alpha^2\Gamma(3/\beta)}{\Gamma(1/\beta)}$$

31.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\mu_3' - 3\mu_2'\mu_1' + 2\mu_1'^3}{(\mu_2' - \mu_1'^2)^{1.5}} = 0$$

31.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\mu_4' - 4\mu_1'\mu_3' + 6\mu_1'^2\mu_2' - 3\mu_1'^4}{(\mu_2' - \mu_1'^2)^2} = \frac{\Gamma(5/\beta)\Gamma(1/\beta)}{\Gamma(3/\beta)^2}$$

31.12 Parametric median

$$\text{Median}(X) = \mu$$

31.13 Parametric mode

$$\text{Mode}(X) = \mu$$

31.14 Additional information and definitions

- μ : Location parameter
- α : Scale parameter
- u : Uniform[0,1] random varible
- $\mathrm{P}^{-1}(a, u)$: Inverse of regularized lower incomplete gamma function
- $\gamma(a, x)$: Lower incomplete gamma function
- $\Gamma(x)$: Gamma function

31.15 Spreadsheet documents

- Excel file from GitHub repository
- Google spreadsheet document

32 Generalized Pareto Distribution

32.1 Distribution definition

$$X \sim \text{GeneralizedPareto}\,(c, \mu, \sigma)$$

32.2 Distribution domain

$$\text{if } c \geqslant 0:\; x \in (\mu, \infty)\,, \quad \text{if } c < 0:\; x \in \left(-\infty, \mu - \frac{\sigma}{c}\right)$$

32.3 Parameters domain and parameters constraints

$$c \in \mathbb{R}, \mu \in \mathbb{R}, \sigma \in \mathbb{R}^{+}$$

32.4 Cumulative distribution function

$$F_X\,(x) = 1 - (1 + cz(x))^{-1/c}$$

32.5 Probability density function

$$f_X\,(x) = \frac{1}{\sigma}(1 + cz(x))^{-(1/c+1)}$$

32.6 Percent point function/Sample

$$F_X^{-1}\,(u) = \mu + \frac{\sigma(u^{-c} - 1)}{c}$$

32.7 Parametric centered moments

$$\mu'_k = E[X^k] = \int_{-\infty}^{\infty} x^k f_X\,(x)\,dx = \frac{(-1)^k}{c^k} \sum_{i=0}^{k} \binom{k}{i} \frac{(-1)^i}{1 - ci} \quad \text{if } < \frac{1}{k}$$

32.8 Parametric mean

$$\text{Mean}(X) = \mu + \sigma\mu'_1 = \mu + \frac{\sigma}{1 - c} \quad \text{if } c < 1$$

32.9 Parametric variance

$$\text{Variance}(X) = \sigma^2(\mu'_2 - \mu'^2_1) = \frac{\sigma^2}{(1-c)^2(1-2c)} \quad \text{if } c < 1/2$$

32.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\mu'_3 - 3\mu'_2\mu'_1 + 2\mu'^3_1}{(\mu'_2 - \mu'^2_1)^{1.5}} = \frac{2(1+c)\sqrt{1-2c}}{(1-3c)} \quad \text{if } c < 1/3$$

32.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\mu'_4 - 4\mu'_1\mu'_3 + 6\mu'^2_1\mu'_2 - 3\mu'^4_1}{(\mu'_2 - \mu'^2_1)^2} = \frac{3(1-2c)(2c^2 + c + 3)}{(1-3c)(1-4c)} \quad \text{if } c < 1/4$$

32.12 Parametric median

$$\text{Median}(X) = \mu$$

32.13 Parametric mode

$$\text{Mode}(X) = \mu + \frac{\sigma(2^c - 1)}{c}$$

32.14 Additional information and definitions

- μ : Location parameter
- σ : Scale parameter
- $z(x) = (x - \mu)/\sigma$
- u : Uniform[0,1] random varible

32.15 Spreadsheet documents

- Excel file from GitHub repository
- Google spreadsheet document

33 Gibrat Distribution

33.1 Distribution definition

$$X \sim \mathrm{Gibrat}\left(\mathrm{Loc}, \mathrm{Sc}\right)$$

33.2 Distribution domain

$$x \in \left(\mathrm{Loc}, \infty\right)$$

33.3 Parameters domain and parameters constraints

$$\mathrm{Loc} \in \mathbb{R}, \mathrm{Sc} \in \mathbb{R}^{+}$$

33.4 Cumulative distribution function

$$F_X\left(x\right) = \Phi\left(\ln x\right) = \frac{1}{2}\left(1 + \mathrm{erf}\left(\frac{\ln z(x)}{\sqrt{2}}\right)\right)$$

33.5 Probability density function

$$f_X\left(x\right) = \frac{1}{\mathrm{Sc}}\frac{1}{x\sqrt{2\pi}}\exp\left(-\frac{1}{2}\left(\ln z(x)\right)^2\right)$$

33.6 Percent point function/Sample

$$F_X^{-1}\left(u\right) = \mathrm{Loc} + \mathrm{Sc} \times \exp\left(\Phi^{-1}\left(u\right)\right)$$

33.7 Parametric centered moments

$$\mu'_k = E[X^k] = \int_{\mathrm{Loc}}^{\infty} x^k f_X\left(x\right)dx = \exp\left(\frac{k^2}{2}\right)$$

33.8 Parametric mean

$$\mathrm{Mean}(X) = \mathrm{Loc} + \mathrm{Sc}\cdot\tilde{\mu}'_1 = \mathrm{Loc} + \mathrm{Sc}\cdot\sqrt{e}$$

33.9 Parametric variance

$$\mathrm{Variance}(X) = \mathrm{Sc}^2\cdot(\tilde{\mu}'_2 - \tilde{\mu}'^2_1) = \mathrm{Sc}^2\left[e^2 - e\right]$$

33.10 Parametric skewness

$$\mathrm{Skewness}(X) = \frac{\mu'_3 - 3\mu'_2\mu'_1 + 2\mu'^3_1}{(\mu'_2 - \mu'^2_1)^{1.5}} = \sqrt{e-1}\,(2+e)$$

33.11 Parametric kurtosis

$$\mathrm{Kurtosis}(X) = \frac{\mu'_4 - 4\mu'_1\mu'_3 + 6\mu'^2_1\mu'_2 - 3\mu'^4_1}{(\mu'_2 - \mu'^2_1)^2} = e^4 + 2e^3 + 3e^2 - 3$$

33.12 Parametric median

$$\mathrm{Median}(X) = \mathrm{Loc} + \mathrm{Sc} \times \exp\left(\Phi^{-1}\left(1/2\right)\right)$$

33.13 Parametric mode

$$\mathrm{Mode}(X) = \mathrm{Loc} + \frac{\mathrm{Sc}}{e}$$

33.14 Additional information and definitions

- Loc : Location parameter
- Sc : Scale parameter
- $z(x) = (x - \text{Loc}) / \text{Sc}$
- u : Uniform[0,1] random varible
- $\Phi(x)$: CDF normal standard distribution
- $\Phi^{-1}(x)$: PPF normal standard distribution
- $\text{erf}(x)$: Error function

33.15 Spreadsheet documents

- Excel file from GitHub repository
- Google spreadsheet document

34 Gumbel Left Distribution

34.1 Distribution definition

$$X \sim \text{GumbelLeft}\left(\mu, \sigma\right)$$

34.2 Distribution domain

$$x \in (-\infty, \infty)$$

34.3 Parameters domain and parameters constraints

$$\mu \in \mathbb{R}, \sigma \in \mathbb{R}^{+}$$

34.4 Cumulative distribution function

$$F_X\left(x\right) = 1 - \exp\left(-e^{z(x)}\right)$$

34.5 Probability density function

$$f_X\left(x\right) = \frac{1}{\sigma}\exp\left(z(x) - e^{z(x)}\right)$$

34.6 Percent point function/Sample

$$F_X^{-1}\left(u\right) = \mu + \sigma \ln\left(-\ln\left(1-u\right)\right)$$

34.7 Parametric centered moments

$$\tilde{\mu}'_k = E[\tilde{X}^k] = \int_{-\infty}^{\infty} x^k f_{\tilde{X}}\left(x\right) dx$$

34.8 Parametric mean

$$\text{Mean}(X) = \mu + \sigma\tilde{\mu}'_1 = \mu - \gamma\sigma$$

34.9 Parametric variance

$$\text{Variance}(X) = \sigma^2(\tilde{\mu}'_2 - \tilde{\mu}'^2_1) = \sigma^2\frac{\pi^2}{6}$$

34.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\tilde{\mu}'_3 - 3\tilde{\mu}'_2\tilde{\mu}'_1 + 2\tilde{\mu}'^3_1}{(\tilde{\mu}'_2 - \tilde{\mu}'^2_1)^{1.5}} = -\frac{12\sqrt{6}\zeta(3)}{\pi^3}$$

34.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\tilde{\mu}'_4 - 4\tilde{\mu}'_1\tilde{\mu}'_3 + 6\tilde{\mu}'^2_1\tilde{\mu}'_2 - 3\tilde{\mu}'^4_1}{(\tilde{\mu}'_2 - \tilde{\mu}'^2_1)^2} = 3 + \frac{12}{5}$$

34.12 Parametric median

$$\text{Median}(X) = \mu + \sigma \ln\left(-\ln\left(\frac{1}{2}\right)\right)$$

34.13 Parametric mode

$$\text{Mode}(X) = \mu$$

34.14 Additional information and definitions

- $\tilde{X} \sim \text{GumbelLeft}\left(0, 1\right)$
- μ : Location parameter
- σ : Scale parameter
- $z\left(x\right) = \left(x - \mu\right)/\sigma$
- u : Uniform[0,1] random varible
- γ : Euler-Mascheroni constant = 0.5772156649
- $\zeta(3)$: Apéry's constant = 1.2020569031

34.15 Spreadsheet documents

- Excel file from GitHub repository
- Google spreadsheet document

35 Gumbel Right Distribution

35.1 Distribution definition

$$X \sim \text{GumbelRight}\left(\mu, \sigma\right)$$

35.2 Distribution domain

$$x \in (-\infty, \infty)$$

35.3 Parameters domain and parameters constraints

$$\mu \in \mathbb{R}, \sigma \in \mathbb{R}^{+}$$

35.4 Cumulative distribution function

$$F_X\left(x\right) = \exp\left(-e^{-z(x)}\right)$$

35.5 Probability density function

$$f_X\left(x\right) = \frac{1}{\sigma}\exp\left(-\left(z(x) + e^{-z(x)}\right)\right)$$

35.6 Percent point function/Sample

$$F_X^{-1}\left(u\right) = \tilde{\mu} - \sigma\ln\left(-\ln\left(u\right)\right)$$

35.7 Parametric centered moments

$$\tilde{\mu}_k' = E[\tilde{X}^k] = \int_{-\infty}^{\infty} x^k f_{\tilde{X}}\left(x\right)dx$$

35.8 Parametric mean

$$\text{Mean}(X) = \mu + \sigma\tilde{\mu}_1' = \mu + \gamma\sigma$$

35.9 Parametric variance

$$\text{Variance}(X) = \sigma^2(\tilde{\mu}_2' - \tilde{\mu}_1'^2) = \sigma^2\frac{\pi^2}{6}$$

35.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\tilde{\mu}_3' - 3\tilde{\mu}_2'\tilde{\mu}_1' + 2\tilde{\mu}_1'^3}{(\tilde{\mu}_2' - \tilde{\mu}_1'^2)^{1.5}} = \frac{12\sqrt{6}\zeta(3)}{\pi^3}$$

35.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\tilde{\mu}_4' - 4\tilde{\mu}_1'\tilde{\mu}_3' + 6\tilde{\mu}_1'^2\tilde{\mu}_2' - 3\tilde{\mu}_1'^4}{(\tilde{\mu}_2' - \tilde{\mu}_1'^2)^2} = 3 + \frac{12}{5}$$

35.12 Parametric median

$$\text{Median}(X) = \mu - \sigma\ln\left(-\ln\left(\frac{1}{2}\right)\right)$$

35.13 Parametric mode

$$\text{Mode}(X) = \mu$$

35.14 Additional information and definitions

- $\tilde{X} \sim \text{GumbelRight}\,(0, 1)$
- μ : Location parameter
- σ : Scale parameter
- $z\,(x) = (x - \mu)\,/\sigma$
- u : Uniform[0,1] random varible
- γ : Euler-Mascheroni constant = 0.5772156649
- $\zeta(3)$: Apéry's constant = 1.2020569031

35.15 Spreadsheet documents

- Excel file from GitHub repository
- Google spreadsheet document

36 Half Normal Distribution

36.1 Distribution definition

$$X \sim \mathrm{HalfNormal}\left(\mu, \sigma\right)$$

36.2 Distribution domain

$$x \in (\mu, \infty)$$

36.3 Parameters domain and parameters constraints

$$\mu \in \mathbb{R}, \sigma \in \mathbb{R}^{+}$$

36.4 Cumulative distribution function

$$F_X\left(x\right) = 2\Phi\left(z(x)\right) - 1 = \mathrm{erf}\left(\frac{z(x)}{\sqrt{2}}\right)$$

36.5 Probability density function

$$f_X\left(x\right) = \frac{\sqrt{2}}{\sigma\sqrt{\pi}}\exp\left(-\frac{z(x)^2}{2}\right)$$

36.6 Percent point function/Sample

$$F_X^{-1}\left(u\right) = \mu + \sigma\Phi^{-1}\left(\frac{1+u}{2}\right) = \tilde{\mu} + \sigma\sqrt{2}\,\mathrm{erf}^{-1}(u)$$

36.7 Parametric centered moments

$$\tilde{\mu}'_k = E[\tilde{X}^k] = \int_0^{\infty} x^k f_{\tilde{X}}\left(x\right) dx = \frac{2^{n/2}\Gamma(\frac{n+1}{2})}{\sqrt{\pi}}$$

36.8 Parametric mean

$$\mathrm{Mean}(X) = \tilde{\mu} + \sigma\tilde{\mu}'_1 = \tilde{\mu} + \sigma\sqrt{\frac{2}{\pi}}$$

36.9 Parametric variance

$$\mathrm{Variance}(X) = \sigma^2(\tilde{\mu}'_2 - \tilde{\mu}'^2_1) = \sigma^2\left(1 - \frac{2}{\pi}\right)$$

36.10 Parametric skewness

$$\mathrm{Skewness}(X) = \frac{\tilde{\mu}'_3 - 3\tilde{\mu}'_2\tilde{\mu}'_1 + 2\tilde{\mu}'^3_1}{(\tilde{\mu}'_2 - \tilde{\mu}'^2_1)^{1.5}} = \frac{\sqrt{2}(4-\pi)}{(\pi-2)^{3/2}} = 0.9952717$$

36.11 Parametric kurtosis

$$\mathrm{Kurtosis}(X) = \frac{\tilde{\mu}'_4 - 4\tilde{\mu}'_1\tilde{\mu}'_3 + 6\tilde{\mu}'^2_1\tilde{\mu}'_2 - 3\tilde{\mu}'^4_1}{(\tilde{\mu}'_2 - \tilde{\mu}'^2_1)^2} = 3 + \frac{8(\pi-3)}{(\pi-2)^2} = 3.869177$$

36.12 Parametric median

$$\mathrm{Median}(X) = \mu + \sigma\sqrt{2}\,\mathrm{erf}^{-1}(1/2)$$

36.13 Parametric mode

$$\mathrm{Mode}(X) = \mu$$

36.14 Additional information and definitions

- $\tilde{X} \sim \text{HalfNormal}\,(0, 1)$
- μ : Location parameter
- σ : Scale parameter
- $z\,(x) = (x - \mu)\,/\sigma$
- u : Uniform[0,1] random varible
- $\Phi\,(x)$: CDF normal standard distribution
- $\Phi^{-1}\,(x)$: PPF normal standard distribution
- $\text{erf}(x)$: Error function
- $\Gamma\,(x)$: Gamma function

36.15 Spreadsheet documents

- Excel file from GitHub repository
- Google spreadsheet document

37 Hyperbolic Secant Distribution

37.1 Distribution definition

$$X \sim \text{HyperbolicSecant}\,(\mu, \sigma)$$

37.2 Distribution domain

$$x \in (-\infty, \infty)$$

37.3 Parameters domain and parameters constraints

$$\mu \in \mathbb{R}, \sigma \in \mathbb{R}^{+}$$

37.4 Cumulative distribution function

$$F_X\,(x) = \frac{2}{\pi}\arctan\Big[\exp\Big(\frac{\pi}{2}\,z(x)\Big)\Big]$$

37.5 Probability density function

$$f_X\,(x) = \frac{1}{2\sigma}\,\text{sech}\Big(\frac{\pi}{2}\,z(x)\Big)$$

37.6 Percent point function/Sample

$$F_X^{-1}\,(u) = \mu + \sigma\frac{2}{\pi}\,\ln\Big[\tan\left(\frac{\pi}{2}\,u\right)\Big]$$

37.7 Parametric centered moments

$$\tilde{\mu}'_k = E[\tilde{X}^k] = \int_{-\infty}^{\infty} x^k f_{\tilde{X}}\,(x)\,dx = \frac{1+(-1)^k}{2\pi 2^{2k}}k!\left[\zeta\left(k+1,\frac{1}{4}\right) - \zeta\left(k+1,\frac{3}{4}\right)\right]$$

37.8 Parametric mean

$$\text{Mean}(X) = \mu + \sigma\tilde{\mu}'_1 = \mu$$

37.9 Parametric variance

$$\text{Variance}(X) = \sigma^2(\tilde{\mu}'_2 - \tilde{\mu}'^2_1) = \sigma^2$$

37.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\tilde{\mu}'_3 - 3\tilde{\mu}'_2\tilde{\mu}'_1 + 2\tilde{\mu}'^3_1}{(\tilde{\mu}'_2 - \tilde{\mu}'^2_1)^{1.5}} = 0$$

37.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\tilde{\mu}'_4 - 4\tilde{\mu}'_1\tilde{\mu}'_3 + 6\tilde{\mu}'^2_1\tilde{\mu}'_2 - 3\tilde{\mu}'^4_1}{(\tilde{\mu}'_2 - \tilde{\mu}'^2_1)^2} = 3$$

37.12 Parametric median

$$\text{Median}(X) = \mu$$

37.13 Parametric mode

$$\text{Mode}(X) = \mu$$

37.14 Additional information and definitions

- $\tilde{X} \sim \text{HyperbolicSecant}\,(0, 1)$
- μ : Location parameter
- σ : Scale parameter
- $z\,(x) = (x - \mu)\,/\sigma$
- u : Uniform[0,1] random varible
- $\zeta(a, s)$: Hurwitz zeta function

37.15 Spreadsheet documents

- Excel file from GitHub repository
- Google spreadsheet document

38 Inverse Gamma Distribution

38.1 Distribution definition

$$X \sim \text{InverseGamma}\,(\alpha, \beta)$$

38.2 Distribution domain

$$x \in (0, \infty)$$

38.3 Parameters domain and parameters constraints

$$\alpha \in \mathbb{R}^{+}, \beta \in \mathbb{R}^{+}$$

38.4 Cumulative distribution function

$$F_X\,(x) = 1 - \frac{\gamma(\alpha, \beta/x)}{\Gamma(\alpha)} = 1 - \mathrm{P}\left(\alpha, \frac{\beta}{x}\right)$$

38.5 Probability density function

$$f_X\,(x) = \frac{\beta^{\alpha}}{\Gamma(\alpha)} x^{-\alpha-1} \exp\left(-\frac{\beta}{x}\right)$$

38.6 Percent point function/Sample

$$F_X^{-1}\,(u) = \frac{\beta}{\mathrm{P}^{-1}\,(\alpha, 1-u)}$$

38.7 Parametric centered moments

$$\tilde{\mu}'_k = E[\tilde{X}^k] = \int_0^{\infty} x^k f_{\tilde{X}}\,(x)\,dx = \frac{\Gamma(\alpha-k)}{\Gamma(\alpha)} = \frac{1}{(\alpha-1)\cdots(\alpha-k)} \quad \text{if } \alpha > k$$

38.8 Parametric mean

$$\text{Mean}(X) = \beta\tilde{\mu}'_1$$

38.9 Parametric variance

$$\text{Variance}(X) = \beta^2(\tilde{\mu}'_2 - \tilde{\mu}'^2_1)$$

38.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\tilde{\mu}'_3 - 3\tilde{\mu}'_2\tilde{\mu}'_1 + 2\tilde{\mu}'^3_1}{(\tilde{\mu}'_2 - \tilde{\mu}'^2_1)^{1.5}}$$

38.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\tilde{\mu}'_4 - 4\tilde{\mu}'_1\tilde{\mu}'_3 + 6\tilde{\mu}'^2_1\tilde{\mu}'_2 - 3\tilde{\mu}'^4_1}{(\tilde{\mu}'_2 - \tilde{\mu}'^2_1)^2}$$

38.12 Parametric median

$$\text{Median}(X) = \frac{\beta}{\mathrm{P}^{-1}\left(\alpha, \frac{1}{2}\right)}$$

38.13 Parametric mode

$$\text{Mode}(X) = \frac{\beta}{\alpha+1}$$

38.14 Additional information and definitions

- $\tilde{X} \sim \text{InverseGamma}\left(\alpha, 1\right)$
- β : Scale parameter
- u : Uniform[0,1] random varible
- $\text{P}\left(a, x\right) = \frac{\gamma(a,x)}{\Gamma(a)}$: Regularized lower incomplete gamma function
- $\text{P}^{-1}\left(a, u\right)$: Inverse of regularized lower incomplete gamma function
- $\gamma\left(a, x\right)$: Lower incomplete gamma function
- $\Gamma\left(x\right)$: Gamma function

38.15 Spreadsheet documents

- Excel file from GitHub repository
- Google spreadsheet document

39 Inverse Gamma 3P Distribution

39.1 Distribution definition

$$X \sim \text{InverseGamma}_{3\text{P}}\left(\alpha, \text{Loc}, \beta\right)$$

39.2 Distribution domain

$$x \in (\text{Loc}, \infty)$$

39.3 Parameters domain and parameters constraints

$$\alpha \in \mathbb{R}^{+}, \text{Loc} \in \mathbb{R}, \beta \in \mathbb{R}^{+}$$

39.4 Cumulative distribution function

$$F_X\left(x\right) = 1 - \frac{\gamma(\alpha, \beta/(x-\text{Loc}))}{\Gamma(\alpha)} = 1 - \text{P}\left(\alpha, \frac{\beta}{x-\text{Loc}}\right)$$

39.5 Probability density function

$$f_X\left(x\right) = \frac{\beta^{\alpha}}{\Gamma(\alpha)}(x-\text{Loc})^{-\alpha-1}\exp\left(-\frac{\beta}{x-\text{Loc}}\right)$$

39.6 Percent point function/Sample

$$F_X^{-1}\left(u\right) = \text{Loc} + \frac{\beta}{\text{P}^{-1}\left(\alpha, 1-u\right)}$$

39.7 Parametric centered moments

$$\tilde{\mu}'_k = E[\tilde{X}^k] = \int_0^{\infty} x^k f_{\tilde{X}}\left(x\right)dx = \frac{\Gamma(\alpha-k)}{\Gamma(\alpha)} = \frac{1}{(\alpha-1)\cdots(\alpha-k)} \quad \text{if } \alpha > k$$

39.8 Parametric mean

$$\text{Mean}(X) = \text{Loc} + \beta\mu'_1$$

39.9 Parametric variance

$$\text{Variance}(X) = \beta^2(\mu'_2 - \mu'^2_1)$$

39.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\mu'_3 - 3\mu'_2\mu'_1 + 2\mu'^3_1}{(\mu'_2 - \mu'^2_1)^{1.5}}$$

39.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\mu'_4 - 4\mu'_1\mu'_3 + 6\mu'^2_1\mu'_2 - 3\mu'^4_1}{(\mu'_2 - \mu'^2_1)^2}$$

39.12 Parametric median

$$\text{Median}(X) = \text{Loc} + \frac{\beta}{\text{P}^{-1}\left(\alpha, \frac{1}{2}\right)}$$

39.13 Parametric mode

$$\text{Mode}(X) = \text{Loc} + \frac{\beta}{\alpha+1}$$

39.14 Additional information and definitions

- $\tilde{X} \sim \text{InverseGamma}_{\text{3P}}(\alpha, 0, 1)$
- Loc : Location parameter
- β : Scale parameter
- u : Uniform[0,1] random varible
- $\mathrm{P}(a, x) = \frac{\gamma(a,x)}{\Gamma(a)}$: Regularized lower incomplete gamma function
- $\mathrm{P}^{-1}(a, u)$: Inverse of regularized lower incomplete gamma function
- $\gamma(a, x)$: Lower incomplete gamma function
- $\Gamma(x)$: Gamma function

39.15 Spreadsheet documents

- Excel file from GitHub repository
- Google spreadsheet document

40 Inverse Gaussian Distribution

40.1 Distribution definition

$$X \sim \text{InverseGaussian}\left(\mu, \lambda\right)$$

40.2 Distribution domain

$$x \in (0, \infty)$$

40.3 Parameters domain and parameters constraints

$$\mu \in \mathbb{R}^{+}, \lambda \in \mathbb{R}^{+}$$

40.4 Cumulative distribution function

$$F_X\left(x\right) = \Phi\left(\sqrt{\frac{\lambda}{x}}\left(\frac{x}{\mu} - 1\right)\right) + \exp\left(\frac{2\lambda}{\mu}\right)\Phi\left(-\sqrt{\frac{\lambda}{x}}\left(\frac{x}{\mu} + 1\right)\right)$$

40.5 Probability density function

$$f_X\left(x\right) = \sqrt{\frac{\lambda}{2\pi x^3}}\exp\left[-\frac{\lambda(x-\mu)^2}{2\mu^2 x}\right]$$

40.6 Percent point function/Sample

$$\text{Sample}_X = \begin{cases} x_0 & \text{if } u_2 \leqslant \frac{\mu}{\mu + x_0} \\ \frac{\mu^2}{x_0} & \text{if } u_2 \geqslant \frac{\mu}{\mu + x_0} \end{cases}$$

40.7 Parametric centered moments

$$\mu'_k = E[X^k] = \int_0^{\infty} x^k f_X\left(x\right) dx$$

40.8 Parametric mean

$$\text{Mean}(X) = \mu'_1 = \mu$$

40.9 Parametric variance

$$\text{Variance}(X) = \mu'_2 - \mu'^2_1 = \frac{\mu^3}{\lambda}$$

40.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\mu'_3 - 3\mu'_2\mu'_1 + 2\mu'^3_1}{(\mu'_2 - \mu'^2_1)^{1.5}} = 3\left(\frac{\mu}{\lambda}\right)^{1/2}$$

40.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\mu'_4 - 4\mu'_1\mu'_3 + 6\mu'^2_1\mu'_2 - 3\mu'^4_1}{(\mu'_2 - \mu'^2_1)^2} = 3 + \frac{15\mu}{\lambda}$$

40.12 Parametric median

$$\text{Median}(X) = F_X^{-1}\left(\frac{1}{2}\right)$$

40.13 Parametric mode

$$\text{Mode}(X) = \mu \left[\left(1 + \frac{9\mu^2}{4\lambda^2}\right)^{\frac{1}{2}} - \frac{3\mu}{2\lambda} \right]$$

40.14 Additional information and definitions

- Computing an analytic expression for the inverse of the cumulative distribution function is not feasible. Nonetheless, it is possible to generate a random sample from the distribution.
- $\Phi\left(x\right)$: CDF normal standard distribution
- $\Phi^{-1}\left(x\right)$: PPF normal standard distribution
- $x_0 = \mu + \frac{\mu^2[\Phi^{-1}(u_1)]^2}{2\lambda} - \frac{\mu}{2\lambda}\sqrt{4\mu\lambda[\Phi^{-1}\left(u_1\right)]^2 + \mu^2([\Phi^{-1}\left(u_1\right)]^2)^2}$
- u_1 : Uniform[0,1] random varible
- u_2 : Uniform[0,1] random varible

40.15 Spreadsheet documents

- Excel file from GitHub repository
- Google spreadsheet document

41 Inverse Gaussian 3P Distribution

41.1 Distribution definition

$$X \sim \text{InverseGaussian}_{3\text{P}}\,(\mu, \lambda, \text{Loc})$$

41.2 Distribution domain

$$x \in (0, \infty)$$

41.3 Parameters domain and parameters constraints

$$\mu \in \mathbb{R}^{+}, \lambda \in \mathbb{R}^{+}, \text{Loc} \in \mathbb{R}$$

41.4 Cumulative distribution function

$$F_X\,(x) = \Phi\left(\sqrt{\frac{\lambda}{x-\text{Loc}}}\left(\frac{x-\text{Loc}}{\mu}-1\right)\right) + \exp\left(\frac{2\lambda}{\mu}\right)\Phi\left(-\sqrt{\frac{\lambda}{x-\text{Loc}}}\left(\frac{x-\text{Loc}}{\mu}+1\right)\right)$$

41.5 Probability density function

$$f_X\,(x) = \sqrt{\frac{\lambda}{2\pi(x-\text{Loc})^3}}\exp\left[-\frac{\lambda(x-\mu-\text{Loc})^2}{2\mu^2(x-\text{Loc})}\right]$$

41.6 Percent point function/Sample

$$\text{Sample}_X = \begin{cases} \text{Loc} + x_0 & \text{if} \quad u_2 \leqslant \frac{\mu}{\mu+x_0} \\ \text{Loc} + \frac{\mu^2}{x_0} & \text{if} \quad u_2 \geqslant \frac{\mu}{\mu+x_0} \end{cases}$$

41.7 Parametric centered moments

$$\mu'_k = E[X^k] = \int_{\text{Loc}}^{\infty} x^k f_X\,(x)\,dx$$

41.8 Parametric mean

$$\text{Mean}(X) = \mu'_1 = \text{Loc} + \mu$$

41.9 Parametric variance

$$\text{Variance}(X) = \mu'_2 - \mu'^2_1 = \frac{\mu^3}{\lambda}$$

41.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\mu'_3 - 3\mu'_2\mu'_1 + 2\mu'^3_1}{(\mu'_2 - \mu'^2_1)^{1.5}} = 3\left(\frac{\mu}{\lambda}\right)^{1/2}$$

41.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\mu'_4 - 4\mu'_1\mu'_3 + 6\mu'^2_1\mu'_2 - 3\mu'^4_1}{(\mu'_2 - \mu'^2_1)^2} = 3 + \frac{15\mu}{\lambda}$$

41.12 Parametric median

$$\text{Median}(X) = F_X^{-1}\left(\frac{1}{2}\right)$$

41.13 Parametric mode

$$\text{Mode}(X) = \text{Loc} + \mu\left[\left(1 + \frac{9\mu^2}{4\lambda^2}\right)^{\frac{1}{2}} - \frac{3\mu}{2\lambda}\right]$$

41.14 Additional information and definitions

- Computing an analytic expression for the inverse of the cumulative distribution function is not feasible. Nonetheless, it is possible to generate a random sample from the distribution.
- Loc : Location parameter
- $\Phi(x)$: CDF normal standard distribution
- $\Phi^{-1}(x)$: PPF normal standard distribution
- $x_0 = \mu + \frac{\mu^2[\Phi^{-1}(u_1)]^2}{2\lambda} - \frac{\mu}{2\lambda}\sqrt{4\mu\lambda[\Phi^{-1}(u_1)]^2 + \mu^2([\Phi^{-1}(u_1)]^2)^2}$
- u_1 : Uniform[0,1] random varible
- u_2 : Uniform[0,1] random varible

41.15 Spreadsheet documents

- Excel file from GitHub repository
- Google spreadsheet document

42 Johnson SB Distribution

42.1 Distribution definition

$$X \sim \text{JohnsonSB}\left(\xi, \lambda, \gamma, \delta\right)$$

42.2 Distribution domain

$$x \in (\xi, \xi + \lambda)$$

42.3 Parameters domain and parameters constraints

$$\xi \in \mathbb{R}, \lambda \in \mathbb{R}^{+}, \gamma \in \mathbb{R}, \delta \in \mathbb{R}^{+}$$

42.4 Cumulative distribution function

$$F_X\left(x\right) = \Phi\left(\gamma + \delta \ln \frac{z(x)}{1 - z(x)}\right)$$

42.5 Probability density function

$$f_X\left(x\right) = \frac{\delta}{\lambda\sqrt{2\pi}z(1 - z(x))} \exp\left[-\frac{1}{2}\left(\gamma + \delta \ln \frac{z(x)}{1 - z(x)}\right)^2\right]$$

42.6 Percent point function/Sample

$$F_X^{-1}\left(u\right) = \frac{\lambda \exp\left(\frac{\Phi^{-1}(u) - \gamma}{\delta}\right)}{1 + \exp\left(\frac{\Phi^{-1}(u) - \gamma}{\delta}\right)} + \xi$$

42.7 Parametric centered moments

$$\mu_k' = E[X^k] = \int_{\xi}^{\xi+\lambda} x^k f_X\left(x\right) dx$$

42.8 Parametric mean

$$\text{Mean}(X) = \mu_1'$$

42.9 Parametric variance

$$\text{Variance}(X) = \mu_2' - \mu_1'^2$$

42.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\mu_3' - 3\mu_2'\mu_1' + 2\mu_1'^3}{(\mu_2' - \mu_1'^2)^{1.5}}$$

42.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\mu_4' - 4\mu_1'\mu_3' + 6\mu_1'^2\mu_2' - 3\mu_1'^4}{(\mu_2' - \mu_1'^2)^2}$$

42.12 Parametric median

$$\text{Median}(X) = \frac{\lambda \exp\left(\frac{\Phi^{-1}(1/2) - \gamma}{\delta}\right)}{1 + \exp\left(\frac{\Phi^{-1}(1/2) - \gamma}{\delta}\right)} + \xi$$

42.13 Parametric mode

$$\text{Mode}(X) = \arg\max_{x} f_X\left(x\right)$$

42.14 Additional information and definitions

- ξ : Location parameter
- λ : Scale parameter
- $z\left(x\right) = \left(x - \xi\right)/\lambda$
- u : Uniform[0,1] random varible
- $\Phi\left(x\right)$: CDF normal standard distribution
- $\Phi^{-1}\left(x\right)$: PPF normal standard distribution

42.15 Spreadsheet documents

- Excel file from GitHub repository
- Google spreadsheet document

43 Johnson SU Distribution

43.1 Distribution definition

$$X \sim \text{JohnsonSU}\,(\xi, \lambda, \gamma, \delta)$$

43.2 Distribution domain

$$x \in (-\infty, \infty)$$

43.3 Parameters domain and parameters constraints

$$\xi \in \mathbb{R}, \lambda \in \mathbb{R}^+, \gamma \in \mathbb{R}, \delta \in \mathbb{R}^+$$

43.4 Cumulative distribution function

$$F_X\,(x) = \Phi\left(\gamma + \delta \sinh^{-1}(z(x))\right)$$

43.5 Probability density function

$$f_X\,(x) = \frac{\delta}{\lambda\sqrt{2\pi}\sqrt{z(x)^2+1}} \exp\left[-\frac{1}{2}\left(\gamma + \delta \sinh^{-1}(z(x))\right)^2\right]$$

43.6 Percent point function/Sample

$$F_X^{-1}\,(u) = \lambda \sinh\left(\frac{\Phi^{-1}(u)-\gamma}{\delta}\right) + \xi$$

43.7 Parametric centered moments

$$\mu'_k = E[X^k] = \int_{-\infty}^{\infty} x^k f_X\,(x)\,dx$$

43.8 Parametric mean

$$\text{Mean}(X) = \mu'_1 = \xi - \lambda \exp\frac{\delta^{-2}}{2} \sinh\left(\frac{\gamma}{\delta}\right)$$

43.9 Parametric variance

$$\text{Variance}(X) = \mu'_2 - \mu'^2_1 = \frac{\lambda^2}{2}(\exp(\delta^{-2})-1)\left(\exp(\delta^{-2})\cosh\left(\frac{2\gamma}{\delta}\right)+1\right)$$

43.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\mu'_3 - 3\mu'_2\mu'_1 + 2\mu'^3_1}{(\mu'_2 - \mu'^2_1)^{1.5}} = -\frac{\lambda^3\sqrt{e^{\delta^{-2}}}(e^{\delta^{-2}}-1)^2(e^{\delta^{-2}})(e^{\delta^{-2}}+2)\sinh(\frac{3\gamma}{\delta}) + 3\sinh(\frac{2\gamma}{\delta}))}{4\text{Variance}(X)^{1.5}}$$

43.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\mu'_4 - 4\mu'_1\mu'_3 + 6\mu'^2_1\mu'_2 - 3\mu'^4_1}{(\mu'_2 - \mu'^2_1)^2} = \frac{\lambda^4(e^{\delta^{-2}}-1)^2(K_1+K_2+K_3)}{8\text{Variance}(X)^2}$$

43.12 Parametric median

$$\text{Median}(X) = \xi + \lambda \sinh\left(-\frac{\gamma}{\delta}\right)$$

43.13 Parametric mode

$$\text{Mode}(X) = \arg\max_x f_X\,(x)$$

43.14 Additional information and definitions

- ξ : Location parameter
- λ : Scale parameter
- $z\left(x\right) = \left(x - \xi\right)/\lambda$
- u : Uniform[0,1] random varible
- $\Phi\left(x\right)$: CDF normal standard distribution
- $\Phi^{-1}\left(x\right)$: PPF normal standard distribution
- $K_1 = \left(e^{\delta^{-2}}\right)^2 \left(\left(e^{\delta^{-2}}\right)^4 + 2\left(e^{\delta^{-2}}\right)^3 + 3\left(e^{\delta^{-2}}\right)^2 - 3\right)\cosh\left(\frac{4\gamma}{\delta}\right)$
- $K_2 = 4\left(e^{\delta^{-2}}\right)^2 \left(\left(e^{\delta^{-2}}\right) + 2\right)\cosh\left(\frac{3\gamma}{\delta}\right)$
- $K_3 = 3\left(2\left(e^{\delta^{-2}}\right) + 1\right)$

43.15 Spreadsheet documents

- Excel file from GitHub repository
- Google spreadsheet document

44 Kumaraswamy Distribution

44.1 Distribution definition

$$X \sim \text{Kumaraswamy}\left(\alpha, \beta, \min, \max\right)$$

44.2 Distribution domain

$$x \in (\min, \max)$$

44.3 Parameters domain and parameters constraints

$$\alpha \in \mathbb{R}^{+}, \beta \in \mathbb{R}^{+}, \min \in \mathbb{R}, \max \in \mathbb{R}$$

44.4 Cumulative distribution function

$$F_X\left(x\right) = 1 - (1 - z(x)^{\alpha})^{\beta}$$

44.5 Probability density function

$$f_X\left(x\right) = \alpha\beta z(x)^{\alpha-1}(1 - z(x)^{\alpha})^{\beta-1}$$

44.6 Percent point function/Sample

$$F_X^{-1}\left(u\right) = \min + (\max - \min) \times (1 - (1-u)^{\frac{1}{\beta}})^{\frac{1}{\alpha}}$$

44.7 Parametric centered moments

$$\tilde{\mu}'_k = E[\tilde{X}^k] = \int_0^1 x^k f_{\tilde{X}}\left(x\right) dx = \beta \text{Beta}(1 + \frac{k}{\alpha}, \beta)$$

44.8 Parametric mean

$$\text{Mean}(X) = \min + (\max - \min) \times \tilde{\mu}'_1$$

44.9 Parametric variance

$$\text{Variance}(X) = (\max - \min)^2 \left(\tilde{\mu}'_2 - \tilde{\mu}'^2_1\right)$$

44.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\tilde{\mu}'_3 - 3\tilde{\mu}'_2\tilde{\mu}'_1 + 2\tilde{\mu}'^3_1}{(\tilde{\mu}'_2 - \tilde{\mu}'^2_1)^{1.5}}$$

44.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\tilde{\mu}'_4 - 4\tilde{\mu}'_1\tilde{\mu}'_3 + 6\tilde{\mu}'^2_1\tilde{\mu}'_2 - 3\tilde{\mu}'^4_1}{(\tilde{\mu}'_2 - \tilde{\mu}'^2_1)^2}$$

44.12 Parametric median

$$\text{Median}(X) = \min + (\max - \min) \times \left(1 - 2^{-1/b}\right)^{1/a}$$

44.13 Parametric mode

$$\text{Mode}(X) = \min + (\max - \min) \times \left(\frac{a-1}{ab-1}\right)^{1/a}$$

44.14 Additional information and definitions

- $\tilde{X} \sim \text{Kumaraswamy}\left(\alpha, \beta, 0, 1\right)$
- $z\left(x\right) = \left(x - \min\right) / \left(\max - \min\right)$
- u : Uniform[0,1] random varible
- $\text{Beta}\left(x, y\right)$: Beta function

44.15 Spreadsheet documents

- Excel file from GitHub repository
- Google spreadsheet document

45 Laplace Distribution

45.1 Distribution definition

$$X \sim \text{Laplace}\,(\mu, b)$$

45.2 Distribution domain

$$x \in (-\infty, \infty)$$

45.3 Parameters domain and parameters constraints

$$\mu \in \mathbb{R}^+, b \in \mathbb{R}^+$$

45.4 Cumulative distribution function

$$F_X\,(x) = \tfrac{1}{2} + \tfrac{1}{2}\text{sign}(x-\mu)\left(1 - \exp\left(-\frac{|x-\mu|}{b}\right)\right)$$

45.5 Probability density function

$$f_X\,(x) = \frac{1}{2b}\exp\left(-\frac{|x-\mu|}{b}\right)$$

45.6 Percent point function/Sample

$$F_X^{-1}\,(u) = \mu - b \times \text{sign}\left(u - \frac{1}{2}\right)\,\ln\left(1 - 2\left|p - \frac{1}{2}\right|\right)$$

45.7 Parametric centered moments

$$\mu_k' = E[X^k] = \int_{-\infty}^{\infty} x^k f_X\,(x)\,dx = \left(\frac{1}{2}\right)\sum_{k=0}^{r}\left[\frac{r!}{(r-k)!}b^k\mu^{(r-k)}\{1+(-1)^k\}\right]$$

45.8 Parametric mean

$$\text{Mean}(X) = \mu_1' = \mu$$

45.9 Parametric variance

$$\text{Variance}(X) = \mu_2' - \mu_1'^2 = 2b^2$$

45.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\mu_3' - 3\mu_2'\mu_1' + 2\mu_1'^3}{(\mu_2' - \mu_1'^2)^{1.5}} = 0$$

45.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\mu_4' - 4\mu_1'\mu_3' + 6\mu_1'^2\mu_2' - 3\mu_1'^4}{(\mu_2' - \mu_1'^2)^2} = 6$$

45.12 Parametric median

$$\text{Median}(X) = \mu$$

45.13 Parametric mode

$$\text{Mode}(X) = \mu$$

45.14 Additional information and definitions

- μ : Location parameter
- b : Scale parameter
- u : Uniform[0,1] random varible

45.15 Spreadsheet documents

- Excel file from GitHub repository
- Google spreadsheet document

46 Levy Distribution

46.1 Distribution definition

$$X \sim \text{Levy}\left(\mu, c\right)$$

46.2 Distribution domain

$$x \in [\mu, \infty)$$

46.3 Parameters domain and parameters constraints

$$\mu \in \mathbb{R}, c \in \mathbb{R}^{+}$$

46.4 Cumulative distribution function

$$F_X\left(x\right) = 1 - \text{erf}\left(\sqrt{\frac{c}{2(x-\mu)}}\right)$$

46.5 Probability density function

$$f_X\left(x\right) = \sqrt{\frac{c}{2\pi}} \; \frac{e^{-\frac{c}{2(x-\mu)}}}{(x-\mu)^{3/2}}$$

46.6 Percent point function/Sample

$$F_X^{-1}\left(u\right) = \mu + \frac{c}{2\left(\text{erf}^{-1}(1-u)\right)^2}$$

46.7 Parametric centered moments

$$\mu'_k = E[X^k] = \int_{\mu}^{\infty} x^k f_X\left(x\right) dx$$

46.8 Parametric mean

$$\text{Mean}(X) = \mu'_1 = \infty$$

46.9 Parametric variance

$$\text{Variance}(X) = \mu'_2 - \mu'^2_1 = \infty$$

46.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\mu'_3 - 3\mu'_2\mu'_1 + 2\mu'^3_1}{(\mu'_2 - \mu'^2_1)^{1.5}} = \text{undefined}$$

46.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\mu'_4 - 4\mu'_1\mu'_3 + 6\mu'^2_1\mu'_2 - 3\mu'^4_1}{(\mu'_2 - \mu'^2_1)^2} = \text{undefined}$$

46.12 Parametric median

$$\text{Median}(X) = \mu + \frac{c}{2(\text{erf}^{-1}(1/2))^2}$$

46.13 Parametric mode

$$\text{Mode}(X) = \mu + \frac{c}{3}$$

46.14 Additional information and definitions

- μ : Location parameter
- c : Scale parameter
- u : Uniform[0,1] random varible
- $\text{erf}(x)$: Error function
- $\text{erf}^{-1}(x)$: Inverse of error function

46.15 Spreadsheet documents

- Excel file from GitHub repository
- Google spreadsheet document

47 Loggamma Distribution

47.1 Distribution definition

$$X \sim \mathrm{LogGamma}\left(c, \mu, \sigma\right)$$

47.2 Distribution domain

$$x \in (0, \infty)$$

47.3 Parameters domain and parameters constraints

$$c \in \mathbb{R}^+, \mu \in \mathbb{R}, \sigma \in \mathbb{R}^+$$

47.4 Cumulative distribution function

$$F_X\left(x\right) = \frac{\gamma\left(c, e^x\right)}{\Gamma\left(c\right)} = \mathrm{P}\left(c, e^{z(x)}\right)$$

47.5 Probability density function

$$f_X\left(x\right) = \frac{\exp\left(cz(x) - e^{z(x)}\right)}{\sigma\Gamma\left(c\right)}$$

47.6 Percent point function/Sample

$$F_X^{-1}\left(u\right) = \mu + \sigma \ln\left(\mathrm{P}^{-1}\left(u, c\right)\right)$$

47.7 Parametric centered moments

$$\mu'_k = E[X^k] = \int_{-\infty}^{\infty} x^k f_X\left(x\right) dx$$

47.8 Parametric mean

$$\mathrm{Mean}(X) = \mu'_1 = \mu + \sigma\psi_0$$

47.9 Parametric variance

$$\mathrm{Variance}(X) = \mu'_2 - \mu'^2_1 = \alpha^2\psi_1\left(c\right)$$

47.10 Parametric skewness

$$\mathrm{Skewness}(X) = \frac{\mu'_3 - 3\mu'_2\mu'_1 + 2\mu'^3_1}{(\mu'_2 - \mu'^2_1)^{1.5}} = \frac{\psi_2\left(c\right)}{\psi_1\left(c\right)}$$

47.11 Parametric kurtosis

$$\mathrm{Kurtosis}(X) = \frac{\mu'_4 - 4\mu'_1\mu'_3 + 6\mu'^2_1\mu'_2 - 3\mu'^4_1}{(\mu'_2 - \mu'^2_1)^2} = \frac{\psi_3\left(c\right)}{\psi_1\left(c\right)}$$

47.12 Parametric median

$$\mathrm{Median}(X) = \mu + \sigma \ln\left(\mathrm{P}^{-1}\left(1/2, c\right)\right)$$

47.13 Parametric mode

$$\mathrm{Mode}(X) = \mu + \sigma \ln(c)$$

47.14 Additional information and definitions

- μ : Location parameter
- σ : Scale parameter
- $z(x) = (x - \mu)/\sigma$
- u : Uniform[0,1] random varible
- $\mathrm{P}(a, x) = \frac{\gamma(a,x)}{\Gamma(a)}$: Regularized lower incomplete gamma function
- $\mathrm{P}^{-1}(a, u)$: Inverse of regularized lower incomplete gamma function
- $\gamma(a, x)$: Lower incomplete gamma function
- $\Gamma(x)$: Gamma function
- $\psi_0(x)$: Digamma function
- $\psi_n(x)$: Polygamma function of order $n \in \mathbb{N}$

47.15 Spreadsheet documents

- Excel file from GitHub repository
- Google spreadsheet document

48 Logistic Distribution

48.1 Distribution definition

$$X \sim \text{Logistic}\left(\mu, \sigma\right)$$

48.2 Distribution domain

$$x \in (-\infty, \infty)$$

48.3 Parameters domain and parameters constraints

$$\mu \in \mathbb{R}, \sigma \in \mathbb{R}^{+}$$

48.4 Cumulative distribution function

$$F_X\left(x\right) = \frac{1}{1 + e^{-(x-\mu)/\sigma}}$$

48.5 Probability density function

$$f_X\left(x\right) = \frac{e^{-(x-\mu)/\sigma}}{\sigma\left(1 + e^{-(x-\mu)/\sigma}\right)^2}$$

48.6 Percent point function/Sample

$$F_X^{-1}\left(u\right) = \mu + \sigma \log\left(\frac{u}{1-u}\right)$$

48.7 Parametric centered moments

$$\mu'_k = E[X^k] = \int_{-\infty}^{\infty} x^k f_X\left(x\right) dx$$

48.8 Parametric mean

$$\text{Mean}(X) = \mu'_1 = \mu$$

48.9 Parametric variance

$$\text{Variance}(X) = \mu'_2 - \mu'^2_1 = \frac{\sigma^2\pi^2}{3}$$

48.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\mu'_3 - 3\mu'_2\mu'_1 + 2\mu'^3_1}{(\mu'_2 - \mu'^2_1)^{1.5}} = 0$$

48.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\mu'_4 - 4\mu'_1\mu'_3 + 6\mu'^2_1\mu'_2 - 3\mu'^4_1}{(\mu'_2 - \mu'^2_1)^2} = 3 + 6/5$$

48.12 Parametric median

$$\text{Median}(X) = \mu$$

48.13 Parametric mode

$$\text{Mode}(X) = \mu$$

48.14 Additional information and definitions

- μ : Location parameter
- σ : Scale parameter
- u : Uniform[0,1] random varible

48.15 Spreadsheet documents

- Excel file from GitHub repository
- Google spreadsheet document

49 Loglogistic Distribution

49.1 Distribution definition

$$X \sim \text{LogLogistic}(\alpha, \beta)$$

49.2 Distribution domain

$$x \in [0, \infty)$$

49.3 Parameters domain and parameters constraints

$$\alpha \in \mathbb{R}^+, \beta \in \mathbb{R}^+$$

49.4 Cumulative distribution function

$$F_X(x) = \frac{1}{1 + (x/\alpha)^{-\beta}}$$

49.5 Probability density function

$$f_X(x) = \frac{(\beta/\alpha)(x/\alpha)^{\beta-1}}{\left(1 + (x/\alpha)^{\beta}\right)^2}$$

49.6 Percent point function/Sample

$$F_X^{-1}(u) = \alpha \left(\frac{u}{1-u}\right)^{1/\beta}$$

49.7 Parametric centered moments

$$\mu'_k = E[X^k] = \int_0^\infty x^k f_X(x)\, dx = \alpha^k \text{Beta}(1 - k/\beta, 1 + k/\beta) = \alpha^k \frac{k\pi/\beta}{\sin(k\pi/\beta)}$$

49.8 Parametric mean

$$\text{Mean}(X) = \mu'_1$$

49.9 Parametric variance

$$\text{Variance}(X) = \mu'_2 - \mu'^2_1$$

49.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\mu'_3 - 3\mu'_2\mu'_1 + 2\mu'^3_1}{(\mu'_2 - \mu'^2_1)^{1.5}}$$

49.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\mu'_4 - 4\mu'_1\mu'_3 + 6\mu'^2_1\mu'_2 - 3\mu'^4_1}{(\mu'_2 - \mu'^2_1)^2}$$

49.12 Parametric median

$$\text{Median}(X) = \alpha$$

49.13 Parametric mode

$$\text{Mode}(X) = \alpha \left(\frac{\beta - 1}{\beta + 1}\right)^{1/\beta}$$

49.14 Additional information and definitions

- α : Scale parameter
- u : Uniform[0,1] random varible
- Beta (x, y) : Beta function

49.15 Spreadsheet documents

- Excel file from GitHub repository
- Google spreadsheet document

50 Loglogistic 3P Distribution

50.1 Distribution definition

$$X \sim \text{LogLogistic}_{3\text{P}}\left(\text{Loc}, \alpha, \beta\right)$$

50.2 Distribution domain

$$x \in [\text{Loc}, \infty)$$

50.3 Parameters domain and parameters constraints

$$\text{Loc} \in \mathbb{R}, \alpha \in \mathbb{R}^{+}, \beta \in \mathbb{R}^{+}$$

50.4 Cumulative distribution function

$$F_X\left(x\right) = \frac{1}{1 + ((x - \text{Loc})/\alpha)^{-\beta}}$$

50.5 Probability density function

$$f_X\left(x\right) = \frac{(\beta/\alpha)((x - \text{Loc})/\alpha)^{\beta-1}}{\left(1 + ((x - \text{Loc})/\alpha)^{\beta}\right)^2}$$

50.6 Percent point function/Sample

$$F_X^{-1}\left(u\right) = \text{Loc} + \alpha\left(\frac{u}{1-u}\right)^{1/\beta}$$

50.7 Parametric centered moments

$$\tilde{\mu}'_k = E[\tilde{X}^k] = \int_0^{\infty} x^k f_{\tilde{X}}\left(x\right) dx = \alpha^k \text{Beta}(1 - k/\beta, 1 + k/\beta) = \alpha^k\, \frac{k\pi/\beta}{\sin(k\pi/\beta)}$$

50.8 Parametric mean

$$\text{Mean}(X) = \text{Loc} + \tilde{\mu}'_1$$

50.9 Parametric variance

$$\text{Variance}(X) = \tilde{\mu}'_2 - \tilde{\mu}'^2_1$$

50.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\tilde{\mu}'_3 - 3\tilde{\mu}'_2\tilde{\mu}'_1 + 2\tilde{\mu}'^3_1}{(\tilde{\mu}'_2 - \tilde{\mu}'^2_1)^{1.5}}$$

50.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\tilde{\mu}'_4 - 4\tilde{\mu}'_1\tilde{\mu}'_3 + 6\tilde{\mu}'^2_1\tilde{\mu}'_2 - 3\tilde{\mu}'^4_1}{(\tilde{\mu}'_2 - \tilde{\mu}'^2_1)^2}$$

50.12 Parametric median

$$\text{Median}(X) = \text{Loc} + \alpha$$

50.13 Parametric mode

$$\text{Mode}(X) = \text{Loc} + \alpha\left(\frac{\beta - 1}{\beta + 1}\right)^{1/\beta}$$

50.14 Additional information and definitions

- $\tilde{X} \sim \text{LogLogistic}\,(\alpha, \beta)$
- Loc : Location parameter
- α : Scale parameter
- u : Uniform[0,1] random varible
- $\text{Beta}\,(x, y)$: Beta function

50.15 Spreadsheet documents

- Excel file from GitHub repository
- Google spreadsheet document

51 Lognormal Distribution

51.1 Distribution definition

$$X \sim \text{LogNormal}\left(\mu, \sigma\right)$$

51.2 Distribution domain

$$x \in (-\infty, \infty)$$

51.3 Parameters domain and parameters constraints

$$\mu \in \mathbb{R}, \sigma \in \mathbb{R}^{+}$$

51.4 Cumulative distribution function

$$F_X\left(x\right) = \frac{1}{2}\left[1 + \text{erf}\left(\frac{\ln(x) - \mu}{\sigma\sqrt{2}}\right)\right]$$

51.5 Probability density function

$$f_X\left(x\right) = \frac{1}{x\sigma\sqrt{2\pi}}\ \exp\left(-\frac{\left(\ln\left(x\right) - \mu\right)^2}{2\sigma^2}\right)$$

51.6 Percent point function/Sample

$$F_X^{-1}\left(u\right) = \exp(\mu + \sqrt{2\sigma^2}\,\text{erf}^{-1}(2u-1))$$

51.7 Parametric centered moments

$$\mu_k' = E[X^k] = \int_{-\infty}^{\infty} x^k f_X\left(x\right) dx = e^{k\mu + k^2\sigma^2/2}$$

51.8 Parametric mean

$$\text{Mean}(X) = \mu_1'$$

51.9 Parametric variance

$$\text{Variance}(X) = \mu_2' - \mu_1'^2$$

51.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\mu_3' - 3\mu_2'\mu_1' + 2\mu_1'^3}{(\mu_2' - \mu_1'^2)^{1.5}}$$

51.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\mu_4' - 4\mu_1'\mu_3' + 6\mu_1'^2\mu_2' - 3\mu_1'^4}{(\mu_2' - \mu_1'^2)^2}$$

51.12 Parametric median

$$\text{Median}(X) = \exp(\mu)$$

51.13 Parametric mode

$$\text{Mode}(X) = \exp(\mu - \sigma^2)$$

51.14 Additional information and definitions

- μ : Location parameter
- σ : Scale parameter
- u : Uniform[0,1] random varible

51.15 Spreadsheet documents

- Excel file from GitHub repository
- Google spreadsheet document

52 Maxwell Distribution

52.1 Distribution definition

$$X \sim \text{Maxwell}\,(\alpha, \text{Loc})$$

52.2 Distribution domain

$$x \in (0, \infty)$$

52.3 Parameters domain and parameters constraints

$$\alpha \in \mathbb{R}^{+}, \text{Loc} \in \mathbb{R}$$

52.4 Cumulative distribution function

$$F_X\,(x) = \text{erf}\left(\frac{x - \text{Loc}}{\sqrt{2}\alpha}\right) - \sqrt{\frac{2}{\pi}}\frac{(x - \text{Loc})e^{-(x-\text{Loc})^2/\left(2\alpha^2\right)}}{\alpha}$$

52.5 Probability density function

$$f_X\,(x) = \sqrt{\frac{2}{\pi}}\frac{(x - \text{Loc})^2 e^{-(x-\text{Loc})^2/\left(2\alpha^2\right)}}{\alpha^3}$$

52.6 Percent point function/Sample

$$F_X^{-1}\,(u) = \text{Loc} + \alpha\sqrt{2\text{P}^{-1}\,(1.5, u)}$$

52.7 Parametric centered moments

$$\mu_k' = E[X^k] = \int_{-\infty}^{\infty} x^k f_X\,(x)\,dx$$

52.8 Parametric mean

$$\text{Mean}(X) = \mu_1' = \text{Loc} + 2\alpha\sqrt{\frac{2}{\pi}}$$

52.9 Parametric variance

$$\text{Variance}(X) = \mu_2' - \mu_1'^2 = \frac{\alpha^2(3\pi - 8)}{\pi}$$

52.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\mu_3' - 3\mu_2'\mu_1' + 2\mu_1'^3}{(\mu_2' - \mu_1'^2)^{1.5}} = \frac{2\sqrt{2}(16 - 5\pi)}{(3\pi - 8)^{3/2}}$$

52.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\mu_4' - 4\mu_1'\mu_3' + 6\mu_1'^2\mu_2' - 3\mu_1'^4}{(\mu_2' - \mu_1'^2)^2} = 4\frac{\left(-96 + 40\pi - 3\pi^2\right)}{(3\pi - 8)^2} + 3$$

52.12 Parametric median

$$\text{Median}(X) = \text{Loc} + \alpha\sqrt{2\text{P}^{-1}\left(1.5, \frac{1}{2}\right)}$$

52.13 Parametric mode

$$\text{Mode}(X) = \text{Loc} + \alpha\sqrt{2}$$

52.14 Additional information and definitions

- Loc : Location parameter
- α : Scale parameter
- u : Uniform[0,1] random varible
- $\mathrm{P}^{-1}(a, u)$: Inverse of regularized lower incomplete gamma function

52.15 Spreadsheet documents

- Excel file from GitHub repository
- Google spreadsheet document

53 Moyal Distribution

53.1 Distribution definition

$$X \sim \mathrm{Moyal}\left(\mu, \sigma\right)$$

53.2 Distribution domain

$$x \in (-\infty, \infty)$$

53.3 Parameters domain and parameters constraints

$$\mu \in \mathbb{R}, \sigma \in \mathbb{R}^{+}$$

53.4 Cumulative distribution function

$$F_X\left(x\right) = 1 - \mathrm{P}\left(\frac{1}{2}, \frac{e^{-z(x)}}{2}\right) = 1 - \mathrm{erf}\left(\frac{\exp\left(-0.5z(x)\right)}{\sqrt{2}}\right)$$

53.5 Probability density function

$$f_X\left(x\right) = \frac{1}{\sqrt{2\pi}}\exp\left(-\frac{1}{2}\left(z(x) + e^{-z(x)}\right)\right)$$

53.6 Percent point function/Sample

$$F_X^{-1}\left(u\right) = \mu + \sigma \ln\left[\Phi^{-1}\left(\left(\frac{1-u}{2}\right)^2\right)\right] = \mu + \sigma \ln\left[2\mathrm{P}^{-1}\left(\frac{1}{2}, 1-u\right)\right]$$

53.7 Parametric centered moments

$$\mu'_k = E[X^k] = \int_{-\infty}^{\infty} x^k f_X\left(x\right) dx$$

53.8 Parametric mean

$$\mathrm{Mean}(X) = \mu'_1 = \mu + \sigma(\ln(2) + \gamma)$$

53.9 Parametric variance

$$\mathrm{Variance}(X) = \mu'_2 - \mu'^2_1 = \sigma^2\left(\frac{\pi^2}{2}\right)$$

53.10 Parametric skewness

$$\mathrm{Skewness}(X) = \frac{\mu'_3 - 3\mu'_2\mu'_1 + 2\mu'^3_1}{(\mu'_2 - \mu'^2_1)^{1.5}} = \frac{28\sqrt{2}\zeta(3)}{\pi^3}$$

53.11 Parametric kurtosis

$$\mathrm{Kurtosis}(X) = \frac{\mu'_4 - 4\mu'_1\mu'_3 + 6\mu'^2_1\mu'_2 - 3\mu'^4_1}{(\mu'_2 - \mu'^2_1)^2} = 7$$

53.12 Parametric median

$$\mathrm{Median}(X) = \mu + \sigma \ln\left[2\mathrm{P}^{-1}\left(\frac{1}{2}, \frac{1}{2}\right)\right]$$

53.13 Parametric mode

$$\mathrm{Mode}(X) = \mu$$

53.14 Additional information and definitions

- μ : Location parameter
- σ : Scale parameter
- $z(x) = (x - \mu)/\sigma$
- $\mathrm{P}(a, x) = \frac{\gamma(a,x)}{\Gamma(a)}$: Regularized lower incomplete gamma function
- $\mathrm{P}^{-1}(a, u)$: Inverse of regularized lower incomplete gamma function
- $\gamma(a, x)$: Lower incomplete gamma function
- $\Gamma(x)$: Gamma function
- $\mathrm{erf}(x)$: Error function
- $\Phi^{-1}(x)$: PPF normal standard distribution
- γ : Euler-Mascheroni constant = 0.5772156649
- $\zeta(3)$: Apéry's constant = 1.2020569031

53.15 Spreadsheet documents

- Excel file from GitHub repository
- Google spreadsheet document

54 Nakagami Distribution

54.1 Distribution definition

$$X \sim \text{Nakagami}\,(m, \Omega)$$

54.2 Distribution domain

$$x \in (0, \infty)$$

54.3 Parameters domain and parameters constraints

$$m \in \mathbb{R}^+_{\geqslant \frac{1}{2}}, \Omega \in \mathbb{R}^+$$

54.4 Cumulative distribution function

$$F_X\,(x) = \frac{\gamma\left(m, \frac{m}{\Omega}x^2\right)}{\Gamma(m)} = \mathrm{P}\left(m, \frac{m}{\Omega}x^2\right)$$

54.5 Probability density function

$$f_X\,(x) = \frac{2m^m}{\Gamma(m)\Omega^m} x^{2m-1} \exp\left(-\frac{m}{\Omega}x^2\right)$$

54.6 Percent point function/Sample

$$F_X^{-1}\,(u) = \sqrt{\frac{\Omega}{m}\mathrm{P}^{-1}\,(m, u)}$$

54.7 Parametric centered moments

$$\mu'_k = E[X^k] = \int_{-\infty}^{\infty} x^k f_X\,(x)\,dx$$

54.8 Parametric mean

$$\text{Mean}(X) = \mu'_1 = \frac{\Gamma(m+\frac{1}{2})}{\Gamma(m)}\left(\frac{\Omega}{m}\right)^{1/2}$$

54.9 Parametric variance

$$\text{Variance}(X) = \mu'_2 - \mu'^2_1 = \Omega\left(1 - \frac{1}{m}\left(\frac{\Gamma(m+\frac{1}{2})}{\Gamma(m)}\right)^2\right)$$

54.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\mu'_3 - 3\mu'_2\mu'_1 + 2\mu'^3_1}{(\mu'_2 - \mu'^2_1)^{1.5}} = \frac{\frac{\Gamma(m+\frac{1}{2})}{\Gamma(m)\sqrt{m}}\left(1 - 4m\left(1 - \frac{1}{m}\left(\frac{\Gamma(m+\frac{1}{2})}{\Gamma(m)}\right)^2\right)\right)}{2m\left(1 - \frac{1}{m}\left(\frac{\Gamma(m+\frac{1}{2})}{\Gamma(m)}\right)^2\right)^{3/2}}$$

54.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\mu'_4 - 4\mu'_1\mu'_3 + 6\mu'^2_1\mu'_2 - 3\mu'^4_1}{(\mu'_2 - \mu'^2_1)^2} = 3 + \frac{-6\left(\frac{\Gamma(m+\frac{1}{2})}{\Gamma(m)\sqrt{m}}\right)^4 m + (8m-2)\left(\frac{\Gamma(m+\frac{1}{2})}{\Gamma(m)\sqrt{m}}\right)^2 - 2m + 1}{m\left(1 - \frac{1}{m}\left(\frac{\Gamma(m+\frac{1}{2})}{\Gamma(m)}\right)^2\right)^2}$$

54.12 Parametric median

$$\text{Median}(X) = \sqrt{\frac{\Omega}{m}\mathrm{P}^{-1}\left(m, \frac{1}{2}\right)}$$

54.13 Parametric mode

$$\text{Mode}(X) = \frac{\sqrt{2}}{2}\left(\frac{(2m-1)\Omega}{m}\right)^{1/2}$$

54.14 Additional information and definitions

- u : Uniform[0,1] random varible
- $\mathrm{P}\left(a, x\right) = \frac{\gamma(a,x)}{\Gamma(a)}$: Regularized lower incomplete gamma function
- $\mathrm{P}^{-1}\left(a, u\right)$: Inverse of regularized lower incomplete gamma function

54.15 Spreadsheet documents

- Excel file from GitHub repository
- Google spreadsheet document

55 Non Central Chi Square Distribution

55.1 Distribution definition

$$X \sim \text{NonCentralChiSquare}\left(\lambda, n\right)$$

55.2 Distribution domain

$$x \in [0, +\infty)$$

55.3 Parameters domain and parameters constraints

$$\lambda \in \mathbb{R}^+, n \in \mathbb{R}^+$$

55.4 Cumulative distribution function

$$F_X\left(x\right) = 1 - Q_{\frac{n}{2}}\left(\sqrt{\lambda}, \sqrt{x}\right)$$

55.5 Probability density function

$$f_X\left(x\right) = \frac{1}{2} e^{-(x+\lambda)/2} \left(\frac{x}{\lambda}\right)^{n/4-1/2} I_{n/2-1}(\sqrt{\lambda x})$$

55.6 Percent point function/Sample

$$\text{Sample}_X = \sum_{i=1}^{n} \left(\sqrt{\frac{\lambda}{n}} + \Phi^{-1}\left(u_i\right)\right)^2$$

55.7 Parametric centered moments

$$\mu'_k = E[X^k] = \int_0^\infty x^k f_X\left(x\right) dx = 2^{k-1}(k-1)!(n+k\lambda) + \sum_{j=1}^{k-1} \frac{(k-1)!2^{j-1}}{(k-j)!}(n+j\lambda)\mu'_{k-j}$$

55.8 Parametric mean

$$\text{Mean}(X) = \mu'_1 = n + \lambda$$

55.9 Parametric variance

$$\text{Variance}(X) = \mu'_2 - \mu'^2_1 = 2(n + 2\lambda)$$

55.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\mu'_3 - 3\mu'_2\mu'_1 + 2\mu'^3_1}{(\mu'_2 - \mu'^2_1)^{1.5}} = \frac{2^{3/2}(n+3\lambda)}{(n+2\lambda)^{3/2}}$$

55.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\mu'_4 - 4\mu'_1\mu'_3 + 6\mu'^2_1\mu'_2 - 3\mu'^4_1}{(\mu'_2 - \mu'^2_1)^2} = \frac{12(n+4\lambda)}{(n+2\lambda)^2}$$

55.12 Parametric median

$$\text{Median}(X) = F_X^{-1}\left(\frac{1}{2}\right)$$

55.13 Parametric mode

$$\text{Mode}(X) = \arg\max_x f_X\left(x\right)$$

55.14 Additional information and definitions

- Computing an analytic expression for the inverse of the cumulative distribution function is not feasible. Nonetheless, it is possible to generate a random sample from the distribution.
- u_i : Uniform[0,1] random varible
- $\Phi^{-1}(x)$: PPF normal standard distribution
- $I_\alpha(x)$: Modified Bessel function of the first kind of order $\alpha \in \mathbb{N}$

55.15 Spreadsheet documents

- Excel file from GitHub repository
- Google spreadsheet document

56 Non Central F Distribution

56.1 Distribution definition

$$X \sim \text{NonCentralF}\left(\lambda, n_1, n_2\right)$$

56.2 Distribution domain

$$x \in [0, \infty)$$

56.3 Parameters domain and parameters constraints

$$\lambda \in \mathbb{R}^+, n_1 \in \mathbb{R}^+, n_2 \in \mathbb{R}^+$$

56.4 Cumulative distribution function

$$F_X\left(x\right) = \sum_{j=0}^{\infty} \left(\frac{\left(\frac{1}{2}\lambda\right)^j}{j!} e^{-\lambda/2} \right) I_{n_1 x/(n_2+n_1 x)} \left(\frac{n_1}{2} + j, \frac{n_2}{2} \right)$$

56.5 Probability density function

$$f_X\left(x\right) = \sum_{k=0}^{\infty} \frac{e^{-\lambda/2}(\lambda/2)^k}{\text{Beta}\left(\frac{n_2}{2}, \frac{n_1}{2}+k\right) k!} \left(\frac{n_1}{n_2}\right)^{\frac{n_1}{2}+k} \left(\frac{n_2}{n_2+n_1 x}\right)^{\frac{n_1+n_2}{2}+k} x^{n_1/2-1+k}$$

56.6 Percent point function/Sample

$$\text{Sample}_X = \frac{\left(\sum_{i=1}^{n_1} \left(\sqrt{\frac{\lambda}{n_1}} + \Phi^{-1}\left(u_i\right)\right)^2\right)/n_1}{\left(2\text{P}^{-1}\left(\frac{n_2}{2}, u\right)\right)/n_2}$$

56.7 Parametric centered moments

$$\mu_k' = E[X^k] = \int_0^{\infty} x^k f_X\left(x\right) dx = e^{-\lambda/2} \left(\frac{n1}{n2}\right)^k \frac{\Gamma\left(n_1/2-k\right)}{\Gamma\left(n_1/2\right)} \sum_{r=0}^{\infty} \left(\frac{1}{r!}\right) \left(\frac{\lambda}{2}\right)^r \frac{\Gamma\left(\frac{n_1}{2}+r+k\right)}{\Gamma\left(\frac{n_1}{2}+r\right)}$$

56.8 Parametric mean

$$\text{Mean}(X) = \mu_1'$$

56.9 Parametric variance

$$\text{Variance}(X) = \mu_2' - \mu_1'^2$$

56.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\mu_3' - 3\mu_2'\mu_1' + 2\mu_1'^3}{(\mu_2' - \mu_1'^2)^{1.5}}$$

56.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\mu_4' - 4\mu_1'\mu_3' + 6\mu_1'^2\mu_2' - 3\mu_1'^4}{(\mu_2' - \mu_1'^2)^2}$$

56.12 Parametric median

$$\text{Median}(X) = F_X^{-1}\left(\frac{1}{2}\right)$$

56.13 Parametric mode

$$\text{Mode}(X) = \arg\max_{x} f_X(x)$$

56.14 Additional information and definitions

- Computing an analytic expression for the inverse of the cumulative distribution function is not feasible. Nonetheless, it is possible to generate a random sample from the distribution.
- u : Uniform[0,1] random varible
- u_i : Uniform[0,1] random varible
- $\Phi^{-1}(x)$: PPF normal standard distribution
- $\mathrm{P}^{-1}(a, u)$: Inverse of regularized lower incomplete gamma function
- $I(x, a, b)$: Regularized incomplete beta function
- $\mathrm{Beta}(x, y)$: Beta function

56.15 Spreadsheet documents

- Excel file from GitHub repository
- Google spreadsheet document

57 Non Central T Student Distribution

57.1 Distribution definition

$$X \sim \mathrm{NonCentralTStudent}\left(\lambda, n, \mathrm{Loc}, \mathrm{Sc}\right)$$

57.2 Distribution domain

$$x \in (-\infty, \infty)$$

57.3 Parameters domain and parameters constraints

$$\lambda \in \mathbb{R}, n \in \mathbb{R}^{+}, \mathrm{Sc} \in \mathbb{R}^{+}, \mathrm{Loc} \in \mathbb{R}$$

57.4 Cumulative distribution function

$$F_X\left(x\right) = \begin{cases} \frac{1}{2}\sum_{j=0}^{\infty}\frac{1}{j!}(-\lambda\sqrt{2})^j e^{\frac{-\lambda^2}{2}}\frac{\Gamma(\frac{j+1}{2})}{\sqrt{\pi}} I_{n/(n+z(x)^2)}\left(\frac{n}{2},\frac{j+1}{2}\right) & \text{if } \; z(x) \geq 0 \\ 1-\frac{1}{2}\sum_{j=0}^{\infty}\frac{1}{j!}(-\lambda\sqrt{2})^j e^{\frac{-\lambda^2}{2}}\frac{\Gamma(\frac{j+1}{2})}{\sqrt{\pi}} I_{n/(n+z(x)^2)}\left(\frac{n}{2},\frac{j+1}{2}\right) & \text{if } \; z(x) < 0 \end{cases}$$

57.5 Probability density function

$$f_X\left(x\right) = \frac{1}{\mathrm{Sc}}\frac{n^{n/2}\Gamma\left(n+1\right)}{2^n e^{\lambda^2/2}\left(n+z(x)^2\right)^{n/2}\Gamma\left(n/2\right)} \times \left\{ \frac{\sqrt{2}\lambda z(x) {}_1F_1\left(\frac{n}{2}+1, \frac{3}{2}, \frac{\lambda^2 z(x)^2}{2(n+z(x)^2)}\right)}{\left(n+z(x)^2\right)\Gamma\left(\frac{n+1}{2}\right)} - \frac{{}_1F_1\left(\frac{n+1}{2}, \frac{1}{2}, \frac{\lambda^2 z(x)^2}{2(n+z(x)^2)}\right)}{\sqrt{n+z(x)^2}\,\Gamma\left(\frac{n}{2}+1\right)} \right\}$$

57.6 Percent point function/Sample

$$\mathrm{Sample}_X = \mathrm{Loc} + \mathrm{Sc}\frac{\left(\lambda + \Phi^{-1}\left(u\right)\right)}{\left(\sqrt{2\mathrm{P}^{-1}\left(\frac{n}{2}, u\right)}\right)/n}$$

57.7 Parametric centered moments

$$\tilde{\mu}'_k = E[\tilde{X}^k] = \int_0^{\infty} x^k f_{\tilde{X}}\left(x\right)dx = \frac{e^{-\lambda^2/2}}{\sqrt{n\pi}\Gamma\left(n/2\right)}\Gamma\left(\frac{n-k}{2}\right)n^{k/2}\sum_{r=0}^{\infty}\frac{\lambda^r 2^{r/2}}{r!}\Gamma\left(\frac{r+k+1}{2}\right)$$

57.8 Parametric mean

$$\mathrm{Mean}(X) = \mathrm{Loc} + \mathrm{Sc}\tilde{\mu}'_1$$

57.9 Parametric variance

$$\mathrm{Variance}(X) = \mathrm{Sc}^2(\tilde{\mu}'_2 - \tilde{\mu}'^2_1)$$

57.10 Parametric skewness

$$\mathrm{Skewness}(X) = \frac{\tilde{\mu}'_3 - 3\tilde{\mu}'_2\tilde{\mu}'_1 + 2\tilde{\mu}'^3_1}{(\tilde{\mu}'_2 - \tilde{\mu}'^2_1)^{1.5}}$$

57.11 Parametric kurtosis

$$\mathrm{Kurtosis}(X) = \frac{\tilde{\mu}'_4 - 4\tilde{\mu}'_1\tilde{\mu}'_3 + 6\tilde{\mu}'^2_1\tilde{\mu}'_2 - 3\tilde{\mu}'^4_1}{(\tilde{\mu}'_2 - \tilde{\mu}'^2_1)^2}$$

57.12 Parametric median

$$\mathrm{Median}(X) = F_X^{-1}\left(\frac{1}{2}\right)$$

57.13 Parametric mode

$$\text{Mode}(X) = \arg\max_{x} f_X\left(x\right)$$

57.14 Additional information and definitions

- $\tilde{X} \sim \text{NonCentralTStudent}\left(\lambda, n, 0, 1\right)$
- Computing an analytic expression for the inverse of the cumulative distribution function is not feasible. Nonetheless, it is possible to generate a random sample from the distribution.
- Loc : Location parameter
- Sc : Scale parameter
- $z\left(x\right) = \left(x - \text{Loc}\right)/\text{Sc}$
- u : Uniform[0,1] random varible
- $\text{P}^{-1}\left(a, u\right)$: Inverse of regularized lower incomplete gamma function
- $I_\alpha\left(x\right)$: Modified Bessel function of the first kind of order $\alpha \in \mathbb{N}$
- ${}_1F_1(a, b, z)$: Kummer's confluent hypergeometric function

57.15 Spreadsheet documents

- Excel file from GitHub repository
- Google spreadsheet document

58 Normal Distribution

58.1 Distribution definition

$$X \sim \mathrm{Normal}\left(\mu, \sigma\right)$$

58.2 Distribution domain

$$x \in (-\infty, \infty)$$

58.3 Parameters domain and parameters constraints

$$\mu \in \mathbb{R}, \sigma \in \mathbb{R}^{+}$$

58.4 Cumulative distribution function

$$F_X\left(x\right) = \frac{1}{2}\left[1 + \operatorname{erf}\left(\frac{x-\mu}{\sigma\sqrt{2}}\right)\right] = \Phi\left(\frac{x-\mu}{\sigma}\right)$$

58.5 Probability density function

$$f_X\left(x\right) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} = \phi\left(\frac{x-\mu}{\sigma}\right)$$

58.6 Percent point function/Sample

$$F_X^{-1}\left(u\right) = \mu + \sigma\sqrt{2}\operatorname{erf}^{-1}(2u-1) = \mu + \sigma\Phi^{-1}\left(u\right)$$

58.7 Parametric centered moments

$$\mu_k' = E[X^k] = \int_{-\infty}^{\infty} x^k f_X\left(x\right) dx = \sigma^k \cdot (-i\sqrt{2})^k U\left(-\frac{k}{2}, \frac{1}{2}, -\frac{1}{2}\left(\frac{\mu}{\sigma}\right)^2\right)$$

58.8 Parametric mean

$$\mathrm{Mean}(X) = \mu_1' = \mu$$

58.9 Parametric variance

$$\mathrm{Variance}(X) = \mu_2' - \mu_1'^2 = \sigma^2$$

58.10 Parametric skewness

$$\mathrm{Skewness}(X) = \frac{\mu_3' - 3\mu_2'\mu_1' + 2\mu_1'^3}{(\mu_2' - \mu_1'^2)^{1.5}} = 0$$

58.11 Parametric kurtosis

$$\mathrm{Kurtosis}(X) = \frac{\mu_4' - 4\mu_1'\mu_3' + 6\mu_1'^2\mu_2' - 3\mu_1'^4}{(\mu_2' - \mu_1'^2)^2} = 3$$

58.12 Parametric median

$$\mathrm{Median}(X) = \mu$$

58.13 Parametric mode

$$\mathrm{Mode}(X) = \mu$$

58.14 Additional information and definitions

- μ : Location parameter
- σ : Scale parameter
- u : Uniform[0,1] random varible
- $U(a, b, z)$: Tricomi's confluent hypergeometric function
- $\Phi\left(x\right)$: CDF normal standard distribution
- $\Phi^{-1}\left(x\right)$: PPF normal standard distribution
- $\phi\left(x\right)$: PDF normal standard distribution
- $\operatorname{erf}(x)$: Error function
- $\operatorname{erf}^{-1}(x)$: Inverse of error function

58.15 Spreadsheet documents

- Excel file from GitHub repository
- Google spreadsheet document

59 Pareto First Kind Distribution

59.1 Distribution definition

$$X \sim \text{ParetoFirstKind}\left(x_{\mathrm{m}}, \alpha, \text{Loc}\right)$$

59.2 Distribution domain

$$x \in [\text{Loc} + x_{\mathrm{m}}, \infty)$$

59.3 Parameters domain and parameters constraints

$$x_{\mathrm{m}} \in \mathbb{R}^{+}, \alpha \in \mathbb{R}^{+}, \text{Loc} \in \mathbb{R}$$

59.4 Cumulative distribution function

$$F_X\left(x\right) = 1 - \left(\frac{x_{\mathrm{m}}}{x - \text{Loc}}\right)^{\alpha}$$

59.5 Probability density function

$$f_X\left(x\right) = \frac{\alpha x_{\mathrm{m}}^{\alpha}}{(x - \text{Loc})^{\alpha+1}}$$

59.6 Percent point function/Sample

$$F_X^{-1}\left(u\right) = \text{Loc} + x_{\mathrm{m}}(1-u)^{-\frac{1}{\alpha}}$$

59.7 Parametric centered moments

$$\tilde{\mu}'_k = E[\tilde{X}^k] = \int_{x_{\mathrm{m}}}^{\infty} x^k f_{\tilde{X}}\left(x\right) dx = \begin{cases} \infty & \text{if } \alpha \leq k \\ \frac{\alpha x_{\mathrm{m}}^k}{\alpha - k} & \text{if } \alpha > k \end{cases}$$

59.8 Parametric mean

$$\text{Mean}(X) = \text{Loc} + \tilde{\mu}'_1 = \text{Loc} + \frac{\alpha x_{\mathrm{m}}}{\alpha - 1} \quad \text{if } \alpha > 1$$

59.9 Parametric variance

$$\text{Variance}(X) = (\tilde{\mu}'_2 - \tilde{\mu}'^2_1) = \frac{x_{\mathrm{m}}^2 \alpha}{(\alpha-1)^2(\alpha-2)} \quad \text{if } \alpha > 2$$

59.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\tilde{\mu}'_3 - 3\tilde{\mu}'_2\tilde{\mu}'_1 + 2\tilde{\mu}'^3_1}{(\tilde{\mu}'_2 - \tilde{\mu}'^2_1)^{1.5}} = \frac{2(1+\alpha)}{\alpha - 3}\sqrt{\frac{\alpha-2}{\alpha}} \quad \text{if } \alpha > 3$$

59.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\tilde{\mu}'_4 - 4\tilde{\mu}'_1\tilde{\mu}'_3 + 6\tilde{\mu}'^2_1\tilde{\mu}'_2 - 3\tilde{\mu}'^4_1}{(\tilde{\mu}'_2 - \tilde{\mu}'^2_1)^2} = \frac{6(\alpha^3 + \alpha^2 - 6\alpha - 2)}{\alpha(\alpha-3)(\alpha-4)} \quad \text{if } \alpha > 4$$

59.12 Parametric median

$$\text{Median}(X) = \text{Loc} + x_{\mathrm{m}}\sqrt[\alpha]{2}$$

59.13 Parametric mode

$$\text{Mode}(X) = \text{Loc} + x_{\mathrm{m}}$$

59.14 Additional information and definitions

- $\tilde{X} \sim \text{ParetoFirstKind}\,(x_{\text{m}}, \alpha, 0)$
- Loc : Location parameter
- x_{m} : Scale parameter
- u : Uniform[0,1] random varible

59.15 Spreadsheet documents

- Excel file from GitHub repository
- Google spreadsheet document

60 Pareto Second Kind Distribution

60.1 Distribution definition

$$X \sim \text{ParetoSecondKind}\left(x_{\mathrm{m}}, \alpha, \mathrm{Loc}\right)$$

60.2 Distribution domain

$$x \in (\mathrm{Loc}, \infty)$$

60.3 Parameters domain and parameters constraints

$$x_{\mathrm{m}} \in \mathbb{R}^{+}, \alpha \in \mathbb{R}^{+}, \mathrm{Loc} \in \mathbb{R}$$

60.4 Cumulative distribution function

$$F_X\left(x\right) = 1 - \left[1 + \frac{x - \mathrm{Loc}}{x_{\mathrm{m}}}\right]^{-\alpha}$$

60.5 Probability density function

$$f_X\left(x\right) = \frac{\alpha}{x_{\mathrm{m}}}\left[1 + \frac{x - \mathrm{Loc}}{x_{\mathrm{m}}}\right]^{-(\alpha+1)}$$

60.6 Percent point function/Sample

$$F_X^{-1}\left(u\right) = \mathrm{Loc} + x_{\mathrm{m}}\left[(1-p)^{-\frac{1}{\alpha}} - 1\right]$$

60.7 Parametric centered moments

$$\tilde{\mu}'_k = E[\tilde{X}^k] = \int_0^{\infty} x^k f_{\tilde{X}}\left(x\right) dx = \frac{x_{\mathrm{m}}^k \Gamma(\alpha - k)\Gamma(1+k)}{\Gamma(\alpha)}$$

60.8 Parametric mean

$$\mathrm{Mean}(X) = \tilde{\mu}'_1 = \frac{x_{\mathrm{m}}}{\alpha - 1} \quad \text{if } \alpha > 1$$

60.9 Parametric variance

$$\mathrm{Variance}(X) = \tilde{\mu}'_2 - \tilde{\mu}'^2_1 = \frac{x_{\mathrm{m}}^2 \alpha}{(\alpha-1)^2(\alpha-2)} \quad \text{if } \alpha > 2$$

60.10 Parametric skewness

$$\mathrm{Skewness}(X) = \frac{\tilde{\mu}'_3 - 3\tilde{\mu}'_2\tilde{\mu}'_1 + 2\tilde{\mu}'^3_1}{(\tilde{\mu}'_2 - \tilde{\mu}'^2_1)^{1.5}} = \frac{2(1+\alpha)}{\alpha - 3}\sqrt{\frac{\alpha - 2}{\alpha}} \quad \text{if } \alpha > 3$$

60.11 Parametric kurtosis

$$\mathrm{Kurtosis}(X) = \frac{\tilde{\mu}'_4 - 4\tilde{\mu}'_1\tilde{\mu}'_3 + 6\tilde{\mu}'^2_1\tilde{\mu}'_2 - 3\tilde{\mu}'^4_1}{(\tilde{\mu}'_2 - \tilde{\mu}'^2_1)^2} = \frac{6(\alpha^3 + \alpha^2 - 6\alpha - 2)}{\alpha(\alpha-3)(\alpha-4)} \quad \text{if } \alpha > 4$$

60.12 Parametric median

$$\mathrm{Median}(X) = x_{\mathrm{m}}\left(\sqrt[\alpha]{2} - 1\right)$$

60.13 Parametric mode

$$\mathrm{Mode}(X) = 0$$

60.14 Additional information and definitions

- $X \sim \text{ParetoSecondKind}\left(x_{\text{m}}, \alpha, 0\right)$
- x_{m} : Scale parameter
- u : Uniform[0,1] random varible
- $\Gamma\left(x\right)$: Gamma function

60.15 Spreadsheet documents

- Excel file from GitHub repository
- Google spreadsheet document

61 Pert Distribution

61.1 Distribution definition

$$X \sim \text{Pert}\,(a, b, c)$$

61.2 Distribution domain

$$x \in [a, c]$$

61.3 Parameters domain and parameters constraints

$$a \in \mathbb{R}, b \in \mathbb{R}, c \in \mathbb{R}, a < b < c$$

61.4 Cumulative distribution function

$$F_X\,(x) = I(z(x), \alpha_1, \alpha_2)$$

61.5 Probability density function

$$f_X\,(x) = \frac{(x-a)^{\alpha_1 - 1}(c-x)^{\alpha_2 - 1}}{\text{Beta}(\alpha_1, \alpha_2)(c-a)^{\alpha_1 + \alpha_2 - 1}}$$

61.6 Percent point function/Sample

$$F_X^{-1}\,(u) = a + (c-a) \cdot I^{-1}\,(u, \alpha_1, \alpha_2)$$

61.7 Parametric centered moments

$$\mu'_k = E[X^k] = \int_a^c x^k f_X\,(x)\,dx$$

61.8 Parametric mean

$$\text{Mean}(X) = \mu'_1 = \frac{a + 4b + c}{6}$$

61.9 Parametric variance

$$\text{Variance}(X) = \mu'_2 - \mu'^2_1 = \frac{(\text{Mean}(X) - a)(c - \text{Mean}(X))}{7}$$

61.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\mu'_3 - 3\mu'_2\mu'_1 + 2\mu'^3_1}{(\mu'_2 - \mu'^2_1)^{1.5}} = \frac{2\,(\alpha_2 - \alpha_1)\sqrt{\alpha_1 + \alpha_2 + 1}}{(\alpha_1 + \alpha_2 + 2)\sqrt{\alpha_1\alpha_2}}$$

61.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\mu'_4 - 4\mu'_1\mu'_3 + 6\mu'^2_1\mu'_2 - 3\mu'^4_1}{(\mu'_2 - \mu'^2_1)^2} = \frac{6[(\alpha_1 - \alpha_2)^2(\alpha_1 + \alpha_2 + 1) - \alpha_1\alpha_2(\alpha_1 + \alpha_2 + 2)]}{\alpha_1\alpha_2(\alpha_1 + \alpha_2 + 2)(\alpha_1 + \alpha_2 + 3)} + 3$$

61.12 Parametric median

$$\text{Median}(X) = a + (c-a) \cdot I^{-1}\left(\frac{1}{2}, \alpha_1, \alpha_2\right)$$

61.13 Parametric mode

$$\text{Mode}(X) = b$$

61.14 Additional information and definitions

- $z(x) = (x-a)/(c-a)$
- u : Uniform[0,1] random varible
- $\alpha_1 = \frac{4b+c-5a}{c-a}, \alpha_2 = \frac{5c-a-4b}{c-a}$
- $I(x,a,b)$: Regularized incomplete beta function
- $I^{-1}(x,a,b)$: Inverse of regularized incomplete beta function
- $\mathrm{Beta}(x,y)$: Beta function

61.15 Spreadsheet documents

- Excel file from GitHub repository
- Google spreadsheet document

62 Power Function Distribution

62.1 Distribution definition

$$X \sim \text{PowerFunction}\left(\alpha, a, b\right)$$

62.2 Distribution domain

$$x \in [a, b]$$

62.3 Parameters domain and parameters constraints

$$\alpha \in \mathbb{R}^{+}, a \in \mathbb{R}, b \in \mathbb{R}, a < b$$

62.4 Cumulative distribution function

$$F_X\left(x\right) = \left(\frac{x-a}{b-a}\right)^{\alpha}$$

62.5 Probability density function

$$f_X\left(x\right) = \frac{\alpha(x-a)^{\alpha-1}}{(b-a)^{\alpha}}$$

62.6 Percent point function/Sample

$$F_X^{-1}\left(u\right) = \left[a + u(b-a)\right]^{-\alpha}$$

62.7 Parametric centered moments

$$\mu'_k = E[X^k] = \int_a^b x^k f_X\left(x\right) dx$$

62.8 Parametric mean

$$\text{Mean}(X) = \mu'_1 = \frac{a + b\alpha}{\alpha + 1}$$

62.9 Parametric variance

$$\text{Variance}(X) = \mu'_2 - \mu'^2_1 = \frac{2a^2 + 2ab\alpha + b^2\alpha(\alpha+1)}{(\alpha+1)(\alpha+2)} - \text{Mean}(X)^2$$

62.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\mu'_3 - 3\mu'_2\mu'_1 + 2\mu'^3_1}{(\mu'_2 - \mu'^2_1)^{1.5}} = 2\left(1-\alpha\right)\sqrt{\frac{\alpha+2}{\alpha\left(\alpha+3\right)}}$$

62.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\mu'_4 - 4\mu'_1\mu'_3 + 6\mu'^2_1\mu'_2 - 3\mu'^4_1}{(\mu'_2 - \mu'^2_1)^2} = \frac{6\left(\alpha^3 - \alpha^2 - 6\alpha + 2\right)}{\alpha\left(\alpha+3\right)\left(\alpha+4\right)} + 3$$

62.12 Parametric median

$$\text{Median}(X) = \left[a + \frac{1}{2}(b-a)\right]^{-\alpha}$$

62.13 Parametric mode

$$\text{Mode}(X) = \text{undefined}$$

62.14 Additional information and definitions

- a : Location parameter
- $b - a$: Scale parameter
- u : Uniform[0,1] random varible

62.15 Spreadsheet documents

- Excel file from GitHub repository
- Google spreadsheet document

63 Rayleigh Distribution

63.1 Distribution definition

$$X \sim \text{Rayleigh}\,(\gamma, \sigma)$$

63.2 Distribution domain

$$x \in [\gamma, \infty)$$

63.3 Parameters domain and parameters constraints

$$\gamma \in \mathbb{R}, \sigma \in \mathbb{R}^{+}$$

63.4 Cumulative distribution function

$$F_X\,(x) = 1 - e^{-z(x)^2/2}$$

63.5 Probability density function

$$f_X\,(x) = z(x) \times e^{-z(x)^2/2}/\sigma$$

63.6 Percent point function/Sample

$$F_X^{-1}\,(u) = \gamma + \sigma\sqrt{-2\log\left(1-u\right)}$$

63.7 Parametric centered moments

$$\mu_k' = E[\tilde{X}^k] = \int_0^\infty x^k f_{\tilde{X}}\,(x)\,dx = \sqrt{2^k}\Gamma\left(\frac{k}{2}+1\right)$$

63.8 Parametric mean

$$\text{Mean}(X) = \gamma + \sigma \cdot \mu_1' = \gamma + \sigma\sqrt{\frac{\pi}{2}}$$

63.9 Parametric variance

$$\text{Variance}(X) = \sigma^2(\mu_2' - \mu_1'^2) = \sigma^2\frac{4-\pi}{2}$$

63.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\mu_3' - 3\mu_2'\mu_1' + 2\mu_1'^3}{(\mu_2' - \mu_1'^2)^{1.5}} = \frac{2\left(\pi-3\right)\sqrt{\pi}}{\left(4-\pi\right)^{3/2}}$$

63.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\mu_4' - 4\mu_1'\mu_3' + 6\mu_1'^2\mu_2' - 3\mu_1'^4}{(\mu_2' - \mu_1'^2)^2} = 3 + \frac{24\pi - 6\pi^2 - 16}{\left(4-\pi\right)^2}$$

63.12 Parametric median

$$\text{Median}(X) = \gamma + \sigma\sqrt{-2\log\left(\frac{1}{2}\right)}$$

63.13 Parametric mode

$$\text{Mode}(X) = \gamma + \sigma$$

63.14 Additional information and definitions

- $\tilde{X} \sim \text{Rayleigh}\,(0,1)$
- γ : Location parameter
- σ : Scale parameter
- $z\,(x) = (x-\gamma)\,/\sigma$
- u : Uniform[0,1] random varible
- $\Gamma\,(x)$: Gamma function

63.15 Spreadsheet documents

- Excel file from GitHub repository
- Google spreadsheet document

64 Reciprocal Distribution

64.1 Distribution definition

$$X \sim \text{Reciprocal}\,(a, b)$$

64.2 Distribution domain

$$x \in [a, b]$$

64.3 Parameters domain and parameters constraints

$$a \in \mathbb{R}^+, b \in \mathbb{R}^+, a < b$$

64.4 Cumulative distribution function

$$F_X\,(x) = \frac{\ln(x) - \ln(a)}{\ln(b) - \ln(a)}$$

64.5 Probability density function

$$f_X\,(x) = \frac{1}{x\,(\ln(b) - \ln(a))}$$

64.6 Percent point function/Sample

$$F_X^{-1}\,(u) = \exp(\ln(a) + u \times (\ln(b) - \ln(a)))$$

64.7 Parametric centered moments

$$\mu_k' = E[X^k] = \int_a^b x^k f_X\,(x)\,dx = \frac{b^k - a^k}{k\,(\ln(b) - \ln(a))}$$

64.8 Parametric mean

$$\text{Mean}(X) = \mu_1'$$

64.9 Parametric variance

$$\text{Variance}(X) = \mu_2' - \mu_1'^2$$

64.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\mu_3' - 3\mu_2'\mu_1' + 2\mu_1'^3}{(\mu_2' - \mu_1'^2)^{1.5}}$$

64.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\mu_4' - 4\mu_1'\mu_3' + 6\mu_1'^2\mu_2' - 3\mu_1'^4}{(\mu_2' - \mu_1'^2)^2}$$

64.12 Parametric median

$$\text{Median}(X) = \exp\left[\ln(a) + \frac{(\ln(b) - \ln(a))}{2}\right]$$

64.13 Parametric mode

$$\text{Mode}(X) = a$$

64.14 Additional information and definitions

- u : Uniform[0,1] random varible

64.15 Spreadsheet documents

- Excel file from GitHub repository
- Google spreadsheet document

65 Rice Distribution

65.1 Distribution definition

$$X \sim \text{Rice}\,(v, \sigma)$$

65.2 Distribution domain

$$x \in [0, \infty)$$

65.3 Parameters domain and parameters constraints

$$v \in \mathbb{R}^+, \sigma \in \mathbb{R}^+$$

65.4 Cumulative distribution function

$$F_X\,(x) = 1 - Q_1\left(\frac{v}{\sigma}, \frac{x}{\sigma}\right)$$

65.5 Probability density function

$$f_X\,(x) = \frac{x}{\sigma^2} \exp\left(\frac{-(x^2+v^2)}{2\sigma^2}\right) I_0\left(\frac{xv}{\sigma^2}\right)$$

65.6 Percent point function/Sample

$$\text{Sample}_X = \sqrt{\Phi^{-1}(u_1, v, \sigma)^2 + \Phi^{-1}(u_2, 0, \sigma)^2}$$

65.7 Parametric centered moments

$$\mu'_k = E[X^k] = \int_{-\infty}^{\infty} x^k f_X\,(x)\,dx = \sigma^k 2^{k/2}\,\Gamma(1+k/2)\,L_{k/2}(-v^2/2\sigma^2)$$

65.8 Parametric mean

$$\text{Mean}(X) = \mu'_1$$

65.9 Parametric variance

$$\text{Variance}(X) = \mu'_2 - \mu'^2_1$$

65.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\mu'_3 - 3\mu'_2\mu'_1 + 2\mu'^3_1}{(\mu'_2 - \mu'^2_1)^{1.5}}$$

65.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\mu'_4 - 4\mu'_1\mu'_3 + 6\mu'^2_1\mu'_2 - 3\mu'^4_1}{(\mu'_2 - \mu'^2_1)^2}$$

65.12 Parametric median

$$\text{Median}(X) = F_X^{-1}\left(\frac{1}{2}\right)$$

65.13 Parametric mode

$$\text{Mode}(X) = \arg\max_x f_X\,(x)$$

65.14 Additional information and definitions

- Computing an analytic expression for the inverse of the cumulative distribution function is not feasible. Nonetheless, it is possible to generate a random sample from the distribution.
- $\Phi^{-1}(u, mean, variance)$: Inverse of cumulative function from normal distribution
- $L_r(x)$: Laguerre polynomials of order $r \in \mathbb{R}$
- $L_{\frac{1}{2}}(x) = e^{x/2}(x)I_1\left(\frac{x}{2}\right) - e^{x/2}(x-1)I_0\left(\frac{x}{2}\right)$
- $L_{\frac{3}{2}}(x) = \frac{1}{3}e^{x/2}(2x^2 - 6x + 3)I_0(x/2) - \frac{2}{3}e^{x/2}(x-2)xI_1(x/2)$
- $I_\alpha(x)$: Modified Bessel function of the first kind of order $\alpha \in \mathbb{N}$
- $Q_k(a, b)$: Marcum Q-function of order k $\in \mathbb{N}$
- u_1 : Uniform[0,1] random varible
- u_2 : Uniform[0,1] random varible

65.15 Spreadsheet documents

- Excel file from GitHub repository
- Google spreadsheet document

66 Semicircular Distribution

66.1 Distribution definition

$$X \sim \text{Semicircular}\left(\text{Loc}, R\right)$$

66.2 Distribution domain

$$x \in [\text{Loc}, \infty)$$

66.3 Parameters domain and parameters constraints

$$\text{Loc} \in \mathbb{R}, R \in \mathbb{R}^{+}$$

66.4 Cumulative distribution function

$$F_X\left(x\right) = \frac{1}{2} + \frac{z(x)\sqrt{R^2 - z(x)^2}}{\pi R^2} + \frac{\arcsin\left(\frac{z(x)}{R}\right)}{\pi}$$

66.5 Probability density function

$$f_X\left(x\right) = \frac{2}{\pi R^2}\sqrt{R^2 - z(x)^2}$$

66.6 Percent point function/Sample

$$F_X^{-1}\left(u\right) = \text{Loc} + R \times \left(2I^{-1}\left(u, 1.5, 1.5\right) - 1\right)$$

66.7 Parametric centered moments

$$\mu'_k = E[X^k] = \int_{\text{Loc}}^{\infty} x^k f_X\left(x\right) dx$$

66.8 Parametric mean

$$\text{Mean}(X) = \mu'_1 = \text{Loc}$$

66.9 Parametric variance

$$\text{Variance}(X) = \mu'_2 - \mu'^2_1 = \frac{R^2}{4}$$

66.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\mu'_3 - 3\mu'_2\mu'_1 + 2\mu'^3_1}{(\mu'_2 - \mu'^2_1)^{1.5}} = 0$$

66.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\mu'_4 - 4\mu'_1\mu'_3 + 6\mu'^2_1\mu'_2 - 3\mu'^4_1}{(\mu'_2 - \mu'^2_1)^2} = 2$$

66.12 Parametric median

$$\text{Median}(X) = \text{Loc}$$

66.13 Parametric mode

$$\text{Mode}(X) = \text{Loc}$$

66.14 Additional information and definitions

- Loc : Location parameter
- R : Scale parameter
- $z(x) = x - \mathrm{Loc}$
- u : Uniform[0,1] random varible
- $I^{-1}(x, a, b)$: Inverse of regularized incomplete beta function

66.15 Spreadsheet documents

- Excel file from GitHub repository
- Google spreadsheet document

67 T Student Distribution

67.1 Distribution definition

$$X \sim \text{TStudent}\left(\text{df}\right)$$

67.2 Distribution domain

$$x \in (-\infty, \infty)$$

67.3 Parameters domain and parameters constraints

$$\text{df} \in \mathbb{R}^+$$

67.4 Cumulative distribution function

$$F_X\left(x\right) = I\left(\frac{x + \sqrt{x^2 + \text{df}}}{2\sqrt{x^2 + \text{df}}}, \frac{\text{df}}{2}, \frac{\text{df}}{2}\right)$$

67.5 Probability density function

$$f_X\left(x\right) = \frac{\left(1 + x^2/\text{df}\right)^{-(1+\text{df})/2}}{\sqrt{\text{df}} \times \text{Beta}\left(\frac{1}{2}, \frac{\text{df}}{2}\right)}$$

67.6 Percent point function/Sample

$$F_X^{-1}\left(u\right) = \begin{cases} \sqrt{\frac{\text{df}(1 - I^{-1}(u, \text{df}/2, \text{df}/2))}{I^{-1}(u, \text{df}/2, \text{df}/2)}} & \text{if } \ u \geq \frac{1}{2} \\ -\sqrt{\frac{\text{df}(1 - I^{-1}(u, \text{df}/2, \text{df}/2))}{I^{-1}(u, \text{df}/2, \text{df}/2)}} & \text{if } \ u < \frac{1}{2} \end{cases}$$

67.7 Parametric centered moments

$$\mu'_k = E[X^k] = \int_{-\infty}^{\infty} x^k f_X\left(x\right) dx = \begin{cases} 0 & \text{if } \ k \text{ odd } \wedge \ 0 < k < \text{df} \\ \text{df}^{\frac{k}{2}} \ \prod_{i=1}^{k/2} \frac{2i-1}{\text{df}-2i} & \text{if } \ k \text{ even } \wedge \ 0 < k < \text{df} \end{cases}$$

67.8 Parametric mean

$$\text{Mean}(X) = \mu'_1 = 0$$

67.9 Parametric variance

$$\text{Variance}(X) = \mu'_2 - \mu'^2_1 = \begin{cases} \text{df}/(\text{df} + 2) & \text{if } \ \text{df} > 2 \\ \text{undefined} & \text{if } \ \text{df} \leq 2 \end{cases}$$

67.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\mu'_3 - 3\mu'_2\mu'_1 + 2\mu'^3_1}{(\mu'_2 - \mu'^2_1)^{1.5}} = \begin{cases} 0 & \text{if } \ \text{df} > 3 \\ \text{undefined} & \text{if } \ \text{df} \leq 3 \end{cases}$$

67.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\mu'_4 - 4\mu'_1\mu'_3 + 6\mu'^2_1\mu'_2 - 3\mu'^4_1}{(\mu'_2 - \mu'^2_1)^2} = \begin{cases} 3 + 6/(\text{df} - 4) & \text{if } \ \text{df} > 4 \\ \text{undefined} & \text{if } \ \text{df} \leq 4 \end{cases}$$

67.12 Parametric median

$$\text{Median}(X) = 0$$

67.13 Parametric mode

$$\text{Mode}(X) = 0$$

67.14 Additional information and definitions

- u : Uniform[0,1] random varible
- $I(x, a, b)$: Regularized incomplete beta function
- $I^{-1}(x, a, b)$: Inverse of regularized incomplete beta function
- $\mathrm{Beta}(x, y)$: Beta function

67.15 Spreadsheet documents

- Excel file from GitHub repository
- Google spreadsheet document

68 T Student 3P Distribution

68.1 Distribution definition

$$X \sim \text{TStudent}_{\text{3P}}\left(\text{df}, \text{Loc}, \text{Sc}\right)$$

68.2 Distribution domain

$$x \in (-\infty, \infty)$$

68.3 Parameters domain and parameters constraints

$$\text{df} \in \mathbb{R}^{+}, \text{Loc} \in \mathbb{R}, \text{Sc} \in \mathbb{R}^{+}$$

68.4 Cumulative distribution function

$$F_X\left(x\right) = I\left(\frac{z(x) + \sqrt{z(x)^2 + \text{df}}}{2\sqrt{z(x)^2 + \text{df}}}, \frac{\text{df}}{2}, \frac{\text{df}}{2}\right)$$

68.5 Probability density function

$$f_X\left(x\right) = \frac{\left(1 + z(x)^2/\text{df}\right)^{-(1+\text{df})/2}}{\sqrt{\text{df}} \times \text{Beta}\left(\frac{1}{2}, \frac{\text{df}}{2}\right)}$$

68.6 Percent point function/Sample

$$F_X^{-1}\left(u\right) = \begin{cases} \text{Loc} + \text{Sc}\sqrt{\frac{\text{df}(1-I^{-1}(u,\text{df}/2,\text{df}/2))}{I^{-1}(u,\text{df}/2,\text{df}/2)}} & \text{if } \ u \geq \frac{1}{2} \\ \text{Loc} - \text{Sc}\sqrt{\frac{\text{df}(1-I^{-1}(u,\text{df}/2,\text{df}/2))}{I^{-1}(u,\text{df}/2,\text{df}/2)}} & \text{if } \ u < \frac{1}{2} \end{cases}$$

68.7 Parametric centered moments

$$\tilde{\mu}'_k = E[\tilde{X}^k] = \int_0^{\infty} x^k f_{\tilde{X}}\left(x\right) dx = \begin{cases} 0 & \text{if } \ k \text{ odd } \wedge \ 0 < k < \text{df} \\ \text{df}^{\frac{k}{2}} \prod_{i=1}^{k/2} \frac{2i-1}{\text{df}-2i} & \text{if } \ k \text{ even } \wedge \ 0 < k < \text{df} \end{cases}$$

68.8 Parametric mean

$$\text{Mean}(X) = \text{Loc} + \text{Sc} \cdot \tilde{\mu}'_1 = \text{Loc}$$

68.9 Parametric variance

$$\text{Variance}(X) = \text{Sc}^2 \times (\tilde{\mu}'_2 - \tilde{\mu}'^2_1) = \begin{cases} \text{Sc}^2 \ \text{df}/(\text{df}+2) & \text{if } \ \text{df} > 2 \\ \text{undefined} & \text{if } \ \text{df} \leq 2 \end{cases}$$

68.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\tilde{\mu}'_3 - 3\tilde{\mu}'_2\tilde{\mu}'_1 + 2\tilde{\mu}'^3_1}{(\tilde{\mu}'_2 - \tilde{\mu}'^2_1)^{1.5}} = \begin{cases} 0 & \text{if } \ \text{df} > 3 \\ \text{undefined} & \text{if } \ \text{df} \leq 3 \end{cases}$$

68.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\tilde{\mu}'_4 - 4\tilde{\mu}'_1\tilde{\mu}'_3 + 6\tilde{\mu}'^2_1\tilde{\mu}'_2 - 3\tilde{\mu}'^4_1}{(\tilde{\mu}'_2 - \tilde{\mu}'^2_1)^2} = \begin{cases} 3 + 6/(\text{df}-4) & \text{if } \ \text{df} > 4 \\ \text{undefined} & \text{if } \ \text{df} \leq 4 \end{cases}$$

68.12 Parametric median

$$\text{Median}(X) = \text{Loc}$$

68.13 Parametric mode

$$\mathrm{Mode}(X) = \mathrm{Loc}$$

68.14 Additional information and definitions

- $\tilde{X} \sim \mathrm{TStudent}\,(\mathrm{df})$
- Loc : Location parameter
- Sc : Scale parameter
- $z\,(x) = (x - \mathrm{Loc})\,/\mathrm{Sc}$
- u : Uniform[0,1] random varible
- $I\,(x, a, b)$: Regularized incomplete beta function
- $I^{-1}\,(x, a, b)$: Inverse of regularized incomplete beta function
- $\mathrm{Beta}\,(x, y)$: Beta function

68.15 Spreadsheet documents

- Excel file from GitHub repository
- Google spreadsheet document

69 Trapezoidal Distribution

69.1 Distribution definition

$$X \sim \text{Trapezoidal}\,(a, b, c, d)$$

69.2 Distribution domain

$$x \in [a, d]$$

69.3 Parameters domain and parameters constraints

$$a \in \mathbb{R}, b \in \mathbb{R}, c \in \mathbb{R}, d \in \mathbb{R}, a < b < c, b < c < d$$

69.4 Cumulative distribution function

$$F_X\,(x) = \begin{cases} \frac{1}{d+c-a-b}\frac{1}{b-a}(x-a)^2 & \text{if } \ a \leq x < b \\ \frac{1}{d+c-a-b}(2x-a-b) & \text{if } \ b \leq x < c \\ 1 - \frac{1}{d+c-a-b}\frac{1}{d-c}(d-x)^2 & \text{if } \ c \leq x \leq d \end{cases}$$

69.5 Probability density function

$$f_X\,(x) = \begin{cases} \frac{2}{d+c-a-b}\frac{x-a}{b-a} & \text{if } \ a \leq x < b \\ \frac{2}{d+c-a-b} & \text{if } \ b \leq x < c \\ \frac{2}{d+c-a-b}\frac{d-x}{d-c} & \text{if } \ c \leq x \leq d \end{cases}$$

69.6 Percent point function/Sample

$$F_X^{-1}\,(u) = \begin{cases} a + \sqrt{u \times (d+c-a-b) \times (b-a)} & \text{if } u \leq A_1 \\ (a + b + u \times (d+c-a-b))/2 & \text{if } A_1 \leq u \leq A_1 + A_2 \\ d - \sqrt{(1-u) \times (d+c-a-b) \times (d-c)} & \text{if } A_1 + A_2 \leq u \leq A_1 + A_2 + A_3 \end{cases}$$

69.7 Parametric centered moments

$$\mu'_k = E[X^k] = \int_a^b x^k f_X\,(x)\,dx = \frac{2}{d+c-b-a}\frac{1}{(k+1)(k+2)}\left(\frac{d^{k+2}-c^{k+2}}{d-c} - \frac{b^{k+2}-a^{k+2}}{b-a}\right)$$

69.8 Parametric mean

$$\text{Mean}(X) = \mu'_1$$

69.9 Parametric variance

$$\text{Variance}(X) = \mu'_2 - \mu'^2_1$$

69.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\mu'_3 - 3\mu'_2\mu'_1 + 2\mu'^3_1}{(\mu'_2 - \mu'^2_1)^{1.5}}$$

69.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\mu'_4 - 4\mu'_1\mu'_3 + 6\mu'^2_1\mu'_2 - 3\mu'^4_1}{(\mu'_2 - \mu'^2_1)^2}$$

69.12 Parametric median

$$\text{Median}(X) = F_X^{-1}\,(1/2)$$

69.13 Parametric mode

$$\text{Mode}(X) \in [b, c]$$

69.14 Additional information and definitions

- u : Uniform[0,1] random varible
- $A_1 = (b-a)/(d+c-a-b)$
- $A_2 = 2(c-b)/(d+c-a-b)$
- $A_3 = (d-c)/(d+c-a-b)$

69.15 Spreadsheet documents

- Excel file from GitHub repository
- Google spreadsheet document

70 Triangular Distribution

70.1 Distribution definition

$$X \sim \text{Triangular}\,(a, b, c)$$

70.2 Distribution domain

$$x \in [a, b]$$

70.3 Parameters domain and parameters constraints

$$a \in \mathbb{R}, b \in \mathbb{R}, c \in \mathbb{R}, a < c < b$$

70.4 Cumulative distribution function

$$F_X\,(x) = \begin{cases} \frac{(x-a)^2}{(b-a)(c-a)} & \text{if} \;\; a < x \leq c \\ 1 - \frac{(b-x)^2}{(b-a)(b-c)} & \text{if} \;\; c < x < b \end{cases}$$

70.5 Probability density function

$$f_X\,(x) = \begin{cases} \frac{2(x-a)}{(b-a)(c-a)} & \text{if} \;\; a \leq x < c, \\ \frac{2(b-x)}{(b-a)(b-c)} & \text{if} \;\; c \leq x \leq b, \end{cases}$$

70.6 Percent point function/Sample

$$F_X^{-1}\,(u) = \begin{cases} a + \sqrt{U(b-a)(c-a)} & \text{if} \;\; 0 < U < \frac{c-a}{b-a} \\ b - \sqrt{(1-U)(b-a)(b-c)} & \text{if} \;\; \frac{c-a}{b-a} \leq U < 1 \end{cases}$$

70.7 Parametric centered moments

$$\mu'_k = E[X^k] = \int_a^b x^k f_X\,(x)\,dx$$

70.8 Parametric mean

$$\text{Mean}(X) = \mu'_1 = \frac{a+b+c}{3}$$

70.9 Parametric variance

$$\text{Variance}(X) = \mu'_2 - \mu'^2_1 = \frac{a^2+b^2+c^2-ab-ac-bc}{18}$$

70.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\mu'_3 - 3\mu'_2\mu'_1 + 2\mu'^3_1}{(\mu'_2 - \mu'^2_1)^{1.5}} = \frac{\sqrt{2}(a+b-2c)(2a-b-c)(a-2b+c)}{5(a^2+b^2+c^2-ab-ac-bc)^{\frac{3}{2}}}$$

70.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\mu'_4 - 4\mu'_1\mu'_3 + 6\mu'^2_1\mu'_2 - 3\mu'^4_1}{(\mu'_2 - \mu'^2_1)^2} = 3 - \frac{3}{5}$$

70.12 Parametric median

$$\text{Median}(X) = \begin{cases} a + \sqrt{\frac{(b-a)(c-a)}{2}} & \text{if } c \geq \frac{a+b}{2} \\ b - \sqrt{\frac{(b-a)(b-c)}{2}} & \text{if } c \leq \frac{a+b}{2} \end{cases}$$

70.13 Parametric mode

$$\text{Mode}(X) \in [b, c]$$

70.14 Additional information and definitions

- u : Uniform[0,1] random varible

70.15 Spreadsheet documents

- Excel file from GitHub repository
- Google spreadsheet document

71 Uniform Distribution

71.1 Distribution definition

$$X \sim \text{Uniform}\,(a, b)$$

71.2 Distribution domain

$$x \in [a, b]$$

71.3 Parameters domain and parameters constraints

$$a \in \mathbb{R}, b \in \mathbb{R}, a < b$$

71.4 Cumulative distribution function

$$F_X\,(x) = \frac{x - a}{b - a}$$

71.5 Probability density function

$$f_X\,(x) = \frac{1}{b - a}$$

71.6 Percent point function/Sample

$$F_X^{-1}\,(u) = a + u \cdot (b - a)$$

71.7 Parametric centered moments

$$\mu'_k = E[X^k] = \int_{-\infty}^{\infty} x^k f_X\,(x)\,dx = \frac{1}{k+1} \sum_{i=0}^{k} a^i b^{k-i}$$

71.8 Parametric mean

$$\text{Mean}(X) = \mu'_1 = \tfrac{1}{2}(a + b)$$

71.9 Parametric variance

$$\text{Variance}(X) = \mu'_2 - \mu'^2_1$$

71.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\mu'_3 - 3\mu'_2\mu'_1 + 2\mu'^3_1}{(\mu'_2 - \mu'^2_1)^{1.5}} = 0$$

71.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\mu'_4 - 4\mu'_1\mu'_3 + 6\mu'^2_1\mu'_2 - 3\mu'^4_1}{(\mu'_2 - \mu'^2_1)^2} = 3 - \frac{6}{5}$$

71.12 Parametric median

$$\text{Median}(X) = \tfrac{1}{2}(a + b)$$

71.13 Parametric mode

$$\text{Mode}(X) \in [a, b]$$

71.14 Additional information and definitions

- u : Uniform[0,1] random varible

71.15 Spreadsheet documents

- Excel file from GitHub repository
- Google spreadsheet document

72 Weibull Distribution

72.1 Distribution definition

$$X \sim \mathrm{Weibull}\left(\alpha, \beta\right)$$

72.2 Distribution domain

$$x \in [0, \infty)$$

72.3 Parameters domain and parameters constraints

$$\alpha \in \mathbb{R}^{+}, \beta \in \mathbb{R}^{+}$$

72.4 Cumulative distribution function

$$F_X\left(x\right) = 1 - e^{-(x/\beta)^{\alpha}}$$

72.5 Probability density function

$$f_X\left(x\right) = \frac{\alpha}{\beta}\left(\frac{x}{\beta}\right)^{\alpha-1} e^{-(x/\beta)^{\alpha}}$$

72.6 Percent point function/Sample

$$F_X^{-1}\left(u\right) = \beta(-\ln(1-u))^{1/\alpha}$$

72.7 Parametric centered moments

$$\mu'_k = E[X^k] = \int_0^{\infty} x^k f_X\left(x\right) dx = \beta^{\alpha}\Gamma\left(1 + \frac{k}{\alpha}\right)$$

72.8 Parametric mean

$$\mathrm{Mean}(X) = \mu'_1 = \beta \cdot \Gamma(1 + 1/\alpha)$$

72.9 Parametric variance

$$\mathrm{Variance}(X) = \mu'_2 - \mu_1'^2 = \beta^2\left[\Gamma\left(1 + 2/\alpha\right) - \left(\Gamma\left(1 + 1/\alpha\right)\right)^2\right]$$

72.10 Parametric skewness

$$\mathrm{Skewness}(X) = \frac{\mu'_3 - 3\mu'_2\mu'_1 + 2\mu_1'^3}{(\mu'_2 - \mu_1'^2)^{1.5}}$$

72.11 Parametric kurtosis

$$\mathrm{Kurtosis}(X) = \frac{\mu'_4 - 4\mu'_1\mu'_3 + 6\mu_1'^2\mu'_2 - 3\mu_1'^4}{(\mu'_2 - \mu_1'^2)^2}$$

72.12 Parametric median

$$\mathrm{Median}(X) = \beta(\ln(2))^{1/\alpha}$$

72.13 Parametric mode

$$\mathrm{Mode}(X) = \begin{cases} \beta\left(\frac{\alpha-1}{\alpha}\right)^{1/\alpha} & \text{if } \alpha > 1 \\ 0 & \text{if } \alpha \leq 1 \end{cases}$$

72.14 Additional information and definitions

- β : Scale parameter
- u : Uniform[0,1] random varible
- $\Gamma(x)$: Gamma function

72.15 Spreadsheet documents

- Excel file from GitHub repository
- Google spreadsheet document

73 Weibull 3P Distribution

73.1 Distribution definition

$$X \sim \text{Weibull}_{\text{3P}}\left(\alpha, \text{Loc}, \beta\right)$$

73.2 Distribution domain

$$x \in [\text{Loc}, \infty)$$

73.3 Parameters domain and parameters constraints

$$\alpha \in \mathbb{R}^{+}, \text{Loc} \in \mathbb{R}, \beta \in \mathbb{R}^{+}$$

73.4 Cumulative distribution function

$$F_X\left(x\right) = 1 - e^{-z(x)^{\alpha}}$$

73.5 Probability density function

$$f_X\left(x\right) = \frac{\alpha}{\beta} z(x)^{\alpha-1} e^{-z(x)^{\alpha}}$$

73.6 Percent point function/Sample

$$F_X^{-1}\left(u\right) = \text{Loc} + \beta(-\ln(1-u))^{1/\alpha}$$

73.7 Parametric centered moments

$$\tilde{\mu}'_k = E[\tilde{X}^k] = \int_0^{\infty} x^k f_{\tilde{X}}\left(x\right) dx = \beta^{\alpha} \Gamma\left(1 + \frac{k}{\alpha}\right)$$

73.8 Parametric mean

$$\text{Mean}(X) = \text{Loc} + \tilde{\mu}'_1 = \text{Loc} + \beta\ \Gamma(1 + 1/\alpha)$$

73.9 Parametric variance

$$\text{Variance}(X) = \tilde{\mu}'_2 - \tilde{\mu}'^2_1 = \beta^2 \left[\Gamma\left(1 + 2/\alpha\right) - \left(\Gamma\left(1 + 1/\alpha\right)\right)^2\right]$$

73.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\tilde{\mu}'_3 - 3\tilde{\mu}'_2\tilde{\mu}'_1 + 2\tilde{\mu}'^3_1}{(\tilde{\mu}'_2 - \tilde{\mu}'^2_1)^{1.5}}$$

73.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\tilde{\mu}'_4 - 4\tilde{\mu}'_1\tilde{\mu}'_3 + 6\tilde{\mu}'^2_1\tilde{\mu}'_2 - 3\tilde{\mu}'^4_1}{(\tilde{\mu}'_2 - \tilde{\mu}'^2_1)^2}$$

73.12 Parametric median

$$\text{Median}(X) = \text{Loc} + \beta(\ln(2))^{1/\alpha}$$

73.13 Parametric mode

$$\text{Mode}(X) = \text{Loc} + \begin{cases} \beta\left(\frac{\alpha-1}{\alpha}\right)^{1/\alpha} & \text{if } \alpha > 1 \\ 0 & \text{if } \alpha \leq 1 \end{cases}$$

73.14 Additional information and definitions

- $\tilde{X} \sim \text{Weibull}\,(\alpha, \beta)$
- Loc : Location parameter
- β : Scale parameter
- $z\,(x) = (x - \text{Loc})\,/\beta$
- u : Uniform[0,1] random varible
- $\Gamma\,(x)$: Gamma function

73.15 Spreadsheet documents

- Excel file from GitHub repository
- Google spreadsheet document

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