

# CONTINUOUS PROBABILITY DISTRIBUTIONS

<https://phitter.io>

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## **Abstract**

This document provides an overview of the continuous probability distributions utilized in Phitter. It includes a detailed description for each distribution, covering aspects such as the definition, domain, parameter definitions and domains, probability density function, cumulative distribution function, percentile point function, raw moments, mean, variance, skewness, kurtosis, median, and mode in a concise and clear manner.

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# 1 Alpha Distribution

## 1.1 Distribution definition

$$X \sim \text{Alpha}(\alpha, \text{Loc}, \text{Sc})$$

## 1.2 Distribution domain

$$x \in (\text{Loc}, \infty)$$

## 1.3 Parameters domain and parameters constraints

$$\alpha \in \mathbb{R}^+, \text{Loc} \in \mathbb{R}, \text{Sc} \in \mathbb{R}^+$$

## 1.4 Cumulative distribution function

$$F_X(x) = \frac{\Phi\left(\alpha - \frac{1}{z(x)}\right)}{\Phi(\alpha)}$$

## 1.5 Probability density function

$$f_X(x) = \frac{1}{\text{Sc} \cdot z(x)^2 \cdot \Phi(\alpha) \cdot \sqrt{2\pi}} \exp\left(-\frac{1}{2} \left(\alpha - \frac{1}{z(x)}\right)^2\right)$$

## 1.6 Percent point function/Sample

$$F_X^{-1}(u) = \text{Loc} + \text{Sc} \times \frac{1}{\alpha - \Phi^{-1}(u\Phi(\alpha))}$$

## 1.7 Parametric centered moments

$$\tilde{\mu}'_k = E[\tilde{X}^k] = \int_0^\infty x^k f_{\tilde{X}}(x) dx$$

## 1.8 Parametric mean

$$\text{Mean}(X) = \text{Loc} + \text{Sc} \cdot \tilde{\mu}'_1$$

## 1.9 Parametric variance

$$\text{Variance}(X) = \text{Sc}^2 \cdot (\tilde{\mu}'_2 - \tilde{\mu}'_1^2)$$

## 1.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\tilde{\mu}'_3 - 3\tilde{\mu}'_2\tilde{\mu}'_1 + 2\tilde{\mu}'_1^3}{(\tilde{\mu}'_2 - \tilde{\mu}'_1^2)^{1.5}}$$

## 1.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\tilde{\mu}'_4 - 4\tilde{\mu}'_1\tilde{\mu}'_3 + 6\tilde{\mu}'_1^2\tilde{\mu}'_2 - 3\tilde{\mu}'_1^4}{(\tilde{\mu}'_2 - \tilde{\mu}'_1^2)^2}$$

## 1.12 Parametric median

$$\text{Median}(X) = \text{Loc} + \frac{\text{Sc}}{\alpha - \Phi^{-1}\left(\frac{1}{2}\Phi(\alpha)\right)}$$

## 1.13 Parametric mode

$$\text{Mode}(X) = \text{Loc} + \text{Sc} \frac{(\sqrt{\alpha^2 + 8} - \alpha)}{4}$$



#### 1.14 Additional information and definitions

- $\tilde{X} \sim \text{Alpha}(\alpha, 0, 1)$
- Loc : Location parameter
- Sc : Scale parameter
- $z(x) = (x - \text{Loc}) / \text{Sc}$
- $u : \text{Uniform}[0,1]$  random variable
- $\Phi(x) : \text{CDF normal standard distribution}$
- $\Phi^{-1}(x) : \text{PPF normal standard distribution}$

#### 1.15 Spreadsheet documents

- [Excel file from GitHub repository](#)
- [Google spreadsheet document](#)

## 2 Arcsine Distribution

### 2.1 Distribution definition

$$X \sim \text{Arcsine}(a, b)$$

### 2.2 Distribution domain

$$x \in (a, b)$$

### 2.3 Parameters domain and parameters constraints

$$a \in \mathbb{R}, b \in \mathbb{R}, a < b$$

### 2.4 Cumulative distribution function

$$F_X(x) = \frac{2}{\pi} \arcsin \left( \sqrt{\frac{x-a}{b-a}} \right)$$

### 2.5 Probability density function

$$f_X(x) = \frac{1}{\pi \sqrt{(x-a)(b-x)}}$$

### 2.6 Percent point function/Sample

$$F_X^{-1}(u) = a + (b-a) \times \sin^2 \left( \frac{\pi}{2} u \right)$$

### 2.7 Parametric centered moments

$$\tilde{\mu}'_k = E[\tilde{X}^k] = \int_0^1 x^k f_{\tilde{X}}(x) dx = \frac{1}{\pi} \text{Beta} \left( \frac{1}{2}, k + \frac{1}{2} \right) = \frac{(2k-1)!!}{2^k k!}$$

### 2.8 Parametric mean

$$\text{Mean}(X) = a + \tilde{\mu}'_1 (b-a) = a + \frac{1}{2} (b-a)$$

### 2.9 Parametric variance

$$\text{Variance}(X) = (b-a)^2 \times (\tilde{\mu}'_2 - \tilde{\mu}'_1^2) = \frac{(b-a)^2}{8}$$

### 2.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\tilde{\mu}'_3 - 3\tilde{\mu}'_2\tilde{\mu}'_1 + 2\tilde{\mu}'_1^3}{(\tilde{\mu}'_2 - \tilde{\mu}'_1^2)^{1.5}} = 0$$

### 2.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\tilde{\mu}'_4 - 4\tilde{\mu}'_1\tilde{\mu}'_3 + 6\tilde{\mu}'_1^2\tilde{\mu}'_2 - 3\tilde{\mu}'_1^4}{(\tilde{\mu}'_2 - \tilde{\mu}'_1^2)^2} = 3 - \frac{3}{2}$$

### 2.12 Parametric median

$$\text{Median}(X) = a + (b-a) \times \sin^2 \left( \frac{\pi}{4} \right)$$

### 2.13 Parametric mode

$$\text{Mode}(X) = \text{undefined}$$

#### 2.14 Additional information and definitions

- $\tilde{X} \sim \text{Arcsine}(0, 1)$
- $u : \text{Uniform}[0, 1]$  random variable
- $\text{Beta}(x, y) : \text{Beta function}$

#### 2.15 Spreadsheet documents

- [Excel file from GitHub repository](#)
- [Google spreadsheet document](#)

### 3 Argus Distribution

#### 3.1 Distribution definition

$$X \sim \text{Argus}(\chi, \text{Loc}, \text{Sc})$$

#### 3.2 Distribution domain

$$x \in (\text{Loc}, \text{Loc} + \text{Sc})$$

#### 3.3 Parameters domain and parameters constraints

$$\chi \in \mathbb{R}^+, \text{Loc} \in \mathbb{R}, \text{Sc} \in \mathbb{R}^+$$

#### 3.4 Cumulative distribution function

$$F_X(x) = 1 - \frac{\Psi\left(\chi\sqrt{1-z(x)^2}\right)}{\Psi(\chi)}$$

#### 3.5 Probability density function

$$f_X(x) = \frac{1}{\text{Sc}} \cdot \frac{\chi^3}{\sqrt{2\pi}\Psi(\chi)} \cdot z(x)\sqrt{1-z(x)^2} \exp\left(-\frac{1}{2}\chi^2(1-z(x)^2)\right)$$

#### 3.6 Percent point function/Sample

$$F_X^{-1}(u) = \text{Loc} + \text{Sc}\sqrt{1 - \frac{2\text{P}^{-1}(\frac{3}{2}, (1-u)\text{P}(\frac{3}{2}, \frac{\chi^2}{2}))}{\chi^2}}$$

#### 3.7 Parametric centered moments

$$\mu'_k = E[X^k] = \int_{\text{Loc}}^{\text{Loc}+\text{Sc}} x^k f_X(x) dx$$

#### 3.8 Parametric mean

$$\text{Mean}(X) = \mu'_1 = \text{Loc} + \text{Sc}\sqrt{\pi/8} \frac{\chi e^{-\frac{\chi^2}{4}} I_1(\frac{\chi^2}{4})}{\Psi(\chi)}$$

#### 3.9 Parametric variance

$$\text{Variance}(X) = \mu'_2 - \mu_1'^2 = \text{Sc}^2 \cdot \left(1 - \frac{3}{\chi^2} + \frac{\chi\phi(\chi)}{\Psi(\chi)}\right) - (\mu - \text{Loc})^2$$

#### 3.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\mu'_3 - 3\mu'_2\mu'_1 + 2\mu_1'^3}{(\mu'_2 - \mu_1'^2)^{1.5}}$$

#### 3.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\mu'_4 - 4\mu'_1\mu'_3 + 6\mu_1'^2\mu'_2 - 3\mu_1'^4}{(\mu'_2 - \mu_1'^2)^2}$$

#### 3.12 Parametric median

$$\text{Median}(X) = \text{Loc} + \text{Sc}\sqrt{1 - \frac{2\text{P}^{-1}(\frac{3}{2}, \frac{1}{2}\text{P}(\frac{3}{2}, \frac{\chi^2}{2}))}{\chi^2}}$$

### 3.13 Parametric mode

$$\text{Mode}(X) = \text{Loc} + \frac{\text{Sc}}{\sqrt{2}\chi} \sqrt{(\chi^2 - 2) + \sqrt{\chi^4 + 4}}$$

### 3.14 Additional information and definitions

- Loc : Location parameter
- Sc : Scale parameter
- $z(x) = (x - \text{Loc}) / \text{Sc}$
- $u$  : Uniform[0,1] random variable
- $\Psi(\chi) = \Phi(\chi) - \chi\phi(\chi) - \frac{1}{2}$
- $\Phi(x)$  : CDF normal standard distribution
- $\phi(x)$  : PDF normal standard distribution
- $I_\alpha(x)$  : Modified Bessel function of the first kind of order  $\alpha \in \mathbb{N}$
- $P(a, x) = \frac{\gamma(a, x)}{\Gamma(a)}$  : Regularized lower incomplete gamma function
- $P^{-1}(a, y)$  : Inverse of regularized lower incomplete gamma function
- $\gamma(a, x)$  : Lower incomplete gamma function
- $\Gamma(x)$  : Gamma function

### 3.15 Spreadsheet documents

- [Excel file from GitHub repository](#)
- [Google spreadsheet document](#)

## 4 Beta Distribution

### 4.1 Distribution definition

$$X \sim \text{Beta}(\alpha, \beta, A, B)$$

### 4.2 Distribution domain

$$x \in (A, B)$$

### 4.3 Parameters domain and parameters constraints

$$\alpha \in \mathbb{R}^+, \beta \in \mathbb{R}^+, A \in \mathbb{R}, B \in \mathbb{R}, A < B$$

### 4.4 Cumulative distribution function

$$F_X(x) = I(z(x), \alpha, \beta)$$

### 4.5 Probability density function

$$f_X(x) = \frac{z(x)^{\alpha-1} (1 - z(x))^{\beta-1}}{\text{Beta}(\alpha, \beta)(B - A)}$$

### 4.6 Percent point function/Sample

$$F_X^{-1}(u) = A + (B - A) \times I^{-1}(u, \alpha, \beta)$$

### 4.7 Parametric centered moments

$$\tilde{\mu}'_k = E[\tilde{X}^k] = \int_0^1 x^k f_{\tilde{X}}(x) dx$$

### 4.8 Parametric mean

$$\text{Mean}(X) = A + (B - A) \cdot \tilde{\mu}'_1 = A + \frac{\alpha(B - A)}{\alpha + \beta}$$

### 4.9 Parametric variance

$$\text{Variance}(X) = (B - A)^2 \cdot (\tilde{\mu}'_2 - \tilde{\mu}'_1^2) = \frac{\alpha\beta(B - A)^2}{(\alpha + \beta)^2(\alpha + \beta + 1)}$$

### 4.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\tilde{\mu}'_3 - 3\tilde{\mu}'_2\tilde{\mu}'_1 + 2\tilde{\mu}'_1^3}{(\tilde{\mu}'_2 - \tilde{\mu}'_1^2)^{1.5}} = \frac{2(\beta - \alpha)\sqrt{\alpha + \beta + 1}}{(\alpha + \beta + 2)\sqrt{\alpha\beta}}$$

### 4.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\tilde{\mu}'_4 - 4\tilde{\mu}'_1\tilde{\mu}'_3 + 6\tilde{\mu}'_1^2\tilde{\mu}'_2 - 3\tilde{\mu}'_1^4}{(\tilde{\mu}'_2 - \tilde{\mu}'_1^2)^2} = 3 + \frac{6[(\alpha - \beta)^2(\alpha + \beta + 1) - \alpha\beta(\alpha + \beta + 2)]}{\alpha\beta(\alpha + \beta + 2)(\alpha + \beta + 3)}$$

### 4.12 Parametric median

$$\text{Median}(X) = A + (B - A) \times I^{-1}\left(\frac{1}{2}, \alpha, \beta\right) \quad \text{if } \alpha, \beta > 1$$

### 4.13 Parametric mode

$$\text{Mode}(X) = A + (B - A) \frac{\alpha - 1}{\alpha + \beta - 2} \quad \text{if } \alpha, \beta > 1$$

#### 4.14 Additional information and definitions

- $\tilde{X} \sim \text{Beta}(\alpha, \beta, 0, 1)$
- $z(x) = (x - A) / (B - A)$
- $u : \text{Uniform}[0,1]$  random variable
- $I(x, a, b) : \text{Regularized incomplete beta function}$
- $I^{-1}(x, a, b) : \text{Inverse of regularized incomplete beta function}$
- $\text{Beta}(x, y) : \text{Beta function}$

#### 4.15 Spreadsheet documents

- [Excel file from GitHub repository](#)
- [Google spreadsheet document](#)

## 5 Beta Prime Distribution

### 5.1 Distribution definition

$$X \sim \text{BetaPrime}(\alpha, \beta)$$

### 5.2 Distribution domain

$$x \in [0, \infty)$$

### 5.3 Parameters domain and parameters constraints

$$\alpha \in \mathbb{R}^+, \beta \in \mathbb{R}^+$$

### 5.4 Cumulative distribution function

$$F_X(x) = I\left(\frac{x}{1+x}, \alpha, \beta\right)$$

### 5.5 Probability density function

$$f_X(x) = \frac{x^{\alpha-1}(1+x)^{-\alpha-\beta}}{\text{Beta}(\alpha, \beta)}$$

### 5.6 Percent point function/Sample

$$F_X^{-1}(u) = \frac{I^{-1}(u, \alpha, \beta)}{1 - I^{-1}(u, \alpha, \beta)}$$

### 5.7 Parametric centered moments

$$\mu'_k = E[X^k] = \int_0^\infty x^k f_X(x) dx = \frac{\Gamma(k+\alpha)\Gamma(\beta-k)}{\Gamma(\alpha)\Gamma(\beta)} \quad \text{if } \beta > k$$

### 5.8 Parametric mean

$$\text{Mean}(X) = \mu'_1 = \frac{\alpha}{\beta-1} \quad \text{if } \beta > 1$$

### 5.9 Parametric variance

$$\text{Variance}(X) = \mu'_2 - \mu_1'^2 = \frac{\alpha(\alpha+\beta-1)}{(\beta-2)(\beta-1)^2} \quad \text{if } \beta > 2$$

### 5.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\mu'_3 - 3\mu'_2\mu'_1 + 2\mu_1'^3}{(\mu'_2 - \mu_1'^2)^{1.5}} = \frac{2(2\alpha+\beta-1)}{\beta-3} \sqrt{\frac{\beta-2}{\alpha(\alpha+\beta-1)}} \quad \text{if } \beta > 3$$

### 5.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\mu'_4 - 4\mu'_1\mu'_3 + 6\mu_1'^2\mu'_2 - 3\mu_1'^4}{(\mu'_2 - \mu_1'^2)^2} \quad \text{if } \beta > 4$$

### 5.12 Parametric median

$$\text{Median}(X) = \frac{I^{-1}\left(\frac{1}{2}, \alpha, \beta\right)}{1 - I^{-1}\left(\frac{1}{2}, \alpha, \beta\right)}$$



### 5.13 Parametric mode

$$\text{Mode}(X) = \frac{\alpha - 1}{\beta + 1}$$

### 5.14 Additional information and definitions

- $u$  : Uniform[0,1] random variable
- $I(x, a, b)$  : Regularized incomplete beta function
- $I^{-1}(x, a, b)$  : Inverse of regularized incomplete beta function
- $\Gamma(x)$  : Gamma function
- Beta  $(x, y)$  : Beta function

### 5.15 Spreadsheet documents

- [Excel file from GitHub repository](#)
- [Google spreadsheet document](#)

## 6 Beta Prime 4P Distribution

### 6.1 Distribution definition

$$X \sim \text{BetaPrime}_{4P}(\alpha, \beta, \text{Loc}, \text{Sc})$$

### 6.2 Distribution domain

$$x \in [\text{Loc}, \infty)$$

### 6.3 Parameters domain and parameters constraints

$$\alpha \in \mathbb{R}^+, \beta \in \mathbb{R}^+, \text{Loc} \in \mathbb{R}, \text{Sc} \in \mathbb{R}^+$$

### 6.4 Cumulative distribution function

$$F_X(x) = I\left(\frac{z(x)}{1+z(x)}, \alpha, \beta\right)$$

### 6.5 Probability density function

$$f_X(x) = \frac{z(x)^{\alpha-1}(1+z(x))^{-\alpha-\beta}}{\text{Sc} \times \text{Beta}(\alpha, \beta)}$$

### 6.6 Percent point function/Sample

$$F_X^{-1}(u) = \text{Loc} + \text{Sc} \frac{I^{-1}(u, \alpha, \beta)}{1 - I^{-1}(u, \alpha, \beta)}$$

### 6.7 Parametric centered moments

$$\tilde{\mu}'_k = E[\tilde{X}^k] = \int_0^\infty x^k f_{\tilde{X}}(x) dx = \frac{\Gamma(k+\alpha)\Gamma(\beta-k)}{\Gamma(\alpha)\Gamma(\beta)} \quad \text{if } \beta > k$$

### 6.8 Parametric mean

$$\text{Mean}(X) = \text{Loc} + \text{Sc} \tilde{\mu}'_1 = \text{Loc} + \text{Sc} \frac{\alpha}{\beta-1} \quad \text{if } \beta > 1$$

### 6.9 Parametric variance

$$\text{Variance}(X) = \text{Sc}^2(\tilde{\mu}'_2 - \tilde{\mu}_1'^2) = \text{Sc}^2 \frac{\alpha(\alpha+\beta-1)}{(\beta-2)(\beta-1)^2} \quad \text{if } \beta > 2$$

### 6.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\tilde{\mu}'_3 - 3\tilde{\mu}'_2\tilde{\mu}'_1 + 2\tilde{\mu}_1'^3}{(\tilde{\mu}'_2 - \tilde{\mu}_1'^2)^{1.5}} = \frac{2(2\alpha+\beta-1)}{\beta-3} \sqrt{\frac{\beta-2}{\alpha(\alpha+\beta-1)}} \quad \text{if } \beta > 3$$

### 6.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\tilde{\mu}'_4 - 4\tilde{\mu}'_1\tilde{\mu}'_3 + 6\tilde{\mu}_1'^2\tilde{\mu}'_2 - 3\tilde{\mu}_1'^4}{(\tilde{\mu}'_2 - \tilde{\mu}_1'^2)^2} \quad \text{if } \beta > 4$$

### 6.12 Parametric median

$$\text{Median}(X) = \text{Loc} + \text{Sc} \frac{I^{-1}\left(\frac{1}{2}, \alpha, \beta\right)}{1 - I^{-1}\left(\frac{1}{2}, \alpha, \beta\right)}$$

### 6.13 Parametric mode

$$\text{Mode}(X) = \text{Loc} + \text{Sc} \frac{\alpha - 1}{\beta + 1}$$

### 6.14 Additional information and definitions

- $\tilde{X} \sim \text{BetaPrime}(\alpha, \beta)$
- Loc : Location parameter
- Sc : Scale parameter
- $z(x) = (x - \text{Loc}) / \text{Sc}$
- $u$  : Uniform[0,1] random variable
- $I(x, a, b)$  : Regularized incomplete beta function
- $I^{-1}(x, a, b)$  : Inverse of regularized incomplete beta function
- $\Gamma(x)$  : Gamma function
- Beta( $x, y$ ) : Beta function

### 6.15 Spreadsheet documents

- [Excel file from GitHub repository](#)
- [Google spreadsheet document](#)

## 7 Bradford Distribution

### 7.1 Distribution definition

$$X \sim \text{Bradford}(c, \min, \max)$$

### 7.2 Distribution domain

$$x \in (\min, \max)$$

### 7.3 Parameters domain and parameters constraints

$$c \in \mathbb{R}^+, \min \in \mathbb{R}, \max \in \mathbb{R}, \min < \max$$

### 7.4 Cumulative distribution function

$$F_X(x) = \frac{\ln(1 + c \cdot z(x))}{k}$$

### 7.5 Probability density function

$$f_X(x) = \frac{c}{k(1 + c \cdot z(x))(\max - \min)}$$

### 7.6 Percent point function/Sample

$$F_X^{-1}(u) = \min + (\max - \min) \times \frac{(1 + c)^u - 1}{c}$$

### 7.7 Parametric centered moments

$$\tilde{\mu}'_k = E[\tilde{X}^k] = \int_0^1 x^k f_{\tilde{X}}(x) dx$$

### 7.8 Parametric mean

$$\text{Mean}(X) = \min + (\max - \min) \cdot \tilde{\mu}'_1 = \min + (\max - \min) \cdot \frac{c - k}{ck}$$

### 7.9 Parametric variance

$$\text{Variance}(X) = (\max - \min)^2 \cdot (\tilde{\mu}'_2 - \tilde{\mu}_1'^2) = (\max - \min)^2 \cdot \frac{(c + 2)k - 2c}{2ck^2}$$

### 7.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\tilde{\mu}'_3 - 3\tilde{\mu}'_2\tilde{\mu}'_1 + 2\tilde{\mu}_1'^3}{(\tilde{\mu}'_2 - \tilde{\mu}_1'^2)^{1.5}} = \frac{\sqrt{2}(12c^2 - 9kc(c + 2) + 2k^2(c(c + 3) + 3))}{\sqrt{c(c(k - 2) + 2k)}(3c(k - 2) + 6k)}$$

### 7.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\tilde{\mu}'_4 - 4\tilde{\mu}'_1\tilde{\mu}'_3 + 6\tilde{\mu}'_2\tilde{\mu}'_2 - 3\tilde{\mu}_1'^4}{(\tilde{\mu}'_2 - \tilde{\mu}_1'^2)^2} = 3 + \frac{c^3(k - 3)(k(3k - 16) + 24) + 12kc^2(k - 4)(k - 3) + 6ck^2(3k - 14) + 1}{3c(c(k - 2) + 2k)^2}$$

### 7.12 Parametric median

$$\text{Median}(X) = \min + (\max - \min) \cdot \frac{(1 + c)^{\frac{1}{2}} - 1}{c}$$

### 7.13 Parametric mode

$$\text{Mode}(X) = \min$$

#### 7.14 Additional information and definitions

- $\tilde{X} \sim \text{Bradford}(c, 0, 1)$
- $k = \ln(1 + c)$
- $z(x) = (x - \min) / (\max - \min)$
- $u : \text{Uniform}[0,1]$  random variable

#### 7.15 Spreadsheet documents

- [Excel file from GitHub repository](#)
- [Google spreadsheet document](#)

## 8 Burr Distribution

### 8.1 Distribution definition

$$X \sim \text{Burr}(A, B, C)$$

### 8.2 Distribution domain

$$x \in [0, \infty)$$

### 8.3 Parameters domain and parameters constraints

$$A \in \mathbb{R}^+, B \in \mathbb{R}, C \in \mathbb{R}^+$$

### 8.4 Cumulative distribution function

$$F_X(x) = 1 - \left[1 + \left(\frac{x}{A}\right)^B\right]^{-C}$$

### 8.5 Probability density function

$$f_X(x) = \frac{BC}{A} \left(\frac{x}{A}\right)^{B-1} \left[1 + \left(\frac{x}{A}\right)^B\right]^{-C-1}$$

### 8.6 Percent point function/Sample

$$F_X^{-1}(u) = A \left[ (1-u)^{-\frac{1}{C}} - 1 \right]^{\frac{1}{B}}$$

### 8.7 Parametric centered moments

$$\mu'_k = E[X^k] = \int_0^\infty x^k f_X(x) dx = A^k C \times \text{Beta}\left(\frac{BC-k}{B}, \frac{B+K}{B}\right)$$

### 8.8 Parametric mean

$$\text{Mean}(X) = \mu'_1$$

### 8.9 Parametric variance

$$\text{Variance}(X) = \mu'_2 - \mu_1'^2$$

### 8.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\mu'_3 - 3\mu'_2\mu'_1 + 2\mu_1'^3}{(\mu'_2 - \mu_1'^2)^{1.5}}$$

### 8.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\mu'_4 - 4\mu'_1\mu'_3 + 6\mu_1'^2\mu'_2 - 3\mu_1'^4}{(\mu'_2 - \mu_1'^2)^2}$$

### 8.12 Parametric median

$$\text{Median}(X) = A \left[ \left(\frac{1}{2}\right)^{-\frac{1}{C}} - 1 \right]^{\frac{1}{B}}$$

### 8.13 Parametric mode

$$\text{Mode}(X) = A \left( \frac{B-1}{BC+1} \right)^{\frac{1}{B}}$$

#### 8.14 Additional information and definitions

- $u$  : Uniform[0,1] random variable
- Beta  $(x, y)$  : Beta function

#### 8.15 Spreadsheet documents

- [Excel file from GitHub repository](#)
- [Google spreadsheet document](#)

## 9 Burr 4P Distribution

### 9.1 Distribution definition

$$X \sim \text{Burr}_{4P}(A, B, C, \text{Loc})$$

### 9.2 Distribution domain

$$x \in [\text{Loc}, \infty)$$

### 9.3 Parameters domain and parameters constraints

$$A \in \mathbb{R}^+, B \in \mathbb{R}, C \in \mathbb{R}^+, \text{Loc} \in \mathbb{R}$$

### 9.4 Cumulative distribution function

$$F_X(x) = 1 - \left[ 1 + \left( \frac{x - \text{Loc}}{A} \right)^B \right]^{-C}$$

### 9.5 Probability density function

$$f_X(x) = \frac{BC}{A} \left( \frac{x - \text{Loc}}{A} \right)^{B-1} \left[ 1 + \left( \frac{x - \text{Loc}}{A} \right)^B \right]^{-C-1}$$

### 9.6 Percent point function/Sample

$$F_X^{-1}(u) = \text{Loc} + A \left[ (1 - u)^{-\frac{1}{C}} - 1 \right]^{\frac{1}{B}}$$

### 9.7 Parametric centered moments

$$\tilde{\mu}'_k = E[\tilde{X}^k] = \int_0^\infty x^k f_{\tilde{X}} = A^k C \times \text{Beta} \left( \frac{BC - k}{B}, \frac{B + K}{B} \right)$$

### 9.8 Parametric mean

$$\text{Mean}(X) = \text{Loc} + \tilde{\mu}'_1$$

### 9.9 Parametric variance

$$\text{Variance}(X) = \tilde{\mu}'_2 - \tilde{\mu}'_1^2$$

### 9.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\tilde{\mu}'_3 - 3\tilde{\mu}'_2\tilde{\mu}'_1 + 2\tilde{\mu}'_1^3}{(\tilde{\mu}'_2 - \tilde{\mu}'_1^2)^{1.5}}$$

### 9.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\tilde{\mu}'_4 - 4\tilde{\mu}'_1\tilde{\mu}'_3 + 6\tilde{\mu}'_1^2\tilde{\mu}'_2 - 3\tilde{\mu}'_1^4}{(\tilde{\mu}'_2 - \tilde{\mu}'_1^2)^2}$$

### 9.12 Parametric median

$$\text{Median}(X) = \text{Loc} + A \left[ \left( \frac{1}{2} \right)^{-\frac{1}{C}} - 1 \right]^{\frac{1}{B}}$$



### 9.13 Parametric mode

$$\text{Mode}(X) = \text{Loc} + A \left( \frac{B-1}{BC+1} \right)^{\frac{1}{B}}$$

### 9.14 Additional information and definitions

- $\tilde{X} \sim \text{Burr}(A, B, C)$
- Loc : Location parameter
- $u$  : Uniform[0,1] random variable
- Beta( $x, y$ ) : Beta function

### 9.15 Spreadsheet documents

- [Excel file from GitHub repository](#)
- [Google spreadsheet document](#)

## 10 Cauchy Distribution

### 10.1 Distribution definition

$$X \sim \text{Cauchy}(x_0, \gamma)$$

### 10.2 Distribution domain

$$x \in (-\infty, +\infty)$$

### 10.3 Parameters domain and parameters constraints

$$x_0 \in \mathbb{R}, \gamma \in \mathbb{R}^+$$

### 10.4 Cumulative distribution function

$$F_X(x) = \frac{1}{\pi} \arctan\left(\frac{x - x_0}{\gamma}\right) + \frac{1}{2}$$

### 10.5 Probability density function

$$f_X(x) = \frac{1}{\pi\gamma \left[1 + \left(\frac{x - x_0}{\gamma}\right)^2\right]}$$

### 10.6 Percent point function/Sample

$$F_X^{-1}(u) = x_0 + \gamma \tan\left[\pi\left(u - \frac{1}{2}\right)\right]$$

### 10.7 Parametric centered moments

$$\mu'_k = E[X^k] = \int_{-\infty}^{\infty} x^k f_X(x) dx$$

### 10.8 Parametric mean

$$\text{Mean}(X) = \text{undefined}$$

### 10.9 Parametric variance

$$\text{Variance}(X) = \text{undefined}$$

### 10.10 Parametric skewness

$$\text{Skewness}(X) = \text{undefined}$$

### 10.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \text{undefined}$$

### 10.12 Parametric median

$$\text{Median}(X) = x_0$$

### 10.13 Parametric mode

$$\text{Mode}(X) = x_0$$

#### 10.14 Additional information and definitions

- $x_0$  : Location parameter
- $\gamma$  : Scale parameter
- $u$  : Uniform[0,1] random variable

#### 10.15 Spreadsheet documents

- [Excel file from GitHub repository](#)
- [Google spreadsheet document](#)

## 11 Chi Square Distribution

### 11.1 Distribution definition

$$X \sim \chi^2(\text{df})$$

### 11.2 Distribution domain

$$x \in (0, \infty)$$

### 11.3 Parameters domain and parameters constraints

$$\text{df} \in \mathbb{N}^+$$

### 11.4 Cumulative distribution function

$$F_X(x) = \frac{\gamma(\frac{\text{df}}{2}, \frac{x}{2})}{\Gamma(\frac{\text{df}}{2})} = P\left(\frac{\text{df}}{2}, \frac{x}{2}\right)$$

### 11.5 Probability density function

$$f_X(x) = \frac{1}{2^{\text{df}/2} \Gamma(\text{df}/2)} x^{\text{df}/2-1} e^{-x/2}$$

### 11.6 Percent point function/Sample

$$F_X^{-1}(u) = 2P^{-1}\left(\frac{\text{df}}{2}, u\right)$$

### 11.7 Parametric centered moments

$$\mu'_k = E[X^k] = \int_0^\infty x^k f_X(x) dx = \text{df}(\text{df} + 2) \cdots (\text{df} + 2k - 2) = 2^k \frac{\Gamma(k + \frac{\text{df}}{2})}{\Gamma(\frac{\text{df}}{2})}$$

### 11.8 Parametric mean

$$\text{Mean}(X) = \mu'_1 = \text{df}$$

### 11.9 Parametric variance

$$\text{Variance}(X) = \mu'_2 - \mu_1'^2 = 2\text{df}$$

### 11.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\mu'_3 - 3\mu'_2\mu'_1 + 2\mu_1'^3}{(\mu'_2 - \mu_1'^2)^{1.5}} = \sqrt{\frac{8}{\text{df}}}$$

### 11.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\mu'_4 - 4\mu'_1\mu'_3 + 6\mu_1'^2\mu'_2 - 3\mu_1'^4}{(\mu'_2 - \mu_1'^2)^2} = 3 + \frac{12}{\text{df}}$$

### 11.12 Parametric median

$$\text{Median}(X) = 2P^{-1}\left(\frac{\text{df}}{2}, \frac{1}{2}\right)$$

### 11.13 Parametric mode

$$\text{Mode}(X) = \max(\text{df} - 2, 0)$$

#### 11.14 Additional information and definitions

- $u$  : Uniform[0,1] random variable
- $P(a, x) = \frac{\gamma(a, x)}{\Gamma(a)}$  : Regularized lower incomplete gamma function
- $P^{-1}(a, u)$  : Inverse of regularized lower incomplete gamma function
- $\gamma(a, x)$  : Lower incomplete gamma function
- $\Gamma(x)$  : Gamma function

#### 11.15 Spreadsheet documents

- [Excel file from GitHub repository](#)
- [Google spreadsheet document](#)

## 12 Chi Square 3P Distribution

### 12.1 Distribution definition

$$X \sim \chi_{3P}^2(\text{df}, \text{Loc}, \text{Sc})$$

### 12.2 Distribution domain

$$x \in (\text{Loc}, \infty)$$

### 12.3 Parameters domain and parameters constraints

$$\text{df} \in \mathbb{N}^+, \text{Loc} \in \mathbb{R}, \text{Sc} \in \mathbb{R}^+$$

### 12.4 Cumulative distribution function

$$F_X(x) = \frac{\gamma(\frac{\text{df}}{2}, \frac{z(x)}{2})}{\Gamma(\frac{\text{df}}{2})} = \text{P}\left(\frac{\text{df}}{2}, \frac{z(x)}{2}\right)$$

### 12.5 Probability density function

$$f_X(x) = \frac{1}{\text{Sc}} \frac{1}{2^{\text{df}/2} \Gamma(\text{df}/2)} x^{\text{df}/2-1} e^{-z(x)/2}$$

### 12.6 Percent point function/Sample

$$F_X^{-1}(u) = 2\text{P}^{-1}\left(\frac{\text{df}}{2}, u\right)$$

### 12.7 Parametric centered moments

$$\tilde{\mu}'_k = E[\tilde{X}^k] = \int_0^\infty x^k f_{\tilde{X}}(x) dx = \text{df}(\text{df}+2) \cdots (\text{df}+2k-2) = 2^k \frac{\Gamma(k + \frac{\text{df}}{2})}{\Gamma(\frac{\text{df}}{2})}$$

### 12.8 Parametric mean

$$\text{Mean}(X) = \text{Loc} + \text{Sc} \cdot \tilde{\mu}'_1 = \text{Loc} + \text{Sc} \cdot \text{df}$$

### 12.9 Parametric variance

$$\text{Variance}(X) = \text{Sc}^2 \cdot (\tilde{\mu}'_2 - \tilde{\mu}'_1^2) = 2 \cdot \text{df} \cdot \text{Sc}^2$$

### 12.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\tilde{\mu}'_3 - 3\tilde{\mu}'_2\tilde{\mu}'_1 + 2\tilde{\mu}'_1^3}{(\tilde{\mu}'_2 - \tilde{\mu}'_1^2)^{1.5}} = \sqrt{\frac{8}{\text{df}}}$$

### 12.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\tilde{\mu}'_4 - 4\tilde{\mu}'_1\tilde{\mu}'_3 + 6\tilde{\mu}'_1^2\tilde{\mu}'_2 - 3\tilde{\mu}'_1^4}{(\tilde{\mu}'_2 - \tilde{\mu}'_1^2)^2} = 3 + \frac{12}{\text{df}}$$

### 12.12 Parametric median

$$\text{Median}(X) = \text{Loc} + \text{Sc} \times 2\text{P}^{-1}\left(\frac{\text{df}}{2}, \frac{1}{2}\right)$$

### 12.13 Parametric mode

$$\text{Mode}(X) = \text{Loc} + \text{Sc} \times \max(\text{df} - 2, 0)$$

#### 12.14 Additional information and definitions

- $\tilde{X} \sim \chi^2(\text{df})$
- Loc : Location parameter
- Sc : Scale parameter
- $z(x) = (x - \text{Loc}) / \text{Sc}$
- $u$  : Uniform[0,1] random variable
- $P(a, x) = \frac{\gamma(a, x)}{\Gamma(a)}$  : Regularized lower incomplete gamma function
- $P^{-1}(a, u)$  : Inverse of regularized lower incomplete gamma function
- $\gamma(a, x)$  : Lower incomplete gamma function
- $\Gamma(x)$  : Gamma function

#### 12.15 Spreadsheet documents

- [Excel file from GitHub repository](#)
- [Google spreadsheet document](#)

## 13 Dagum Distribution

### 13.1 Distribution definition

$$X \sim \text{Dagum}(a, b, p)$$

### 13.2 Distribution domain

$$x \in (0, \infty)$$

### 13.3 Parameters domain and parameters constraints

$$a \in \mathbb{R}^+, b \in \mathbb{R}^+, p \in \mathbb{R}^+$$

### 13.4 Cumulative distribution function

$$F_X(x) = \left(1 + \left(\frac{x}{b}\right)^{-a}\right)^{-p}$$

### 13.5 Probability density function

$$f_X(x) = \frac{ap}{x} \left( \frac{\left(\frac{x}{b}\right)^{ap}}{\left(\left(\frac{x}{b}\right)^a + 1\right)^{p+1}} \right)$$

### 13.6 Percent point function/Sample

$$F_X^{-1}(u) = b(u^{-1/p} - 1)^{-1/a}$$

### 13.7 Parametric centered moments

$$\mu'_k = E[X^k] = \int_0^\infty x^k f_X(x) dx = pb^k \cdot \text{Beta}\left(\frac{ap+k}{a}, \frac{a-k}{a}\right)$$

### 13.8 Parametric mean

$$\text{Mean}(X) = \mu'_1$$

### 13.9 Parametric variance

$$\text{Variance}(X) = \mu'_2 - \mu_1'^2$$

### 13.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\mu'_3 - 3\mu'_2\mu'_1 + 2\mu_1'^3}{(\mu'_2 - \mu_1'^2)^{1.5}}$$

### 13.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\mu'_4 - 4\mu'_1\mu'_3 + 6\mu_1'^2\mu'_2 - 3\mu_1'^4}{(\mu'_2 - \mu_1'^2)^2}$$

### 13.12 Parametric median

$$\text{Median}(X) = b \left( -1 + 2^{\frac{1}{p}} \right)^{-\frac{1}{a}}$$

### 13.13 Parametric mode

$$\text{Mode}(X) = b \left( \frac{ap-1}{a+1} \right)^{\frac{1}{a}}$$



#### 13.14 Additional information and definitions

- $b$  : Scale parameter
- $u$  : Uniform[0,1] random variable
- Beta( $x, y$ ) : Beta function

#### 13.15 Spreadsheet documents

- [Excel file from GitHub repository](#)
- [Google spreadsheet document](#)

## 14 Dagum 4P Distribution

### 14.1 Distribution definition

$$X \sim \text{Dagum}_{4P}(a, b, p, \text{Loc})$$

### 14.2 Distribution domain

$$x \in (\text{Loc}, \infty)$$

### 14.3 Parameters domain and parameters constraints

$$a \in \mathbb{R}^+, b \in \mathbb{R}^+, p \in \mathbb{R}^+, \text{Loc} \in \mathbb{R}$$

### 14.4 Cumulative distribution function

$$F_X(x) = \left(1 + \left(\frac{x - \text{Loc}}{b}\right)^{-a}\right)^{-p}$$

### 14.5 Probability density function

$$f_X(x) = \frac{ap}{x - \text{Loc}} \left( \frac{\left(\frac{x - \text{Loc}}{b}\right)^{ap}}{\left(\left(\frac{x - \text{Loc}}{b}\right)^a + 1\right)^{p+1}} \right)$$

### 14.6 Percent point function/Sample

$$F_X^{-1}(u) = \text{Loc} + b(u^{-1/p} - 1)^{-1/a}$$

### 14.7 Parametric centered moments

$$\tilde{\mu}'_k = E[\tilde{X}^k] = \int_0^\infty x^k f_{\tilde{X}}(x) dx = pb^k \cdot \text{Beta}\left(\frac{ap+k}{a}, \frac{a-k}{a}\right)$$

### 14.8 Parametric mean

$$\text{Mean}(X) = \text{Loc} + \tilde{\mu}'_1$$

### 14.9 Parametric variance

$$\text{Variance}(X) = \tilde{\mu}'_2 - \tilde{\mu}'_1^2$$

### 14.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\tilde{\mu}'_3 - 3\tilde{\mu}'_2\tilde{\mu}'_1 + 2\tilde{\mu}'_1^3}{(\tilde{\mu}'_2 - \tilde{\mu}'_1^2)^{1.5}}$$

### 14.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\tilde{\mu}'_4 - 4\tilde{\mu}'_1\tilde{\mu}'_3 + 6\tilde{\mu}'_1^2\tilde{\mu}'_2 - 3\tilde{\mu}'_1^4}{(\tilde{\mu}'_2 - \tilde{\mu}'_1^2)^2}$$

### 14.12 Parametric median

$$\text{Median}(X) = \text{Loc} + b\left(-1 + 2^{\frac{1}{p}}\right)^{-\frac{1}{a}}$$

### 14.13 Parametric mode

$$\text{Mode}(X) = \text{Loc} + b\left(\frac{ap-1}{a+1}\right)^{\frac{1}{a}}$$

#### 14.14 Additional information and definitions

- $\bar{X} \sim \text{Dagum}(a, b, p)$
- $\text{Loc}$  : Location parameter
- $b$  : Scale parameter
- $u$  : Uniform[0,1] random variable
- $\text{Beta}(x, y)$  : Beta function

#### 14.15 Spreadsheet documents

- [Excel file from GitHub repository](#)
- [Google spreadsheet document](#)

## 15 Erlang Distribution

### 15.1 Distribution definition

$$X \sim \text{Erlang}(k, \beta)$$

### 15.2 Distribution domain

$$x \in [0, \infty)$$

### 15.3 Parameters domain and parameters constraints

$$k \in \mathbb{N}^+, \beta \in \mathbb{R}^+$$

### 15.4 Cumulative distribution function

$$F_X(x) = P(k, \frac{x}{\beta}) = \frac{\gamma(k, \frac{x}{\beta})}{(k-1)!}$$

### 15.5 Probability density function

$$f_X(x) = \frac{x^{k-1} e^{-\frac{x}{\beta}}}{\beta^k (k-1)!}$$

### 15.6 Percent point function/Sample

$$F_X^{-1}(u) = \beta P^{-1}(k, u)$$

### 15.7 Parametric centered moments

$$\mu'_n = E[X^n] = \int_0^\infty x^n f_X(x) dx = \beta^k \frac{\Gamma(n+k)}{\Gamma(k)}$$

### 15.8 Parametric mean

$$\text{Mean}(X) = \mu'_1$$

### 15.9 Parametric variance

$$\text{Variance}(X) = \mu'_2 - \mu_1'^2$$

### 15.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\mu'_3 - 3\mu'_2\mu'_1 + 2\mu_1'^3}{(\mu'_2 - \mu_1'^2)^{1.5}}$$

### 15.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\mu'_4 - 4\mu'_1\mu'_3 + 6\mu_1'^2\mu'_2 - 3\mu_1'^4}{(\mu'_2 - \mu_1'^2)^2}$$

### 15.12 Parametric median

$$\text{Median}(X) = P(k, \frac{1}{2\beta})$$

### 15.13 Parametric mode

$$\text{Mode}(X) = \beta(k-1)$$

#### 15.14 Additional information and definitions

- $\beta$  : Scale parameter
- $u$  : Uniform[0,1] random variable
- $P(a, x) = \frac{\gamma(a, x)}{\Gamma(a)}$  : Regularized lower incomplete gamma function
- $P^{-1}(a, u)$  : Inverse of regularized lower incomplete gamma function
- $\gamma(a, x)$  : Lower incomplete gamma function
- $\Gamma(x)$  : Gamma function

#### 15.15 Spreadsheet documents

- [Excel file from GitHub repository](#)
- [Google spreadsheet document](#)

## 16 Erlang 3P Distribution

### 16.1 Distribution definition

$$X \sim \text{Erlang}_{3P}(k, \beta, \text{Loc})$$

### 16.2 Distribution domain

$$x \in [\text{Loc}, \infty)$$

### 16.3 Parameters domain and parameters constraints

$$k \in \mathbb{N}^+, \beta \in \mathbb{R}^+, \text{Loc} \in \mathbb{R}$$

### 16.4 Cumulative distribution function

$$F_X(x) = P(k, \frac{x - \text{Loc}}{\beta}) = \frac{\gamma(k, \frac{x - \text{Loc}}{\beta})}{(k - 1)!}$$

### 16.5 Probability density function

$$f_X(x) = \frac{(x - \text{Loc})^{k-1} e^{-\frac{x - \text{Loc}}{\beta}}}{\beta^k (k - 1)!}$$

### 16.6 Percent point function/Sample

$$F_X^{-1}(u) = \text{Loc} + \beta P^{-1}(k, u)$$

### 16.7 Parametric centered moments

$$\tilde{\mu}'_n = E[\tilde{X}^n] = \int_0^\infty x^n f_{\tilde{X}}(x) dx = \beta^k \frac{\Gamma(n + k)}{\Gamma(k)}$$

### 16.8 Parametric mean

$$\text{Mean}(X) = \text{Loc} + \tilde{\mu}'_1$$

### 16.9 Parametric variance

$$\text{Variance}(X) = \tilde{\mu}'_2 - \tilde{\mu}'_1{}^2$$

### 16.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\tilde{\mu}'_3 - 3\tilde{\mu}'_2\tilde{\mu}'_1 + 2\tilde{\mu}'_1{}^3}{(\tilde{\mu}'_2 - \tilde{\mu}'_1{}^2)^{1.5}}$$

### 16.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\tilde{\mu}'_4 - 4\tilde{\mu}'_1\tilde{\mu}'_3 + 6\tilde{\mu}'_1{}^2\tilde{\mu}'_2 - 3\tilde{\mu}'_1{}^4}{(\tilde{\mu}'_2 - \tilde{\mu}'_1{}^2)^2}$$

### 16.12 Parametric median

$$\text{Median}(X) = \text{Loc} + P(k, \frac{1}{2\beta})$$

### 16.13 Parametric mode

$$\text{Mode}(X) = \text{Loc} + \beta \cdot (k - 1)$$

#### 16.14 Additional information and definitions

- $\tilde{X} \sim \text{Erlang}(k, \beta)$
- $\text{Loc}$  : Location parameter
- $\beta$  : Scale parameter
- $u$  : Uniform[0,1] random variable
- $P(a, x) = \frac{\gamma(a, x)}{\Gamma(a)}$  : Regularized lower incomplete gamma function
- $P^{-1}(a, u)$  : Inverse of regularized lower incomplete gamma function
- $\gamma(a, x)$  : Lower incomplete gamma function
- $\Gamma(x)$  : Gamma function

#### 16.15 Spreadsheet documents

- [Excel file from GitHub repository](#)
- [Google spreadsheet document](#)

## 17 Error Function Distribution

### 17.1 Distribution definition

$$X \sim \text{ErrorFunction}(h)$$

### 17.2 Distribution domain

$$x \in (-\infty, \infty)$$

### 17.3 Parameters domain and parameters constraints

$$h \in \mathbb{R}^+$$

### 17.4 Cumulative distribution function

$$F_X(x) = \Phi(\sqrt{2}hx)$$

### 17.5 Probability density function

$$f_X(x) = \frac{h}{\sqrt{\pi}} e^{-h^2 x^2}$$

### 17.6 Percent point function/Sample

$$F_X^{-1}(u) = \frac{\Phi^{-1}(u)}{\sqrt{2}h}$$

### 17.7 Parametric centered moments

$$\mu'_k = E[X^k] = \int_{-\infty}^{\infty} x^k f_X(x) dx$$

### 17.8 Parametric mean

$$\text{Mean}(X) = \mu'_1 = 0$$

### 17.9 Parametric variance

$$\text{Variance}(X) = \mu'_2 - \mu'^2_1 = \frac{1}{2h^2}$$

### 17.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\mu'_3 - 3\mu'_2\mu'_1 + 2\mu'^3_1}{(\mu'_2 - \mu'^2_1)^{1.5}} = 0$$

### 17.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\mu'_4 - 4\mu'_1\mu'_3 + 6\mu'^2_1\mu'_2 - 3\mu'^4_1}{(\mu'_2 - \mu'^2_1)^2} = 3$$

### 17.12 Parametric median

$$\text{Median}(X) = 0$$

### 17.13 Parametric mode

$$\text{Mode}(X) = 0$$



#### 17.14 Additional information and definitions

- $h$  : Inverse of scale parameter
- $u$  : Uniform[0,1] random variable
- $\Phi(x)$  : CDF normal standard distribution

#### 17.15 Spreadsheet documents

- [Excel file from GitHub repository](#)
- [Google spreadsheet document](#)

## 18 Exponential Distribution

### 18.1 Distribution definition

$$X \sim \text{Exponential}(\lambda)$$

### 18.2 Distribution domain

$$x \in [0, \infty)$$

### 18.3 Parameters domain and parameters constraints

$$\lambda \in \mathbb{R}^+$$

### 18.4 Cumulative distribution function

$$F_X(x) = 1 - e^{-\lambda x}$$

### 18.5 Probability density function

$$f_X(x) = \lambda e^{-\lambda x}$$

### 18.6 Percent point function/Sample

$$F_X^{-1}(u) = -\frac{\ln(1-u)}{\lambda}$$

### 18.7 Parametric centered moments

$$\mu'_k = E[X^k] = \int_0^\infty x^k f_X(x) dx = \frac{k!}{\lambda^k}$$

### 18.8 Parametric mean

$$\text{Mean}(X) = \mu'_1 = \frac{1}{\lambda}$$

### 18.9 Parametric variance

$$\text{Variance}(X) = \mu'_2 - \mu_1'^2 = \frac{1}{\lambda^2}$$

### 18.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\mu'_3 - 3\mu'_2\mu'_1 + 2\mu_1'^3}{(\mu'_2 - \mu_1'^2)^{1.5}} = 2$$

### 18.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\mu'_4 - 4\mu'_1\mu'_3 + 6\mu_1'^2\mu'_2 - 3\mu_1'^4}{(\mu'_2 - \mu_1'^2)^2} = 9$$

### 18.12 Parametric median

$$\text{Median}(X) = \frac{\ln 2}{\lambda}$$

### 18.13 Parametric mode

$$\text{Mode}(X) = 0$$

#### 18.14 Additional information and definitions

- $\lambda$  : Inverse of scale parameter
- $u$  : Uniform[0,1] random variable

#### 18.15 Spreadsheet documents

- [Excel file from GitHub repository](#)
- [Google spreadsheet document](#)

## 19 Exponential 2P Distribution

### 19.1 Distribution definition

$$X \sim \text{Exponential}_{2P}(\lambda, \text{Loc})$$

### 19.2 Distribution domain

$$x \in [\text{Loc}, \infty)$$

### 19.3 Parameters domain and parameters constraints

$$\lambda \in \mathbb{R}^+, \text{Loc} \in \mathbb{R}$$

### 19.4 Cumulative distribution function

$$F_X(x) = 1 - e^{-\lambda(x-\text{Loc})}$$

### 19.5 Probability density function

$$f_X(x) = \lambda e^{-\lambda(x-\text{Loc})}$$

### 19.6 Percent point function/Sample

$$F_X^{-1}(u) = \text{Loc} - \frac{\ln(1-u)}{\lambda}$$

### 19.7 Parametric centered moments

$$\tilde{\mu}'_k = E[\tilde{X}^k] = \int_0^\infty x^k f_{\tilde{X}}(x) dx = \frac{k!}{\lambda^k}$$

### 19.8 Parametric mean

$$\text{Mean}(X) = \text{Loc} + \tilde{\mu}'_1 = \text{Loc} + \frac{1}{\lambda}$$

### 19.9 Parametric variance

$$\text{Variance}(X) = \tilde{\mu}'_2 - \tilde{\mu}'_1^2 = \frac{1}{\lambda^2}$$

### 19.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\tilde{\mu}'_3 - 3\tilde{\mu}'_2\tilde{\mu}'_1 + 2\tilde{\mu}'_1^3}{(\tilde{\mu}'_2 - \tilde{\mu}'_1^2)^{1.5}} = 2$$

### 19.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\tilde{\mu}'_4 - 4\tilde{\mu}'_1\tilde{\mu}'_3 + 6\tilde{\mu}'_1^2\tilde{\mu}'_2 - 3\tilde{\mu}'_1^4}{(\tilde{\mu}'_2 - \tilde{\mu}'_1^2)^2} = 9$$

### 19.12 Parametric median

$$\text{Median}(X) = \text{Loc} + \frac{\ln 2}{\lambda}$$

### 19.13 Parametric mode

$$\text{Mode}(X) = \text{Loc}$$

#### 19.14 Additional information and definitions

- $\tilde{X} \sim \text{Exponential}(\lambda)$
- Loc : Location parameter
- $\lambda$  : Inverse of scale parameter
- $u$  : Uniform[0,1] random variable

#### 19.15 Spreadsheet documents

- [Excel file from GitHub repository](#)
- [Google spreadsheet document](#)

## 20 F Distribution

### 20.1 Distribution definition

$$X \sim F(df_1, df_2)$$

### 20.2 Distribution domain

$$x \in [0, \infty)$$

### 20.3 Parameters domain and parameters constraints

$$df_1 \in \mathbb{R}^+, df_2 \in \mathbb{R}^+$$

### 20.4 Cumulative distribution function

$$F_X(x) = I_{df_1 x / (df_1 x + df_2)}\left(\frac{df_1}{2}, \frac{df_2}{2}\right)$$

### 20.5 Probability density function

$$f_X(x) = \frac{\sqrt{\frac{(df_1 x)^{df_1} df_2^{df_2}}{(df_1 x + df_2)^{df_1 + df_2}}}}{x \times \text{Beta}\left(\frac{df_1}{2}, \frac{df_2}{2}\right)}$$

### 20.6 Percent point function/Sample

$$F_X^{-1}(u) = \frac{df_2 \times I^{-1}\left(u, \frac{df_1}{2}, \frac{df_2}{2}\right)}{df_1 \times \left(1 - I^{-1}\left(u, \frac{df_1}{2}, \frac{df_2}{2}\right)\right)}$$

### 20.7 Parametric centered moments

$$\mu'_k = E[X^k] = \int_0^\infty x^k f_X(x) dx = \left(\frac{df_2}{df_1}\right)^k \frac{\Gamma\left(\frac{df_1}{2} + k\right) \Gamma\left(\frac{df_2}{2} - k\right)}{\Gamma\left(\frac{df_1}{2}\right) \Gamma\left(\frac{df_2}{2}\right)} \quad \text{if } df_2 > 2k$$

### 20.8 Parametric mean

$$\text{Mean}(X) = \mu'_1 = \frac{df_2}{df_2 - 2} \quad \text{if } df_2 > 2$$

### 20.9 Parametric variance

$$\text{Variance}(X) = \mu'_2 - \mu_1'^2 = \frac{2 df_2^2 (df_1 + df_2 - 2)}{df_1 (df_2 - 2)^2 (df_2 - 4)} \quad \text{if } df_2 > 4$$

### 20.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\mu'_3 - 3\mu'_2\mu'_1 + 2\mu_1'^3}{(\mu'_2 - \mu_1'^2)^{1.5}} = \frac{(2df_1 + df_2 - 2)\sqrt{8(df_2 - 4)}}{(df_2 - 6)\sqrt{df_1(df_1 + df_2 - 2)}} \quad \text{if } df_2 > 6$$

### 20.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\mu'_4 - 4\mu'_1\mu'_3 + 6\mu_1'^2\mu'_2 - 3\mu_1'^4}{(\mu'_2 - \mu_1'^2)^2} = \frac{3(8 + (df_2 - 6) \times \text{Skewness}(X)^2)}{2df_2 - 16} + 3 \quad \text{if } df_2 > 8$$

### 20.12 Parametric median

$$\text{Median}(X) = \frac{df_2 \times I^{-1}\left(\frac{1}{2}, \frac{df_1}{2}, \frac{df_2}{2}\right)}{df_1 \times \left(1 - I^{-1}\left(\frac{1}{2}, \frac{df_1}{2}, \frac{df_2}{2}\right)\right)}$$

### 20.13 Parametric mode

$$\text{Mode}(X) = \frac{\text{df}_2 (\text{df}_1 - 2)}{\text{df}_1 (\text{df}_2 + 2)} \quad \text{if } \text{df}_1 > 2$$

### 20.14 Additional information and definitions

- $u$  : Uniform[0,1] random variable
- $I(x, a, b)$  : Regularized incomplete beta function
- $I^{-1}(x, a, b)$  : Inverse of regularized incomplete beta function
- Beta  $(x, y)$  : Beta function

### 20.15 Spreadsheet documents

- [Excel file from GitHub repository](#)
- [Google spreadsheet document](#)

## 21 F 4P Distribution

### 21.1 Distribution definition

$$X \sim F_{4P}(\text{df}_1, \text{df}_2, \text{Loc}, \text{Sc})$$

### 21.2 Distribution domain

$$x \in [\text{Loc}, \infty)$$

### 21.3 Parameters domain and parameters constraints

$$\text{df}_1 \in \mathbb{R}^+, \text{df}_2 \in \mathbb{R}^+, \text{Loc} \in \mathbb{R}, \text{Sc} \in \mathbb{R}^+$$

### 21.4 Cumulative distribution function

$$F_X(x) = I_{\text{df}_1 z(x)/(\text{df}_1 z(x) + \text{df}_2)}\left(\frac{\text{df}_1}{2}, \frac{\text{df}_2}{2}\right)$$

### 21.5 Probability density function

$$f_X(x) = \frac{1}{\text{Sc}} \times \frac{\sqrt{\frac{(\text{df}_1 z(x))^{\text{df}_1} \text{df}_2^{\text{df}_2}}{(\text{df}_1 z(x) + \text{df}_2)^{\text{df}_1 + \text{df}_2}}}}{z(x) \text{Beta}\left(\frac{\text{df}_1}{2}, \frac{\text{df}_2}{2}\right)}$$

### 21.6 Percent point function/Sample

$$F_X^{-1}(u) = \text{Loc} + \text{Sc} \frac{\text{df}_2 \times I^{-1}\left(u, \frac{\text{df}_1}{2}, \frac{\text{df}_2}{2}\right)}{\text{df}_1 \times \left(1 - I^{-1}\left(u, \frac{\text{df}_1}{2}, \frac{\text{df}_2}{2}\right)\right)}$$

### 21.7 Parametric centered moments

$$\tilde{\mu}'_k = E[\tilde{X}^k] = \int_0^\infty x^k f_{\tilde{X}}(x) dx = \frac{\Gamma\left(\frac{\text{df}_1}{2} + k\right)}{\Gamma\left(\frac{\text{df}_1}{2}\right)} \frac{\Gamma\left(\frac{\text{df}_2}{2} - k\right)}{\Gamma\left(\frac{\text{df}_2}{2}\right)} \left(\frac{\text{df}_2}{\text{df}_1}\right)^k \quad \text{if } \text{df}_2 > 2k$$

### 21.8 Parametric mean

$$\text{Mean}(X) = \text{Loc} + \text{Sc} \tilde{\mu}'_1 = \text{Loc} + \text{Sc} \frac{\text{df}_2}{\text{df}_2 - 2} \quad \text{if } \text{df}_2 > 2$$

### 21.9 Parametric variance

$$\text{Variance}(X) = \text{Sc}^2(\tilde{\mu}'_2 - \tilde{\mu}'_1{}^2) = \text{Sc}^2 \frac{2 \text{df}_2^2 (\text{df}_1 + \text{df}_2 - 2)}{\text{df}_1 (\text{df}_2 - 2)^2 (\text{df}_2 - 4)} \quad \text{if } \text{df}_2 > 4$$

### 21.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\tilde{\mu}'_3 - 3\tilde{\mu}'_2\tilde{\mu}'_1 + 2\tilde{\mu}'_1{}^3}{(\tilde{\mu}'_2 - \tilde{\mu}'_1{}^2)^{1.5}} = \frac{(2\text{df}_1 + \text{df}_2 - 2)\sqrt{8(\text{df}_2 - 4)}}{(\text{df}_2 - 6)\sqrt{\text{df}_1(\text{df}_1 + \text{df}_2 - 2)}} \quad \text{if } \text{df}_2 > 6$$

### 21.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\tilde{\mu}'_4 - 4\tilde{\mu}'_1\tilde{\mu}'_3 + 6\tilde{\mu}'_1{}^2\tilde{\mu}'_2 - 3\tilde{\mu}'_1{}^4}{(\tilde{\mu}'_2 - \tilde{\mu}'_1{}^2)^2} = \frac{3(8 + (\text{df}_2 - 6) \times \text{Skewness}(X)^2)}{2\text{df}_2 - 16} + 3 \quad \text{if } \text{df}_2 > 8$$

### 21.12 Parametric median

$$\text{Median}(X) = \text{Loc} + \text{Sc} \frac{\text{df}_2 \times I^{-1}\left(\frac{1}{2}, \frac{\text{df}_1}{2}, \frac{\text{df}_2}{2}\right)}{\text{df}_1 \times \left(1 - I^{-1}\left(\frac{1}{2}, \frac{\text{df}_1}{2}, \frac{\text{df}_2}{2}\right)\right)}$$



### 21.13 Parametric mode

$$\text{Mode}(X) = \text{Loc} + \text{Sc} \frac{\text{df}_2 (\text{df}_1 - 2)}{\text{df}_1 (\text{df}_2 + 2)} \quad \text{if } \text{df}_1 > 2$$

### 21.14 Additional information and definitions

- $\tilde{X} \sim F(\text{df}_1, \text{df}_2)$
- Loc : Location parameter
- Sc : Scale parameter
- $z(x) = (x - \text{Loc}) / \text{Sc}$
- $u$  : Uniform[0,1] random variable
- $I(x, a, b)$  : Regularized incomplete beta function
- $I^{-1}(x, a, b)$  : Inverse of regularized incomplete beta function
- Beta( $x, y$ ) : Beta function

### 21.15 Spreadsheet documents

- [Excel file from GitHub repository](#)
- [Google spreadsheet document](#)

## 22 Fatigue Life Distribution

### 22.1 Distribution definition

$$X \sim \text{FatigueLife}(\gamma, \text{Loc}, \text{Sc})$$

### 22.2 Distribution domain

$$x \in (\text{Loc}, \infty)$$

### 22.3 Parameters domain and parameters constraints

$$\gamma \in \mathbb{R}^+, \text{Loc} \in \mathbb{R}, \text{Sc} \in \mathbb{R}^+$$

### 22.4 Cumulative distribution function

$$F_X(x) = \Phi\left(\frac{\sqrt{z(x)} - \sqrt{\frac{1}{z(x)}}}{\gamma}\right)$$

### 22.5 Probability density function

$$f_X(x) = \frac{\sqrt{z(x)} + \sqrt{\frac{1}{z(x)}}}{2\gamma z(x)} \phi\left(\frac{\sqrt{z(x)} - \sqrt{\frac{1}{z(x)}}}{\gamma}\right)$$

### 22.6 Percent point function/Sample

$$F_X^{-1}(u) = \text{Loc} + \text{Sc} \frac{1}{4} \left[ \gamma \Phi^{-1}(u) + \sqrt{4 + (\gamma \Phi^{-1}(u))^2} \right]^2$$

### 22.7 Parametric centered moments

$$\mu'_k = E[X^k] = \int_{-\infty}^{\infty} x^k f_X(x) dx$$

### 22.8 Parametric mean

$$\text{Mean}(X) = \text{Loc} + \text{Sc} \cdot \tilde{\mu}'_1 = \text{Loc} + \text{Sc} \left(1 + \frac{\gamma^2}{2}\right)$$

### 22.9 Parametric variance

$$\text{Variance}(X) = \text{Sc}^2 \cdot (\tilde{\mu}'_2 - \tilde{\mu}'_1{}^2) = \text{Sc}^2 \gamma^2 \left(1 + \frac{5\gamma^2}{4}\right)$$

### 22.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\mu'_3 - 3\mu'_2\mu'_1 + 2\mu'_1{}^3}{(\mu'_2 - \mu'_1{}^2)^{1.5}} = \frac{4\gamma(6 + 11\gamma^2)}{(4 + 5\gamma^2)^{1.5}}$$

### 22.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\mu'_4 - 4\mu'_1\mu'_3 + 6\mu'_1{}^2\mu'_2 - 3\mu'_1{}^4}{(\mu'_2 - \mu'_1{}^2)^2} = 3 + \frac{6\gamma^2(93\gamma^2 + 40)}{(5\gamma^2 + 4)^2}$$

### 22.12 Parametric median

$$\text{Median}(X) = \text{Loc} + \text{Sc} \frac{1}{4} \left[ \gamma \Phi^{-1}(1/2) + \sqrt{4 + (\gamma \Phi^{-1}(1/2))^2} \right]^2$$

### 22.13 Parametric mode

$$\text{Mode}(X) = \arg \max_x f_X(x)$$

### 22.14 Additional information and definitions

- Loc : Location parameter
- Sc : Scale parameter
- $z(x) = (x - \text{Loc}) / \text{Sc}$
- $u$  : Uniform[0,1] random variable
- $\Phi(x)$  : CDF normal standard distribution
- $\phi(x)$  : PDF normal standard distribution

### 22.15 Spreadsheet documents

- [Excel file from GitHub repository](#)
- [Google spreadsheet document](#)

## 23 Folded Normal Distribution

### 23.1 Distribution definition

$$X \sim \text{FoldedNormal}(\mu, \sigma)$$

### 23.2 Distribution domain

$$x \in [0, \infty)$$

### 23.3 Parameters domain and parameters constraints

$$\mu \in \mathbb{R}, \sigma \in \mathbb{R}^+$$

### 23.4 Cumulative distribution function

$$F_X(x) = \frac{1}{2} \left[ \operatorname{erf}\left(\frac{x+\mu}{\sigma\sqrt{2}}\right) + \operatorname{erf}\left(\frac{x-\mu}{\sigma\sqrt{2}}\right) \right]$$

### 23.5 Probability density function

$$f_X(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} + \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x+\mu)^2}{2\sigma^2}}$$

### 23.6 Percent point function/Sample

$$\text{Sample}_X(u) = |\mu + \sigma\Phi^{-1}(u)|$$

### 23.7 Parametric centered moments

$$\mu'_k = E[X^k] = \int_0^\infty x^k f_X(x) dx$$

### 23.8 Parametric mean

$$\text{Mean}(X) = \mu'_1 = \sigma\sqrt{\frac{2}{\pi}} e^{(-\mu^2/2\sigma^2)} + \mu(1 - 2\Phi(-\frac{\mu}{\sigma}))$$

### 23.9 Parametric variance

$$\text{Variance}(X) = \mu'_2 - \mu'^2_1 = \mu^2 + \sigma^2 - \text{Mean}(X)^2$$

### 23.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\mu'_3 - 3\mu'_2\mu'_1 + 2\mu'^3_1}{(\mu'_2 - \mu'^2_1)^{1.5}}$$

### 23.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\mu'_4 - 4\mu'_1\mu'_3 + 6\mu'^2_1\mu'_2 - 3\mu'^4_1}{(\mu'_2 - \mu'^2_1)^2}$$

### 23.12 Parametric median

$$\text{Median}(X) = |\mu + \sigma\Phi^{-1}(1/2)|$$

### 23.13 Parametric mode

$$\text{Mode}(X) = \arg \max_x f_X(x)$$

### 23.14 Additional information and definitions

- Computing an analytic expression for the inverse of the cumulative distribution function is not feasible. Nonetheless, it is possible to generate a random sample from the distribution.
- $\mu$  : Location parameter
- $\sigma$  : Scale parameter
- $u$  : Uniform[0,1] random variable
- $\Phi(x)$  : CDF normal standard distribution
- $\phi(x)$  : PDF normal standard distribution
- $\Phi^{-1}(x)$  : PPF normal standard distribution

### 23.15 Spreadsheet documents

- [Excel file from GitHub repository](#)
- [Google spreadsheet document](#)

## 24 Frechet Distribution

### 24.1 Distribution definition

$$X \sim \text{Frechet}(\alpha, \text{Loc}, \text{Sc})$$

### 24.2 Distribution domain

$$x \in [\text{Loc}, \infty)$$

### 24.3 Parameters domain and parameters constraints

$$\alpha \in \mathbb{R}^+, \text{Loc} \in \mathbb{R}, \text{Sc} \in \mathbb{R}^+$$

### 24.4 Cumulative distribution function

$$F_X(x) = e^{(-z(x))^{-\alpha}}$$

### 24.5 Probability density function

$$f_X(x) = \frac{\alpha}{\text{Sc}} (z(x))^{-1-\alpha} e^{-(z(x))^{-\alpha}}$$

### 24.6 Percent point function/Sample

$$F_X^{-1}(u) = \text{Loc} + \text{Sc} (-\ln(u))^{-\frac{1}{\alpha}}$$

### 24.7 Parametric centered moments

$$\mu'_k = E[X^k] = \int_{\text{Loc}}^{\infty} x^k f_X(x) dx = \Gamma\left(1 - \frac{k}{\alpha}\right)$$

### 24.8 Parametric mean

$$\text{Mean}(X) = \text{Loc} + \text{Sc} \cdot \tilde{\mu}'_1 \quad \text{if } \alpha > 1$$

### 24.9 Parametric variance

$$\text{Variance}(X) = \text{Sc}^2 \cdot (\tilde{\mu}'_2 - \tilde{\mu}'_1{}^2) \quad \text{if } \alpha > 2$$

### 24.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\mu'_3 - 3\mu'_2\mu'_1 + 2\mu_1'^3}{(\mu'_2 - \mu_1'^2)^{1.5}} \quad \text{if } \alpha > 3$$

### 24.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\mu'_4 - 4\mu'_1\mu'_3 + 6\mu_1'^2\mu'_2 - 3\mu_1'^4}{(\mu'_2 - \mu_1'^2)^2} \quad \text{if } \alpha > 4$$

### 24.12 Parametric median

$$\text{Median}(X) = \text{Loc} + \frac{\text{Sc}}{\sqrt[\alpha]{\ln(2)}}$$

### 24.13 Parametric mode

$$\text{Mode}(X) = \text{Loc} + \text{Sc} \left( \frac{\alpha}{1 + \alpha} \right)^{1/\alpha}$$

#### 24.14 Additional information and definitions

- $\text{Loc}$  : Location parameter
- $\text{Sc}$  : Scale parameter
- $z(x) = (x - \text{Loc}) / \text{Sc}$
- $u$  : Uniform[0,1] random variable
- $\Gamma(x)$  : Gamma function

#### 24.15 Spreadsheet documents

- [Excel file from GitHub repository](#)
- [Google spreadsheet document](#)

## 25 Gamma Distribution

### 25.1 Distribution definition

$$X \sim \text{Gamma}(\alpha, \beta)$$

### 25.2 Distribution domain

$$x \in (0, \infty)$$

### 25.3 Parameters domain and parameters constraints

$$\alpha \in \mathbb{R}^+, \beta \in \mathbb{R}^+$$

### 25.4 Cumulative distribution function

$$F_X(x) = P\left(\alpha, \frac{x}{\beta}\right) = \frac{1}{\Gamma(\alpha)} \gamma\left(\alpha, \frac{x}{\beta}\right)$$

### 25.5 Probability density function

$$f_X(x) = \frac{1}{\Gamma(\alpha)\beta^\alpha} x^{\alpha-1} e^{-\frac{x}{\beta}}$$

### 25.6 Percent point function/Sample

$$F_X^{-1}(u) = \beta P^{-1}(\alpha, u)$$

### 25.7 Parametric centered moments

$$\mu'_k = E[X^k] = \int_0^\infty x^k f_X(x) dx = \beta^k \frac{\Gamma(k + \alpha)}{\Gamma(\alpha)}$$

### 25.8 Parametric mean

$$\text{Mean}(X) = \mu'_1 = \alpha\beta$$

### 25.9 Parametric variance

$$\text{Variance}(X) = \mu'_2 - \mu_1'^2 = \alpha\beta^2$$

### 25.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\mu'_3 - 3\mu'_2\mu'_1 + 2\mu_1'^3}{(\mu'_2 - \mu_1'^2)^{1.5}} = \frac{2}{\sqrt{\alpha}}$$

### 25.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\mu'_4 - 4\mu'_1\mu'_3 + 6\mu_1'^2\mu'_2 - 3\mu_1'^4}{(\mu'_2 - \mu_1'^2)^2} = 3 + \frac{6}{\alpha}$$

### 25.12 Parametric median

$$\text{Median}(X) = (\alpha - 1)\beta \quad \text{if } \alpha > 1$$

### 25.13 Parametric mode

$$\text{Mode}(X) = \beta P^{-1}\left(\alpha, \frac{1}{2}\right)$$



#### 25.14 Additional information and definitions

- $\beta$  : Scale parameter
- $u$  : Uniform[0,1] random variable
- $P(a, x) = \frac{\gamma(a, x)}{\Gamma(a)}$  : Regularized lower incomplete gamma function
- $P^{-1}(a, u)$  : Inverse of regularized lower incomplete gamma function
- $\gamma(a, x)$  : Lower incomplete gamma function
- $\Gamma(x)$  : Gamma function

#### 25.15 Spreadsheet documents

- [Excel file from GitHub repository](#)
- [Google spreadsheet document](#)

## 26 Gamma 3P Distribution

### 26.1 Distribution definition

$$X \sim \text{Gamma}_{3P}(\alpha, \text{Loc}, \beta)$$

### 26.2 Distribution domain

$$x \in (\text{Loc}, \infty)$$

### 26.3 Parameters domain and parameters constraints

$$\alpha \in \mathbb{R}^+, \text{Loc} \in \mathbb{R}, \beta \in \mathbb{R}^+$$

### 26.4 Cumulative distribution function

$$F_X(x) = P\left(\alpha, \frac{x - \text{Loc}}{\beta}\right) = \frac{1}{\Gamma(\alpha)} \gamma\left(\alpha, \frac{x - \text{Loc}}{\beta}\right)$$

### 26.5 Probability density function

$$f_X(x) = \frac{1}{\Gamma(\alpha)\beta^\alpha} (x - \text{Loc})^{\alpha-1} e^{-\frac{x - \text{Loc}}{\beta}}$$

### 26.6 Percent point function/Sample

$$F_X^{-1}(u) = \text{Loc} + \beta P^{-1}(\alpha, u)$$

### 26.7 Parametric centered moments

$$\tilde{\mu}'_k = E[\tilde{X}^k] = \int_0^\infty x^k f_{\tilde{X}}(x) dx = \beta^k \frac{\Gamma(k + \alpha)}{\Gamma(\alpha)}$$

### 26.8 Parametric mean

$$\text{Mean}(X) = \text{Loc} + \tilde{\mu}'_1 = \text{Loc} + \alpha\beta$$

### 26.9 Parametric variance

$$\text{Variance}(X) = \tilde{\mu}'_2 - \tilde{\mu}'_1{}^2 = \alpha\beta^2$$

### 26.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\tilde{\mu}'_3 - 3\tilde{\mu}'_2\tilde{\mu}'_1 + 2\tilde{\mu}'_1{}^3}{(\tilde{\mu}'_2 - \tilde{\mu}'_1{}^2)^{1.5}} = \frac{2}{\sqrt{\alpha}}$$

### 26.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\tilde{\mu}'_4 - 4\tilde{\mu}'_1\tilde{\mu}'_3 + 6\tilde{\mu}'_1{}^2\tilde{\mu}'_2 - 3\tilde{\mu}'_1{}^4}{(\tilde{\mu}'_2 - \tilde{\mu}'_1{}^2)^2} = 3 + \frac{6}{\alpha}$$

### 26.12 Parametric median

$$\text{Median}(X) = \text{Loc} + (\alpha - 1)\beta \quad \text{if } \alpha > 1$$

### 26.13 Parametric mode

$$\text{Mode}(X) = \text{Loc} + \beta P^{-1}\left(\alpha, \frac{1}{2}\right)$$

#### 26.14 Additional information and definitions

- $\tilde{X} \sim \text{Gamma}(\alpha, \beta)$
- $\text{Loc}$  : Location parameter
- $\beta$  : Scale parameter
- $u$  : Uniform[0,1] random variable
- $P(a, x) = \frac{\gamma(a, x)}{\Gamma(a)}$  : Regularized lower incomplete gamma function
- $P^{-1}(a, u)$  : Inverse of regularized lower incomplete gamma function
- $\gamma(a, x)$  : Lower incomplete gamma function
- $\Gamma(x)$  : Gamma function

#### 26.15 Spreadsheet documents

- [Excel file from GitHub repository](#)
- [Google spreadsheet document](#)

## 27 Generalized Extreme Value Distribution

### 27.1 Distribution definition

$$X \sim \text{GeneralizedExtremeValue}(\xi, \mu, \sigma)$$

### 27.2 Distribution domain

$$\text{if } \xi > 0 : x \in (z(x), \infty), \quad \text{if } \xi = 0 : x \in (-\infty, \infty), \quad \text{if } \xi < 0 : x \in (-\infty, z(x))$$

### 27.3 Parameters domain and parameters constraints

$$\xi \in \mathbb{R}, \mu \in \mathbb{R}, \sigma \in \mathbb{R}^+$$

### 27.4 Cumulative distribution function

$$F_X(x) = \begin{cases} \exp(-\exp(-z(x))) & \text{if } \xi = 0 \\ \exp(-(1 + \xi z(x))^{-1/\xi}) & \text{if } \xi \neq 0 \end{cases}$$

### 27.5 Probability density function

$$f_X(x) = \begin{cases} \frac{1}{\sigma} \exp(-z(x)) \exp(-\exp(-z(x))) & \text{if } \xi = 0 \\ \frac{1}{\sigma} (1 + \xi z(x))^{-(1+1/\xi)} \exp(-(1 + \xi z(x))^{-1/\xi}) & \text{if } \xi \neq 0 \end{cases}$$

### 27.6 Percent point function/Sample

$$F_X^{-1}(u) = \begin{cases} \mu - \sigma \ln(-\ln(u)) & \text{if } \xi = 0 \\ \mu + \frac{\sigma}{\xi} \left( (-\ln(u))^{-\xi} - 1 \right) & \text{if } \xi \neq 0 \end{cases}$$

### 27.7 Parametric centered moments

$$\mu'_k = E[X^k] = \int_{-\infty}^{\infty} x^k f_X(x) dx = \Gamma(1 - k\xi)$$

### 27.8 Parametric mean

$$\text{Mean}(X) = \begin{cases} \mu + \sigma(\mu'_1 - 1)/\xi & \text{if } \xi \neq 0, \xi < 1 \\ \mu + \sigma \gamma & \text{if } \xi = 0 \end{cases}$$

### 27.9 Parametric variance

$$\text{Variance}(X) = \begin{cases} \sigma^2 (\mu'_2 - \mu'^2_1)/\xi^2 & \text{if } \xi \neq 0, \xi < \frac{1}{2} \\ \sigma^2 \frac{\pi^2}{6} & \text{if } \xi = 0 \end{cases}$$

### 27.10 Parametric skewness

$$\text{Skewness}(X) = \begin{cases} \text{sign}(\xi) \frac{\mu'_3 - 3\mu'_2\mu'_1 + 2\mu'^3_1}{(\mu'_2 - \mu'^2_1)^{1.5}} & \text{if } \xi \neq 0, \xi < \frac{1}{3} \\ \frac{12\sqrt{6}\zeta(3)}{\pi^3} & \text{if } \xi = 0 \end{cases}$$

### 27.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \begin{cases} 3 + \frac{\mu'_4 - 4\mu'_1\mu'_3 + 6\mu'^2_1\mu'_2 - 3\mu'^4_1}{(\mu'_2 - \mu'^2_1)^2} & \text{if } \xi \neq 0, \xi < \frac{1}{4} \\ 3 + \frac{12}{5} & \text{if } \xi = 0 \end{cases}$$

### 27.12 Parametric median

$$\text{Median}(X) = \begin{cases} \mu + \sigma \frac{(\ln 2)^{-\xi} - 1}{\xi} & \text{if } \xi \neq 0 \\ \mu - \sigma \ln \ln 2 & \text{if } \xi = 0 \end{cases}$$

### 27.13 Parametric mode

$$\text{Mode}(X) = \begin{cases} \mu + \sigma \frac{(1+\xi)^{-\xi} - 1}{\xi} & \text{if } \xi \neq 0 \\ \mu & \text{if } \xi = 0 \end{cases}$$

### 27.14 Additional information and definitions

- $\mu$  : Location parameter
- $\sigma$  : Scale parameter
- $z(x) = (x - \mu) / \sigma$
- $u$  : Uniform[0,1] random variable
- $\Gamma(x)$  : Gamma function
- $\gamma$  : Euler-Mascheroni constant = 0.5772156649

### 27.15 Spreadsheet documents

- [Excel file from GitHub repository](#)
- [Google spreadsheet document](#)

## 28 Generalized Gamma Distribution

### 28.1 Distribution definition

$$X \sim \text{GeneralizedGamma}(a, d, p)$$

### 28.2 Distribution domain

$$x \in (0, \infty)$$

### 28.3 Parameters domain and parameters constraints

$$a \in \mathbb{R}^+, d \in \mathbb{R}^+, p \in \mathbb{R}^+$$

### 28.4 Cumulative distribution function

$$F_X(x) = P(d/p, (x/a)^p) = \frac{\gamma(d/p, (x/a)^p)}{\Gamma(d/p)}$$

### 28.5 Probability density function

$$f_X(x) = \frac{p/a^d}{\Gamma(d/p)} x^{d-1} e^{-(x/a)^p}$$

### 28.6 Percent point function/Sample

$$F_X^{-1}(u) = aP^{-1}\left(\frac{d}{p}, u\right)^{\frac{1}{p}}$$

### 28.7 Parametric centered moments

$$\mu'_k = E[X^k] = \int_0^\infty x^k f_X(x) dx = a^k \frac{\Gamma(\frac{d+k}{p})}{\Gamma(\frac{d}{p})}$$

### 28.8 Parametric mean

$$\text{Mean}(X) = \mu'_1$$

### 28.9 Parametric variance

$$\text{Variance}(X) = \mu'_2 - \mu_1'^2$$

### 28.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\mu'_3 - 3\mu'_2\mu'_1 + 2\mu_1'^3}{(\mu'_2 - \mu_1'^2)^{1.5}}$$

### 28.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\mu'_4 - 4\mu'_1\mu'_3 + 6\mu_1'^2\mu'_2 - 3\mu_1'^4}{(\mu'_2 - \mu_1'^2)^2}$$

### 28.12 Parametric median

$$\text{Median}(X) = aP^{-1}\left(\frac{d}{p}, \frac{1}{2}\right)^{\frac{1}{p}}$$

### 28.13 Parametric mode

$$\text{Mode}(X) = a\left(\frac{d-1}{p}\right)^{\frac{1}{p}} \quad \text{if } d > 1$$

#### 28.14 Additional information and definitions

- $a$  : Scale parameter
- $u$  : Uniform[0,1] random variable
- $P(a, x) = \frac{\gamma(a, x)}{\Gamma(a)}$  : Regularized lower incomplete gamma function
- $P^{-1}(a, u)$  : Inverse of regularized lower incomplete gamma function
- $\gamma(a, x)$  : Lower incomplete gamma function
- $\Gamma(x)$  : Gamma function

#### 28.15 Spreadsheet documents

- [Excel file from GitHub repository](#)
- [Google spreadsheet document](#)

## 29 Generalized Gamma 4P Distribution

### 29.1 Distribution definition

$$X \sim \text{GeneralizedGamma}_{4P}(a, d, p, \text{Loc})$$

### 29.2 Distribution domain

$$x \in (\text{Loc}, \infty)$$

### 29.3 Parameters domain and parameters constraints

$$a \in \mathbb{R}^+, d \in \mathbb{R}^+, p \in \mathbb{R}^+, \text{Loc} \in \mathbb{R}$$

### 29.4 Cumulative distribution function

$$F_X(x) = P(d/p, ((x - \text{Loc})/a)^p) = \frac{\gamma(d/p, ((x - \text{Loc})/a)^p)}{\Gamma(d/p)}$$

### 29.5 Probability density function

$$f_X(x) = \frac{p/a^d}{\Gamma(d/p)} (x - \text{Loc})^{d-1} e^{-((x - \text{Loc})/a)^p}$$

### 29.6 Percent point function/Sample

$$F_X^{-1}(u) = \text{Loc} + aP^{-1}\left(\frac{d}{p}, u\right)^{\frac{1}{p}}$$

### 29.7 Parametric centered moments

$$\tilde{\mu}'_k = E[\tilde{X}^k] = \int_0^\infty x^k f_{\tilde{X}}(x) dx = a^k \frac{\Gamma(\frac{d+k}{p})}{\Gamma(\frac{d}{p})}$$

### 29.8 Parametric mean

$$\text{Mean}(X) = \text{Loc} + \tilde{\mu}'_1$$

### 29.9 Parametric variance

$$\text{Variance}(X) = \tilde{\mu}'_2 - \tilde{\mu}'_1^2$$

### 29.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\tilde{\mu}'_3 - 3\tilde{\mu}'_2\tilde{\mu}'_1 + 2\tilde{\mu}'_1^3}{(\tilde{\mu}'_2 - \tilde{\mu}'_1^2)^{1.5}}$$

### 29.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\tilde{\mu}'_4 - 4\tilde{\mu}'_1\tilde{\mu}'_3 + 6\tilde{\mu}'_1^2\tilde{\mu}'_2 - 3\tilde{\mu}'_1^4}{(\tilde{\mu}'_2 - \tilde{\mu}'_1^2)^2}$$

### 29.12 Parametric median

$$\text{Median}(X) = \text{Loc} + aP^{-1}\left(\frac{d}{p}, \frac{1}{2}\right)^{\frac{1}{p}}$$

### 29.13 Parametric mode

$$\text{Mode}(X) = \text{Loc} + a\left(\frac{d-1}{p}\right)^{\frac{1}{p}} \quad \text{if } d > 1$$



#### 29.14 Additional information and definitions

- $\tilde{X} \sim \text{GeneralizedGamma}(a, d, p)$
- $\text{Loc}$  : Location parameter
- $a$  : Scale parameter
- $u$  :  $\text{Uniform}[0,1]$  random variable
- $P(a, x) = \frac{\gamma(a, x)}{\Gamma(a)}$  : Regularized lower incomplete gamma function
- $P^{-1}(a, u)$  : Inverse of regularized lower incomplete gamma function
- $\gamma(a, x)$  : Lower incomplete gamma function
- $\Gamma(x)$  : Gamma function

#### 29.15 Spreadsheet documents

- [Excel file from GitHub repository](#)
- [Google spreadsheet document](#)

## 30 Generalized Logistic Distribution

### 30.1 Distribution definition

$$X \sim \text{GeneralizedLogistic}(c, \text{Loc}, \text{Sc})$$

### 30.2 Distribution domain

$$x \in (\text{Loc}, \infty)$$

### 30.3 Parameters domain and parameters constraints

$$c \in \mathbb{R}^+, \text{Loc} \in \mathbb{R}, \text{Sc} \in \mathbb{R}^+$$

### 30.4 Cumulative distribution function

$$F_X(x) = \frac{1}{(1 + \exp(-z(x)))^c}$$

### 30.5 Probability density function

$$f_X(x) = \frac{c \exp(-z(x))}{\text{Sc} (1 + \exp(-z(x)))^{c+1}}$$

### 30.6 Percent point function/Sample

$$F_X^{-1}(u) = \text{Loc} - \text{Sc} \ln(u^{-1/c} - 1)$$

### 30.7 Parametric centered moments

$$\mu'_k = E[X^k] = \int_{-\infty}^{\infty} x^k f_X(x) dx$$

### 30.8 Parametric mean

$$\text{Mean}(X) = \text{Loc} + \text{Sc} \cdot \tilde{\mu}'_1 = \text{Loc} + \text{Sc} (\gamma + \psi_0(c))$$

### 30.9 Parametric variance

$$\text{Variance}(X) = \text{Sc}^2 \cdot (\tilde{\mu}'_2 - \tilde{\mu}'_1{}^2) = \text{Sc}^2 \left( \frac{\pi^2}{6} + \psi_1(c) \right)$$

### 30.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\mu'_3 - 3\mu'_2\mu'_1 + 2\mu_1'^3}{(\mu'_2 - \mu_1'^2)^{1.5}} = \frac{\psi_2(c) + 2\zeta(3)}{\left(\frac{\pi^2}{6} + \psi_1(c)\right)^{3/2}}$$

### 30.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\mu'_4 - 4\mu'_1\mu'_3 + 6\mu_1'^2\mu'_2 - 3\mu_1'^4}{(\mu'_2 - \mu_1'^2)^2} = \frac{\left(\frac{\pi^4}{15} + \psi_3(c)\right)}{\left(\frac{\pi^2}{6} + \psi_1(c)\right)^2}$$

### 30.12 Parametric median

$$\text{Median}(X) = \text{Loc} - \text{Sc} \ln(2^{1/c} - 1)$$

### 30.13 Parametric mode

$$\text{Mode}(X) = \text{Loc} + \text{Sc} \ln(c)$$

### 30.14 Additional information and definitions

- $\text{Loc}$  : Location parameter
- $\text{Sc}$  : Scale parameter
- $z(x) = (x - \text{Loc}) / \text{Sc}$
- $u$  : Uniform[0,1] random variable
- $\gamma$  : Euler-Mascheroni constant = 0.5772156649
- $\psi_0(x)$  : Digamma function
- $\psi_n(x)$  : Polygamma function of order  $n \in \mathbb{N}$

### 30.15 Spreadsheet documents

- [Excel file from GitHub repository](#)
- [Google spreadsheet document](#)

## 31 Generalized Normal Distribution

### 31.1 Distribution definition

$$X \sim \text{GeneralizedNormal}(\beta, \mu, \alpha)$$

### 31.2 Distribution domain

$$x \in (-\infty, +\infty)$$

### 31.3 Parameters domain and parameters constraints

$$\beta \in \mathbb{R}^+, \mu \in \mathbb{R}, \alpha \in \mathbb{R}^+$$

### 31.4 Cumulative distribution function

$$F_X(x) = \frac{1}{2} + \frac{\text{sign}(x - \mu)}{2\Gamma(1/\beta)} \gamma\left(1/\beta, \left|\frac{x - \mu}{\alpha}\right|^\beta\right) = \frac{1}{2} + \frac{\text{sign}(x - \mu)}{2} \text{P}\left(1/\beta, \left|\frac{x - \mu}{\alpha}\right|^\beta\right)$$

### 31.5 Probability density function

$$f_X(x) = \frac{\beta}{2\alpha\Gamma(1/\beta)} \exp\left(-\left(\frac{|x - \mu|}{\alpha}\right)^\beta\right)$$

### 31.6 Percent point function/Sample

$$F_X^{-1}(u) = \text{sign}(u - \frac{1}{2}) \left[ \alpha^\beta \text{P}^{-1}\left(\frac{1}{\beta}, 2|u - \frac{1}{2}|\right) \right]^{1/\beta} + \mu$$

### 31.7 Parametric centered moments

$$\mu'_k = E[X^k] = \int_{-\infty}^{\infty} x^k f_X(x) dx = \begin{cases} 0 & \text{if } k \text{ is odd} \\ \alpha^k \Gamma\left(\frac{k+1}{\beta}\right) / \Gamma\left(\frac{1}{\beta}\right) & \text{if } k \text{ is even} \end{cases}$$

### 31.8 Parametric mean

$$\text{Mean}(X) = \mu + \alpha\mu'_1 = \mu$$

### 31.9 Parametric variance

$$\text{Variance}(X) = \alpha^2(\mu'_2 - \mu'^2_1) = \frac{\alpha^2\Gamma(3/\beta)}{\Gamma(1/\beta)}$$

### 31.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\mu'_3 - 3\mu'_2\mu'_1 + 2\mu'^3_1}{(\mu'_2 - \mu'^2_1)^{1.5}} = 0$$

### 31.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\mu'_4 - 4\mu'_1\mu'_3 + 6\mu'^2_2\mu'_2 - 3\mu'^4_1}{(\mu'_2 - \mu'^2_1)^2} = \frac{\Gamma(5/\beta)\Gamma(1/\beta)}{\Gamma(3/\beta)^2}$$

### 31.12 Parametric median

$$\text{Median}(X) = \mu$$

### 31.13 Parametric mode

$$\text{Mode}(X) = \mu$$

### 31.14 Additional information and definitions

- $\mu$  : Location parameter
- $\alpha$  : Scale parameter
- $u$  : Uniform[0,1] random variable
- $P^{-1}(a, u)$  : Inverse of regularized lower incomplete gamma function
- $\gamma(a, x)$  : Lower incomplete gamma function
- $\Gamma(x)$  : Gamma function

### 31.15 Spreadsheet documents

- [Excel file from GitHub repository](#)
- [Google spreadsheet document](#)

## 32 Generalized Pareto Distribution

### 32.1 Distribution definition

$$X \sim \text{GeneralizedPareto}(c, \mu, \sigma)$$

### 32.2 Distribution domain

$$\text{if } c \geq 0 : x \in (\mu, \infty), \quad \text{if } c < 0 : x \in \left(-\infty, \mu - \frac{\sigma}{c}\right)$$

### 32.3 Parameters domain and parameters constraints

$$c \in \mathbb{R}, \mu \in \mathbb{R}, \sigma \in \mathbb{R}^+$$

### 32.4 Cumulative distribution function

$$F_X(x) = 1 - (1 + cz(x))^{-1/c}$$

### 32.5 Probability density function

$$f_X(x) = \frac{1}{\sigma} (1 + cz(x))^{-(1/c+1)}$$

### 32.6 Percent point function/Sample

$$F_X^{-1}(u) = \mu + \frac{\sigma(u^{-c} - 1)}{c}$$

### 32.7 Parametric centered moments

$$\mu'_k = E[X^k] = \int_{-\infty}^{\infty} x^k f_X(x) dx = \frac{(-1)^k}{c^k} \sum_{i=0}^k \binom{k}{i} \frac{(-1)^i}{1-ci} \quad \text{if } < \frac{1}{k}$$

### 32.8 Parametric mean

$$\text{Mean}(X) = \mu + \sigma \mu'_1 = \mu + \frac{\sigma}{1-c} \quad \text{if } c < 1$$

### 32.9 Parametric variance

$$\text{Variance}(X) = \sigma^2(\mu'_2 - \mu_1'^2) = \frac{\sigma^2}{(1-c)^2(1-2c)} \quad \text{if } c < 1/2$$

### 32.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\mu'_3 - 3\mu'_2\mu'_1 + 2\mu_1'^3}{(\mu'_2 - \mu_1'^2)^{1.5}} = \frac{2(1+c)\sqrt{1-2c}}{(1-3c)} \quad \text{if } c < 1/3$$

### 32.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\mu'_4 - 4\mu'_1\mu'_3 + 6\mu_1'^2\mu'_2 - 3\mu_1'^4}{(\mu'_2 - \mu_1'^2)^2} = \frac{3(1-2c)(2c^2+c+3)}{(1-3c)(1-4c)} \quad \text{if } c < 1/4$$

### 32.12 Parametric median

$$\text{Median}(X) = \mu$$

### 32.13 Parametric mode

$$\text{Mode}(X) = \mu + \frac{\sigma(2^c - 1)}{c}$$

### 32.14 Additional information and definitions

- $\mu$  : Location parameter
- $\sigma$  : Scale parameter
- $z(x) = (x - \mu) / \sigma$
- $u$  : Uniform[0,1] random variable

### 32.15 Spreadsheet documents

- [Excel file from GitHub repository](#)
- [Google spreadsheet document](#)

## 33 Gibrat Distribution

### 33.1 Distribution definition

$$X \sim \text{Gibrat}(\text{Loc}, \text{Sc})$$

### 33.2 Distribution domain

$$x \in (\text{Loc}, \infty)$$

### 33.3 Parameters domain and parameters constraints

$$\text{Loc} \in \mathbb{R}, \text{Sc} \in \mathbb{R}^+$$

### 33.4 Cumulative distribution function

$$F_X(x) = \Phi(\ln x) = \frac{1}{2} \left( 1 + \text{erf} \left( \frac{\ln z(x)}{\sqrt{2}} \right) \right)$$

### 33.5 Probability density function

$$f_X(x) = \frac{1}{\text{Sc}} \frac{1}{x\sqrt{2\pi}} \exp \left( -\frac{1}{2} (\ln z(x))^2 \right)$$

### 33.6 Percent point function/Sample

$$F_X^{-1}(u) = \text{Loc} + \text{Sc} \times \exp \left( \Phi^{-1}(u) \right)$$

### 33.7 Parametric centered moments

$$\mu'_k = E[X^k] = \int_{\text{Loc}}^{\infty} x^k f_X(x) dx = \exp \left( \frac{k^2}{2} \right)$$

### 33.8 Parametric mean

$$\text{Mean}(X) = \text{Loc} + \text{Sc} \cdot \tilde{\mu}'_1 = \text{Loc} + \text{Sc} \cdot \sqrt{e}$$

### 33.9 Parametric variance

$$\text{Variance}(X) = \text{Sc}^2 \cdot (\tilde{\mu}'_2 - \tilde{\mu}'_1{}^2) = \text{Sc}^2 [e^2 - e]$$

### 33.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\mu'_3 - 3\mu'_2\mu'_1 + 2\mu'^3_1}{(\mu'_2 - \mu'^2_1)^{1.5}} = \sqrt{e-1} (2+e)$$

### 33.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\mu'_4 - 4\mu'_1\mu'_3 + 6\mu'^2_1\mu'_2 - 3\mu'^4_1}{(\mu'_2 - \mu'^2_1)^2} = e^4 + 2e^3 + 3e^2 - 3$$

### 33.12 Parametric median

$$\text{Median}(X) = \text{Loc} + \text{Sc} \times \exp \left( \Phi^{-1}(1/2) \right)$$

### 33.13 Parametric mode

$$\text{Mode}(X) = \text{Loc} + \frac{\text{Sc}}{e}$$



### 33.14 Additional information and definitions

- Loc : Location parameter
- Sc : Scale parameter
- $z(x) = (x - \text{Loc}) / \text{Sc}$
- $u$  : Uniform[0,1] random variable
- $\Phi(x)$  : CDF normal standard distribution
- $\Phi^{-1}(x)$  : PPF normal standard distribution
- $\text{erf}(x)$  : Error function

### 33.15 Spreadsheet documents

- [Excel file from GitHub repository](#)
- [Google spreadsheet document](#)

## 34 Gumbel Left Distribution

### 34.1 Distribution definition

$$X \sim \text{GumbelLeft}(\mu, \sigma)$$

### 34.2 Distribution domain

$$x \in (-\infty, \infty)$$

### 34.3 Parameters domain and parameters constraints

$$\mu \in \mathbb{R}, \sigma \in \mathbb{R}^+$$

### 34.4 Cumulative distribution function

$$F_X(x) = 1 - \exp\left(-e^{z(x)}\right)$$

### 34.5 Probability density function

$$f_X(x) = \frac{1}{\sigma} \exp\left(z(x) - e^{z(x)}\right)$$

### 34.6 Percent point function/Sample

$$F_X^{-1}(u) = \mu + \sigma \ln(-\ln(1-u))$$

### 34.7 Parametric centered moments

$$\tilde{\mu}'_k = E[\tilde{X}^k] = \int_{-\infty}^{\infty} x^k f_{\tilde{X}}(x) dx$$

### 34.8 Parametric mean

$$\text{Mean}(X) = \mu + \sigma \tilde{\mu}'_1 = \mu - \gamma \sigma$$

### 34.9 Parametric variance

$$\text{Variance}(X) = \sigma^2(\tilde{\mu}'_2 - \tilde{\mu}'_1{}^2) = \sigma^2 \frac{\pi^2}{6}$$

### 34.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\tilde{\mu}'_3 - 3\tilde{\mu}'_2\tilde{\mu}'_1 + 2\tilde{\mu}'_1{}^3}{(\tilde{\mu}'_2 - \tilde{\mu}'_1{}^2)^{1.5}} = -\frac{12\sqrt{6}\zeta(3)}{\pi^3}$$

### 34.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\tilde{\mu}'_4 - 4\tilde{\mu}'_1\tilde{\mu}'_3 + 6\tilde{\mu}'_1{}^2\tilde{\mu}'_2 - 3\tilde{\mu}'_1{}^4}{(\tilde{\mu}'_2 - \tilde{\mu}'_1{}^2)^2} = 3 + \frac{12}{5}$$

### 34.12 Parametric median

$$\text{Median}(X) = \mu + \sigma \ln\left(-\ln\left(\frac{1}{2}\right)\right)$$

### 34.13 Parametric mode

$$\text{Mode}(X) = \mu$$

### 34.14 Additional information and definitions

- $\tilde{X} \sim \text{GumbelLeft}(0, 1)$
- $\mu$  : Location parameter
- $\sigma$  : Scale parameter
- $z(x) = (x - \mu) / \sigma$
- $u$  : Uniform[0,1] random variable
- $\gamma$  : Euler-Mascheroni constant = 0.5772156649
- $\zeta(3)$  : Apéry's constant = 1.2020569031

### 34.15 Spreadsheet documents

- [Excel file from GitHub repository](#)
- [Google spreadsheet document](#)

## 35 Gumbel Right Distribution

### 35.1 Distribution definition

$$X \sim \text{GumbelRight}(\mu, \sigma)$$

### 35.2 Distribution domain

$$x \in (-\infty, \infty)$$

### 35.3 Parameters domain and parameters constraints

$$\mu \in \mathbb{R}, \sigma \in \mathbb{R}^+$$

### 35.4 Cumulative distribution function

$$F_X(x) = \exp\left(-e^{-z(x)}\right)$$

### 35.5 Probability density function

$$f_X(x) = \frac{1}{\sigma} \exp\left(-\left(z(x) + e^{-z(x)}\right)\right)$$

### 35.6 Percent point function/Sample

$$F_X^{-1}(u) = \tilde{\mu} - \sigma \ln(-\ln(u))$$

### 35.7 Parametric centered moments

$$\tilde{\mu}'_k = E[\tilde{X}^k] = \int_{-\infty}^{\infty} x^k f_{\tilde{X}}(x) dx$$

### 35.8 Parametric mean

$$\text{Mean}(X) = \mu + \sigma \tilde{\mu}'_1 = \mu + \gamma \sigma$$

### 35.9 Parametric variance

$$\text{Variance}(X) = \sigma^2(\tilde{\mu}'_2 - \tilde{\mu}'_1{}^2) = \sigma^2 \frac{\pi^2}{6}$$

### 35.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\tilde{\mu}'_3 - 3\tilde{\mu}'_2\tilde{\mu}'_1 + 2\tilde{\mu}'_1{}^3}{(\tilde{\mu}'_2 - \tilde{\mu}'_1{}^2)^{1.5}} = \frac{12\sqrt{6}\zeta(3)}{\pi^3}$$

### 35.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\tilde{\mu}'_4 - 4\tilde{\mu}'_1\tilde{\mu}'_3 + 6\tilde{\mu}'_1{}^2\tilde{\mu}'_2 - 3\tilde{\mu}'_1{}^4}{(\tilde{\mu}'_2 - \tilde{\mu}'_1{}^2)^2} = 3 + \frac{12}{5}$$

### 35.12 Parametric median

$$\text{Median}(X) = \mu - \sigma \ln\left(-\ln\left(\frac{1}{2}\right)\right)$$

### 35.13 Parametric mode

$$\text{Mode}(X) = \mu$$

### 35.14 Additional information and definitions

- $\tilde{X} \sim \text{GumbelRight}(0, 1)$
- $\mu$  : Location parameter
- $\sigma$  : Scale parameter
- $z(x) = (x - \mu) / \sigma$
- $u$  : Uniform[0,1] random variable
- $\gamma$  : Euler-Mascheroni constant = 0.5772156649
- $\zeta(3)$  : Apéry's constant = 1.2020569031

### 35.15 Spreadsheet documents

- [Excel file from GitHub repository](#)
- [Google spreadsheet document](#)

## 36 Half Normal Distribution

### 36.1 Distribution definition

$$X \sim \text{HalfNormal}(\mu, \sigma)$$

### 36.2 Distribution domain

$$x \in (\mu, \infty)$$

### 36.3 Parameters domain and parameters constraints

$$\mu \in \mathbb{R}, \sigma \in \mathbb{R}^+$$

### 36.4 Cumulative distribution function

$$F_X(x) = 2\Phi(z(x)) - 1 = \text{erf}\left(\frac{z(x)}{\sqrt{2}}\right)$$

### 36.5 Probability density function

$$f_X(x) = \frac{\sqrt{2}}{\sigma\sqrt{\pi}} \exp\left(-\frac{z(x)^2}{2}\right)$$

### 36.6 Percent point function/Sample

$$F_X^{-1}(u) = \mu + \sigma\Phi^{-1}\left(\frac{1+u}{2}\right) = \tilde{\mu} + \sigma\sqrt{2}\text{erf}^{-1}(u)$$

### 36.7 Parametric centered moments

$$\tilde{\mu}'_k = E[\tilde{X}^k] = \int_0^\infty x^k f_{\tilde{X}}(x) dx = \frac{2^{n/2}\Gamma(\frac{n+1}{2})}{\sqrt{\pi}}$$

### 36.8 Parametric mean

$$\text{Mean}(X) = \tilde{\mu} + \sigma\tilde{\mu}'_1 = \tilde{\mu} + \sigma\sqrt{\frac{2}{\pi}}$$

### 36.9 Parametric variance

$$\text{Variance}(X) = \sigma^2(\tilde{\mu}'_2 - \tilde{\mu}'_1^2) = \sigma^2\left(1 - \frac{2}{\pi}\right)$$

### 36.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\tilde{\mu}'_3 - 3\tilde{\mu}'_2\tilde{\mu}'_1 + 2\tilde{\mu}'_1^3}{(\tilde{\mu}'_2 - \tilde{\mu}'_1^2)^{1.5}} = \frac{\sqrt{2}(4-\pi)}{(\pi-2)^{3/2}} = 0.9952717$$

### 36.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\tilde{\mu}'_4 - 4\tilde{\mu}'_1\tilde{\mu}'_3 + 6\tilde{\mu}'_1^2\tilde{\mu}'_2 - 3\tilde{\mu}'_1^4}{(\tilde{\mu}'_2 - \tilde{\mu}'_1^2)^2} = 3 + \frac{8(\pi-3)}{(\pi-2)^2} = 3.869177$$

### 36.12 Parametric median

$$\text{Median}(X) = \mu + \sigma\sqrt{2}\text{erf}^{-1}(1/2)$$

### 36.13 Parametric mode

$$\text{Mode}(X) = \mu$$

### 36.14 Additional information and definitions

- $\tilde{X} \sim \text{HalfNormal}(0, 1)$
- $\mu$  : Location parameter
- $\sigma$  : Scale parameter
- $z(x) = (x - \mu) / \sigma$
- $u$  : Uniform[0,1] random variable
- $\Phi(x)$  : CDF normal standard distribution
- $\Phi^{-1}(x)$  : PPF normal standard distribution
- $\text{erf}(x)$  : Error function
- $\Gamma(x)$  : Gamma function

### 36.15 Spreadsheet documents

- [Excel file from GitHub repository](#)
- [Google spreadsheet document](#)

## 37 Hyperbolic Secant Distribution

### 37.1 Distribution definition

$$X \sim \text{HyperbolicSecant}(\mu, \sigma)$$

### 37.2 Distribution domain

$$x \in (-\infty, \infty)$$

### 37.3 Parameters domain and parameters constraints

$$\mu \in \mathbb{R}, \sigma \in \mathbb{R}^+$$

### 37.4 Cumulative distribution function

$$F_X(x) = \frac{2}{\pi} \arctan \left[ \exp \left( \frac{\pi}{2} z(x) \right) \right]$$

### 37.5 Probability density function

$$f_X(x) = \frac{1}{2\sigma} \operatorname{sech} \left( \frac{\pi}{2} z(x) \right)$$

### 37.6 Percent point function/Sample

$$F_X^{-1}(u) = \mu + \sigma \frac{2}{\pi} \ln \left[ \tan \left( \frac{\pi}{2} u \right) \right]$$

### 37.7 Parametric centered moments

$$\tilde{\mu}'_k = E[\tilde{X}^k] = \int_{-\infty}^{\infty} x^k f_{\tilde{X}}(x) dx = \frac{1 + (-1)^k}{2\pi 2^{2k}} k! \left[ \zeta \left( k+1, \frac{1}{4} \right) - \zeta \left( k+1, \frac{3}{4} \right) \right]$$

### 37.8 Parametric mean

$$\text{Mean}(X) = \mu + \sigma \tilde{\mu}'_1 = \mu$$

### 37.9 Parametric variance

$$\text{Variance}(X) = \sigma^2 (\tilde{\mu}'_2 - \tilde{\mu}'_1{}^2) = \sigma^2$$

### 37.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\tilde{\mu}'_3 - 3\tilde{\mu}'_2\tilde{\mu}'_1 + 2\tilde{\mu}'_1{}^3}{(\tilde{\mu}'_2 - \tilde{\mu}'_1{}^2)^{1.5}} = 0$$

### 37.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\tilde{\mu}'_4 - 4\tilde{\mu}'_1\tilde{\mu}'_3 + 6\tilde{\mu}'_1{}^2\tilde{\mu}'_2 - 3\tilde{\mu}'_1{}^4}{(\tilde{\mu}'_2 - \tilde{\mu}'_1{}^2)^2} = 3$$

### 37.12 Parametric median

$$\text{Median}(X) = \mu$$

### 37.13 Parametric mode

$$\text{Mode}(X) = \mu$$



### 37.14 Additional information and definitions

- $\tilde{X} \sim \text{HyperbolicSecant}(0, 1)$
- $\mu$  : Location parameter
- $\sigma$  : Scale parameter
- $z(x) = (x - \mu) / \sigma$
- $u$  : Uniform[0,1] random variable
- $\zeta(a, s)$  : Hurwitz zeta function

### 37.15 Spreadsheet documents

- [Excel file from GitHub repository](#)
- [Google spreadsheet document](#)

## 38 Inverse Gamma Distribution

### 38.1 Distribution definition

$$X \sim \text{InverseGamma}(\alpha, \beta)$$

### 38.2 Distribution domain

$$x \in (0, \infty)$$

### 38.3 Parameters domain and parameters constraints

$$\alpha \in \mathbb{R}^+, \beta \in \mathbb{R}^+$$

### 38.4 Cumulative distribution function

$$F_X(x) = 1 - \frac{\gamma(\alpha, \beta/x)}{\Gamma(\alpha)} = 1 - P\left(\alpha, \frac{\beta}{x}\right)$$

### 38.5 Probability density function

$$f_X(x) = \frac{\beta^\alpha}{\Gamma(\alpha)} x^{-\alpha-1} \exp\left(-\frac{\beta}{x}\right)$$

### 38.6 Percent point function/Sample

$$F_X^{-1}(u) = \frac{\beta}{P^{-1}(\alpha, 1-u)}$$

### 38.7 Parametric centered moments

$$\tilde{\mu}'_k = E[\tilde{X}^k] = \int_0^\infty x^k f_{\tilde{X}}(x) dx = \frac{\Gamma(\alpha - k)}{\Gamma(\alpha)} = \frac{1}{(\alpha - 1) \cdots (\alpha - k)} \quad \text{if } \alpha > k$$

### 38.8 Parametric mean

$$\text{Mean}(X) = \beta \tilde{\mu}'_1$$

### 38.9 Parametric variance

$$\text{Variance}(X) = \beta^2 (\tilde{\mu}'_2 - \tilde{\mu}'_1^2)$$

### 38.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\tilde{\mu}'_3 - 3\tilde{\mu}'_2\tilde{\mu}'_1 + 2\tilde{\mu}'_1^3}{(\tilde{\mu}'_2 - \tilde{\mu}'_1^2)^{1.5}}$$

### 38.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\tilde{\mu}'_4 - 4\tilde{\mu}'_1\tilde{\mu}'_3 + 6\tilde{\mu}'_1^2\tilde{\mu}'_2 - 3\tilde{\mu}'_1^4}{(\tilde{\mu}'_2 - \tilde{\mu}'_1^2)^2}$$

### 38.12 Parametric median

$$\text{Median}(X) = \frac{\beta}{P^{-1}\left(\alpha, \frac{1}{2}\right)}$$

### 38.13 Parametric mode

$$\text{Mode}(X) = \frac{\beta}{\alpha + 1}$$

### 38.14 Additional information and definitions

- $\tilde{X} \sim \text{InverseGamma}(\alpha, 1)$
- $\beta$  : Scale parameter
- $u$  :  $\text{Uniform}[0,1]$  random variable
- $P(a, x) = \frac{\gamma(a, x)}{\Gamma(a)}$  : Regularized lower incomplete gamma function
- $P^{-1}(a, u)$  : Inverse of regularized lower incomplete gamma function
- $\gamma(a, x)$  : Lower incomplete gamma function
- $\Gamma(x)$  : Gamma function

### 38.15 Spreadsheet documents

- [Excel file from GitHub repository](#)
- [Google spreadsheet document](#)

## 39 Inverse Gamma 3P Distribution

### 39.1 Distribution definition

$$X \sim \text{InverseGamma}_{3P}(\alpha, \text{Loc}, \beta)$$

### 39.2 Distribution domain

$$x \in (\text{Loc}, \infty)$$

### 39.3 Parameters domain and parameters constraints

$$\alpha \in \mathbb{R}^+, \text{Loc} \in \mathbb{R}, \beta \in \mathbb{R}^+$$

### 39.4 Cumulative distribution function

$$F_X(x) = 1 - \frac{\gamma(\alpha, \beta/(x - \text{Loc}))}{\Gamma(\alpha)} = 1 - P\left(\alpha, \frac{\beta}{x - \text{Loc}}\right)$$

### 39.5 Probability density function

$$f_X(x) = \frac{\beta^\alpha}{\Gamma(\alpha)} (x - \text{Loc})^{-\alpha-1} \exp\left(-\frac{\beta}{x - \text{Loc}}\right)$$

### 39.6 Percent point function/Sample

$$F_X^{-1}(u) = \text{Loc} + \frac{\beta}{P^{-1}(\alpha, 1 - u)}$$

### 39.7 Parametric centered moments

$$\tilde{\mu}'_k = E[\tilde{X}^k] = \int_0^\infty x^k f_{\tilde{X}}(x) dx = \frac{\Gamma(\alpha - k)}{\Gamma(\alpha)} = \frac{1}{(\alpha - 1) \cdots (\alpha - k)} \quad \text{if } \alpha > k$$

### 39.8 Parametric mean

$$\text{Mean}(X) = \text{Loc} + \beta \mu'_1$$

### 39.9 Parametric variance

$$\text{Variance}(X) = \beta^2 (\mu'_2 - \mu'^2_1)$$

### 39.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\mu'_3 - 3\mu'_2\mu'_1 + 2\mu'^3_1}{(\mu'_2 - \mu'^2_1)^{1.5}}$$

### 39.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\mu'_4 - 4\mu'_1\mu'_3 + 6\mu'^2_1\mu'_2 - 3\mu'^4_1}{(\mu'_2 - \mu'^2_1)^2}$$

### 39.12 Parametric median

$$\text{Median}(X) = \text{Loc} + \frac{\beta}{P^{-1}\left(\alpha, \frac{1}{2}\right)}$$

### 39.13 Parametric mode

$$\text{Mode}(X) = \text{Loc} + \frac{\beta}{\alpha + 1}$$

### 39.14 Additional information and definitions

- $\tilde{X} \sim \text{InverseGamma}_{3P}(\alpha, 0, 1)$
- $\text{Loc}$  : Location parameter
- $\beta$  : Scale parameter
- $u$  : Uniform[0,1] random variable
- $P(a, x) = \frac{\gamma(a, x)}{\Gamma(a)}$  : Regularized lower incomplete gamma function
- $P^{-1}(a, u)$  : Inverse of regularized lower incomplete gamma function
- $\gamma(a, x)$  : Lower incomplete gamma function
- $\Gamma(x)$  : Gamma function

### 39.15 Spreadsheet documents

- [Excel file from GitHub repository](#)
- [Google spreadsheet document](#)

## 40 Inverse Gaussian Distribution

### 40.1 Distribution definition

$$X \sim \text{InverseGaussian}(\mu, \lambda)$$

### 40.2 Distribution domain

$$x \in (0, \infty)$$

### 40.3 Parameters domain and parameters constraints

$$\mu \in \mathbb{R}^+, \lambda \in \mathbb{R}^+$$

### 40.4 Cumulative distribution function

$$F_X(x) = \Phi\left(\sqrt{\frac{\lambda}{x}}\left(\frac{x}{\mu} - 1\right)\right) + \exp\left(\frac{2\lambda}{\mu}\right)\Phi\left(-\sqrt{\frac{\lambda}{x}}\left(\frac{x}{\mu} + 1\right)\right)$$

### 40.5 Probability density function

$$f_X(x) = \sqrt{\frac{\lambda}{2\pi x^3}} \exp\left[-\frac{\lambda(x - \mu)^2}{2\mu^2 x}\right]$$

### 40.6 Percent point function/Sample

$$\text{Sample}_X = \begin{cases} x_0 & \text{if } u_2 \leq \frac{\mu}{\mu + x_0} \\ \frac{\mu^2}{x_0} & \text{if } u_2 \geq \frac{\mu}{\mu + x_0} \end{cases}$$

### 40.7 Parametric centered moments

$$\mu'_k = E[X^k] = \int_0^\infty x^k f_X(x) dx$$

### 40.8 Parametric mean

$$\text{Mean}(X) = \mu'_1 = \mu$$

### 40.9 Parametric variance

$$\text{Variance}(X) = \mu'_2 - \mu'^2_1 = \frac{\mu^3}{\lambda}$$

### 40.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\mu'_3 - 3\mu'_2\mu'_1 + 2\mu'^3_1}{(\mu'_2 - \mu'^2_1)^{1.5}} = 3\left(\frac{\mu}{\lambda}\right)^{1/2}$$

### 40.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\mu'_4 - 4\mu'_1\mu'_3 + 6\mu'^2_1\mu'_2 - 3\mu'^4_1}{(\mu'_2 - \mu'^2_1)^2} = 3 + \frac{15\mu}{\lambda}$$

### 40.12 Parametric median

$$\text{Median}(X) = F_X^{-1}\left(\frac{1}{2}\right)$$

#### 40.13 Parametric mode

$$\text{Mode}(X) = \mu \left[ \left( 1 + \frac{9\mu^2}{4\lambda^2} \right)^{\frac{1}{2}} - \frac{3\mu}{2\lambda} \right]$$

#### 40.14 Additional information and definitions

- Computing an analytic expression for the inverse of the cumulative distribution function is not feasible. Nonetheless, it is possible to generate a random sample from the distribution.
- $\Phi(x)$  : CDF normal standard distribution
- $\Phi^{-1}(x)$  : PPF normal standard distribution
- $x_0 = \mu + \frac{\mu^2[\Phi^{-1}(u_1)]^2}{2\lambda} - \frac{\mu}{2\lambda} \sqrt{4\mu\lambda[\Phi^{-1}(u_1)]^2 + \mu^2([\Phi^{-1}(u_1)]^2)^2}$
- $u_1$  : Uniform[0,1] random variable
- $u_2$  : Uniform[0,1] random variable

#### 40.15 Spreadsheet documents

- [Excel file from GitHub repository](#)
- [Google spreadsheet document](#)

## 41 Inverse Gaussian 3P Distribution

### 41.1 Distribution definition

$$X \sim \text{InverseGaussian}_{3P}(\mu, \lambda, \text{Loc})$$

### 41.2 Distribution domain

$$x \in (0, \infty)$$

### 41.3 Parameters domain and parameters constraints

$$\mu \in \mathbb{R}^+, \lambda \in \mathbb{R}^+, \text{Loc} \in \mathbb{R}$$

### 41.4 Cumulative distribution function

$$F_X(x) = \Phi\left(\sqrt{\frac{\lambda}{x - \text{Loc}}} \left(\frac{x - \text{Loc}}{\mu} - 1\right)\right) + \exp\left(\frac{2\lambda}{\mu}\right) \Phi\left(-\sqrt{\frac{\lambda}{x - \text{Loc}}} \left(\frac{x - \text{Loc}}{\mu} + 1\right)\right)$$

### 41.5 Probability density function

$$f_X(x) = \sqrt{\frac{\lambda}{2\pi(x - \text{Loc})^3}} \exp\left[-\frac{\lambda(x - \mu - \text{Loc})^2}{2\mu^2(x - \text{Loc})}\right]$$

### 41.6 Percent point function/Sample

$$\text{Sample}_X = \begin{cases} \text{Loc} + x_0 & \text{if } u_2 \leq \frac{\mu}{\mu + x_0} \\ \text{Loc} + \frac{\mu^2}{x_0} & \text{if } u_2 \geq \frac{\mu}{\mu + x_0} \end{cases}$$

### 41.7 Parametric centered moments

$$\mu'_k = E[X^k] = \int_{\text{Loc}}^{\infty} x^k f_X(x) dx$$

### 41.8 Parametric mean

$$\text{Mean}(X) = \mu'_1 = \text{Loc} + \mu$$

### 41.9 Parametric variance

$$\text{Variance}(X) = \mu'_2 - \mu'^2_1 = \frac{\mu^3}{\lambda}$$

### 41.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\mu'_3 - 3\mu'_2\mu'_1 + 2\mu'^3_1}{(\mu'_2 - \mu'^2_1)^{1.5}} = 3\left(\frac{\mu}{\lambda}\right)^{1/2}$$

### 41.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\mu'_4 - 4\mu'_1\mu'_3 + 6\mu'^2_1\mu'_2 - 3\mu'^4_1}{(\mu'_2 - \mu'^2_1)^2} = 3 + \frac{15\mu}{\lambda}$$

### 41.12 Parametric median

$$\text{Median}(X) = F_X^{-1}\left(\frac{1}{2}\right)$$



#### 41.13 Parametric mode

$$\text{Mode}(X) = \text{Loc} + \mu \left[ \left( 1 + \frac{9\mu^2}{4\lambda^2} \right)^{\frac{1}{2}} - \frac{3\mu}{2\lambda} \right]$$

#### 41.14 Additional information and definitions

- Computing an analytic expression for the inverse of the cumulative distribution function is not feasible. Nonetheless, it is possible to generate a random sample from the distribution.
- Loc : Location parameter
- $\Phi(x)$  : CDF normal standard distribution
- $\Phi^{-1}(x)$  : PPF normal standard distribution
- $x_0 = \mu + \frac{\mu^2[\Phi^{-1}(u_1)]^2}{2\lambda} - \frac{\mu}{2\lambda} \sqrt{4\mu\lambda[\Phi^{-1}(u_1)]^2 + \mu^2([\Phi^{-1}(u_1)]^2)^2}$
- $u_1$  : Uniform[0,1] random variable
- $u_2$  : Uniform[0,1] random variable

#### 41.15 Spreadsheet documents

- [Excel file from GitHub repository](#)
- [Google spreadsheet document](#)

## 42 Johnson SB Distribution

### 42.1 Distribution definition

$$X \sim \text{JohnsonSB}(\xi, \lambda, \gamma, \delta)$$

### 42.2 Distribution domain

$$x \in (\xi, \xi + \lambda)$$

### 42.3 Parameters domain and parameters constraints

$$\xi \in \mathbb{R}, \lambda \in \mathbb{R}^+, \gamma \in \mathbb{R}, \delta \in \mathbb{R}^+$$

### 42.4 Cumulative distribution function

$$F_X(x) = \Phi\left(\gamma + \delta \ln \frac{z(x)}{1 - z(x)}\right)$$

### 42.5 Probability density function

$$f_X(x) = \frac{\delta}{\lambda \sqrt{2\pi} z(1 - z(x))} \exp\left[-\frac{1}{2} \left(\gamma + \delta \ln \frac{z(x)}{1 - z(x)}\right)^2\right]$$

### 42.6 Percent point function/Sample

$$F_X^{-1}(u) = \frac{\lambda \exp\left(\frac{\Phi^{-1}(u) - \gamma}{\delta}\right)}{1 + \exp\left(\frac{\Phi^{-1}(u) - \gamma}{\delta}\right)} + \xi$$

### 42.7 Parametric centered moments

$$\mu'_k = E[X^k] = \int_{\xi}^{\xi + \lambda} x^k f_X(x) dx$$

### 42.8 Parametric mean

$$\text{Mean}(X) = \mu'_1$$

### 42.9 Parametric variance

$$\text{Variance}(X) = \mu'_2 - \mu_1'^2$$

### 42.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\mu'_3 - 3\mu'_2\mu'_1 + 2\mu_1'^3}{(\mu'_2 - \mu_1'^2)^{1.5}}$$

### 42.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\mu'_4 - 4\mu'_1\mu'_3 + 6\mu_1'^2\mu'_2 - 3\mu_1'^4}{(\mu'_2 - \mu_1'^2)^2}$$

### 42.12 Parametric median

$$\text{Median}(X) = \frac{\lambda \exp\left(\frac{\Phi^{-1}(1/2) - \gamma}{\delta}\right)}{1 + \exp\left(\frac{\Phi^{-1}(1/2) - \gamma}{\delta}\right)} + \xi$$

### 42.13 Parametric mode

$$\text{Mode}(X) = \arg \max_x f_X(x)$$

### 42.14 Additional information and definitions

- $\xi$  : Location parameter
- $\lambda$  : Scale parameter
- $z(x) = (x - \xi) / \lambda$
- $u$  : Uniform[0,1] random variable
- $\Phi(x)$  : CDF normal standard distribution
- $\Phi^{-1}(x)$  : PPF normal standard distribution

### 42.15 Spreadsheet documents

- [Excel file from GitHub repository](#)
- [Google spreadsheet document](#)

## 43 Johnson SU Distribution

### 43.1 Distribution definition

$$X \sim \text{JohnsonSU}(\xi, \lambda, \gamma, \delta)$$

### 43.2 Distribution domain

$$x \in (-\infty, \infty)$$

### 43.3 Parameters domain and parameters constraints

$$\xi \in \mathbb{R}, \lambda \in \mathbb{R}^+, \gamma \in \mathbb{R}, \delta \in \mathbb{R}^+$$

### 43.4 Cumulative distribution function

$$F_X(x) = \Phi(\gamma + \delta \sinh^{-1}(z(x)))$$

### 43.5 Probability density function

$$f_X(x) = \frac{\delta}{\lambda \sqrt{2\pi} \sqrt{z(x)^2 + 1}} \exp \left[ -\frac{1}{2} (\gamma + \delta \sinh^{-1}(z(x)))^2 \right]$$

### 43.6 Percent point function/Sample

$$F_X^{-1}(u) = \lambda \sinh \left( \frac{\Phi^{-1}(u) - \gamma}{\delta} \right) + \xi$$

### 43.7 Parametric centered moments

$$\mu'_k = E[X^k] = \int_{-\infty}^{\infty} x^k f_X(x) dx$$

### 43.8 Parametric mean

$$\text{Mean}(X) = \mu'_1 = \xi - \lambda \exp \frac{\delta^{-2}}{2} \sinh \left( \frac{\gamma}{\delta} \right)$$

### 43.9 Parametric variance

$$\text{Variance}(X) = \mu'_2 - \mu_1'^2 = \frac{\lambda^2}{2} (\exp(\delta^{-2}) - 1) \left( \exp(\delta^{-2}) \cosh \left( \frac{2\gamma}{\delta} \right) + 1 \right)$$

### 43.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\mu'_3 - 3\mu'_2\mu'_1 + 2\mu_1'^3}{(\mu'_2 - \mu_1'^2)^{1.5}} = -\frac{\lambda^3 \sqrt{e^{\delta^{-2}}} (e^{\delta^{-2}} - 1)^2 (e^{\delta^{-2}} + 2) \sinh(\frac{3\gamma}{\delta}) + 3 \sinh(\frac{2\gamma}{\delta})}{4 \text{Variance}(X)^{1.5}}$$

### 43.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\mu'_4 - 4\mu'_1\mu'_3 + 6\mu_1'^2\mu'_2 - 3\mu_1'^4}{(\mu'_2 - \mu_1'^2)^2} = \frac{\lambda^4 (e^{\delta^{-2}} - 1)^2 (K_1 + K_2 + K_3)}{8 \text{Variance}(X)^2}$$

### 43.12 Parametric median

$$\text{Median}(X) = \xi + \lambda \sinh \left( -\frac{\gamma}{\delta} \right)$$

### 43.13 Parametric mode

$$\text{Mode}(X) = \arg \max_x f_X(x)$$

#### 43.14 Additional information and definitions

- $\xi$  : Location parameter
- $\lambda$  : Scale parameter
- $z(x) = (x - \xi) / \lambda$
- $u$  : Uniform[0,1] random variable
- $\Phi(x)$  : CDF normal standard distribution
- $\Phi^{-1}(x)$  : PPF normal standard distribution
- $K_1 = \left(e^{\delta^{-2}}\right)^2 \left(\left(e^{\delta^{-2}}\right)^4 + 2\left(e^{\delta^{-2}}\right)^3 + 3\left(e^{\delta^{-2}}\right)^2 - 3\right) \cosh\left(\frac{4\gamma}{\delta}\right)$
- $K_2 = 4\left(e^{\delta^{-2}}\right)^2 \left(\left(e^{\delta^{-2}}\right) + 2\right) \cosh\left(\frac{3\gamma}{\delta}\right)$
- $K_3 = 3\left(2\left(e^{\delta^{-2}}\right) + 1\right)$

#### 43.15 Spreadsheet documents

- [Excel file from GitHub repository](#)
- [Google spreadsheet document](#)

## 44 Kumaraswamy Distribution

### 44.1 Distribution definition

$$X \sim \text{Kumaraswamy}(\alpha, \beta, \min, \max)$$

### 44.2 Distribution domain

$$x \in (\min, \max)$$

### 44.3 Parameters domain and parameters constraints

$$\alpha \in \mathbb{R}^+, \beta \in \mathbb{R}^+, \min \in \mathbb{R}, \max \in \mathbb{R}$$

### 44.4 Cumulative distribution function

$$F_X(x) = 1 - (1 - z(x)^\alpha)^\beta$$

### 44.5 Probability density function

$$f_X(x) = \alpha\beta z(x)^{\alpha-1}(1 - z(x)^\alpha)^{\beta-1}$$

### 44.6 Percent point function/Sample

$$F_X^{-1}(u) = \min + (\max - \min) \times (1 - (1 - u)^{\frac{1}{\beta}})^{\frac{1}{\alpha}}$$

### 44.7 Parametric centered moments

$$\tilde{\mu}'_k = E[\tilde{X}^k] = \int_0^1 x^k f_{\tilde{X}}(x) dx = \beta \text{Beta}(1 + \frac{k}{\alpha}, \beta)$$

### 44.8 Parametric mean

$$\text{Mean}(X) = \min + (\max - \min) \times \tilde{\mu}'_1$$

### 44.9 Parametric variance

$$\text{Variance}(X) = (\max - \min)^2 (\tilde{\mu}'_2 - \tilde{\mu}'_1^2)$$

### 44.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\tilde{\mu}'_3 - 3\tilde{\mu}'_2\tilde{\mu}'_1 + 2\tilde{\mu}'_1^3}{(\tilde{\mu}'_2 - \tilde{\mu}'_1^2)^{1.5}}$$

### 44.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\tilde{\mu}'_4 - 4\tilde{\mu}'_1\tilde{\mu}'_3 + 6\tilde{\mu}'_1^2\tilde{\mu}'_2 - 3\tilde{\mu}'_1^4}{(\tilde{\mu}'_2 - \tilde{\mu}'_1^2)^2}$$

### 44.12 Parametric median

$$\text{Median}(X) = \min + (\max - \min) \times \left(1 - 2^{-1/b}\right)^{1/a}$$

### 44.13 Parametric mode

$$\text{Mode}(X) = \min + (\max - \min) \times \left(\frac{a-1}{ab-1}\right)^{1/a}$$

#### 44.14 Additional information and definitions

- $\tilde{X} \sim \text{Kumaraswamy}(\alpha, \beta, 0, 1)$
- $z(x) = (x - \min) / (\max - \min)$
- $u : \text{Uniform}[0,1]$  random variable
- $\text{Beta}(x, y) : \text{Beta function}$

#### 44.15 Spreadsheet documents

- [Excel file from GitHub repository](#)
- [Google spreadsheet document](#)

## 45 Laplace Distribution

### 45.1 Distribution definition

$$X \sim \text{Laplace}(\mu, b)$$

### 45.2 Distribution domain

$$x \in (-\infty, \infty)$$

### 45.3 Parameters domain and parameters constraints

$$\mu \in \mathbb{R}^+, b \in \mathbb{R}^+$$

### 45.4 Cumulative distribution function

$$F_X(x) = \frac{1}{2} + \frac{1}{2}\text{sign}(x - \mu) \left( 1 - \exp\left(-\frac{|x - \mu|}{b}\right) \right)$$

### 45.5 Probability density function

$$f_X(x) = \frac{1}{2b} \exp\left(-\frac{|x - \mu|}{b}\right)$$

### 45.6 Percent point function/Sample

$$F_X^{-1}(u) = \mu - b \times \text{sign}\left(u - \frac{1}{2}\right) \ln\left(1 - 2\left|p - \frac{1}{2}\right|\right)$$

### 45.7 Parametric centered moments

$$\mu'_k = E[X^k] = \int_{-\infty}^{\infty} x^k f_X(x) dx = \left(\frac{1}{2}\right) \sum_{k=0}^r \left[ \frac{r!}{(r-k)!} b^k \mu^{(r-k)} \{1 + (-1)^k\} \right]$$

### 45.8 Parametric mean

$$\text{Mean}(X) = \mu'_1 = \mu$$

### 45.9 Parametric variance

$$\text{Variance}(X) = \mu'_2 - \mu'^2_1 = 2b^2$$

### 45.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\mu'_3 - 3\mu'_2\mu'_1 + 2\mu'^3_1}{(\mu'_2 - \mu'^2_1)^{1.5}} = 0$$

### 45.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\mu'_4 - 4\mu'_1\mu'_3 + 6\mu'^2_1\mu'_2 - 3\mu'^4_1}{(\mu'_2 - \mu'^2_1)^2} = 6$$

### 45.12 Parametric median

$$\text{Median}(X) = \mu$$

### 45.13 Parametric mode

$$\text{Mode}(X) = \mu$$



#### 45.14 Additional information and definitions

- $\mu$  : Location parameter
- $b$  : Scale parameter
- $u$  : Uniform[0,1] random variable

#### 45.15 Spreadsheet documents

- [Excel file from GitHub repository](#)
- [Google spreadsheet document](#)

## 46 Levy Distribution

### 46.1 Distribution definition

$$X \sim \text{Levy}(\mu, c)$$

### 46.2 Distribution domain

$$x \in [\mu, \infty)$$

### 46.3 Parameters domain and parameters constraints

$$\mu \in \mathbb{R}, c \in \mathbb{R}^+$$

### 46.4 Cumulative distribution function

$$F_X(x) = 1 - \operatorname{erf}\left(\sqrt{\frac{c}{2(x-\mu)}}\right)$$

### 46.5 Probability density function

$$f_X(x) = \sqrt{\frac{c}{2\pi}} \frac{e^{-\frac{c}{2(x-\mu)}}}{(x-\mu)^{3/2}}$$

### 46.6 Percent point function/Sample

$$F_X^{-1}(u) = \mu + \frac{c}{2(\operatorname{erf}^{-1}(1-u))^2}$$

### 46.7 Parametric centered moments

$$\mu'_k = E[X^k] = \int_{\mu}^{\infty} x^k f_X(x) dx$$

### 46.8 Parametric mean

$$\text{Mean}(X) = \mu'_1 = \infty$$

### 46.9 Parametric variance

$$\text{Variance}(X) = \mu'_2 - \mu'^2_1 = \infty$$

### 46.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\mu'_3 - 3\mu'_2\mu'_1 + 2\mu'^3_1}{(\mu'_2 - \mu'^2_1)^{1.5}} = \text{undefined}$$

### 46.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\mu'_4 - 4\mu'_1\mu'_3 + 6\mu'^2_1\mu'_2 - 3\mu'^4_1}{(\mu'_2 - \mu'^2_1)^2} = \text{undefined}$$

### 46.12 Parametric median

$$\text{Median}(X) = \mu + \frac{c}{2(\operatorname{erf}^{-1}(1/2))^2}$$

### 46.13 Parametric mode

$$\text{Mode}(X) = \mu + \frac{c}{3}$$

#### 46.14 Additional information and definitions

- $\mu$  : Location parameter
- $c$  : Scale parameter
- $u$  : Uniform[0,1] random variable
- $\text{erf}(x)$  : Error function
- $\text{erf}^{-1}(x)$  : Inverse of error function

#### 46.15 Spreadsheet documents

- [Excel file from GitHub repository](#)
- [Google spreadsheet document](#)

## 47 Loggamma Distribution

### 47.1 Distribution definition

$$X \sim \text{LogGamma}(c, \mu, \sigma)$$

### 47.2 Distribution domain

$$x \in (0, \infty)$$

### 47.3 Parameters domain and parameters constraints

$$c \in \mathbb{R}^+, \mu \in \mathbb{R}, \sigma \in \mathbb{R}^+$$

### 47.4 Cumulative distribution function

$$F_X(x) = \frac{\gamma(c, e^x)}{\Gamma(c)} = P(c, e^{z(x)})$$

### 47.5 Probability density function

$$f_X(x) = \frac{\exp(cz(x) - e^{z(x)})}{\sigma \Gamma(c)}$$

### 47.6 Percent point function/Sample

$$F_X^{-1}(u) = \mu + \sigma \ln(P^{-1}(u, c))$$

### 47.7 Parametric centered moments

$$\mu'_k = E[X^k] = \int_{-\infty}^{\infty} x^k f_X(x) dx$$

### 47.8 Parametric mean

$$\text{Mean}(X) = \mu'_1 = \mu + \sigma \psi_0$$

### 47.9 Parametric variance

$$\text{Variance}(X) = \mu'_2 - \mu'^2_1 = \sigma^2 \psi_1(c)$$

### 47.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\mu'_3 - 3\mu'_2\mu'_1 + 2\mu'^3_1}{(\mu'_2 - \mu'^2_1)^{1.5}} = \frac{\psi_2(c)}{\psi_1(c)}$$

### 47.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\mu'_4 - 4\mu'_1\mu'_3 + 6\mu'^2_1\mu'_2 - 3\mu'^4_1}{(\mu'_2 - \mu'^2_1)^2} = \frac{\psi_3(c)}{\psi_1(c)}$$

### 47.12 Parametric median

$$\text{Median}(X) = \mu + \sigma \ln(P^{-1}(1/2, c))$$

### 47.13 Parametric mode

$$\text{Mode}(X) = \mu + \sigma \ln(c)$$

#### 47.14 Additional information and definitions

- $\mu$  : Location parameter
- $\sigma$  : Scale parameter
- $z(x) = (x - \mu) / \sigma$
- $u$  : Uniform[0,1] random variable
- $P(a, x) = \frac{\gamma(a, x)}{\Gamma(a)}$  : Regularized lower incomplete gamma function
- $P^{-1}(a, u)$  : Inverse of regularized lower incomplete gamma function
- $\gamma(a, x)$  : Lower incomplete gamma function
- $\Gamma(x)$  : Gamma function
- $\psi_0(x)$  : Digamma function
- $\psi_n(x)$  : Polygamma function of order  $n \in \mathbb{N}$

#### 47.15 Spreadsheet documents

- [Excel file from GitHub repository](#)
- [Google spreadsheet document](#)

## 48 Logistic Distribution

### 48.1 Distribution definition

$$X \sim \text{Logistic}(\mu, \sigma)$$

### 48.2 Distribution domain

$$x \in (-\infty, \infty)$$

### 48.3 Parameters domain and parameters constraints

$$\mu \in \mathbb{R}, \sigma \in \mathbb{R}^+$$

### 48.4 Cumulative distribution function

$$F_X(x) = \frac{1}{1 + e^{-(x-\mu)/\sigma}}$$

### 48.5 Probability density function

$$f_X(x) = \frac{e^{-(x-\mu)/\sigma}}{\sigma \left(1 + e^{-(x-\mu)/\sigma}\right)^2}$$

### 48.6 Percent point function/Sample

$$F_X^{-1}(u) = \mu + \sigma \log\left(\frac{u}{1-u}\right)$$

### 48.7 Parametric centered moments

$$\mu'_k = E[X^k] = \int_{-\infty}^{\infty} x^k f_X(x) dx$$

### 48.8 Parametric mean

$$\text{Mean}(X) = \mu'_1 = \mu$$

### 48.9 Parametric variance

$$\text{Variance}(X) = \mu'_2 - \mu_1'^2 = \frac{\sigma^2 \pi^2}{3}$$

### 48.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\mu'_3 - 3\mu'_2\mu'_1 + 2\mu_1'^3}{(\mu'_2 - \mu_1'^2)^{1.5}} = 0$$

### 48.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\mu'_4 - 4\mu'_1\mu'_3 + 6\mu_1'^2\mu'_2 - 3\mu_1'^4}{(\mu'_2 - \mu_1'^2)^2} = 3 + 6/5$$

### 48.12 Parametric median

$$\text{Median}(X) = \mu$$

### 48.13 Parametric mode

$$\text{Mode}(X) = \mu$$

#### 48.14 Additional information and definitions

- $\mu$  : Location parameter
- $\sigma$  : Scale parameter
- $u$  : Uniform[0,1] random variable

#### 48.15 Spreadsheet documents

- [Excel file from GitHub repository](#)
- [Google spreadsheet document](#)

## 49 Loglogistic Distribution

### 49.1 Distribution definition

$$X \sim \text{LogLogistic}(\alpha, \beta)$$

### 49.2 Distribution domain

$$x \in [0, \infty)$$

### 49.3 Parameters domain and parameters constraints

$$\alpha \in \mathbb{R}^+, \beta \in \mathbb{R}^+$$

### 49.4 Cumulative distribution function

$$F_X(x) = \frac{1}{1 + (x/\alpha)^{-\beta}}$$

### 49.5 Probability density function

$$f_X(x) = \frac{(\beta/\alpha)(x/\alpha)^{\beta-1}}{(1 + (x/\alpha)^{\beta})^2}$$

### 49.6 Percent point function/Sample

$$F_X^{-1}(u) = \alpha \left( \frac{u}{1-u} \right)^{1/\beta}$$

### 49.7 Parametric centered moments

$$\mu'_k = E[X^k] = \int_0^\infty x^k f_X(x) dx = \alpha^k \text{Beta}(1 - k/\beta, 1 + k/\beta) = \alpha^k \frac{k\pi/\beta}{\sin(k\pi/\beta)}$$

### 49.8 Parametric mean

$$\text{Mean}(X) = \mu'_1$$

### 49.9 Parametric variance

$$\text{Variance}(X) = \mu'_2 - \mu_1'^2$$

### 49.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\mu'_3 - 3\mu'_2\mu'_1 + 2\mu_1'^3}{(\mu'_2 - \mu_1'^2)^{1.5}}$$

### 49.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\mu'_4 - 4\mu'_1\mu'_3 + 6\mu_1'^2\mu'_2 - 3\mu_1'^4}{(\mu'_2 - \mu_1'^2)^2}$$

### 49.12 Parametric median

$$\text{Median}(X) = \alpha$$

### 49.13 Parametric mode

$$\text{Mode}(X) = \alpha \left( \frac{\beta-1}{\beta+1} \right)^{1/\beta}$$



#### 49.14 Additional information and definitions

- $\alpha$  : Scale parameter
- $u$  : Uniform[0,1] random variable
- Beta( $x, y$ ) : Beta function

#### 49.15 Spreadsheet documents

- [Excel file from GitHub repository](#)
- [Google spreadsheet document](#)

## 50 Loglogistic 3P Distribution

### 50.1 Distribution definition

$$X \sim \text{LogLogistic}_{3P}(\text{Loc}, \alpha, \beta)$$

### 50.2 Distribution domain

$$x \in [\text{Loc}, \infty)$$

### 50.3 Parameters domain and parameters constraints

$$\text{Loc} \in \mathbb{R}, \alpha \in \mathbb{R}^+, \beta \in \mathbb{R}^+$$

### 50.4 Cumulative distribution function

$$F_X(x) = \frac{1}{1 + ((x - \text{Loc})/\alpha)^{-\beta}}$$

### 50.5 Probability density function

$$f_X(x) = \frac{(\beta/\alpha)((x - \text{Loc})/\alpha)^{\beta-1}}{(1 + ((x - \text{Loc})/\alpha)^\beta)^2}$$

### 50.6 Percent point function/Sample

$$F_X^{-1}(u) = \text{Loc} + \alpha \left( \frac{u}{1-u} \right)^{1/\beta}$$

### 50.7 Parametric centered moments

$$\tilde{\mu}'_k = E[\tilde{X}^k] = \int_0^\infty x^k f_{\tilde{X}}(x) dx = \alpha^k \text{Beta}(1 - k/\beta, 1 + k/\beta) = \alpha^k \frac{k\pi/\beta}{\sin(k\pi/\beta)}$$

### 50.8 Parametric mean

$$\text{Mean}(X) = \text{Loc} + \tilde{\mu}'_1$$

### 50.9 Parametric variance

$$\text{Variance}(X) = \tilde{\mu}'_2 - \tilde{\mu}'_1^2$$

### 50.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\tilde{\mu}'_3 - 3\tilde{\mu}'_2\tilde{\mu}'_1 + 2\tilde{\mu}'_1^3}{(\tilde{\mu}'_2 - \tilde{\mu}'_1^2)^{1.5}}$$

### 50.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\tilde{\mu}'_4 - 4\tilde{\mu}'_1\tilde{\mu}'_3 + 6\tilde{\mu}'_1^2\tilde{\mu}'_2 - 3\tilde{\mu}'_1^4}{(\tilde{\mu}'_2 - \tilde{\mu}'_1^2)^2}$$

### 50.12 Parametric median

$$\text{Median}(X) = \text{Loc} + \alpha$$

### 50.13 Parametric mode

$$\text{Mode}(X) = \text{Loc} + \alpha \left( \frac{\beta - 1}{\beta + 1} \right)^{1/\beta}$$

#### 50.14 Additional information and definitions

- $\tilde{X} \sim \text{LogLogistic}(\alpha, \beta)$
- $\text{Loc}$  : Location parameter
- $\alpha$  : Scale parameter
- $u$  : Uniform[0,1] random variable
- $\text{Beta}(x, y)$  : Beta function

#### 50.15 Spreadsheet documents

- [Excel file from GitHub repository](#)
- [Google spreadsheet document](#)

## 51 Lognormal Distribution

### 51.1 Distribution definition

$$X \sim \text{LogNormal}(\mu, \sigma)$$

### 51.2 Distribution domain

$$x \in (-\infty, \infty)$$

### 51.3 Parameters domain and parameters constraints

$$\mu \in \mathbb{R}, \sigma \in \mathbb{R}^+$$

### 51.4 Cumulative distribution function

$$F_X(x) = \frac{1}{2} \left[ 1 + \operatorname{erf} \left( \frac{\ln(x) - \mu}{\sigma\sqrt{2}} \right) \right]$$

### 51.5 Probability density function

$$f_X(x) = \frac{1}{x\sigma\sqrt{2\pi}} \exp \left( -\frac{(\ln(x) - \mu)^2}{2\sigma^2} \right)$$

### 51.6 Percent point function/Sample

$$F_X^{-1}(u) = \exp(\mu + \sqrt{2\sigma^2} \operatorname{erf}^{-1}(2u - 1))$$

### 51.7 Parametric centered moments

$$\mu'_k = E[X^k] = \int_{-\infty}^{\infty} x^k f_X(x) dx = e^{k\mu + k^2\sigma^2/2}$$

### 51.8 Parametric mean

$$\text{Mean}(X) = \mu'_1 = e^{\mu + \frac{\sigma^2}{2}}$$

### 51.9 Parametric variance

$$\text{Variance}(X) = \mu'_2 - \mu_1'^2 = e^{2\mu + \sigma^2}(e^{\sigma^2} - 1)$$

### 51.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\mu'_3 - 3\mu'_2\mu'_1 + 2\mu_1'^3}{(\mu'_2 - \mu_1'^2)^{1.5}} = (e^{\sigma^2} + 2)\sqrt{e^{\sigma^2} - 1}$$

### 51.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\mu'_4 - 4\mu'_1\mu'_3 + 6\mu_1'^2\mu'_2 - 3\mu_1'^4}{(\mu'_2 - \mu_1'^2)^2} = e^{4\sigma^2} + 2e^{3\sigma^2} + 3e^{2\sigma^2} - 3$$

### 51.12 Parametric median

$$\text{Median}(X) = \exp(\mu)$$

### 51.13 Parametric mode

$$\text{Mode}(X) = \exp(\mu - \sigma^2)$$

#### 51.14 Additional information and definitions

- $\mu$  : Location parameter
- $\sigma$  : Scale parameter
- $u$  : Uniform[0,1] random variable

#### 51.15 Spreadsheet documents

- [Excel file from GitHub repository](#)
- [Google spreadsheet document](#)

## 52 Maxwell Distribution

### 52.1 Distribution definition

$$X \sim \text{Maxwell}(\alpha, \text{Loc})$$

### 52.2 Distribution domain

$$x \in (0, \infty)$$

### 52.3 Parameters domain and parameters constraints

$$\alpha \in \mathbb{R}^+, \text{Loc} \in \mathbb{R}$$

### 52.4 Cumulative distribution function

$$F_X(x) = \text{erf}\left(\frac{x - \text{Loc}}{\sqrt{2}\alpha}\right) - \sqrt{\frac{2}{\pi}} \frac{(x - \text{Loc})e^{-(x - \text{Loc})^2/(2\alpha^2)}}{\alpha}$$

### 52.5 Probability density function

$$f_X(x) = \sqrt{\frac{2}{\pi}} \frac{(x - \text{Loc})^2 e^{-(x - \text{Loc})^2/(2\alpha^2)}}{\alpha^3}$$

### 52.6 Percent point function/Sample

$$F_X^{-1}(u) = \text{Loc} + \alpha \sqrt{2\text{P}^{-1}(1.5, u)}$$

### 52.7 Parametric centered moments

$$\mu'_k = E[X^k] = \int_{-\infty}^{\infty} x^k f_X(x) dx$$

### 52.8 Parametric mean

$$\text{Mean}(X) = \mu'_1 = \text{Loc} + 2\alpha \sqrt{\frac{2}{\pi}}$$

### 52.9 Parametric variance

$$\text{Variance}(X) = \mu'_2 - \mu_1'^2 = \frac{\alpha^2(3\pi - 8)}{\pi}$$

### 52.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\mu'_3 - 3\mu'_2\mu'_1 + 2\mu_1'^3}{(\mu'_2 - \mu_1'^2)^{1.5}} = \frac{2\sqrt{2}(16 - 5\pi)}{(3\pi - 8)^{3/2}}$$

### 52.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\mu'_4 - 4\mu'_1\mu'_3 + 6\mu_1'^2\mu'_2 - 3\mu_1'^4}{(\mu'_2 - \mu_1'^2)^2} = 4 \frac{(-96 + 40\pi - 3\pi^2)}{(3\pi - 8)^2} + 3$$

### 52.12 Parametric median

$$\text{Median}(X) = \text{Loc} + \alpha \sqrt{2\text{P}^{-1}\left(1.5, \frac{1}{2}\right)}$$

### 52.13 Parametric mode

$$\text{Mode}(X) = \text{Loc} + \alpha\sqrt{2}$$

### 52.14 Additional information and definitions

- Loc : Location parameter
- $\alpha$  : Scale parameter
- $u$  : Uniform[0,1] random variable
- $P^{-1}(a, u)$  : Inverse of regularized lower incomplete gamma function

### 52.15 Spreadsheet documents

- [Excel file from GitHub repository](#)
- [Google spreadsheet document](#)

## 53 Moyal Distribution

### 53.1 Distribution definition

$$X \sim \text{Moyal}(\mu, \sigma)$$

### 53.2 Distribution domain

$$x \in (-\infty, \infty)$$

### 53.3 Parameters domain and parameters constraints

$$\mu \in \mathbb{R}, \sigma \in \mathbb{R}^+$$

### 53.4 Cumulative distribution function

$$F_X(x) = 1 - P\left(\frac{1}{2}, \frac{e^{-z(x)}}{2}\right) = 1 - \text{erf}\left(\frac{\exp(-0.5z(x))}{\sqrt{2}}\right)$$

### 53.5 Probability density function

$$f_X(x) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2}\left(z(x) + e^{-z(x)}\right)\right)$$

### 53.6 Percent point function/Sample

$$F_X^{-1}(u) = \mu + \sigma \ln\left[\Phi^{-1}\left(\left(\frac{1-u}{2}\right)^2\right)\right] = \mu + \sigma \ln\left[2P^{-1}\left(\frac{1}{2}, 1-u\right)\right]$$

### 53.7 Parametric centered moments

$$\mu'_k = E[X^k] = \int_{-\infty}^{\infty} x^k f_X(x) dx$$

### 53.8 Parametric mean

$$\text{Mean}(X) = \mu'_1 = \mu + \sigma(\ln(2) + \gamma)$$

### 53.9 Parametric variance

$$\text{Variance}(X) = \mu'_2 - \mu'^2_1 = \sigma^2\left(\frac{\pi^2}{2}\right)$$

### 53.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\mu'_3 - 3\mu'_2\mu'_1 + 2\mu'^3_1}{(\mu'_2 - \mu'^2_1)^{1.5}} = \frac{28\sqrt{2}\zeta(3)}{\pi^3}$$

### 53.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\mu'_4 - 4\mu'_1\mu'_3 + 6\mu'^2_1\mu'_2 - 3\mu'^4_1}{(\mu'_2 - \mu'^2_1)^2} = 7$$

### 53.12 Parametric median

$$\text{Median}(X) = \mu + \sigma \ln\left[2P^{-1}\left(\frac{1}{2}, \frac{1}{2}\right)\right]$$

### 53.13 Parametric mode

$$\text{Mode}(X) = \mu$$



### 53.14 Additional information and definitions

- $\mu$  : Location parameter
- $\sigma$  : Scale parameter
- $z(x) = (x - \mu) / \sigma$
- $P(a, x) = \frac{\gamma(a, x)}{\Gamma(a)}$  : Regularized lower incomplete gamma function
- $P^{-1}(a, u)$  : Inverse of regularized lower incomplete gamma function
- $\gamma(a, x)$  : Lower incomplete gamma function
- $\Gamma(x)$  : Gamma function
- $\text{erf}(x)$  : Error function
- $\Phi^{-1}(x)$  : PPF normal standard distribution
- $\gamma$  : Euler-Mascheroni constant = 0.5772156649
- $\zeta(3)$  : Apéry's constant = 1.2020569031

### 53.15 Spreadsheet documents

- [Excel file from GitHub repository](#)
- [Google spreadsheet document](#)

## 54 Nakagami Distribution

### 54.1 Distribution definition

$$X \sim \text{Nakagami}(m, \Omega)$$

### 54.2 Distribution domain

$$x \in (0, \infty)$$

### 54.3 Parameters domain and parameters constraints

$$m \in \mathbb{R}_{\geq \frac{1}{2}}^+, \Omega \in \mathbb{R}^+$$

### 54.4 Cumulative distribution function

$$F_X(x) = \frac{\gamma\left(m, \frac{m}{\Omega}x^2\right)}{\Gamma(m)} = P\left(m, \frac{m}{\Omega}x^2\right)$$

### 54.5 Probability density function

$$f_X(x) = \frac{2m^m}{\Gamma(m)\Omega^m} x^{2m-1} \exp\left(-\frac{m}{\Omega}x^2\right)$$

### 54.6 Percent point function/Sample

$$F_X^{-1}(u) = \sqrt{\frac{\Omega}{m} P^{-1}(m, u)}$$

### 54.7 Parametric centered moments

$$\mu'_k = E[X^k] = \int_{-\infty}^{\infty} x^k f_X(x) dx$$

### 54.8 Parametric mean

$$\text{Mean}(X) = \mu'_1 = \frac{\Gamma(m + \frac{1}{2})}{\Gamma(m)} \left(\frac{\Omega}{m}\right)^{1/2}$$

### 54.9 Parametric variance

$$\text{Variance}(X) = \mu'_2 - \mu_1'^2 = \Omega \left(1 - \frac{1}{m} \left(\frac{\Gamma(m + \frac{1}{2})}{\Gamma(m)}\right)^2\right)$$

### 54.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\mu'_3 - 3\mu'_2\mu'_1 + 2\mu_1'^3}{(\mu'_2 - \mu_1'^2)^{1.5}} = \frac{\frac{\Gamma(m + \frac{1}{2})}{\Gamma(m)\sqrt{m}} \left(1 - 4m \left(1 - \frac{1}{m} \left(\frac{\Gamma(m + \frac{1}{2})}{\Gamma(m)}\right)^2\right)\right)}{2m \left(1 - \frac{1}{m} \left(\frac{\Gamma(m + \frac{1}{2})}{\Gamma(m)}\right)^2\right)^{3/2}}$$

### 54.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\mu'_4 - 4\mu'_1\mu'_3 + 6\mu_1'^2\mu'_2 - 3\mu_1'^4}{(\mu'_2 - \mu_1'^2)^2} = 3 + \frac{-6 \left(\frac{\Gamma(m + \frac{1}{2})}{\Gamma(m)\sqrt{m}}\right)^4 m + (8m - 2) \left(\frac{\Gamma(m + \frac{1}{2})}{\Gamma(m)\sqrt{m}}\right)^2 - 2m + 1}{m \left(1 - \frac{1}{m} \left(\frac{\Gamma(m + \frac{1}{2})}{\Gamma(m)}\right)^2\right)^2}$$

#### 54.12 Parametric median

$$\text{Median}(X) = \sqrt{\frac{\Omega}{m} P^{-1}\left(m, \frac{1}{2}\right)}$$

#### 54.13 Parametric mode

$$\text{Mode}(X) = \frac{\sqrt{2}}{2} \left( \frac{(2m-1)\Omega}{m} \right)^{1/2}$$

#### 54.14 Additional information and definitions

- $u$  : Uniform[0,1] random variable
- $P(a, x) = \frac{\gamma(a, x)}{\Gamma(a)}$  : Regularized lower incomplete gamma function
- $P^{-1}(a, u)$  : Inverse of regularized lower incomplete gamma function

#### 54.15 Spreadsheet documents

- [Excel file from GitHub repository](#)
- [Google spreadsheet document](#)

## 55 Non Central Chi Square Distribution

### 55.1 Distribution definition

$$X \sim \text{NonCentralChiSquare}(\lambda, n)$$

### 55.2 Distribution domain

$$x \in [0, +\infty)$$

### 55.3 Parameters domain and parameters constraints

$$\lambda \in \mathbb{R}^+, n \in \mathbb{R}^+$$

### 55.4 Cumulative distribution function

$$F_X(x) = 1 - Q_{\frac{n}{2}}(\sqrt{\lambda}, \sqrt{x})$$

### 55.5 Probability density function

$$f_X(x) = \frac{1}{2} e^{-(x+\lambda)/2} \left(\frac{x}{\lambda}\right)^{n/4-1/2} I_{n/2-1}(\sqrt{\lambda x})$$

### 55.6 Percent point function/Sample

$$\text{Sample}_X = \sum_{i=1}^n \left( \sqrt{\frac{\lambda}{n}} + \Phi^{-1}(u_i) \right)^2$$

### 55.7 Parametric centered moments

$$\mu'_k = E[X^k] = \int_0^\infty x^k f_X(x) dx = 2^{k-1} (k-1)! (n+k\lambda) + \sum_{j=1}^{k-1} \frac{(k-1)! 2^{j-1}}{(k-j)!} (n+j\lambda) \mu'_{k-j}$$

### 55.8 Parametric mean

$$\text{Mean}(X) = \mu'_1 = n + \lambda$$

### 55.9 Parametric variance

$$\text{Variance}(X) = \mu'_2 - \mu_1'^2 = 2(n + 2\lambda)$$

### 55.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\mu'_3 - 3\mu'_2\mu'_1 + 2\mu_1'^3}{(\mu'_2 - \mu_1'^2)^{1.5}} = \frac{2^{3/2}(n + 3\lambda)}{(n + 2\lambda)^{3/2}}$$

### 55.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\mu'_4 - 4\mu'_1\mu'_3 + 6\mu_1'^2\mu'_2 - 3\mu_1'^4}{(\mu'_2 - \mu_1'^2)^2} = \frac{12(n + 4\lambda)}{(n + 2\lambda)^2}$$

### 55.12 Parametric median

$$\text{Median}(X) = F_X^{-1}\left(\frac{1}{2}\right)$$

### 55.13 Parametric mode

$$\text{Mode}(X) = \arg \max_x f_X(x)$$

#### 55.14 Additional information and definitions

- Computing an analytic expression for the inverse of the cumulative distribution function is not feasible. Nonetheless, it is possible to generate a random sample from the distribution.
- $u_i$  : Uniform[0,1] random variable
- $\Phi^{-1}(x)$  : PPF normal standard distribution
- $I_\alpha(x)$  : Modified Bessel function of the first kind of order  $\alpha \in \mathbb{N}$

#### 55.15 Spreadsheet documents

- [Excel file from GitHub repository](#)
- [Google spreadsheet document](#)

## 56 Non Central F Distribution

### 56.1 Distribution definition

$$X \sim \text{NonCentralF}(\lambda, n_1, n_2)$$

### 56.2 Distribution domain

$$x \in [0, \infty)$$

### 56.3 Parameters domain and parameters constraints

$$\lambda \in \mathbb{R}^+, n_1 \in \mathbb{R}^+, n_2 \in \mathbb{R}^+$$

### 56.4 Cumulative distribution function

$$F_X(x) = \sum_{j=0}^{\infty} \left( \frac{\left(\frac{1}{2}\lambda\right)^j}{j!} e^{-\lambda/2} \right) I_{n_1 x / (n_2 + n_1 x)} \left( \frac{n_1}{2} + j, \frac{n_2}{2} \right)$$

### 56.5 Probability density function

$$f_X(x) = \sum_{k=0}^{\infty} \frac{e^{-\lambda/2} (\lambda/2)^k}{\text{Beta}\left(\frac{n_2}{2}, \frac{n_1}{2} + k\right) k!} \left( \frac{n_1}{n_2} \right)^{\frac{n_1}{2} + k} \left( \frac{n_2}{n_2 + n_1 x} \right)^{\frac{n_1 + n_2}{2} + k} x^{n_1/2 - 1 + k}$$

### 56.6 Percent point function/Sample

$$\text{Sample}_X = \frac{\left( \sum_{i=1}^{n_1} \left( \sqrt{\frac{\lambda}{n_1}} + \Phi^{-1}(u_i) \right)^2 \right) / n_1}{(2P^{-1}\left(\frac{n_2}{2}, u\right)) / n_2}$$

### 56.7 Parametric centered moments

$$\mu'_k = E[X^k] = \int_0^{\infty} x^k f_X(x) dx = e^{-\lambda/2} \left( \frac{n_1}{n_2} \right)^k \frac{\Gamma(n_1/2 - k)}{\Gamma(n_1/2)} \sum_{r=0}^{\infty} \left( \frac{1}{r!} \right) \left( \frac{\lambda}{2} \right)^r \frac{\Gamma\left(\frac{n_1}{2} + r + k\right)}{\Gamma\left(\frac{n_1}{2} + r\right)}$$

### 56.8 Parametric mean

$$\text{Mean}(X) = \mu'_1$$

### 56.9 Parametric variance

$$\text{Variance}(X) = \mu'_2 - \mu'^2_1$$

### 56.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\mu'_3 - 3\mu'_2\mu'_1 + 2\mu'^3_1}{(\mu'_2 - \mu'^2_1)^{1.5}}$$

### 56.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\mu'_4 - 4\mu'_1\mu'_3 + 6\mu'^2_1\mu'_2 - 3\mu'^4_1}{(\mu'_2 - \mu'^2_1)^2}$$

### 56.12 Parametric median

$$\text{Median}(X) = F_X^{-1}\left(\frac{1}{2}\right)$$

### 56.13 Parametric mode

$$\text{Mode}(X) = \arg \max_x f_X(x)$$

### 56.14 Additional information and definitions

- Computing an analytic expression for the inverse of the cumulative distribution function is not feasible. Nonetheless, it is possible to generate a random sample from the distribution.
- $u$  : Uniform[0,1] random variable
- $u_i$  : Uniform[0,1] random variable
- $\Phi^{-1}(x)$  : PPF normal standard distribution
- $P^{-1}(a, u)$  : Inverse of regularized lower incomplete gamma function
- $I(x, a, b)$  : Regularized incomplete beta function
- Beta  $(x, y)$  : Beta function

### 56.15 Spreadsheet documents

- [Excel file from GitHub repository](#)
- [Google spreadsheet document](#)

## 57 Non Central T Student Distribution

### 57.1 Distribution definition

$$X \sim \text{NonCentralTStudent}(\lambda, n, \text{Loc}, \text{Sc})$$

### 57.2 Distribution domain

$$x \in (-\infty, \infty)$$

### 57.3 Parameters domain and parameters constraints

$$\lambda \in \mathbb{R}, n \in \mathbb{R}^+, \text{Sc} \in \mathbb{R}^+, \text{Loc} \in \mathbb{R}$$

### 57.4 Cumulative distribution function

$$F_X(x) = \begin{cases} \frac{1}{2} \sum_{j=0}^{\infty} \frac{1}{j!} (-\lambda\sqrt{2})^j e^{\frac{-\lambda^2}{2}} \frac{\Gamma(\frac{j+1}{2})}{\sqrt{\pi}} I_{n/(n+z(x)^2)}\left(\frac{n}{2}, \frac{j+1}{2}\right) & \text{if } z(x) \geq 0 \\ 1 - \frac{1}{2} \sum_{j=0}^{\infty} \frac{1}{j!} (-\lambda\sqrt{2})^j e^{\frac{-\lambda^2}{2}} \frac{\Gamma(\frac{j+1}{2})}{\sqrt{\pi}} I_{n/(n+z(x)^2)}\left(\frac{n}{2}, \frac{j+1}{2}\right) & \text{if } z(x) < 0 \end{cases}$$

### 57.5 Probability density function

$$f_X(x) = \frac{1}{\text{Sc}} \frac{n^{n/2} \Gamma(n+1)}{2^n e^{\lambda^2/2} (n+z(x)^2)^{n/2} \Gamma(n/2)} \times \left\{ \frac{\sqrt{2} \lambda z(x) {}_1F_1\left(\frac{n}{2}+1, \frac{3}{2}, \frac{\lambda^2 z(x)^2}{2(n+z(x)^2)}\right)}{(n+z(x)^2) \Gamma\left(\frac{n+1}{2}\right)} - \frac{{}_1F_1\left(\frac{n+1}{2}, \frac{1}{2}, \frac{\lambda^2 z(x)^2}{2(n+z(x)^2)}\right)}{\sqrt{n+z(x)^2} \Gamma\left(\frac{n}{2}+1\right)} \right\}$$

### 57.6 Percent point function/Sample

$$\text{Sample}_X = \text{Loc} + \text{Sc} \frac{(\lambda + \Phi^{-1}(u))}{\left(\sqrt{2\text{P}^{-1}\left(\frac{n}{2}, u\right)}\right)/n}$$

### 57.7 Parametric centered moments

$$\tilde{\mu}'_k = E[\tilde{X}^k] = \int_0^\infty x^k f_{\tilde{X}}(x) dx = \frac{e^{-\lambda^2/2}}{\sqrt{n\pi}\Gamma(n/2)} \Gamma\left(\frac{n-k}{2}\right) n^{k/2} \sum_{r=0}^{\infty} \frac{\lambda^r 2^{r/2}}{r!} \Gamma\left(\frac{r+k+1}{2}\right)$$

### 57.8 Parametric mean

$$\text{Mean}(X) = \text{Loc} + \text{Sc} \tilde{\mu}'_1$$

### 57.9 Parametric variance

$$\text{Variance}(X) = \text{Sc}^2 (\tilde{\mu}'_2 - \tilde{\mu}_1'^2)$$

### 57.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\tilde{\mu}'_3 - 3\tilde{\mu}'_2\tilde{\mu}'_1 + 2\tilde{\mu}_1'^3}{(\tilde{\mu}'_2 - \tilde{\mu}_1'^2)^{1.5}}$$

### 57.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\tilde{\mu}'_4 - 4\tilde{\mu}'_1\tilde{\mu}'_3 + 6\tilde{\mu}_1'^2\tilde{\mu}'_2 - 3\tilde{\mu}_1'^4}{(\tilde{\mu}'_2 - \tilde{\mu}_1'^2)^2}$$

### 57.12 Parametric median

$$\text{Median}(X) = F_X^{-1}\left(\frac{1}{2}\right)$$



### 57.13 Parametric mode

$$\text{Mode}(X) = \arg \max_x f_X(x)$$

### 57.14 Additional information and definitions

- $\tilde{X} \sim \text{NonCentralTStudent}(\lambda, n, 0, 1)$
- Computing an analytic expression for the inverse of the cumulative distribution function is not feasible. Nonetheless, it is possible to generate a random sample from the distribution.
- Loc : Location parameter
- Sc : Scale parameter
- $z(x) = (x - \text{Loc}) / \text{Sc}$
- $u$  : Uniform[0,1] random variable
- $P^{-1}(a, u)$  : Inverse of regularized lower incomplete gamma function
- $I_\alpha(x)$  : Modified Bessel function of the first kind of order  $\alpha \in \mathbb{N}$
- ${}_1F_1(a, b, z)$  : Kummer's confluent hypergeometric function

### 57.15 Spreadsheet documents

- [Excel file from GitHub repository](#)
- [Google spreadsheet document](#)

## 58 Normal Distribution

### 58.1 Distribution definition

$$X \sim \text{Normal}(\mu, \sigma)$$

### 58.2 Distribution domain

$$x \in (-\infty, \infty)$$

### 58.3 Parameters domain and parameters constraints

$$\mu \in \mathbb{R}, \sigma \in \mathbb{R}^+$$

### 58.4 Cumulative distribution function

$$F_X(x) = \frac{1}{2} \left[ 1 + \operatorname{erf} \left( \frac{x - \mu}{\sigma \sqrt{2}} \right) \right] = \Phi \left( \frac{x - \mu}{\sigma} \right)$$

### 58.5 Probability density function

$$f_X(x) = \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{1}{2} \left( \frac{x - \mu}{\sigma} \right)^2} = \phi \left( \frac{x - \mu}{\sigma} \right)$$

### 58.6 Percent point function/Sample

$$F_X^{-1}(u) = \mu + \sigma \sqrt{2} \operatorname{erf}^{-1}(2u - 1) = \mu + \sigma \Phi^{-1}(u)$$

### 58.7 Parametric centered moments

$$\mu'_k = E[X^k] = \int_{-\infty}^{\infty} x^k f_X(x) dx = \sigma^k \cdot (-i\sqrt{2})^k U \left( -\frac{k}{2}, \frac{1}{2}, -\frac{1}{2} \left( \frac{\mu}{\sigma} \right)^2 \right)$$

### 58.8 Parametric mean

$$\text{Mean}(X) = \mu'_1 = \mu$$

### 58.9 Parametric variance

$$\text{Variance}(X) = \mu'_2 - \mu_1'^2 = \sigma^2$$

### 58.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\mu'_3 - 3\mu'_2\mu'_1 + 2\mu_1'^3}{(\mu'_2 - \mu_1'^2)^{1.5}} = 0$$

### 58.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\mu'_4 - 4\mu'_1\mu'_3 + 6\mu_1'^2\mu'_2 - 3\mu_1'^4}{(\mu'_2 - \mu_1'^2)^2} = 3$$

### 58.12 Parametric median

$$\text{Median}(X) = \mu$$

### 58.13 Parametric mode

$$\text{Mode}(X) = \mu$$

#### 58.14 Additional information and definitions

- $\mu$  : Location parameter
- $\sigma$  : Scale parameter
- $u$  : Uniform[0,1] random variable
- $U(a, b, z)$  : Tricomi's confluent hypergeometric function
- $\Phi(x)$  : CDF normal standard distribution
- $\Phi^{-1}(x)$  : PPF normal standard distribution
- $\phi(x)$  : PDF normal standard distribution
- $\text{erf}(x)$  : Error function
- $\text{erf}^{-1}(x)$  : Inverse of error function

#### 58.15 Spreadsheet documents

- [Excel file from GitHub repository](#)
- [Google spreadsheet document](#)

## 59 Pareto First Kind Distribution

### 59.1 Distribution definition

$$X \sim \text{ParetoFirstKind}(x_m, \alpha, \text{Loc})$$

### 59.2 Distribution domain

$$x \in [\text{Loc} + x_m, \infty)$$

### 59.3 Parameters domain and parameters constraints

$$x_m \in \mathbb{R}^+, \alpha \in \mathbb{R}^+, \text{Loc} \in \mathbb{R}$$

### 59.4 Cumulative distribution function

$$F_X(x) = 1 - \left( \frac{x_m}{x - \text{Loc}} \right)^\alpha$$

### 59.5 Probability density function

$$f_X(x) = \frac{\alpha x_m^\alpha}{(x - \text{Loc})^{\alpha+1}}$$

### 59.6 Percent point function/Sample

$$F_X^{-1}(u) = \text{Loc} + x_m(1 - u)^{-\frac{1}{\alpha}}$$

### 59.7 Parametric centered moments

$$\tilde{\mu}'_k = E[\tilde{X}^k] = \int_{x_m}^{\infty} x^k f_{\tilde{X}}(x) dx = \begin{cases} \infty & \text{if } \alpha \leq k \\ \frac{\alpha x_m^k}{\alpha - k} & \text{if } \alpha > k \end{cases}$$

### 59.8 Parametric mean

$$\text{Mean}(X) = \text{Loc} + \tilde{\mu}'_1 = \text{Loc} + \frac{\alpha x_m}{\alpha - 1} \quad \text{if } \alpha > 1$$

### 59.9 Parametric variance

$$\text{Variance}(X) = (\tilde{\mu}'_2 - \tilde{\mu}'_1^2) = \frac{x_m^2 \alpha}{(\alpha - 1)^2(\alpha - 2)} \quad \text{if } \alpha > 2$$

### 59.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\tilde{\mu}'_3 - 3\tilde{\mu}'_2\tilde{\mu}'_1 + 2\tilde{\mu}'_1^3}{(\tilde{\mu}'_2 - \tilde{\mu}'_1^2)^{1.5}} = \frac{2(1 + \alpha)}{\alpha - 3} \sqrt{\frac{\alpha - 2}{\alpha}} \quad \text{if } \alpha > 3$$

### 59.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\tilde{\mu}'_4 - 4\tilde{\mu}'_1\tilde{\mu}'_3 + 6\tilde{\mu}'_1^2\tilde{\mu}'_2 - 3\tilde{\mu}'_1^4}{(\tilde{\mu}'_2 - \tilde{\mu}'_1^2)^2} = \frac{6(\alpha^3 + \alpha^2 - 6\alpha - 2)}{\alpha(\alpha - 3)(\alpha - 4)} \quad \text{if } \alpha > 4$$

### 59.12 Parametric median

$$\text{Median}(X) = \text{Loc} + x_m \sqrt[\alpha]{2}$$

### 59.13 Parametric mode

$$\text{Mode}(X) = \text{Loc} + x_m$$

#### 59.14 Additional information and definitions

- $\tilde{X} \sim \text{ParetoFirstKind}(x_m, \alpha, 0)$
- $\text{Loc}$  : Location parameter
- $x_m$  : Scale parameter
- $u$  : Uniform[0,1] random variable

#### 59.15 Spreadsheet documents

- [Excel file from GitHub repository](#)
- [Google spreadsheet document](#)

## 60 Pareto Second Kind Distribution

### 60.1 Distribution definition

$$X \sim \text{ParetoSecondKind}(x_m, \alpha, \text{Loc})$$

### 60.2 Distribution domain

$$x \in (\text{Loc}, \infty)$$

### 60.3 Parameters domain and parameters constraints

$$x_m \in \mathbb{R}^+, \alpha \in \mathbb{R}^+, \text{Loc} \in \mathbb{R}$$

### 60.4 Cumulative distribution function

$$F_X(x) = 1 - \left[1 + \frac{x - \text{Loc}}{x_m}\right]^{-\alpha}$$

### 60.5 Probability density function

$$f_X(x) = \frac{\alpha}{x_m} \left[1 + \frac{x - \text{Loc}}{x_m}\right]^{-(\alpha+1)}$$

### 60.6 Percent point function/Sample

$$F_X^{-1}(u) = \text{Loc} + x_m \left[(1 - p)^{-\frac{1}{\alpha}} - 1\right]$$

### 60.7 Parametric centered moments

$$\tilde{\mu}'_k = E[\tilde{X}^k] = \int_0^\infty x^k f_{\tilde{X}}(x) dx = \frac{x_m^k \Gamma(\alpha - k) \Gamma(1 + k)}{\Gamma(\alpha)}$$

### 60.8 Parametric mean

$$\text{Mean}(X) = \tilde{\mu}'_1 = \frac{x_m}{\alpha - 1} \quad \text{if } \alpha > 1$$

### 60.9 Parametric variance

$$\text{Variance}(X) = \tilde{\mu}'_2 - \tilde{\mu}'_1^2 = \frac{x_m^2 \alpha}{(\alpha - 1)^2 (\alpha - 2)} \quad \text{if } \alpha > 2$$

### 60.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\tilde{\mu}'_3 - 3\tilde{\mu}'_2\tilde{\mu}'_1 + 2\tilde{\mu}'_1^3}{(\tilde{\mu}'_2 - \tilde{\mu}'_1^2)^{1.5}} = \frac{2(1 + \alpha)}{\alpha - 3} \sqrt{\frac{\alpha - 2}{\alpha}} \quad \text{if } \alpha > 3$$

### 60.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\tilde{\mu}'_4 - 4\tilde{\mu}'_1\tilde{\mu}'_3 + 6\tilde{\mu}'_1^2\tilde{\mu}'_2 - 3\tilde{\mu}'_1^4}{(\tilde{\mu}'_2 - \tilde{\mu}'_1^2)^2} = \frac{6(\alpha^3 + \alpha^2 - 6\alpha - 2)}{\alpha(\alpha - 3)(\alpha - 4)} \quad \text{if } \alpha > 4$$

### 60.12 Parametric median

$$\text{Median}(X) = x_m \left(\sqrt[3]{2} - 1\right)$$

### 60.13 Parametric mode

$$\text{Mode}(X) = 0$$

#### 60.14 Additional information and definitions

- $X \sim \text{ParetoSecondKind}(x_m, \alpha, 0)$
- $x_m$  : Scale parameter
- $u$  : Uniform[0,1] random variable
- $\Gamma(x)$  : Gamma function

#### 60.15 Spreadsheet documents

- [Excel file from GitHub repository](#)
- [Google spreadsheet document](#)

## 61 Pert Distribution

### 61.1 Distribution definition

$$X \sim \text{Pert}(a, b, c)$$

### 61.2 Distribution domain

$$x \in [a, c]$$

### 61.3 Parameters domain and parameters constraints

$$a \in \mathbb{R}, b \in \mathbb{R}, c \in \mathbb{R}, a < b < c$$

### 61.4 Cumulative distribution function

$$F_X(x) = I(z(x), \alpha_1, \alpha_2)$$

### 61.5 Probability density function

$$f_X(x) = \frac{(x-a)^{\alpha_1-1}(c-x)^{\alpha_2-1}}{\text{Beta}(\alpha_1, \alpha_2)(c-a)^{\alpha_1+\alpha_2-1}}$$

### 61.6 Percent point function/Sample

$$F_X^{-1}(u) = a + (c-a) \cdot I^{-1}(u, \alpha_1, \alpha_2)$$

### 61.7 Parametric centered moments

$$\mu'_k = E[X^k] = \int_a^c x^k f_X(x) dx$$

### 61.8 Parametric mean

$$\text{Mean}(X) = \mu'_1 = \frac{a + 4b + c}{6}$$

### 61.9 Parametric variance

$$\text{Variance}(X) = \mu'_2 - \mu_1'^2 = \frac{(\text{Mean}(X) - a)(c - \text{Mean}(X))}{7}$$

### 61.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\mu'_3 - 3\mu'_2\mu'_1 + 2\mu_1'^3}{(\mu'_2 - \mu_1'^2)^{1.5}} = \frac{2(\alpha_2 - \alpha_1)\sqrt{\alpha_1 + \alpha_2 + 1}}{(\alpha_1 + \alpha_2 + 2)\sqrt{\alpha_1\alpha_2}}$$

### 61.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\mu'_4 - 4\mu'_1\mu'_3 + 6\mu_1'^2\mu'_2 - 3\mu_1'^4}{(\mu'_2 - \mu_1'^2)^2} = \frac{6[(\alpha_1 - \alpha_2)^2(\alpha_1 + \alpha_2 + 1) - \alpha_1\alpha_2(\alpha_1 + \alpha_2 + 2)]}{\alpha_1\alpha_2(\alpha_1 + \alpha_2 + 2)(\alpha_1 + \alpha_2 + 3)} + 3$$

### 61.12 Parametric median

$$\text{Median}(X) = a + (c-a) \cdot I^{-1}\left(\frac{1}{2}, \alpha_1, \alpha_2\right)$$

### 61.13 Parametric mode

$$\text{Mode}(X) = b$$



#### 61.14 Additional information and definitions

- $z(x) = (x - a) / (c - a)$
- $u$  : Uniform[0,1] random variable
- $\alpha_1 = \frac{4b+c-5a}{c-a}, \alpha_2 = \frac{5c-a-4b}{c-a}$
- $I(x, a, b)$  : Regularized incomplete beta function
- $I^{-1}(x, a, b)$  : Inverse of regularized incomplete beta function
- Beta( $x, y$ ) : Beta function

#### 61.15 Spreadsheet documents

- [Excel file from GitHub repository](#)
- [Google spreadsheet document](#)

## 62 Power Function Distribution

### 62.1 Distribution definition

$$X \sim \text{PowerFunction}(\alpha, a, b)$$

### 62.2 Distribution domain

$$x \in [a, b]$$

### 62.3 Parameters domain and parameters constraints

$$\alpha \in \mathbb{R}^+, a \in \mathbb{R}, b \in \mathbb{R}, a < b$$

### 62.4 Cumulative distribution function

$$F_X(x) = \left( \frac{x-a}{b-a} \right)^\alpha$$

### 62.5 Probability density function

$$f_X(x) = \frac{\alpha(x-a)^{\alpha-1}}{(b-a)^\alpha}$$

### 62.6 Percent point function/Sample

$$F_X^{-1}(u) = [a + u(b-a)]^{-\alpha}$$

### 62.7 Parametric centered moments

$$\mu'_k = E[X^k] = \int_a^b x^k f_X(x) dx$$

### 62.8 Parametric mean

$$\text{Mean}(X) = \mu'_1 = \frac{a+b\alpha}{\alpha+1}$$

### 62.9 Parametric variance

$$\text{Variance}(X) = \mu'_2 - \mu'^2_1 = \frac{2a^2 + 2ab\alpha + b^2\alpha(\alpha+1)}{(\alpha+1)(\alpha+2)} - \text{Mean}(X)^2$$

### 62.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\mu'_3 - 3\mu'_2\mu'_1 + 2\mu'^3_1}{(\mu'_2 - \mu'^2_1)^{1.5}} = 2(1-\alpha) \sqrt{\frac{\alpha+2}{\alpha(\alpha+3)}}$$

### 62.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\mu'_4 - 4\mu'_1\mu'_3 + 6\mu'^2_1\mu'_2 - 3\mu'^4_1}{(\mu'_2 - \mu'^2_1)^2} = \frac{6(\alpha^3 - \alpha^2 - 6\alpha + 2)}{\alpha(\alpha+3)(\alpha+4)} + 3$$

### 62.12 Parametric median

$$\text{Median}(X) = \left[ a + \frac{1}{2}(b-a) \right]^{-\alpha}$$

### 62.13 Parametric mode

$$\text{Mode}(X) = \text{undefined}$$

#### 62.14 Additional information and definitions

- $a$  : Location parameter
- $b - a$  : Scale parameter
- $u$  : Uniform[0,1] random variable

#### 62.15 Spreadsheet documents

- [Excel file from GitHub repository](#)
- [Google spreadsheet document](#)

## 63 Rayleigh Distribution

### 63.1 Distribution definition

$$X \sim \text{Rayleigh}(\gamma, \sigma)$$

### 63.2 Distribution domain

$$x \in [\gamma, \infty)$$

### 63.3 Parameters domain and parameters constraints

$$\gamma \in \mathbb{R}, \sigma \in \mathbb{R}^+$$

### 63.4 Cumulative distribution function

$$F_X(x) = 1 - e^{-z(x)^2/2}$$

### 63.5 Probability density function

$$f_X(x) = z(x) \times e^{-z(x)^2/2}/\sigma$$

### 63.6 Percent point function/Sample

$$F_X^{-1}(u) = \gamma + \sigma \sqrt{-2 \log(1-u)}$$

### 63.7 Parametric centered moments

$$\mu'_k = E[\tilde{X}^k] = \int_0^\infty x^k f_{\tilde{X}}(x) dx = \sqrt{2k} \Gamma\left(\frac{k}{2} + 1\right)$$

### 63.8 Parametric mean

$$\text{Mean}(X) = \gamma + \sigma \cdot \mu'_1 = \gamma + \sigma \sqrt{\frac{\pi}{2}}$$

### 63.9 Parametric variance

$$\text{Variance}(X) = \sigma^2(\mu'_2 - \mu_1'^2) = \sigma^2 \frac{4 - \pi}{2}$$

### 63.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\mu'_3 - 3\mu'_2\mu'_1 + 2\mu_1'^3}{(\mu'_2 - \mu_1'^2)^{1.5}} = \frac{2(\pi - 3)\sqrt{\pi}}{(4 - \pi)^{3/2}}$$

### 63.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\mu'_4 - 4\mu'_1\mu'_3 + 6\mu_1'^2\mu'_2 - 3\mu_1'^4}{(\mu'_2 - \mu_1'^2)^2} = 3 + \frac{24\pi - 6\pi^2 - 16}{(4 - \pi)^2}$$

### 63.12 Parametric median

$$\text{Median}(X) = \gamma + \sigma \sqrt{-2 \log\left(\frac{1}{2}\right)}$$

### 63.13 Parametric mode

$$\text{Mode}(X) = \gamma + \sigma$$

#### 63.14 Additional information and definitions

- $\tilde{X} \sim \text{Rayleigh}(0, 1)$
- $\gamma$  : Location parameter
- $\sigma$  : Scale parameter
- $z(x) = (x - \gamma) / \sigma$
- $u$  : Uniform[0,1] random variable
- $\Gamma(x)$  : Gamma function

#### 63.15 Spreadsheet documents

- [Excel file from GitHub repository](#)
- [Google spreadsheet document](#)

## 64 Reciprocal Distribution

### 64.1 Distribution definition

$$X \sim \text{Reciprocal}(a, b)$$

### 64.2 Distribution domain

$$x \in [a, b]$$

### 64.3 Parameters domain and parameters constraints

$$a \in \mathbb{R}^+, b \in \mathbb{R}^+, a < b$$

### 64.4 Cumulative distribution function

$$F_X(x) = \frac{\ln(x) - \ln(a)}{\ln(b) - \ln(a)}$$

### 64.5 Probability density function

$$f_X(x) = \frac{1}{x(\ln(b) - \ln(a))}$$

### 64.6 Percent point function/Sample

$$F_X^{-1}(u) = \exp(\ln(a) + u \times (\ln(b) - \ln(a)))$$

### 64.7 Parametric centered moments

$$\mu'_k = E[X^k] = \int_a^b x^k f_X(x) dx = \frac{b^k - a^k}{k(\ln(b) - \ln(a))}$$

### 64.8 Parametric mean

$$\text{Mean}(X) = \mu'_1$$

### 64.9 Parametric variance

$$\text{Variance}(X) = \mu'_2 - \mu'^2_1$$

### 64.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\mu'_3 - 3\mu'_2\mu'_1 + 2\mu'^3_1}{(\mu'_2 - \mu'^2_1)^{1.5}}$$

### 64.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\mu'_4 - 4\mu'_1\mu'_3 + 6\mu'^2_1\mu'_2 - 3\mu'^4_1}{(\mu'_2 - \mu'^2_1)^2}$$

### 64.12 Parametric median

$$\text{Median}(X) = \exp \left[ \ln(a) + \frac{(\ln(b) - \ln(a))}{2} \right]$$

### 64.13 Parametric mode

$$\text{Mode}(X) = a$$

#### 64.14 Additional information and definitions

- $u$  : Uniform[0,1] random variable

#### 64.15 Spreadsheet documents

- [Excel file from GitHub repository](#)
- [Google spreadsheet document](#)

## 65 Rice Distribution

### 65.1 Distribution definition

$$X \sim \text{Rice}(v, \sigma)$$

### 65.2 Distribution domain

$$x \in [0, \infty)$$

### 65.3 Parameters domain and parameters constraints

$$v \in \mathbb{R}^+, \sigma \in \mathbb{R}^+$$

### 65.4 Cumulative distribution function

$$F_X(x) = 1 - Q_1\left(\frac{v}{\sigma}, \frac{x}{\sigma}\right)$$

### 65.5 Probability density function

$$f_X(x) = \frac{x}{\sigma^2} \exp\left(-\frac{(x^2 + v^2)}{2\sigma^2}\right) I_0\left(\frac{xv}{\sigma^2}\right)$$

### 65.6 Percent point function/Sample

$$\text{Sample}_X = \sqrt{\Phi^{-1}(u_1, v, \sigma)^2 + \Phi^{-1}(u_2, 0, \sigma)^2}$$

### 65.7 Parametric centered moments

$$\mu'_k = E[X^k] = \int_{-\infty}^{\infty} x^k f_X(x) dx = \sigma^k 2^{k/2} \Gamma(1 + k/2) L_{k/2}(-v^2/2\sigma^2)$$

### 65.8 Parametric mean

$$\text{Mean}(X) = \mu'_1$$

### 65.9 Parametric variance

$$\text{Variance}(X) = \mu'_2 - \mu_1'^2$$

### 65.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\mu'_3 - 3\mu'_2\mu'_1 + 2\mu_1'^3}{(\mu'_2 - \mu_1'^2)^{1.5}}$$

### 65.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\mu'_4 - 4\mu'_1\mu'_3 + 6\mu_1'^2\mu'_2 - 3\mu_1'^4}{(\mu'_2 - \mu_1'^2)^2}$$

### 65.12 Parametric median

$$\text{Median}(X) = F_X^{-1}\left(\frac{1}{2}\right)$$

### 65.13 Parametric mode

$$\text{Mode}(X) = \arg \max_x f_X(x)$$



#### 65.14 Additional information and definitions

- Computing an analytic expression for the inverse of the cumulative distribution function is not feasible. Nonetheless, it is possible to generate a random sample from the distribution.
- $\Phi^{-1}(u, mean, variance)$  : Inverse of cumulative function from normal distribution
- $L_r(x)$  : Laguerre polynomials of order  $r \in \mathbb{R}$
- $L_{\frac{1}{2}}(x) = e^{x/2}(x)I_1\left(\frac{x}{2}\right) - e^{x/2}(x-1)I_0\left(\frac{x}{2}\right)$
- $L_{\frac{3}{2}}(x) = \frac{1}{3}e^{x/2}(2x^2 - 6x + 3)I_0(x/2) - \frac{2}{3}e^{x/2}(x-2)xI_1(x/2)$
- $I_\alpha(x)$  : Modified Bessel function of the first kind of order  $\alpha \in \mathbb{N}$
- $Q_k(a, b)$  : Marcum Q-function of order  $k \in \mathbb{N}$
- $u_1$  : Uniform[0,1] random variable
- $u_2$  : Uniform[0,1] random variable

#### 65.15 Spreadsheet documents

- [Excel file from GitHub repository](#)
- [Google spreadsheet document](#)

## 66 Semicircular Distribution

### 66.1 Distribution definition

$$X \sim \text{Semicircular}(\text{Loc}, R)$$

### 66.2 Distribution domain

$$x \in [\text{Loc}, \infty)$$

### 66.3 Parameters domain and parameters constraints

$$\text{Loc} \in \mathbb{R}, R \in \mathbb{R}^+$$

### 66.4 Cumulative distribution function

$$F_X(x) = \frac{1}{2} + \frac{z(x)\sqrt{R^2 - z(x)^2}}{\pi R^2} + \frac{\arcsin\left(\frac{z(x)}{R}\right)}{\pi}$$

### 66.5 Probability density function

$$f_X(x) = \frac{2}{\pi R^2} \sqrt{R^2 - z(x)^2}$$

### 66.6 Percent point function/Sample

$$F_X^{-1}(u) = \text{Loc} + R \times (2I^{-1}(u, 1.5, 1.5) - 1)$$

### 66.7 Parametric centered moments

$$\mu'_k = E[X^k] = \int_{\text{Loc}}^{\infty} x^k f_X(x) dx$$

### 66.8 Parametric mean

$$\text{Mean}(X) = \mu'_1 = \text{Loc}$$

### 66.9 Parametric variance

$$\text{Variance}(X) = \mu'_2 - \mu_1'^2 = \frac{R^2}{4}$$

### 66.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\mu'_3 - 3\mu'_2\mu'_1 + 2\mu_1'^3}{(\mu'_2 - \mu_1'^2)^{1.5}} = 0$$

### 66.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\mu'_4 - 4\mu'_1\mu'_3 + 6\mu_1'^2\mu'_2 - 3\mu_1'^4}{(\mu'_2 - \mu_1'^2)^2} = 2$$

### 66.12 Parametric median

$$\text{Median}(X) = \text{Loc}$$

### 66.13 Parametric mode

$$\text{Mode}(X) = \text{Loc}$$

#### 66.14 Additional information and definitions

- $\text{Loc}$  : Location parameter
- $R$  : Scale parameter
- $z(x) = x - \text{Loc}$
- $u$  : Uniform[0,1] random variable
- $I^{-1}(x, a, b)$  : Inverse of regularized incomplete beta function

#### 66.15 Spreadsheet documents

- [Excel file from GitHub repository](#)
- [Google spreadsheet document](#)

## 67 T Student Distribution

### 67.1 Distribution definition

$$X \sim \text{TStudent}(\text{df})$$

### 67.2 Distribution domain

$$x \in (-\infty, \infty)$$

### 67.3 Parameters domain and parameters constraints

$$\text{df} \in \mathbb{R}^+$$

### 67.4 Cumulative distribution function

$$F_X(x) = I\left(\frac{x + \sqrt{x^2 + \text{df}}}{2\sqrt{x^2 + \text{df}}}, \frac{\text{df}}{2}, \frac{\text{df}}{2}\right)$$

### 67.5 Probability density function

$$f_X(x) = \frac{(1 + x^2/\text{df})^{-(1+\text{df})/2}}{\sqrt{\text{df}} \times \text{Beta}\left(\frac{1}{2}, \frac{\text{df}}{2}\right)}$$

### 67.6 Percent point function/Sample

$$F_X^{-1}(u) = \begin{cases} \sqrt{\frac{\text{df}(1-I^{-1}(u, \text{df}/2, \text{df}/2))}{I^{-1}(u, \text{df}/2, \text{df}/2)}} & \text{if } u \geq \frac{1}{2} \\ -\sqrt{\frac{\text{df}(1-I^{-1}(u, \text{df}/2, \text{df}/2))}{I^{-1}(u, \text{df}/2, \text{df}/2)}} & \text{if } u < \frac{1}{2} \end{cases}$$

### 67.7 Parametric centered moments

$$\mu'_k = E[X^k] = \int_{-\infty}^{\infty} x^k f_X(x) dx = \begin{cases} 0 & \text{if } k \text{ odd} \wedge 0 < k < \text{df} \\ \text{df}^{k/2} \prod_{i=1}^{k/2} \frac{2i-1}{\text{df}-2i} & \text{if } k \text{ even} \wedge 0 < k < \text{df} \end{cases}$$

### 67.8 Parametric mean

$$\text{Mean}(X) = \mu'_1 = 0$$

### 67.9 Parametric variance

$$\text{Variance}(X) = \mu'_2 - \mu_1'^2 = \begin{cases} \text{df}/(\text{df} + 2) & \text{if } \text{df} > 2 \\ \text{undefined} & \text{if } \text{df} \leq 2 \end{cases}$$

### 67.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\mu'_3 - 3\mu'_2\mu'_1 + 2\mu_1'^3}{(\mu'_2 - \mu_1'^2)^{1.5}} = \begin{cases} 0 & \text{if } \text{df} > 3 \\ \text{undefined} & \text{if } \text{df} \leq 3 \end{cases}$$

### 67.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\mu'_4 - 4\mu'_1\mu'_3 + 6\mu_1'^2\mu'_2 - 3\mu_1'^4}{(\mu'_2 - \mu_1'^2)^2} = \begin{cases} 3 + 6/(\text{df} - 4) & \text{if } \text{df} > 4 \\ \text{undefined} & \text{if } \text{df} \leq 4 \end{cases}$$

### 67.12 Parametric median

$$\text{Median}(X) = 0$$

### 67.13 Parametric mode

$$\text{Mode}(X) = 0$$

#### 67.14 Additional information and definitions

- $u$  : Uniform[0,1] random variable
- $I(x, a, b)$  : Regularized incomplete beta function
- $I^{-1}(x, a, b)$  : Inverse of regularized incomplete beta function
- Beta  $(x, y)$  : Beta function

#### 67.15 Spreadsheet documents

- [Excel file from GitHub repository](#)
- [Google spreadsheet document](#)

## 68 T Student 3P Distribution

### 68.1 Distribution definition

$$X \sim \text{TStudent}_{3P}(\text{df}, \text{Loc}, \text{Sc})$$

### 68.2 Distribution domain

$$x \in (-\infty, \infty)$$

### 68.3 Parameters domain and parameters constraints

$$\text{df} \in \mathbb{R}^+, \text{Loc} \in \mathbb{R}, \text{Sc} \in \mathbb{R}^+$$

### 68.4 Cumulative distribution function

$$F_X(x) = I\left(\frac{z(x) + \sqrt{z(x)^2 + \text{df}}}{2\sqrt{z(x)^2 + \text{df}}}, \frac{\text{df}}{2}, \frac{\text{df}}{2}\right)$$

### 68.5 Probability density function

$$f_X(x) = \frac{(1 + z(x)^2/\text{df})^{-(1+\text{df})/2}}{\sqrt{\text{df}} \times \text{Beta}\left(\frac{1}{2}, \frac{\text{df}}{2}\right)}$$

### 68.6 Percent point function/Sample

$$F_X^{-1}(u) = \begin{cases} \text{Loc} + \text{Sc} \sqrt{\frac{\text{df}(1-I^{-1}(u, \text{df}/2, \text{df}/2))}{I^{-1}(u, \text{df}/2, \text{df}/2)}} & \text{if } u \geq \frac{1}{2} \\ \text{Loc} - \text{Sc} \sqrt{\frac{\text{df}(1-I^{-1}(u, \text{df}/2, \text{df}/2))}{I^{-1}(u, \text{df}/2, \text{df}/2)}} & \text{if } u < \frac{1}{2} \end{cases}$$

### 68.7 Parametric centered moments

$$\tilde{\mu}'_k = E[\tilde{X}^k] = \int_0^\infty x^k f_{\tilde{X}}(x) dx = \begin{cases} 0 & \text{if } k \text{ odd} \wedge 0 < k < \text{df} \\ \text{df}^{\frac{k}{2}} \prod_{i=1}^{k/2} \frac{2i-1}{\text{df}-2i} & \text{if } k \text{ even} \wedge 0 < k < \text{df} \end{cases}$$

### 68.8 Parametric mean

$$\text{Mean}(X) = \text{Loc} + \text{Sc} \cdot \tilde{\mu}'_1 = \text{Loc}$$

### 68.9 Parametric variance

$$\text{Variance}(X) = \text{Sc}^2 \times (\tilde{\mu}'_2 - \tilde{\mu}'_1{}^2) = \begin{cases} \text{Sc}^2 \text{df}/(\text{df} + 2) & \text{if } \text{df} > 2 \\ \text{undefined} & \text{if } \text{df} \leq 2 \end{cases}$$

### 68.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\tilde{\mu}'_3 - 3\tilde{\mu}'_2\tilde{\mu}'_1 + 2\tilde{\mu}'_1{}^3}{(\tilde{\mu}'_2 - \tilde{\mu}'_1{}^2)^{1.5}} = \begin{cases} 0 & \text{if } \text{df} > 3 \\ \text{undefined} & \text{if } \text{df} \leq 3 \end{cases}$$

### 68.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\tilde{\mu}'_4 - 4\tilde{\mu}'_1\tilde{\mu}'_3 + 6\tilde{\mu}'_1{}^2\tilde{\mu}'_2 - 3\tilde{\mu}'_1{}^4}{(\tilde{\mu}'_2 - \tilde{\mu}'_1{}^2)^2} = \begin{cases} 3 + 6/(\text{df} - 4) & \text{if } \text{df} > 4 \\ \text{undefined} & \text{if } \text{df} \leq 4 \end{cases}$$

### 68.12 Parametric median

$$\text{Median}(X) = \text{Loc}$$

### 68.13 Parametric mode

$$\text{Mode}(X) = \text{Loc}$$

### 68.14 Additional information and definitions

- $\tilde{X} \sim \text{TStudent}(\text{df})$
- $\text{Loc}$  : Location parameter
- $\text{Sc}$  : Scale parameter
- $z(x) = (x - \text{Loc}) / \text{Sc}$
- $u$  :  $\text{Uniform}[0,1]$  random variable
- $I(x, a, b)$  : Regularized incomplete beta function
- $I^{-1}(x, a, b)$  : Inverse of regularized incomplete beta function
- $\text{Beta}(x, y)$  : Beta function

### 68.15 Spreadsheet documents

- [Excel file from GitHub repository](#)
- [Google spreadsheet document](#)

## 69 Trapezoidal Distribution

### 69.1 Distribution definition

$$X \sim \text{Trapezoidal}(a, b, c, d)$$

### 69.2 Distribution domain

$$x \in [a, d]$$

### 69.3 Parameters domain and parameters constraints

$$a \in \mathbb{R}, b \in \mathbb{R}, c \in \mathbb{R}, d \in \mathbb{R}, a < b < c, b < c < d$$

### 69.4 Cumulative distribution function

$$F_X(x) = \begin{cases} \frac{1}{d+c-a-b} \frac{1}{b-a} (x-a)^2 & \text{if } a \leq x < b \\ \frac{1}{d+c-a-b} (2x-a-b) & \text{if } b \leq x < c \\ 1 - \frac{1}{d+c-a-b} \frac{1}{d-c} (d-x)^2 & \text{if } c \leq x \leq d \end{cases}$$

### 69.5 Probability density function

$$f_X(x) = \begin{cases} \frac{2}{d+c-a-b} \frac{x-a}{b-a} & \text{if } a \leq x < b \\ \frac{2}{d+c-a-b} & \text{if } b \leq x < c \\ \frac{2}{d+c-a-b} \frac{d-x}{d-c} & \text{if } c \leq x \leq d \end{cases}$$

### 69.6 Percent point function/Sample

$$F_X^{-1}(u) = \begin{cases} a + \sqrt{u \times (d+c-a-b) \times (b-a)} & \text{if } u \leq A_1 \\ (a+b+u \times (d+c-a-b))/2 & \text{if } A_1 \leq u \leq A_1 + A_2 \\ d - \sqrt{(1-u) \times (d+c-a-b) \times (d-c)} & \text{if } A_1 + A_2 \leq u \leq A_1 + A_2 + A_3 \end{cases}$$

### 69.7 Parametric centered moments

$$\mu'_k = E[X^k] = \int_a^b x^k f_X(x) dx = \frac{2}{d+c-b-a} \frac{1}{(k+1)(k+2)} \left( \frac{d^{k+2} - c^{k+2}}{d-c} - \frac{b^{k+2} - a^{k+2}}{b-a} \right)$$

### 69.8 Parametric mean

$$\text{Mean}(X) = \mu'_1$$

### 69.9 Parametric variance

$$\text{Variance}(X) = \mu'_2 - \mu_1'^2$$

### 69.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\mu'_3 - 3\mu'_2\mu'_1 + 2\mu_1'^3}{(\mu'_2 - \mu_1'^2)^{1.5}}$$

### 69.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\mu'_4 - 4\mu'_1\mu'_3 + 6\mu_1'^2\mu'_2 - 3\mu_1'^4}{(\mu'_2 - \mu_1'^2)^2}$$

### 69.12 Parametric median

$$\text{Median}(X) = F_X^{-1}(1/2)$$



### 69.13 Parametric mode

$$\text{Mode}(X) \in [b, c]$$

### 69.14 Additional information and definitions

- $u$  : Uniform[0,1] random variable
- $A_1 = (b - a)/(d + c - a - b)$
- $A_2 = 2(c - b)/(d + c - a - b)$
- $A_3 = (d - c)/(d + c - a - b)$

### 69.15 Spreadsheet documents

- [Excel file from GitHub repository](#)
- [Google spreadsheet document](#)

## 70 Triangular Distribution

### 70.1 Distribution definition

$$X \sim \text{Triangular}(a, b, c)$$

### 70.2 Distribution domain

$$x \in [a, b]$$

### 70.3 Parameters domain and parameters constraints

$$a \in \mathbb{R}, b \in \mathbb{R}, c \in \mathbb{R}, a < c < b$$

### 70.4 Cumulative distribution function

$$F_X(x) = \begin{cases} \frac{(x-a)^2}{(b-a)(c-a)} & \text{if } a < x \leq c \\ 1 - \frac{(b-x)^2}{(b-a)(b-c)} & \text{if } c < x < b \end{cases}$$

### 70.5 Probability density function

$$f_X(x) = \begin{cases} \frac{2(x-a)}{(b-a)(c-a)} & \text{if } a \leq x < c, \\ \frac{2(b-x)}{(b-a)(b-c)} & \text{if } c \leq x \leq b, \end{cases}$$

### 70.6 Percent point function/Sample

$$F_X^{-1}(u) = \begin{cases} a + \sqrt{U(b-a)(c-a)} & \text{if } 0 < U < \frac{c-a}{b-a} \\ b - \sqrt{(1-U)(b-a)(b-c)} & \text{if } \frac{c-a}{b-a} \leq U < 1 \end{cases}$$

### 70.7 Parametric centered moments

$$\mu'_k = E[X^k] = \int_a^b x^k f_X(x) dx$$

### 70.8 Parametric mean

$$\text{Mean}(X) = \mu'_1 = \frac{a+b+c}{3}$$

### 70.9 Parametric variance

$$\text{Variance}(X) = \mu'_2 - \mu_1'^2 = \frac{a^2 + b^2 + c^2 - ab - ac - bc}{18}$$

### 70.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\mu'_3 - 3\mu'_2\mu'_1 + 2\mu_1'^3}{(\mu_2' - \mu_1'^2)^{1.5}} = \frac{\sqrt{2}(a+b-2c)(2a-b-c)(a-2b+c)}{5(a^2+b^2+c^2-ab-ac-bc)^{\frac{3}{2}}}$$

### 70.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\mu'_4 - 4\mu'_1\mu'_3 + 6\mu_1'^2\mu_2' - 3\mu_1'^4}{(\mu_2' - \mu_1'^2)^2} = 3 - \frac{3}{5}$$

### 70.12 Parametric median

$$\text{Median}(X) = \begin{cases} a + \sqrt{\frac{(b-a)(c-a)}{2}} & \text{if } c \geq \frac{a+b}{2} \\ b - \sqrt{\frac{(b-a)(b-c)}{2}} & \text{if } c \leq \frac{a+b}{2} \end{cases}$$

### 70.13 Parametric mode

$$\text{Mode}(X) \in [b, c]$$

### 70.14 Additional information and definitions

- $u$  : Uniform[0,1] random variable

### 70.15 Spreadsheet documents

- [Excel file from GitHub repository](#)
- [Google spreadsheet document](#)

## 71 Uniform Distribution

### 71.1 Distribution definition

$$X \sim \text{Uniform}(a, b)$$

### 71.2 Distribution domain

$$x \in [a, b]$$

### 71.3 Parameters domain and parameters constraints

$$a \in \mathbb{R}, b \in \mathbb{R}, a < b$$

### 71.4 Cumulative distribution function

$$F_X(x) = \frac{x - a}{b - a}$$

### 71.5 Probability density function

$$f_X(x) = \frac{1}{b - a}$$

### 71.6 Percent point function/Sample

$$F_X^{-1}(u) = a + u \cdot (b - a)$$

### 71.7 Parametric centered moments

$$\mu'_k = E[X^k] = \int_{-\infty}^{\infty} x^k f_X(x) dx = \frac{1}{k+1} \sum_{i=0}^k a^i b^{k-i}$$

### 71.8 Parametric mean

$$\text{Mean}(X) = \mu'_1 = \frac{1}{2}(a + b)$$

### 71.9 Parametric variance

$$\text{Variance}(X) = \mu'_2 - \mu_1'^2$$

### 71.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\mu'_3 - 3\mu'_2\mu'_1 + 2\mu_1'^3}{(\mu'_2 - \mu_1'^2)^{1.5}} = 0$$

### 71.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\mu'_4 - 4\mu'_1\mu'_3 + 6\mu_1'^2\mu'_2 - 3\mu_1'^4}{(\mu'_2 - \mu_1'^2)^2} = 3 - \frac{6}{5}$$

### 71.12 Parametric median

$$\text{Median}(X) = \frac{1}{2}(a + b)$$

### 71.13 Parametric mode

$$\text{Mode}(X) \in [a, b]$$

### 71.14 Additional information and definitions

- $u : \text{Uniform}[0,1]$  random variable

### 71.15 Spreadsheet documents

- [Excel file from GitHub repository](#)
- [Google spreadsheet document](#)

## 72 Weibull Distribution

### 72.1 Distribution definition

$$X \sim \text{Weibull}(\alpha, \beta)$$

### 72.2 Distribution domain

$$x \in [0, \infty)$$

### 72.3 Parameters domain and parameters constraints

$$\alpha \in \mathbb{R}^+, \beta \in \mathbb{R}^+$$

### 72.4 Cumulative distribution function

$$F_X(x) = 1 - e^{-(x/\beta)^\alpha}$$

### 72.5 Probability density function

$$f_X(x) = \frac{\alpha}{\beta} \left(\frac{x}{\beta}\right)^{\alpha-1} e^{-(x/\beta)^\alpha}$$

### 72.6 Percent point function/Sample

$$F_X^{-1}(u) = \beta(-\ln(1-u))^{1/\alpha}$$

### 72.7 Parametric centered moments

$$\mu'_k = E[X^k] = \int_0^\infty x^k f_X(x) dx = \beta^\alpha \Gamma\left(1 + \frac{k}{\alpha}\right)$$

### 72.8 Parametric mean

$$\text{Mean}(X) = \mu'_1 = \beta \cdot \Gamma(1 + 1/\alpha)$$

### 72.9 Parametric variance

$$\text{Variance}(X) = \mu'_2 - \mu_1'^2 = \beta^2 \left[ \Gamma(1 + 2/\alpha) - (\Gamma(1 + 1/\alpha))^2 \right]$$

### 72.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\mu'_3 - 3\mu'_2\mu'_1 + 2\mu_1'^3}{(\mu'_2 - \mu_1'^2)^{1.5}}$$

### 72.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\mu'_4 - 4\mu'_1\mu'_3 + 6\mu_1'^2\mu'_2 - 3\mu_1'^4}{(\mu'_2 - \mu_1'^2)^2}$$

### 72.12 Parametric median

$$\text{Median}(X) = \beta(\ln(2))^{1/\alpha}$$

### 72.13 Parametric mode

$$\text{Mode}(X) = \begin{cases} \beta \left(\frac{\alpha-1}{\alpha}\right)^{1/\alpha} & \text{if } \alpha > 1 \\ 0 & \text{if } \alpha \leq 1 \end{cases}$$

#### 72.14 Additional information and definitions

- $\beta$  : Scale parameter
- $u$  : Uniform[0,1] random variable
- $\Gamma(x)$  : Gamma function

#### 72.15 Spreadsheet documents

- [Excel file from GitHub repository](#)
- [Google spreadsheet document](#)

## 73 Weibull 3P Distribution

### 73.1 Distribution definition

$$X \sim \text{Weibull}_{3P}(\alpha, \text{Loc}, \beta)$$

### 73.2 Distribution domain

$$x \in [\text{Loc}, \infty)$$

### 73.3 Parameters domain and parameters constraints

$$\alpha \in \mathbb{R}^+, \text{Loc} \in \mathbb{R}, \beta \in \mathbb{R}^+$$

### 73.4 Cumulative distribution function

$$F_X(x) = 1 - e^{-z(x)^\alpha}$$

### 73.5 Probability density function

$$f_X(x) = \frac{\alpha}{\beta} z(x)^{\alpha-1} e^{-z(x)^\alpha}$$

### 73.6 Percent point function/Sample

$$F_X^{-1}(u) = \text{Loc} + \beta(-\ln(1-u))^{1/\alpha}$$

### 73.7 Parametric centered moments

$$\tilde{\mu}'_k = E[\tilde{X}^k] = \int_0^\infty x^k f_{\tilde{X}}(x) dx = \beta^\alpha \Gamma\left(1 + \frac{k}{\alpha}\right)$$

### 73.8 Parametric mean

$$\text{Mean}(X) = \text{Loc} + \tilde{\mu}'_1 = \text{Loc} + \beta \Gamma(1 + 1/\alpha)$$

### 73.9 Parametric variance

$$\text{Variance}(X) = \tilde{\mu}'_2 - \tilde{\mu}'_1^2 = \beta^2 \left[ \Gamma(1 + 2/\alpha) - (\Gamma(1 + 1/\alpha))^2 \right]$$

### 73.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\tilde{\mu}'_3 - 3\tilde{\mu}'_2\tilde{\mu}'_1 + 2\tilde{\mu}'_1^3}{(\tilde{\mu}'_2 - \tilde{\mu}'_1^2)^{1.5}}$$

### 73.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\tilde{\mu}'_4 - 4\tilde{\mu}'_1\tilde{\mu}'_3 + 6\tilde{\mu}'_1^2\tilde{\mu}'_2 - 3\tilde{\mu}'_1^4}{(\tilde{\mu}'_2 - \tilde{\mu}'_1^2)^2}$$

### 73.12 Parametric median

$$\text{Median}(X) = \text{Loc} + \beta(\ln(2))^{1/\alpha}$$

### 73.13 Parametric mode

$$\text{Mode}(X) = \text{Loc} + \begin{cases} \beta \left( \frac{\alpha-1}{\alpha} \right)^{1/\alpha} & \text{if } \alpha > 1 \\ 0 & \text{if } \alpha \leq 1 \end{cases}$$



### 73.14 Additional information and definitions

- $\tilde{X} \sim \text{Weibull}(\alpha, \beta)$
- $\text{Loc}$  : Location parameter
- $\beta$  : Scale parameter
- $z(x) = (x - \text{Loc}) / \beta$
- $u$  : Uniform[0,1] random variable
- $\Gamma(x)$  : Gamma function

### 73.15 Spreadsheet documents

- [Excel file from GitHub repository](#)
- [Google spreadsheet document](#)

## References

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- [2] Pauli Virtanen, Ralf Gommers, Travis E. Oliphant, Matt Haberland, Tyler Reddy, David Cournapeau, Evgeni Burovski, Pearu Peterson, Warren Weckesser, Jonathan Bright, Stéfan J. van der Walt, Matthew Brett, Joshua Wilson, K. Jarrod Millman, Nikolay Mayorov, Andrew R. J. Nelson, Eric Jones, Robert Kern, Eric Larson, C J Carey, İlhan Polat, Yu Feng, Eric W. Moore, Jake VanderPlas, Denis Laxalde, Josef Perktold, Robert Cimrman, Ian Henriksen, E. A. Quintero, Charles R. Harris, Anne M. Archibald, Antônio H. Ribeiro, Fabian Pedregosa, Paul van Mulbregt, and SciPy 1.0 Contributors. SciPy 1.0: Fundamental Algorithms for Scientific Computing in Python. *Nature Methods*, 17:261–272, 2020.