

DISCRETE PROBABILITY DISTRIBUTIONS

<https://phitter.io>

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Abstract

This document provides an overview of the continuous probability distributions utilized in Phitter. It includes a detailed description for each distribution, covering aspects such as the definition, domain, parameter definitions and domains, probability density function, cumulative distribution function, percentile point function, raw moments, mean, variance, skewness, kurtosis, median, and mode in a concise and clear manner.

Contents

1	Bernoulli Distribution	5
1.1	Distribution definition	5
1.2	Distribution domain	5
1.3	Parameters domain and parameters constraints	5
1.4	Cumulative distribution function	5
1.5	Probability density function	5
1.6	Percent point function/Sample	5
1.7	Parametric centered moments	5
1.8	Parametric mean	5
1.9	Parametric variance	5
1.10	Parametric skewness	5
1.11	Parametric kurtosis	5
1.12	Parametric median	5
1.13	Parametric mode	5
1.14	Additional information and definitions	6
1.15	Spreadsheet documents	6
2	Binomial Distribution	7
2.1	Distribution definition	7
2.2	Distribution domain	7
2.3	Parameters domain and parameters constraints	7
2.4	Cumulative distribution function	7
2.5	Probability density function	7
2.6	Percent point function/Sample	7
2.7	Parametric centered moments	7
2.8	Parametric mean	7
2.9	Parametric variance	7
2.10	Parametric skewness	7
2.11	Parametric kurtosis	7
2.12	Parametric median	7
2.13	Parametric mode	7
2.14	Additional information and definitions	8
2.15	Spreadsheet documents	8
3	Geometric Distribution	9
3.1	Distribution definition	9
3.2	Distribution domain	9
3.3	Parameters domain and parameters constraints	9
3.4	Cumulative distribution function	9
3.5	Probability density function	9
3.6	Percent point function/Sample	9
3.7	Parametric centered moments	9
3.8	Parametric mean	9
3.9	Parametric variance	9
3.10	Parametric skewness	9
3.11	Parametric kurtosis	9
3.12	Parametric median	9
3.13	Parametric mode	9
3.14	Additional information and definitions	10
3.15	Spreadsheet documents	10

4	Hypergeometric Distribution	11
4.1	Distribution definition	11
4.2	Distribution domain	11
4.3	Parameters domain and parameters constraints	11
4.4	Cumulative distribution function	11
4.5	Probability density function	11
4.6	Percent point function/Sample	11
4.7	Parametric centered moments	11
4.8	Parametric mean	11
4.9	Parametric variance	11
4.10	Parametric skewness	11
4.11	Parametric kurtosis	11
4.12	Parametric median	11
4.13	Parametric mode	11
4.14	Additional information and definitions	12
4.15	Spreadsheet documents	12
5	Logarithmic Distribution	13
5.1	Distribution definition	13
5.2	Distribution domain	13
5.3	Parameters domain and parameters constraints	13
5.4	Cumulative distribution function	13
5.5	Probability density function	13
5.6	Percent point function/Sample	13
5.7	Parametric centered moments	13
5.8	Parametric mean	13
5.9	Parametric variance	13
5.10	Parametric skewness	13
5.11	Parametric kurtosis	13
5.12	Parametric median	13
5.13	Parametric mode	13
5.14	Additional information and definitions	14
5.15	Spreadsheet documents	14
6	Negative Binomial Distribution	15
6.1	Distribution definition	15
6.2	Distribution domain	15
6.3	Parameters domain and parameters constraints	15
6.4	Cumulative distribution function	15
6.5	Probability density function	15
6.6	Percent point function/Sample	15
6.7	Parametric centered moments	15
6.8	Parametric mean	15
6.9	Parametric variance	15
6.10	Parametric skewness	15
6.11	Parametric kurtosis	15
6.12	Parametric median	15
6.13	Parametric mode	15
6.14	Additional information and definitions	16
6.15	Spreadsheet documents	16
7	Poisson Distribution	17
7.1	Distribution definition	17
7.2	Distribution domain	17
7.3	Parameters domain and parameters constraints	17
7.4	Cumulative distribution function	17
7.5	Probability density function	17

7.6	Percent point function/Sample	17
7.7	Parametric centered moments	17
7.8	Parametric mean	17
7.9	Parametric variance	17
7.10	Parametric skewness	17
7.11	Parametric kurtosis	17
7.12	Parametric median	17
7.13	Parametric mode	17
7.14	Additional information and definitions	18
7.15	Spreadsheet documents	18
8	Uniform Distribution	19
8.1	Distribution definition	19
8.2	Distribution domain	19
8.3	Parameters domain and parameters constraints	19
8.4	Cumulative distribution function	19
8.5	Probability density function	19
8.6	Percent point function/Sample	19
8.7	Parametric centered moments	19
8.8	Parametric mean	19
8.9	Parametric variance	19
8.10	Parametric skewness	19
8.11	Parametric kurtosis	19
8.12	Parametric median	19
8.13	Parametric mode	19
8.14	Additional information and definitions	20
8.15	Spreadsheet documents	20

1 Bernoulli Distribution

1.1 Distribution definition

$$X \sim \text{Bernoulli}(p)$$

1.2 Distribution domain

$$x \in \{0, 1\}$$

1.3 Parameters domain and parameters constraints

$$p \in (0, 1) \subseteq \mathbb{R}$$

1.4 Cumulative distribution function

$$F_X(x) = \begin{cases} 1 - p & \text{if } x = 0 \\ 1 & \text{if } x = 1 \end{cases}$$

1.5 Probability density function

$$f_X(x) = p^x(1 - p)^{1-x}$$

1.6 Percent point function/Sample

$$F_X^{-1}(u) = \begin{cases} 0 & \text{if } u \leq p \\ 1 & \text{if } u > p \end{cases}$$

1.7 Parametric centered moments

$$E[X^k] = \mu'_k = \sum_{x=0}^1 x^k f_X(x) = p$$

1.8 Parametric mean

$$\text{Mean}(X) = \mu'_1 = p$$

1.9 Parametric variance

$$\text{Variance}(X) = (\mu'_2 - \mu_1'^2) = p(1 - p)$$

1.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\mu'_3 - 3\mu'_2\mu'_1 + 2\mu_1'^3}{(\mu'_2 - \mu_1'^2)^{1.5}} = \frac{1 - 2p}{\sqrt{p(1 - p)}}$$

1.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\mu'_4 - 4\mu'_1\mu'_3 + 6\mu_1'^2\mu'_2 - 3\mu_1'^4}{(\mu'_2 - \mu_1'^2)^2} = 3 + \frac{1 - 6p(1 - p)}{p(1 - p)}$$

1.12 Parametric median

$$\text{Median}(X) = \begin{cases} 0 & \text{if } p < 1/2 \\ [0, 1] & \text{if } p = 1/2 \\ 1 & \text{if } p > 1/2 \end{cases}$$

1.13 Parametric mode

$$\text{Mode}(X) = \begin{cases} 0 & \text{if } p < 1/2 \\ 0, 1 & \text{if } p = 1/2 \\ 1 & \text{if } p > 1/2 \end{cases}$$

1.14 Additional information and definitions

- u : Uniform[0,1] random variable

1.15 Spreadsheet documents

- [Excel file from GitHub repository](#)
- [Google spreadsheet document](#)

2 Binomial Distribution

2.1 Distribution definition

$$X \sim \text{Binomial}(n, p)$$

2.2 Distribution domain

$$x \in \mathbb{N} \equiv \{0, 1, 2, \dots\}$$

2.3 Parameters domain and parameters constraints

$$n \in \mathbb{N}, p \in (0, 1) \subseteq \mathbb{R}$$

2.4 Cumulative distribution function

$$F_X(x) = \sum_{i=0}^x \binom{n}{i} p^i (1-p)^{n-i} = I(1-p, n-x+1, x)$$

2.5 Probability density function

$$f_X(x) = \binom{n}{x} p^x (1-p)^{n-x}$$

2.6 Percent point function/Sample

$$F_X^{-1}(u) = \arg \min_x |F_X(x) - u|$$

2.7 Parametric centered moments

$$E[X^k] = \mu'_k = \sum_{x=0}^{\infty} x^k f_X(x) = \sum_{i=0}^k \frac{n!}{(n-i)!} S(k, i) p^i$$

2.8 Parametric mean

$$\text{Mean}(X) = \mu'_1 = np$$

2.9 Parametric variance

$$\text{Variance}(X) = (\mu'_2 - \mu_1'^2) = np(1-p)$$

2.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\mu'_3 - 3\mu'_2\mu'_1 + 2\mu_1'^3}{(\mu'_2 - \mu_1'^2)^{1.5}} = \frac{1-2p}{\sqrt{np(1-p)}}$$

2.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\mu'_4 - 4\mu'_1\mu'_3 + 6\mu_1'^2\mu'_2 - 3\mu_1'^4}{(\mu'_2 - \mu_1'^2)^2} = 3 + \frac{1-6p(1-p)}{np(1-p)}$$

2.12 Parametric median

$$\text{Median}(X) = \lfloor np \rfloor \vee \lceil np \rceil$$

2.13 Parametric mode

$$\text{Mode}(X) = \lfloor (n+1)p \rfloor \vee \lceil (n+1)p \rceil - 1$$

2.14 Additional information and definitions

- Computing an analytic expression for the inverse of the cumulative distribution function is not feasible. However, it is possible to calculate the Percentile Point Function by approximating it to the nearest integer.
- u : Uniform[0,1] random variable
- $\lfloor x \rfloor$: Floor function
- $\lceil x \rceil$: Ceiling Function
- $I(x, a, b)$: Regularized incomplete beta function
- $S(a, b)$: Stirling numbers of the second kind $= \frac{1}{b!} \sum_{j=0}^b (-1)^{b-j} \binom{b}{j} j^a$

2.15 Spreadsheet documents

- [Excel file from GitHub repository](#)
- [Google spreadsheet document](#)

3 Geometric Distribution

3.1 Distribution definition

$$X \sim \text{Geometric}(p)$$

3.2 Distribution domain

$$x \in \mathbb{N} \equiv \{0, 1, 2, \dots\}$$

3.3 Parameters domain and parameters constraints

$$p \in (0, 1) \subseteq \mathbb{R}$$

3.4 Cumulative distribution function

$$F_X(x) = 1 - (1 - p)^{\lfloor x \rfloor}$$

3.5 Probability density function

$$f_X(x) = (1 - p)^{x-1} p$$

3.6 Percent point function/Sample

$$F_X^{-1}(u) = \left\lceil \frac{\ln(1 - u)}{\ln(1 - p)} \right\rceil$$

3.7 Parametric centered moments

$$E[X^k] = \mu'_k = \sum_{x=0}^{\infty} x^k f_X(x) = \sum_{x=0}^{\infty} (1 - p)^x p \cdot x^k$$

3.8 Parametric mean

$$\text{Mean}(X) = \mu'_1 = \frac{1}{p}$$

3.9 Parametric variance

$$\text{Variance}(X) = (\mu'_2 - \mu_1'^2) = \frac{1 - p}{p^2}$$

3.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\mu'_3 - 3\mu'_2\mu'_1 + 2\mu_1'^3}{(\mu'_2 - \mu_1'^2)^{1.5}} = \frac{2 - p}{\sqrt{1 - p}}$$

3.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\mu'_4 - 4\mu'_1\mu'_3 + 6\mu_1'^2\mu'_2 - 3\mu_1'^4}{(\mu'_2 - \mu_1'^2)^2} = 9 + \frac{p^2}{1 - p}$$

3.12 Parametric median

$$\text{Median}(X) = \left\lceil \frac{-1}{\log_2(1 - p)} \right\rceil$$

3.13 Parametric mode

$$\text{Mode}(X) = 1$$

3.14 Additional information and definitions

- u : Uniform[0,1] random variable
- $\lfloor x \rfloor$: Floor function
- $\lceil x \rceil$: Ceiling Function

3.15 Spreadsheet documents

- [Excel file from GitHub repository](#)
- [Google spreadsheet document](#)

4 Hypergeometric Distribution

4.1 Distribution definition

$$X \sim \text{Hypergeometric}(N, K, n)$$

4.2 Distribution domain

$$x \in \{\max(0, n + K - N), \min(n, K)\}$$

4.3 Parameters domain and parameters constraints

$$N \in \mathbb{N}, K \in \{0 \dots N\}, n \in \{0 \dots N\}$$

4.4 Cumulative distribution function

$$F_X(x) = \sum_{i=0}^x \binom{K}{i} \binom{N-K}{n-i} / \binom{N}{n}$$

4.5 Probability density function

$$f_X(x) = \binom{K}{x} \binom{N-K}{n-x} / \binom{N}{n}$$

4.6 Percent point function/Sample

$$F_X^{-1}(u) = \arg \min_x |F_X(x) - u|$$

4.7 Parametric centered moments

$$E[X^k] = \mu'_k = \sum_{x=\max(0, n+K-N)}^{\min(n, K)} x^k f_X(x)$$

4.8 Parametric mean

$$\text{Mean}(X) = \mu'_1 = \frac{nK}{N}$$

4.9 Parametric variance

$$\text{Variance}(X) = (\mu'_2 - \mu_1'^2) = n \frac{K}{N} \frac{N-K}{N} \frac{N-n}{N-1}$$

4.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\mu'_3 - 3\mu'_2\mu'_1 + 2\mu_1'^3}{(\mu'_2 - \mu_1'^2)^{1.5}} = \frac{(N-2K)(N-1)^{\frac{1}{2}}(N-2n)}{[nK(N-K)(N-n)]^{\frac{1}{2}}(N-2)}$$

4.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\mu'_4 - 4\mu'_1\mu'_3 + 6\mu_1'^2\mu'_2 - 3\mu_1'^4}{(\mu'_2 - \mu_1'^2)^2} = 3 + \frac{1}{nK(N-K)(N-n)(N-2)(N-3)}$$

4.12 Parametric median

$$\text{Median}(X) = F_X^{-1}(0.5)$$

4.13 Parametric mode

$$\text{Mode}(X) = \left\lfloor \frac{(n+1)(K+1)}{N+2} \right\rfloor$$

4.14 Additional information and definitions

- Computing an analytic expression for the inverse of the cumulative distribution function is not feasible. However, it is possible to calculate the Percentile Point Function by approximating it to the nearest integer.
- u : Uniform[0,1] random variable
- $\lfloor x \rfloor$: Floor function
- $\lceil x \rceil$: Ceiling Function

4.15 Spreadsheet documents

- [Excel file from GitHub repository](#)
- [Google spreadsheet document](#)

5 Logarithmic Distribution

5.1 Distribution definition

$$X \sim \text{Logarithmic}(p)$$

5.2 Distribution domain

$$x \in \mathbb{N}_{\geq 1} \equiv \{1, 2, \dots\}$$

5.3 Parameters domain and parameters constraints

$$p \in (0, 1) \subseteq \mathbb{R}$$

5.4 Cumulative distribution function

$$F_X(x) = \sum_{i=0}^x \frac{1}{-\ln(1-p)} \frac{p^i}{i}$$

5.5 Probability density function

$$f_X(x) = \frac{1}{-\ln(1-p)} \frac{p^x}{x}$$

5.6 Percent point function/Sample

$$F_X^{-1}(u) = \arg \min_x |F_X(x) - u|$$

5.7 Parametric centered moments

$$E[X^k] = \mu'_k = \sum_{x=0}^{\infty} x^k f_X(x) = \frac{(k-1)!}{-\ln(1-p)} \left(\frac{p}{1-p} \right)^k$$

5.8 Parametric mean

$$\text{Mean}(X) = \mu'_1 = \frac{1}{-\ln(1-p)} \frac{p}{1-p}$$

5.9 Parametric variance

$$\text{Variance}(X) = (\mu'_2 - \mu_1'^2) = -\frac{p^2 + p \ln(1-p)}{(1-p)^2 (\ln(1-p))^2}$$

5.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\mu'_3 - 3\mu'_2\mu'_1 + 2\mu_1'^3}{(\mu'_2 - \mu_1'^2)^{1.5}}$$

5.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\mu'_4 - 4\mu'_1\mu'_3 + 6\mu_1'^2\mu'_2 - 3\mu_1'^4}{(\mu'_2 - \mu_1'^2)^2}$$

5.12 Parametric median

$$\text{Median}(X) = F_X^{-1}(0.5)$$

5.13 Parametric mode

$$\text{Mode}(X) = 1$$

5.14 Additional information and definitions

- Computing an analytic expression for the inverse of the cumulative distribution function is not feasible. However, it is possible to calculate the Percentile Point Function by approximating it to the nearest integer.
- u : Uniform[0,1] random variable

5.15 Spreadsheet documents

- [Excel file from GitHub repository](#)
- [Google spreadsheet document](#)

6 Negative Binomial Distribution

6.1 Distribution definition

$$X \sim \text{NegativeBinomial}(r, p)$$

6.2 Distribution domain

$$x \in \mathbb{N} \equiv \{0, 1, 2, \dots\}$$

6.3 Parameters domain and parameters constraints

$$r \in \mathbb{N}_{\geq 1}, p \in (0, 1) \subseteq \mathbb{R}$$

6.4 Cumulative distribution function

$$F_X(x) = I(p, r, x + 1)$$

6.5 Probability density function

$$f_X(x) = \binom{r+x-1}{x} p^r (1-p)^x$$

6.6 Percent point function/Sample

$$F_X^{-1}(u) = \arg \min_x |F_X(x) - u|$$

6.7 Parametric centered moments

$$E[X^k] = \mu'_k = \sum_{x=0}^{\infty} x^k f_X(x)$$

6.8 Parametric mean

$$\text{Mean}(X) = \mu'_1 = \frac{r(1-p)}{p}$$

6.9 Parametric variance

$$\text{Variance}(X) = (\mu'_2 - \mu'^2_1) = \frac{r(1-p)}{p^2}$$

6.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\mu'_3 - 3\mu'_2\mu'_1 + 2\mu'^3_1}{(\mu'_2 - \mu'^2_1)^{1.5}} = \frac{2-p}{\sqrt{r(1-p)}}$$

6.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\mu'_4 - 4\mu'_1\mu'_3 + 6\mu'^2_1\mu'_2 - 3\mu'^4_1}{(\mu'_2 - \mu'^2_1)^2} = 3 + \frac{6}{r} + \frac{p^2}{r(1-p)}$$

6.12 Parametric median

$$\text{Median}(X) = F_X^{-1}(0.5)$$

6.13 Parametric mode

$$\text{Mode}(X) = \lfloor (r-1)(1-p)/p \rfloor$$

6.14 Additional information and definitions

- Computing an analytic expression for the inverse of the cumulative distribution function is not feasible. However, it is possible to calculate the Percentile Point Function by approximating it to the nearest integer.
- u : Uniform[0,1] random variable
- $I(x, a, b)$: Regularized incomplete beta function
- $\lfloor x \rfloor$: Floor function

6.15 Spreadsheet documents

- [Excel file from GitHub repository](#)
- [Google spreadsheet document](#)

7 Poisson Distribution

7.1 Distribution definition

$$X \sim \text{Poisson}(\lambda)$$

7.2 Distribution domain

$$x \in \mathbb{N} \equiv \{0, 1, 2, \dots\}$$

7.3 Parameters domain and parameters constraints

$$\lambda \in \mathbb{R}^+$$

7.4 Cumulative distribution function

$$F_X(x) = e^{-\lambda} \sum_{i=0}^x \frac{\lambda^i}{i!} = 1 - \frac{\gamma(x+1, \lambda)}{x!} = 1 - P(x-1, \lambda)$$

7.5 Probability density function

$$f_X(x) = \frac{\lambda^x e^{-\lambda}}{x!}$$

7.6 Percent point function/Sample

$$F_X^{-1}(u) = \arg \min_x |F_X(x) - u|$$

7.7 Parametric centered moments

$$E[X^k] = \mu'_k = \sum_{x=0}^{\infty} x^k f_X(x)$$

7.8 Parametric mean

$$\text{Mean}(X) = \mu'_1 = \lambda$$

7.9 Parametric variance

$$\text{Variance}(X) = (\mu'_2 - \mu_1'^2) = \lambda$$

7.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\mu'_3 - 3\mu'_2\mu'_1 + 2\mu_1'^3}{(\mu'_2 - \mu_1'^2)^{1.5}} = \lambda^{-1/2}$$

7.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\mu'_4 - 4\mu'_1\mu'_3 + 6\mu_1'^2\mu'_2 - 3\mu_1'^4}{(\mu'_2 - \mu_1'^2)^2} = 3 + \lambda^{-1}$$

7.12 Parametric median

$$\text{Median}(X) = \lfloor \lambda + 1/3 - 0.02/\lambda \rfloor$$

7.13 Parametric mode

$$\text{Mode}(X) = \lfloor \lambda \rfloor$$

7.14 Additional information and definitions

- Computing an analytic expression for the inverse of the cumulative distribution function is not feasible. However, it is possible to calculate the Percentile Point Function by approximating it to the nearest integer.
- u : Uniform[0,1] random variable
- $\lfloor x \rfloor$: Floor function
- $P(a, x) = \frac{\gamma(a, x)}{\Gamma(a)}$: Regularized lower incomplete gamma function
- $\gamma(a, x)$: Lower incomplete Gamma function

7.15 Spreadsheet documents

- [Excel file from GitHub repository](#)
- [Google spreadsheet document](#)

8 Uniform Distribution

8.1 Distribution definition

$$X \sim \text{Uniform}(a, b)$$

8.2 Distribution domain

$$x \in \{a, a+1, \dots, b-1, b\}$$

8.3 Parameters domain and parameters constraints

$$a \in \mathbb{N}, b \in \mathbb{N}, a < b$$

8.4 Cumulative distribution function

$$F_X(x) = \frac{x - a + 1}{b - a + 1}$$

8.5 Probability density function

$$f_X(x) = \frac{1}{b - a + 1}$$

8.6 Percent point function/Sample

$$F_X^{-1}(u) = \lceil u(b - a + 1) + a - 1 \rceil$$

8.7 Parametric centered moments

$$E[X^k] = \mu'_k = \sum_{x=a}^b x^k f_X(x) = \frac{1}{b - a + 1} \sum_{x=a}^b x^k$$

8.8 Parametric mean

$$\text{Mean}(X) = \mu'_1 = \frac{a + b}{2}$$

8.9 Parametric variance

$$\text{Variance}(X) = (\mu'_2 - \mu_1'^2) = \frac{(b - a + 1)^2 - 1}{12}$$

8.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\mu'_3 - 3\mu'_2\mu'_1 + 2\mu_1'^3}{(\mu'_2 - \mu_1'^2)^{1.5}} = 0$$

8.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\mu'_4 - 4\mu'_1\mu'_3 + 6\mu_1'^2\mu'_2 - 3\mu_1'^4}{(\mu'_2 - \mu_1'^2)^2} = 3 - \frac{6((b - a + 1)^2 + 1)}{5((b - a + 1)^2 - 1)}$$

8.12 Parametric median

$$\text{Median}(X) = \frac{a + b}{2}$$

8.13 Parametric mode

$$\text{Mode}(X) \in [a, b]$$

8.14 Additional information and definitions

- u : Uniform[0,1] random variable
- $\lceil x \rceil$: Ceiling Function

8.15 Spreadsheet documents

- [Excel file from GitHub repository](#)
- [Google spreadsheet document](#)

References

- [1] Eric Jones, Travis Oliphant, Pearu Peterson, et al. SciPy: Open source scientific tools for Python, 2001–.
- [2] Pauli Virtanen, Ralf Gommers, Travis E. Oliphant, Matt Haberland, Tyler Reddy, David Cournapeau, Evgeni Burovski, Pearu Peterson, Warren Weckesser, Jonathan Bright, Stéfan J. van der Walt, Matthew Brett, Joshua Wilson, K. Jarrod Millman, Nikolay Mayorov, Andrew R. J. Nelson, Eric Jones, Robert Kern, Eric Larson, C J Carey, İlhan Polat, Yu Feng, Eric W. Moore, Jake VanderPlas, Denis Laxalde, Josef Perktold, Robert Cimrman, Ian Henriksen, E. A. Quintero, Charles R. Harris, Anne M. Archibald, Antônio H. Ribeiro, Fabian Pedregosa, Paul van Mulbregt, and SciPy 1.0 Contributors. SciPy 1.0: Fundamental Algorithms for Scientific Computing in Python. *Nature Methods*, 17:261–272, 2020.