

CONTINUOUS PROBABILITY DISTRIBUTIONS

<https://phitter.io>

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Abstract

This document provides an overview of the continuous probability distributions utilized in Phitter. It includes a detailed description for each distribution, covering aspects such as the definition, domain, parameter definitions and domains, probability density function, cumulative distribution function, percentile point function, raw moments, mean, variance, skewness, kurtosis, median, and mode in a concise and clear manner.

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1 Alpha Distribution

1.1 Distribution definition

$$X \sim \text{Alpha}(\alpha, \text{Loc}, \text{Sc})$$

1.2 Distribution domain

$$x \in (\text{Loc}, \infty)$$

1.3 Parameters domain and parameters constraints

$$\alpha \in \mathbb{R}^+, \text{Loc} \in \mathbb{R}, \text{Sc} \in \mathbb{R}^+$$

1.4 Cumulative distribution function

$$F_X(x) = \frac{\Phi\left(\alpha - \frac{1}{z(x)}\right)}{\Phi(\alpha)}$$

1.5 Probability density function

$$f_X(x) = \frac{1}{\text{Sc} \cdot z(x)^2 \cdot \Phi(\alpha) \cdot \sqrt{2\pi}} \exp\left(-\frac{1}{2} \left(\alpha - \frac{1}{z(x)}\right)^2\right)$$

1.6 Percent point function/Sample

$$F_X^{-1}(u) = \text{Loc} + \text{Sc} \times \frac{1}{\alpha - \Phi^{-1}(u\Phi(\alpha))}$$

1.7 Parametric centered moments

$$\tilde{\mu}'_k = E[\tilde{X}^k] = \int_0^\infty x^k f_{\tilde{X}}(x) dx$$

1.8 Parametric mean

$$\text{Mean}(X) = \text{Loc} + \text{Sc} \cdot \tilde{\mu}'_1$$

1.9 Parametric variance

$$\text{Variance}(X) = \text{Sc}^2 \cdot (\tilde{\mu}'_2 - \tilde{\mu}'_1^2)$$

1.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\tilde{\mu}'_3 - 3\tilde{\mu}'_2\tilde{\mu}'_1 + 2\tilde{\mu}'_1^3}{(\tilde{\mu}'_2 - \tilde{\mu}'_1^2)^{1.5}}$$

1.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\tilde{\mu}'_4 - 4\tilde{\mu}'_1\tilde{\mu}'_3 + 6\tilde{\mu}'_1^2\tilde{\mu}'_2 - 3\tilde{\mu}'_1^4}{(\tilde{\mu}'_2 - \tilde{\mu}'_1^2)^2}$$

1.12 Parametric median

$$\text{Median}(X) = \text{Loc} + \frac{\text{Sc}}{\alpha - \Phi^{-1}\left(\frac{1}{2}\Phi(\alpha)\right)}$$

1.13 Parametric mode

$$\text{Mode}(X) = \text{Loc} + \text{Sc} \frac{(\sqrt{\alpha^2 + 8} - \alpha)}{4}$$

1.14 Additional information and definitions

- $\tilde{X} \sim \text{Alpha}(\alpha, 0, 1)$
- Loc : Location parameter
- Sc : Scale parameter
- $z(x) = (x - \text{Loc}) / \text{Sc}$
- $u : \text{Uniform}[0,1]$ random variable
- $\Phi(x) : \text{CDF normal standard distribution}$
- $\Phi^{-1}(x) : \text{PPF normal standard distribution}$

1.15 Spreadsheet documents

- [Excel file from GitHub repository](#)
- [Google spreadsheet document](#)

2 Arcsine Distribution

2.1 Distribution definition

$$X \sim \text{Arcsine}(a, b)$$

2.2 Distribution domain

$$x \in (a, b)$$

2.3 Parameters domain and parameters constraints

$$a \in \mathbb{R}, b \in \mathbb{R}, a < b$$

2.4 Cumulative distribution function

$$F_X(x) = \frac{2}{\pi} \arcsin \left(\sqrt{\frac{x-a}{b-a}} \right)$$

2.5 Probability density function

$$f_X(x) = \frac{1}{\pi \sqrt{(x-a)(b-x)}}$$

2.6 Percent point function/Sample

$$F_X^{-1}(u) = a + (b-a) \times \sin^2 \left(\frac{\pi}{2} u \right)$$

2.7 Parametric centered moments

$$\tilde{\mu}'_k = E[\tilde{X}^k] = \int_0^1 x^k f_{\tilde{X}}(x) dx = \frac{1}{\pi} \text{Beta} \left(\frac{1}{2}, k + \frac{1}{2} \right) = \frac{(2k-1)!!}{2^k k!}$$

2.8 Parametric mean

$$\text{Mean}(X) = a + \tilde{\mu}'_1 (b-a) = a + \frac{1}{2} (b-a)$$

2.9 Parametric variance

$$\text{Variance}(X) = (b-a)^2 \times (\tilde{\mu}'_2 - \tilde{\mu}_1'^2) = \frac{(b-a)^2}{8}$$

2.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\tilde{\mu}'_3 - 3\tilde{\mu}'_2\tilde{\mu}'_1 + 2\tilde{\mu}_1'^3}{(\tilde{\mu}'_2 - \tilde{\mu}_1'^2)^{1.5}} = 0$$

2.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\tilde{\mu}'_4 - 4\tilde{\mu}'_1\tilde{\mu}'_3 + 6\tilde{\mu}_1'^2\tilde{\mu}'_2 - 3\tilde{\mu}_1'^4}{(\tilde{\mu}'_2 - \tilde{\mu}_1'^2)^2} = 3 - \frac{3}{2}$$

2.12 Parametric median

$$\text{Median}(X) = a + (b-a) \times \sin^2 \left(\frac{\pi}{4} \right)$$

2.13 Parametric mode

$$\text{Mode}(X) = \text{undefined}$$

2.14 Additional information and definitions

- $\tilde{X} \sim \text{Arcsine}(0, 1)$
- $u : \text{Uniform}[0, 1]$ random variable
- $\text{Beta}(x, y) : \text{Beta function}$

2.15 Spreadsheet documents

- [Excel file from GitHub repository](#)
- [Google spreadsheet document](#)

3 Argus Distribution

3.1 Distribution definition

$$X \sim \text{Argus}(\chi, \text{Loc}, \text{Sc})$$

3.2 Distribution domain

$$x \in (\text{Loc}, \text{Loc} + \text{Sc})$$

3.3 Parameters domain and parameters constraints

$$\chi \in \mathbb{R}^+, \text{Loc} \in \mathbb{R}, \text{Sc} \in \mathbb{R}^+$$

3.4 Cumulative distribution function

$$F_X(x) = 1 - \frac{\Psi\left(\chi\sqrt{1-z(x)^2}\right)}{\Psi(\chi)}$$

3.5 Probability density function

$$f_X(x) = \frac{1}{\text{Sc}} \cdot \frac{\chi^3}{\sqrt{2\pi}\Psi(\chi)} \cdot z(x)\sqrt{1-z(x)^2} \exp\left(-\frac{1}{2}\chi^2(1-z(x)^2)\right)$$

3.6 Percent point function/Sample

$$F_X^{-1}(u) = \text{Loc} + \text{Sc}\sqrt{1 - \frac{2\text{P}^{-1}(\frac{3}{2}, (1-u)\text{P}(\frac{3}{2}, \frac{\chi^2}{2}))}{\chi^2}}$$

3.7 Parametric centered moments

$$\mu'_k = E[X^k] = \int_{\text{Loc}}^{\text{Loc}+\text{Sc}} x^k f_X(x) dx$$

3.8 Parametric mean

$$\text{Mean}(X) = \mu'_1 = \text{Loc} + \text{Sc}\sqrt{\pi/8} \frac{\chi e^{-\frac{\chi^2}{4}} I_1(\frac{\chi^2}{4})}{\Psi(\chi)}$$

3.9 Parametric variance

$$\text{Variance}(X) = \mu'_2 - \mu_1'^2 = \text{Sc}^2 \cdot \left(1 - \frac{3}{\chi^2} + \frac{\chi\phi(\chi)}{\Psi(\chi)}\right) - (\mu - \text{Loc})^2$$

3.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\mu'_3 - 3\mu'_2\mu'_1 + 2\mu_1'^3}{(\mu'_2 - \mu_1'^2)^{1.5}}$$

3.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\mu'_4 - 4\mu'_1\mu'_3 + 6\mu_1'^2\mu'_2 - 3\mu_1'^4}{(\mu'_2 - \mu_1'^2)^2}$$

3.12 Parametric median

$$\text{Median}(X) = \text{Loc} + \text{Sc}\sqrt{1 - \frac{2\text{P}^{-1}(\frac{3}{2}, \frac{1}{2}\text{P}(\frac{3}{2}, \frac{\chi^2}{2}))}{\chi^2}}$$

3.13 Parametric mode

$$\text{Mode}(X) = \text{Loc} + \frac{\text{Sc}}{\sqrt{2}\chi} \sqrt{(\chi^2 - 2) + \sqrt{\chi^4 + 4}}$$

3.14 Additional information and definitions

- Loc : Location parameter
- Sc : Scale parameter
- $z(x) = (x - \text{Loc}) / \text{Sc}$
- u : Uniform[0,1] random variable
- $\Psi(\chi) = \Phi(\chi) - \chi\phi(\chi) - \frac{1}{2}$
- $\Phi(x)$: CDF normal standard distribution
- $\phi(x)$: PDF normal standard distribution
- $I_\alpha(x)$: Modified Bessel function of the first kind of order $\alpha \in \mathbb{N}$
- $P(a, x) = \frac{\gamma(a, x)}{\Gamma(a)}$: Regularized lower incomplete gamma function
- $P^{-1}(a, y)$: Inverse of regularized lower incomplete gamma function
- $\gamma(a, x)$: Lower incomplete gamma function
- $\Gamma(x)$: Gamma function

3.15 Spreadsheet documents

- [Excel file from GitHub repository](#)
- [Google spreadsheet document](#)

4 Beta Distribution

4.1 Distribution definition

$$X \sim \text{Beta}(\alpha, \beta, A, B)$$

4.2 Distribution domain

$$x \in (A, B)$$

4.3 Parameters domain and parameters constraints

$$\alpha \in \mathbb{R}^+, \beta \in \mathbb{R}^+, A \in \mathbb{R}, B \in \mathbb{R}, A < B$$

4.4 Cumulative distribution function

$$F_X(x) = I(z(x), \alpha, \beta)$$

4.5 Probability density function

$$f_X(x) = \frac{z(x)^{\alpha-1} (1 - z(x))^{\beta-1}}{\text{Beta}(\alpha, \beta)(B - A)}$$

4.6 Percent point function/Sample

$$F_X^{-1}(u) = A + (B - A) \times I^{-1}(u, \alpha, \beta)$$

4.7 Parametric centered moments

$$\tilde{\mu}'_k = E[\tilde{X}^k] = \int_0^1 x^k f_{\tilde{X}}(x) dx$$

4.8 Parametric mean

$$\text{Mean}(X) = A + (B - A) \cdot \tilde{\mu}'_1 = A + \frac{\alpha(B - A)}{\alpha + \beta}$$

4.9 Parametric variance

$$\text{Variance}(X) = (B - A)^2 \cdot (\tilde{\mu}'_2 - \tilde{\mu}'_1^2) = \frac{\alpha\beta(B - A)^2}{(\alpha + \beta)^2(\alpha + \beta + 1)}$$

4.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\tilde{\mu}'_3 - 3\tilde{\mu}'_2\tilde{\mu}'_1 + 2\tilde{\mu}'_1^3}{(\tilde{\mu}'_2 - \tilde{\mu}'_1^2)^{1.5}} = \frac{2(\beta - \alpha)\sqrt{\alpha + \beta + 1}}{(\alpha + \beta + 2)\sqrt{\alpha\beta}}$$

4.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\tilde{\mu}'_4 - 4\tilde{\mu}'_1\tilde{\mu}'_3 + 6\tilde{\mu}'_1^2\tilde{\mu}'_2 - 3\tilde{\mu}'_1^4}{(\tilde{\mu}'_2 - \tilde{\mu}'_1^2)^2} = 3 + \frac{6[(\alpha - \beta)^2(\alpha + \beta + 1) - \alpha\beta(\alpha + \beta + 2)]}{\alpha\beta(\alpha + \beta + 2)(\alpha + \beta + 3)}$$

4.12 Parametric median

$$\text{Median}(X) = A + (B - A) \times I^{-1}\left(\frac{1}{2}, \alpha, \beta\right) \quad \text{if } \alpha, \beta > 1$$

4.13 Parametric mode

$$\text{Mode}(X) = A + (B - A) \frac{\alpha - 1}{\alpha + \beta - 2} \quad \text{if } \alpha, \beta > 1$$

4.14 Additional information and definitions

- $\tilde{X} \sim \text{Beta}(\alpha, \beta, 0, 1)$
- $z(x) = (x - A) / (B - A)$
- $u : \text{Uniform}[0,1]$ random variable
- $I(x, a, b) : \text{Regularized incomplete beta function}$
- $I^{-1}(x, a, b) : \text{Inverse of regularized incomplete beta function}$
- $\text{Beta}(x, y) : \text{Beta function}$

4.15 Spreadsheet documents

- [Excel file from GitHub repository](#)
- [Google spreadsheet document](#)

5 Beta Prime Distribution

5.1 Distribution definition

$$X \sim \text{BetaPrime}(\alpha, \beta)$$

5.2 Distribution domain

$$x \in [0, \infty)$$

5.3 Parameters domain and parameters constraints

$$\alpha \in \mathbb{R}^+, \beta \in \mathbb{R}^+$$

5.4 Cumulative distribution function

$$F_X(x) = I\left(\frac{x}{1+x}, \alpha, \beta\right)$$

5.5 Probability density function

$$f_X(x) = \frac{x^{\alpha-1}(1+x)^{-\alpha-\beta}}{\text{Beta}(\alpha, \beta)}$$

5.6 Percent point function/Sample

$$F_X^{-1}(u) = \frac{I^{-1}(u, \alpha, \beta)}{1 - I^{-1}(u, \alpha, \beta)}$$

5.7 Parametric centered moments

$$\mu'_k = E[X^k] = \int_0^\infty x^k f_X(x) dx = \frac{\Gamma(k+\alpha)\Gamma(\beta-k)}{\Gamma(\alpha)\Gamma(\beta)} \quad \text{if } \beta > k$$

5.8 Parametric mean

$$\text{Mean}(X) = \mu'_1 = \frac{\alpha}{\beta-1} \quad \text{if } \beta > 1$$

5.9 Parametric variance

$$\text{Variance}(X) = \mu'_2 - \mu_1'^2 = \frac{\alpha(\alpha+\beta-1)}{(\beta-2)(\beta-1)^2} \quad \text{if } \beta > 2$$

5.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\mu'_3 - 3\mu'_2\mu'_1 + 2\mu_1'^3}{(\mu'_2 - \mu_1'^2)^{1.5}} = \frac{2(2\alpha+\beta-1)}{\beta-3} \sqrt{\frac{\beta-2}{\alpha(\alpha+\beta-1)}} \quad \text{if } \beta > 3$$

5.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\mu'_4 - 4\mu'_1\mu'_3 + 6\mu_1'^2\mu'_2 - 3\mu_1'^4}{(\mu'_2 - \mu_1'^2)^2} \quad \text{if } \beta > 4$$

5.12 Parametric median

$$\text{Median}(X) = \frac{I^{-1}\left(\frac{1}{2}, \alpha, \beta\right)}{1 - I^{-1}\left(\frac{1}{2}, \alpha, \beta\right)}$$

5.13 Parametric mode

$$\text{Mode}(X) = \frac{\alpha - 1}{\beta + 1}$$

5.14 Additional information and definitions

- u : Uniform[0,1] random variable
- $I(x, a, b)$: Regularized incomplete beta function
- $I^{-1}(x, a, b)$: Inverse of regularized incomplete beta function
- $\Gamma(x)$: Gamma function
- Beta (x, y) : Beta function

5.15 Spreadsheet documents

- [Excel file from GitHub repository](#)
- [Google spreadsheet document](#)

6 Beta Prime 4P Distribution

6.1 Distribution definition

$$X \sim \text{BetaPrime}_{4P}(\alpha, \beta, \text{Loc}, \text{Sc})$$

6.2 Distribution domain

$$x \in [\text{Loc}, \infty)$$

6.3 Parameters domain and parameters constraints

$$\alpha \in \mathbb{R}^+, \beta \in \mathbb{R}^+, \text{Loc} \in \mathbb{R}, \text{Sc} \in \mathbb{R}^+$$

6.4 Cumulative distribution function

$$F_X(x) = I\left(\frac{z(x)}{1+z(x)}, \alpha, \beta\right)$$

6.5 Probability density function

$$f_X(x) = \frac{z(x)^{\alpha-1}(1+z(x))^{-\alpha-\beta}}{\text{Sc} \times \text{Beta}(\alpha, \beta)}$$

6.6 Percent point function/Sample

$$F_X^{-1}(u) = \text{Loc} + \text{Sc} \frac{I^{-1}(u, \alpha, \beta)}{1 - I^{-1}(u, \alpha, \beta)}$$

6.7 Parametric centered moments

$$\tilde{\mu}'_k = E[\tilde{X}^k] = \int_0^\infty x^k f_{\tilde{X}}(x) dx = \frac{\Gamma(k+\alpha) \Gamma(\beta-k)}{\Gamma(\alpha) \Gamma(\beta)} \quad \text{if } \beta > k$$

6.8 Parametric mean

$$\text{Mean}(X) = \text{Loc} + \text{Sc} \tilde{\mu}'_1 = \text{Loc} + \text{Sc} \frac{\alpha}{\beta-1} \quad \text{if } \beta > 1$$

6.9 Parametric variance

$$\text{Variance}(X) = \text{Sc}^2(\tilde{\mu}'_2 - \tilde{\mu}_1'^2) = \text{Sc}^2 \frac{\alpha(\alpha+\beta-1)}{(\beta-2)(\beta-1)^2} \quad \text{if } \beta > 2$$

6.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\tilde{\mu}'_3 - 3\tilde{\mu}'_2\tilde{\mu}'_1 + 2\tilde{\mu}_1'^3}{(\tilde{\mu}'_2 - \tilde{\mu}_1'^2)^{1.5}} = \frac{2(2\alpha+\beta-1)}{\beta-3} \sqrt{\frac{\beta-2}{\alpha(\alpha+\beta-1)}} \quad \text{if } \beta > 3$$

6.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\tilde{\mu}'_4 - 4\tilde{\mu}'_1\tilde{\mu}'_3 + 6\tilde{\mu}_1'^2\tilde{\mu}'_2 - 3\tilde{\mu}_1'^4}{(\tilde{\mu}'_2 - \tilde{\mu}_1'^2)^2} \quad \text{if } \beta > 4$$

6.12 Parametric median

$$\text{Median}(X) = \text{Loc} + \text{Sc} \frac{I^{-1}\left(\frac{1}{2}, \alpha, \beta\right)}{1 - I^{-1}\left(\frac{1}{2}, \alpha, \beta\right)}$$

6.13 Parametric mode

$$\text{Mode}(X) = \text{Loc} + \text{Sc} \frac{\alpha - 1}{\beta + 1}$$

6.14 Additional information and definitions

- $\tilde{X} \sim \text{BetaPrime}(\alpha, \beta)$
- Loc : Location parameter
- Sc : Scale parameter
- $z(x) = (x - \text{Loc}) / \text{Sc}$
- u : Uniform[0,1] random variable
- $I(x, a, b)$: Regularized incomplete beta function
- $I^{-1}(x, a, b)$: Inverse of regularized incomplete beta function
- $\Gamma(x)$: Gamma function
- Beta(x, y) : Beta function

6.15 Spreadsheet documents

- [Excel file from GitHub repository](#)
- [Google spreadsheet document](#)

7 Bradford Distribution

7.1 Distribution definition

$$X \sim \text{Bradford}(c, \min, \max)$$

7.2 Distribution domain

$$x \in (\min, \max)$$

7.3 Parameters domain and parameters constraints

$$c \in \mathbb{R}^+, \min \in \mathbb{R}, \max \in \mathbb{R}, \min < \max$$

7.4 Cumulative distribution function

$$F_X(x) = \frac{\ln(1 + c \cdot z(x))}{k}$$

7.5 Probability density function

$$f_X(x) = \frac{c}{k(1 + c \cdot z(x))(\max - \min)}$$

7.6 Percent point function/Sample

$$F_X^{-1}(u) = \min + (\max - \min) \times \frac{(1 + c)^u - 1}{c}$$

7.7 Parametric centered moments

$$\tilde{\mu}'_k = E[\tilde{X}^k] = \int_0^1 x^k f_{\tilde{X}}(x) dx$$

7.8 Parametric mean

$$\text{Mean}(X) = \min + (\max - \min) \cdot \tilde{\mu}'_1 = \min + (\max - \min) \cdot \frac{c - k}{ck}$$

7.9 Parametric variance

$$\text{Variance}(X) = (\max - \min)^2 \cdot (\tilde{\mu}'_2 - \tilde{\mu}_1'^2) = (\max - \min)^2 \cdot \frac{(c + 2)k - 2c}{2ck^2}$$

7.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\tilde{\mu}'_3 - 3\tilde{\mu}'_2\tilde{\mu}'_1 + 2\tilde{\mu}_1'^3}{(\tilde{\mu}'_2 - \tilde{\mu}_1'^2)^{1.5}} = \frac{\sqrt{2}(12c^2 - 9kc(c + 2) + 2k^2(c(c + 3) + 3))}{\sqrt{c(c(k - 2) + 2k)}(3c(k - 2) + 6k)}$$

7.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\tilde{\mu}'_4 - 4\tilde{\mu}'_1\tilde{\mu}'_3 + 6\tilde{\mu}'_2\tilde{\mu}'_2 - 3\tilde{\mu}_1'^4}{(\tilde{\mu}'_2 - \tilde{\mu}_1'^2)^2} = 3 + \frac{c^3(k - 3)(k(3k - 16) + 24) + 12kc^2(k - 4)(k - 3) + 6ck^2(3k - 14) + 1}{3c(c(k - 2) + 2k)^2}$$

7.12 Parametric median

$$\text{Median}(X) = \min + (\max - \min) \cdot \frac{(1 + c)^{\frac{1}{2}} - 1}{c}$$

7.13 Parametric mode

$$\text{Mode}(X) = \min$$

7.14 Additional information and definitions

- $\tilde{X} \sim \text{Bradford}(c, 0, 1)$
- $k = \ln(1 + c)$
- $z(x) = (x - \min) / (\max - \min)$
- $u : \text{Uniform}[0,1]$ random variable

7.15 Spreadsheet documents

- [Excel file from GitHub repository](#)
- [Google spreadsheet document](#)

8 Burr Distribution

8.1 Distribution definition

$$X \sim \text{Burr}(A, B, C)$$

8.2 Distribution domain

$$x \in [0, \infty)$$

8.3 Parameters domain and parameters constraints

$$A \in \mathbb{R}^+, B \in \mathbb{R}, C \in \mathbb{R}^+$$

8.4 Cumulative distribution function

$$F_X(x) = 1 - \left[1 + \left(\frac{x}{A}\right)^B\right]^{-C}$$

8.5 Probability density function

$$f_X(x) = \frac{BC}{A} \left(\frac{x}{A}\right)^{B-1} \left[1 + \left(\frac{x}{A}\right)^B\right]^{-C-1}$$

8.6 Percent point function/Sample

$$F_X^{-1}(u) = A \left[(1-u)^{-\frac{1}{C}} - 1 \right]^{\frac{1}{B}}$$

8.7 Parametric centered moments

$$\mu'_k = E[X^k] = \int_0^\infty x^k f_X(x) dx = A^k C \times \text{Beta}\left(\frac{BC-k}{B}, \frac{B+K}{B}\right)$$

8.8 Parametric mean

$$\text{Mean}(X) = \mu'_1$$

8.9 Parametric variance

$$\text{Variance}(X) = \mu'_2 - \mu_1'^2$$

8.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\mu'_3 - 3\mu'_2\mu'_1 + 2\mu_1'^3}{(\mu'_2 - \mu_1'^2)^{1.5}}$$

8.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\mu'_4 - 4\mu'_1\mu'_3 + 6\mu_1'^2\mu'_2 - 3\mu_1'^4}{(\mu'_2 - \mu_1'^2)^2}$$

8.12 Parametric median

$$\text{Median}(X) = A \left[\left(\frac{1}{2}\right)^{-\frac{1}{C}} - 1 \right]^{\frac{1}{B}}$$

8.13 Parametric mode

$$\text{Mode}(X) = A \left(\frac{B-1}{BC+1} \right)^{\frac{1}{B}}$$

8.14 Additional information and definitions

- u : Uniform[0,1] random variable
- Beta (x, y) : Beta function

8.15 Spreadsheet documents

- [Excel file from GitHub repository](#)
- [Google spreadsheet document](#)

9 Burr 4P Distribution

9.1 Distribution definition

$$X \sim \text{Burr}_{4P}(A, B, C, \text{Loc})$$

9.2 Distribution domain

$$x \in [\text{Loc}, \infty)$$

9.3 Parameters domain and parameters constraints

$$A \in \mathbb{R}^+, B \in \mathbb{R}, C \in \mathbb{R}^+, \text{Loc} \in \mathbb{R}$$

9.4 Cumulative distribution function

$$F_X(x) = 1 - \left[1 + \left(\frac{x - \text{Loc}}{A} \right)^B \right]^{-C}$$

9.5 Probability density function

$$f_X(x) = \frac{BC}{A} \left(\frac{x - \text{Loc}}{A} \right)^{B-1} \left[1 + \left(\frac{x - \text{Loc}}{A} \right)^B \right]^{-C-1}$$

9.6 Percent point function/Sample

$$F_X^{-1}(u) = \text{Loc} + A \left[(1 - u)^{-\frac{1}{C}} - 1 \right]^{\frac{1}{B}}$$

9.7 Parametric centered moments

$$\tilde{\mu}'_k = E[\tilde{X}^k] = \int_0^\infty x^k f_{\tilde{X}} = A^k C \times \text{Beta} \left(\frac{BC - k}{B}, \frac{B + K}{B} \right)$$

9.8 Parametric mean

$$\text{Mean}(X) = \text{Loc} + \tilde{\mu}'_1$$

9.9 Parametric variance

$$\text{Variance}(X) = \tilde{\mu}'_2 - \tilde{\mu}'_1^2$$

9.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\tilde{\mu}'_3 - 3\tilde{\mu}'_2\tilde{\mu}'_1 + 2\tilde{\mu}'_1^3}{(\tilde{\mu}'_2 - \tilde{\mu}'_1^2)^{1.5}}$$

9.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\tilde{\mu}'_4 - 4\tilde{\mu}'_1\tilde{\mu}'_3 + 6\tilde{\mu}'_1^2\tilde{\mu}'_2 - 3\tilde{\mu}'_1^4}{(\tilde{\mu}'_2 - \tilde{\mu}'_1^2)^2}$$

9.12 Parametric median

$$\text{Median}(X) = \text{Loc} + A \left[\left(\frac{1}{2} \right)^{-\frac{1}{C}} - 1 \right]^{\frac{1}{B}}$$

9.13 Parametric mode

$$\text{Mode}(X) = \text{Loc} + A \left(\frac{B-1}{BC+1} \right)^{\frac{1}{B}}$$

9.14 Additional information and definitions

- $\tilde{X} \sim \text{Burr}(A, B, C)$
- Loc : Location parameter
- u : Uniform[0,1] random variable
- Beta(x, y) : Beta function

9.15 Spreadsheet documents

- [Excel file from GitHub repository](#)
- [Google spreadsheet document](#)

10 Cauchy Distribution

10.1 Distribution definition

$$X \sim \text{Cauchy}(x_0, \gamma)$$

10.2 Distribution domain

$$x \in (-\infty, +\infty)$$

10.3 Parameters domain and parameters constraints

$$x_0 \in \mathbb{R}, \gamma \in \mathbb{R}^+$$

10.4 Cumulative distribution function

$$F_X(x) = \frac{1}{\pi} \arctan\left(\frac{x - x_0}{\gamma}\right) + \frac{1}{2}$$

10.5 Probability density function

$$f_X(x) = \frac{1}{\pi\gamma \left[1 + \left(\frac{x - x_0}{\gamma}\right)^2\right]}$$

10.6 Percent point function/Sample

$$F_X^{-1}(u) = x_0 + \gamma \tan\left[\pi\left(u - \frac{1}{2}\right)\right]$$

10.7 Parametric centered moments

$$\mu'_k = E[X^k] = \int_{-\infty}^{\infty} x^k f_X(x) dx$$

10.8 Parametric mean

$$\text{Mean}(X) = \text{undefined}$$

10.9 Parametric variance

$$\text{Variance}(X) = \text{undefined}$$

10.10 Parametric skewness

$$\text{Skewness}(X) = \text{undefined}$$

10.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \text{undefined}$$

10.12 Parametric median

$$\text{Median}(X) = x_0$$

10.13 Parametric mode

$$\text{Mode}(X) = x_0$$

10.14 Additional information and definitions

- x_0 : Location parameter
- γ : Scale parameter
- u : Uniform[0,1] random variable

10.15 Spreadsheet documents

- [Excel file from GitHub repository](#)
- [Google spreadsheet document](#)

11 Chi Square Distribution

11.1 Distribution definition

$$X \sim \chi^2(\text{df})$$

11.2 Distribution domain

$$x \in (0, \infty)$$

11.3 Parameters domain and parameters constraints

$$\text{df} \in \mathbb{N}^+$$

11.4 Cumulative distribution function

$$F_X(x) = \frac{\gamma(\frac{\text{df}}{2}, \frac{x}{2})}{\Gamma(\frac{\text{df}}{2})} = \text{P}\left(\frac{\text{df}}{2}, \frac{x}{2}\right)$$

11.5 Probability density function

$$f_X(x) = \frac{1}{2^{\text{df}/2} \Gamma(\text{df}/2)} x^{\text{df}/2-1} e^{-x/2}$$

11.6 Percent point function/Sample

$$F_X^{-1}(u) = 2\text{P}^{-1}\left(\frac{\text{df}}{2}, u\right)$$

11.7 Parametric centered moments

$$\mu'_k = E[X^k] = \int_0^\infty x^k f_X(x) dx = \text{df}(\text{df} + 2) \cdots (\text{df} + 2k - 2) = 2^k \frac{\Gamma(k + \frac{\text{df}}{2})}{\Gamma(\frac{\text{df}}{2})}$$

11.8 Parametric mean

$$\text{Mean}(X) = \mu'_1 = \text{df}$$

11.9 Parametric variance

$$\text{Variance}(X) = \mu'_2 - \mu_1'^2 = 2\text{df}$$

11.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\mu'_3 - 3\mu'_2\mu'_1 + 2\mu_1'^3}{(\mu'_2 - \mu_1'^2)^{1.5}} = \sqrt{\frac{8}{\text{df}}}$$

11.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\mu'_4 - 4\mu'_1\mu'_3 + 6\mu_1'^2\mu'_2 - 3\mu_1'^4}{(\mu'_2 - \mu_1'^2)^2} = 3 + \frac{12}{\text{df}}$$

11.12 Parametric median

$$\text{Median}(X) = 2\text{P}^{-1}\left(\frac{\text{df}}{2}, \frac{1}{2}\right)$$

11.13 Parametric mode

$$\text{Mode}(X) = \max(\text{df} - 2, 0)$$

11.14 Additional information and definitions

- u : Uniform[0,1] random variable
- $P(a, x) = \frac{\gamma(a, x)}{\Gamma(a)}$: Regularized lower incomplete gamma function
- $P^{-1}(a, u)$: Inverse of regularized lower incomplete gamma function
- $\gamma(a, x)$: Lower incomplete gamma function
- $\Gamma(x)$: Gamma function

11.15 Spreadsheet documents

- [Excel file from GitHub repository](#)
- [Google spreadsheet document](#)

12 Chi Square 3P Distribution

12.1 Distribution definition

$$X \sim \chi_{3P}^2(\text{df}, \text{Loc}, \text{Sc})$$

12.2 Distribution domain

$$x \in (\text{Loc}, \infty)$$

12.3 Parameters domain and parameters constraints

$$\text{df} \in \mathbb{N}^+, \text{Loc} \in \mathbb{R}, \text{Sc} \in \mathbb{R}^+$$

12.4 Cumulative distribution function

$$F_X(x) = \frac{\gamma(\frac{\text{df}}{2}, \frac{z(x)}{2})}{\Gamma(\frac{\text{df}}{2})} = \text{P}\left(\frac{\text{df}}{2}, \frac{z(x)}{2}\right)$$

12.5 Probability density function

$$f_X(x) = \frac{1}{\text{Sc}} \frac{1}{2^{\text{df}/2} \Gamma(\text{df}/2)} x^{\text{df}/2-1} e^{-z(x)/2}$$

12.6 Percent point function/Sample

$$F_X^{-1}(u) = 2\text{P}^{-1}\left(\frac{\text{df}}{2}, u\right)$$

12.7 Parametric centered moments

$$\tilde{\mu}'_k = E[\tilde{X}^k] = \int_0^\infty x^k f_{\tilde{X}}(x) dx = \text{df}(\text{df}+2) \cdots (\text{df}+2k-2) = 2^k \frac{\Gamma(k + \frac{\text{df}}{2})}{\Gamma(\frac{\text{df}}{2})}$$

12.8 Parametric mean

$$\text{Mean}(X) = \text{Loc} + \text{Sc} \cdot \tilde{\mu}'_1 = \text{Loc} + \text{Sc} \cdot \text{df}$$

12.9 Parametric variance

$$\text{Variance}(X) = \text{Sc}^2 \cdot (\tilde{\mu}'_2 - \tilde{\mu}'_1^2) = 2 \cdot \text{df} \cdot \text{Sc}^2$$

12.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\tilde{\mu}'_3 - 3\tilde{\mu}'_2\tilde{\mu}'_1 + 2\tilde{\mu}'_1^3}{(\tilde{\mu}'_2 - \tilde{\mu}'_1^2)^{1.5}} = \sqrt{\frac{8}{\text{df}}}$$

12.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\tilde{\mu}'_4 - 4\tilde{\mu}'_1\tilde{\mu}'_3 + 6\tilde{\mu}'_1^2\tilde{\mu}'_2 - 3\tilde{\mu}'_1^4}{(\tilde{\mu}'_2 - \tilde{\mu}'_1^2)^2} = 3 + \frac{12}{\text{df}}$$

12.12 Parametric median

$$\text{Median}(X) = \text{Loc} + \text{Sc} \times 2\text{P}^{-1}\left(\frac{\text{df}}{2}, \frac{1}{2}\right)$$

12.13 Parametric mode

$$\text{Mode}(X) = \text{Loc} + \text{Sc} \times \max(\text{df} - 2, 0)$$

12.14 Additional information and definitions

- $\tilde{X} \sim \chi^2(\text{df})$
- Loc : Location parameter
- Sc : Scale parameter
- $z(x) = (x - \text{Loc}) / \text{Sc}$
- u : Uniform[0,1] random variable
- $P(a, x) = \frac{\gamma(a, x)}{\Gamma(a)}$: Regularized lower incomplete gamma function
- $P^{-1}(a, u)$: Inverse of regularized lower incomplete gamma function
- $\gamma(a, x)$: Lower incomplete gamma function
- $\Gamma(x)$: Gamma function

12.15 Spreadsheet documents

- [Excel file from GitHub repository](#)
- [Google spreadsheet document](#)

13 Dagum Distribution

13.1 Distribution definition

$$X \sim \text{Dagum}(a, b, p)$$

13.2 Distribution domain

$$x \in (0, \infty)$$

13.3 Parameters domain and parameters constraints

$$a \in \mathbb{R}^+, b \in \mathbb{R}^+, p \in \mathbb{R}^+$$

13.4 Cumulative distribution function

$$F_X(x) = \left(1 + \left(\frac{x}{b}\right)^{-a}\right)^{-p}$$

13.5 Probability density function

$$f_X(x) = \frac{ap}{x} \left(\frac{\left(\frac{x}{b}\right)^{ap}}{\left(\left(\frac{x}{b}\right)^a + 1\right)^{p+1}} \right)$$

13.6 Percent point function/Sample

$$F_X^{-1}(u) = b(u^{-1/p} - 1)^{-1/a}$$

13.7 Parametric centered moments

$$\mu'_k = E[X^k] = \int_0^\infty x^k f_X(x) dx = pb^k \cdot \text{Beta}\left(\frac{ap+k}{a}, \frac{a-k}{a}\right)$$

13.8 Parametric mean

$$\text{Mean}(X) = \mu'_1$$

13.9 Parametric variance

$$\text{Variance}(X) = \mu'_2 - \mu_1'^2$$

13.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\mu'_3 - 3\mu'_2\mu'_1 + 2\mu_1'^3}{(\mu'_2 - \mu_1'^2)^{1.5}}$$

13.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\mu'_4 - 4\mu'_1\mu'_3 + 6\mu_1'^2\mu'_2 - 3\mu_1'^4}{(\mu'_2 - \mu_1'^2)^2}$$

13.12 Parametric median

$$\text{Median}(X) = b \left(-1 + 2^{\frac{1}{p}} \right)^{-\frac{1}{a}}$$

13.13 Parametric mode

$$\text{Mode}(X) = b \left(\frac{ap-1}{a+1} \right)^{\frac{1}{a}}$$

13.14 Additional information and definitions

- b : Scale parameter
- u : Uniform[0,1] random variable
- $\text{Beta}(x, y)$: Beta function

13.15 Spreadsheet documents

- [Excel file from GitHub repository](#)
- [Google spreadsheet document](#)

14 Dagum 4P Distribution

14.1 Distribution definition

$$X \sim \text{Dagum}_{4P}(a, b, p, \text{Loc})$$

14.2 Distribution domain

$$x \in (\text{Loc}, \infty)$$

14.3 Parameters domain and parameters constraints

$$a \in \mathbb{R}^+, b \in \mathbb{R}^+, p \in \mathbb{R}^+, \text{Loc} \in \mathbb{R}$$

14.4 Cumulative distribution function

$$F_X(x) = \left(1 + \left(\frac{x - \text{Loc}}{b}\right)^{-a}\right)^{-p}$$

14.5 Probability density function

$$f_X(x) = \frac{ap}{x - \text{Loc}} \left(\frac{\left(\frac{x - \text{Loc}}{b}\right)^{ap}}{\left(\left(\frac{x - \text{Loc}}{b}\right)^a + 1\right)^{p+1}} \right)$$

14.6 Percent point function/Sample

$$F_X^{-1}(u) = \text{Loc} + b(u^{-1/p} - 1)^{-1/a}$$

14.7 Parametric centered moments

$$\tilde{\mu}'_k = E[\tilde{X}^k] = \int_0^\infty x^k f_{\tilde{X}}(x) dx = pb^k \cdot \text{Beta}\left(\frac{ap+k}{a}, \frac{a-k}{a}\right)$$

14.8 Parametric mean

$$\text{Mean}(X) = \text{Loc} + \tilde{\mu}'_1$$

14.9 Parametric variance

$$\text{Variance}(X) = \tilde{\mu}'_2 - \tilde{\mu}'_1^2$$

14.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\tilde{\mu}'_3 - 3\tilde{\mu}'_2\tilde{\mu}'_1 + 2\tilde{\mu}'_1^3}{(\tilde{\mu}'_2 - \tilde{\mu}'_1^2)^{1.5}}$$

14.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\tilde{\mu}'_4 - 4\tilde{\mu}'_1\tilde{\mu}'_3 + 6\tilde{\mu}'_1^2\tilde{\mu}'_2 - 3\tilde{\mu}'_1^4}{(\tilde{\mu}'_2 - \tilde{\mu}'_1^2)^2}$$

14.12 Parametric median

$$\text{Median}(X) = \text{Loc} + b\left(-1 + 2^{\frac{1}{p}}\right)^{-\frac{1}{a}}$$

14.13 Parametric mode

$$\text{Mode}(X) = \text{Loc} + b\left(\frac{ap-1}{a+1}\right)^{\frac{1}{a}}$$

14.14 Additional information and definitions

- $\bar{X} \sim \text{Dagum}(a, b, p)$
- Loc : Location parameter
- b : Scale parameter
- u : Uniform[0,1] random variable
- $\text{Beta}(x, y)$: Beta function

14.15 Spreadsheet documents

- [Excel file from GitHub repository](#)
- [Google spreadsheet document](#)

15 Erlang Distribution

15.1 Distribution definition

$$X \sim \text{Erlang}(k, \beta)$$

15.2 Distribution domain

$$x \in [0, \infty)$$

15.3 Parameters domain and parameters constraints

$$k \in \mathbb{N}^+, \beta \in \mathbb{R}^+$$

15.4 Cumulative distribution function

$$F_X(x) = P(k, \frac{x}{\beta}) = \frac{\gamma(k, \frac{x}{\beta})}{(k-1)!}$$

15.5 Probability density function

$$f_X(x) = \frac{x^{k-1} e^{-\frac{x}{\beta}}}{\beta^k (k-1)!}$$

15.6 Percent point function/Sample

$$F_X^{-1}(u) = \beta P^{-1}(k, u)$$

15.7 Parametric centered moments

$$\mu'_n = E[X^n] = \int_0^\infty x^n f_X(x) dx = \beta^k \frac{\Gamma(n+k)}{\Gamma(k)}$$

15.8 Parametric mean

$$\text{Mean}(X) = \mu'_1$$

15.9 Parametric variance

$$\text{Variance}(X) = \mu'_2 - \mu_1'^2$$

15.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\mu'_3 - 3\mu'_2\mu'_1 + 2\mu_1'^3}{(\mu'_2 - \mu_1'^2)^{1.5}}$$

15.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\mu'_4 - 4\mu'_1\mu'_3 + 6\mu_1'^2\mu'_2 - 3\mu_1'^4}{(\mu'_2 - \mu_1'^2)^2}$$

15.12 Parametric median

$$\text{Median}(X) = P(k, \frac{1}{2\beta})$$

15.13 Parametric mode

$$\text{Mode}(X) = \beta(k-1)$$

15.14 Additional information and definitions

- β : Scale parameter
- u : Uniform[0,1] random variable
- $P(a, x) = \frac{\gamma(a, x)}{\Gamma(a)}$: Regularized lower incomplete gamma function
- $P^{-1}(a, u)$: Inverse of regularized lower incomplete gamma function
- $\gamma(a, x)$: Lower incomplete gamma function
- $\Gamma(x)$: Gamma function

15.15 Spreadsheet documents

- [Excel file from GitHub repository](#)
- [Google spreadsheet document](#)

16 Erlang 3P Distribution

16.1 Distribution definition

$$X \sim \text{Erlang}_{3P}(k, \beta, \text{Loc})$$

16.2 Distribution domain

$$x \in [\text{Loc}, \infty)$$

16.3 Parameters domain and parameters constraints

$$k \in \mathbb{N}^+, \beta \in \mathbb{R}^+, \text{Loc} \in \mathbb{R}$$

16.4 Cumulative distribution function

$$F_X(x) = P(k, \frac{x - \text{Loc}}{\beta}) = \frac{\gamma(k, \frac{x - \text{Loc}}{\beta})}{(k - 1)!}$$

16.5 Probability density function

$$f_X(x) = \frac{(x - \text{Loc})^{k-1} e^{-\frac{x - \text{Loc}}{\beta}}}{\beta^k (k - 1)!}$$

16.6 Percent point function/Sample

$$F_X^{-1}(u) = \text{Loc} + \beta P^{-1}(k, u)$$

16.7 Parametric centered moments

$$\tilde{\mu}'_n = E[\tilde{X}^n] = \int_0^\infty x^n f_{\tilde{X}}(x) dx = \beta^k \frac{\Gamma(n + k)}{\Gamma(k)}$$

16.8 Parametric mean

$$\text{Mean}(X) = \text{Loc} + \tilde{\mu}'_1$$

16.9 Parametric variance

$$\text{Variance}(X) = \tilde{\mu}'_2 - \tilde{\mu}'_1^2$$

16.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\tilde{\mu}'_3 - 3\tilde{\mu}'_2\tilde{\mu}'_1 + 2\tilde{\mu}'_1^3}{(\tilde{\mu}'_2 - \tilde{\mu}'_1^2)^{1.5}}$$

16.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\tilde{\mu}'_4 - 4\tilde{\mu}'_1\tilde{\mu}'_3 + 6\tilde{\mu}'_1^2\tilde{\mu}'_2 - 3\tilde{\mu}'_1^4}{(\tilde{\mu}'_2 - \tilde{\mu}'_1^2)^2}$$

16.12 Parametric median

$$\text{Median}(X) = \text{Loc} + P(k, \frac{1}{2\beta})$$

16.13 Parametric mode

$$\text{Mode}(X) = \text{Loc} + \beta \cdot (k - 1)$$

16.14 Additional information and definitions

- $\tilde{X} \sim \text{Erlang}(k, \beta)$
- Loc : Location parameter
- β : Scale parameter
- u : Uniform[0,1] random variable
- $P(a, x) = \frac{\gamma(a, x)}{\Gamma(a)}$: Regularized lower incomplete gamma function
- $P^{-1}(a, u)$: Inverse of regularized lower incomplete gamma function
- $\gamma(a, x)$: Lower incomplete gamma function
- $\Gamma(x)$: Gamma function

16.15 Spreadsheet documents

- [Excel file from GitHub repository](#)
- [Google spreadsheet document](#)

17 Error Function Distribution

17.1 Distribution definition

$$X \sim \text{ErrorFunction}(h)$$

17.2 Distribution domain

$$x \in (-\infty, \infty)$$

17.3 Parameters domain and parameters constraints

$$h \in \mathbb{R}^+$$

17.4 Cumulative distribution function

$$F_X(x) = \Phi(\sqrt{2}hx)$$

17.5 Probability density function

$$f_X(x) = \frac{h}{\sqrt{\pi}} e^{-h^2 x^2}$$

17.6 Percent point function/Sample

$$F_X^{-1}(u) = \frac{\Phi^{-1}(u)}{\sqrt{2}h}$$

17.7 Parametric centered moments

$$\mu'_k = E[X^k] = \int_{-\infty}^{\infty} x^k f_X(x) dx$$

17.8 Parametric mean

$$\text{Mean}(X) = \mu'_1 = 0$$

17.9 Parametric variance

$$\text{Variance}(X) = \mu'_2 - \mu'^2_1 = \frac{1}{2h^2}$$

17.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\mu'_3 - 3\mu'_2\mu'_1 + 2\mu'^3_1}{(\mu'_2 - \mu'^2_1)^{1.5}} = 0$$

17.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\mu'_4 - 4\mu'_1\mu'_3 + 6\mu'^2_1\mu'_2 - 3\mu'^4_1}{(\mu'_2 - \mu'^2_1)^2} = 3$$

17.12 Parametric median

$$\text{Median}(X) = 0$$

17.13 Parametric mode

$$\text{Mode}(X) = 0$$

17.14 Additional information and definitions

- h : Inverse of scale parameter
- u : Uniform[0,1] random variable
- $\Phi(x)$: CDF normal standard distribution

17.15 Spreadsheet documents

- [Excel file from GitHub repository](#)
- [Google spreadsheet document](#)

18 Exponential Distribution

18.1 Distribution definition

$$X \sim \text{Exponential}(\lambda)$$

18.2 Distribution domain

$$x \in [0, \infty)$$

18.3 Parameters domain and parameters constraints

$$\lambda \in \mathbb{R}^+$$

18.4 Cumulative distribution function

$$F_X(x) = 1 - e^{-\lambda x}$$

18.5 Probability density function

$$f_X(x) = \lambda e^{-\lambda x}$$

18.6 Percent point function/Sample

$$F_X^{-1}(u) = -\frac{\ln(1-u)}{\lambda}$$

18.7 Parametric centered moments

$$\mu'_k = E[X^k] = \int_0^\infty x^k f_X(x) dx = \frac{k!}{\lambda^k}$$

18.8 Parametric mean

$$\text{Mean}(X) = \mu'_1 = \frac{1}{\lambda}$$

18.9 Parametric variance

$$\text{Variance}(X) = \mu'_2 - \mu_1'^2 = \frac{1}{\lambda^2}$$

18.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\mu'_3 - 3\mu'_2\mu'_1 + 2\mu_1'^3}{(\mu'_2 - \mu_1'^2)^{1.5}} = 2$$

18.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\mu'_4 - 4\mu'_1\mu'_3 + 6\mu_1'^2\mu'_2 - 3\mu_1'^4}{(\mu'_2 - \mu_1'^2)^2} = 9$$

18.12 Parametric median

$$\text{Median}(X) = \frac{\ln 2}{\lambda}$$

18.13 Parametric mode

$$\text{Mode}(X) = 0$$

18.14 Additional information and definitions

- λ : Inverse of scale parameter
- u : Uniform[0,1] random variable

18.15 Spreadsheet documents

- [Excel file from GitHub repository](#)
- [Google spreadsheet document](#)

19 Exponential 2P Distribution

19.1 Distribution definition

$$X \sim \text{Exponential}_{2P}(\lambda, \text{Loc})$$

19.2 Distribution domain

$$x \in [\text{Loc}, \infty)$$

19.3 Parameters domain and parameters constraints

$$\lambda \in \mathbb{R}^+, \text{Loc} \in \mathbb{R}$$

19.4 Cumulative distribution function

$$F_X(x) = 1 - e^{-\lambda(x-\text{Loc})}$$

19.5 Probability density function

$$f_X(x) = \lambda e^{-\lambda(x-\text{Loc})}$$

19.6 Percent point function/Sample

$$F_X^{-1}(u) = \text{Loc} - \frac{\ln(1-u)}{\lambda}$$

19.7 Parametric centered moments

$$\tilde{\mu}'_k = E[\tilde{X}^k] = \int_0^\infty x^k f_{\tilde{X}}(x) dx = \frac{k!}{\lambda^k}$$

19.8 Parametric mean

$$\text{Mean}(X) = \text{Loc} + \tilde{\mu}'_1 = \text{Loc} + \frac{1}{\lambda}$$

19.9 Parametric variance

$$\text{Variance}(X) = \tilde{\mu}'_2 - \tilde{\mu}'_1^2 = \frac{1}{\lambda^2}$$

19.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\tilde{\mu}'_3 - 3\tilde{\mu}'_2\tilde{\mu}'_1 + 2\tilde{\mu}'_1^3}{(\tilde{\mu}'_2 - \tilde{\mu}'_1^2)^{1.5}} = 2$$

19.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\tilde{\mu}'_4 - 4\tilde{\mu}'_1\tilde{\mu}'_3 + 6\tilde{\mu}'_1^2\tilde{\mu}'_2 - 3\tilde{\mu}'_1^4}{(\tilde{\mu}'_2 - \tilde{\mu}'_1^2)^2} = 9$$

19.12 Parametric median

$$\text{Median}(X) = \text{Loc} + \frac{\ln 2}{\lambda}$$

19.13 Parametric mode

$$\text{Mode}(X) = \text{Loc}$$

19.14 Additional information and definitions

- $\tilde{X} \sim \text{Exponential}(\lambda)$
- Loc : Location parameter
- λ : Inverse of scale parameter
- u : Uniform[0,1] random variable

19.15 Spreadsheet documents

- [Excel file from GitHub repository](#)
- [Google spreadsheet document](#)

20 F Distribution

20.1 Distribution definition

$$X \sim F(df_1, df_2)$$

20.2 Distribution domain

$$x \in [0, \infty)$$

20.3 Parameters domain and parameters constraints

$$df_1 \in \mathbb{R}^+, df_2 \in \mathbb{R}^+$$

20.4 Cumulative distribution function

$$F_X(x) = I_{df_1 x / (df_1 x + df_2)}\left(\frac{df_1}{2}, \frac{df_2}{2}\right)$$

20.5 Probability density function

$$f_X(x) = \frac{\sqrt{\frac{(df_1 x)^{df_1} df_2^{df_2}}{(df_1 x + df_2)^{df_1 + df_2}}}}{x \times \text{Beta}\left(\frac{df_1}{2}, \frac{df_2}{2}\right)}$$

20.6 Percent point function/Sample

$$F_X^{-1}(u) = \frac{df_2 \times I^{-1}\left(u, \frac{df_1}{2}, \frac{df_2}{2}\right)}{df_1 \times \left(1 - I^{-1}\left(u, \frac{df_1}{2}, \frac{df_2}{2}\right)\right)}$$

20.7 Parametric centered moments

$$\mu'_k = E[X^k] = \int_0^\infty x^k f_X(x) dx = \left(\frac{df_2}{df_1}\right)^k \frac{\Gamma\left(\frac{df_1}{2} + k\right) \Gamma\left(\frac{df_2}{2} - k\right)}{\Gamma\left(\frac{df_1}{2}\right) \Gamma\left(\frac{df_2}{2}\right)} \quad \text{if } df_2 > 2k$$

20.8 Parametric mean

$$\text{Mean}(X) = \mu'_1 = \frac{df_2}{df_2 - 2} \quad \text{if } df_2 > 2$$

20.9 Parametric variance

$$\text{Variance}(X) = \mu'_2 - \mu_1'^2 = \frac{2 df_2^2 (df_1 + df_2 - 2)}{df_1 (df_2 - 2)^2 (df_2 - 4)} \quad \text{if } df_2 > 4$$

20.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\mu'_3 - 3\mu'_2\mu'_1 + 2\mu_1'^3}{(\mu'_2 - \mu_1'^2)^{1.5}} = \frac{(2df_1 + df_2 - 2)\sqrt{8(df_2 - 4)}}{(df_2 - 6)\sqrt{df_1(df_1 + df_2 - 2)}} \quad \text{if } df_2 > 6$$

20.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\mu'_4 - 4\mu'_1\mu'_3 + 6\mu_1'^2\mu'_2 - 3\mu_1'^4}{(\mu'_2 - \mu_1'^2)^2} = \frac{3(8 + (df_2 - 6) \times \text{Skewness}(X)^2)}{2df_2 - 16} + 3 \quad \text{if } df_2 > 8$$

20.12 Parametric median

$$\text{Median}(X) = \frac{df_2 \times I^{-1}\left(\frac{1}{2}, \frac{df_1}{2}, \frac{df_2}{2}\right)}{df_1 \times \left(1 - I^{-1}\left(\frac{1}{2}, \frac{df_1}{2}, \frac{df_2}{2}\right)\right)}$$

20.13 Parametric mode

$$\text{Mode}(X) = \frac{\text{df}_2 (\text{df}_1 - 2)}{\text{df}_1 (\text{df}_2 + 2)} \quad \text{if } \text{df}_1 > 2$$

20.14 Additional information and definitions

- u : Uniform[0,1] random variable
- $I(x, a, b)$: Regularized incomplete beta function
- $I^{-1}(x, a, b)$: Inverse of regularized incomplete beta function
- Beta (x, y) : Beta function

20.15 Spreadsheet documents

- [Excel file from GitHub repository](#)
- [Google spreadsheet document](#)

21 F 4P Distribution

21.1 Distribution definition

$$X \sim F_{4P}(\text{df}_1, \text{df}_2, \text{Loc}, \text{Sc})$$

21.2 Distribution domain

$$x \in [\text{Loc}, \infty)$$

21.3 Parameters domain and parameters constraints

$$\text{df}_1 \in \mathbb{R}^+, \text{df}_2 \in \mathbb{R}^+, \text{Loc} \in \mathbb{R}, \text{Sc} \in \mathbb{R}^+$$

21.4 Cumulative distribution function

$$F_X(x) = I_{\text{df}_1 z(x)/(\text{df}_1 z(x) + \text{df}_2)}\left(\frac{\text{df}_1}{2}, \frac{\text{df}_2}{2}\right)$$

21.5 Probability density function

$$f_X(x) = \frac{1}{\text{Sc}} \times \frac{\sqrt{\frac{(\text{df}_1 z(x))^{\text{df}_1} \text{df}_2^{\text{df}_2}}{(\text{df}_1 z(x) + \text{df}_2)^{\text{df}_1 + \text{df}_2}}}}{z(x) \text{Beta}\left(\frac{\text{df}_1}{2}, \frac{\text{df}_2}{2}\right)}$$

21.6 Percent point function/Sample

$$F_X^{-1}(u) = \text{Loc} + \text{Sc} \frac{\text{df}_2 \times I^{-1}\left(u, \frac{\text{df}_1}{2}, \frac{\text{df}_2}{2}\right)}{\text{df}_1 \times \left(1 - I^{-1}\left(u, \frac{\text{df}_1}{2}, \frac{\text{df}_2}{2}\right)\right)}$$

21.7 Parametric centered moments

$$\tilde{\mu}'_k = E[\tilde{X}^k] = \int_0^\infty x^k f_{\tilde{X}}(x) dx = \frac{\Gamma\left(\frac{\text{df}_1}{2} + k\right)}{\Gamma\left(\frac{\text{df}_1}{2}\right)} \frac{\Gamma\left(\frac{\text{df}_2}{2} - k\right)}{\Gamma\left(\frac{\text{df}_2}{2}\right)} \left(\frac{\text{df}_2}{\text{df}_1}\right)^k \quad \text{if } \text{df}_2 > 2k$$

21.8 Parametric mean

$$\text{Mean}(X) = \text{Loc} + \text{Sc} \tilde{\mu}'_1 = \text{Loc} + \text{Sc} \frac{\text{df}_2}{\text{df}_2 - 2} \quad \text{if } \text{df}_2 > 2$$

21.9 Parametric variance

$$\text{Variance}(X) = \text{Sc}^2(\tilde{\mu}'_2 - \tilde{\mu}'_1{}^2) = \text{Sc}^2 \frac{2 \text{df}_2^2 (\text{df}_1 + \text{df}_2 - 2)}{\text{df}_1 (\text{df}_2 - 2)^2 (\text{df}_2 - 4)} \quad \text{if } \text{df}_2 > 4$$

21.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\tilde{\mu}'_3 - 3\tilde{\mu}'_2\tilde{\mu}'_1 + 2\tilde{\mu}'_1{}^3}{(\tilde{\mu}'_2 - \tilde{\mu}'_1{}^2)^{1.5}} = \frac{(2\text{df}_1 + \text{df}_2 - 2)\sqrt{8(\text{df}_2 - 4)}}{(\text{df}_2 - 6)\sqrt{\text{df}_1(\text{df}_1 + \text{df}_2 - 2)}} \quad \text{if } \text{df}_2 > 6$$

21.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\tilde{\mu}'_4 - 4\tilde{\mu}'_1\tilde{\mu}'_3 + 6\tilde{\mu}'_1{}^2\tilde{\mu}'_2 - 3\tilde{\mu}'_1{}^4}{(\tilde{\mu}'_2 - \tilde{\mu}'_1{}^2)^2} = \frac{3(8 + (\text{df}_2 - 6) \times \text{Skewness}(X)^2)}{2\text{df}_2 - 16} + 3 \quad \text{if } \text{df}_2 > 8$$

21.12 Parametric median

$$\text{Median}(X) = \text{Loc} + \text{Sc} \frac{\text{df}_2 \times I^{-1}\left(\frac{1}{2}, \frac{\text{df}_1}{2}, \frac{\text{df}_2}{2}\right)}{\text{df}_1 \times \left(1 - I^{-1}\left(\frac{1}{2}, \frac{\text{df}_1}{2}, \frac{\text{df}_2}{2}\right)\right)}$$

21.13 Parametric mode

$$\text{Mode}(X) = \text{Loc} + \text{Sc} \frac{\text{df}_2 (\text{df}_1 - 2)}{\text{df}_1 (\text{df}_2 + 2)} \quad \text{if } \text{df}_1 > 2$$

21.14 Additional information and definitions

- $\tilde{X} \sim F(\text{df}_1, \text{df}_2)$
- Loc : Location parameter
- Sc : Scale parameter
- $z(x) = (x - \text{Loc}) / \text{Sc}$
- u : Uniform[0,1] random variable
- $I(x, a, b)$: Regularized incomplete beta function
- $I^{-1}(x, a, b)$: Inverse of regularized incomplete beta function
- Beta(x, y) : Beta function

21.15 Spreadsheet documents

- [Excel file from GitHub repository](#)
- [Google spreadsheet document](#)

22 Fatigue Life Distribution

22.1 Distribution definition

$$X \sim \text{FatigueLife}(\gamma, \text{Loc}, \text{Sc})$$

22.2 Distribution domain

$$x \in (\text{Loc}, \infty)$$

22.3 Parameters domain and parameters constraints

$$\gamma \in \mathbb{R}^+, \text{Loc} \in \mathbb{R}, \text{Sc} \in \mathbb{R}^+$$

22.4 Cumulative distribution function

$$F_X(x) = \Phi\left(\frac{\sqrt{z(x)} - \sqrt{\frac{1}{z(x)}}}{\gamma}\right)$$

22.5 Probability density function

$$f_X(x) = \frac{\sqrt{z(x)} + \sqrt{\frac{1}{z(x)}}}{2\gamma z(x)} \phi\left(\frac{\sqrt{z(x)} - \sqrt{\frac{1}{z(x)}}}{\gamma}\right)$$

22.6 Percent point function/Sample

$$F_X^{-1}(u) = \text{Loc} + \text{Sc} \frac{1}{4} \left[\gamma \Phi^{-1}(u) + \sqrt{4 + (\gamma \Phi^{-1}(u))^2} \right]^2$$

22.7 Parametric centered moments

$$\mu'_k = E[X^k] = \int_{-\infty}^{\infty} x^k f_X(x) dx$$

22.8 Parametric mean

$$\text{Mean}(X) = \text{Loc} + \text{Sc} \cdot \tilde{\mu}'_1 = \text{Loc} + \text{Sc} \left(1 + \frac{\gamma^2}{2}\right)$$

22.9 Parametric variance

$$\text{Variance}(X) = \text{Sc}^2 \cdot (\tilde{\mu}'_2 - \tilde{\mu}'_1{}^2) = \text{Sc}^2 \gamma^2 \left(1 + \frac{5\gamma^2}{4}\right)$$

22.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\mu'_3 - 3\mu'_2\mu'_1 + 2\mu'_1{}^3}{(\mu'_2 - \mu'_1{}^2)^{1.5}} = \frac{4\gamma(6 + 11\gamma^2)}{(4 + 5\gamma^2)^{1.5}}$$

22.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\mu'_4 - 4\mu'_1\mu'_3 + 6\mu'_1{}^2\mu'_2 - 3\mu'_1{}^4}{(\mu'_2 - \mu'_1{}^2)^2} = 3 + \frac{6\gamma^2(93\gamma^2 + 40)}{(5\gamma^2 + 4)^2}$$

22.12 Parametric median

$$\text{Median}(X) = \text{Loc} + \text{Sc} \frac{1}{4} \left[\gamma \Phi^{-1}(1/2) + \sqrt{4 + (\gamma \Phi^{-1}(1/2))^2} \right]^2$$

22.13 Parametric mode

$$\text{Mode}(X) = \arg \max_x f_X(x)$$

22.14 Additional information and definitions

- Loc : Location parameter
- Sc : Scale parameter
- $z(x) = (x - \text{Loc}) / \text{Sc}$
- u : Uniform[0,1] random variable
- $\Phi(x)$: CDF normal standard distribution
- $\phi(x)$: PDF normal standard distribution

22.15 Spreadsheet documents

- [Excel file from GitHub repository](#)
- [Google spreadsheet document](#)

23 Folded Normal Distribution

23.1 Distribution definition

$$X \sim \text{FoldedNormal}(\mu, \sigma)$$

23.2 Distribution domain

$$x \in [0, \infty)$$

23.3 Parameters domain and parameters constraints

$$\mu \in \mathbb{R}, \sigma \in \mathbb{R}^+$$

23.4 Cumulative distribution function

$$F_X(x) = \frac{1}{2} \left[\operatorname{erf}\left(\frac{x+\mu}{\sigma\sqrt{2}}\right) + \operatorname{erf}\left(\frac{x-\mu}{\sigma\sqrt{2}}\right) \right]$$

23.5 Probability density function

$$f_X(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} + \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x+\mu)^2}{2\sigma^2}}$$

23.6 Percent point function/Sample

$$\text{Sample}_X(u) = |\mu + \sigma\Phi^{-1}(u)|$$

23.7 Parametric centered moments

$$\mu'_k = E[X^k] = \int_0^\infty x^k f_X(x) dx$$

23.8 Parametric mean

$$\text{Mean}(X) = \mu'_1 = \sigma\sqrt{\frac{2}{\pi}} e^{(-\mu^2/2\sigma^2)} + \mu(1 - 2\Phi(-\frac{\mu}{\sigma}))$$

23.9 Parametric variance

$$\text{Variance}(X) = \mu'_2 - \mu'^2_1 = \mu^2 + \sigma^2 - \text{Mean}(X)^2$$

23.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\mu'_3 - 3\mu'_2\mu'_1 + 2\mu'^3_1}{(\mu'_2 - \mu'^2_1)^{1.5}}$$

23.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\mu'_4 - 4\mu'_1\mu'^2_3 + 6\mu'^2_1\mu'_2 - 3\mu'^4_1}{(\mu'_2 - \mu'^2_1)^2}$$

23.12 Parametric median

$$\text{Median}(X) = |\mu + \sigma\Phi^{-1}(1/2)|$$

23.13 Parametric mode

$$\text{Mode}(X) = \arg \max_x f_X(x)$$

23.14 Additional information and definitions

- Computing an analytic expression for the inverse of the cumulative distribution function is not feasible. Nonetheless, it is possible to generate a random sample from the distribution.
- μ : Location parameter
- σ : Scale parameter
- u : Uniform[0,1] random variable
- $\Phi(x)$: CDF normal standard distribution
- $\phi(x)$: PDF normal standard distribution
- $\Phi^{-1}(x)$: PPF normal standard distribution

23.15 Spreadsheet documents

- [Excel file from GitHub repository](#)
- [Google spreadsheet document](#)

24 Frechet Distribution

24.1 Distribution definition

$$X \sim \text{Frechet}(\alpha, \text{Loc}, \text{Sc})$$

24.2 Distribution domain

$$x \in [\text{Loc}, \infty)$$

24.3 Parameters domain and parameters constraints

$$\alpha \in \mathbb{R}^+, \text{Loc} \in \mathbb{R}, \text{Sc} \in \mathbb{R}^+$$

24.4 Cumulative distribution function

$$F_X(x) = e^{(-z(x))^{-\alpha}}$$

24.5 Probability density function

$$f_X(x) = \frac{\alpha}{\text{Sc}} (z(x))^{-1-\alpha} e^{-(z(x))^{-\alpha}}$$

24.6 Percent point function/Sample

$$F_X^{-1}(u) = \text{Loc} + \text{Sc} (-\ln(u))^{-\frac{1}{\alpha}}$$

24.7 Parametric centered moments

$$\mu'_k = E[X^k] = \int_{\text{Loc}}^{\infty} x^k f_X(x) dx = \Gamma\left(1 - \frac{k}{\alpha}\right)$$

24.8 Parametric mean

$$\text{Mean}(X) = \text{Loc} + \text{Sc} \cdot \tilde{\mu}'_1 \quad \text{if } \alpha > 1$$

24.9 Parametric variance

$$\text{Variance}(X) = \text{Sc}^2 \cdot (\tilde{\mu}'_2 - \tilde{\mu}'_1{}^2) \quad \text{if } \alpha > 2$$

24.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\mu'_3 - 3\mu'_2\mu'_1 + 2\mu'^3_1}{(\mu'_2 - \mu'^2_1)^{1.5}} \quad \text{if } \alpha > 3$$

24.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\mu'_4 - 4\mu'_1\mu'_3 + 6\mu'^2_1\mu'_2 - 3\mu'^4_1}{(\mu'_2 - \mu'^2_1)^2} \quad \text{if } \alpha > 4$$

24.12 Parametric median

$$\text{Median}(X) = \text{Loc} + \frac{\text{Sc}}{\sqrt[\alpha]{\ln(2)}}$$

24.13 Parametric mode

$$\text{Mode}(X) = \text{Loc} + \text{Sc} \left(\frac{\alpha}{1 + \alpha} \right)^{1/\alpha}$$

24.14 Additional information and definitions

- Loc : Location parameter
- Sc : Scale parameter
- $z(x) = (x - \text{Loc}) / \text{Sc}$
- u : Uniform[0,1] random variable
- $\Gamma(x)$: Gamma function

24.15 Spreadsheet documents

- [Excel file from GitHub repository](#)
- [Google spreadsheet document](#)

25 Gamma Distribution

25.1 Distribution definition

$$X \sim \text{Gamma}(\alpha, \beta)$$

25.2 Distribution domain

$$x \in (0, \infty)$$

25.3 Parameters domain and parameters constraints

$$\alpha \in \mathbb{R}^+, \beta \in \mathbb{R}^+$$

25.4 Cumulative distribution function

$$F_X(x) = P\left(\alpha, \frac{x}{\beta}\right) = \frac{1}{\Gamma(\alpha)} \gamma\left(\alpha, \frac{x}{\beta}\right)$$

25.5 Probability density function

$$f_X(x) = \frac{1}{\Gamma(\alpha)\beta^\alpha} x^{\alpha-1} e^{-\frac{x}{\beta}}$$

25.6 Percent point function/Sample

$$F_X^{-1}(u) = \beta P^{-1}(\alpha, u)$$

25.7 Parametric centered moments

$$\mu'_k = E[X^k] = \int_0^\infty x^k f_X(x) dx = \beta^k \frac{\Gamma(k + \alpha)}{\Gamma(\alpha)}$$

25.8 Parametric mean

$$\text{Mean}(X) = \mu'_1 = \alpha\beta$$

25.9 Parametric variance

$$\text{Variance}(X) = \mu'_2 - \mu_1'^2 = \alpha\beta^2$$

25.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\mu'_3 - 3\mu'_2\mu'_1 + 2\mu_1'^3}{(\mu'_2 - \mu_1'^2)^{1.5}} = \frac{2}{\sqrt{\alpha}}$$

25.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\mu'_4 - 4\mu'_1\mu'_3 + 6\mu_1'^2\mu'_2 - 3\mu_1'^4}{(\mu'_2 - \mu_1'^2)^2} = 3 + \frac{6}{\alpha}$$

25.12 Parametric median

$$\text{Median}(X) = (\alpha - 1)\beta \quad \text{if } \alpha > 1$$

25.13 Parametric mode

$$\text{Mode}(X) = \beta P^{-1}\left(\alpha, \frac{1}{2}\right)$$

25.14 Additional information and definitions

- β : Scale parameter
- u : Uniform[0,1] random variable
- $P(a, x) = \frac{\gamma(a, x)}{\Gamma(a)}$: Regularized lower incomplete gamma function
- $P^{-1}(a, u)$: Inverse of regularized lower incomplete gamma function
- $\gamma(a, x)$: Lower incomplete gamma function
- $\Gamma(x)$: Gamma function

25.15 Spreadsheet documents

- [Excel file from GitHub repository](#)
- [Google spreadsheet document](#)

26 Gamma 3P Distribution

26.1 Distribution definition

$$X \sim \text{Gamma}_{3P}(\alpha, \text{Loc}, \beta)$$

26.2 Distribution domain

$$x \in (\text{Loc}, \infty)$$

26.3 Parameters domain and parameters constraints

$$\alpha \in \mathbb{R}^+, \text{Loc} \in \mathbb{R}, \beta \in \mathbb{R}^+$$

26.4 Cumulative distribution function

$$F_X(x) = P\left(\alpha, \frac{x - \text{Loc}}{\beta}\right) = \frac{1}{\Gamma(\alpha)} \gamma\left(\alpha, \frac{x - \text{Loc}}{\beta}\right)$$

26.5 Probability density function

$$f_X(x) = \frac{1}{\Gamma(\alpha)\beta^\alpha} (x - \text{Loc})^{\alpha-1} e^{-\frac{x - \text{Loc}}{\beta}}$$

26.6 Percent point function/Sample

$$F_X^{-1}(u) = \text{Loc} + \beta P^{-1}(\alpha, u)$$

26.7 Parametric centered moments

$$\tilde{\mu}'_k = E[\tilde{X}^k] = \int_0^\infty x^k f_{\tilde{X}}(x) dx = \beta^k \frac{\Gamma(k + \alpha)}{\Gamma(\alpha)}$$

26.8 Parametric mean

$$\text{Mean}(X) = \text{Loc} + \tilde{\mu}'_1 = \text{Loc} + \alpha\beta$$

26.9 Parametric variance

$$\text{Variance}(X) = \tilde{\mu}'_2 - \tilde{\mu}'_1{}^2 = \alpha\beta^2$$

26.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\tilde{\mu}'_3 - 3\tilde{\mu}'_2\tilde{\mu}'_1 + 2\tilde{\mu}'_1{}^3}{(\tilde{\mu}'_2 - \tilde{\mu}'_1{}^2)^{1.5}} = \frac{2}{\sqrt{\alpha}}$$

26.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\tilde{\mu}'_4 - 4\tilde{\mu}'_1\tilde{\mu}'_3 + 6\tilde{\mu}'_1{}^2\tilde{\mu}'_2 - 3\tilde{\mu}'_1{}^4}{(\tilde{\mu}'_2 - \tilde{\mu}'_1{}^2)^2} = 3 + \frac{6}{\alpha}$$

26.12 Parametric median

$$\text{Median}(X) = \text{Loc} + (\alpha - 1)\beta \quad \text{if } \alpha > 1$$

26.13 Parametric mode

$$\text{Mode}(X) = \text{Loc} + \beta P^{-1}\left(\alpha, \frac{1}{2}\right)$$

26.14 Additional information and definitions

- $\tilde{X} \sim \text{Gamma}(\alpha, \beta)$
- Loc : Location parameter
- β : Scale parameter
- u : Uniform[0,1] random variable
- $P(a, x) = \frac{\gamma(a, x)}{\Gamma(a)}$: Regularized lower incomplete gamma function
- $P^{-1}(a, u)$: Inverse of regularized lower incomplete gamma function
- $\gamma(a, x)$: Lower incomplete gamma function
- $\Gamma(x)$: Gamma function

26.15 Spreadsheet documents

- [Excel file from GitHub repository](#)
- [Google spreadsheet document](#)

27 Generalized Extreme Value Distribution

27.1 Distribution definition

$$X \sim \text{GeneralizedExtremeValue}(\xi, \mu, \sigma)$$

27.2 Distribution domain

$$\text{if } \xi > 0 : x \in (z(x), \infty), \quad \text{if } \xi = 0 : x \in (-\infty, \infty), \quad \text{if } \xi < 0 : x \in (-\infty, z(x))$$

27.3 Parameters domain and parameters constraints

$$\xi \in \mathbb{R}, \mu \in \mathbb{R}, \sigma \in \mathbb{R}^+$$

27.4 Cumulative distribution function

$$F_X(x) = \begin{cases} \exp(-\exp(-z(x))) & \text{if } \xi = 0 \\ \exp(-(1 + \xi z(x))^{-1/\xi}) & \text{if } \xi \neq 0 \end{cases}$$

27.5 Probability density function

$$f_X(x) = \begin{cases} \frac{1}{\sigma} \exp(-z(x)) \exp(-\exp(-z(x))) & \text{if } \xi = 0 \\ \frac{1}{\sigma} (1 + \xi z(x))^{-(1+1/\xi)} \exp(-(1 + \xi z(x))^{-1/\xi}) & \text{if } \xi \neq 0 \end{cases}$$

27.6 Percent point function/Sample

$$F_X^{-1}(u) = \begin{cases} \mu - \sigma \ln(-\ln(u)) & \text{if } \xi = 0 \\ \mu + \frac{\sigma}{\xi} \left((-\ln(u))^{-\xi} - 1 \right) & \text{if } \xi \neq 0 \end{cases}$$

27.7 Parametric centered moments

$$\mu'_k = E[X^k] = \int_{-\infty}^{\infty} x^k f_X(x) dx = \Gamma(1 - k\xi)$$

27.8 Parametric mean

$$\text{Mean}(X) = \begin{cases} \mu + \sigma(\mu'_1 - 1)/\xi & \text{if } \xi \neq 0, \xi < 1 \\ \mu + \sigma \gamma & \text{if } \xi = 0 \end{cases}$$

27.9 Parametric variance

$$\text{Variance}(X) = \begin{cases} \sigma^2 (\mu'_2 - \mu'^2_1)/\xi^2 & \text{if } \xi \neq 0, \xi < \frac{1}{2} \\ \sigma^2 \frac{\pi^2}{6} & \text{if } \xi = 0 \end{cases}$$

27.10 Parametric skewness

$$\text{Skewness}(X) = \begin{cases} \text{sign}(\xi) \frac{\mu'_3 - 3\mu'_2\mu'_1 + 2\mu'^3_1}{(\mu'_2 - \mu'^2_1)^{1.5}} & \text{if } \xi \neq 0, \xi < \frac{1}{3} \\ \frac{12\sqrt{6}\zeta(3)}{\pi^3} & \text{if } \xi = 0 \end{cases}$$

27.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \begin{cases} 3 + \frac{\mu'_4 - 4\mu'_3\mu'_1 + 6\mu'^2_2 - 3\mu'^4_1}{(\mu'_2 - \mu'^2_1)^2} & \text{if } \xi \neq 0, \xi < \frac{1}{4} \\ 3 + \frac{12}{5} & \text{if } \xi = 0 \end{cases}$$

27.12 Parametric median

$$\text{Median}(X) = \begin{cases} \mu + \sigma \frac{(\ln 2)^{-\xi} - 1}{\xi} & \text{if } \xi \neq 0 \\ \mu - \sigma \ln \ln 2 & \text{if } \xi = 0 \end{cases}$$

27.13 Parametric mode

$$\text{Mode}(X) = \begin{cases} \mu + \sigma \frac{(1+\xi)^{-\xi} - 1}{\xi} & \text{if } \xi \neq 0 \\ \mu & \text{if } \xi = 0 \end{cases}$$

27.14 Additional information and definitions

- μ : Location parameter
- σ : Scale parameter
- $z(x) = (x - \mu) / \sigma$
- u : Uniform[0,1] random variable
- $\Gamma(x)$: Gamma function
- γ : Euler-Mascheroni constant = 0.5772156649

27.15 Spreadsheet documents

- [Excel file from GitHub repository](#)
- [Google spreadsheet document](#)

28 Generalized Gamma Distribution

28.1 Distribution definition

$$X \sim \text{GeneralizedGamma}(a, d, p)$$

28.2 Distribution domain

$$x \in (0, \infty)$$

28.3 Parameters domain and parameters constraints

$$a \in \mathbb{R}^+, d \in \mathbb{R}^+, p \in \mathbb{R}^+$$

28.4 Cumulative distribution function

$$F_X(x) = P(d/p, (x/a)^p) = \frac{\gamma(d/p, (x/a)^p)}{\Gamma(d/p)}$$

28.5 Probability density function

$$f_X(x) = \frac{p/a^d}{\Gamma(d/p)} x^{d-1} e^{-(x/a)^p}$$

28.6 Percent point function/Sample

$$F_X^{-1}(u) = aP^{-1}\left(\frac{d}{p}, u\right)^{\frac{1}{p}}$$

28.7 Parametric centered moments

$$\mu'_k = E[X^k] = \int_0^\infty x^k f_X(x) dx = a^k \frac{\Gamma(\frac{d+k}{p})}{\Gamma(\frac{d}{p})}$$

28.8 Parametric mean

$$\text{Mean}(X) = \mu'_1$$

28.9 Parametric variance

$$\text{Variance}(X) = \mu'_2 - \mu_1'^2$$

28.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\mu'_3 - 3\mu'_2\mu'_1 + 2\mu_1'^3}{(\mu'_2 - \mu_1'^2)^{1.5}}$$

28.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\mu'_4 - 4\mu'_1\mu'_3 + 6\mu_1'^2\mu'_2 - 3\mu_1'^4}{(\mu'_2 - \mu_1'^2)^2}$$

28.12 Parametric median

$$\text{Median}(X) = aP^{-1}\left(\frac{d}{p}, \frac{1}{2}\right)^{\frac{1}{p}}$$

28.13 Parametric mode

$$\text{Mode}(X) = a\left(\frac{d-1}{p}\right)^{\frac{1}{p}} \quad \text{if } d > 1$$

28.14 Additional information and definitions

- a : Scale parameter
- u : Uniform[0,1] random variable
- $P(a, x) = \frac{\gamma(a, x)}{\Gamma(a)}$: Regularized lower incomplete gamma function
- $P^{-1}(a, u)$: Inverse of regularized lower incomplete gamma function
- $\gamma(a, x)$: Lower incomplete gamma function
- $\Gamma(x)$: Gamma function

28.15 Spreadsheet documents

- [Excel file from GitHub repository](#)
- [Google spreadsheet document](#)

29 Generalized Gamma 4P Distribution

29.1 Distribution definition

$$X \sim \text{GeneralizedGamma}_{4P}(a, d, p, \text{Loc})$$

29.2 Distribution domain

$$x \in (\text{Loc}, \infty)$$

29.3 Parameters domain and parameters constraints

$$a \in \mathbb{R}^+, d \in \mathbb{R}^+, p \in \mathbb{R}^+, \text{Loc} \in \mathbb{R}$$

29.4 Cumulative distribution function

$$F_X(x) = P(d/p, ((x - \text{Loc})/a)^p) = \frac{\gamma(d/p, ((x - \text{Loc})/a)^p)}{\Gamma(d/p)}$$

29.5 Probability density function

$$f_X(x) = \frac{p/a^d}{\Gamma(d/p)} (x - \text{Loc})^{d-1} e^{-((x - \text{Loc})/a)^p}$$

29.6 Percent point function/Sample

$$F_X^{-1}(u) = \text{Loc} + aP^{-1}\left(\frac{d}{p}, u\right)^{\frac{1}{p}}$$

29.7 Parametric centered moments

$$\tilde{\mu}'_k = E[\tilde{X}^k] = \int_0^\infty x^k f_{\tilde{X}}(x) dx = a^k \frac{\Gamma(\frac{d+k}{p})}{\Gamma(\frac{d}{p})}$$

29.8 Parametric mean

$$\text{Mean}(X) = \text{Loc} + \tilde{\mu}'_1$$

29.9 Parametric variance

$$\text{Variance}(X) = \tilde{\mu}'_2 - \tilde{\mu}'_1^2$$

29.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\tilde{\mu}'_3 - 3\tilde{\mu}'_2\tilde{\mu}'_1 + 2\tilde{\mu}'_1^3}{(\tilde{\mu}'_2 - \tilde{\mu}'_1^2)^{1.5}}$$

29.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\tilde{\mu}'_4 - 4\tilde{\mu}'_1\tilde{\mu}'_3 + 6\tilde{\mu}'_1^2\tilde{\mu}'_2 - 3\tilde{\mu}'_1^4}{(\tilde{\mu}'_2 - \tilde{\mu}'_1^2)^2}$$

29.12 Parametric median

$$\text{Median}(X) = \text{Loc} + aP^{-1}\left(\frac{d}{p}, \frac{1}{2}\right)^{\frac{1}{p}}$$

29.13 Parametric mode

$$\text{Mode}(X) = \text{Loc} + a\left(\frac{d-1}{p}\right)^{\frac{1}{p}} \quad \text{if } d > 1$$

29.14 Additional information and definitions

- $\tilde{X} \sim \text{GeneralizedGamma}(a, d, p)$
- Loc : Location parameter
- a : Scale parameter
- u : Uniform[0,1] random variable
- $P(a, x) = \frac{\gamma(a, x)}{\Gamma(a)}$: Regularized lower incomplete gamma function
- $P^{-1}(a, u)$: Inverse of regularized lower incomplete gamma function
- $\gamma(a, x)$: Lower incomplete gamma function
- $\Gamma(x)$: Gamma function

29.15 Spreadsheet documents

- [Excel file from GitHub repository](#)
- [Google spreadsheet document](#)

30 Generalized Logistic Distribution

30.1 Distribution definition

$$X \sim \text{GeneralizedLogistic}(c, \text{Loc}, \text{Sc})$$

30.2 Distribution domain

$$x \in (\text{Loc}, \infty)$$

30.3 Parameters domain and parameters constraints

$$c \in \mathbb{R}^+, \text{Loc} \in \mathbb{R}, \text{Sc} \in \mathbb{R}^+$$

30.4 Cumulative distribution function

$$F_X(x) = \frac{1}{(1 + \exp(-z(x)))^c}$$

30.5 Probability density function

$$f_X(x) = \frac{c \exp(-z(x))}{\text{Sc} (1 + \exp(-z(x)))^{c+1}}$$

30.6 Percent point function/Sample

$$F_X^{-1}(u) = \text{Loc} - \text{Sc} \ln(u^{-1/c} - 1)$$

30.7 Parametric centered moments

$$\mu'_k = E[X^k] = \int_{-\infty}^{\infty} x^k f_X(x) dx$$

30.8 Parametric mean

$$\text{Mean}(X) = \text{Loc} + \text{Sc} \cdot \tilde{\mu}'_1 = \text{Loc} + \text{Sc} (\gamma + \psi_0(c))$$

30.9 Parametric variance

$$\text{Variance}(X) = \text{Sc}^2 \cdot (\tilde{\mu}'_2 - \tilde{\mu}'_1{}^2) = \text{Sc}^2 \left(\frac{\pi^2}{6} + \psi_1(c) \right)$$

30.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\mu'_3 - 3\mu'_2\mu'_1 + 2\mu_1'^3}{(\mu'_2 - \mu_1'^2)^{1.5}} = \frac{\psi_2(c) + 2\zeta(3)}{\left(\frac{\pi^2}{6} + \psi_1(c)\right)^{3/2}}$$

30.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\mu'_4 - 4\mu'_1\mu'_3 + 6\mu_1'^2\mu'_2 - 3\mu_1'^4}{(\mu'_2 - \mu_1'^2)^2} = \frac{\left(\frac{\pi^4}{15} + \psi_3(c)\right)}{\left(\frac{\pi^2}{6} + \psi_1(c)\right)^2}$$

30.12 Parametric median

$$\text{Median}(X) = \text{Loc} - \text{Sc} \ln(2^{1/c} - 1)$$

30.13 Parametric mode

$$\text{Mode}(X) = \text{Loc} + \text{Sc} \ln(c)$$

30.14 Additional information and definitions

- Loc : Location parameter
- Sc : Scale parameter
- $z(x) = (x - \text{Loc}) / \text{Sc}$
- u : Uniform[0,1] random variable
- γ : Euler-Mascheroni constant = 0.5772156649
- $\psi_0(x)$: Digamma function
- $\psi_n(x)$: Polygamma function of order $n \in \mathbb{N}$

30.15 Spreadsheet documents

- [Excel file from GitHub repository](#)
- [Google spreadsheet document](#)

31 Generalized Normal Distribution

31.1 Distribution definition

$$X \sim \text{GeneralizedNormal}(\beta, \mu, \alpha)$$

31.2 Distribution domain

$$x \in (-\infty, +\infty)$$

31.3 Parameters domain and parameters constraints

$$\beta \in \mathbb{R}^+, \mu \in \mathbb{R}, \alpha \in \mathbb{R}^+$$

31.4 Cumulative distribution function

$$F_X(x) = \frac{1}{2} + \frac{\text{sign}(x - \mu)}{2\Gamma(1/\beta)} \gamma\left(1/\beta, \left|\frac{x - \mu}{\alpha}\right|^\beta\right) = \frac{1}{2} + \frac{\text{sign}(x - \mu)}{2} \text{P}\left(1/\beta, \left|\frac{x - \mu}{\alpha}\right|^\beta\right)$$

31.5 Probability density function

$$f_X(x) = \frac{\beta}{2\alpha\Gamma(1/\beta)} \exp\left(-\left(\frac{|x - \mu|}{\alpha}\right)^\beta\right)$$

31.6 Percent point function/Sample

$$F_X^{-1}(u) = \text{sign}(u - \frac{1}{2}) \left[\alpha^\beta \text{P}^{-1}\left(\frac{1}{\beta}, 2|u - \frac{1}{2}|\right) \right]^{1/\beta} + \mu$$

31.7 Parametric centered moments

$$\mu'_k = E[X^k] = \int_{-\infty}^{\infty} x^k f_X(x) dx = \begin{cases} 0 & \text{if } k \text{ is odd} \\ \alpha^k \Gamma\left(\frac{k+1}{\beta}\right) / \Gamma\left(\frac{1}{\beta}\right) & \text{if } k \text{ is even} \end{cases}$$

31.8 Parametric mean

$$\text{Mean}(X) = \mu + \alpha \mu'_1 = \mu$$

31.9 Parametric variance

$$\text{Variance}(X) = \alpha^2(\mu'_2 - \mu_1'^2) = \frac{\alpha^2 \Gamma(3/\beta)}{\Gamma(1/\beta)}$$

31.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\mu'_3 - 3\mu'_2\mu'_1 + 2\mu_1'^3}{(\mu'_2 - \mu_1'^2)^{1.5}} = 0$$

31.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\mu'_4 - 4\mu'_1\mu'_3 + 6\mu_1'^2\mu'_2 - 3\mu_1'^4}{(\mu'_2 - \mu_1'^2)^2} = \frac{\Gamma(5/\beta)\Gamma(1/\beta)}{\Gamma(3/\beta)^2}$$

31.12 Parametric median

$$\text{Median}(X) = \mu$$

31.13 Parametric mode

$$\text{Mode}(X) = \mu$$

31.14 Additional information and definitions

- μ : Location parameter
- α : Scale parameter
- u : Uniform[0,1] random variable
- $P^{-1}(a, u)$: Inverse of regularized lower incomplete gamma function
- $\gamma(a, x)$: Lower incomplete gamma function
- $\Gamma(x)$: Gamma function

31.15 Spreadsheet documents

- [Excel file from GitHub repository](#)
- [Google spreadsheet document](#)

32 Generalized Pareto Distribution

32.1 Distribution definition

$$X \sim \text{GeneralizedPareto}(c, \mu, \sigma)$$

32.2 Distribution domain

$$\text{if } c \geq 0 : x \in (\mu, \infty), \quad \text{if } c < 0 : x \in \left(-\infty, \mu - \frac{\sigma}{c}\right)$$

32.3 Parameters domain and parameters constraints

$$c \in \mathbb{R}, \mu \in \mathbb{R}, \sigma \in \mathbb{R}^+$$

32.4 Cumulative distribution function

$$F_X(x) = 1 - (1 + cz(x))^{-1/c}$$

32.5 Probability density function

$$f_X(x) = \frac{1}{\sigma} (1 + cz(x))^{-(1/c+1)}$$

32.6 Percent point function/Sample

$$F_X^{-1}(u) = \mu + \frac{\sigma(u^{-c} - 1)}{c}$$

32.7 Parametric centered moments

$$\mu'_k = E[X^k] = \int_{-\infty}^{\infty} x^k f_X(x) dx = \frac{(-1)^k}{c^k} \sum_{i=0}^k \binom{k}{i} \frac{(-1)^i}{1-ci} \quad \text{if } < \frac{1}{k}$$

32.8 Parametric mean

$$\text{Mean}(X) = \mu + \sigma \mu'_1 = \mu + \frac{\sigma}{1-c} \quad \text{if } c < 1$$

32.9 Parametric variance

$$\text{Variance}(X) = \sigma^2(\mu'_2 - \mu_1'^2) = \frac{\sigma^2}{(1-c)^2(1-2c)} \quad \text{if } c < 1/2$$

32.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\mu'_3 - 3\mu'_2\mu'_1 + 2\mu_1'^3}{(\mu'_2 - \mu_1'^2)^{1.5}} = \frac{2(1+c)\sqrt{1-2c}}{(1-3c)} \quad \text{if } c < 1/3$$

32.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\mu'_4 - 4\mu'_1\mu'_3 + 6\mu_1'^2\mu'_2 - 3\mu_1'^4}{(\mu'_2 - \mu_1'^2)^2} = \frac{3(1-2c)(2c^2+c+3)}{(1-3c)(1-4c)} \quad \text{if } c < 1/4$$

32.12 Parametric median

$$\text{Median}(X) = \mu$$

32.13 Parametric mode

$$\text{Mode}(X) = \mu + \frac{\sigma(2^c - 1)}{c}$$

32.14 Additional information and definitions

- μ : Location parameter
- σ : Scale parameter
- $z(x) = (x - \mu) / \sigma$
- u : Uniform[0,1] random variable

32.15 Spreadsheet documents

- [Excel file from GitHub repository](#)
- [Google spreadsheet document](#)

33 Gibrat Distribution

33.1 Distribution definition

$$X \sim \text{Gibrat}(\text{Loc}, \text{Sc})$$

33.2 Distribution domain

$$x \in (\text{Loc}, \infty)$$

33.3 Parameters domain and parameters constraints

$$\text{Loc} \in \mathbb{R}, \text{Sc} \in \mathbb{R}^+$$

33.4 Cumulative distribution function

$$F_X(x) = \Phi(\ln x) = \frac{1}{2} \left(1 + \text{erf} \left(\frac{\ln z(x)}{\sqrt{2}} \right) \right)$$

33.5 Probability density function

$$f_X(x) = \frac{1}{\text{Sc}} \frac{1}{x\sqrt{2\pi}} \exp \left(-\frac{1}{2} (\ln z(x))^2 \right)$$

33.6 Percent point function/Sample

$$F_X^{-1}(u) = \text{Loc} + \text{Sc} \times \exp \left(\Phi^{-1}(u) \right)$$

33.7 Parametric centered moments

$$\mu'_k = E[X^k] = \int_{\text{Loc}}^{\infty} x^k f_X(x) dx = \exp \left(\frac{k^2}{2} \right)$$

33.8 Parametric mean

$$\text{Mean}(X) = \text{Loc} + \text{Sc} \cdot \tilde{\mu}'_1 = \text{Loc} + \text{Sc} \cdot \sqrt{e}$$

33.9 Parametric variance

$$\text{Variance}(X) = \text{Sc}^2 \cdot (\tilde{\mu}'_2 - \tilde{\mu}'_1{}^2) = \text{Sc}^2 [e^2 - e]$$

33.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\mu'_3 - 3\mu'_2\mu'_1 + 2\mu'^3_1}{(\mu'_2 - \mu'^2_1)^{1.5}} = \sqrt{e-1} (2+e)$$

33.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\mu'_4 - 4\mu'_1\mu'_3 + 6\mu'^2_1\mu'_2 - 3\mu'^4_1}{(\mu'_2 - \mu'^2_1)^2} = e^4 + 2e^3 + 3e^2 - 3$$

33.12 Parametric median

$$\text{Median}(X) = \text{Loc} + \text{Sc} \times \exp \left(\Phi^{-1}(1/2) \right)$$

33.13 Parametric mode

$$\text{Mode}(X) = \text{Loc} + \frac{\text{Sc}}{e}$$

33.14 Additional information and definitions

- Loc : Location parameter
- Sc : Scale parameter
- $z(x) = (x - \text{Loc}) / \text{Sc}$
- u : Uniform[0,1] random variable
- $\Phi(x)$: CDF normal standard distribution
- $\Phi^{-1}(x)$: PPF normal standard distribution
- $\text{erf}(x)$: Error function

33.15 Spreadsheet documents

- [Excel file from GitHub repository](#)
- [Google spreadsheet document](#)

34 Gumbel Left Distribution

34.1 Distribution definition

$$X \sim \text{GumbelLeft}(\mu, \sigma)$$

34.2 Distribution domain

$$x \in (-\infty, \infty)$$

34.3 Parameters domain and parameters constraints

$$\mu \in \mathbb{R}, \sigma \in \mathbb{R}^+$$

34.4 Cumulative distribution function

$$F_X(x) = 1 - \exp\left(-e^{z(x)}\right)$$

34.5 Probability density function

$$f_X(x) = \frac{1}{\sigma} \exp\left(z(x) - e^{z(x)}\right)$$

34.6 Percent point function/Sample

$$F_X^{-1}(u) = \mu + \sigma \ln(-\ln(1-u))$$

34.7 Parametric centered moments

$$\tilde{\mu}'_k = E[\tilde{X}^k] = \int_{-\infty}^{\infty} x^k f_{\tilde{X}}(x) dx$$

34.8 Parametric mean

$$\text{Mean}(X) = \mu + \sigma \tilde{\mu}'_1 = \mu - \gamma \sigma$$

34.9 Parametric variance

$$\text{Variance}(X) = \sigma^2(\tilde{\mu}'_2 - \tilde{\mu}'_1{}^2) = \sigma^2 \frac{\pi^2}{6}$$

34.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\tilde{\mu}'_3 - 3\tilde{\mu}'_2\tilde{\mu}'_1 + 2\tilde{\mu}'_1{}^3}{(\tilde{\mu}'_2 - \tilde{\mu}'_1{}^2)^{1.5}} = -\frac{12\sqrt{6}\zeta(3)}{\pi^3}$$

34.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\tilde{\mu}'_4 - 4\tilde{\mu}'_1\tilde{\mu}'_3 + 6\tilde{\mu}'_1{}^2\tilde{\mu}'_2 - 3\tilde{\mu}'_1{}^4}{(\tilde{\mu}'_2 - \tilde{\mu}'_1{}^2)^2} = 3 + \frac{12}{5}$$

34.12 Parametric median

$$\text{Median}(X) = \mu + \sigma \ln\left(-\ln\left(\frac{1}{2}\right)\right)$$

34.13 Parametric mode

$$\text{Mode}(X) = \mu$$

34.14 Additional information and definitions

- $\tilde{X} \sim \text{GumbelLeft}(0, 1)$
- μ : Location parameter
- σ : Scale parameter
- $z(x) = (x - \mu) / \sigma$
- u : Uniform[0,1] random variable
- γ : Euler-Mascheroni constant = 0.5772156649
- $\zeta(3)$: Apéry's constant = 1.2020569031

34.15 Spreadsheet documents

- [Excel file from GitHub repository](#)
- [Google spreadsheet document](#)

35 Gumbel Right Distribution

35.1 Distribution definition

$$X \sim \text{GumbelRight}(\mu, \sigma)$$

35.2 Distribution domain

$$x \in (-\infty, \infty)$$

35.3 Parameters domain and parameters constraints

$$\mu \in \mathbb{R}, \sigma \in \mathbb{R}^+$$

35.4 Cumulative distribution function

$$F_X(x) = \exp\left(-e^{-z(x)}\right)$$

35.5 Probability density function

$$f_X(x) = \frac{1}{\sigma} \exp\left(-\left(z(x) + e^{-z(x)}\right)\right)$$

35.6 Percent point function/Sample

$$F_X^{-1}(u) = \tilde{\mu} - \sigma \ln(-\ln(u))$$

35.7 Parametric centered moments

$$\tilde{\mu}'_k = E[\tilde{X}^k] = \int_{-\infty}^{\infty} x^k f_{\tilde{X}}(x) dx$$

35.8 Parametric mean

$$\text{Mean}(X) = \mu + \sigma \tilde{\mu}'_1 = \mu + \gamma \sigma$$

35.9 Parametric variance

$$\text{Variance}(X) = \sigma^2(\tilde{\mu}'_2 - \tilde{\mu}'_1{}^2) = \sigma^2 \frac{\pi^2}{6}$$

35.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\tilde{\mu}'_3 - 3\tilde{\mu}'_2\tilde{\mu}'_1 + 2\tilde{\mu}'_1{}^3}{(\tilde{\mu}'_2 - \tilde{\mu}'_1{}^2)^{1.5}} = \frac{12\sqrt{6}\zeta(3)}{\pi^3}$$

35.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\tilde{\mu}'_4 - 4\tilde{\mu}'_1\tilde{\mu}'_3 + 6\tilde{\mu}'_1{}^2\tilde{\mu}'_2 - 3\tilde{\mu}'_1{}^4}{(\tilde{\mu}'_2 - \tilde{\mu}'_1{}^2)^2} = 3 + \frac{12}{5}$$

35.12 Parametric median

$$\text{Median}(X) = \mu - \sigma \ln\left(-\ln\left(\frac{1}{2}\right)\right)$$

35.13 Parametric mode

$$\text{Mode}(X) = \mu$$

35.14 Additional information and definitions

- $\tilde{X} \sim \text{GumbelRight}(0, 1)$
- μ : Location parameter
- σ : Scale parameter
- $z(x) = (x - \mu) / \sigma$
- u : Uniform[0,1] random variable
- γ : Euler-Mascheroni constant = 0.5772156649
- $\zeta(3)$: Apéry's constant = 1.2020569031

35.15 Spreadsheet documents

- [Excel file from GitHub repository](#)
- [Google spreadsheet document](#)

36 Half Normal Distribution

36.1 Distribution definition

$$X \sim \text{HalfNormal}(\mu, \sigma)$$

36.2 Distribution domain

$$x \in (\mu, \infty)$$

36.3 Parameters domain and parameters constraints

$$\mu \in \mathbb{R}, \sigma \in \mathbb{R}^+$$

36.4 Cumulative distribution function

$$F_X(x) = 2\Phi(z(x)) - 1 = \text{erf}\left(\frac{z(x)}{\sqrt{2}}\right)$$

36.5 Probability density function

$$f_X(x) = \frac{\sqrt{2}}{\sigma\sqrt{\pi}} \exp\left(-\frac{z(x)^2}{2}\right)$$

36.6 Percent point function/Sample

$$F_X^{-1}(u) = \mu + \sigma\Phi^{-1}\left(\frac{1+u}{2}\right) = \tilde{\mu} + \sigma\sqrt{2}\text{erf}^{-1}(u)$$

36.7 Parametric centered moments

$$\tilde{\mu}'_k = E[\tilde{X}^k] = \int_0^\infty x^k f_{\tilde{X}}(x) dx = \frac{2^{n/2}\Gamma(\frac{n+1}{2})}{\sqrt{\pi}}$$

36.8 Parametric mean

$$\text{Mean}(X) = \tilde{\mu} + \sigma\tilde{\mu}'_1 = \tilde{\mu} + \sigma\sqrt{\frac{2}{\pi}}$$

36.9 Parametric variance

$$\text{Variance}(X) = \sigma^2(\tilde{\mu}'_2 - \tilde{\mu}'_1^2) = \sigma^2\left(1 - \frac{2}{\pi}\right)$$

36.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\tilde{\mu}'_3 - 3\tilde{\mu}'_2\tilde{\mu}'_1 + 2\tilde{\mu}'_1^3}{(\tilde{\mu}'_2 - \tilde{\mu}'_1^2)^{1.5}} = \frac{\sqrt{2}(4-\pi)}{(\pi-2)^{3/2}} = 0.9952717$$

36.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\tilde{\mu}'_4 - 4\tilde{\mu}'_1\tilde{\mu}'_3 + 6\tilde{\mu}'_1^2\tilde{\mu}'_2 - 3\tilde{\mu}'_1^4}{(\tilde{\mu}'_2 - \tilde{\mu}'_1^2)^2} = 3 + \frac{8(\pi-3)}{(\pi-2)^2} = 3.869177$$

36.12 Parametric median

$$\text{Median}(X) = \mu + \sigma\sqrt{2}\text{erf}^{-1}(1/2)$$

36.13 Parametric mode

$$\text{Mode}(X) = \mu$$

36.14 Additional information and definitions

- $\tilde{X} \sim \text{HalfNormal}(0, 1)$
- μ : Location parameter
- σ : Scale parameter
- $z(x) = (x - \mu) / \sigma$
- u : Uniform[0,1] random variable
- $\Phi(x)$: CDF normal standard distribution
- $\Phi^{-1}(x)$: PPF normal standard distribution
- $\text{erf}(x)$: Error function
- $\Gamma(x)$: Gamma function

36.15 Spreadsheet documents

- [Excel file from GitHub repository](#)
- [Google spreadsheet document](#)

37 Hyperbolic Secant Distribution

37.1 Distribution definition

$$X \sim \text{HyperbolicSecant}(\mu, \sigma)$$

37.2 Distribution domain

$$x \in (-\infty, \infty)$$

37.3 Parameters domain and parameters constraints

$$\mu \in \mathbb{R}, \sigma \in \mathbb{R}^+$$

37.4 Cumulative distribution function

$$F_X(x) = \frac{2}{\pi} \arctan \left[\exp \left(\frac{\pi}{2} z(x) \right) \right]$$

37.5 Probability density function

$$f_X(x) = \frac{1}{2\sigma} \operatorname{sech} \left(\frac{\pi}{2} z(x) \right)$$

37.6 Percent point function/Sample

$$F_X^{-1}(u) = \mu + \sigma \frac{2}{\pi} \ln \left[\tan \left(\frac{\pi}{2} u \right) \right]$$

37.7 Parametric centered moments

$$\tilde{\mu}'_k = E[\tilde{X}^k] = \int_{-\infty}^{\infty} x^k f_{\tilde{X}}(x) dx = \frac{1 + (-1)^k}{2\pi 2^{2k}} k! \left[\zeta \left(k+1, \frac{1}{4} \right) - \zeta \left(k+1, \frac{3}{4} \right) \right]$$

37.8 Parametric mean

$$\text{Mean}(X) = \mu + \sigma \tilde{\mu}'_1 = \mu$$

37.9 Parametric variance

$$\text{Variance}(X) = \sigma^2 (\tilde{\mu}'_2 - \tilde{\mu}'_1{}^2) = \sigma^2$$

37.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\tilde{\mu}'_3 - 3\tilde{\mu}'_2\tilde{\mu}'_1 + 2\tilde{\mu}'_1{}^3}{(\tilde{\mu}'_2 - \tilde{\mu}'_1{}^2)^{1.5}} = 0$$

37.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\tilde{\mu}'_4 - 4\tilde{\mu}'_1\tilde{\mu}'_3 + 6\tilde{\mu}'_1{}^2\tilde{\mu}'_2 - 3\tilde{\mu}'_1{}^4}{(\tilde{\mu}'_2 - \tilde{\mu}'_1{}^2)^2} = 3$$

37.12 Parametric median

$$\text{Median}(X) = \mu$$

37.13 Parametric mode

$$\text{Mode}(X) = \mu$$

37.14 Additional information and definitions

- $\tilde{X} \sim \text{HyperbolicSecant}(0, 1)$
- μ : Location parameter
- σ : Scale parameter
- $z(x) = (x - \mu) / \sigma$
- u : Uniform[0,1] random variable
- $\zeta(a, s)$: Hurwitz zeta function

37.15 Spreadsheet documents

- [Excel file from GitHub repository](#)
- [Google spreadsheet document](#)

38 Inverse Gamma Distribution

38.1 Distribution definition

$$X \sim \text{InverseGamma}(\alpha, \beta)$$

38.2 Distribution domain

$$x \in (0, \infty)$$

38.3 Parameters domain and parameters constraints

$$\alpha \in \mathbb{R}^+, \beta \in \mathbb{R}^+$$

38.4 Cumulative distribution function

$$F_X(x) = 1 - \frac{\gamma(\alpha, \beta/x)}{\Gamma(\alpha)} = 1 - P\left(\alpha, \frac{\beta}{x}\right)$$

38.5 Probability density function

$$f_X(x) = \frac{\beta^\alpha}{\Gamma(\alpha)} x^{-\alpha-1} \exp\left(-\frac{\beta}{x}\right)$$

38.6 Percent point function/Sample

$$F_X^{-1}(u) = \frac{\beta}{P^{-1}(\alpha, 1-u)}$$

38.7 Parametric centered moments

$$\tilde{\mu}'_k = E[\tilde{X}^k] = \int_0^\infty x^k f_{\tilde{X}}(x) dx = \frac{\Gamma(\alpha - k)}{\Gamma(\alpha)} = \frac{1}{(\alpha - 1) \cdots (\alpha - k)} \quad \text{if } \alpha > k$$

38.8 Parametric mean

$$\text{Mean}(X) = \beta \tilde{\mu}'_1$$

38.9 Parametric variance

$$\text{Variance}(X) = \beta^2 (\tilde{\mu}'_2 - \tilde{\mu}'_1^2)$$

38.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\tilde{\mu}'_3 - 3\tilde{\mu}'_2\tilde{\mu}'_1 + 2\tilde{\mu}'_1^3}{(\tilde{\mu}'_2 - \tilde{\mu}'_1^2)^{1.5}}$$

38.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\tilde{\mu}'_4 - 4\tilde{\mu}'_1\tilde{\mu}'_3 + 6\tilde{\mu}'_1^2\tilde{\mu}'_2 - 3\tilde{\mu}'_1^4}{(\tilde{\mu}'_2 - \tilde{\mu}'_1^2)^2}$$

38.12 Parametric median

$$\text{Median}(X) = \frac{\beta}{P^{-1}\left(\alpha, \frac{1}{2}\right)}$$

38.13 Parametric mode

$$\text{Mode}(X) = \frac{\beta}{\alpha + 1}$$

38.14 Additional information and definitions

- $\tilde{X} \sim \text{InverseGamma}(\alpha, 1)$
- β : Scale parameter
- u : $\text{Uniform}[0,1]$ random variable
- $P(a, x) = \frac{\gamma(a, x)}{\Gamma(a)}$: Regularized lower incomplete gamma function
- $P^{-1}(a, u)$: Inverse of regularized lower incomplete gamma function
- $\gamma(a, x)$: Lower incomplete gamma function
- $\Gamma(x)$: Gamma function

38.15 Spreadsheet documents

- [Excel file from GitHub repository](#)
- [Google spreadsheet document](#)

39 Inverse Gamma 3P Distribution

39.1 Distribution definition

$$X \sim \text{InverseGamma}_{3P}(\alpha, \text{Loc}, \beta)$$

39.2 Distribution domain

$$x \in (\text{Loc}, \infty)$$

39.3 Parameters domain and parameters constraints

$$\alpha \in \mathbb{R}^+, \text{Loc} \in \mathbb{R}, \beta \in \mathbb{R}^+$$

39.4 Cumulative distribution function

$$F_X(x) = 1 - \frac{\gamma(\alpha, \beta/(x - \text{Loc}))}{\Gamma(\alpha)} = 1 - P\left(\alpha, \frac{\beta}{x - \text{Loc}}\right)$$

39.5 Probability density function

$$f_X(x) = \frac{\beta^\alpha}{\Gamma(\alpha)} (x - \text{Loc})^{-\alpha-1} \exp\left(-\frac{\beta}{x - \text{Loc}}\right)$$

39.6 Percent point function/Sample

$$F_X^{-1}(u) = \text{Loc} + \frac{\beta}{P^{-1}(\alpha, 1 - u)}$$

39.7 Parametric centered moments

$$\tilde{\mu}'_k = E[\tilde{X}^k] = \int_0^\infty x^k f_{\tilde{X}}(x) dx = \frac{\Gamma(\alpha - k)}{\Gamma(\alpha)} = \frac{1}{(\alpha - 1) \cdots (\alpha - k)} \quad \text{if } \alpha > k$$

39.8 Parametric mean

$$\text{Mean}(X) = \text{Loc} + \beta \mu'_1$$

39.9 Parametric variance

$$\text{Variance}(X) = \beta^2 (\mu'_2 - \mu_1'^2)$$

39.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\mu'_3 - 3\mu'_2\mu'_1 + 2\mu_1'^3}{(\mu'_2 - \mu_1'^2)^{1.5}}$$

39.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\mu'_4 - 4\mu'_1\mu'_3 + 6\mu_1'^2\mu'_2 - 3\mu_1'^4}{(\mu'_2 - \mu_1'^2)^2}$$

39.12 Parametric median

$$\text{Median}(X) = \text{Loc} + \frac{\beta}{P^{-1}(\alpha, \frac{1}{2})}$$

39.13 Parametric mode

$$\text{Mode}(X) = \text{Loc} + \frac{\beta}{\alpha + 1}$$

39.14 Additional information and definitions

- $\tilde{X} \sim \text{InverseGamma}_{3P}(\alpha, 0, 1)$
- Loc : Location parameter
- β : Scale parameter
- u : Uniform[0,1] random variable
- $P(a, x) = \frac{\gamma(a, x)}{\Gamma(a)}$: Regularized lower incomplete gamma function
- $P^{-1}(a, u)$: Inverse of regularized lower incomplete gamma function
- $\gamma(a, x)$: Lower incomplete gamma function
- $\Gamma(x)$: Gamma function

39.15 Spreadsheet documents

- [Excel file from GitHub repository](#)
- [Google spreadsheet document](#)

40 Inverse Gaussian Distribution

40.1 Distribution definition

$$X \sim \text{InverseGaussian}(\mu, \lambda)$$

40.2 Distribution domain

$$x \in (0, \infty)$$

40.3 Parameters domain and parameters constraints

$$\mu \in \mathbb{R}^+, \lambda \in \mathbb{R}^+$$

40.4 Cumulative distribution function

$$F_X(x) = \Phi\left(\sqrt{\frac{\lambda}{x}}\left(\frac{x}{\mu} - 1\right)\right) + \exp\left(\frac{2\lambda}{\mu}\right)\Phi\left(-\sqrt{\frac{\lambda}{x}}\left(\frac{x}{\mu} + 1\right)\right)$$

40.5 Probability density function

$$f_X(x) = \sqrt{\frac{\lambda}{2\pi x^3}} \exp\left[-\frac{\lambda(x - \mu)^2}{2\mu^2 x}\right]$$

40.6 Percent point function/Sample

$$\text{Sample}_X = \begin{cases} x_0 & \text{if } u_2 \leq \frac{\mu}{\mu + x_0} \\ \frac{\mu^2}{x_0} & \text{if } u_2 \geq \frac{\mu}{\mu + x_0} \end{cases}$$

40.7 Parametric centered moments

$$\mu'_k = E[X^k] = \int_0^\infty x^k f_X(x) dx$$

40.8 Parametric mean

$$\text{Mean}(X) = \mu'_1 = \mu$$

40.9 Parametric variance

$$\text{Variance}(X) = \mu'_2 - \mu'^2_1 = \frac{\mu^3}{\lambda}$$

40.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\mu'_3 - 3\mu'_2\mu'_1 + 2\mu'^3_1}{(\mu'_2 - \mu'^2_1)^{1.5}} = 3\left(\frac{\mu}{\lambda}\right)^{1/2}$$

40.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\mu'_4 - 4\mu'_1\mu'_3 + 6\mu'^2_1\mu'_2 - 3\mu'^4_1}{(\mu'_2 - \mu'^2_1)^2} = 3 + \frac{15\mu}{\lambda}$$

40.12 Parametric median

$$\text{Median}(X) = F_X^{-1}\left(\frac{1}{2}\right)$$

40.13 Parametric mode

$$\text{Mode}(X) = \mu \left[\left(1 + \frac{9\mu^2}{4\lambda^2} \right)^{\frac{1}{2}} - \frac{3\mu}{2\lambda} \right]$$

40.14 Additional information and definitions

- Computing an analytic expression for the inverse of the cumulative distribution function is not feasible. Nonetheless, it is possible to generate a random sample from the distribution.
- $\Phi(x)$: CDF normal standard distribution
- $\Phi^{-1}(x)$: PPF normal standard distribution
- $x_0 = \mu + \frac{\mu^2[\Phi^{-1}(u_1)]^2}{2\lambda} - \frac{\mu}{2\lambda} \sqrt{4\mu\lambda[\Phi^{-1}(u_1)]^2 + \mu^2([\Phi^{-1}(u_1)]^2)^2}$
- u_1 : Uniform[0,1] random variable
- u_2 : Uniform[0,1] random variable

40.15 Spreadsheet documents

- [Excel file from GitHub repository](#)
- [Google spreadsheet document](#)

41 Inverse Gaussian 3P Distribution

41.1 Distribution definition

$$X \sim \text{InverseGaussian}_{3P}(\mu, \lambda, \text{Loc})$$

41.2 Distribution domain

$$x \in (0, \infty)$$

41.3 Parameters domain and parameters constraints

$$\mu \in \mathbb{R}^+, \lambda \in \mathbb{R}^+, \text{Loc} \in \mathbb{R}$$

41.4 Cumulative distribution function

$$F_X(x) = \Phi\left(\sqrt{\frac{\lambda}{x - \text{Loc}}} \left(\frac{x - \text{Loc}}{\mu} - 1\right)\right) + \exp\left(\frac{2\lambda}{\mu}\right) \Phi\left(-\sqrt{\frac{\lambda}{x - \text{Loc}}} \left(\frac{x - \text{Loc}}{\mu} + 1\right)\right)$$

41.5 Probability density function

$$f_X(x) = \sqrt{\frac{\lambda}{2\pi(x - \text{Loc})^3}} \exp\left[-\frac{\lambda(x - \mu - \text{Loc})^2}{2\mu^2(x - \text{Loc})}\right]$$

41.6 Percent point function/Sample

$$\text{Sample}_X = \begin{cases} \text{Loc} + x_0 & \text{if } u_2 \leq \frac{\mu}{\mu + x_0} \\ \text{Loc} + \frac{\mu^2}{x_0} & \text{if } u_2 \geq \frac{\mu}{\mu + x_0} \end{cases}$$

41.7 Parametric centered moments

$$\mu'_k = E[X^k] = \int_{\text{Loc}}^{\infty} x^k f_X(x) dx$$

41.8 Parametric mean

$$\text{Mean}(X) = \mu'_1 = \text{Loc} + \mu$$

41.9 Parametric variance

$$\text{Variance}(X) = \mu'_2 - \mu_1'^2 = \frac{\mu^3}{\lambda}$$

41.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\mu'_3 - 3\mu'_2\mu'_1 + 2\mu_1'^3}{(\mu'_2 - \mu_1'^2)^{1.5}} = 3\left(\frac{\mu}{\lambda}\right)^{1/2}$$

41.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\mu'_4 - 4\mu'_1\mu'_3 + 6\mu_1'^2\mu'_2 - 3\mu_1'^4}{(\mu'_2 - \mu_1'^2)^2} = 3 + \frac{15\mu}{\lambda}$$

41.12 Parametric median

$$\text{Median}(X) = F_X^{-1}\left(\frac{1}{2}\right)$$

41.13 Parametric mode

$$\text{Mode}(X) = \text{Loc} + \mu \left[\left(1 + \frac{9\mu^2}{4\lambda^2} \right)^{\frac{1}{2}} - \frac{3\mu}{2\lambda} \right]$$

41.14 Additional information and definitions

- Computing an analytic expression for the inverse of the cumulative distribution function is not feasible. Nonetheless, it is possible to generate a random sample from the distribution.
- Loc : Location parameter
- $\Phi(x)$: CDF normal standard distribution
- $\Phi^{-1}(x)$: PPF normal standard distribution
- $x_0 = \mu + \frac{\mu^2[\Phi^{-1}(u_1)]^2}{2\lambda} - \frac{\mu}{2\lambda} \sqrt{4\mu\lambda[\Phi^{-1}(u_1)]^2 + \mu^2([\Phi^{-1}(u_1)]^2)^2}$
- u_1 : Uniform[0,1] random variable
- u_2 : Uniform[0,1] random variable

41.15 Spreadsheet documents

- [Excel file from GitHub repository](#)
- [Google spreadsheet document](#)

42 Johnson SB Distribution

42.1 Distribution definition

$$X \sim \text{JohnsonSB}(\xi, \lambda, \gamma, \delta)$$

42.2 Distribution domain

$$x \in (\xi, \xi + \lambda)$$

42.3 Parameters domain and parameters constraints

$$\xi \in \mathbb{R}, \lambda \in \mathbb{R}^+, \gamma \in \mathbb{R}, \delta \in \mathbb{R}^+$$

42.4 Cumulative distribution function

$$F_X(x) = \Phi\left(\gamma + \delta \ln \frac{z(x)}{1 - z(x)}\right)$$

42.5 Probability density function

$$f_X(x) = \frac{\delta}{\lambda \sqrt{2\pi} z(1 - z(x))} \exp\left[-\frac{1}{2} \left(\gamma + \delta \ln \frac{z(x)}{1 - z(x)}\right)^2\right]$$

42.6 Percent point function/Sample

$$F_X^{-1}(u) = \frac{\lambda \exp\left(\frac{\Phi^{-1}(u) - \gamma}{\delta}\right)}{1 + \exp\left(\frac{\Phi^{-1}(u) - \gamma}{\delta}\right)} + \xi$$

42.7 Parametric centered moments

$$\mu'_k = E[X^k] = \int_{\xi}^{\xi + \lambda} x^k f_X(x) dx$$

42.8 Parametric mean

$$\text{Mean}(X) = \mu'_1$$

42.9 Parametric variance

$$\text{Variance}(X) = \mu'_2 - \mu_1'^2$$

42.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\mu'_3 - 3\mu'_2\mu'_1 + 2\mu_1'^3}{(\mu'_2 - \mu_1'^2)^{1.5}}$$

42.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\mu'_4 - 4\mu'_1\mu'_3 + 6\mu_1'^2\mu'_2 - 3\mu_1'^4}{(\mu'_2 - \mu_1'^2)^2}$$

42.12 Parametric median

$$\text{Median}(X) = \frac{\lambda \exp\left(\frac{\Phi^{-1}(1/2) - \gamma}{\delta}\right)}{1 + \exp\left(\frac{\Phi^{-1}(1/2) - \gamma}{\delta}\right)} + \xi$$

42.13 Parametric mode

$$\text{Mode}(X) = \arg \max_x f_X(x)$$

42.14 Additional information and definitions

- ξ : Location parameter
- λ : Scale parameter
- $z(x) = (x - \xi) / \lambda$
- u : Uniform[0,1] random variable
- $\Phi(x)$: CDF normal standard distribution
- $\Phi^{-1}(x)$: PPF normal standard distribution

42.15 Spreadsheet documents

- [Excel file from GitHub repository](#)
- [Google spreadsheet document](#)

43 Johnson SU Distribution

43.1 Distribution definition

$$X \sim \text{JohnsonSU}(\xi, \lambda, \gamma, \delta)$$

43.2 Distribution domain

$$x \in (-\infty, \infty)$$

43.3 Parameters domain and parameters constraints

$$\xi \in \mathbb{R}, \lambda \in \mathbb{R}^+, \gamma \in \mathbb{R}, \delta \in \mathbb{R}^+$$

43.4 Cumulative distribution function

$$F_X(x) = \Phi(\gamma + \delta \sinh^{-1}(z(x)))$$

43.5 Probability density function

$$f_X(x) = \frac{\delta}{\lambda \sqrt{2\pi} \sqrt{z(x)^2 + 1}} \exp \left[-\frac{1}{2} (\gamma + \delta \sinh^{-1}(z(x)))^2 \right]$$

43.6 Percent point function/Sample

$$F_X^{-1}(u) = \lambda \sinh \left(\frac{\Phi^{-1}(u) - \gamma}{\delta} \right) + \xi$$

43.7 Parametric centered moments

$$\mu'_k = E[X^k] = \int_{-\infty}^{\infty} x^k f_X(x) dx$$

43.8 Parametric mean

$$\text{Mean}(X) = \mu'_1 = \xi - \lambda \exp \frac{\delta^{-2}}{2} \sinh \left(\frac{\gamma}{\delta} \right)$$

43.9 Parametric variance

$$\text{Variance}(X) = \mu'_2 - \mu_1'^2 = \frac{\lambda^2}{2} (\exp(\delta^{-2}) - 1) \left(\exp(\delta^{-2}) \cosh \left(\frac{2\gamma}{\delta} \right) + 1 \right)$$

43.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\mu'_3 - 3\mu'_2\mu'_1 + 2\mu_1'^3}{(\mu'_2 - \mu_1'^2)^{1.5}} = -\frac{\lambda^3 \sqrt{e^{\delta^{-2}}} (e^{\delta^{-2}} - 1)^2 (e^{\delta^{-2}} + 2) \sinh(\frac{3\gamma}{\delta}) + 3 \sinh(\frac{2\gamma}{\delta})}{4 \text{Variance}(X)^{1.5}}$$

43.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\mu'_4 - 4\mu'_1\mu'_3 + 6\mu_1'^2\mu'_2 - 3\mu_1'^4}{(\mu'_2 - \mu_1'^2)^2} = \frac{\lambda^4 (e^{\delta^{-2}} - 1)^2 (K_1 + K_2 + K_3)}{8 \text{Variance}(X)^2}$$

43.12 Parametric median

$$\text{Median}(X) = \xi + \lambda \sinh \left(-\frac{\gamma}{\delta} \right)$$

43.13 Parametric mode

$$\text{Mode}(X) = \arg \max_x f_X(x)$$

43.14 Additional information and definitions

- ξ : Location parameter
- λ : Scale parameter
- $z(x) = (x - \xi) / \lambda$
- u : Uniform[0,1] random variable
- $\Phi(x)$: CDF normal standard distribution
- $\Phi^{-1}(x)$: PPF normal standard distribution
- $K_1 = \left(e^{\delta^{-2}}\right)^2 \left(\left(e^{\delta^{-2}}\right)^4 + 2\left(e^{\delta^{-2}}\right)^3 + 3\left(e^{\delta^{-2}}\right)^2 - 3\right) \cosh\left(\frac{4\gamma}{\delta}\right)$
- $K_2 = 4\left(e^{\delta^{-2}}\right)^2 \left(\left(e^{\delta^{-2}}\right) + 2\right) \cosh\left(\frac{3\gamma}{\delta}\right)$
- $K_3 = 3\left(2\left(e^{\delta^{-2}}\right) + 1\right)$

43.15 Spreadsheet documents

- [Excel file from GitHub repository](#)
- [Google spreadsheet document](#)

44 Kumaraswamy Distribution

44.1 Distribution definition

$$X \sim \text{Kumaraswamy}(\alpha, \beta, \min, \max)$$

44.2 Distribution domain

$$x \in (\min, \max)$$

44.3 Parameters domain and parameters constraints

$$\alpha \in \mathbb{R}^+, \beta \in \mathbb{R}^+, \min \in \mathbb{R}, \max \in \mathbb{R}$$

44.4 Cumulative distribution function

$$F_X(x) = 1 - (1 - z(x)^\alpha)^\beta$$

44.5 Probability density function

$$f_X(x) = \alpha\beta z(x)^{\alpha-1}(1 - z(x)^\alpha)^{\beta-1}$$

44.6 Percent point function/Sample

$$F_X^{-1}(u) = \min + (\max - \min) \times (1 - (1 - u)^{\frac{1}{\beta}})^{\frac{1}{\alpha}}$$

44.7 Parametric centered moments

$$\tilde{\mu}'_k = E[\tilde{X}^k] = \int_0^1 x^k f_{\tilde{X}}(x) dx = \beta \text{Beta}(1 + \frac{k}{\alpha}, \beta)$$

44.8 Parametric mean

$$\text{Mean}(X) = \min + (\max - \min) \times \tilde{\mu}'_1$$

44.9 Parametric variance

$$\text{Variance}(X) = (\max - \min)^2 (\tilde{\mu}'_2 - \tilde{\mu}'_1^2)$$

44.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\tilde{\mu}'_3 - 3\tilde{\mu}'_2\tilde{\mu}'_1 + 2\tilde{\mu}'_1^3}{(\tilde{\mu}'_2 - \tilde{\mu}'_1^2)^{1.5}}$$

44.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\tilde{\mu}'_4 - 4\tilde{\mu}'_1\tilde{\mu}'_3 + 6\tilde{\mu}'_1^2\tilde{\mu}'_2 - 3\tilde{\mu}'_1^4}{(\tilde{\mu}'_2 - \tilde{\mu}'_1^2)^2}$$

44.12 Parametric median

$$\text{Median}(X) = \min + (\max - \min) \times \left(1 - 2^{-1/b}\right)^{1/a}$$

44.13 Parametric mode

$$\text{Mode}(X) = \min + (\max - \min) \times \left(\frac{a-1}{ab-1}\right)^{1/a}$$

44.14 Additional information and definitions

- $\tilde{X} \sim \text{Kumaraswamy}(\alpha, \beta, 0, 1)$
- $z(x) = (x - \min) / (\max - \min)$
- $u : \text{Uniform}[0,1]$ random variable
- $\text{Beta}(x, y) : \text{Beta function}$

44.15 Spreadsheet documents

- [Excel file from GitHub repository](#)
- [Google spreadsheet document](#)

45 Laplace Distribution

45.1 Distribution definition

$$X \sim \text{Laplace}(\mu, b)$$

45.2 Distribution domain

$$x \in (-\infty, \infty)$$

45.3 Parameters domain and parameters constraints

$$\mu \in \mathbb{R}^+, b \in \mathbb{R}^+$$

45.4 Cumulative distribution function

$$F_X(x) = \frac{1}{2} + \frac{1}{2}\text{sign}(x - \mu) \left(1 - \exp\left(-\frac{|x - \mu|}{b}\right) \right)$$

45.5 Probability density function

$$f_X(x) = \frac{1}{2b} \exp\left(-\frac{|x - \mu|}{b}\right)$$

45.6 Percent point function/Sample

$$F_X^{-1}(u) = \mu - b \times \text{sign}\left(u - \frac{1}{2}\right) \ln\left(1 - 2\left|p - \frac{1}{2}\right|\right)$$

45.7 Parametric centered moments

$$\mu'_k = E[X^k] = \int_{-\infty}^{\infty} x^k f_X(x) dx = \left(\frac{1}{2}\right) \sum_{k=0}^r \left[\frac{r!}{(r-k)!} b^k \mu^{(r-k)} \{1 + (-1)^k\} \right]$$

45.8 Parametric mean

$$\text{Mean}(X) = \mu'_1 = \mu$$

45.9 Parametric variance

$$\text{Variance}(X) = \mu'_2 - \mu'^2_1 = 2b^2$$

45.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\mu'_3 - 3\mu'_2\mu'_1 + 2\mu'^3_1}{(\mu'_2 - \mu'^2_1)^{1.5}} = 0$$

45.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\mu'_4 - 4\mu'_1\mu'_3 + 6\mu'^2_1\mu'_2 - 3\mu'^4_1}{(\mu'_2 - \mu'^2_1)^2} = 6$$

45.12 Parametric median

$$\text{Median}(X) = \mu$$

45.13 Parametric mode

$$\text{Mode}(X) = \mu$$

45.14 Additional information and definitions

- μ : Location parameter
- b : Scale parameter
- u : Uniform[0,1] random variable

45.15 Spreadsheet documents

- [Excel file from GitHub repository](#)
- [Google spreadsheet document](#)

46 Levy Distribution

46.1 Distribution definition

$$X \sim \text{Levy}(\mu, c)$$

46.2 Distribution domain

$$x \in [\mu, \infty)$$

46.3 Parameters domain and parameters constraints

$$\mu \in \mathbb{R}, c \in \mathbb{R}^+$$

46.4 Cumulative distribution function

$$F_X(x) = 1 - \operatorname{erf}\left(\sqrt{\frac{c}{2(x-\mu)}}\right)$$

46.5 Probability density function

$$f_X(x) = \sqrt{\frac{c}{2\pi}} \frac{e^{-\frac{c}{2(x-\mu)}}}{(x-\mu)^{3/2}}$$

46.6 Percent point function/Sample

$$F_X^{-1}(u) = \mu + \frac{c}{2(\operatorname{erf}^{-1}(1-u))^2}$$

46.7 Parametric centered moments

$$\mu'_k = E[X^k] = \int_{\mu}^{\infty} x^k f_X(x) dx$$

46.8 Parametric mean

$$\text{Mean}(X) = \mu'_1 = \infty$$

46.9 Parametric variance

$$\text{Variance}(X) = \mu'_2 - \mu'^2_1 = \infty$$

46.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\mu'_3 - 3\mu'_2\mu'_1 + 2\mu'^3_1}{(\mu'_2 - \mu'^2_1)^{1.5}} = \text{undefined}$$

46.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\mu'_4 - 4\mu'_1\mu'_3 + 6\mu'^2_1\mu'_2 - 3\mu'^4_1}{(\mu'_2 - \mu'^2_1)^2} = \text{undefined}$$

46.12 Parametric median

$$\text{Median}(X) = \mu + \frac{c}{2(\operatorname{erf}^{-1}(1/2))^2}$$

46.13 Parametric mode

$$\text{Mode}(X) = \mu + \frac{c}{3}$$

46.14 Additional information and definitions

- μ : Location parameter
- c : Scale parameter
- u : Uniform[0,1] random variable
- $\text{erf}(x)$: Error function
- $\text{erf}^{-1}(x)$: Inverse of error function

46.15 Spreadsheet documents

- [Excel file from GitHub repository](#)
- [Google spreadsheet document](#)

47 Loggamma Distribution

47.1 Distribution definition

$$X \sim \text{LogGamma}(c, \mu, \sigma)$$

47.2 Distribution domain

$$x \in (0, \infty)$$

47.3 Parameters domain and parameters constraints

$$c \in \mathbb{R}^+, \mu \in \mathbb{R}, \sigma \in \mathbb{R}^+$$

47.4 Cumulative distribution function

$$F_X(x) = \frac{\gamma(c, e^x)}{\Gamma(c)} = P(c, e^{z(x)})$$

47.5 Probability density function

$$f_X(x) = \frac{\exp(cz(x) - e^{z(x)})}{\sigma \Gamma(c)}$$

47.6 Percent point function/Sample

$$F_X^{-1}(u) = \mu + \sigma \ln(P^{-1}(u, c))$$

47.7 Parametric centered moments

$$\mu'_k = E[X^k] = \int_{-\infty}^{\infty} x^k f_X(x) dx$$

47.8 Parametric mean

$$\text{Mean}(X) = \mu'_1 = \mu + \sigma \psi_0$$

47.9 Parametric variance

$$\text{Variance}(X) = \mu'_2 - \mu'^2_1 = \sigma^2 \psi_1(c)$$

47.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\mu'_3 - 3\mu'_2\mu'_1 + 2\mu'^3_1}{(\mu'_2 - \mu'^2_1)^{1.5}} = \frac{\psi_2(c)}{\psi_1(c)}$$

47.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\mu'_4 - 4\mu'_1\mu'_3 + 6\mu'^2_1\mu'_2 - 3\mu'^4_1}{(\mu'_2 - \mu'^2_1)^2} = \frac{\psi_3(c)}{\psi_1(c)}$$

47.12 Parametric median

$$\text{Median}(X) = \mu + \sigma \ln(P^{-1}(1/2, c))$$

47.13 Parametric mode

$$\text{Mode}(X) = \mu + \sigma \ln(c)$$

47.14 Additional information and definitions

- μ : Location parameter
- σ : Scale parameter
- $z(x) = (x - \mu) / \sigma$
- u : Uniform[0,1] random variable
- $P(a, x) = \frac{\gamma(a, x)}{\Gamma(a)}$: Regularized lower incomplete gamma function
- $P^{-1}(a, u)$: Inverse of regularized lower incomplete gamma function
- $\gamma(a, x)$: Lower incomplete gamma function
- $\Gamma(x)$: Gamma function
- $\psi_0(x)$: Digamma function
- $\psi_n(x)$: Polygamma function of order $n \in \mathbb{N}$

47.15 Spreadsheet documents

- [Excel file from GitHub repository](#)
- [Google spreadsheet document](#)

48 Logistic Distribution

48.1 Distribution definition

$$X \sim \text{Logistic}(\mu, \sigma)$$

48.2 Distribution domain

$$x \in (-\infty, \infty)$$

48.3 Parameters domain and parameters constraints

$$\mu \in \mathbb{R}, \sigma \in \mathbb{R}^+$$

48.4 Cumulative distribution function

$$F_X(x) = \frac{1}{1 + e^{-(x-\mu)/\sigma}}$$

48.5 Probability density function

$$f_X(x) = \frac{e^{-(x-\mu)/\sigma}}{\sigma (1 + e^{-(x-\mu)/\sigma})^2}$$

48.6 Percent point function/Sample

$$F_X^{-1}(u) = \mu + \sigma \log\left(\frac{u}{1-u}\right)$$

48.7 Parametric centered moments

$$\mu'_k = E[X^k] = \int_{-\infty}^{\infty} x^k f_X(x) dx$$

48.8 Parametric mean

$$\text{Mean}(X) = \mu'_1 = \mu$$

48.9 Parametric variance

$$\text{Variance}(X) = \mu'_2 - \mu_1'^2 = \frac{\sigma^2 \pi^2}{3}$$

48.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\mu'_3 - 3\mu'_2\mu'_1 + 2\mu_1'^3}{(\mu'_2 - \mu_1'^2)^{1.5}} = 0$$

48.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\mu'_4 - 4\mu'_1\mu'_3 + 6\mu_1'^2\mu'_2 - 3\mu_1'^4}{(\mu'_2 - \mu_1'^2)^2} = 3 + 6/5$$

48.12 Parametric median

$$\text{Median}(X) = \mu$$

48.13 Parametric mode

$$\text{Mode}(X) = \mu$$

48.14 Additional information and definitions

- μ : Location parameter
- σ : Scale parameter
- u : Uniform[0,1] random variable

48.15 Spreadsheet documents

- [Excel file from GitHub repository](#)
- [Google spreadsheet document](#)

49 Loglogistic Distribution

49.1 Distribution definition

$$X \sim \text{LogLogistic}(\alpha, \beta)$$

49.2 Distribution domain

$$x \in [0, \infty)$$

49.3 Parameters domain and parameters constraints

$$\alpha \in \mathbb{R}^+, \beta \in \mathbb{R}^+$$

49.4 Cumulative distribution function

$$F_X(x) = \frac{1}{1 + (x/\alpha)^{-\beta}}$$

49.5 Probability density function

$$f_X(x) = \frac{(\beta/\alpha)(x/\alpha)^{\beta-1}}{(1 + (x/\alpha)^{\beta})^2}$$

49.6 Percent point function/Sample

$$F_X^{-1}(u) = \alpha \left(\frac{u}{1-u} \right)^{1/\beta}$$

49.7 Parametric centered moments

$$\mu'_k = E[X^k] = \int_0^\infty x^k f_X(x) dx = \alpha^k \text{Beta}(1 - k/\beta, 1 + k/\beta) = \alpha^k \frac{k\pi/\beta}{\sin(k\pi/\beta)}$$

49.8 Parametric mean

$$\text{Mean}(X) = \mu'_1$$

49.9 Parametric variance

$$\text{Variance}(X) = \mu'_2 - \mu_1'^2$$

49.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\mu'_3 - 3\mu'_2\mu'_1 + 2\mu_1'^3}{(\mu'_2 - \mu_1'^2)^{1.5}}$$

49.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\mu'_4 - 4\mu'_1\mu'_3 + 6\mu_1'^2\mu'_2 - 3\mu_1'^4}{(\mu'_2 - \mu_1'^2)^2}$$

49.12 Parametric median

$$\text{Median}(X) = \alpha$$

49.13 Parametric mode

$$\text{Mode}(X) = \alpha \left(\frac{\beta-1}{\beta+1} \right)^{1/\beta}$$

49.14 Additional information and definitions

- α : Scale parameter
- u : Uniform[0,1] random variable
- Beta(x, y) : Beta function

49.15 Spreadsheet documents

- [Excel file from GitHub repository](#)
- [Google spreadsheet document](#)

50 Loglogistic 3P Distribution

50.1 Distribution definition

$$X \sim \text{LogLogistic}_{3P}(\text{Loc}, \alpha, \beta)$$

50.2 Distribution domain

$$x \in [\text{Loc}, \infty)$$

50.3 Parameters domain and parameters constraints

$$\text{Loc} \in \mathbb{R}, \alpha \in \mathbb{R}^+, \beta \in \mathbb{R}^+$$

50.4 Cumulative distribution function

$$F_X(x) = \frac{1}{1 + ((x - \text{Loc})/\alpha)^{-\beta}}$$

50.5 Probability density function

$$f_X(x) = \frac{(\beta/\alpha)((x - \text{Loc})/\alpha)^{\beta-1}}{(1 + ((x - \text{Loc})/\alpha)^\beta)^2}$$

50.6 Percent point function/Sample

$$F_X^{-1}(u) = \text{Loc} + \alpha \left(\frac{u}{1-u} \right)^{1/\beta}$$

50.7 Parametric centered moments

$$\tilde{\mu}'_k = E[\tilde{X}^k] = \int_0^\infty x^k f_{\tilde{X}}(x) dx = \alpha^k \text{Beta}(1 - k/\beta, 1 + k/\beta) = \alpha^k \frac{k\pi/\beta}{\sin(k\pi/\beta)}$$

50.8 Parametric mean

$$\text{Mean}(X) = \text{Loc} + \tilde{\mu}'_1$$

50.9 Parametric variance

$$\text{Variance}(X) = \tilde{\mu}'_2 - \tilde{\mu}'_1^2$$

50.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\tilde{\mu}'_3 - 3\tilde{\mu}'_2\tilde{\mu}'_1 + 2\tilde{\mu}'_1^3}{(\tilde{\mu}'_2 - \tilde{\mu}'_1^2)^{1.5}}$$

50.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\tilde{\mu}'_4 - 4\tilde{\mu}'_1\tilde{\mu}'_3 + 6\tilde{\mu}'_1^2\tilde{\mu}'_2 - 3\tilde{\mu}'_1^4}{(\tilde{\mu}'_2 - \tilde{\mu}'_1^2)^2}$$

50.12 Parametric median

$$\text{Median}(X) = \text{Loc} + \alpha$$

50.13 Parametric mode

$$\text{Mode}(X) = \text{Loc} + \alpha \left(\frac{\beta - 1}{\beta + 1} \right)^{1/\beta}$$

50.14 Additional information and definitions

- $\tilde{X} \sim \text{LogLogistic}(\alpha, \beta)$
- Loc : Location parameter
- α : Scale parameter
- u : Uniform[0,1] random variable
- $\text{Beta}(x, y)$: Beta function

50.15 Spreadsheet documents

- [Excel file from GitHub repository](#)
- [Google spreadsheet document](#)

51 Lognormal Distribution

51.1 Distribution definition

$$X \sim \text{LogNormal}(\mu, \sigma)$$

51.2 Distribution domain

$$x \in (-\infty, \infty)$$

51.3 Parameters domain and parameters constraints

$$\mu \in \mathbb{R}, \sigma \in \mathbb{R}^+$$

51.4 Cumulative distribution function

$$F_X(x) = \frac{1}{2} \left[1 + \operatorname{erf} \left(\frac{\ln(x) - \mu}{\sigma\sqrt{2}} \right) \right]$$

51.5 Probability density function

$$f_X(x) = \frac{1}{x\sigma\sqrt{2\pi}} \exp \left(-\frac{(\ln(x) - \mu)^2}{2\sigma^2} \right)$$

51.6 Percent point function/Sample

$$F_X^{-1}(u) = \exp(\mu + \sqrt{2\sigma^2} \operatorname{erf}^{-1}(2u - 1))$$

51.7 Parametric centered moments

$$\mu'_k = E[X^k] = \int_{-\infty}^{\infty} x^k f_X(x) dx = e^{k\mu + k^2\sigma^2/2}$$

51.8 Parametric mean

$$\text{Mean}(X) = \mu'_1 = e^{\mu + \frac{\sigma^2}{2}}$$

51.9 Parametric variance

$$\text{Variance}(X) = \mu'_2 - \mu_1'^2 = e^{2\mu + \sigma^2}(e^{\sigma^2} - 1)$$

51.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\mu'_3 - 3\mu'_2\mu'_1 + 2\mu_1'^3}{(\mu'_2 - \mu_1'^2)^{1.5}} = (e^{\sigma^2} + 2)\sqrt{e^{\sigma^2} - 1}$$

51.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\mu'_4 - 4\mu'_1\mu'_3 + 6\mu_1'^2\mu'_2 - 3\mu_1'^4}{(\mu'_2 - \mu_1'^2)^2} = e^{4\sigma^2} + 2e^{3\sigma^2} + 3e^{2\sigma^2} - 3$$

51.12 Parametric median

$$\text{Median}(X) = \exp(\mu)$$

51.13 Parametric mode

$$\text{Mode}(X) = \exp(\mu - \sigma^2)$$

51.14 Additional information and definitions

- μ : Location parameter
- σ : Scale parameter
- u : Uniform[0,1] random variable

51.15 Spreadsheet documents

- [Excel file from GitHub repository](#)
- [Google spreadsheet document](#)

52 Maxwell Distribution

52.1 Distribution definition

$$X \sim \text{Maxwell}(\alpha, \text{Loc})$$

52.2 Distribution domain

$$x \in (0, \infty)$$

52.3 Parameters domain and parameters constraints

$$\alpha \in \mathbb{R}^+, \text{Loc} \in \mathbb{R}$$

52.4 Cumulative distribution function

$$F_X(x) = \text{erf}\left(\frac{x - \text{Loc}}{\sqrt{2}\alpha}\right) - \sqrt{\frac{2}{\pi}} \frac{(x - \text{Loc})e^{-(x - \text{Loc})^2/(2\alpha^2)}}{\alpha}$$

52.5 Probability density function

$$f_X(x) = \sqrt{\frac{2}{\pi}} \frac{(x - \text{Loc})^2 e^{-(x - \text{Loc})^2/(2\alpha^2)}}{\alpha^3}$$

52.6 Percent point function/Sample

$$F_X^{-1}(u) = \text{Loc} + \alpha \sqrt{2P^{-1}(1.5, u)}$$

52.7 Parametric centered moments

$$\mu'_k = E[X^k] = \int_{-\infty}^{\infty} x^k f_X(x) dx$$

52.8 Parametric mean

$$\text{Mean}(X) = \mu'_1 = \text{Loc} + 2\alpha \sqrt{\frac{2}{\pi}}$$

52.9 Parametric variance

$$\text{Variance}(X) = \mu'_2 - \mu_1'^2 = \frac{\alpha^2(3\pi - 8)}{\pi}$$

52.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\mu'_3 - 3\mu'_2\mu'_1 + 2\mu_1'^3}{(\mu'_2 - \mu_1'^2)^{1.5}} = \frac{2\sqrt{2}(16 - 5\pi)}{(3\pi - 8)^{3/2}}$$

52.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\mu'_4 - 4\mu'_1\mu'_3 + 6\mu_1'^2\mu'_2 - 3\mu_1'^4}{(\mu'_2 - \mu_1'^2)^2} = 4 \frac{(-96 + 40\pi - 3\pi^2)}{(3\pi - 8)^2} + 3$$

52.12 Parametric median

$$\text{Median}(X) = \text{Loc} + \alpha \sqrt{2P^{-1}\left(1.5, \frac{1}{2}\right)}$$

52.13 Parametric mode

$$\text{Mode}(X) = \text{Loc} + \alpha\sqrt{2}$$

52.14 Additional information and definitions

- Loc : Location parameter
- α : Scale parameter
- u : Uniform[0,1] random variable
- $P^{-1}(a, u)$: Inverse of regularized lower incomplete gamma function

52.15 Spreadsheet documents

- [Excel file from GitHub repository](#)
- [Google spreadsheet document](#)

53 Moyal Distribution

53.1 Distribution definition

$$X \sim \text{Moyal}(\mu, \sigma)$$

53.2 Distribution domain

$$x \in (-\infty, \infty)$$

53.3 Parameters domain and parameters constraints

$$\mu \in \mathbb{R}, \sigma \in \mathbb{R}^+$$

53.4 Cumulative distribution function

$$F_X(x) = 1 - P\left(\frac{1}{2}, \frac{e^{-z(x)}}{2}\right) = 1 - \text{erf}\left(\frac{\exp(-0.5z(x))}{\sqrt{2}}\right)$$

53.5 Probability density function

$$f_X(x) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2}\left(z(x) + e^{-z(x)}\right)\right)$$

53.6 Percent point function/Sample

$$F_X^{-1}(u) = \mu + \sigma \ln\left[\Phi^{-1}\left(\left(\frac{1-u}{2}\right)^2\right)\right] = \mu + \sigma \ln\left[2P^{-1}\left(\frac{1}{2}, 1-u\right)\right]$$

53.7 Parametric centered moments

$$\mu'_k = E[X^k] = \int_{-\infty}^{\infty} x^k f_X(x) dx$$

53.8 Parametric mean

$$\text{Mean}(X) = \mu'_1 = \mu + \sigma(\ln(2) + \gamma)$$

53.9 Parametric variance

$$\text{Variance}(X) = \mu'_2 - \mu'^2_1 = \sigma^2\left(\frac{\pi^2}{2}\right)$$

53.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\mu'_3 - 3\mu'_2\mu'_1 + 2\mu'^3_1}{(\mu'_2 - \mu'^2_1)^{1.5}} = \frac{28\sqrt{2}\zeta(3)}{\pi^3}$$

53.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\mu'_4 - 4\mu'_1\mu'_3 + 6\mu'^2_1\mu'_2 - 3\mu'^4_1}{(\mu'_2 - \mu'^2_1)^2} = 7$$

53.12 Parametric median

$$\text{Median}(X) = \mu + \sigma \ln\left[2P^{-1}\left(\frac{1}{2}, \frac{1}{2}\right)\right]$$

53.13 Parametric mode

$$\text{Mode}(X) = \mu$$

53.14 Additional information and definitions

- μ : Location parameter
- σ : Scale parameter
- $z(x) = (x - \mu) / \sigma$
- $P(a, x) = \frac{\gamma(a, x)}{\Gamma(a)}$: Regularized lower incomplete gamma function
- $P^{-1}(a, u)$: Inverse of regularized lower incomplete gamma function
- $\gamma(a, x)$: Lower incomplete gamma function
- $\Gamma(x)$: Gamma function
- $\text{erf}(x)$: Error function
- $\Phi^{-1}(x)$: PPF normal standard distribution
- γ : Euler-Mascheroni constant = 0.5772156649
- $\zeta(3)$: Apéry's constant = 1.2020569031

53.15 Spreadsheet documents

- [Excel file from GitHub repository](#)
- [Google spreadsheet document](#)

54 Nakagami Distribution

54.1 Distribution definition

$$X \sim \text{Nakagami}(m, \Omega)$$

54.2 Distribution domain

$$x \in (0, \infty)$$

54.3 Parameters domain and parameters constraints

$$m \in \mathbb{R}_{\geq \frac{1}{2}}^+, \Omega \in \mathbb{R}^+$$

54.4 Cumulative distribution function

$$F_X(x) = \frac{\gamma\left(m, \frac{m}{\Omega}x^2\right)}{\Gamma(m)} = P\left(m, \frac{m}{\Omega}x^2\right)$$

54.5 Probability density function

$$f_X(x) = \frac{2m^m}{\Gamma(m)\Omega^m} x^{2m-1} \exp\left(-\frac{m}{\Omega}x^2\right)$$

54.6 Percent point function/Sample

$$F_X^{-1}(u) = \sqrt{\frac{\Omega}{m} P^{-1}(m, u)}$$

54.7 Parametric centered moments

$$\mu'_k = E[X^k] = \int_{-\infty}^{\infty} x^k f_X(x) dx$$

54.8 Parametric mean

$$\text{Mean}(X) = \mu'_1 = \frac{\Gamma(m + \frac{1}{2})}{\Gamma(m)} \left(\frac{\Omega}{m}\right)^{1/2}$$

54.9 Parametric variance

$$\text{Variance}(X) = \mu'_2 - \mu_1'^2 = \Omega \left(1 - \frac{1}{m} \left(\frac{\Gamma(m + \frac{1}{2})}{\Gamma(m)}\right)^2\right)$$

54.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\mu'_3 - 3\mu'_2\mu'_1 + 2\mu_1'^3}{(\mu'_2 - \mu_1'^2)^{1.5}} = \frac{\frac{\Gamma(m + \frac{1}{2})}{\Gamma(m)\sqrt{m}} \left(1 - 4m \left(1 - \frac{1}{m} \left(\frac{\Gamma(m + \frac{1}{2})}{\Gamma(m)}\right)^2\right)\right)}{2m \left(1 - \frac{1}{m} \left(\frac{\Gamma(m + \frac{1}{2})}{\Gamma(m)}\right)^2\right)^{3/2}}$$

54.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\mu'_4 - 4\mu'_1\mu'_3 + 6\mu_1'^2\mu'_2 - 3\mu_1'^4}{(\mu'_2 - \mu_1'^2)^2} = 3 + \frac{-6 \left(\frac{\Gamma(m + \frac{1}{2})}{\Gamma(m)\sqrt{m}}\right)^4 m + (8m - 2) \left(\frac{\Gamma(m + \frac{1}{2})}{\Gamma(m)\sqrt{m}}\right)^2 - 2m + 1}{m \left(1 - \frac{1}{m} \left(\frac{\Gamma(m + \frac{1}{2})}{\Gamma(m)}\right)^2\right)^2}$$

54.12 Parametric median

$$\text{Median}(X) = \sqrt{\frac{\Omega}{m} P^{-1}\left(m, \frac{1}{2}\right)}$$

54.13 Parametric mode

$$\text{Mode}(X) = \frac{\sqrt{2}}{2} \left(\frac{(2m-1)\Omega}{m} \right)^{1/2}$$

54.14 Additional information and definitions

- u : Uniform[0,1] random variable
- $P(a, x) = \frac{\gamma(a, x)}{\Gamma(a)}$: Regularized lower incomplete gamma function
- $P^{-1}(a, u)$: Inverse of regularized lower incomplete gamma function

54.15 Spreadsheet documents

- [Excel file from GitHub repository](#)
- [Google spreadsheet document](#)

55 Non Central Chi Square Distribution

55.1 Distribution definition

$$X \sim \text{NonCentralChiSquare}(\lambda, n)$$

55.2 Distribution domain

$$x \in [0, +\infty)$$

55.3 Parameters domain and parameters constraints

$$\lambda \in \mathbb{R}^+, n \in \mathbb{R}^+$$

55.4 Cumulative distribution function

$$F_X(x) = 1 - Q_{\frac{n}{2}}(\sqrt{\lambda}, \sqrt{x})$$

55.5 Probability density function

$$f_X(x) = \frac{1}{2} e^{-(x+\lambda)/2} \left(\frac{x}{\lambda}\right)^{n/4-1/2} I_{n/2-1}(\sqrt{\lambda x})$$

55.6 Percent point function/Sample

$$\text{Sample}_X = \sum_{i=1}^n \left(\sqrt{\frac{\lambda}{n}} + \Phi^{-1}(u_i) \right)^2$$

55.7 Parametric centered moments

$$\mu'_k = E[X^k] = \int_0^\infty x^k f_X(x) dx = 2^{k-1} (k-1)! (n+k\lambda) + \sum_{j=1}^{k-1} \frac{(k-1)! 2^{j-1}}{(k-j)!} (n+j\lambda) \mu'_{k-j}$$

55.8 Parametric mean

$$\text{Mean}(X) = \mu'_1 = n + \lambda$$

55.9 Parametric variance

$$\text{Variance}(X) = \mu'_2 - \mu_1'^2 = 2(n + 2\lambda)$$

55.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\mu'_3 - 3\mu'_2\mu'_1 + 2\mu_1'^3}{(\mu'_2 - \mu_1'^2)^{1.5}} = \frac{2^{3/2}(n + 3\lambda)}{(n + 2\lambda)^{3/2}}$$

55.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\mu'_4 - 4\mu'_1\mu'_3 + 6\mu_1'^2\mu'_2 - 3\mu_1'^4}{(\mu'_2 - \mu_1'^2)^2} = \frac{12(n + 4\lambda)}{(n + 2\lambda)^2}$$

55.12 Parametric median

$$\text{Median}(X) = F_X^{-1}\left(\frac{1}{2}\right)$$

55.13 Parametric mode

$$\text{Mode}(X) = \arg \max_x f_X(x)$$

55.14 Additional information and definitions

- Computing an analytic expression for the inverse of the cumulative distribution function is not feasible. Nonetheless, it is possible to generate a random sample from the distribution.
- u_i : Uniform[0,1] random variable
- $\Phi^{-1}(x)$: PPF normal standard distribution
- $I_\alpha(x)$: Modified Bessel function of the first kind of order $\alpha \in \mathbb{N}$

55.15 Spreadsheet documents

- [Excel file from GitHub repository](#)
- [Google spreadsheet document](#)

56 Non Central F Distribution

56.1 Distribution definition

$$X \sim \text{NonCentralF}(\lambda, n_1, n_2)$$

56.2 Distribution domain

$$x \in [0, \infty)$$

56.3 Parameters domain and parameters constraints

$$\lambda \in \mathbb{R}^+, n_1 \in \mathbb{R}^+, n_2 \in \mathbb{R}^+$$

56.4 Cumulative distribution function

$$F_X(x) = \sum_{j=0}^{\infty} \left(\frac{(\frac{1}{2}\lambda)^j}{j!} e^{-\lambda/2} \right) I_{n_1 x / (n_2 + n_1 x)} \left(\frac{n_1}{2} + j, \frac{n_2}{2} \right)$$

56.5 Probability density function

$$f_X(x) = \sum_{k=0}^{\infty} \frac{e^{-\lambda/2} (\lambda/2)^k}{\text{Beta}(\frac{n_2}{2}, \frac{n_1}{2} + k) k!} \left(\frac{n_1}{n_2} \right)^{\frac{n_1}{2} + k} \left(\frac{n_2}{n_2 + n_1 x} \right)^{\frac{n_1 + n_2}{2} + k} x^{n_1/2 - 1 + k}$$

56.6 Percent point function/Sample

$$\text{Sample}_X = \frac{\left(\sum_{i=1}^{n_1} \left(\sqrt{\frac{\lambda}{n_1}} + \Phi^{-1}(u_i) \right)^2 \right) / n_1}{(2P^{-1}(\frac{n_2}{2}, u)) / n_2}$$

56.7 Parametric centered moments

$$\mu'_k = E[X^k] = \int_0^{\infty} x^k f_X(x) dx = e^{-\lambda/2} \left(\frac{n_1}{n_2} \right)^k \frac{\Gamma(n_1/2 - k)}{\Gamma(n_1/2)} \sum_{r=0}^{\infty} \left(\frac{1}{r!} \right) \left(\frac{\lambda}{2} \right)^r \frac{\Gamma(\frac{n_1}{2} + r + k)}{\Gamma(\frac{n_1}{2} + r)}$$

56.8 Parametric mean

$$\text{Mean}(X) = \mu'_1$$

56.9 Parametric variance

$$\text{Variance}(X) = \mu'_2 - \mu'^2_1$$

56.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\mu'_3 - 3\mu'_2\mu'_1 + 2\mu'^3_1}{(\mu'_2 - \mu'^2_1)^{1.5}}$$

56.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\mu'_4 - 4\mu'_1\mu'_3 + 6\mu'^2_1\mu'_2 - 3\mu'^4_1}{(\mu'_2 - \mu'^2_1)^2}$$

56.12 Parametric median

$$\text{Median}(X) = F_X^{-1}\left(\frac{1}{2}\right)$$

56.13 Parametric mode

$$\text{Mode}(X) = \arg \max_x f_X(x)$$

56.14 Additional information and definitions

- Computing an analytic expression for the inverse of the cumulative distribution function is not feasible. Nonetheless, it is possible to generate a random sample from the distribution.
- u : Uniform[0,1] random variable
- u_i : Uniform[0,1] random variable
- $\Phi^{-1}(x)$: PPF normal standard distribution
- $P^{-1}(a, u)$: Inverse of regularized lower incomplete gamma function
- $I(x, a, b)$: Regularized incomplete beta function
- Beta (x, y) : Beta function

56.15 Spreadsheet documents

- [Excel file from GitHub repository](#)
- [Google spreadsheet document](#)

57 Non Central T Student Distribution

57.1 Distribution definition

$$X \sim \text{NonCentralTStudent}(\lambda, n, \text{Loc}, \text{Sc})$$

57.2 Distribution domain

$$x \in (-\infty, \infty)$$

57.3 Parameters domain and parameters constraints

$$\lambda \in \mathbb{R}, n \in \mathbb{R}^+, \text{Sc} \in \mathbb{R}^+, \text{Loc} \in \mathbb{R}$$

57.4 Cumulative distribution function

$$F_X(x) = \begin{cases} \frac{1}{2} \sum_{j=0}^{\infty} \frac{1}{j!} (-\lambda\sqrt{2})^j e^{\frac{-\lambda^2}{2}} \frac{\Gamma(\frac{j+1}{2})}{\sqrt{\pi}} I_{n/(n+z(x)^2)}\left(\frac{n}{2}, \frac{j+1}{2}\right) & \text{if } z(x) \geq 0 \\ 1 - \frac{1}{2} \sum_{j=0}^{\infty} \frac{1}{j!} (-\lambda\sqrt{2})^j e^{\frac{-\lambda^2}{2}} \frac{\Gamma(\frac{j+1}{2})}{\sqrt{\pi}} I_{n/(n+z(x)^2)}\left(\frac{n}{2}, \frac{j+1}{2}\right) & \text{if } z(x) < 0 \end{cases}$$

57.5 Probability density function

$$f_X(x) = \frac{1}{\text{Sc}} \frac{n^{n/2} \Gamma(n+1)}{2^n e^{\lambda^2/2} (n+z(x)^2)^{n/2} \Gamma(n/2)} \times \left\{ \frac{\sqrt{2} \lambda z(x) {}_1F_1\left(\frac{n}{2}+1, \frac{3}{2}, \frac{\lambda^2 z(x)^2}{2(n+z(x)^2)}\right)}{(n+z(x)^2) \Gamma\left(\frac{n+1}{2}\right)} - \frac{{}_1F_1\left(\frac{n+1}{2}, \frac{1}{2}, \frac{\lambda^2 z(x)^2}{2(n+z(x)^2)}\right)}{\sqrt{n+z(x)^2} \Gamma\left(\frac{n}{2}+1\right)} \right\}$$

57.6 Percent point function/Sample

$$\text{Sample}_X = \text{Loc} + \text{Sc} \frac{(\lambda + \Phi^{-1}(u))}{\left(\sqrt{2\text{P}^{-1}\left(\frac{n}{2}, u\right)}\right)/n}$$

57.7 Parametric centered moments

$$\tilde{\mu}'_k = E[\tilde{X}^k] = \int_0^\infty x^k f_{\tilde{X}}(x) dx = \frac{e^{-\lambda^2/2}}{\sqrt{n\pi}\Gamma(n/2)} \Gamma\left(\frac{n-k}{2}\right) n^{k/2} \sum_{r=0}^{\infty} \frac{\lambda^r 2^{r/2}}{r!} \Gamma\left(\frac{r+k+1}{2}\right)$$

57.8 Parametric mean

$$\text{Mean}(X) = \text{Loc} + \text{Sc} \tilde{\mu}'_1$$

57.9 Parametric variance

$$\text{Variance}(X) = \text{Sc}^2 (\tilde{\mu}'_2 - \tilde{\mu}_1'^2)$$

57.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\tilde{\mu}'_3 - 3\tilde{\mu}'_2\tilde{\mu}'_1 + 2\tilde{\mu}_1'^3}{(\tilde{\mu}'_2 - \tilde{\mu}_1'^2)^{1.5}}$$

57.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\tilde{\mu}'_4 - 4\tilde{\mu}'_1\tilde{\mu}'_3 + 6\tilde{\mu}_1'^2\tilde{\mu}'_2 - 3\tilde{\mu}_1'^4}{(\tilde{\mu}'_2 - \tilde{\mu}_1'^2)^2}$$

57.12 Parametric median

$$\text{Median}(X) = F_X^{-1}\left(\frac{1}{2}\right)$$

57.13 Parametric mode

$$\text{Mode}(X) = \arg \max_x f_X(x)$$

57.14 Additional information and definitions

- $\tilde{X} \sim \text{NonCentralTStudent}(\lambda, n, 0, 1)$
- Computing an analytic expression for the inverse of the cumulative distribution function is not feasible. Nonetheless, it is possible to generate a random sample from the distribution.
- Loc : Location parameter
- Sc : Scale parameter
- $z(x) = (x - \text{Loc}) / \text{Sc}$
- u : Uniform[0,1] random variable
- $P^{-1}(a, u)$: Inverse of regularized lower incomplete gamma function
- $I_\alpha(x)$: Modified Bessel function of the first kind of order $\alpha \in \mathbb{N}$
- ${}_1F_1(a, b, z)$: Kummer's confluent hypergeometric function

57.15 Spreadsheet documents

- [Excel file from GitHub repository](#)
- [Google spreadsheet document](#)

58 Normal Distribution

58.1 Distribution definition

$$X \sim \text{Normal}(\mu, \sigma)$$

58.2 Distribution domain

$$x \in (-\infty, \infty)$$

58.3 Parameters domain and parameters constraints

$$\mu \in \mathbb{R}, \sigma \in \mathbb{R}^+$$

58.4 Cumulative distribution function

$$F_X(x) = \frac{1}{2} \left[1 + \operatorname{erf} \left(\frac{x - \mu}{\sigma \sqrt{2}} \right) \right] = \Phi \left(\frac{x - \mu}{\sigma} \right)$$

58.5 Probability density function

$$f_X(x) = \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{1}{2} \left(\frac{x - \mu}{\sigma} \right)^2} = \phi \left(\frac{x - \mu}{\sigma} \right)$$

58.6 Percent point function/Sample

$$F_X^{-1}(u) = \mu + \sigma \sqrt{2} \operatorname{erf}^{-1}(2u - 1) = \mu + \sigma \Phi^{-1}(u)$$

58.7 Parametric centered moments

$$\mu'_k = E[X^k] = \int_{-\infty}^{\infty} x^k f_X(x) dx = \sigma^k \cdot (-i\sqrt{2})^k U \left(-\frac{k}{2}, \frac{1}{2}, -\frac{1}{2} \left(\frac{\mu}{\sigma} \right)^2 \right)$$

58.8 Parametric mean

$$\text{Mean}(X) = \mu'_1 = \mu$$

58.9 Parametric variance

$$\text{Variance}(X) = \mu'_2 - \mu_1'^2 = \sigma^2$$

58.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\mu'_3 - 3\mu'_2\mu'_1 + 2\mu_1'^3}{(\mu'_2 - \mu_1'^2)^{1.5}} = 0$$

58.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\mu'_4 - 4\mu'_1\mu'_3 + 6\mu_1'^2\mu'_2 - 3\mu_1'^4}{(\mu'_2 - \mu_1'^2)^2} = 3$$

58.12 Parametric median

$$\text{Median}(X) = \mu$$

58.13 Parametric mode

$$\text{Mode}(X) = \mu$$

58.14 Additional information and definitions

- μ : Location parameter
- σ : Scale parameter
- u : Uniform[0,1] random variable
- $U(a, b, z)$: Tricomi's confluent hypergeometric function
- $\Phi(x)$: CDF normal standard distribution
- $\Phi^{-1}(x)$: PPF normal standard distribution
- $\phi(x)$: PDF normal standard distribution
- $\text{erf}(x)$: Error function
- $\text{erf}^{-1}(x)$: Inverse of error function

58.15 Spreadsheet documents

- [Excel file from GitHub repository](#)
- [Google spreadsheet document](#)

59 Pareto First Kind Distribution

59.1 Distribution definition

$$X \sim \text{ParetoFirstKind}(x_m, \alpha, \text{Loc})$$

59.2 Distribution domain

$$x \in [\text{Loc} + x_m, \infty)$$

59.3 Parameters domain and parameters constraints

$$x_m \in \mathbb{R}^+, \alpha \in \mathbb{R}^+, \text{Loc} \in \mathbb{R}$$

59.4 Cumulative distribution function

$$F_X(x) = 1 - \left(\frac{x_m}{x - \text{Loc}} \right)^\alpha$$

59.5 Probability density function

$$f_X(x) = \frac{\alpha x_m^\alpha}{(x - \text{Loc})^{\alpha+1}}$$

59.6 Percent point function/Sample

$$F_X^{-1}(u) = \text{Loc} + x_m(1 - u)^{-\frac{1}{\alpha}}$$

59.7 Parametric centered moments

$$\tilde{\mu}'_k = E[\tilde{X}^k] = \int_{x_m}^{\infty} x^k f_{\tilde{X}}(x) dx = \begin{cases} \infty & \text{if } \alpha \leq k \\ \frac{\alpha x_m^k}{\alpha - k} & \text{if } \alpha > k \end{cases}$$

59.8 Parametric mean

$$\text{Mean}(X) = \text{Loc} + \tilde{\mu}'_1 = \text{Loc} + \frac{\alpha x_m}{\alpha - 1} \quad \text{if } \alpha > 1$$

59.9 Parametric variance

$$\text{Variance}(X) = (\tilde{\mu}'_2 - \tilde{\mu}'_1^2) = \frac{x_m^2 \alpha}{(\alpha - 1)^2(\alpha - 2)} \quad \text{if } \alpha > 2$$

59.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\tilde{\mu}'_3 - 3\tilde{\mu}'_2\tilde{\mu}'_1 + 2\tilde{\mu}'_1^3}{(\tilde{\mu}'_2 - \tilde{\mu}'_1^2)^{1.5}} = \frac{2(1 + \alpha)}{\alpha - 3} \sqrt{\frac{\alpha - 2}{\alpha}} \quad \text{if } \alpha > 3$$

59.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\tilde{\mu}'_4 - 4\tilde{\mu}'_1\tilde{\mu}'_3 + 6\tilde{\mu}'_1^2\tilde{\mu}'_2 - 3\tilde{\mu}'_1^4}{(\tilde{\mu}'_2 - \tilde{\mu}'_1^2)^2} = \frac{6(\alpha^3 + \alpha^2 - 6\alpha - 2)}{\alpha(\alpha - 3)(\alpha - 4)} \quad \text{if } \alpha > 4$$

59.12 Parametric median

$$\text{Median}(X) = \text{Loc} + x_m \sqrt[\alpha]{2}$$

59.13 Parametric mode

$$\text{Mode}(X) = \text{Loc} + x_m$$

59.14 Additional information and definitions

- $\tilde{X} \sim \text{ParetoFirstKind}(x_m, \alpha, 0)$
- Loc : Location parameter
- x_m : Scale parameter
- u : Uniform[0,1] random variable

59.15 Spreadsheet documents

- [Excel file from GitHub repository](#)
- [Google spreadsheet document](#)

60 Pareto Second Kind Distribution

60.1 Distribution definition

$$X \sim \text{ParetoSecondKind}(x_m, \alpha, \text{Loc})$$

60.2 Distribution domain

$$x \in (\text{Loc}, \infty)$$

60.3 Parameters domain and parameters constraints

$$x_m \in \mathbb{R}^+, \alpha \in \mathbb{R}^+, \text{Loc} \in \mathbb{R}$$

60.4 Cumulative distribution function

$$F_X(x) = 1 - \left[1 + \frac{x - \text{Loc}}{x_m} \right]^{-\alpha}$$

60.5 Probability density function

$$f_X(x) = \frac{\alpha}{x_m} \left[1 + \frac{x - \text{Loc}}{x_m} \right]^{-(\alpha+1)}$$

60.6 Percent point function/Sample

$$F_X^{-1}(u) = \text{Loc} + x_m \left[(1 - p)^{-\frac{1}{\alpha}} - 1 \right]$$

60.7 Parametric centered moments

$$\tilde{\mu}'_k = E[\tilde{X}^k] = \int_0^\infty x^k f_{\tilde{X}}(x) dx = \frac{x_m^k \Gamma(\alpha - k) \Gamma(1 + k)}{\Gamma(\alpha)}$$

60.8 Parametric mean

$$\text{Mean}(X) = \tilde{\mu}'_1 = \frac{x_m}{\alpha - 1} \quad \text{if } \alpha > 1$$

60.9 Parametric variance

$$\text{Variance}(X) = \tilde{\mu}'_2 - \tilde{\mu}'_1^2 = \frac{x_m^2 \alpha}{(\alpha - 1)^2 (\alpha - 2)} \quad \text{if } \alpha > 2$$

60.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\tilde{\mu}'_3 - 3\tilde{\mu}'_2\tilde{\mu}'_1 + 2\tilde{\mu}'_1^3}{(\tilde{\mu}'_2 - \tilde{\mu}'_1^2)^{1.5}} = \frac{2(1 + \alpha)}{\alpha - 3} \sqrt{\frac{\alpha - 2}{\alpha}} \quad \text{if } \alpha > 3$$

60.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\tilde{\mu}'_4 - 4\tilde{\mu}'_1\tilde{\mu}'_3 + 6\tilde{\mu}'_1^2\tilde{\mu}'_2 - 3\tilde{\mu}'_1^4}{(\tilde{\mu}'_2 - \tilde{\mu}'_1^2)^2} = \frac{6(\alpha^3 + \alpha^2 - 6\alpha - 2)}{\alpha(\alpha - 3)(\alpha - 4)} \quad \text{if } \alpha > 4$$

60.12 Parametric median

$$\text{Median}(X) = x_m \left(\sqrt[3]{2} - 1 \right)$$

60.13 Parametric mode

$$\text{Mode}(X) = 0$$

60.14 Additional information and definitions

- $X \sim \text{ParetoSecondKind}(x_m, \alpha, 0)$
- x_m : Scale parameter
- u : Uniform[0,1] random variable
- $\Gamma(x)$: Gamma function

60.15 Spreadsheet documents

- [Excel file from GitHub repository](#)
- [Google spreadsheet document](#)

61 Pert Distribution

61.1 Distribution definition

$$X \sim \text{Pert}(a, b, c)$$

61.2 Distribution domain

$$x \in [a, c]$$

61.3 Parameters domain and parameters constraints

$$a \in \mathbb{R}, b \in \mathbb{R}, c \in \mathbb{R}, a < b < c$$

61.4 Cumulative distribution function

$$F_X(x) = I(z(x), \alpha_1, \alpha_2)$$

61.5 Probability density function

$$f_X(x) = \frac{(x-a)^{\alpha_1-1}(c-x)^{\alpha_2-1}}{\text{Beta}(\alpha_1, \alpha_2)(c-a)^{\alpha_1+\alpha_2-1}}$$

61.6 Percent point function/Sample

$$F_X^{-1}(u) = a + (c-a) \cdot I^{-1}(u, \alpha_1, \alpha_2)$$

61.7 Parametric centered moments

$$\mu'_k = E[X^k] = \int_a^c x^k f_X(x) dx$$

61.8 Parametric mean

$$\text{Mean}(X) = \mu'_1 = \frac{a+4b+c}{6}$$

61.9 Parametric variance

$$\text{Variance}(X) = \mu'_2 - \mu_1'^2 = \frac{(\text{Mean}(X) - a)(c - \text{Mean}(X))}{7}$$

61.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\mu'_3 - 3\mu'_2\mu'_1 + 2\mu_1'^3}{(\mu'_2 - \mu_1'^2)^{1.5}} = \frac{2(\alpha_2 - \alpha_1)\sqrt{\alpha_1 + \alpha_2 + 1}}{(\alpha_1 + \alpha_2 + 2)\sqrt{\alpha_1\alpha_2}}$$

61.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\mu'_4 - 4\mu'_1\mu'_3 + 6\mu_1'^2\mu'_2 - 3\mu_1'^4}{(\mu'_2 - \mu_1'^2)^2} = \frac{6[(\alpha_1 - \alpha_2)^2(\alpha_1 + \alpha_2 + 1) - \alpha_1\alpha_2(\alpha_1 + \alpha_2 + 2)]}{\alpha_1\alpha_2(\alpha_1 + \alpha_2 + 2)(\alpha_1 + \alpha_2 + 3)} + 3$$

61.12 Parametric median

$$\text{Median}(X) = a + (c-a) \cdot I^{-1}\left(\frac{1}{2}, \alpha_1, \alpha_2\right)$$

61.13 Parametric mode

$$\text{Mode}(X) = b$$

61.14 Additional information and definitions

- $z(x) = (x - a) / (c - a)$
- u : Uniform[0,1] random variable
- $\alpha_1 = \frac{4b+c-5a}{c-a}, \alpha_2 = \frac{5c-a-4b}{c-a}$
- $I(x, a, b)$: Regularized incomplete beta function
- $I^{-1}(x, a, b)$: Inverse of regularized incomplete beta function
- Beta(x, y) : Beta function

61.15 Spreadsheet documents

- [Excel file from GitHub repository](#)
- [Google spreadsheet document](#)

62 Power Function Distribution

62.1 Distribution definition

$$X \sim \text{PowerFunction}(\alpha, a, b)$$

62.2 Distribution domain

$$x \in [a, b]$$

62.3 Parameters domain and parameters constraints

$$\alpha \in \mathbb{R}^+, a \in \mathbb{R}, b \in \mathbb{R}, a < b$$

62.4 Cumulative distribution function

$$F_X(x) = \left(\frac{x-a}{b-a} \right)^\alpha$$

62.5 Probability density function

$$f_X(x) = \frac{\alpha(x-a)^{\alpha-1}}{(b-a)^\alpha}$$

62.6 Percent point function/Sample

$$F_X^{-1}(u) = [a + u(b-a)]^{-\alpha}$$

62.7 Parametric centered moments

$$\mu'_k = E[X^k] = \int_a^b x^k f_X(x) dx$$

62.8 Parametric mean

$$\text{Mean}(X) = \mu'_1 = \frac{a+b\alpha}{\alpha+1}$$

62.9 Parametric variance

$$\text{Variance}(X) = \mu'_2 - \mu'^2_1 = \frac{2a^2 + 2ab\alpha + b^2\alpha(\alpha+1)}{(\alpha+1)(\alpha+2)} - \text{Mean}(X)^2$$

62.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\mu'_3 - 3\mu'_2\mu'_1 + 2\mu'^3_1}{(\mu'_2 - \mu'^2_1)^{1.5}} = 2(1-\alpha) \sqrt{\frac{\alpha+2}{\alpha(\alpha+3)}}$$

62.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\mu'_4 - 4\mu'_1\mu'_3 + 6\mu'^2_1\mu'_2 - 3\mu'^4_1}{(\mu'_2 - \mu'^2_1)^2} = \frac{6(\alpha^3 - \alpha^2 - 6\alpha + 2)}{\alpha(\alpha+3)(\alpha+4)} + 3$$

62.12 Parametric median

$$\text{Median}(X) = \left[a + \frac{1}{2}(b-a) \right]^{-\alpha}$$

62.13 Parametric mode

$$\text{Mode}(X) = \text{undefined}$$

62.14 Additional information and definitions

- a : Location parameter
- $b - a$: Scale parameter
- u : Uniform[0,1] random variable

62.15 Spreadsheet documents

- [Excel file from GitHub repository](#)
- [Google spreadsheet document](#)

63 Rayleigh Distribution

63.1 Distribution definition

$$X \sim \text{Rayleigh}(\gamma, \sigma)$$

63.2 Distribution domain

$$x \in [\gamma, \infty)$$

63.3 Parameters domain and parameters constraints

$$\gamma \in \mathbb{R}, \sigma \in \mathbb{R}^+$$

63.4 Cumulative distribution function

$$F_X(x) = 1 - e^{-z(x)^2/2}$$

63.5 Probability density function

$$f_X(x) = z(x) \times e^{-z(x)^2/2}/\sigma$$

63.6 Percent point function/Sample

$$F_X^{-1}(u) = \gamma + \sigma \sqrt{-2 \log(1-u)}$$

63.7 Parametric centered moments

$$\mu'_k = E[\tilde{X}^k] = \int_0^\infty x^k f_{\tilde{X}}(x) dx = \sqrt{2k} \Gamma\left(\frac{k}{2} + 1\right)$$

63.8 Parametric mean

$$\text{Mean}(X) = \gamma + \sigma \cdot \mu'_1 = \gamma + \sigma \sqrt{\frac{\pi}{2}}$$

63.9 Parametric variance

$$\text{Variance}(X) = \sigma^2(\mu'_2 - \mu_1'^2) = \sigma^2 \frac{4 - \pi}{2}$$

63.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\mu'_3 - 3\mu'_2\mu'_1 + 2\mu_1'^3}{(\mu'_2 - \mu_1'^2)^{1.5}} = \frac{2(\pi - 3)\sqrt{\pi}}{(4 - \pi)^{3/2}}$$

63.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\mu'_4 - 4\mu'_1\mu'_3 + 6\mu_1'^2\mu'_2 - 3\mu_1'^4}{(\mu'_2 - \mu_1'^2)^2} = 3 + \frac{24\pi - 6\pi^2 - 16}{(4 - \pi)^2}$$

63.12 Parametric median

$$\text{Median}(X) = \gamma + \sigma \sqrt{-2 \log\left(\frac{1}{2}\right)}$$

63.13 Parametric mode

$$\text{Mode}(X) = \gamma + \sigma$$

63.14 Additional information and definitions

- $\tilde{X} \sim \text{Rayleigh}(0, 1)$
- γ : Location parameter
- σ : Scale parameter
- $z(x) = (x - \gamma) / \sigma$
- u : Uniform[0,1] random variable
- $\Gamma(x)$: Gamma function

63.15 Spreadsheet documents

- [Excel file from GitHub repository](#)
- [Google spreadsheet document](#)

64 Reciprocal Distribution

64.1 Distribution definition

$$X \sim \text{Reciprocal}(a, b)$$

64.2 Distribution domain

$$x \in [a, b]$$

64.3 Parameters domain and parameters constraints

$$a \in \mathbb{R}^+, b \in \mathbb{R}^+, a < b$$

64.4 Cumulative distribution function

$$F_X(x) = \frac{\ln(x) - \ln(a)}{\ln(b) - \ln(a)}$$

64.5 Probability density function

$$f_X(x) = \frac{1}{x(\ln(b) - \ln(a))}$$

64.6 Percent point function/Sample

$$F_X^{-1}(u) = \exp(\ln(a) + u \times (\ln(b) - \ln(a)))$$

64.7 Parametric centered moments

$$\mu'_k = E[X^k] = \int_a^b x^k f_X(x) dx = \frac{b^k - a^k}{k(\ln(b) - \ln(a))}$$

64.8 Parametric mean

$$\text{Mean}(X) = \mu'_1$$

64.9 Parametric variance

$$\text{Variance}(X) = \mu'_2 - \mu'^2_1$$

64.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\mu'_3 - 3\mu'_2\mu'_1 + 2\mu'^3_1}{(\mu'_2 - \mu'^2_1)^{1.5}}$$

64.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\mu'_4 - 4\mu'_1\mu'_3 + 6\mu'^2_1\mu'_2 - 3\mu'^4_1}{(\mu'_2 - \mu'^2_1)^2}$$

64.12 Parametric median

$$\text{Median}(X) = \exp \left[\ln(a) + \frac{(\ln(b) - \ln(a))}{2} \right]$$

64.13 Parametric mode

$$\text{Mode}(X) = a$$

64.14 Additional information and definitions

- u : Uniform[0,1] random variable

64.15 Spreadsheet documents

- [Excel file from GitHub repository](#)
- [Google spreadsheet document](#)

65 Rice Distribution

65.1 Distribution definition

$$X \sim \text{Rice}(v, \sigma)$$

65.2 Distribution domain

$$x \in [0, \infty)$$

65.3 Parameters domain and parameters constraints

$$v \in \mathbb{R}^+, \sigma \in \mathbb{R}^+$$

65.4 Cumulative distribution function

$$F_X(x) = 1 - Q_1\left(\frac{v}{\sigma}, \frac{x}{\sigma}\right)$$

65.5 Probability density function

$$f_X(x) = \frac{x}{\sigma^2} \exp\left(-\frac{(x^2 + v^2)}{2\sigma^2}\right) I_0\left(\frac{xv}{\sigma^2}\right)$$

65.6 Percent point function/Sample

$$\text{Sample}_X = \sqrt{\Phi^{-1}(u_1, v, \sigma)^2 + \Phi^{-1}(u_2, 0, \sigma)^2}$$

65.7 Parametric centered moments

$$\mu'_k = E[X^k] = \int_{-\infty}^{\infty} x^k f_X(x) dx = \sigma^k 2^{k/2} \Gamma(1 + k/2) L_{k/2}(-v^2/2\sigma^2)$$

65.8 Parametric mean

$$\text{Mean}(X) = \mu'_1$$

65.9 Parametric variance

$$\text{Variance}(X) = \mu'_2 - \mu_1'^2$$

65.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\mu'_3 - 3\mu'_2\mu'_1 + 2\mu_1'^3}{(\mu'_2 - \mu_1'^2)^{1.5}}$$

65.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\mu'_4 - 4\mu'_1\mu'_3 + 6\mu_1'^2\mu'_2 - 3\mu_1'^4}{(\mu'_2 - \mu_1'^2)^2}$$

65.12 Parametric median

$$\text{Median}(X) = F_X^{-1}\left(\frac{1}{2}\right)$$

65.13 Parametric mode

$$\text{Mode}(X) = \arg \max_x f_X(x)$$

65.14 Additional information and definitions

- Computing an analytic expression for the inverse of the cumulative distribution function is not feasible. Nonetheless, it is possible to generate a random sample from the distribution.
- $\Phi^{-1}(u, mean, variance)$: Inverse of cumulative function from normal distribution
- $L_r(x)$: Laguerre polynomials of order $r \in \mathbb{R}$
- $L_{\frac{1}{2}}(x) = e^{x/2}(x)I_1\left(\frac{x}{2}\right) - e^{x/2}(x-1)I_0\left(\frac{x}{2}\right)$
- $L_{\frac{3}{2}}(x) = \frac{1}{3}e^{x/2}(2x^2 - 6x + 3)I_0(x/2) - \frac{2}{3}e^{x/2}(x-2)xI_1(x/2)$
- $I_\alpha(x)$: Modified Bessel function of the first kind of order $\alpha \in \mathbb{N}$
- $Q_k(a, b)$: Marcum Q-function of order $k \in \mathbb{N}$
- u_1 : Uniform[0,1] random variable
- u_2 : Uniform[0,1] random variable

65.15 Spreadsheet documents

- [Excel file from GitHub repository](#)
- [Google spreadsheet document](#)

66 Semicircular Distribution

66.1 Distribution definition

$$X \sim \text{Semicircular}(\text{Loc}, R)$$

66.2 Distribution domain

$$x \in [\text{Loc}, \infty)$$

66.3 Parameters domain and parameters constraints

$$\text{Loc} \in \mathbb{R}, R \in \mathbb{R}^+$$

66.4 Cumulative distribution function

$$F_X(x) = \frac{1}{2} + \frac{z(x)\sqrt{R^2 - z(x)^2}}{\pi R^2} + \frac{\arcsin\left(\frac{z(x)}{R}\right)}{\pi}$$

66.5 Probability density function

$$f_X(x) = \frac{2}{\pi R^2} \sqrt{R^2 - z(x)^2}$$

66.6 Percent point function/Sample

$$F_X^{-1}(u) = \text{Loc} + R \times (2I^{-1}(u, 1.5, 1.5) - 1)$$

66.7 Parametric centered moments

$$\mu'_k = E[X^k] = \int_{\text{Loc}}^{\infty} x^k f_X(x) dx$$

66.8 Parametric mean

$$\text{Mean}(X) = \mu'_1 = \text{Loc}$$

66.9 Parametric variance

$$\text{Variance}(X) = \mu'_2 - \mu_1'^2 = \frac{R^2}{4}$$

66.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\mu'_3 - 3\mu'_2\mu'_1 + 2\mu_1'^3}{(\mu'_2 - \mu_1'^2)^{1.5}} = 0$$

66.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\mu'_4 - 4\mu'_1\mu'_3 + 6\mu_1'^2\mu'_2 - 3\mu_1'^4}{(\mu'_2 - \mu_1'^2)^2} = 2$$

66.12 Parametric median

$$\text{Median}(X) = \text{Loc}$$

66.13 Parametric mode

$$\text{Mode}(X) = \text{Loc}$$

66.14 Additional information and definitions

- Loc : Location parameter
- R : Scale parameter
- $z(x) = x - \text{Loc}$
- u : Uniform[0,1] random variable
- $I^{-1}(x, a, b)$: Inverse of regularized incomplete beta function

66.15 Spreadsheet documents

- [Excel file from GitHub repository](#)
- [Google spreadsheet document](#)

67 T Student Distribution

67.1 Distribution definition

$$X \sim \text{TStudent}(\text{df})$$

67.2 Distribution domain

$$x \in (-\infty, \infty)$$

67.3 Parameters domain and parameters constraints

$$\text{df} \in \mathbb{R}^+$$

67.4 Cumulative distribution function

$$F_X(x) = I\left(\frac{x + \sqrt{x^2 + \text{df}}}{2\sqrt{x^2 + \text{df}}}, \frac{\text{df}}{2}, \frac{\text{df}}{2}\right)$$

67.5 Probability density function

$$f_X(x) = \frac{(1 + x^2/\text{df})^{-(1+\text{df})/2}}{\sqrt{\text{df}} \times \text{Beta}\left(\frac{1}{2}, \frac{\text{df}}{2}\right)}$$

67.6 Percent point function/Sample

$$F_X^{-1}(u) = \begin{cases} \sqrt{\frac{\text{df}(1-I^{-1}(u, \text{df}/2, \text{df}/2))}{I^{-1}(u, \text{df}/2, \text{df}/2)}} & \text{if } u \geq \frac{1}{2} \\ -\sqrt{\frac{\text{df}(1-I^{-1}(u, \text{df}/2, \text{df}/2))}{I^{-1}(u, \text{df}/2, \text{df}/2)}} & \text{if } u < \frac{1}{2} \end{cases}$$

67.7 Parametric centered moments

$$\mu'_k = E[X^k] = \int_{-\infty}^{\infty} x^k f_X(x) dx = \begin{cases} 0 & \text{if } k \text{ odd} \wedge 0 < k < \text{df} \\ \text{df}^{k/2} \prod_{i=1}^{k/2} \frac{2i-1}{\text{df}-2i} & \text{if } k \text{ even} \wedge 0 < k < \text{df} \end{cases}$$

67.8 Parametric mean

$$\text{Mean}(X) = \mu'_1 = 0$$

67.9 Parametric variance

$$\text{Variance}(X) = \mu'_2 - \mu_1'^2 = \begin{cases} \text{df}/(\text{df} + 2) & \text{if } \text{df} > 2 \\ \text{undefined} & \text{if } \text{df} \leq 2 \end{cases}$$

67.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\mu'_3 - 3\mu'_2\mu'_1 + 2\mu_1'^3}{(\mu'_2 - \mu_1'^2)^{1.5}} = \begin{cases} 0 & \text{if } \text{df} > 3 \\ \text{undefined} & \text{if } \text{df} \leq 3 \end{cases}$$

67.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\mu'_4 - 4\mu'_1\mu'_3 + 6\mu_1'^2\mu'_2 - 3\mu_1'^4}{(\mu'_2 - \mu_1'^2)^2} = \begin{cases} 3 + 6/(\text{df} - 4) & \text{if } \text{df} > 4 \\ \text{undefined} & \text{if } \text{df} \leq 4 \end{cases}$$

67.12 Parametric median

$$\text{Median}(X) = 0$$

67.13 Parametric mode

$$\text{Mode}(X) = 0$$

67.14 Additional information and definitions

- u : Uniform[0,1] random variable
- $I(x, a, b)$: Regularized incomplete beta function
- $I^{-1}(x, a, b)$: Inverse of regularized incomplete beta function
- Beta (x, y) : Beta function

67.15 Spreadsheet documents

- [Excel file from GitHub repository](#)
- [Google spreadsheet document](#)

68 T Student 3P Distribution

68.1 Distribution definition

$$X \sim \text{TStudent}_{3P}(\text{df}, \text{Loc}, \text{Sc})$$

68.2 Distribution domain

$$x \in (-\infty, \infty)$$

68.3 Parameters domain and parameters constraints

$$\text{df} \in \mathbb{R}^+, \text{Loc} \in \mathbb{R}, \text{Sc} \in \mathbb{R}^+$$

68.4 Cumulative distribution function

$$F_X(x) = I\left(\frac{z(x) + \sqrt{z(x)^2 + \text{df}}}{2\sqrt{z(x)^2 + \text{df}}}, \frac{\text{df}}{2}, \frac{\text{df}}{2}\right)$$

68.5 Probability density function

$$f_X(x) = \frac{(1 + z(x)^2/\text{df})^{-(1+\text{df})/2}}{\sqrt{\text{df}} \times \text{Beta}\left(\frac{1}{2}, \frac{\text{df}}{2}\right)}$$

68.6 Percent point function/Sample

$$F_X^{-1}(u) = \begin{cases} \text{Loc} + \text{Sc} \sqrt{\frac{\text{df}(1-I^{-1}(u, \text{df}/2, \text{df}/2))}{I^{-1}(u, \text{df}/2, \text{df}/2)}} & \text{if } u \geq \frac{1}{2} \\ \text{Loc} - \text{Sc} \sqrt{\frac{\text{df}(1-I^{-1}(u, \text{df}/2, \text{df}/2))}{I^{-1}(u, \text{df}/2, \text{df}/2)}} & \text{if } u < \frac{1}{2} \end{cases}$$

68.7 Parametric centered moments

$$\tilde{\mu}'_k = E[\tilde{X}^k] = \int_0^\infty x^k f_{\tilde{X}}(x) dx = \begin{cases} 0 & \text{if } k \text{ odd} \wedge 0 < k < \text{df} \\ \text{df}^{\frac{k}{2}} \prod_{i=1}^{k/2} \frac{2i-1}{\text{df}-2i} & \text{if } k \text{ even} \wedge 0 < k < \text{df} \end{cases}$$

68.8 Parametric mean

$$\text{Mean}(X) = \text{Loc} + \text{Sc} \cdot \tilde{\mu}'_1 = \text{Loc}$$

68.9 Parametric variance

$$\text{Variance}(X) = \text{Sc}^2 \times (\tilde{\mu}'_2 - \tilde{\mu}'_1{}^2) = \begin{cases} \text{Sc}^2 \text{df}/(\text{df} + 2) & \text{if } \text{df} > 2 \\ \text{undefined} & \text{if } \text{df} \leq 2 \end{cases}$$

68.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\tilde{\mu}'_3 - 3\tilde{\mu}'_2\tilde{\mu}'_1 + 2\tilde{\mu}'_1{}^3}{(\tilde{\mu}'_2 - \tilde{\mu}'_1{}^2)^{1.5}} = \begin{cases} 0 & \text{if } \text{df} > 3 \\ \text{undefined} & \text{if } \text{df} \leq 3 \end{cases}$$

68.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\tilde{\mu}'_4 - 4\tilde{\mu}'_1\tilde{\mu}'_3 + 6\tilde{\mu}'_1{}^2\tilde{\mu}'_2 - 3\tilde{\mu}'_1{}^4}{(\tilde{\mu}'_2 - \tilde{\mu}'_1{}^2)^2} = \begin{cases} 3 + 6/(\text{df} - 4) & \text{if } \text{df} > 4 \\ \text{undefined} & \text{if } \text{df} \leq 4 \end{cases}$$

68.12 Parametric median

$$\text{Median}(X) = \text{Loc}$$

68.13 Parametric mode

$$\text{Mode}(X) = \text{Loc}$$

68.14 Additional information and definitions

- $\tilde{X} \sim \text{TStudent}(\text{df})$
- Loc : Location parameter
- Sc : Scale parameter
- $z(x) = (x - \text{Loc}) / \text{Sc}$
- u : $\text{Uniform}[0,1]$ random variable
- $I(x, a, b)$: Regularized incomplete beta function
- $I^{-1}(x, a, b)$: Inverse of regularized incomplete beta function
- $\text{Beta}(x, y)$: Beta function

68.15 Spreadsheet documents

- [Excel file from GitHub repository](#)
- [Google spreadsheet document](#)

69 Trapezoidal Distribution

69.1 Distribution definition

$$X \sim \text{Trapezoidal}(a, b, c, d)$$

69.2 Distribution domain

$$x \in [a, d]$$

69.3 Parameters domain and parameters constraints

$$a \in \mathbb{R}, b \in \mathbb{R}, c \in \mathbb{R}, d \in \mathbb{R}, a < b < c, b < c < d$$

69.4 Cumulative distribution function

$$F_X(x) = \begin{cases} \frac{1}{d+c-a-b} \frac{1}{b-a} (x-a)^2 & \text{if } a \leq x < b \\ \frac{1}{d+c-a-b} (2x-a-b) & \text{if } b \leq x < c \\ 1 - \frac{1}{d+c-a-b} \frac{1}{d-c} (d-x)^2 & \text{if } c \leq x \leq d \end{cases}$$

69.5 Probability density function

$$f_X(x) = \begin{cases} \frac{2}{d+c-a-b} \frac{x-a}{b-a} & \text{if } a \leq x < b \\ \frac{2}{d+c-a-b} & \text{if } b \leq x < c \\ \frac{2}{d+c-a-b} \frac{d-x}{d-c} & \text{if } c \leq x \leq d \end{cases}$$

69.6 Percent point function/Sample

$$F_X^{-1}(u) = \begin{cases} a + \sqrt{u \times (d+c-a-b) \times (b-a)} & \text{if } u \leq A_1 \\ (a+b+u \times (d+c-a-b))/2 & \text{if } A_1 \leq u \leq A_1 + A_2 \\ d - \sqrt{(1-u) \times (d+c-a-b) \times (d-c)} & \text{if } A_1 + A_2 \leq u \leq A_1 + A_2 + A_3 \end{cases}$$

69.7 Parametric centered moments

$$\mu'_k = E[X^k] = \int_a^b x^k f_X(x) dx = \frac{2}{d+c-b-a} \frac{1}{(k+1)(k+2)} \left(\frac{d^{k+2} - c^{k+2}}{d-c} - \frac{b^{k+2} - a^{k+2}}{b-a} \right)$$

69.8 Parametric mean

$$\text{Mean}(X) = \mu'_1$$

69.9 Parametric variance

$$\text{Variance}(X) = \mu'_2 - \mu_1'^2$$

69.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\mu'_3 - 3\mu'_2\mu'_1 + 2\mu_1'^3}{(\mu'_2 - \mu_1'^2)^{1.5}}$$

69.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\mu'_4 - 4\mu'_1\mu'_3 + 6\mu_1'^2\mu'_2 - 3\mu_1'^4}{(\mu'_2 - \mu_1'^2)^2}$$

69.12 Parametric median

$$\text{Median}(X) = F_X^{-1}(1/2)$$

69.13 Parametric mode

$$\text{Mode}(X) \in [b, c]$$

69.14 Additional information and definitions

- u : Uniform[0,1] random variable
- $A_1 = (b - a)/(d + c - a - b)$
- $A_2 = 2(c - b)/(d + c - a - b)$
- $A_3 = (d - c)/(d + c - a - b)$

69.15 Spreadsheet documents

- [Excel file from GitHub repository](#)
- [Google spreadsheet document](#)

70 Triangular Distribution

70.1 Distribution definition

$$X \sim \text{Triangular}(a, b, c)$$

70.2 Distribution domain

$$x \in [a, b]$$

70.3 Parameters domain and parameters constraints

$$a \in \mathbb{R}, b \in \mathbb{R}, c \in \mathbb{R}, a < c < b$$

70.4 Cumulative distribution function

$$F_X(x) = \begin{cases} \frac{(x-a)^2}{(b-a)(c-a)} & \text{if } a < x \leq c \\ 1 - \frac{(b-x)^2}{(b-a)(b-c)} & \text{if } c < x < b \end{cases}$$

70.5 Probability density function

$$f_X(x) = \begin{cases} \frac{2(x-a)}{(b-a)(c-a)} & \text{if } a \leq x < c, \\ \frac{2(b-x)}{(b-a)(b-c)} & \text{if } c \leq x \leq b, \end{cases}$$

70.6 Percent point function/Sample

$$F_X^{-1}(u) = \begin{cases} a + \sqrt{U(b-a)(c-a)} & \text{if } 0 < U < \frac{c-a}{b-a} \\ b - \sqrt{(1-U)(b-a)(b-c)} & \text{if } \frac{c-a}{b-a} \leq U < 1 \end{cases}$$

70.7 Parametric centered moments

$$\mu'_k = E[X^k] = \int_a^b x^k f_X(x) dx$$

70.8 Parametric mean

$$\text{Mean}(X) = \mu'_1 = \frac{a+b+c}{3}$$

70.9 Parametric variance

$$\text{Variance}(X) = \mu'_2 - \mu_1'^2 = \frac{a^2 + b^2 + c^2 - ab - ac - bc}{18}$$

70.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\mu'_3 - 3\mu'_2\mu'_1 + 2\mu_1'^3}{(\mu_2' - \mu_1'^2)^{1.5}} = \frac{\sqrt{2}(a+b-2c)(2a-b-c)(a-2b+c)}{5(a^2+b^2+c^2-ab-ac-bc)^{\frac{3}{2}}}$$

70.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\mu'_4 - 4\mu'_1\mu'_3 + 6\mu_1'^2\mu_2' - 3\mu_1'^4}{(\mu_2' - \mu_1'^2)^2} = 3 - \frac{3}{5}$$

70.12 Parametric median

$$\text{Median}(X) = \begin{cases} a + \sqrt{\frac{(b-a)(c-a)}{2}} & \text{if } c \geq \frac{a+b}{2} \\ b - \sqrt{\frac{(b-a)(b-c)}{2}} & \text{if } c \leq \frac{a+b}{2} \end{cases}$$

70.13 Parametric mode

$$\text{Mode}(X) \in [b, c]$$

70.14 Additional information and definitions

- u : Uniform[0,1] random variable

70.15 Spreadsheet documents

- [Excel file from GitHub repository](#)
- [Google spreadsheet document](#)

71 Uniform Distribution

71.1 Distribution definition

$$X \sim \text{Uniform}(a, b)$$

71.2 Distribution domain

$$x \in [a, b]$$

71.3 Parameters domain and parameters constraints

$$a \in \mathbb{R}, b \in \mathbb{R}, a < b$$

71.4 Cumulative distribution function

$$F_X(x) = \frac{x - a}{b - a}$$

71.5 Probability density function

$$f_X(x) = \frac{1}{b - a}$$

71.6 Percent point function/Sample

$$F_X^{-1}(u) = a + u \cdot (b - a)$$

71.7 Parametric centered moments

$$\mu'_k = E[X^k] = \int_{-\infty}^{\infty} x^k f_X(x) dx = \frac{1}{k+1} \sum_{i=0}^k a^i b^{k-i}$$

71.8 Parametric mean

$$\text{Mean}(X) = \mu'_1 = \frac{1}{2}(a + b)$$

71.9 Parametric variance

$$\text{Variance}(X) = \mu'_2 - \mu_1'^2$$

71.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\mu'_3 - 3\mu'_2\mu'_1 + 2\mu_1'^3}{(\mu'_2 - \mu_1'^2)^{1.5}} = 0$$

71.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\mu'_4 - 4\mu'_1\mu'_3 + 6\mu_1'^2\mu'_2 - 3\mu_1'^4}{(\mu'_2 - \mu_1'^2)^2} = 3 - \frac{6}{5}$$

71.12 Parametric median

$$\text{Median}(X) = \frac{1}{2}(a + b)$$

71.13 Parametric mode

$$\text{Mode}(X) \in [a, b]$$

71.14 Additional information and definitions

- $u : \text{Uniform}[0,1]$ random variable

71.15 Spreadsheet documents

- [Excel file from GitHub repository](#)
- [Google spreadsheet document](#)

72 Weibull Distribution

72.1 Distribution definition

$$X \sim \text{Weibull}(\alpha, \beta)$$

72.2 Distribution domain

$$x \in [0, \infty)$$

72.3 Parameters domain and parameters constraints

$$\alpha \in \mathbb{R}^+, \beta \in \mathbb{R}^+$$

72.4 Cumulative distribution function

$$F_X(x) = 1 - e^{-(x/\beta)^\alpha}$$

72.5 Probability density function

$$f_X(x) = \frac{\alpha}{\beta} \left(\frac{x}{\beta}\right)^{\alpha-1} e^{-(x/\beta)^\alpha}$$

72.6 Percent point function/Sample

$$F_X^{-1}(u) = \beta(-\ln(1-u))^{1/\alpha}$$

72.7 Parametric centered moments

$$\mu'_k = E[X^k] = \int_0^\infty x^k f_X(x) dx = \beta^\alpha \Gamma\left(1 + \frac{k}{\alpha}\right)$$

72.8 Parametric mean

$$\text{Mean}(X) = \mu'_1 = \beta \cdot \Gamma(1 + 1/\alpha)$$

72.9 Parametric variance

$$\text{Variance}(X) = \mu'_2 - \mu_1'^2 = \beta^2 \left[\Gamma(1 + 2/\alpha) - (\Gamma(1 + 1/\alpha))^2 \right]$$

72.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\mu'_3 - 3\mu'_2\mu'_1 + 2\mu_1'^3}{(\mu'_2 - \mu_1'^2)^{1.5}}$$

72.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\mu'_4 - 4\mu'_1\mu'_3 + 6\mu_1'^2\mu'_2 - 3\mu_1'^4}{(\mu'_2 - \mu_1'^2)^2}$$

72.12 Parametric median

$$\text{Median}(X) = \beta(\ln(2))^{1/\alpha}$$

72.13 Parametric mode

$$\text{Mode}(X) = \begin{cases} \beta \left(\frac{\alpha-1}{\alpha}\right)^{1/\alpha} & \text{if } \alpha > 1 \\ 0 & \text{if } \alpha \leq 1 \end{cases}$$

72.14 Additional information and definitions

- β : Scale parameter
- u : Uniform[0,1] random variable
- $\Gamma(x)$: Gamma function

72.15 Spreadsheet documents

- [Excel file from GitHub repository](#)
- [Google spreadsheet document](#)

73 Weibull 3P Distribution

73.1 Distribution definition

$$X \sim \text{Weibull}_{3P}(\alpha, \text{Loc}, \beta)$$

73.2 Distribution domain

$$x \in [\text{Loc}, \infty)$$

73.3 Parameters domain and parameters constraints

$$\alpha \in \mathbb{R}^+, \text{Loc} \in \mathbb{R}, \beta \in \mathbb{R}^+$$

73.4 Cumulative distribution function

$$F_X(x) = 1 - e^{-z(x)^\alpha}$$

73.5 Probability density function

$$f_X(x) = \frac{\alpha}{\beta} z(x)^{\alpha-1} e^{-z(x)^\alpha}$$

73.6 Percent point function/Sample

$$F_X^{-1}(u) = \text{Loc} + \beta(-\ln(1-u))^{1/\alpha}$$

73.7 Parametric centered moments

$$\tilde{\mu}'_k = E[\tilde{X}^k] = \int_0^\infty x^k f_{\tilde{X}}(x) dx = \beta^\alpha \Gamma\left(1 + \frac{k}{\alpha}\right)$$

73.8 Parametric mean

$$\text{Mean}(X) = \text{Loc} + \tilde{\mu}'_1 = \text{Loc} + \beta \Gamma(1 + 1/\alpha)$$

73.9 Parametric variance

$$\text{Variance}(X) = \tilde{\mu}'_2 - \tilde{\mu}'_1^2 = \beta^2 \left[\Gamma(1 + 2/\alpha) - (\Gamma(1 + 1/\alpha))^2 \right]$$

73.10 Parametric skewness

$$\text{Skewness}(X) = \frac{\tilde{\mu}'_3 - 3\tilde{\mu}'_2\tilde{\mu}'_1 + 2\tilde{\mu}'_1^3}{(\tilde{\mu}'_2 - \tilde{\mu}'_1^2)^{1.5}}$$

73.11 Parametric kurtosis

$$\text{Kurtosis}(X) = \frac{\tilde{\mu}'_4 - 4\tilde{\mu}'_1\tilde{\mu}'_3 + 6\tilde{\mu}'_1^2\tilde{\mu}'_2 - 3\tilde{\mu}'_1^4}{(\tilde{\mu}'_2 - \tilde{\mu}'_1^2)^2}$$

73.12 Parametric median

$$\text{Median}(X) = \text{Loc} + \beta(\ln(2))^{1/\alpha}$$

73.13 Parametric mode

$$\text{Mode}(X) = \text{Loc} + \begin{cases} \beta \left(\frac{\alpha-1}{\alpha}\right)^{1/\alpha} & \text{if } \alpha > 1 \\ 0 & \text{if } \alpha \leq 1 \end{cases}$$

73.14 Additional information and definitions

- $\tilde{X} \sim \text{Weibull}(\alpha, \beta)$
- Loc : Location parameter
- β : Scale parameter
- $z(x) = (x - \text{Loc}) / \beta$
- u : Uniform[0,1] random variable
- $\Gamma(x)$: Gamma function

73.15 Spreadsheet documents

- [Excel file from GitHub repository](#)
- [Google spreadsheet document](#)

References

- [1] Eric Jones, Travis Oliphant, Pearu Peterson, et al. SciPy: Open source scientific tools for Python, 2001–.
- [2] Pauli Virtanen, Ralf Gommers, Travis E. Oliphant, Matt Haberland, Tyler Reddy, David Cournapeau, Evgeni Burovski, Pearu Peterson, Warren Weckesser, Jonathan Bright, Stéfan J. van der Walt, Matthew Brett, Joshua Wilson, K. Jarrod Millman, Nikolay Mayorov, Andrew R. J. Nelson, Eric Jones, Robert Kern, Eric Larson, C J Carey, İlhan Polat, Yu Feng, Eric W. Moore, Jake VanderPlas, Denis Laxalde, Josef Perktold, Robert Cimrman, Ian Henriksen, E. A. Quintero, Charles R. Harris, Anne M. Archibald, Antônio H. Ribeiro, Fabian Pedregosa, Paul van Mulbregt, and SciPy 1.0 Contributors. SciPy 1.0: Fundamental Algorithms for Scientific Computing in Python. *Nature Methods*, 17:261–272, 2020.