Maths and stuff

1 Evolution Equations

The heat treatment process is simulated with the transformation of austenite into bainite and martensite....

The initial value problem represented by the rate equations is solved by implicit time integration which leads to the evolution equation $\beta^{n+1} = \beta^n + \Delta\beta^{n+1}$, with $\Delta\beta^{n+1} = \Delta t\,\dot{\beta}^{n+1}$.
After bringing these equations into residual form

$$\begin{aligned}\underline{\mathbf{R}_{\boldsymbol{\beta}}} &= \underline{\boldsymbol{\beta}}^{n+1} - \underline{\boldsymbol{\beta}}^{n} - \underline{\boldsymbol{\Delta\beta}} \\ &= \begin{bmatrix} R_{\beta_B} \\ R_{\beta_M} \end{bmatrix} = \begin{bmatrix} \beta_B^{n+1} - \beta_B^n - \Delta\beta_B^{n+1} \\ \beta_M^{n+1} - \beta_M^n - \Delta\beta_M^{n+1} \end{bmatrix} \qquad (1) \\ \text{with } \underline{\boldsymbol{\Delta\beta}} &= f(\underline{\boldsymbol{\beta}}^{n+1}) \qquad (2)\end{aligned}$$

the backward Newton method is used to solve the equation system 4.1. The update of the backward Newton

$$\underline{\boldsymbol{\beta}}^{k+1} = \underline{\boldsymbol{\beta}}^{k} - \frac{\mathbf{R}_{\boldsymbol{\beta}}}{\underline{\mathbf{R}_{\boldsymbol{\beta'}}}} \qquad (3)$$

with the iteration counter k requires the derivation of the tangent

$$\underline{\mathbf{R}_{\boldsymbol{\beta'}}} = \begin{bmatrix} \frac{\partial R_{\beta_B}}{\partial \beta_B} & \frac{\partial R_{\beta_B}}{\partial \beta_M} \\ \frac{\partial R_{\beta_M}}{\partial \beta_B} & \frac{\partial R_{\beta_M}}{\partial \beta_M} \end{bmatrix} \text{ with} \qquad (4)$$

$$\frac{\partial R_{\beta_B}}{\partial \beta_B} = 1 - \Delta t\, b^{\frac{1}{N}}\, N \left[B^{1-\frac{1}{N}} - \left(1 - \frac{1}{N}\right) B^{-\frac{1}{N}} \right] \qquad (5)$$

$$\frac{\partial R_{\beta_B}}{\partial \beta_M} = -\Delta t\, b^{\frac{1}{N}}\, N \left[B^{1-\frac{1}{N}} - \left(1 - \frac{1}{N}\right) B^{-\frac{1}{N}} \left(1 - \frac{\beta_A}{1-\beta_M}\right) \right] \qquad (6)$$

$$\frac{\partial R_{\beta_M}}{\partial \beta_B} = -k\,\Delta T\,\zeta_{A\to M} \qquad (7)$$

$$\frac{\partial R_{\beta_M}}{\partial \beta_M} = 1 - k\,\Delta T\,\zeta_{A\to M} \quad . \qquad (8)$$

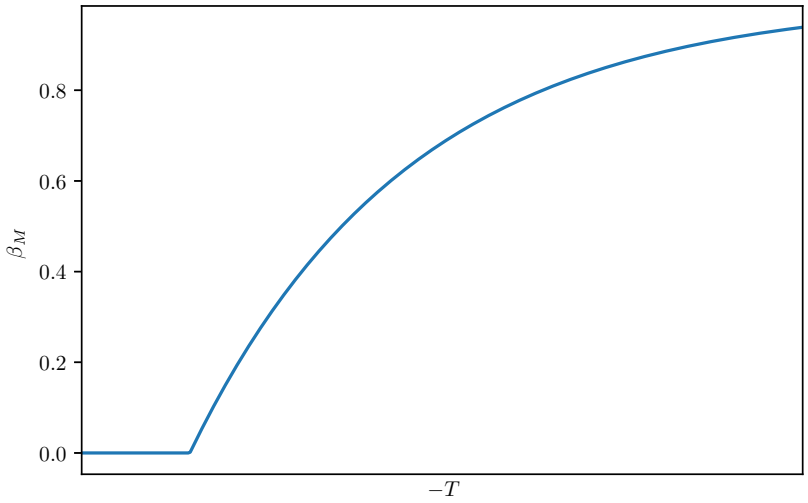

Figure 1: Development of the martensite volume fraction under cooling.