

# Deepening Representations

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## Abstract

There is a vast literature in cognitive science and empirical philosophy of mind concerning whether connectionist models can provide explanations in terms of representational content. Critics have claimed that connectionist networks cannot provide properly psychological explanations, since there is no principled way to group tokened representations into types that share the same content. Despite this once vigorous debate, relatively little has been said about representational content in state-of-the-art deep neural networks (DNNs). While these networks can be viewed as extensions of early connectionist models, their added complexity introduces a number of complications. This paper brings modern advances in machine learning and computer vision to bear on the question of content in connectionist networks. I consider what I call the *cluster approach*, which I take to be the strongest account of content in connectionist systems. I show that technical and methodological problems arise when generalizing this approach to DNNs. I then argue that we can retain the spirit of the cluster approach by understanding the tools of representational similarity analysis (RSA) in cognitive neuroscience as a form of hypothesis testing in search of real, causal patterns in the underlying neural data.

## 1 Introduction

In the results of the 14th biennial Critical Assessment of protein Structure Prediction (CASP), AlphaFold — a deep learning system designed by Google’s DeepMind research group — was recognized as a solution to the "protein folding problem" (Jumper et al., 2020). This achievement was hailed as a major scientific advance for both biology and artificial intelligence (AI). The problem of predicting a protein’s structure from its amino acid chain has stood as a grand challenge in biology for the last fifty years. There are an estimated  $10^{300}$  possible conformations of a typical protein. This means that it would take longer than the age of the known universe to enumerate each conformation by brute force calculation (Levinthal, 1969). Yet, AlphaFold can accurately determine structures in, at most, a matter of days. The successful prediction of spontaneous folding depends on AlphaFold’s capacity

to acquire and exploit representations of amino acid residue pairs in order to generate and update an abstract spacial graph model of a protein’s possible structure.

State-of-the-art machine learning systems like AlphaFold are ubiquitous in modern life. These systems find applications in everything from automated decision assistance in medicine and criminal justice to playing complex board games like chess and Go (Silver et al., 2016). Our epistemic networks are increasingly intertwined with machine learning algorithms, and scientists rely on them in complex computational models. Understanding how information is encoded in abstract representations is crucial for understanding the intelligent capacities of these systems. This imperative suggests revisiting connectionist approaches that explain intelligence in terms of the capacities to learn abstract representations from low-level perceptual inputs (Hassabis et al., 2017; Buckner, 2018).

Connectionists hope to explain cognition using artificial neural networks as models (Rumelhart et al., 1986). Connectionist networks consist in large, parallel collections of artificial neurons called units, or nodes, together with weights that measure the strength of connections between nodes. We can view these networks as models of the neurons and synaptic links in the brain at some level of abstraction. Mounting empirical evidence suggests that such models are useful tools for learning about regions of interest in the brain. Even so, connectionism is not without its skeptics (Fodor and Pylyshyn, 1988; Fodor and Lepore, 1999). One standard complaint against connectionism charges that connectionist networks cannot provide a psychological explanation of behavior, since there is no principled way to separate internal representations into groups which share the same content. If connectionism is to succeed as a representational theory of cognition, we must have a plausible theory of representation that preserves sameness of content in the face of individual variance in physical implementation.

This paper scrutinizes an explicit proposal about the type of entity to be considered as content-bearing in connectionist systems, namely that the vehicles of content in connectionist systems are clusters of neural activity distributed across a model’s internal mechanism (Tiffany, 1999; Gardenfors, 2000; Shea, 2007). Call this proposal the *cluster approach* to content. This approach adapts the idea of a state space semantics deployed in earlier accounts of semantic similarity in connectionist networks (Horgan and Tienson, 1996; Churchland, 1998; Tiffany, 1999). The basic idea of what Horgan and Tienson (1996) call *dynamical cognition* is that we can use the tools of dynamical systems theory to provide a mathematical characterization of a cognitive system (see also van Gelder, 1995). The set of all possible states of a system defines that system’s state space. Applied to connectionist networks, state space conceives of the internal, representational landscape of a network as a mathematically defined space whose axes are constituted by the possible activation levels of the neurons distributed across the network. Individual points in state space correspond to a global pattern of activity in the network. So, points in this space can be construed as token representations. We can then identify clusters of points, or regions in state space, as corresponding to representation types (Horgan and Tienson, 1996; Gardenfors, 2000; Shea, 2007). The cluster

approach has become the *de facto* solution to the problem of fixing the semantic content of representations for connectionists over the course of the last two decades. This approach comes with considerable advantages, so it merits careful consideration. As such, this paper begins by first motivating the strengths of the approach before revealing a pressing technical problem for generalizing its results. I close with a proposal for moving forward with representational explanations of connectionist networks inspired by recent work in computational neuroscience.

Despite its successes, the cluster approach builds on an outdated picture of neural networks. The growing ubiquity of AI has brought increased philosophical attention to deep learning (Lecun et al., 2015; Buckner, 2019). Since deep neural networks (hereafter, DNNs) are extensions of early connectionist networks, the cluster approach to state space semantics would seem the obvious choice for analyzing representational content in these models. Yet, there is a glaring problem with applying the cluster approach to DNNs that has thus far gone unnoticed in the literature on representations. The complexity of these networks means they process representations in many hundreds or even thousands of dimensions. The ‘curse of dimensionality’ refers to a set of problems that emerge when analyzing data in high-dimensional spaces that otherwise would not occur in low-dimensional spaces like those of ordinary three-dimensional experience. One phenomenon subsumed under the curse of dimensionality is the problem of measuring relative distance in high-dimensional Euclidean space. As dimensionality increases, state space becomes increasingly sparse and nearest neighbor searches from a given starting point become unstable. Sparseness means that measures of distance between points in the space expand and the ratios of relative distance between different points converge. As the number of dimensions approaches infinity, the proportional difference between the nearest and farthest point distances from an arbitrary starting point in the state space approaches zero. This means that the relevant contrast between points in the state space of a model vanishes as dimensionality increases. The curse of dimensionality suggests that the concept of proximity may not even be qualitatively meaningful in the high-dimensional state spaces that are characteristic of DNNs. It follows from this vanishing relative contrast between points that we cannot rely on relative distance in state space to identify contentful clusters of neural activity in high-dimensional models.

Through the course of this paper I aim to convince the reader that attempts to individuate content in connectionist models using the cluster approach fail to generalize well to deep learning models with high dimensionality. I then offer a positive proposal for ascribing content to DNNs inspired by recent developments in computational neuroscience. The structure of the paper is as follows: §2 introduces the cluster approach to content in connectionist networks. In §3, I introduce a host of technical problems for generalizing the cluster approach to DNNs posed by the curse of dimensionality. Finally, §4 argues that we can look to empirical work in neuroscience that identifies predictive patterns in the representational structure of computational models as a guide to attributing content to connectionist models.

## 2 Connectionist Representations: The cluster approach to content

Connectionism broadly-construed takes artificial neural networks as idealized models of cognition. A neural network consists in multiple, interconnected layers of units joined together by a pattern of weights which determine the strength of activity passed from one unit to the next. The layers of a network are typically divided into three classes: an input layer that receives encoded information for processing, an output layer that produces the result of processing, and a hidden layer (or layers) that lies in between. The process of training a simple classifier network involves introducing a large number of antecedently labeled example inputs and using a supervised algorithm called error backpropagation to fine-tune the associative links between units such that the network learns to successfully generalize to novel inputs.

Identifying representations in a connectionist model usually involves measuring the distributed patterns of activation across the hidden units of a network associated with different input samples. Each token representation in a neural network will be one such pattern of neural activity distributed across the network’s hidden layer(s). Such a pattern, or activation vector, can be characterized as an  $n$ -tuple, where  $n$  is the number of neurons in the population. We can usefully conceptualize these activations as points in a geometric space for the purpose of comparing informational content. *State space* refers to an abstract,  $n$ -dimensional space whose axes are constituted by the possible activation of units in the hidden layers of the model such that any activation vector corresponds to a point in that space. So, a point in state space corresponds with a token representation.

This leads quite seductively to the thought that points in state space themselves are the vehicles of content in neural networks and that each point has unique content. This is clearly problematic since it rules out the possibility of identifying vehicles of content that can be realized despite variations in physical implementation. This assumption confuses a subtle point. As Shea (2007) points out, representations are “concrete particulars” with both content and non-semantic properties. Vehicles of content are also representations, but ones individuated “according to the non-semantic properties which group different tokens together as being of the same representation type for the purpose of assigning content to them” (Shea, 2007, p. 4). In other words, the above assumption appears to wrongly identify the non-semantic properties of representational tokens as those that one should use to organize representations into types which identify sameness of content. Intuitively, we do think it possible for different biological brains to encode the same content despite vast differences in their neural architectures. The representational types in connectionist networks should be comparable across different networks in this way. Therefore, we must be open to representational accounts of connectionist systems wherein the vehicles of content are more coarse-grained.

The *cluster approach* to content answers these demands. The cluster approach says that sameness of content can be measured by analyzing the internal structure of clusters of neural activations in the model’s state space. Rather than identifying individual hidden units or the

relational structure of points in state space as the vehicles of content, the cluster approach identifies clusters of activity in state space as the correct vehicles of content. On this view, representations can be grouped into identical types in virtue of their falling within discrete regions in state space where contentful clusters of neural activity emerge through training. Clusters of points are identified relative to the overall volume filled by the activation of points produced by all training samples. Since the size of a cluster is relative to the overall size of the semantic space, clusters can be realized in nets with different numbers of hidden units. The idea is just that a set of training samples that produces a cluster of points in state space A might likewise produce a cluster of points in state space B. In such a case, two different state space clusters in two different neural networks can represent the very same property to their respective networks.<sup>1</sup>

In this vein, Gardenfors (2000) provides a detailed account of how information can be represented using geometric structures in a *conceptual space*. A conceptual space, for Gardenfors, is just an abstract, metric space whose axes are constituted by a number of quality dimensions, which represent the various qualities of objects. This same apparatus is often described as a *feature space*, because the coordinate axes are defined by a set of possible features of stimuli. In such a space, natural properties are represented by convex regions<sup>2</sup>. A conceptual space, or feature space, is distinguished from a representational state space, since the former is just a way of describing the possible properties of a set of stimuli. A conceptual space is a representational state space when it corresponds to the how a system represents a set of stimuli under the relevant task conditions such that neural activity reliably varies along the dimensions of the defined conceptual space (Goddard et al., 2018). Since many feature dimensions are determined by our perceptually (e.g. color, shape, or spacial properties), the criterion that natural properties correspond to convex regions in a conceptual space turns out to be perceptually grounded (Gardenfors, 2000, p. 77). This makes it highly serviceable for modeling dynamical cognition when the dimensions of neural variation can be properly identified for a particular task. This framework can be fruitfully applied to a connectionist network in order to characterize the representational structure of

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<sup>1</sup>Laakso and Cottrell (2000) demonstrated that it is possible to compare clustering patterns in pairs of networks with different numbers of hidden units. To do this, they first calculated the hidden unit activations for each input sample in a network. They then computed the Euclidean distances between every pair of input sample activation points in each network. The set of distance measurements for a given network can then be coded as vector which captures the internal structure of that network’s state space. Since each network was trained on the same number of input samples, each set of distance measures of a network will have equal cardinality. So, reproducing this procedure for every element in a pair of networks results in a pair of vectors of length  $n(n-1)/2$ , where  $n$  is the number of input samples. To determine the overall similarity of the state space of network A to that of network B, Laakso and Cottrell computed the statistical correlation between these vector codings. Using statistical correlations in this way purports to achieve scale invariance, since it is insensitive to the absolute magnitude of the distances being compared.

<sup>2</sup>A subset C of a conceptual space S is said to be convex if, for all points x and y in C, all points between x and y are also in C (Gardenfors, 2000, p. 66-71)

its state space in terms of convex regions, or clusters.

So trained neural networks can be described by clusters in state space, but why should we ascribe content to them? Shea (2007), which provides among the clearest articulations of the cluster approach, argues that we should do so “if clusters are to be invoked, as is common empirical practice, in an explanation of a network’s ability to generalize its correct performance to new samples; thus that clusters are vehicles of content to the extent that they form the basis of generalization” (Shea, 2007, p. 12-13). If those clusters play a role in the mechanism of operation that results in successful generalization, then those clusters are content bearing.

This appears to be born out by empirical evidence. Training tends to cause networks to develop tight clusters in state space. This is because the goal of training can be characterized as producing a function for correctly sorting inputs into clusters at the output layer. The development of hidden layer clustering serves as an important intermediary in achieving this goal. These clusters can be individuated by taking a trained network and plotting the distribution of activation points in its state space. Convex regions in state space containing clusters of activity can then be identified by measuring and comparing the relative distances of groups of proximal points in relation to the overall volume filled by activity in the space.<sup>3</sup>

The cluster approach is a significant departure from earlier accounts of content in connectionist networks.<sup>4</sup> According to Shea, content ascriptions are always made on the basis of the successful performance of a network. Regions in state space do not themselves antecedently represent anything. The argument for taking clusters as the vehicles of content is based on giving an explanation for successful generalization. It is only after a network has

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<sup>3</sup>This can also be achieved using Voronoi tessellation (Gardenfors, 2000). Given a set of sites in a space  $P = \{p_1, \dots, p_n\}$ , Voronoi tessellation partitions a space such that an arbitrary point  $q$  belongs to a region  $i$  when bisector  $d(q, p_i)$  is less than  $d(q, p_j)$ . However, when it comes to a similarity space, this explicitly assumes that we have information about prototypical representations, where each site represents a concept prototype (Gardenfors, 2000, p. 87).

<sup>4</sup>Churchland (1998) defends a version of state space semantics based on relative similarity. This similarity approach takes the overall pattern of simultaneous activation levels across the neurons of a model’s hidden layers as the vehicles of content, but holds that a point in a model’s state space acquires a specific semantic content as a function of its position relative to all of the other contentful points within that space. So, Churchland replaces the notion of semantic identity with one of relative similarity. Fodor and Lepore (1999) observe, however, that the similarity approach amounts to an unacceptable form of content holism. The content of a particular activation vector depends on its relation to all other contentful points in the space. But the identity of those contentful points each depends on their relations to every other point, and regress threatens to collapse the distinction between tokens and types. It follows that individual representations cannot be compared between two different models, since comparisons are only possible between entire state spaces. Put another way, if representational content is adjudicated according to similarity in semantic space, then the type-token distinction seemingly breaks down, and there is no principled way of characterizing a model that separates points in state space into types (Fodor, 2000, p. 50). Yet, the ability of neural networks to successfully generalize to novel samples seems to indicate that a representational explanation of their performance might still be in reach.

been trained and can successfully generalize performance to novel inputs that it represents. Shea remarks that clusters emerge through the process of training and that these clusters are content bearing because of the role they play in producing explanations of network performance. So the process of learning transforms the state space of a model from content-less, mathematically defined space to one that tracks properties that are relevant for classification. The process of learning is essentially one by which the model learns to represent fundamental patterns in the distal features of its environment and generate successful behavior on the basis of those patterns. On this he says: “The proposal is not to endow networks with original intentionality. Hidden layer clusters are only contentful in virtue of the contents ascribed to outputs. The modeler takes the outputs to represent some properties  $O_1, \dots, O_n$  and trains the network to be good at classifying by these properties.” (Shea, 2007, p. 15-16) He remarks that any good, naturalistic theory of content (whether causal, informational, or teleological) would ascribe content to outputs on these grounds, and the content of hidden layer clusters stems from the intermediary roles they play in explaining contentful outputs.

Shea goes on to give four conditions which he takes to be jointly sufficient for attributing content to patterns of activation found in a hidden layer of a connectionist network.

Jointly sufficient conditions for attributing content to representations generated by a connectionist model:

- i the network can correctly classify some set of samples which differ from those in the training set (i.e., “new samples”);
- ii the new samples fall into hidden layer clusters formed by samples in the training set;
- iii each new sample shares a property with the training samples in its cluster; and
- iv that property is relevant to the classificatory task.

So, the cluster approach says that a tokened representation in a neural network associated with a novel input stimulus belongs to a contentful type just in case it falls into some hidden layer cluster formed by samples in the training set that share a property that is causally or constitutively relevant to the classificatory task at the output layer (Shea, 2007). Such a cluster can be identified with a convex region in state space. Belonging to such a region indicates that membership in a representation type that tracks a natural property of perceptual inputs (Gardenfors, 2000). If this is right, then we can use this kind of cluster analysis to verify that the relations between our putative vehicles of content match our intuitions about concept similarity. We should expect, for instance, that the cluster of activations corresponding to “dog” input stimuli be nearer in state space to that of “cat” inputs than that of “fish” inputs. Furthermore, if clusters are vehicles of content, then different networks can have type-identical representations. Again, this is empirically testable

since we can verify whether a group of sample inputs which activate a cluster in some network A also activate a cluster in another network B. By dispensing with an overly fine-grained notion of content, the cluster approach avoids the worries plaguing earlier theories of content in connectionist systems. Instead, clusters in state space group many different states of a network’s internal mechanism into types, and a straightforward, representational explanation of neural network performance is in hand. This approach overcomes Fodor’s famous objections to state space semantics, and serves as the gold standard for connectionists today.

### 3 Deepening Connectionism: the curse of dimensionality

The field of AI has shifted rapidly since the introduction of the cluster approach. Over the course of the last half-dozen years, deep learning has become by far the most successful approach to machine learning (ML) (Lecun et al., 2015). In contrast with earlier connectionist networks, state-of-the-art DNNs typically contain many more layers with different kinds of processing nodes that deploy varying activation functions. The deepening of models affords enormous gains in their computational power, efficiency, and ability to solve complex decision problems when compared to shallow networks. Deep neural networks have achieved human-level benchmarks in novel image classification tasks. Their success underwrites advances in autonomous vehicles, facial recognition, and assisted diagnoses. Moreover, DNNs are among the strongest of our current candidates for a partial explanatory model of the human visual system (Yamins et al., 2014; Yamins and DiCarlo, 2016). Yamins and co-authors focus their attention primarily on hierarchically structured deep convolutional neural networks (DCNNs) as explanatory models of the primate ventral visual pathway (Cao and Yamins, 2021). These networks are not only appealing for their impressive performance in object categorization tasks. They also incorporate several hallmark features of neural computation, such as nonlinear transduction and max pooling of inputs (Yamins and DiCarlo, 2016).<sup>5</sup> Attention

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<sup>5</sup>The architecture of a generic DCNN involves a repeated process of transforming input signals through convolution and passing that transformed input through a series of units whose function is to detect the presence of certain features and make a “decision” as to which feature is most likely present at a given location of the input sample. Convolutional nodes take a multidimensional array of input data and a learned array of parameters that, when applied to the input data, amplify the presence of a certain feature while minimizing others. In image classification tasks, this involves scanning a narrow pixel window of the sample image. Those pixels are read as arrays of RGB color vectors which the convolutional node transforms into a matrix of RGB vectors that amplifies the presence of a certain feature, like contrasts or shadings at early stages, or higher-level features like vertical lines, angles, or curves at subsequent layers. These convolutional units, often called “filters” or “kernels,” typically then pass their output to a “rectified linear unit” which activates according to a simple function called rectification if the output of the preceding convolution operation exceeds a certain threshold. A decision is then made by a nonlinear downsampling node whose function is to agglomerate the activations of several different filters responsible for overlapping regions of the input sample and pass along only the activation of the most salient feature in that region. This is most commonly achieved through



to these networks is essential to a healthy synthesis of work in AI and cognitive neuroscience (Hassabis et al., 2017; Botvinick et al., 2017; Buckner, 2019).

However, the pioneering work which undergirds the cluster approach to state space semantics builds on exclusively shallow classifier networks. For instance, the networks Shea cites as empirical support for his proposal all had a single hidden layer with only a few units. Conversely, state-of-the-art deep learning models used in modern ML tasks typically compute over many thousands of individual nodes across many more hidden layers. Even a relatively old five-layer deep convolutional neural network deployed in Krizhevsky et al. (2012) contains 650,000 individual nodes with over 60 million parameters. Consequently, these models will have state spaces that are orders of magnitude more complex than those in a relatively simple three-layer network. Though it would seem natural to extend the cluster approach to representational explanations of deep learning models, their added complexity introduces pressing technical and methodological problems.

A major impediment to making good sense of the role internal representations play in explanations of decisions made by DNNs arises from the fact that their object recognition tasks operate in a high-dimensional state space. Category clusters in such a space can be characterized instead as multidimensional manifolds embedded in this space. A neural object manifold is a subspace of lower-dimensionality embedded in a high-dimensional activation space. There are good reasons to think that manifolds play a representational role. Typical explanations of perceptual similarity in such models *do* invoke similar notions to those employed by the cluster approach, albeit with additional complexity. Here the goal of training an image classifier can be understood as locating a global output function for transforming and drawing a discrete boundary — in the form of a linear hyperplane — between category manifolds embedded in the network’s state space (DiCarlo and Cox, 2007). But the high-dimensional nature of DNNs poses a significant difficulty for visualization and defies easy understanding. Dimensionality raises important methodological concerns for the cluster approach, since manifolds will not be so well-behaved as their low-dimensional equivalents.

Our ordinary intuitions often fail when dealing with high dimensionality. In practice, manifolds in highly complex models become hopelessly entangled. This entanglement suggests that category manifolds do not satisfy the convexity criterion (Gardenfors, 2000), since, for many category manifolds, there will be a pair of members  $x$  and  $y$  such that some point along the bisector  $d(x, y)$  is not a member of the manifold. Moreover, general problems with working with high-dimensional data draws into question whether independently measuring the representational similarity of category manifolds remains tenable in our ever deepening models. For example, Gardenfors (2000) relies on Voronoi tessellation as a procedure for identifying convex regions in a feature space corresponding to natural properties. However,

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a downsampling function called max pooling, which only sends to the next layer activation from its most highly activated input. The final result of this process is a simplified representation of abstracted features of the source image, which can then be similarly processed by subsequent layers responsible for detecting increasingly complex features.

Voronoi tessellation is not guaranteed to partition a feature space into convex regions when using a non-Euclidean distance metric. Worse yet, constructing Voronoi diagrams can often become prohibitively expensive in even moderately high-dimensional spaces. This raises a potential concern regarding the solvency of the cluster approach, since existing accounts are not attuned to the counter-intuitive behavior of high-dimensional feature spaces.

I should note here that the problem of dimensionality is not limited to DNNs, since dimensionality is strictly speaking a feature of layers within a network, while depth is a structural feature of the network as a whole.<sup>6</sup> Under the assumption that distributed patterns of activity are representations, dimensionality is itself a problem that must be dealt with, whether one is considering artificial neural networks or neural population codes, since each activity pattern will be a point in a high-dimensional space. Therefore, dimensionality could pose a problem even for shallow networks with a single densely packed layer. For instance, the computational vision model, HMAX (Riesenhuber and Poggio, 1999), is a simple hierarchical feedforward network with four layers where the final layer contains 484 nodes, resulting in a moderately high dimensionality. Similarly, any ImageNET trained network will have a high-dimensional image space, regardless of depth, due to the 224 x 224 pixel images encoded at the input layer. But why any modeler would ever want to design a shallow network with many, many nodes is mysterious given the enormous advantages of increased depth. The focus on depth here is justified by the pragmatic success of task-optimized DNNs as both impressive feats of engineering in AI and as potential explanatory models of regions of interest in the brain.

Empirical work in high-dimensional data analysis has repeatedly drawn attention to what has become known as ‘the curse of dimensionality.’ The curse of dimensionality refers to a collection of difficulties posed for similarity measurements, nearest neighbor searches, outlier detection, and other important data analysis tasks that arise in high-dimensional vector spaces. A related problem concerns the general behavior of distance measurements in high-dimensional state space: it may turn out that such measurements become less and less qualitatively meaningful as dimensionality increases.

This requires some elaboration. High-dimensional multivariate data analysis requires a notion of distance that generalizes to many dimensions. Such distance measurements typically rely on what is called the  $L_p$ -norm (or alternatively a Minkowski metric). Here  $L_p$  is a norm picking out a function from a real vector space to the nonnegative real numbers that satisfies certain properties. Formally,

$$L_p(x, y) = \sum_{i=1}^n (\|x^i - y^i\|^p)^{1/p}$$

This function expresses that, for the points  $x$  and  $y$  in the  $n$ -dimensional real vector space  $R^n$ , and for the value  $p$  in the set of real numbers greater than or equal to 1,  $L_p$  of  $x$  and

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<sup>6</sup>I am grateful to an anonymous reviewer for pressing me on this point.

$y$  equals the  $p^{th}$ -root of the sum of the differences between each variable of  $x$  and  $y$  raised to the power of  $p$ .<sup>7</sup> For instance, when  $p$  takes a value of 2,  $L_p$  is the Euclidean norm and distance between points in the vector space is measured using Euclidean distance. When measuring Euclidean distance, this essentially amounts to using the Pythagorean theorem, which states that the length of the hypotenuse of a triangle equals the square root of the sum of the squares of the other two sides, generalized to a space of arbitrarily many dimensions. We can imagine drawing a bisector between points  $x$  and  $y$  in  $R^n$  and then triangulating the length of that line by first summing the squares of the difference between each dimensional variable (from 1 to  $n$ ) of  $x$  and  $y$  and then taking the square root of that total. The choice of norm constrains similarity analyses by effecting how modelers carve up the high-dimensional space. While Euclidean distance is the most commonly used metric norm,  $p$  can take a wide range of real values.

This is where the curse of dimensionality rears its head. Making inferences about similarity in ML models becomes difficult in high dimensions, since a training set of fixed size occupies a shrinking fraction of state space as dimensionality increases (Domingos, 2012). Moreover, high-dimensional feature space is sparse. The vast majority of the volume of a multivariate normal distribution lies not near the mean, but in a very small, increasingly distant fraction of the space around the mean. Formal results show that, for a wide range of norms, the proportional difference between the nearest and farthest points from a given sample point in a space vanishes as dimensionality increases (Beyer et al., 1999; Pestov, 2000). This would be a non-issue if there were only two elements to search through. But there are many irrelevant features in most ML tasks, and the high noise-to-signal ratio makes the search unstable. This makes similarity comparisons on the basis of relative distance in state space become increasingly dubious, since nearest neighbor searches become effectively random as models become more complex. The relative contrast between proximal and distal points tends to become less and less meaningful in high-dimensional spaces. Essentially, the space expands, and the points of interest grow farther and farther apart from one another such that the relative contrast between them becomes qualitatively meaningless. The distances between points which intuitively should be nearby quickly become nearly indistinguishable from the distances between points farther away from each other.

Using Beyer’s theorem, Zimek et al. (2012) demonstrate that using distance measurements to identify clusters becomes increasingly difficult in high-dimensional spaces. Their results show that, for Euclidean distances, the volume of a hypersphere exhibits unusual behavior that makes similarity judgments less meaningful in high-dimensional state spaces. They observed that small changes in its radius could decide whether every point or no point fell within the volume of a hypersphere (Zimek et al., 2012, p. 369-370). This would be as if,

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<sup>7</sup>Some suggest that fractional values for  $p$  might make analyses of high-dimensional data sets more tractable (Aggarwal et al., 2001). However, these values do not strictly speaking qualify as distance metrics because they violate the triangle inequality.

while trying to visualize clusters in a two dimensional scatter plot, very minuscule changes in the radius of a cluster made the difference between every single activation in the network falling within the same cluster and no activations falling within that same cluster.

Zimek et al. (2012) point out that this concentration effect is not always observed when data is multiply distributed. Nearest-neighbor searches can be both theoretically and practically meaningful when the search is limited to objects from the same cluster as the query point, and it is assumed that other clusters are well separated from the cluster in question (Zimek et al., 2012, p. 367). They appeal here to the concept of pairwise stability of clusters, which holds whenever the mean distance between points of different clusters dominates the mean distance between points belonging to the same cluster (see also Bennett et al., 1999). When clusters are pairwise stable, the nearest neighbor to any point belonging to a given cluster will tend to also be a member of that same cluster (Zimek et al., 2012; Bennett et al., 1999). But this is not necessarily helpful when category cluster membership is precisely what is at issue. Additionally, this assumes that all dimensions of a dataset carry information that is relevant for clustering. However, this is often not the case, and irrelevant features in the data can impede the separation of distributions.

Durrant and Kabán (2009) identify the prevalence of irrelevant attributes in feature space as a major contributing factor to the curse of dimensionality. The presence of irrelevant attributes adds noise that masks attributes relevant for categorization. The ratio of noise to correlated attributes in the dataset are a determinative factor in whether distance concentration occurs (Durrant and Kabán, 2009; Zimek et al., 2012). Durrant and Kabán (2009) provide accordingly a negative requirement for avoiding the concentration of distance measures in high-dimensional Euclidean space: the contribution of relevant attributes must grow no slower than that of noise (Durrant and Kabán, 2009, p. 390). Successful concept acquisition requires parsing through relevant and irrelevant attributes and learning to abstract away from features of category exemplars that do not contribute to successful categorization. When inputs have many irrelevant features, the distance concentration effect poses a problem for category clustering in high dimensions. This points to a connection between the curse of dimensionality and the problem of “nuisance variation” which plagues perceptual similarity tasks in ML. Nuisance factors are repeated and systematic sources of variance that do not contribute to successful decisions. ML systems and computer vision models need to learn how to adjust to nuisance variation in order to solve perceptual similarity tasks. DNNs’ resilience in the face of nuisance variation is frequently explained in terms of their capacity to draw exponentially more linear regions in state space compared to shallower networks in order to identify far more complex decision boundaries (Montúfar et al., 2014). The question is how can we effectively interpret the representational structure underlying these complex decision boundaries?

One suggestion is to opt for alternative measures of similarity. This is reflected in the common practice in cognitive neuroscience of using correlation distance to measure similarity between points in state space associated with different brain states (Kriegeskorte et al.,

2008a; Kriegeskorte and Kievit, 2013; Nili et al., 2014). Correlation distance computes similarity by subtracting the Pearson’s correlation coefficient  $\rho$  between two activation vectors from 1. Pearson’s correlation is a measure of normalized covariance between two variables. It can be calculated by dividing the covariance of two variables by the product of their standard deviations. Correlation can be given a geometric interpretation due to its relation to cosine distance. The cosine of the angle between two vectors  $X$  and  $Y$  can be obtained by normalizing them by their  $L_2$ -norms and then calculating their inner product. The cosine distance between  $X$  and  $Y$  can then be represented as:

$$d_{cos}(X, Y) = 1 - \frac{\langle X, Y \rangle}{\|X\| \|Y\|} = 1 - \cos(\angle X, Y).$$

The correlation distance between two  $n$ -dimensional vectors is equivalent to their cosine distance after subtracting the mean value from each activity pattern. If  $\bar{V} = V - \bar{V}$  represents the difference between some vector and its sample mean, then correlation distance can be defined as

$$d_{\rho}(X, Y) = 1 - \frac{\langle \bar{X}, \bar{Y} \rangle}{\|\bar{X}\| \|\bar{Y}\|} = 1 - \cos(\angle \bar{X}, \bar{Y}).$$

Unlike Euclidean distance, normalization makes correlation distance scale invariant. Pearson’s  $\rho$  between two identical vectors is 1 regardless of vector scale. So, correlation distance is 0 when the normalized patterns of activity are identical. Correlation distance normally takes a real value between 0 and 1 where lower values indicate that the two patterns of activity are closer together in state space. Representational similarity analysis (RSA) — which I will discuss in further detail below — uses these correlation measures to then construct a simplified model of the underlying representational space (Kriegeskorte et al., 2008a; Kriegeskorte and Kievit, 2013). RSA takes the pairwise distance between activation vectors evoked by different experimental conditions and arranges them into a two-dimensional representational dissimilarity matrix. This same method of analyzing the representational geometry of state space has been used to compare representations in the primate visual system to those in DNNs (Khaligh-Razavi and Kriegeskorte, 2014; Yamins et al., 2014). But if the underlying space is not particularly well-structured (e.g. it suffers from the distance concentration effect), then these problems with structure are likely to manifest in the reconstructed space as well. Moreover, as Carlson et al. (2018) point out, by moving in this way from data to phenomenon modelers make certain choices in reconstructing the brain’s representational space. These choices embody assumptions that could potentially distort the true nature of that space (Carlson et al., 2018, p. 92). In this case, the decision to use correlation distance as a similarity measure effects how representations are reconstructed from the data.

The above point is made especially clear by Bobadilla-Suarez et al. (2020), who suggest that the choice of similarity measure can have significant implications for what the underlying representational space might look like. In that paper, the authors consider three distinct

families of common similarity measures and analyze how they impact the structure of the underlying representational space. These families of similarity measures are organized according to the properties of the data which they are sensitive to. First, this includes whether the measure of similarity in question depends on the angle between vectors (such as correlation or cosine distance) in state space or on vector magnitudes. The latter are then subdivided into distributional measures that are sensitive to covariance across dimensions (like Mahalanobis distance) and those that are not (like  $L_p$ -norms/Minkowski metrics). They evaluated the performance of these different similarity measures across different regions of interest in the brain and for different categorization tasks. They did this using a decoding approach, relying on confusability in a machine learning classifier as a proxy for baseline neural similarity. This is motivated by the intuition that brain states that are similar should be more likely to be confusable in decoding. They then tested the predicted similarity relations generated by each measure against a confusability matrix generated from classifier decisions. Their results indicated that “similarity profiles differed between studies, suggesting that the operable neural similarity measures can change as a function of task or stimuli” (Bobadilla-Suarez et al., 2020, p. 377). Moreover, competing metrics tended to outperform correlation distance.

Another suggestion is to use some kind of feature selection to reduce the dimensionality of the representations under investigation. There is an extensive literature in the fields of data science, computational neuroscience, machine learning, and high-dimensional statistics that considers how techniques for dimensionality reduction might shed light on representational structure by uncovering simplified low-dimensional manifolds embedded in high-dimensional activation space (see e.g. van der Maaten and Hinton (2008)). Dimensionality reduction involves reducing the number of variables required to accurately describe high-dimensional patterns of activity. There are a range of different techniques used for dimensionality reduction. A handful of standard techniques among these includes principle component analysis (PCA), multidimensional scaling (MDS), random projection, factor analysis, ISOMAP, t-SNE, and UMAP (van der Maaten and Hinton, 2008; Ganguli and Sompolinsky, 2012; Lin et al., 2019; McInnes et al., 2020). This list is far from exhaustive, and modelers’ choice of dimensionality reduction technique typically depends on the size, dimensionality, and qualitative nature of their data set as well as various pragmatic considerations (Gracia et al., 2014; Nick et al., 2015). Methods of dimensionality reduction that are optimized for feature extraction and cluster analysis may provide an efficient way of visualizing high-dimensional representations. This is reflected in the common practice of using PCA or MDS to analyze and interpret neural manifolds in cognitive neuroscience (see for instance Kriegeskorte et al., 2008b; Mante et al., 2013). PCA, for instance, reduces the axes of the underlying space to the dimensions with the highest variation with change in stimuli. These highly variable dimensions are interpreted as those which are functionally relevant for the task at hand. This makes it essentially a kind of data-driven functional decomposition. However, neural activity is noisy and highly correlated, and PCA runs the risk of excluding functionally relevant

dimensions, or overemphasizing irrelevant ones. In general, dimensionality reduction can introduce similar interpretive difficulties, like overlap in visualized points where none exists in the native dimensionality of state space. Perhaps more problematically, these choices also embody implicit assumptions on the part of the modeler about the content and structure of neural representations. These assumptions threaten to introduce bias in the transition from data to phenomena (Carlson et al., 2018).

Collectively, these issues present sufficient cause for concern that traditional methods for identifying category clusters in state space as the vehicles of content may be beyond reach in DNNs with many thousands of dimensions. Shea’s proposal explicitly calls for identifying clusters by first plotting the distribution of points in state space before identifying regions “containing clusters of proximal points which are relatively distant from the other points in state space (relative to the overall volume filled by activation points produced by all training samples)” (Shea, 2007, p. 9-10). Similarly, Gardenfors (2000) relies on Voronoi tessellation with Euclidean distance to partition a feature space into convex regions. But in high-dimensional space, *all* of the points in a distribution will tend to be relatively distant from one another. The intuition-breaking behavior of high-dimensional space means that attempting to identify the border of a cluster in these ways might be effectively impossible since small variations in the radius of the cluster could possibly mean the difference between including or excluding every single activation in the sample set. In practice, modelers make a host of choices to structure the data that underlies a state space in order to analyze and interpret the representational structure of a neural population or computational model. However, these choices embody implicit assumptions of scientists. These assumptions constrain the range of hypotheses tested against the data, threaten to warp the underlying structure of state space, and possibly bias conclusions drawn from data.

Nevertheless, I think that the general thrust of the cluster approach seems on the right track, since manifolds are typically invoked in quasi-representational explanations of the mechanism of operation of DNNs. But if the cluster approach is to provide the kind of theory of representation connectionists desire, then we will need a method for measuring representational similarity that is generalizable to deep learning models, else the prospect of an adequate state space semantics collapses. Clearly, there is representational structure latent in object manifolds. But the question remains, how can we make the task of interpreting that structure tractable without introducing undesirable conceptual baggage that biases the range of hypotheses modelers consider? The following section looks to the philosophy of science for answers.

## 4 Content in Deep Neural Networks

Thus far, I have presented a set of difficulties for generalizing a connectionist theory of representational content to state-of-the-art DNNs. The curse of dimensionality suggests that we cannot make meaningful, hypothesis-neutral judgments about cluster membership

in high-dimensional space. Nevertheless, manifolds — the high-dimensional equivalents of clusters — are typically invoked in existing explanations of the successful performance of deep neural networks. This seems to suggest that representational explanations of DNNs are still within reach. However, we still need a method for individuating type-identical content when dimensionality precludes grouping representations by directly plotting points in state space. Fortunately, conceptual tools from the philosophy of science can help us move beyond these methodological worries.

#### *4.1 Representational Vehicles Exploit Real Patterns in Nature*

The most influential accounts of representational content — especially Fodor (1990), Dretske (1988), and Millikan (1984) — focus primarily on the intentionality of sensory-perceptual states. Their hope was to incorporate intelligent behavior into the scientific image of the world in a way that preserved the behavior-guiding role played by the contents of internal states. In seeing that my dog Frasier is now sitting beside me, there is something that my accompanying perceptual state is directed at — my furry, tail-wagging companion. The idea goes that there is some internal state that represents Frasier to the mechanism in my brain responsible for processing sensory inputs and producing the right behavior, namely petting him. However, the sense of representation in play here is far more deflationary and typically cashed out in terms of encoded information that can be read out by downstream processes to produce behavior. First, this involves identifying physical constituents of a mechanism that we can group into types. It also requires a principled criterion for fixing the meaning of these groups according to the information which they purport to encode. To do this we need to say how a system could manage to carve out features of its environment that are alike in kind and represent them for processing in a physical mechanism. Historically, this has meant showing that representations reliably instantiate the right kind of causal relation with what they represent such that they function to track the relevant kind of phenomena (Dretske, 1988). We can say that a state of a system tracks a kind when the state has been causally selected to control that system’s interactions with phenomena of that kind in virtue of the information that it encodes.

So, we can think of tokened representations as type-identical when they function to track stimuli of the same kind. I offer that we can think of these kinds as stable property clusters bound together by causal informational relations (Slater, 2015). We can bootstrap Slater’s (2015) stable property cluster account of kinds with Andersen’s information-theoretic revitalization of Dennett’s “real patterns” (see also Potochnik, 2017) to get some purchase on how a neural network could come to track such a kind (Andersen, 2017; Dennett, 1991; Stinson, 2020). According to Andersen (2017), a real pattern is one that “can be reliably picked out and tracked through time and which allows one to make predictions that are better than chance.” Real patterns are counterfactually robust: the microphysical state underlying a tokening of a pattern could have been slightly different, while still tokening the same



pattern. The basic idea is that kinds or patterns can be reliably picked out and tracked by information-theoretic means and make useful predictions. This ensures that patterns are not met with jury-rigged kinds. Nevertheless, they remain metaphysically innocuous. Since real patterns make useful predictions and are stable under counterfactual perturbation, a collection of phenomena that manifest a real pattern will constitute a kind.

To get a handle on what constitutes a real pattern, consider a digital chess program in which two computational engines are playing each other in a game of chess. The state of this game at any one moment in time can be described as no more than a complex array of pixels, or a bit map. The bit map gives a complete, accurate description of the state of the board at any one instant in the game much like a complete microphysical description of an actual chess board would. In principle, we could compute the entirety of the current board-state using nothing but the bit map. If we know enough details about the algorithms our chess-playing engines implement, then we could use the bit map to accurately predict future board-states. But this would be extremely computationally costly. It is much more efficient to characterize our program at a higher-level of description in terms of chess positions. At this level of description, familiar patterns emerge from the complicated array of flashing pixels. We can identify them as knights, rooks, pawns, and all of the recognizable features that constitute a board-state in a game of chess. Once recognized as a game of chess, enormously more efficient ways of predicting future board-states become available to us with relatively little loss of accuracy (depending on how adept you are at chess). Recognizing these real patterns means the difference between computing millions of pixels and merely inferring in your head what is likely to be the best move in an ongoing game of chess.

Applying these patterns to representations requires some disambiguation. In doing so, I do not mean to endorse Dennett’s instrumentalism about content. Real patterns — as discussed by Andersen (2017) and Potochnik (2017) — refer to *causal* patterns that structure our natural world. These patterns are idealized descriptions of underlying phenomena that allow us to successfully navigate the messy complexities inherent in nature. When thinking about the content of our internal representations, we can think of them as picking out patterns in our environment. These patterns allow for more efficient neural computations. We can say that a tokened representation belongs to a type when a system’s internal mechanism has learned to exploit a real pattern in nature for the purpose of representing and controlling its behavior with respect to that associated kind of phenomena. This is compatible with the commitment to some non-semantic way of individuating the physical particulars instantiated in a system that encode representations of patterns embodied by distal features of the environment. These representations are veridical when they allow for the successful prediction and manipulation of the features of the environment they function to track. Like other causal-informational patterns, representational content is counterfactually robust. The internal state of a mechanism underlying a tokened representation could have been slightly different while still tokening the same content, or type. This yields a modest picture of concept acquisition in terms of subjective category representations, or “conceptualizations”

(Gauker, 2011, p.6), which nevertheless play a useful role in explaining how different neural networks can learn the same concepts. Different networks may have slightly different conceptualizations based on differences in their initialization conditions or input encoding, but the category groupings correspond to the same real patterns with only slight variations or exceptions in how they are represented to the network. This modest realism about the content and structure of representations is also compatible with understanding representations themselves as causal patterns that arise from the transformations of input signals into output signals and thus the legitimate subjects of idealized models (Kriegeskorte and Kievit, 2013; Diedrichsen and Kriegeskorte, 2017).

There are many ways to describe a single pattern. In nature there are often trade-offs — though they needn't be strict ones — between computational efficiency and accuracy. Whether we prefer efficient pattern descriptions with high levels of noise or more computationally expensive patterns with lower noise tolerance may depend on how easily and reliably we can identify a pattern and the costs to us for getting things wrong. Dennett refers to specifications of such preferences as "design decisions" (Dennett, 1991). While these design decisions are not available to us when it comes our sensory-perceptual system and mental representations, they are incorporated into the phylogeny of our cognitive apparatus. When it comes to neural networks, many of these design choices are plausibly assimilated through training. In deep neural networks, the learning process can be interpreted as searching through the feature space to identify patterns with the goal of maximizing the ratio of computational efficiency to error function much in the same way that evolutionary competition selects for efficiency gains in biological systems (Buckner, 2019, p. 10). Since these patterns are considered real features of the natural world, representations manage to carve out real joints in nature through this selective process. As such, the vehicles of content latch on to efficient, counterfactually robust patterns that allow them to make successful decisions. These patterns also help us make predictions and sound inductive inferences about the behavior of a system.

Understanding representational content in terms of a system learning to exploit real patterns seems to vindicate the cluster approach. After all, the cluster approach understands training as transforming state space into a mechanism that causally tracks properties relevant for classification. Clusters get their content in virtue of the causal role they play in producing successful behavior. But this understanding also opens new avenues for ascribing content to models where the curse of dimensionality precludes directly mapping out vehicles of content in state space. We just need to identify patterns that are represented to a network's mechanism of operation that allow us to make accurate predictions and inductive inferences about the network's successful performance. In practice, this involves examining idealized models that make patterns in the geometric structure of state space more salient.

## 4.2 *Real Patterns and Idealized Models*

Idealization has enjoyed significant philosophical attention, especially in the context of scientific models (Cartwright, 1994; Weisberg, 2007; Rohwer and Rice, 2013; Elliott-Graves and Weisberg, 2014). Real patterns can help to illuminate the positive representational role of idealization in analyzing the structure of high-dimensional state space. Here it is useful to draw on Potochnik’s (2017) extensive account of the relationship between patterns and idealization in science. She describes how the practice of science by limited humans in a complex world leads to widespread idealization. Potochnik argues that scientific enterprise can be understood as a search for causal patterns in the face of “causal complexity”. Causal complexity refers to the fact that phenomena under investigation are causally influenced by a multitude of factors beyond the variables targeted by the investigation, including by controls implemented to highlight the targeted variables and many other influences (Potochnik, 2017, p. 35). Science aims to represent real patterns embodied by causally complex phenomena. This is because patterns play a central role in promoting understanding and manipulability. Causal complexity means that phenomena can embody many patterns. The more causal influences there are on a target phenomenon, the more variables there are to produce patterns. Scientists thus have a choice not only about what phenomena should be the targets of study, but also about which patterns should be the focus of their study. Just as competing pragmatic and epistemic considerations inform the design decisions specifying how a single pattern is described, human interests inform which patterns will be useful for making scientific progress. Real patterns that are of interest to scientists tend to be simple and general enough to facilitate understanding, but fine-grained enough to provide the basis for manipulability.<sup>8</sup>

Specifying patterns that are productive for science motivates idealization. Idealizations are assumptions made without regard for whether they are true and often with full knowledge that they are false (Cartwright, 1994; Potochnik, 2017, p. 42). Physics often assumes frictionless planes even though no such planes exists. But idealizations in science often take subtler forms. A linear regression model draws a curve that purports to represent the relationship between variables and treats divergence from this curve as error or noise. This is done with full knowledge that divergence in the data also represents observed facts. Idealizations of this kind interpolate and extrapolate beyond the data to identify the pattern they instantiate. Elgin (2004) suggests that these “felicitous falsehoods” play a cognitively valuable role in science. They “impose an order on things, highlight certain aspects of the phenomena, reveal connections, patterns and discrepancies, and make possible insights that we could not otherwise obtain” (Elgin, 2004, p. 127). So idealizations, understood as deliber-

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<sup>8</sup>The emphasis on manipulability is meant to capture that information-theoretic real patterns are by and large causal patterns (Woodward, 2003). Causation, however, is a metaphysically fraught topic. Those unfriendly to manipulability accounts of causation should still feel free to regard real patterns as both cognitively and pragmatically valuable for their role in facilitating understanding and manipulability.

ate false assumptions, can contribute to successful scientific representations by highlighting real patterns in what they are intended to represent.

This picture also distinguishes idealizations from abstractions. The former intentionally represents a target system as different than it actually is, while the latter merely omits certain details. Abstractions represent by ignoring some features of the target phenomenon that are inconsequential for the representation. This generates a picture of idealizations and abstractions as distinct but compatible practices of scientific investigation. These practices often become intertwined in scientific models.

Cao and Yamins (2021) augmented notion of model-mechanism-mapping (3M++) provides a relevant illustration of this tendency. On the standard account of 3M, in order for abstract dynamical models to have explanatory force in systems and cognitive neuroscience there must be a plausible mapping between elements in a model and the parts of the mechanism they represent (Kaplan and Craver, 2011). 3M++ involves two expansions of this standard account. These are an “adequate abstraction” expansion followed by a “transform similarity” expansion. Adequate abstraction allows for meaningful mechanistic comparisons between systems with very different low-level implementation details. Computational models are built out of transistors, not cells. In comparing a target system, like the visual pathway, to a computational model, scientists apply abstractions that course-grain the target system and its computational model. These abstractions relinquish certain details in order to meaningfully compare the abstract dynamics of target system and model. The enduring details are those scientists take to be functionally relevant for providing an adequate explanation of the phenomena of interest. Simply describing the system in terms of a state space constitutes one such kind of abstraction. But with complex biological systems scientists often need to make difficult choices about what to measure and what features of the abstract dynamics are functionally relevant. These choices constitute, either implicitly or explicitly, a commitment to some degree of abstraction as adequate for explaining their target phenomenon. Precisely how much abstraction qualifies as empirically or predictively adequate should be informed by principled theoretical and pragmatic reasons. Transform similarity allows for a class of systems with the underlying components to be considered a single explanatory target, even when the actual target systems differ in some substantial way (Cao and Yamins, 2021, p. 10). This notion is used to address the fact that there is no one-to-one mapping between neurons in different brains. The basic idea is that we can define a transform class that consists of a set of linear maps between populations of neural activity in different individuals. This transform class posits that activity of any neuron in a given region of one individual can be reproduced by a linear combination of neural activity in the corresponding region of another individual.<sup>9</sup>

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<sup>9</sup>Stated more formally, this means that for all input stimuli  $x$ , the neuronal responses to  $x$  by neuron  $i$  in individual  $T$  can be written as  $n_i^T(x) = \sum_{j=1}^M a_{ji} n_j^S(x)$  where  $a_{ji}$  are constants that represent the contribution each source neuron makes to replicating the activity of the target neuron (Cao and Yamins, 2021, p. 7).

According to the 3M++ criteria, mappings between models and their targets are mediated by a common abstraction because information preserving similarity transforms are only defined in an abstract state space (Cao and Yamins, 2021, p. 10-11). For a fully-runnable computational model this merely requires reading off activity of each unit in the model for each stimulus response. Course graining the target system requires a bit more work and involves treating each neuron as a function that produces a spike-rate for each input to obtain a vector for each stimulus. Once the low-level details have been stripped away, we can use such a transform to generate a mapping from the model dynamics to the target. The relation of transform similarity represents the contributions of multiple source neurons as identical to the contribution of a single model unit. The result of this transform represents the state space of the target system as if it was isomorphic to the model’s state space. Transform similarity thus constitutes a kind of idealization that operates in tandem with prior abstractions in order for a computational model to represent and explain a complex target phenomenon.

Idealizations make a cognitively valuable contribution to scientific models. They play a positive representational role by representing a system as if it were some way that it is not. Potochnik (2017) calls this *representation as-if*. The positive content of idealizations is to center the relevance of some real pattern. Potochnik suggests that "idealizations contribute to understanding by representing as-if to the end of depicting a causal pattern, thereby highlighting certain aspects of that phenomenon (to the exclusion of others) and revealing connections with other, possibly disparate phenomena that embody the same pattern or, in some cases, that are exceptions to that pattern" (Potochnik, 2017, p. 97). I argue that this should be seen as the explanatory goal of 3M++ as well as of representational models in cognitive neuroscience, which compare the representational geometry of state spaces derived from multiple modalities. Moreover, what causal pattern occupies the focal point of a particular scientific explanation is, to some degree, sensitive to the interests and goals of scientists. Therefore, it should not come as a surprise that even more data-driven methods of functionally decomposing a high-dimensional state space are not hypothesis-neutral, nor should this be seen as a devastating flaw. Real patterns that are of fundamental interests to the explanatory goals of scientists already constrain the hypothesis-space. This is legitimate so long as those interests are sufficiently well-motivated by empirical and theoretical considerations. Carlson et al. (2018) make a similar remark that hypothesis-driven approaches that use dimensionality reduction can be defensible, "so long as they are carefully constrained" (Carlson et al., 2018, p. 95). As long as these conditions are satisfied, different ways of decomposing state space in order to make real patterns salient to scientists are sound methods of testing whether a target phenomenon really does embody the causal pattern of interest.

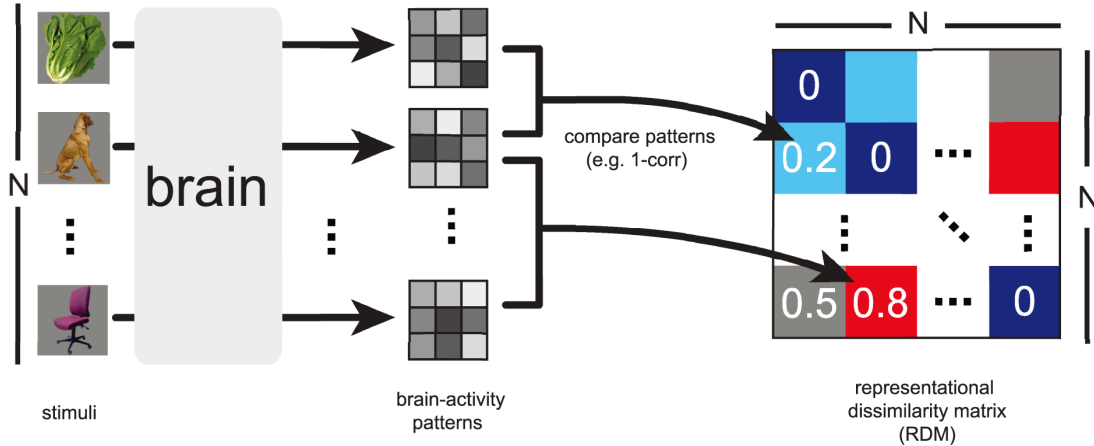
With this on board, the actual methods and practices of computational neuroscience and machine learning can serve as a guide to representational explanations. Recent work in these areas has uncovered a surprising fact about deep convolutional neural networks (DCNNs). Models optimized merely to classify images predict spiking responses in the highest level

of the ventral stream, the inferior temporal cortex (IT) (Yamins et al., 2014). That such task-optimized models manage to predict something about the brain supports the notion that these neural networks form partial explanatory models of the brain (Cao and Yamins, 2021). These developments point in the direction of progress for cognitive science. The quantitative methods used to establish predictive relationships between DCNNs and primate brain activity can be assimilated for the purpose of ascribing content to high-dimensional connectionist models.

### 4.3 *Representational Models*

We want to understand the structure and content of representations encoded in DNNs, at least in part, to better understand how these models can teach us about neural representations in the brain. In service of this goal it will be helpful to distinguish between decoding and encoding models that are used to analyze neural representations. Multivariate pattern analysis (MVPA) refers to a set of techniques for analyzing patterns in neural data (Haxby, 2012; Ritchie et al., 2019). MVPA can be separated into decoding and encoding methods (Naselaris et al., 2011; Naselaris and Kay, 2015; Kriegeskorte and Douglas, 2019). Researches often appeal to linear decodability to reveal the content of neural representations (Eliasmith, 2013). Neuroscientists rely on computational models to make comparisons or assist in decoding subsets of neural populations, or regions of interest, in the brain. The idea is that a biologically plausible linear classifier can stand in as a proxy for the downstream processes that read out information encoded by neural representations. But as computational models grow in scale and complexity, so do the number of idiosyncrasies between models and their target system. Defining a precise mapping from computational model to neural population becomes increasingly intractable (Kriegeskorte et al., 2008a). Moreover, decodability does not always license inferences about representations, especially when decoding is itself assisted by machine learning techniques (Ritchie et al., 2019). Both linear and non-linear classifiers are unconstrained by the information that the brain actually exploits. While brute force classifiers may prove effective at distinguishing activity patterns associated with different experimental conditions, we need an additional guarantee that a decoding exploits the same information encoded by neural representations.

By contrast, encoding models aim to predict the response patterns of neural activity from descriptions of the experimental conditions (Kriegeskorte and Douglas, 2019). Encoding models work in the opposite direction of decoding models. Whereas decoding models aim to detect the presence of specific information in the brain, encoding models map sensory stimuli to sensory regions of the brain and make predictions about the representational space. Encoding models thus work in the same direction as the flow of information in the brain to make comprehensive predictions about the representational space (Naselaris and Kay, 2015; Yamins and DiCarlo, 2016; Diedrichsen and Kriegeskorte, 2017). An encoding model can consist in a DNN model trained on some sensory-perceptual task (Kriegeskorte

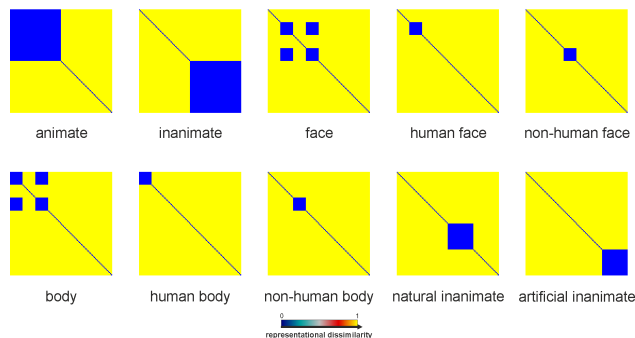


**Figure 1:** Computing representational dissimilarity matrices (RDMs) (reproduced from Nili et al. (2014)).

and Douglas, 2019). Since there is no one-to-one correspondence between the model and measurement channels in the brain, we need to apply the principles of abstraction and idealization in order to test the predictions of an encoding model. This can either involve testing predictions of raw measurements using a linear transform or by testing predictions about the similarity structure in a representational space (Naselaris et al., 2011; Kriegeskorte and Douglas, 2019; Cao and Yamins, 2021). The latter case is well suited to the goal of analyzing representational content. We can use the techniques of representational similarity analysis (RSA) to make hypotheses and test predictions about the similarity relations of representations associated with different experimental conditions. RSA provides a framework for comparing represented information that seeks out such predictive relations by establishing a second-order isomorphism between the representational structure of a model and its target brain region (Kriegeskorte et al., 2008a).

RSA examines the representational structure of an activation space using representational dissimilarity matrices (RDMs). RDMs visualize pairs of neural activations in a square, symmetric matrix of pairwise stimulus correlations. An RDM contains a cell for each unique pair of experimental conditions. The matrix can be arranged according to an observer’s (or a group of observers’) similarity judgments of these conditions. Each cell contains a value measuring the similarity between the activation vectors associated with two stimuli. For a given set of stimuli, the matrix describes how similar or dissimilar the representations are according to a chosen similarity measure (e.g correlation distance, Mahalanobis distance) for every possible pair of stimuli. Entries along the diagonal represent comparisons between identical stimuli and take a value of 0. The value of each off-diagonal entry represents the

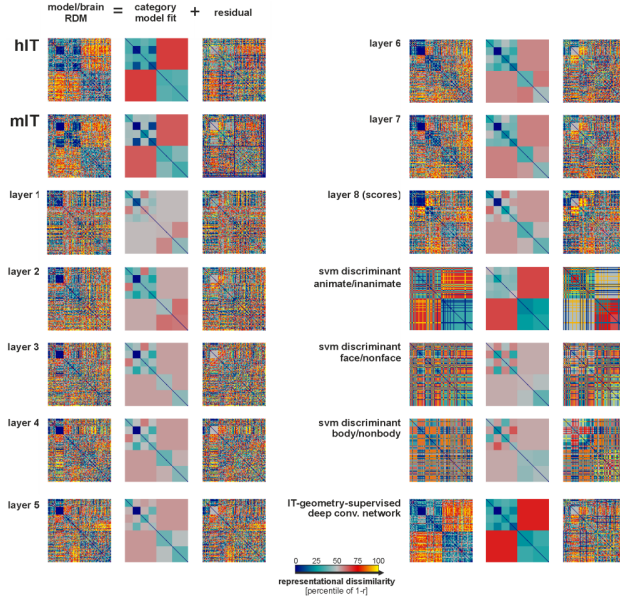
dissimilarity between the activation patterns corresponding with two different stimuli. Lower value entries indicate that a pair of stimuli produce more similar representations, while a value of 1 indicates no correlation whatsoever. For any given pair of stimuli  $i$  and  $j$ , each entry in the matrix is computed by taking the activation vector of  $i$  and the activation vector of  $j$  (or their associated neural population code when computing a RDM for the primate IT), computing the similarity in activation space between those two vectors to yield a real value between typically between 0 and 1 (Figure 1) (Kriegeskorte et al., 2008a; Nili et al., 2014). The result is a two-dimensional map of the similarity relations between a set of activation vectors. RDMs measuring IT neural population responses exhibit a clear block-diagonal structure characteristic of the IT’s high performance at object categorization. Predictively adequate connectionist models will naturally exhibit a similar block-diagonal structure in their own RDMs. When cells are arranged according to observer similarity judgments, strong structural correlation provides evidence that the activation space implements a psychological space (Kriegeskorte et al., 2008a). Thus, RDMs can be interpreted as a simplified description of the representational geometry of a given activation space.



**Figure 2:** Khaligh-Razavi and Kriegeskorte (2014) created ten different category-cluster RDMs as predictors of clustering. These prediction RDMs were then fit to each computational model using a linear regression to model the semantic structure of their representations (reproduced from Khaligh-Razavi and Kriegeskorte (2014)).

A clear example of this framework in action can be found in Khaligh-Razavi and Kriegeskorte (2014). Khaligh-Razavi and Kriegeskorte (2014) analyzed brain responses in both monkey IT and human IT for a set of color images of objects spanning a range of animate and inanimate categories. They then used RDMs to compare these representations to those generated by 37 different computational models of varying designs. To measure the strength of clustering they created ten category-cluster RDMs as predictors, which they subsequently fit to each IT and model RDM (Figure 2). These grouped the set of experimental conditions into groups according to a number of intuitive categories. The category-clusters represented animate, inanimate, face, human face, non-human face, body, human body, non-human body, natural inanimate, and artificial inanimate (Khaligh-Razavi and Kriegeskorte, 2014).



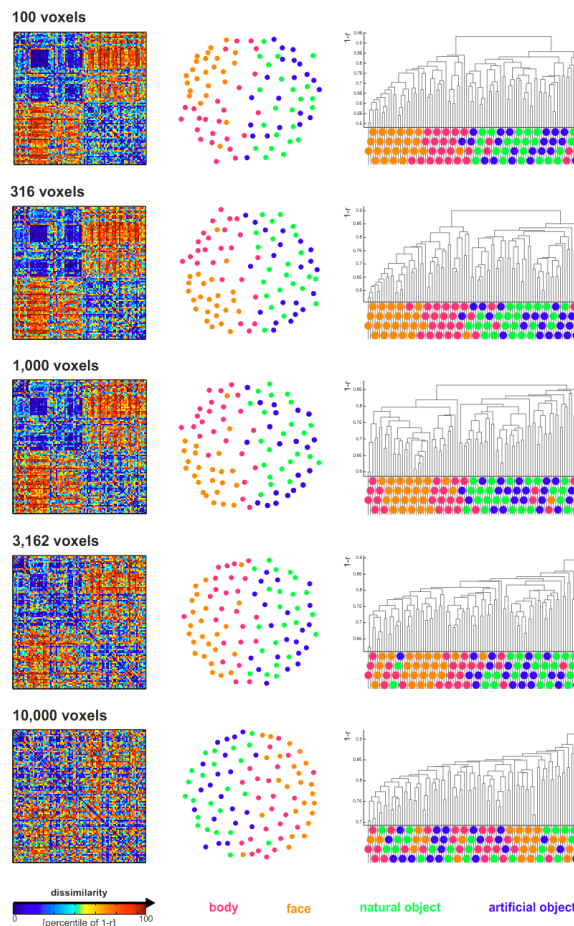


**Figure 3:** Category-cluster analysis of a supervised DCNN model reveals that a similar representational structure to human and monkey IT neuronal population responses emerges across the various layers of the model. Using a linear-regression, they fit their predicted category-cluster RDMs (Figure 2) to each layer of the DCNN in order to visualize the prominence of distinct category clusters at each hidden layer of the model. The result of this fitting is shown in the middle column. The final weighted combination of layers (bottom right) shows a clustering structure remarkably similar to that of the hIT and mIT (reproduced from Khaligh-Razavi and Kriegeskorte (2014)).

Though models designed to emulate the structure of the ventral stream (such as HMAX and VisNet) were included among the 37 computational models, they found that these were outperformed in predictive accuracy by a task-optimized DCNN. The representations generated by the supervised DCNN best predicted the the category clustering found in the IT RDMs (Figure 3).

One way of interpreting this procedure is that the modelers identified real patterns in their experimental conditions and constructed a model of a conceptual space around these patterns to generate the category-cluster RDMs. By fitting this conceptual model to an encoding model (in this case, a DCNN), we can identify the real patterns that arise as plausible candidates for the content of representations, since they are both highly efficient, counterfactually robust, and tolerant of nuisance variation. The modelers then used these patterns to predict the representational similarity structure of human and monkey IT RDMs. What we find is that — much like the primate visual system — higher levels of processing in the DCNN begin to approximate the very same patterns. Each subsequent layer represents and

processes higher level properties of the input stimuli with greater tolerance for noise and nuisance variation than the layers preceding it. Comparing brain and model RDMs establishes a kind of second-order isomorphism between model and brain region representations that very closely mirrors the category-cluster patterns constructed by a well-defined conceptual space. This suggests that both our computational model and its target system represent the same real patterns at later stages of processing. This allows these systems to efficiently identify new conditions and generalize successful performance to these conditions. If this interpretation is right, then DCNNs turn out to be predictive of neural activity in the primate visual system precisely because they learn to represent and exploit the same kinds of high-level patterns.



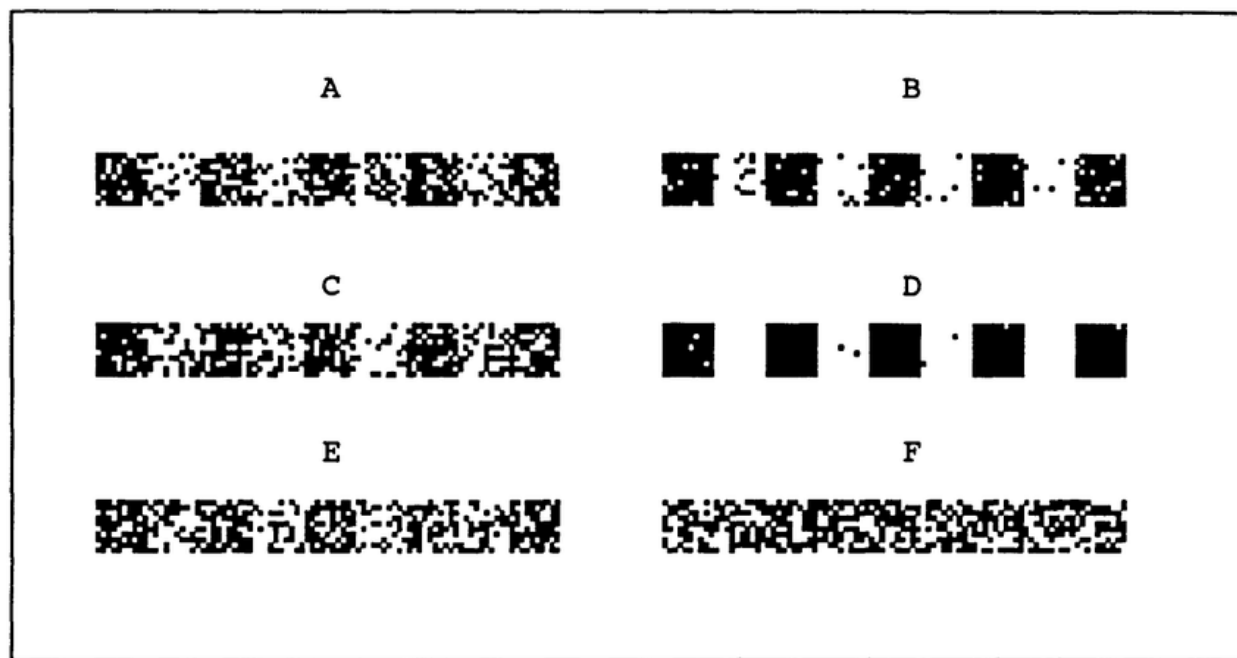
**Figure 4:** RDMs (left), MDS arrangements (middle), and dendrogram trees (right) were computed on the same region while varying the number of voxels selected for inclusion. The similarity structure and categorical clustering of human IT varies based on the number of voxels. Figure reproduced from Kriegeskorte et al. (2008b) supplemental data.

However, as discussed in §3, the modelers’ choice of similarity measure is a crucial decision for RSA. Which measure of similarity one chooses has important implications for how the representational space is reconstructed from the underlying data. While it has been commonplace to use correlation distance because it is scale invariant, there are open questions as to whether this is a principled reason for choosing it as a similarity measure (Walther et al., 2016; Bobadilla-Suarez et al., 2020). Results from Walther et al. (2016) and Bobadilla-Suarez et al. (2020) both seem to suggest that other measures of similarity perform better for certain kinds of data. Moreover, modelers typically opt for some form of noise normalization to better capture the dimensions of variation in the representational space. This typically precedes some kind of feature selection and subsequent dimensionality reduction. Dimensionality reduction can prove helpful for navigating intractably complex activation spaces and representing only the dimensions that are functionally relevant to a representational space with lower intrinsic dimensionality. However, these data-driven methods are not guaranteed to generate a hypothesis-neutral transition from data to phenomena (Goddard et al., 2018; Carlson et al., 2018). How the modeler chooses to carve up the data effects the structure of the representational space.

Figure 4, reproduced from Kriegeskorte et al. (2008b), illustrates this effect. Kriegeskorte et al. (2008b) used data-driven methods to identify the responsiveness of voxels in the human IT cortex to visual stimuli. Sets of voxels were then selected for inclusion in the similarity analysis according to their responsiveness. This marks an implicit decision to treat voxels that are highly responsive to stimuli as functionally relevant to the representational space and to treat less responsive voxels as noise. The number of voxels selected for inclusion impacts the structure and discernibility of patterns in the representational space. As the number of selected voxels increases the categorical structure found in the RDM becomes less distinct. At 10,000 voxels the organization between faces and bodies appears almost completely obliterated. Of course, we have good, theory-driven reasons to suspect that structure is there. Stripping away irrelevant features reveals that structure. However, selecting for certain sets of voxels over others, even when using data-driven methods, runs the risk of biasing results towards a favored hypothesis.

To deal with these worries, we should emphasize an important difference between encoding and decoding models. Encoding model approaches like RSA differ from decoding approaches to MVPA in that a model constitutes a testable hypothesis about the representational structure of a sensory region of interest in the brain. Kriegeskorte et al. (2008a) emphasize the fact that RSA is hypothesis-driven when we test whether a predefined model fits the representational space of a region of interest in the brain. This kind of hypothesis testing can be cashed out in terms of an exploratory search for causal patterns in the underlying neural data. Importantly, the focal pattern of interest constrains the hypothesis space. Which patterns are useful for drawing out representational explanations will depend on a host of goals and interests of particular researchers. Because the underlying neural data is exceptionally complex and noisy, testing for and representing salient, causal patterns in the

data often requires various abstractions and idealizations. These might include opting to select for specific features, dimensionality reduction, normalization, or choosing a particular similarity measure. Modelers must make choices like these to structure and represent the state space in such a way that emphasizes the presence of real patterns to facilitate understanding. Idealizations and abstractions of this kind are often inseparable from the actual practices involved in studying neural representations, even when those methods purport to be data-driven and thus “objective.” If we understand encoding approaches as an exploratory search for real patterns in neural data that allow us to make testable predictions about the structure of representational space, then the rampant idealization found in RSA becomes innocuous. After all, there are many ways to represent a focal pattern that a phenomenon embodies. A real pattern is present in the data so long as there is a way of representing the information carried by the pattern that is more efficient than a bit map of the data.



**Figure 5:** Reproduced from (Dennett, 1991, p. 31).

We can conceive of the patterns embodied by an encoding model much in the same way that Dennett (1991) thinks of the simple pattern he calls “bar code” (see Figure 5). Figure 5 pictures six frames labeled A-F, each consisting in ten rows of ninety pixels. Each of the first five frames embodies the same real pattern, bar code, consisting of ten rows of ten black dots followed by ten rows of ten white dots, etc. In fact, all six instances were generated with the same algorithm, and the ratios of noise allowed to interfere with the production of each frame varied from 1% to 50%. While A-E embody bar code, F poses a curious case. F appears

indistinguishable from pseudo-random noise. Dennett (1991) suggests that the notion of an indiscernible pattern is self-contradictory and that F is therefore degenerate. However, there is one sense in which F does embody a pattern. All six frames were produced by the same algorithm. This algorithm produces a dot matrix of the pattern bar code, but simulates noise by changing the color of a certain percentage of randomly selected pixels. This simplified description constitutes a distinct real pattern embodied by A-F (Potochnik calls this pattern “bc-algorithm”) (Potochnik, 2017, p. 28). Though this pattern is not readily discernible by simply looking at F, we know it is there because we have privileged access to the program that produced each frame. Since encoding models are fully runnable computational models, we have, to some extent, a similar kind of privileged access to how an instance of a pattern embodied by our model is produced. We also have theoretical reasons to suspect that there are real patterns in the neural data based on what we know about sensory regions of the brain and the nature of object recognition tasks. This motivates exploratory searches that involve abstractions and idealizations in order to make those patterns salient.

This view of encoding models as a search for real patterns in neural data aligns with the suggestion that hypothesis-laden methods in neuroscience should be properly conceived of as exploratory analyses (Goddard et al., 2018; Carlson et al., 2018). According to Goddard et al. (2018), an exploratory analysis “gives you evidence that, for the stimuli, parameters, and contexts that were examined, there is a feature space that can capture the variation in those parameters” (Goddard et al., 2018, p. 61). It is a real virtue of encoding model approaches that they use hypothesis-driven methods to carve up an activation space such that it captures the variation associated with different experimental conditions. Ritchie et al. (2019) suggest that we can make more secure inferences about neural representations when information encoded by neural activity can be used to predict observer behavior based on a psychological model (Ritchie et al., 2019, p. 599-601). The idea is that if a state space implements a psychological similarity space, then we can apply psychological hypotheses directly to that state space. RSA can be understood as reconstructing a representational space according to a particular functional decomposition of neural activity. That functional decomposition consists of a set of theoretically and pragmatically motivated idealizations and abstractions that structure the state space into a more well-defined representational space. We can then use a task-optimized DNN as an encoding model that generates testable predictions about the structure of that representational space. These predictions are linked to the target representational space because the behavior of both model and target system are optimized to solve the same task. By testing model predictions against the reconstructed neural data we can get confirmatory evidence that certain categorical features are actually represented in that space.

My claim then is that we are justified in using connectionist models as experimental tools in virtue of the real patterns of representational similarity identified by RSA. We can make hypotheses that groups of stimuli should be represented as more or less similar to each other on the basis of some shared semantic feature. We can then use an encoding model, like a

DCNN, to generate testable predictions about how these patterns are to be represented in a sensory region of the brain. These predictions can then be verified by comparing the structure visualized in different RDMs. This approach has a significant advantage over traditional cluster analysis. RDMs serve as idealized descriptions of the similarity structure of a region of interest in the brain or an associated model. Importantly, this description abstracts away from the actual layout of the target state space. The cluster approach attempts to explicitly identify regions or manifolds within a space as the vehicles of content in virtue of clustering of neural activity. It then groups representations into types on the basis of their falling within the bounds of an identified cluster. But, as we have seen, drawing well-defined borders around activity in a state space such that the identified regions capture any coherent notion of semantic content becomes intractable in even moderately high-dimensional spaces. RDMs represent an idealized reconstruction of the geometry of state space by computing the similarity between ordered pairs of individual activation points. Rather than attempting to identify semantic categories with discrete, well defined regions in a high-dimensional state space, RDMs visualize the statistical correlation between individual representations and model how real patterns of representational similarity arise from the underlying neural data.

By identifying category clusters represented in the RDM as the markers of content we abstract away from specific architectures and physical implementation. We can instead view an RDM as an idealized model of a network’s representational space. We can use this model to make hypotheses and test predictions about representational information and discover confirmatory evidence about the content of neural representations. With this in mind we can modify a naive cluster approach like Shea’s to arrive at a new set of jointly sufficient conditions for ascribing content to a connectionist model:

Modified jointly sufficient conditions for attributing content to representations generated by a connectionist model:

- i the network can correctly classify some set of samples which differ from those in its learning history
- ii the new samples generate representations which conform to predicted clustering patterns in the model’s RDM;
- iii our predictions for novel samples are made on the basis of some property shared in common with samples from the network’s learning history and their subsequent clustering pattern in the model’s RDM; and
- iv that property is relevant to the classificatory task.

As seen above, a virtue of this proposal is that RDMs not only enable comparisons between different models, but they can also be used to compare representations obtained from

multiple modalities (e.g. fMRI activation patterns and computational models). This enables us to make meaningful comparisons between representations produced by neural networks and regions of interest in the brain. For instance, we can compute RDMs from both a DCNN model and corresponding activity in a human IT cortex across a range of stimuli. We can then compute the statistical correlation between the model RDM and the biological RDM to determine the overall correlation of their representations. This provides a promising avenue for using connectionist models to investigate the representational mechanisms in the brain. The further upshot of this view is that satisfying the above conditions constitutes *evidence* that a pattern of neural activity has a particular representational content. I have laid out the case that understanding representational content in terms of real patterns is fruitful for hypothesis-driven, experimental approaches to cognitive neuroscience. Researches can use connectionist models to make testable predictions about real patterns in representational geometry of sensory regions in the brain. This emphasis on exploratory searches for real patterns means that the relationship between machine learning and cognitive neuroscience turns out to be highly experimental rather than purely conceptual. Moreover, connectionist models lend themselves to experimental methods that are hypothesis-driven and that make use of idealizations and abstractions that are both theoretically and pragmatically motivated. Against the backdrop of this complex web of ancillary assumptions, researches can use connectionist models to acquire confirmatory or dis-confirmatory evidence that a set of neural data embodies a particular real pattern.

## 5 Conclusion

I have just suggested a framework for generalizing the basic principles of the cluster approach to state-of-the-art deep learning models. The upshot of this is that it motivates a rigorous, empirical approach to studying localized aspects of cognition with connectionist models. If we want to understand the various mechanisms underlying cognition, we should begin by isolating regions of the brain which instantiate those mechanisms. We can extend the principles of the above framework to test different models for similarity to the region of interest in the brain. By testing models with varying designs and degrees of neurophysiological inspiration we can begin to uncover the features of the underlying mechanism which are essential for reproducing the cognitive phenomenon of interest. Through such an iterative process we approach a genuinely explanatory mechanistic model of that cognitive phenomenon.

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