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# Finding optimal kidney exchange cycles

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May 2, 2016

We created 4 different algorithms, each of which covered different areas of the problem space. The primary motivation for the ensemble of algorithms was the wide range of graph densities in our instance set; thus, our objective for each algorithm was to target a specific subset of the problem space. We also took advantage of the highly optimized mixed linear integer programming (MILP) library, Gurobi [1].

The first step in solving an instance was splitting the graph into its strongly connected components, each of which could be solved independently. For instances with densities below 25%, we designed a MILP which copied the graph $|V|$ times and made the restriction that the $l^{th}$ copy of the graph contained only a single cycle beginning and ending at the $l^{th}$ vertex. We then enforced flow and cycle length constraints across every graph copy. This solved most medium-low density graphs to optimality. This algorithm was inspired by the polynomial-sized Extended Edge model presented in Constantino et al. (2013) [2].

For graphs of less than 40% density, we found that the optimal $k \leq 3$-cycle allocation achieved optimality in much less time than the previously described algorithm. Here, we exhaustively enumerated every possible $k \leq 3$-cycle, formulated it as a MILP optimization, and enforced vertex constraints.

For the highly dense graphs of greater than 40% density, we created two different algorithms. The first was a $k = 2$-cycle MILP which reinterpreted the directed graph as a purely undirected graph and then optimized over the set of undirected edges (2-cycles). The second was a randomized, greedy cycle finder inspired by the description presented in Abbassi et al. (2008) [3] that achieved better scores than the $k = 2$-cycle cover on select instances.

# References

[1] Gurobi Optimization, Inc. "Gurobi Optimizer", `http://www.gurobi.com`, 2015.

[2] Constantino, Miguel, et al. "New insights on integer-programming models for the kidney exchange problem." *European Journal of Operational Research* 231.1 (2013): 57-68.

[3] Abbassi, Zeinab, and Laks V. S. Lakshmanan. "Offline Matching Approximation Algorithms in Exchange Markets." *Proceeding of the 17th International Conference on World Wide Web - WWW'08* (2008): Web.