Aufgabe 3:

a)

$$\int_{x_{min}}^{x_{max}} 1\,dx = x_{max} - x_{min} \overset{!}{=} 1 \qquad N = \frac{1}{x_{max} - x_{min}}$$

$$A(x) = \int_{x_{min}}^{x} 1\,dx = x - x_{min}$$

$$\frac{A(x)}{A} = \frac{x - x_{min}}{x_{max} - x_{min}} = r(x)$$

$$x = r \cdot (x_{max} - x_{min}) + x_{min}$$

b)

$$A(x) = \int_0^x N e^{-t/\tau}\,dx = N\tau(1 - e^{-x/\tau})$$

$$N\tau(1 - e^{-x/\tau}) = r(x)$$

$$e^{-x/\tau} = 1 - \frac{r(x)}{N\tau}$$

$$x = -\tau \ln\left(1 - \frac{r(x)}{N\tau}\right)$$

c)

$$\int_{x_{min}}^{x} N x^{-n}\,dx = \frac{N x^{-n+1}}{-n+1} - \frac{N x^{-n+1}}{-n+1}$$

$$\frac{N x_0^{-n+1}}{-n+1} - \frac{N x_{min}^{-n+1}}{-n+1} = r$$

$$\Leftrightarrow \quad x_0^{-n+1} = \exp((-n+1)\ln(x_0)) = \frac{(-n+1)r}{N} + x_{min}^{-n+1}$$

$$x_0 = \exp\left(\ln\left(\frac{(-n+1)r}{N} + x_{min}^{(-n+1)}\right)/(-n+1)\right) = \left(\frac{(-n+1)r}{N} + x_{min}^{(-n+1)}\right)^{\frac{1}{(-n+1)}}$$

d)

$$A(x) = \int_{-\infty}^{x} \frac{1}{\pi}\frac{1}{1+x^2}\,dx = \frac{1}{\pi}\tan^{-1}(x)\Big|_{-\infty}^{x_0} = \frac{1}{\pi}\tan^{-1}(x_0) + \frac{1}{2}$$

$$\frac{1}{\pi}\tan^{-1}(x_0) + \frac{1}{2} = r$$

$$x = \tan\left(\pi\left(r - \frac{1}{2}\right)\right)$$