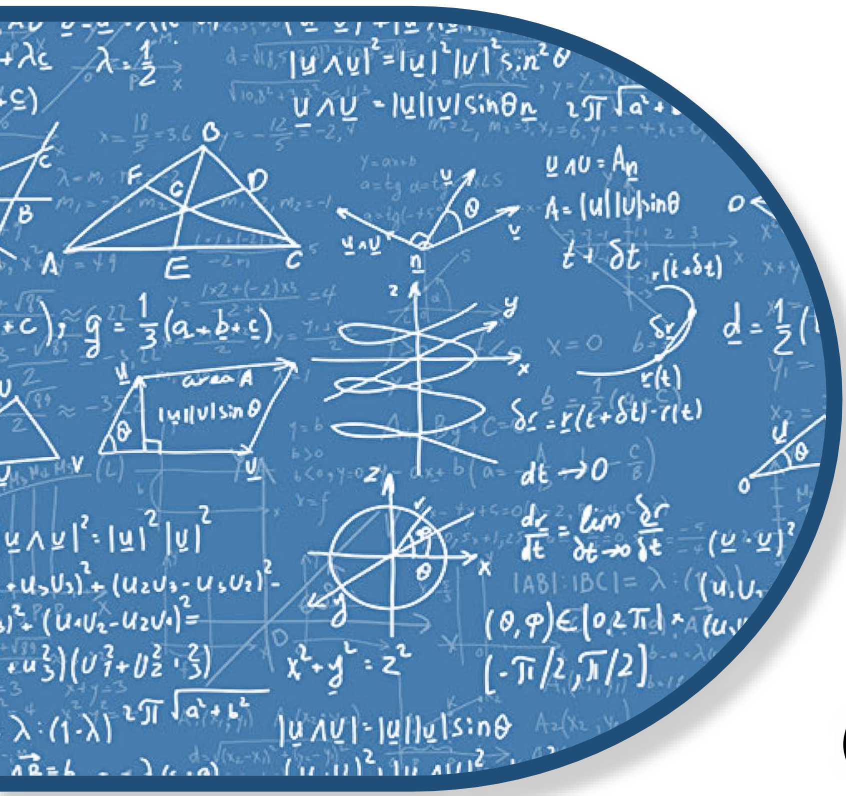
التكنولوجيا التطبيقية
APPLIED TECHNOLOGY
وزارة التربية والتعليم والتعليم الفني

MINISTRY OF EDUCATION AND TECHNICAL EDUCATION
وزارة التربية والتعليم والتعليم الفنى



الرياضيات البحتة

الصف الثاني

التخصصات الصناعية (**لغة انجليزية**)

PURE MATHEMATICS

Second Grade

For industrial (**English**)

2nd.

2023 - 2024

Unit 1 : Differentiation and Its Applications–Integration

Unit 1 : Differentiation and Its Applications

Dear student by the end of this unit you will recognize the following:

- The derivative of the constant function.
- The derivative of the function $f(x) = x^n$.
- The derivative of the function $f(x) = x$.
- The derivative of the function $f(x) = ax^n$.
- The derivative of the product of two functions.
- The derivative of the quotient of two functions.
- The derivative of the function $y = \left(f(x)\right)^n$.
- The derivative of trig. functions
- To find the equation of the tangent, the equation of the normal to a curve at a point on it.
- Some rules of indefinite integration.

Lesson One

Rules of differentiation (derivative)

Let $y = f(x)$, the first derivative of this function is symbolized as :

$y^{\backslash}$, $f^{\backslash}(x)$, $\frac{dy}{dx}$, $\frac{d}{dx}(f(x))$

1. $f(x) = a \in R$ where a is constant (independent on x) $\Rightarrow$ $f^{\backslash}(x) = 0$
2. $f(x) = x^n$, $n \in R$ $\Rightarrow$ $f^{\backslash}(x) = nx^{n-1}$
3. $f(x) = x$ $\Rightarrow$ $f^{\backslash}(x) = 1$

Important note:

First derivative of $f(x)$ is equal to rate of change.

i.e. $f'(x) = \lim_{h\to 0}\frac{f(x+h) - f(x)}{h}$ = slope of tangent = $\tan\theta$

(where θ is inclination angle of the tangent with +ve X-axis)

Example (1)

Find the first derivative of of :

- $y = 2x^7$ $\Rightarrow$ $\frac{dy}{dx} = 14x^6$
- $y = \frac{3}{x^4} = 3x^{-4}$ $\Rightarrow$ $y^{\backslash} = -12x^{-5}$
- $f(x) = \sqrt[4]{x} = (x)^{\frac{1}{4}}$ $\Rightarrow$ $f^{\backslash}(x) = \frac{1}{4}\ 12x^{-\frac{3}{4}}$

Example (2)

Find the slope of the tangent to the curve of the function f where $f(x) = x^2 - x + 1$ at the point (-2 , 7)

Solution

$f^\backslash(x) = 2x - 1$

at $x = -2$

$f^\backslash(-2) = -5$ $\quad\therefore$ slope of the tangent = –5

Example (3)

Prove that the tangent to the curve of the function $y = 3x^2 - 5x$ at the point (1 , -2) makes an angle of measure 45° with the positive direction of X-axis

Solution

$\because y = 3x^2 - 5x \qquad \therefore\ y^\backslash = 6x - 5$

at $x = 1 \qquad \therefore\ y^\backslash = 1$

i.e. $\tan\theta = 1 \qquad \therefore\ \theta = \tan^{-1} 1 = 45°$

Example (4)

Find the points on the curve $y = x^3 - 3x^2$ at which the slope of the tangent equals zero (the tangent // X-axis)

Solution

$$\frac{dy}{dx} = 3x^2 - 6x = 0 \quad \Rightarrow \quad 3x(x-2) = 0$$

$3x = 0$ $\quad or \quad$ $x - 2 = 0$

$x = 0$	$x = 2$
$\therefore y = (0)^3 - 3(0)^2$	$y = (2)^3 - 3(2)^2$
$y = 0$	$y = -4$

Points : (0 , 0) and (2 , – 4)

Example (5) If $y = mx^2 - 3x$ and $\frac{dy}{dx} = 7$ at $x = 1$ find m .

Solution:

$$\frac{dy}{dx} = 2mx - 3 = 7$$

$$2mx = 10$$

$$2m(1) = 10 \quad \Rightarrow \quad m = 5$$

Example (6)

Find the slope of the tangent to the curve $y = x - \frac{2}{x}$ at $x = 1$

Solution

$$y = x - 2x^{-1}$$

$$\therefore \frac{dy}{dx} = 1 + 2x^{-2}$$

at $x = 1 \quad \Rightarrow \quad \frac{dy}{dx} = 1 + 2 = 3$

$\therefore$ the slope of the tangent = 3

EXERCISE (1)

1. Find the points on the curve $y = x^3 - 4x + 2$ at which the slope of the tangent = –1

2. Find the points on the curve $y = 2x^3 - x + 3$ at which the slope of the tangent = 5

3. Find the first derivative of :
   a) $y = x^5 - 3x^2 + 5x - 9$
   b) $y = 2x^2 + 5$

4. Find the slope of the tangent to the curve $y = x^2 - \frac{1}{x}$ at $x = 1$

5. Find the measure of the angle which the tangent to the curve $y = 2x^3 + 3x$ makes with the positive direction of X-axis at the point (-1 , 0).

6. Find the points on the curve $y = x^3 - 4x + 1$ at which the tangent makes an angle of measure 135° with the positive direction of X-axis.

7. If the slope of the tangent to the curve $y = x^2 + ax$ at the point (1 , 3) is equal to 6 then, find the value of a .

Lesson Two

Derivative Of The Product Of Two Functions

If $y = f(x)\,.\,g(x)$ then, $\frac{dy}{dx} = f(x)\,.\,g^{\backslash}(x) + g(x)\,.\,f^{\backslash}(x)$

i.e. $\frac{dy}{dx} =$ first × (second)\ + second × (first)\

Example (1)

Find the first derivative of the function $y = (3x^2 - 2x + 1)(x^4 - 5x^2 + 2)$ when $x = 0$

Solution:

$$\frac{dy}{dx} = (3x^2 - 2x + 1)(4x^3 - 10x) + (x^4 - 5x^2 + 2)(6x - 2)$$

when $x = 0$

$$\frac{dy}{dx} = (0 - 0 + 1)(0 - 0) + (0 - 0 + 2)(0 - 2)$$

$$\frac{dy}{dx} = -\ 4$$

Example (2)

Find the slope of the tangent to the curve of the function $y = (2x + 5)(3x^2 - 1)$ at $x = -1$

Solution:

$$\frac{dy}{dx} = (2x + 5)(6x) + (3x^2 - 1)(2)$$

At $x = -1$

$$\frac{dy}{dx} = (-2 + 5)(-6) + (3 - 1)(2) = -14$$

∴ Slope of the tangent = –14

EXERCISE (2)

1. Find the points on the curve $y = (x^2 + 5)(x - 1)$ at which the slope of the tangent is equal to - 2 .

2. Find the first derivative of :

   a) $y = (x^5 - 3x)(5x - 9)$

   b) $y = (x^2 + 5)(x^2 - 3)$

3. Find the slope of the tangent to the curve $y = x^2(5x - 1)$ at $x = 1$

4. Find the measure of the angle which the tangent to the curve $y = (x^3 + 3x)(x - 5)$ makes with the positive direction of X-axis at the point (-1 , 0).

Lesson Three

Derivative Of The Quotient Of Two Functions

If $y = \frac{f(x)}{g(x)}$ where $g(x) \neq 0$

then, $\frac{dy}{dx} = \frac{\mathrm{g(x)\,.\,f^{\backslash}(x)\; - \; f(x).\,g^{\backslash}(x)}}{[\,\mathrm{g(x)}\,]^2}$

i.e. $\frac{\mathrm{dy}}{\mathrm{dx}} = \frac{\text{denominator.(numerator)}^{\backslash} - \text{numerator.(denominator)}^{\backslash}}{[\text{denominator}]^2}$

Example (1)

Find the first derivative of the function $y = \frac{3x-2}{2x+3}$ when $x = 1$

Solution

$$\frac{dy}{dx} = \frac{(2x+3)(3) - (3x-2)(2)}{(2x+3)^2}$$

when $x = 1 \Rightarrow \frac{dy}{dx} = \frac{(\,2+3)\,(3)\; - \;(3-2)(2)}{(\;2+3\;)^2} \quad = \frac{13}{25}$

Example (2)

Find the first derivative of the function $y = \frac{5x+11}{x\;+2}$ when $x = 0$

Solution

$$\frac{dy}{dx} = \frac{(x+2)(5) - (5x+11)(1)}{(x+2)^2}$$

when $x = 0 \qquad \frac{dy}{dx} = \frac{(0+2)\,(\,5\,) - (\,0+11)\,(1)}{(0+2)^2} = \frac{-1}{4}$

Example (3)

If $f(x) = \frac{x^2}{3x+2}$ find $f^\backslash(-1)$

Solution:

$$f^\backslash(x) = \frac{(3x+2)(2x) - x^2(3)}{(3x+2)^2}$$

$$f^\backslash(-1) = \frac{(-3+2)(-2) - 1(3)}{(-3+2)^2} = -1$$

Example (4)

Find the measure of the angle which the tangent to the curve $y = \frac{x-1}{x^2-x+1}$ makes with the positive direction of X-axis at the point (1 , 0).

Solution:

$$y^\backslash = \frac{(x^2 - x + 1)(1) - (x-1)(2x-1)}{(x^2 - x + 1)^2}$$

At $x = 1$ $\quad y^\backslash = \frac{(1-1+1)\,(1) - (1-1)\,(2-1)}{(1-1+1)^2} = 1$

$\tan\theta = 1$

$\theta = 45°$

EXERCISE (3)

1. Find the first derivative of the function $y = \frac{2x+3}{x-2}$ when $x = 3$
2. Find the slope of the tangent to the curve $y = x - \frac{2}{x}$ at $x = 1$
3. Find the measure of the angle which the tangent to the curve $y = \frac{1+x}{1-x}$ makes with the positive direction of X-axis at the point (-1 , 0).
4. Find the points on the curve $y = \frac{x-2}{x-1}$ at which the slope of the tangent is equal to 1 .

Lesson Four

Derivative Of Composite Functions

Corollary:

If $y = (f(x))^n$, then: $\boxed{\dfrac{dy}{dx} = n(f(x))^{n-1}.\ f^{\backslash}(x)}$

Example (1)

If $y = (x^3 + 3x^2 - 1)^7$ then, find $\dfrac{dy}{dx}$ at $x = 0$

Solution:

$$\frac{dy}{dx} = 7(x^3 + 3x^2 - 1)^6.(3x^2 + 6x)$$

At $x = 0$

$$\frac{dy}{dx} = 7(0 + 0 - 1)^6.(0 + 0) = 0$$

Example (2)

Find the slope of the tangent to the curve of the function $y = x\,(2x - 1)^5 + 3$ at the point (0 , 3).

Solution:

$$\frac{dy}{dx} = (x)[5(2x - 1)^4.(2)] + 1(2x - 1)^5$$

$at\ \ x = 0$

$$\frac{dy}{dx} = (0)[5(0 - 1)^4.(2)] + 1(0 - 1)^5 = -1$$

∴ slope of the tangent = –1

Example (3)

If $y = z^3$, $z = 2x^2 - 3x + 1$ $find$ $\frac{dy}{dx}$ at $x = 2$

Solution

$$y = (2x^2 - 3x + 1)^3$$

$$\therefore \frac{dy}{dx} = 3(2x^2 - 3x + 1)^2 . (4x - 3)$$

At $x = 2$

$$\frac{dy}{dx} = 3(8 - 6 + 1)^2 . (8 - 3) = 135$$

Example (4)

If $y = 2z^3$, $z = \sqrt{x^2 - 2x}$ $find$ $\frac{dy}{dx}$ at $x = 3$

Solution:

$$y = 2(\sqrt{x^2 - 2x}\,)^3 = 2(x^2 - 2x)^{\frac{3}{2}}$$

$$\therefore \frac{dy}{dx} = 3(x^2 - 2x)^{\frac{1}{2}} . (2x - 2)$$

At $x = 3$ $\Rightarrow$ $\frac{dy}{dx} = 3(9 - 6)^{\frac{1}{2}} . (6 - 2) = 12\sqrt{3}$

Example (5)

If $y = (3z - 1)^4$, $z = x(3x - 2)$ $find$ $\frac{dy}{dx}$ at $x = 1$

Solution:

$$y = [3(3x^2 - 2x) - 1]^4 = [9x^2 - 6x - 1]^4$$

$$\therefore \frac{dy}{dx} = 4[9x^2 - 6x - 1]^3 . (18x - 6)$$

At $x = 1$ $\Rightarrow$ $\frac{dy}{dx} = 4[9 - 6 - 1]^3 . (18 - 6) = 384$

EXERCISE (4)

1. $Find\ \frac{dy}{dx}$ if $y = (2x^2 - 3x - 5)^5$ at $x = 2$

2. If $f(x) = \frac{3x^2+1}{x^2+2}$ $, find\ f^{\backslash}(x)\ at\ x = 1$

3. If $y = z^2$, $z = x^2 + 1,\quad then\ find\ \frac{dy}{dx}\ at\ x = -1$

4. If $y = z^3$, $z = \frac{x}{x+1}\ then,\ find\ \frac{dy}{dx}\ at\ x = 1$

5. If $y = z + 2$, $z = x^2 + 2x$, $then\ find\ \frac{dy}{dx}\ at\ x = 2$

6. If $y = z^3 + z$, $z = x^2 - x\quad then,\ find\ \frac{dy}{dx}\ at\ x = 2$

Lesson Five

Derivative Of Trigonometric Functions

1. $If\ \ y = sinx \qquad \Rightarrow \qquad \frac{dy}{dx} = cosx$
2. $If\ \ y = cos\ x \qquad \Rightarrow \qquad \frac{dy}{dx} = -sinx$
3. $If\ \ y = tanx \qquad \Rightarrow \qquad \frac{dy}{dx} = sec^2x$

Corollaries:

1. $If\ \ y = sin(ax+b) \qquad \Rightarrow \qquad \frac{dy}{dx} = a\ cos(ax+b)$
2. $If\ \ y = cos(ax+b) \qquad \Rightarrow \qquad \frac{dy}{dx} = -a\ sin(ax+b)$
3. $If\ \ y = tan(ax+b) \qquad \Rightarrow \qquad \frac{dy}{dx} = a\ sec^2(ax+b)$

Example (1)

Find the first derivative of the function $y = sin4x$

Solution:

$$\frac{dy}{dx} = \ 4\ cos\ 4x$$

Example (2)

Find the first derivative of the function $y = \cos\ (2x+5)$

Solution:

$$\frac{dy}{dx} = \ -2\ sin(2x+5)$$

Example (3)

Find the first derivative of the function $y = x\ sin2x$

Solution:

$$\frac{dy}{dx} = x\ (2\ cos2x)\ + sin2x$$

$$= 2xcos2x + sin2x$$

Example (4)

Find the first derivative of the function $y = \frac{sin2x}{cos3x}$ $at\ \ x = \pi$

Solution:

$$\frac{dy}{dx} = \frac{cos3x\ (2cos2x) - sin2x\ \ (-3sin3x)}{(cos3x)^2}$$

$at\ \ x = \pi$

$$\frac{dy}{dx} = \frac{\cos\ 3\pi\ \ (2\cos\ 2\pi\ ) \ - \sin\ 2\pi\ (-3\ \sin\ 3\pi)}{(cos3\pi)^2} = \frac{(-1)(2)-0(0)}{(-1)^2} = -2$$

Example (5)

Find the first derivative of the function $y = 2cosx - tan5x$

Solution:

$$\frac{dy}{dx} = -2sinx - 5sec^2 5x$$

EXERCISE (5)

Find the first derivative of each of the following functions:

1. $y = tan2x$
2. $y = sin2x + sin3x$
3. $y = xsinx - x^2$
4. $y = sin2x + cos3x$
5. $y = 2x + sin2x \quad at \quad x = 0$
6. $y = \cos\ 2x + \sin\ 5x$
7. $y = \tan\ (3x + 2)$
8. $y = cosx.sinx \quad at \ \ x = \pi$
9. $y = (3x - \cos\ 2x)^3$
10. $y = x^3 \cos\ 2x$

Lesson six

Derivative Of Trigonometric Functions (follow)

Example (1)

If $y = \sin\left(2x + \frac{\pi}{3}\right)$ $find\ y^{\backslash}\ at\ x = \pi$

Solution:

$$y^{\backslash} = 2\cos\left(2x + \frac{\pi}{3}\right) = 2\cos\left(2\pi + \frac{\pi}{3}\right) = 2 \times \frac{1}{2} = 1$$

Example (2) If $y = 2x \sin x \cos x$, prove that: $y^{\backslash} = \sin 2x + 2x \cos 2x$

Solution

Note:

Sin2x = 2sinx . cosx

$$y = x \sin 2x$$

$$y^{\backslash} = x(2\cos 2x) + \sin 2x = 2x \cos 2x + \sin 2x$$

Example (3)

If $y = \frac{\sin x}{\sin x + \cos x}$, prove that: $y^{\backslash} = \frac{1}{1 + \sin 2x}$

Solution

$$y^{\backslash} = \frac{\cos x\ (\sin x + \cos x) - \sin x\ (\cos x - \sin x)}{(\sin x + \cos x)^2}$$

$$= \frac{\cos x \sin x + \cos^2 x - \sin x \cos x + \sin^2 x}{\sin^2 x + \cos^2 x + 2\sin x \cos x}$$

$$= \frac{1}{1 + \sin 2x}$$

Example (4)

If $y = \sqrt{1 + Cos2x}$, find $\frac{dy}{dx}$ $at\ x = \frac{\pi}{4}$

Solution:

$$y = (1 + Cos2x)^{\frac{1}{2}}$$

$$\therefore y^{\backslash} = \frac{1}{2}(1 + \text{Cos } 2x)^{\frac{-1}{2}} \quad (-2 \text{ Sin } 2x)$$

When $x = \frac{\pi}{4}$ $\Rightarrow$ $y^{\backslash} = -1$

EXERCISE (6)

1. If $y = Cosx\,.sinx$, find $\frac{dy}{dx}$ $at\;\; x = \frac{\pi}{4}$
2. If $y = (sinx + cosx)^2$, prove that: $y^{\backslash} = 2cos2x$
3. Find $\frac{d}{dx}(2x^2 sinx\; cosx)$ $at\;\; x = \pi$
4. If $y = sinx - \frac{1}{3}sin^3x$, prove that: $y^{\backslash} = cos^3x$
5. If $y = \frac{x}{sinx}$, find $\frac{dy}{dx}$ $at\;\; x = \frac{\pi}{2}$
6. If $y = x\;cos2x$, find $\frac{dy}{dx}$ $at\;\; x = \frac{\pi}{2}$
7. Find $\frac{d}{dx}\left(\frac{x}{cos3x}\right)$ $at\;\; x = \frac{\pi}{3}$
8. Find the slope of the tangent to the curve $y = sin2x.cosx$ $at\;\; x = \pi$

Lesson seven

Equation of Tangent

Slope of the tangent to a curve:

Remember: *Let* m_1, m_2 *be the slopes of two straight lines* L_1, L_2

1. $If\ \ L_1 // L_2\ \ then,\ \ m_1 = m_2$
2. $If\ \ L_1 \perp L_2\ \ then,\ \ m_1 m_2 = -1$
3. Equation of the tangent to a curve at any point (x_1, y_1) is

$$\boxed{(y - y_1) = m\,(x - x_1)}$$

Example (1) Find the equation of the tangent to the curve

$y = 2x^3 - 4x^2 + 3$ at the point (2,3) which lies on the curve.

Solution:

$y^{\backslash} = 6x^2 - 8x$

When $x = 2$, $y^{\backslash} = 8$

Equation of the tangent : $y - 3 = 8(x - 2)$

$$y - 8x + 13 = 0$$

Example (2) Find the equation of the tangent to the curve $y = \frac{x-1}{x^2-x+1}$

at the point (1,0) which lies on the curve.

Solution:

$$y^{\backslash} = \frac{1(x^2 - x + 1) - (x - 1)(2x - 1)}{(x^2 - x + 1)^2}$$

When $x = 1$,

$$y^{\backslash} = \frac{1(1 - 1 + 1) - (1 - 1)(2 - 1)}{(1 - 1 + 1)^2} \qquad m = 1$$

Equation of the tangent : $y - 0 = 1(x - 1)$

$$y = x - 1$$

Example (3)

Find the equation of the tangent to the curve $y = (2x + 5)(3x^2 - 1)$ at $x = 0$.

Solution:

$y^\backslash = (2x + 5)(6x) + (3x^2 - 1)(2)$

at $x = 0$,

$\therefore y^\backslash = -2 =$ the slop of the tangent

and $y = (0 + 5)(0 - 1) = -5$

$\Rightarrow$ point $(0, -5)$

Equation of the tangent : $y + 5 = -2(x - 0)$

$y + 2x + 5 = 0$

EXERCISE (7)

1. Find the equation of the tangent to the curve $y = x - \frac{2}{x}$ at $x = 1$

2. Find the equation of the tangent to the curve $y = \frac{x-2}{x-3}$ at $x = 2$

3. Find the equation of the tangent to the curve
   $y = (x^2 + 5)(x - 1)$ at $x = 2$

4. Find the equation of the tangent to the curve
   $y = x^2(5x - 1)$ at $x = 3$

Lesson eight

Equation Of Normal

Equation of the normal to a curve at any point (x_1, y_1) is :

$$\boxed{y - y_1 = \frac{-1}{m}(x - x_1)}$$

Example (1) Find the equation of the normal to the curve $y = 2x^3 - 4x^2$ at the point $(2,0)$ which lies on the curve

Solution:

$y^\backslash = 6x^2 - 8x$

When $x = 2$, $y^\backslash = 8$

Equation of the normal : $y - 0 = \frac{-1}{8}(x - 2)$

$$8y + x - 2 = 0$$

Example (2)

Find the equation of the normal to the curve $y = \frac{x - 1}{x^2 - x + 1}$

at the point $(1,0)$ which lies on the curve.

Solution:

$$y^\backslash = \frac{1(x^2 - x + 1) - (x - 1)(2x - 1)}{(x^2 - x + 1)^2}$$

When $x = 1$,

$$y^\backslash = \frac{1(1 - 1 + 1) - (1 - 1)(2 - 1)}{(1 - 1 + 1)^2}$$

, $m = 1$

Equation of the normal : $y - 0 = -1(x - 1)$

$$y + x - 1 = 0$$

Example (3)

Find the equation of the normal to the curve $y = (2x+5)(3x^2-1)$ at $x = 0$.

Solution:

$y^{\backslash} = (2x+5)(6x) + (3x^2-1)(2)$

When $x = 0$,

$\therefore y^{\backslash} = -2 = m$ (slope of tangent)

and $y = (0+5)(0-1) = -5$

$\Rightarrow$ point $(0, -5)$

Equation of the normal : $y + 5 = \frac{1}{2}(x-0)$

$2y - x + 10 = 0$

EXERCISE (8)

1. Find the equation of the normal to the curve $y = x - \frac{2}{x}$ at $x = 2$
2. Find the equation of the normal to the curve $y = \frac{x-2}{x-3}$ at $x = 2$
3. Find the equation of the normal to the curve
   $y = (x^2 + 5)(x - 1)$ at $x = 1$
4. Find the equation of the normal to the curve
   $y = x^2(5x - 1)$ at $x = 1$

Lesson nine

Indefinite Integration

Rules:

1. $\int x^n \ dx = \frac{x^{n+1}}{n+1} \ + \ c$
2. $\int a \ dx = ax \ + \ c$
3. $\int (ax+b)^n \ dx \ = \frac{(ax \ + \ b)^{n+1}}{a(n+1)} \ + \ c$
4. $\int \ [\,f(x)]^n . \ \left[f^{\backslash}(x)\right] \ dx \ = \ \frac{[\,f(x)\,]^{\,n+1}}{n+1} + c$

Example (1)

Find: (a) $\int 5x \ \ dx$ (b) $\int (3x^2 - x + 1) \ dx$

Solution:

a) $\int 5x \ \ dx \ = \frac{5}{2}x^2 + c$

b) $\int (3x^2 - x + 1) \ \ dx = x^3 - \frac{1}{2}x^2 + x + c$

Example (2)

Find: $\int (x-2)(x-1)(x+1) \quad dx$

Solution:

$$\int (x-2)(x-1)(x+1).dx = \int (x-2)(x^2-1) \quad dx$$

$$= \int (x^3-2x^2-x+2) \quad dx$$

$$= \frac{x^4}{4} - \frac{2x^3}{3} - \frac{x^2}{2} + 2x + c$$

Example (3) Find: $\int \frac{x^2+m^2x}{x} \quad dx \quad , m \text{ is constant}$

Solution:

$$\int \frac{x^2+\ m^2x}{x} \quad dx \quad = \int (x+m^2)\, dx \quad = \ \frac{1}{2}x^2 + m^2x \ + c$$

Example (4) Find: $\int (x+\frac{1}{x})^2 \quad dx$

Solution:

$$\int (x+\frac{1}{x})^2 \quad dx \quad = \ \int \left(x^2+2+\frac{1}{x^2}\right) dx$$

$$= \int (x^2+2+x^{-2}) \ dx \quad = \ \frac{1}{3}x^3 + 2x - x^{-1} \ + \ c$$

Example (5) Find: $\int (7x-3\,)^8 \quad dx$

Solution:

$$\int (7x-3\,)^8 \quad dx \quad = \ \frac{(7x-3\,)^9}{9 \text{ x } 7} \ + \ c$$

$$= \frac{(7x-3\,)^9}{63} \ + \ c \ = \ \frac{1}{63} \ (7x-3\,)^9 + \ c$$

<u>Example (6)</u> Find : $\int (2x+11)\left(\sqrt{2x+11}\right)dx$

Solution:

$$\int (2x+11)(2x+11)^{\frac{1}{2}} \quad dx$$

$$= \int (2x+11)^{\frac{3}{2}} \; dx = \frac{(2x+11)^{\frac{5}{2}}}{\frac{5}{2} \text{ x } 2} + c = \frac{1}{5} \; (\; 2x+11 \;)^{\frac{5}{2}} \; + c$$

<u>Example (7)</u> Find: $\int 2x\,(x^2+2)^5\,dx$

Solution:

$$\int 2x\,(x^2+2)^5\,dx \; = \; \frac{(\;x^2 \; + \; 2)^6}{6} \; + \; c$$

EXERCISE (9)

Find

1. $\int x(x+3)\,dx$
2. $\int x^2(4x+2)\,dx$
3. $\int (x-4)(x+4)\,dx$
4. $\int \frac{x^2+2x-3}{x-1}\,dx$
5. $\int \frac{x^3+27}{x+3}\,dx$
6. $\int (2x-4)^5\,dx$
7. $\int x^2(4x^3-1)^7\,dx$

Lesson ten

Indefinite Integration(follow)

Integrals of some trig. functions:

1. $\int sinx\ \ dx = -cosx + c$
2. $\int cosx\ \ \ dx = sinx + c$
3. $\int sec^2x\ \ \ dx = tanx + c$

Corollaries:

1. $\int \sin(ax+b)\ \ dx = -\frac{1}{a}cos(ax+b) + c$
2. $\int \cos(ax+b)\ \ dx = \frac{1}{a}\ sin\ (ax+b) + c$
3. $\int sec^2(ax+b)\ \ dx = \frac{1}{a}tan(ax+b) + c$

Remember:

$\sin^2x + \cos^2x = 1$

$1 + \tan^2x = \sec^2x$

$1 + \cot^2x = \csc^2x$

$\text{Sin}2x = 2\sin x \cdot \cos x$

$$\tan 2x = \frac{2\tan x}{1 - \tan^2 x}$$

$\cos 2x = \cos^2x - \sin^2x$

$= 2\cos^2x - 1$

$= 1 - 2\sin^2x$

$cos^2x = \frac{1}{2} + \frac{1}{2}cos2x$

$sin^2x = \frac{1}{2} - \frac{1}{2}cos2x$

Example (1): $\int (cosx - sinx)\ dx = sinx + cosx + c$

Example (2): $\int (sec^2x + cosx)\ dx = tanx + sinx + c$

Example (3): $\int (1 + sinx)^2\ dx$

$$= \int (1 + 2sinx + sin^2x)\ dx$$

$$= \int \left(1 + 2sinx + \frac{1}{2} - \frac{1}{2}cos2x\right)\ dx = \int \left(\frac{3}{2} + 2sinx - \frac{1}{2}cos2x\right)\ dx$$

$$= \frac{3}{2}x\ - 2cosx\ - \frac{1}{4}sin2x\ + c$$

Example (4): $\int (sinx + cos)^2\ dx$

$$= \int (sin^2x + cos^2x + 2sinx\ cosx)\ dx$$

$$= \int (1 + sin2x\ ).dx = x - \frac{1}{2}cos2x + c$$

Example (5): $\int \frac{sin^3x \ + \ cos^3x}{sinx \ + \ cosx} \ dx$

$$= \int \frac{(sinx + cosx) \ (sin^2x - sinx \ cosx + cos^2x)}{sinx + cosx} \ dx$$

$$= \int (1 - sinx \ cosx) \, . \, dx$$

$$= \int (1 - \tfrac{1}{2} sin2x) \quad dx = x + \tfrac{1}{4} cos2x + c$$

EXERCISE (10)

Find

1. $\int (7sinx - 2cosx)\, dx$

2. $\int (\, sin2x - 5cosx\,)\ \ dx$

3. $\int (\, sinx + \ sec^2x\,)\ \ dx$

4. $\int \sin\ (7x + 3)\ \ dx$

5. $\int \cos\ (5 - 2x)\ \ \ dx$

6. $\int 2cos^2x\ \ \ dx$

7. $\int\ (cos^2x - sin^2x)\ \ dx$

Unit TEST

FIRST QUESTION:

1. Find the measure of the angle which the tangent to the curve $y = 2x^3 + 3x$ makes with the positive direction of X-axis at the point (-1 , 0).
2. Find the slope of the tangent to the curve of the function

   $y = x^2(5x - 1)$ at $x = 1.$

SECOND QUESTION:

1. Find the first derivative of the function $y = \frac{2x+3}{x-2}$ when $x = 3$
2. If $y = z + 2$, $z = x^2 + 2x$ $\quad then, \; find \; \frac{dy}{dx} \; at \; x = 2$

THIRD QUESTION:

1. Find the first derivative of the function $y = xsinx - x^2$
2. $Find\ the\ equation\ of\ the\ tangent\ to\ the\ curve$

   $y = 2x^3 - 4x^2 + 3 \quad at\ the\ point\ (2,3)\ which\ lies\ on\ the\ curve.$

FOURTH QUESTION:

$Find$

1. $\int x^2 \; (4x + 2) \; dx$
2. $\int \cos \; (5 - 2x) \; dx$

EVALUATION