3

$$P = (P_0, P_1, P_2, P_3) = (\vec{E}, \vec{p}c)$$

$$P^\mu = mU^\mu = (\gamma mc, \gamma m\vec{v}) = \left(\frac{\gamma mc^2}{c}, \gamma m\vec{v}\right) =$$

$$= \left(\frac{E}{c}, \vec{p}\right)$$

$$P \cdot P = P_\mu P^\mu = \left(\frac{E}{c}, \vec{p}\right) \cdot \left(\frac{\vec{E}}{c}, \vec{p}\right) = \frac{E^2}{c^2} - \vec{p}^2 = m^2c^2$$

for $c = 1$:- $\quad E^2 = m^2 + \vec{p}^2$

$$P_{before} \longrightarrow P_\gamma = \left(\frac{hc}{\lambda}, \frac{hc}{\lambda}, 0, 0\right), \quad P_m = (mc^2, 0, 0, 0)$$

$$P_{after} \longrightarrow P'_\gamma = \left(\frac{hc}{\lambda'}, \frac{hc}{\lambda'}\cos\theta, \frac{hc}{\lambda'}\sin\theta, 0\right)$$

4 momentum conservation law:

$$P_\gamma + P_m = P'_\gamma + P'_m$$

$$(P_\gamma + P_m - P'_\gamma)^2 = P'^2_m$$

$$\cancel{P_\gamma^2} + P_m^2 + \cancel{P_\gamma'^2} + 2P_m(P_\gamma - P_\gamma') - 2P_\gamma P_\gamma' = P_m'^2$$

$$m^2 + 2m\left(\frac{1}{\lambda} - \frac{1}{\lambda'}\right) - 2\frac{1}{\lambda}\cdot\frac{1}{\lambda'}(1-\cos\theta) = m^2c^4$$

Multiplying by $\lambda\lambda'/_{2m}$ and solving for λ':

$$\boxed{\lambda' = \lambda + \frac{(1-\cos\theta)}{m}}$$

(5) $s=(p_1+p_2)^2$, $t=(p_1-p_3)^2$, $u=(p_1-p_4)^2$

$$\begin{cases} s=p_1^2+p_2^2+2p_1p_2 \\ t=p_1^2+p_3^2-2p_1p_3 \\ u=p_1^2+p_4^2-2p_1p_4 \end{cases}$$

For the conservation law: $p_1+p_2=p_3+p_4$

$$s+t+u=p_1^2+p_2^2+p_3^2+p_1^2+p_4^2+2p_1p_2-2p_1p_3-2p_1p_4+$$
$$+p_1^2$$

where: $p_2=p_3+p_4-p_1$

$$s+t+u=p_1^2+(p_3+p_4-p_1)^2+p_3^2+p_1^2+p_4^2+p_1^2+$$
$$+2p_1(p_3+p_4-p_1)-2p_1p_3-2p_1p_4$$

$\cdot\ (p_3+p_4-p_1)^2=p_3^2+p_4^2+p_1^2+2p_3p_4-2p_1p_4-2p_1p_3$

kajoma

$$s + t + u = p_1^2 + p_2^2 + p_1^2 + p_3^2 + p_1^2 + p_4^2 + 2p_1p_2 - 2p_1p_3 - 2p_1p_4$$

$$= p_1^2 + p_2^2 + p_3^2 + p_4^2 + p_4^2 + p_1^2 + 2p_1(p_3 + p_4 - p_1) - 2p_1p_3 - 2p_1p_4$$

$$s + t + u = p_1^2 + p_2^2 + \cancel{p_1^2} + p_3^2 + \cancel{p_1^2} + p_4^2 + \cancel{2p_1p_3} + \cancel{2p_1p_4} - \cancel{2p_1^2} - \cancel{2p_1p_3} - \cancel{2p_1p_4}$$

$$= p_1^2 + p_2^2 + p_3^2 + p_4^2$$

$p^2 = m^2c^2$ $\therefore$ for $c = 1$; $p^2 = m^2$

Therefore:

$$\boxed{s + t + u = m_1^2 + m_2^2 + m_3^2 + m_4^2}$$

(13)

$$\pi \longrightarrow \mu + \nu$$

$E_\pi = E_\mu + E_\nu$

$p_\pi = p_\mu + p_\nu$ $\quad \therefore \; p_\nu = -p_\mu = |p_\mu| = |p_\nu|$

$$p_\pi = 0$$

$$p^2 = p_\mu p^\mu = E^2 - \vec{p}^{\,2} c^2 = m^2 c^4 \quad \rightarrow \text{ invariant}$$

$$\pi \rightarrow \quad E_\pi^2 - p_\pi c^2 = m^2 c^4$$

$$E_\pi^2 = m_\pi^2 c^4 \;\; \therefore$$

$$E_\pi = m_\pi c^2 \,//$$

$$\mu \rightarrow \quad E_\mu^2 - p_\mu^2 c^2 = m_\mu^2 c^4$$

$$E_\mu^2 = c^2 (p_\mu^2 + m_\mu^2 c^2)$$

$$E_\mu = c\sqrt{p_\mu^2 + m_\mu c^2} \,//$$

$$\nu \rightarrow \quad E_\nu^2 - p_\nu^2 c^2 = \cancel{m_\nu}^{\,m_\nu = 0} c^2$$

$$E_\nu = p_\nu c = -p_\mu c = |p_\mu| c$$

$$p_\pi = p_\mu + p_\nu \therefore p_\nu = p_\pi - p_\mu \therefore p_\mu = p_\pi - p_\nu$$

$$\cdot\ p_\pi^2 = (p_\mu + p_\nu)^2 = (p_\mu^2 + 2p_\mu p_\nu + p_\nu^2) \quad (1)$$

$$\cdot\ p_\nu^2 = (p_\mu - p_\mu)^2 = p_\pi^2 - 2p_\mu p_\pi + p_\mu^2 \quad (2)$$

$$p_\nu^2 = 0 \ ; \ p_\pi^2 = E_\pi^2 - p_\pi^2 c^2 = m_\pi^2 c^2 \ ; \ p_\mu^2 = m_\mu^2 c^2$$

$$\cdot\ p_\mu p_\pi = \left(\frac{E_\mu}{c}, p_\mu\right)\left(\frac{E_\pi}{c}, p_\pi\right) = m_\pi^2 c^4$$

$$\cancel{= E_\pi^2 - p_\pi^2 c^2 = m^2 c^4}$$

$$= \frac{E_\mu}{c}\frac{E_\pi}{c} - \underbrace{p_\mu \cancel{p_\pi}^{\,0}}_{p_\mu = 0} = \frac{E_\mu}{\cancel{c}} \cdot \frac{m_\pi c^{\cancel{2}}}{c} = m_\pi E_\mu$$

Na equação 1:

$$0 = m_\pi^2 c^2 + m_\mu^2 c^2 - 2E_\mu m_\pi$$

$\therefore$

$$\boxed{E_\mu = \frac{m_\pi^2 c^2 + m_\mu^2 c^2}{2m_\pi}}$$

$$p_\mu^2 = (p_\mu - p_\nu)^2 \qquad E_\pi = m_\pi c^2$$

$$m_\mu^2 c^2 = m_\pi^2 c^2 - 2 p_\pi p_\nu + \cancel{p_\nu^2}^{\,0}$$

$$m_\mu^2 c^2 = m_\pi^2 c^2 - 2\underbrace{\left(\frac{E_\pi}{c}\frac{E_\nu}{c}\right)}_{m_\pi E_\nu} = m_\pi^2 c^2 - 2 m_\pi E_\nu$$

$$E_\nu = |p_\nu| \cdot c = |p_\mu| \cdot c$$

$$m_\mu^2 c^2 = m_\pi^2 c^2 - 2 m_\pi (|p_\mu| \cdot c)$$

$$|p_\mu| = \frac{(m_\pi^2 - m_\mu^2)\, c}{2 m_\pi}$$

For the μ 4-velocity, we have the relation:

$$\frac{p_\mu}{E_\mu} = \frac{v_\mu}{c^2} \; ; \; v_\mu = \frac{p_\mu c^2}{E_\mu}$$

$$v_\mu = \left(\frac{m_\pi^2 - m_\mu^2}{\cancel{2 m_\pi}}\right) c \cdot \frac{\cancel{2 m_\pi}}{(m_\pi^2 c^2 + m_\mu^2 c^2)} c^2$$

$$v_\mu = \frac{m_\pi^2 - m_\mu^2}{m_\pi^2 c^2 + m_\nu^2 c^2}$$

$$\boxed{v_\mu = \frac{m_\pi^2 - m_\mu^2}{m_\pi^2 + m_\nu^2}\, c}$$

(14) $p + p = p + p + p + \bar{p}$

Before : $p^{\mu}_{tot} = \left(\frac{E + mc^2}{c}, |p|, 0, 0 \right)$.

After : $p^{\mu}_{tot} = \left(4mc, 0, 0, 0 \right)$.

↳ all four particles at rest -

Total Energy conservation : $P_a^{tot^2} = P_b^{tot^2}$

$$\left(\frac{E}{c} + mc \right)^2 - \vec{p}^{\,2} = (4mc)^2$$

$$\frac{E^2}{c^2} + 2mE + m^2c^2 - \vec{p}^{\,2} = 16mc^2$$

But, $p_\mu p^\mu = \frac{E^2}{c^2} - \vec{p}^{\,2} = m^2c^2$

$$2mE + m^2c^2 + m^2c^2 = 16m^2c^2$$

$$2mE = 14m^2c^2$$

$$\boxed{E = 7mc^2}$$

$P_e = (E_e, \vec{P}_e c)$

elétron próton

$\approx p c$

7) HERA: $E_e = 27,6\ GeV$; $E_p = 920\ GeV$

$\approx p$

a)

$\sqrt{s} = E_1 + E_2 \Rightarrow \sqrt{s} = 2E_{cm}$

$$\sqrt{s} \cong \sqrt{4 E_e E_p} = \sqrt{4 \cdot 27,6 \cdot 920\ GeV^2} \cong 318,7\ GeV$$

No referencial do CM: $|\vec{p}_1^{\,cm}| = |\vec{p}_2^{\,cm}| = |\vec{p}_{cm}|$

ou $E_1^{cm} = E_2^{cm} = E^{cm}$

Ou seja, no CM as energias das duas partículas são iguais, logo:

$$E_e^{CM} = E_p^{CM} = \frac{\sqrt{s}}{2} \cong 160\ GeV$$