Solution of Exercise 3

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1 Theory Exercise

1.1 Question 1

State the objective function of a GAN (the one proposed by Goodfellow et al.)

$$\min_G \max_D V(D,G) = \mathbf{E}_{x \sim p_{data}(x)}[\log D(x)] + \mathbf{E}_{z \sim p_z(z)}[\log(1 - D(G(z)))]. \quad (1)$$

where the Generator G is trying to minimise the value function and Discriminator D is trying to maximize the value function.

1.2 Question 2

Describe each term in the objective function of the GAN

In the equation 1, $V()$ is value function, $G()$ is generator and gives generted fake data as output, $D()$ is discriminator and gives a scalar value for real or fake data, E is expectation, x is input real data, z is input noise, p_{data} is input data distribution, $p_z(z)$ is a prior on input noise variables.

1.3 Question 3

Can you reformulate the objective function of GAN in terms of Categorical Cross-Entropy? Justify your answer.

The original objective function is of the following form:

$$\min_G \max_D V(D,G) = crossEntropy(1., D(data)) + crossEntropy(0., D(G(z))) \quad (2)$$

We can transform it to categorical cross entropy by changing the binary labels to one hot vector:

$$\min_G \max_D V(D,G) = CrossEntropy(\mathbf{1}_r, D(data)) + crossEntropy(\mathbf{1}_f, D(G(z))) \quad (3)$$

1.4 Question 4

Describe the Generator and Discriminator. Be as formal as possible

Generator: It is a mapping of input noise to output (fake data x). It defines a prior on input noise variables $p_z(z)$ and then maps it into data space as $G(z; \theta_g)$, where G is a differentiable function (in vanilla GAN it is represented by a multilayer perceptron with parameters θ_g). It's objective is to learn generator's distribution p_g over data x which corresponds to real data distribution over x $p_{data}(x)$.

Discriminator: It assigns a probability $D(x)$ that the input came from the data distribution $p_{data}(x)$ rather than generator's distribution p_g. In vanilla GAN $D(x; \theta_d)$ is used as disciminator which is a multilayer perceptron that outputs a single scalar. $D()$ tries to maximize the probability of assigning the correct label (real or fake) to both training examples and samples from G.

1.5 Question 5

What are the problems that GANs face while training. Describe them if any.

Theoretically GANs can converge if we could modify the density function directly but rather than that we do the following:

- we modify G (sample generation function) and D (Density ratio),
- we represent G and D as highly non-convex parameteric functions

These two points results in the following problems:

1. **Oscillation:** Samples of different categories keep getting produced as one trains more and more but doesn't produce nice consistent set of samples
2. **Mode Collaps:** produces sample with one particular image or same theme samples like using the same dog image at different places with different background or using same theme like a beach scene

1.6 Question 6

How can you evaluate GANs? Please provide objective answers

GANs are evaluated by estimating the probability of test data under generator's distribution p_g. This is done by using Kernel density estimation methods. Following steps are followed:

- samples are generated with G
- probability density function is assumed as gaussian
- fitting a Gaussian Parzen window (KDE) on these samples

- σ parameter of the Gaussians is obtained by cross validation on the validation set
- mean log-likelihood of samples on test set is reported under above distribution