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T3. Memristive properties of gas-discharge lamps

One of the rather familiar electronic devices that exhibit properties similar to those of a memristor (dependence of resistance on current at previous moments of time) is a **gas-discharge lamp**. Resistance R of such lamp is determined by the concentration of free electrons n_e, which changes with current flow:

$$\begin{cases} R = \dfrac{F}{n_{\mathrm{e}}} \\ \dfrac{\mathrm{d}n_{\mathrm{e}}}{\mathrm{d}t} = -\beta n_{\mathrm{e}} + \alpha V I, \end{cases}$$

Here F is determined by the geometry of the tube (in the simplest model, conductivity is proportional to concentration), the $-\beta n_{\mathrm{e}}$ term is responsible for the neutralization of free electrons as the result of collisions, and the last tern $\alpha V I$ is responsible for the thermal emission of electrons.

A1	What is the minimum voltage $V_{\min}$ that must be applied to a discharge lamp for it to light up?

The lamp is connected to a source with EMF $\mathcal{E}$ and internal resistance r.

A2	Find the expressions for the equilibrium current through the lamp and the voltage across the lamp.

Suppose now the lamp is supplied with an alternating voltage $V(t) = V_0 \cos \omega t$. The internal resistance of the voltage source is still equal to r.

B1	Express the current and voltage on the lamp at some time through the concentration of electrons n_{e}.

B2	Considering n_{e} to be nearly constant, average the equation for the concentration over time and find the equilibrium value of the electron concentration n_0.

In fact, the electron concentration depends on time, and in a first approximation this dependence can be represented as:

$$n_{\mathrm{e}}(t) = n_0 + n_1 \cos(2\omega t + \phi),$$

where $n_1 \ll n_0$.

B3	Find n_1 and ϕ.

A change in concentration results in a change in lamp resistance. We define its maximum relative change as:

$$\delta \equiv \frac{R_{\max} - R_{\min}}{R_{\mathrm{avg}}}.$$

B4	Determine δ for the lamp assuming $\delta \ll 1$. What does this smallness condition correspond to in terms of the initial variables of the problem?

T4. Thermistor hysteresis

Thermistors also exhibit memristor properties – its temperature depends on the current at previous points in time:

$$C\frac{\mathrm{d}T}{\mathrm{d}t} = -\kappa(T - T_0) + VI,$$

where T_0 is the room temperature, coefficient κ determines the rate of heat exchange with the environment, and C is the heat capacity of the thermistor. The resistance of the thermistor depends on the temperature according to the formula:

$$R(T) = R_0 \exp\left[\frac{E_g}{k_B T}\right],$$

whete E_g is activation energy (depends on thermistor's semiconducting structure), k_B is Boltzmann constant.

An alternating voltage $V(t) = V_0 \cos\omega t$ was applied to the thermistor, and it reached a temperature of T_1 on average.

A1	By averaging the equation for temperature, obtain the equation that relates V_0, T_1, and the other variables of the problem.

The table below shows the dependence of the average temperature of the thermistor on the applied voltage. The room temperature is $t_0 = 20\,^\circ\mathrm{C}$, and the resistance of the thermistor at this temperature is $R = 8.60\ \Omega$.

$V_0,\ V$	2.0	2.5	3.0	3.5	4.0	4.5	5.0
$t_1, ^\circ\mathrm{C}$	20.7	21.1	21.6	22.2	22.9	23.8	24.8
$V_0,\ V$	5.5	6.0	6.5	7.0	7.5	8.0	8.5
$t_1, ^\circ\mathrm{C}$	26.0	27.4	29.1	31.0	33.3	36.2	40.0

A2	Calculate the activation energy E_g. Express your answer in electronvolts ($1\ eV = 1.60\cdot 10^{-19}\ J$). Boltzmann constatn is equal to $k_B = 1.38 \cdot 10^{-23}\ J/K$.

A3	Calculate R_0 and κ.

Let's now consider the voltage dependence of the current through the thermistor, taking into account in the first approximation the time dependence of temperature (and resistance). In the first approximation we can write:

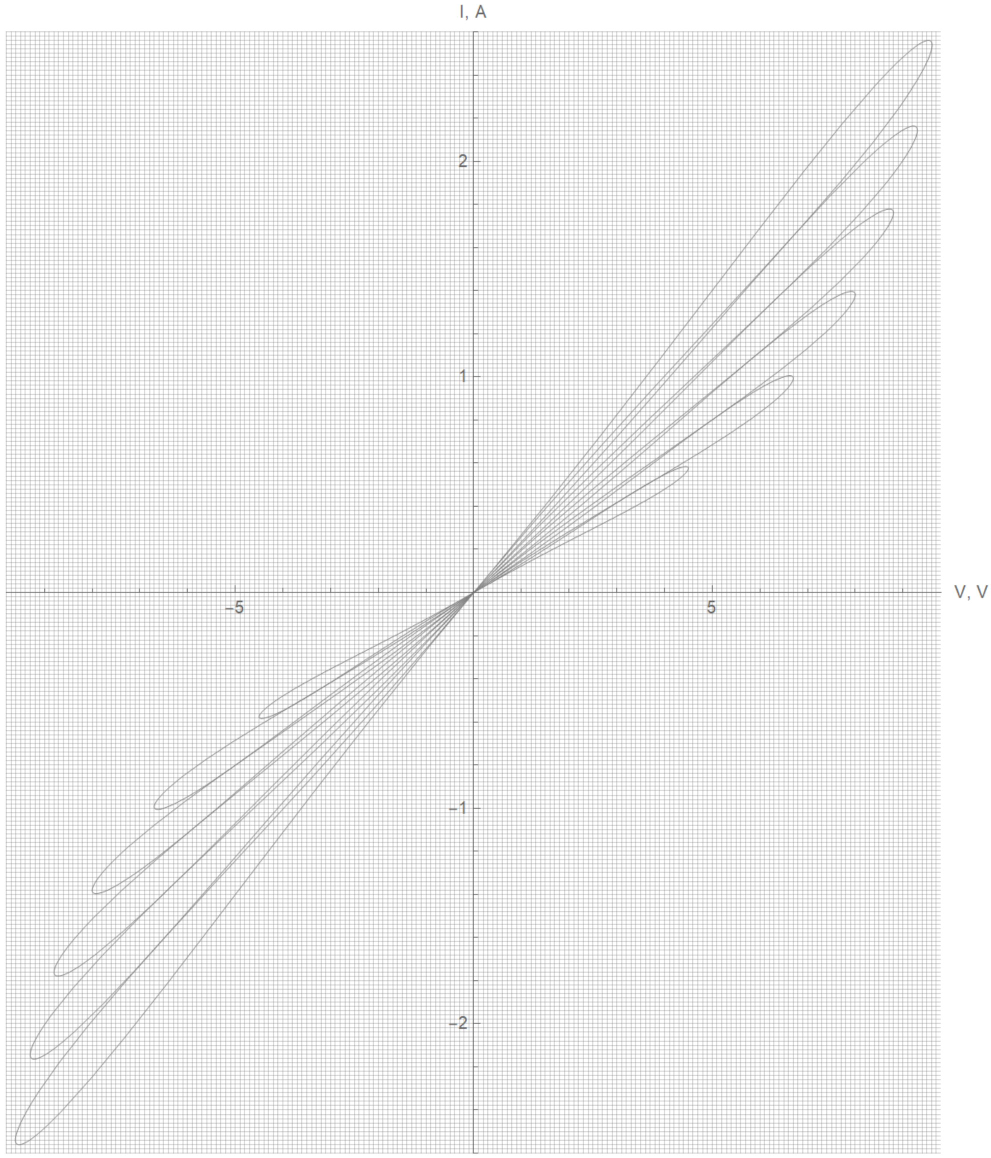
$$T(t) = T_1 + T_2 \cos[2\omega t + \phi],$$

where $T_2 \ll (T_1 - T_0)$.

B1	Find expressions for T_2 and ϕ.

The figure below shows several hysteresis loops of a thermistor. Input voltage frequency is equal to:

$$\nu \equiv \frac{\omega}{2\pi} = 0.2\ Hz.$$



C1	Calculate thermistor's heat capacity C.