(a). 1.

Obj: $\min_{x_{ij}} \sum_{i=1}^{n}\sum_{j=1}^{n} D_{ij}\, x_{ij}$

n cities in total, Distance: $D_{ij}$.

$x_{ij}$ for $i=1,\dots n$, & $j=1,\dots n$ denoting whether City $i$ & $j$ are connected along the tour.

→ explaination:

We're try to minimise the total tour distance. The $x_{ij}$ are binary $\{0,1\}$.

When $x_{ij}=0$, which mean it doesn't take the road between $i \to j$. and times the distance $D_{ij}$. Otherwise, $x_{ij}=1$, which mean it take the road between $i \to j$. and times the distance $D_{ij}$.

S.t.

① $x_{ij} = x_{ji}$ $\forall$ $i=1\dots n$ & $j=1\dots n$.

means the distance between two road are the same no matter what direction, $i \to j$ or $j \to i$

② $x_{ii} = 0$ $\forall$ $i=1,\dots n$.

means the from $i \leftrightarrow i$'s distance is 0

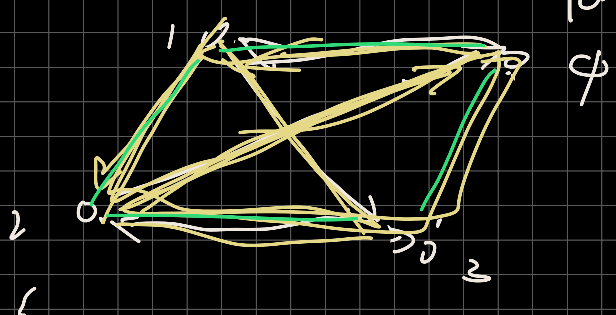
③ $\sum_{j=1}^{n} x_{ij} = 2 \quad \forall\ i = 1, \dots n.$

Sine $x_{ij} = x_{ji} \Rightarrow \sum_{j=1}^{n} x_{ij} + x_{ji} = 2.$

means. if $i$, fixed, after visiting $i$, he/she can visiting only 1 city next. i.e. $i \to j$, $i \cancel{\to} z$. And vice verso.

④ $\sum_{i \in S} \sum_{j \in S} x_{ij} \le 2(|S| - 1) \quad \forall$ non-empty subset of cities $S \subset \{1, 2, \dots n\}$

Total Route be taken cannot exceed 2 times $(|S|-1)$. $|S|$ means the # of cities you pick up. This is for subtour elimination. for every subset of nodes, it forbids the # of selected route within the subset to be equal or large the # of nodes in S.




⑤ $x_{ij} \in \{0, 1\}. \ \forall i, j$

means its binary, sales-person can chose picks the route (1) or not (0)

2. $n^2 + n + n + n^2 + n!(n-r)!$

$= 2n^2 + 2n + n!(n-r)!$

$r \in 1 \dots n.$

$n!(n-r)!$ is S's combination.