Integer Programming Example : Scheduling

$i = 1, 2, \ldots, n$ patients

$t = 1, 2, \ldots, T$ periods

} Index Sets

α_{it} patient i's preference for time period t

c_t # surgeons available at time t

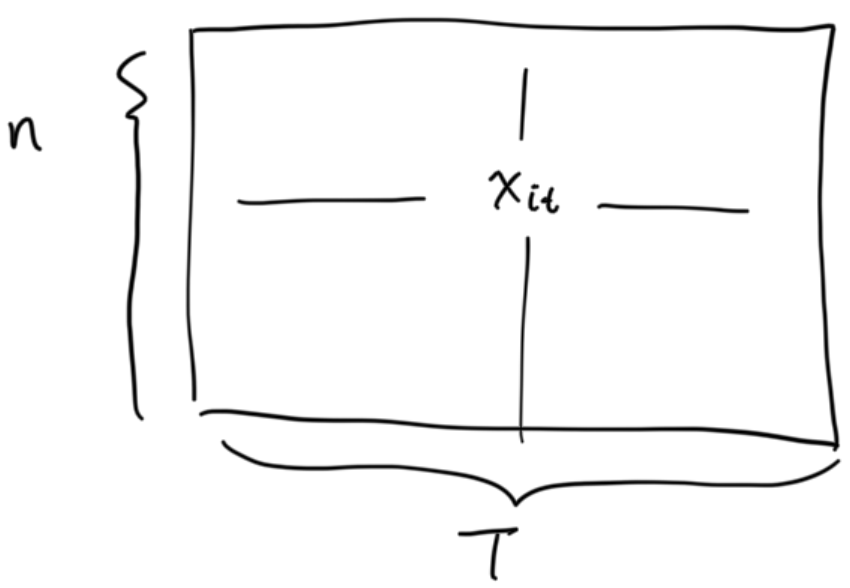
} Parameters

Decisions :

x_{it} : whether we schedule patient i to period t or not.

$\begin{cases} 1 \rightarrow \text{yes, } i \text{ scheduled to } t \\ 0 \rightarrow \text{o/w.} \end{cases}$

(i.e., x_{it} is a binary decision var)



Objective [we want to maximize # of patients scheduled into their preferred times]

$$\text{maximize} \quad \sum_{i=1}^{n} \sum_{t=1}^{T} \alpha_{it} x_{it}$$

$$= \alpha_{11}x_{11} + \alpha_{12}x_{12} + \alpha_{13}\cdot x_{13} + \cdots + \alpha_{1T}x_{1T}$$

$$+ \boxed{\alpha_{21}x_{21} + \alpha_{22}x_{22} + \alpha_{23}x_{23} + \cdots + \alpha_{2T}x_{2T}} = \alpha_{23}\cdot x_{23}$$

$$\vdots$$

$$+ \alpha_{n1}x_{n1} + \cdots$$

$x_{n1}\ x_{nT}$

Constraints :

(1). All patients need to be scheduled exactly once

(2). # patients scheduled into any time period is no more than # surgeons available for that period.

i : $x_{i1}, x_{i2}, \dots, x_{iT}$

$$x_{i1} + x_{i2} + \cdots + x_{iT} = 1$$

(1) For all $i = 1, \dots, n$

$$\sum_{t=1}^{T} x_{it} = 1 \quad \text{or} \quad \sum_{t=1}^{I} x_{it} = 1 \quad \forall\, i = 1, \dots, n.$$

For time period t : $x_{1t}, x_{2t}, \dots, x_{nt}$

$$x_{1t} + x_{2t} + \cdots + x_{nt} \le C_t$$

or

(2) $$\sum_{i=1}^{n} x_{it} \le C_t \quad , \quad \forall\, t = 1, 2, 3, \dots, T$$

Extension :

$$U_i = \begin{cases} 0 & \text{patient } i \text{ does not require urgent care} \\ 1 & \text{o/w} \end{cases}$$

✓ $\boxed{x_{i1} + x_{i2} \geq u_i \quad \text{for} \quad i = 1, \dots, n.}$

$$u_i x_{i3} + u_i x_{i4} + u_i x_{i5} + u_i x_{i6} = 0 \quad \text{for} \quad i = 1, \dots, n$$

or $\boxed{u_i \cdot (x_{i3} + x_{i4} + \dots + x_{i6}) = 0 \quad \text{for} \quad i = 1, \dots, n.}$

$$\begin{cases} u_i = 1 & x_{i3} + \dots + x_{i6} = 0 \\ u_i = 0 & 0 = 0 \end{cases}$$