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MATHEMATICAL MODELING (CO2011)

Assignment ***(Semester 201, Duration: 06 weeks)***

"Dynamical systems in forecasting Greenhouse Micro-climate"

(Version 0.1)

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Mục lục

1 Introduction

Nowadays, many kinds of fruits and vegetables are sold all year round despite growing seasons due to advanced technologies. Crops are often grown in modern greenhouses equipped with automatic or semi-automatic control systems. The systems are used to adjust the weather and climate factors inside the greenhouses to create the best environment for the crops.

Figure 1: *Greenhouses in Lâm Đồng (Việt Nam).*

The main climatic components of a greenhouse include temperature, vapor pressure of water and, in particular, the CO_2 concentration, which directly affects the crop yield. These components are normally affected by one or more different objects present inside the greenhouse, the structure of the greenhouse and the air currents inside the greenhouse. As depicted in Figure 2,

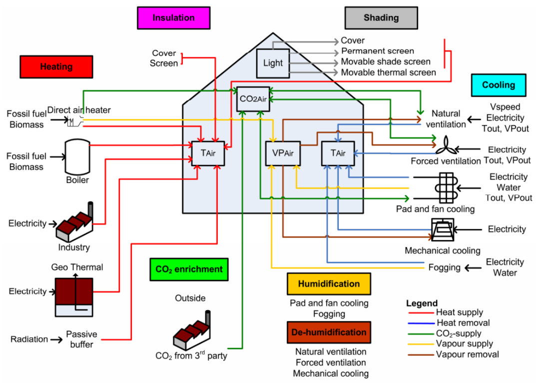

Figure 2: *Factors inside greenhouses and functional.*

the concentration of CO_2 in the greenhouse air is due to many factors, for example, the natural winds that enter and exit the greenhouse through the greenhouse openings.

2 CO_2 exchange

In this section, the concentration of CO_2 in the greenhouse air will be described in more detail. For general purposes, we consider a greenhouse equipped with thermal screens. Thermal screens are made of many different materials such as metal or elastic-plastic. They are used to protect crops from damage caused by direct sunlight as well as from freezing in winter.

A thermal screen divides a greenhouse into two different compartments, above and below the screen. The upper compartment is often narrower than the lower one (see Figure 3). This results

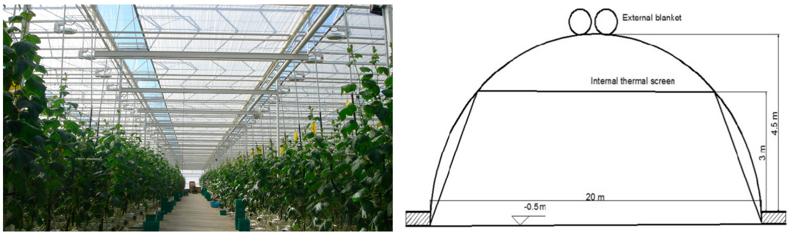

Figure 3: *Thermal screens work to shielding trees and regulating the greenhouse climate.*

in different concentrations of CO_2 in the greenhouse air above and below the screen. A schematic summary of the circulation of CO_2 in the greenhouse is then shown in Figure 4.

For the lower compartment of the greenhouse, the amount of CO_2 is mainly brought in from sources such as the natural airflow through the pad system and exited through the fan system (see Figure 5) and Figure 6). In addition, CO_2 in this space is also received from direct air heaters (see Figure 7) and from the third party. A portion of CO_2 in the lower compartment is also lost to the upper part of the greenhouse under the direction of the difference in temperature and air density between the two compartments while a large amount of CO_2 in the lower compartment of the greenhouse is absorbed into plants for photosynthesis. For the upper part of the greenhouse, the amount of CO_2 is mainly received from the exchange with the lower compartment and is released to the outside through the roof openings (if any) as in Figure 10.

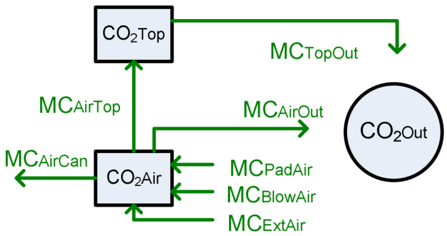

Figure 4: *The CO_2 flow inside and outside a greenhouse.*

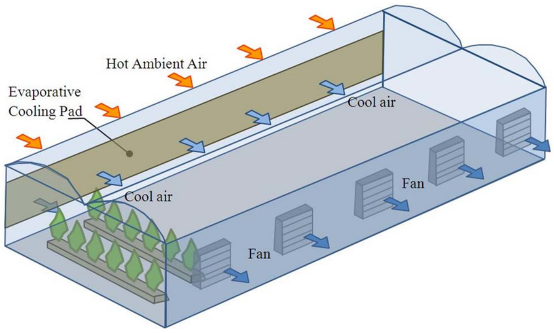

Figure 5: *The movement of CO_2 through the pad system and fan system.*

3 Dynamical system models and assumptions

In this section, a dynamical system representing the CO_2 concentration in the greenhouse will be discussed. The model has been studied by many authors [Mar94; Van11; De 96]. From Figure

Figure 6: *Pad system (left) and fan system (right).*

Figure 7: *Heating system.*

4, the two differential equations represents the fluctuation of CO_2 concentration in the lower and upper compartments of the greenhouse..

$$\begin{cases} cap_{CO_2\,Air}\dot{CO_2}_{Air} = MC_{BlowAir} + MC_{ExtAir} + MC_{PadAir} \\ \qquad\qquad\qquad\qquad -MC_{AirCan} - MC_{AirTop} - MC_{AirOut}, & (1) \\ cap_{CO_2\,Top}\dot{CO_2}_{Top} = MC_{AirTop} - MC_{TopOut}. & (2) \end{cases}$$

Assuming that the amount of CO_2 in the air in the lower and upper space of the greenhouse is not affected by other elements except for those shown in Figure 4. Furthermore, the greenhouse is a perfect tank in the sense that the concentration of CO_2 is uniformly distributed in each compartment of the greenhouse. The notations cap_A, $CO_{2\,A}$, $\dot{CO}_{2\,A}$ and MC_{AB} are respectively the capacity to store CO_2 in A (m), the CO_2 concentration in A (mg m^{-3}), the rate of change of CO_2 concentration in A (mg m^{-3} s^{-1}), and the net CO_2 flux from A to B (mg m^{-2} s $^{-1}$), where Air and Top represent the lower and upper compartments, $Blow$ represents the direct air heater, Ext represents the source from the third party, Pad represents the pad system, Can represents the total foliage of the plants inside the greenhouse, and Out represents the space outside the greenhouse.

Here are formulas to calculate MC_{AB}. First, we consider the amount of CO_2 going from the heater into the greenhouse air as follows.

$$MC_{BlowAir} = \frac{\eta_{HeatCO_2} U_{Blow} P_{Blow}}{A_{Flr}}. \tag{3}$$

In this formula, η_{HeatCO_2} is the amount of CO_2 generated when 1 Joule of sensible heat is generated by the heater (mg $\{CO_2\}$ J^{-1}). The dimensionless parameter U_{Blow}, which is to control the amount of CO_2 generated by the heater, is a number in $[0,1]$. The factor P_{Blow} is the CO_2-generation capacity of the heater (W) and A_{Flr} is the area of the greenhouse (m^2).

Similarly, the amount of CO_2 that is pumped into the greenhouse by the third party that supplies CO_2 is given by

$$MC_{ExtAir} = \frac{U_{ExtCO_2} \phi_{ExtCO_2}}{A_{Flr}}. \tag{4}$$

The notations U_{ExtCO_2} and ϕ_{ExtCO_2} are respectively a dimensionless parameter in $[0, 1]$ that adjusts the rate at which the gas is injected into the greenhouse and that represents the third party's ability to pump CO_2 (mg s^{-1}).

On the other hand, the amount of CO_2 that enters the greenhouse through the pad system is due to the difference in the concentration of CO_2 inside and outside the greenhouse and the ability of the pad system for the air to go through as in Figure 6. Furthermore, the pad can be adjusted to let in more air. The following formula is used to calculate MC_{PadAir}.

$$MC_{PadAir} = f_{Pad}(CO_{2\,Out} - CO_{2\,Air}) = \frac{U_{Pad}\phi_{Pad}}{A_{Flr}}(CO_{2\,Out} - CO_{2\,Air}). \tag{5}$$

The flux f_{Pad} (m s^{-1}) is the product of the dimensionless parameter U_{Pad} in $[0, 1]$, which represents the permeability of the pad and ϕ_{Pad}, which is the ability for the airflow to pass through (m^3 s^{-1}) divided by the area of the greenhouse floor.

The net flux of CO_2 from the lower compartment to the upper compartment of the greenhouse is more complicated and it depends on the difference in temperature and air density between the two compartments.

$$MC_{AirTop} = f_{ThScr}(CO_{2\,Air} - CO_{2\,Top}), \tag{6}$$

where the airflow rate through the thermal screen f_{ThScr} (m s^{-1}) is the sum of the penetration rate through the screen and the rate at the open regions that are not covered by the thermal screen (see Figure 8).

$$f_{ThScr} = U_{ThScr}K_{ThScr}|T_{Air} - T_{Top}|^{\frac{2}{3}} + (1 - U_{ThScr})\left[\frac{g(1 - U_{ThScr})}{2\rho^{Mean}_{Air}}|\rho_{Air} - \rho_{Top}|\right]^{\frac{1}{2}}. \tag{7}$$

The dimensionless parameter $U_{ThScr} \in [0, 1]$ represents the places covered by the thermal

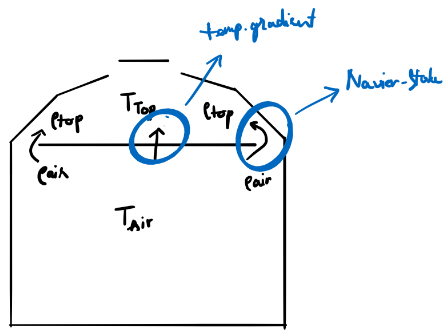

Figure 8: *Movement of air through the thermal screen.*

screen. The flux through those places depends on the difference between the temperature above the screen T_{Top} (K) and the temperature below the screen T_{Air} (K) and the permeability of the screen K_{ThScr} (m K$^{-\frac{2}{3}}$ s^{-1}). At the places that are not covered by the thermal screen, the flux is given by a Navier–Stokes equation depending on the difference of the air density below the screen ρ_{Air} and the air density above the screen ρ_{Top} (kg m^{-3}). Note that the coefficient $2/3$ in the formula (7) comes from the experiment in the work of [Bal89]. In that work, the authors used measured data on the air-exchange rate through different kinds of thermal screens, which are made of 12 different materials, to train the model $K_{ThScr}|T_{Air} - T_{Top}|^m$ where m is an adjustable parameter to find that m is nearly $0.66 \approx 2/3$. Particularly, the Navier–Stokes formula comes from the study of [MG], in which the authors considered the theoretical model of air exchange through cracks in the screen surface caused by the air-density difference

$$\phi_{crack} = \frac{L \cdot SO}{\rho_{mean}} \left[\frac{1}{2}\rho_{mean} \cdot SO \cdot g \cdot (\rho_1 - \rho_2)\right]^{\frac{1}{2}}, \tag{8}$$

where ϕ_{crack} (m^3 s^{-1}) is the rate of airflow through the screen, L (m) is the length of the opening on the screen, SO is the percentage of the opening on the screen (m), ρ_{mean} (kg m^{-3}) is the average density of the air density upper the screen ρ_1 (kg m^{-3}) and the air density beneath the screen ρ_2 (kg m^{-3}), and g is the gravitational acceleration (m s^{-2}). The following and experimental works have also shown that the Navier–Stokes formula (8) gives good results once compared with the measured data.

Similarly, for the net CO_2 flux from the inside to the outside of the greenhouse, let consider the following formula

$$MC_{AirOut} = (f_{VentSide} + f_{VentForced})(CO_{2\,Air} - CO_{2\,Out}). \tag{9}$$

Here, $f_{VentSide}$ is the flux due to the fan system on the sidewalls of the greenhouse and $f_{VentForced}$ is the flux due to the fan system inside the greenhouse. Both of them are measured in m s^{-1}. In this case, the Bernoulli principle plays an important role represented by the pressure difference from the outside of the greenhouse, which is caused by the natural airflow through the roof surface, and the pressure from the inside of the greenhouse caused by the lateral airflow as in (see Figure 9). The Stack effect, also known as the Chimney effect, should also be considered (see Figure 10). The Stack effect is an effect where in winter, cold air flows from outside into the greenhouse and is gradually heated by the heating system and tends to go up and escape back to the outside through the roof openings, in the summer it is the opposite.

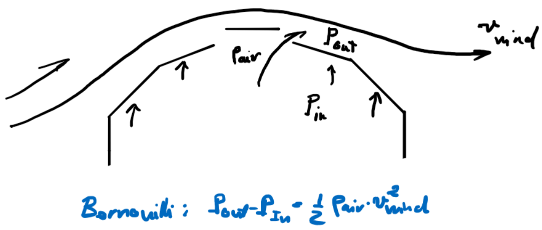

Figure 9: *Airflow through the greenhouse openings.*

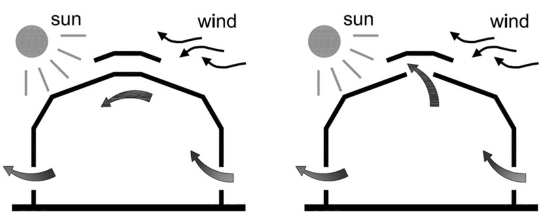

Figure 10: *No Stack effect (left) and Stack effect (right).*

To generalize the model for many different types of greenhouses, the following general formula

$f_{VentRoofSide}$ (m s^{-1}) is used to set the formula for $f_{VentSide}$ [Kit+96].

$$f_{VentRoofSide} = \frac{C_d}{A_{Flr}}\left[\frac{U_{Roof}^2 U_{Side}^2 A_{Roof}^2 A_{Side}^2}{U_{Roof}^2 A_{Roof}^2 + U_{Side}^2 A_{Side}^2} \cdot \frac{2gh_{SideRoof}(T_{Air} - T_{Out})}{T_{Air}^{Mean}} + \left(\frac{U_{Roof}A_{Roof} + U_{Side}A_{Side}}{2}\right)^2 C_w v_{Wind}^2\right]^{\frac{1}{2}}. \quad (10)$$

The formula (10) is the sum of the two components multiplied by the ratio between the dimensionless discharged coefficient C_d and the area of the greenhouse floor A_{Flr} (m^2). The first component, depending on the temperature difference between the outside and the inside of the greenhouse (in the space under the thermal screen), represents the Stack effect when the ventilation area on the roof A_{Roof} (m^2) is non-zero. The second component is given by the pressure difference inside and outside the greenhouse and is calculated as the total area of the ventilation openings on the greenhouse divided by two times the natural wind speed v_{Wind} (m s^{-1}) squared and the global wind pressure coefficient C_w (dimensionless). The coefficients C_d and C_w are theoretical coefficients depending on the structure and shape of the greenhouse and can be estimated by using experimental data.

In addition, this topic also explores insect screens on ventilation openings and ventilators and the leakage coefficient of the greenhouse. In the presence of an insect screen, the movement speed of the air currents through the ventilation areas will be reduced by a factor

$$\eta_{InsScr} = \zeta_{InsScr}(2 - \zeta_{InsScr}), \quad (11)$$

where ζ_{InsScr} is the porosity (dimensionless), which is the ratio of the area of the holes in the screen to the total area of the screen. Given the leakage coefficient $c_{leakage}$, which depends on the greenhouse type and is dimensionless, the air-exchange rate is added an amount of approximately 50% of the leakage rate

$$f_{leakage} = \begin{cases} 0.25 \cdot c_{leakage}, & v_{Wind} < 0.25, \\ v_{Wind} \cdot c_{leakage}, & v_{Wind} \geq 0.25. \end{cases} \quad (12)$$

Implicitly, we assumed the uniform distribution of the leakage rate.

Let η_{Side_Thr} be the Stack-effect threshold. If η_{Side}, the ratio between the sidewalls ventilation area and the total ventilation area, exceeds the threshold, the Stack effect does not occur and vice versa. Then, $f_{VentSide}$ is given by the following

$$f_{VentSide} = \begin{cases} \eta_{InsScr} f''_{VentSide} + 0.5 f_{leakage}, & \eta_{Side} \geq \eta_{Side\_Thr}, \\ \eta_{InsScr}\left[U_{ThScr} f''_{VentSide} \right. & \\ \quad \left. +(1 - U_{ThScr}) f_{VentRoofSide}\eta_{Side}\right] + 0.5 f_{leakage}, & \eta_{Side} < \eta_{Side\_Thr}. \end{cases} \quad (13)$$

In which, $f''_{VentSide} = \dfrac{C_d U_{Side} A_{Side} v_{Wind}}{2A_{Flr}}\sqrt{C_w}$ when $A_{Roof} = 0$. Moreover, the Stack effect does not occur where is covered by the thermal screen. In case when you can't get the value of η_{Side_Thr}, you can use the η_{Roof_Thr} instead.

The flux $f_{VentForced}$ by the fan system inside the greenhouse is calculated as follows.

$$f_{VentForced} = \frac{\eta_{InsScr} U_{VentForced} \phi_{VentForced}}{A_{Flr}}. \quad (14)$$

The dimensionless parameter $U_{VentForced} \in [0, 1]$ is to adjust the wind speed $\phi_{VentForced}$ due to the system (m^3 s^{-1}).

Similarly to MC_{AirOut}, the net CO_2 flux from the greenhouse to outside the greenhouse through the roof openings is calculated by using the formula

$$MC_{TopOut} = f_{VentRoof}(CO_{2\,Top} - CO_{2\,Out}), \tag{15}$$

where $f_{VentRoof}$ is the flux rate through the roof openings and is given by

$$f_{VentRoof} = \begin{cases} \eta_{InsScr} f''_{VentRoof} + 0.5 f_{leakage}, & \eta_{Roof} \geq \eta_{Roof\_Thr}, \\ \eta_{InsScr} \Big[U_{ThScr} f''_{VentRoof} & \\ \quad + (1 - U_{ThScr}) f_{VentRoofSide} \eta_{Side} \Big] + 0.5 f_{leakage}, & \eta_{Roof} < \eta_{Roof\_Thr}. \end{cases} \tag{16}$$

However, this $f_{VentRoof}$ differs from $f_{VentSide}$ as in (13), when the ratio η_{Roof} of the roof-opening area to the total ventilation area exceeds the Stack-effect threshold η_{Roof_Thr}, the Stack effect does not occur and we cannot reuse the formula $f_{VentRoofSide}$ in (10) where $A_{Side} = 0$ to calculate $f''_{VentRoof}$, which is introduced in [BB95].

$$f''_{VentRoof} = \frac{C_d U_{Roof} A_{Roof}}{2 A_{Flr}} \left[\frac{g h_{Vent} (T_{Air} - T_{Out})}{2 T^{Mean}_{Air}} + C_w v^2_{Wind} \right]^{\frac{1}{2}}. \tag{17}$$

Finally, we need to describe the amount of CO_2 that is absorbed into the leaves due to photosynthesis.

$$MC_{AirCan} = M_{CH_2O} h_{C_{Buf}} (P - R). \tag{18}$$

Here, M_{CH_2O} is the molar mass of CH_2O (mg μmol^{-1}), P is the photosynthetic rate (μmol $\{CO_2\}$ m^{-2} s^{-1}), R is the respiration rate (μmol $\{CO_2\}$ m^{-2} s^{-1}), and

$$h_{C_{Buf}} = \begin{cases} 0, & C_{Buf} > C^{Max}_{Buf}, \\ 1, & C_{Buf} \leq C^{Max}_{Buf}, \end{cases} \tag{19}$$

shows the cessation of photosynthesis when CH_2O is C_{Buf} (mg $\{CH_2O\}$m^{-2}) has reached C^{Max}_{Buf} (mg $\{CH_2O\}$ m^{-2}), which is the limit of the carbohydrates storage of the plants. Usually, the respiration rate is negligible compared to the photosynthetic rate and can be omitted or calculated as about 1% of the photosynthetic rate. The photosynthetic rate will be described in more detail in the next section.

To simplify the assignment, $h_{C_{Buf}}$ will always have a value of 1, meaning that C_{Buf} will have no effect on the CO_2 fluctuation.

4 Photosynthesis of C3 plants

Note: This section will not be implemented, instead we will use Equation (9.10) and relevant ones in reference [Van11] to solve Equation (18) in this exercise with assumption that PAR_{Can} is a constant (e.g. 100, ...). However, the final exam will may have questions for this section.

In this topic, we only consider plants belonging to the C3 group including tomatoes, cucumbers, etc. Photosynthesis is the process of using CO_2, water, and energy from sunlight to form organic compounds that feed plants. This process is mainly done by chlorophyll contained in chloroplasts, a special kind of organelles of leaf and plant cells. Photosynthesis has two phases consisting of the light-dependence phase and the light-independence (or dark) phase. In the light-dependence phase, the leaves absorb sunlight and convert to energy in the thylakoid component of the chloroplasts to provide energy for the dark phase. Products of the light-dependence phase are NADPH (Nicotinamide Adenine Dinucleotide Phosphate) and ATP (Adenosine Triphosphate). In the dark phase, also known as the Calvin cycle, a sequence of biochemical reactions as CO_2 assimilation, CO_2 reduction, and the regeneration of CO_2 receptor, which is the so-called Rubisco enzyme contained in the stroma of the chloroplasts, without the need of light.

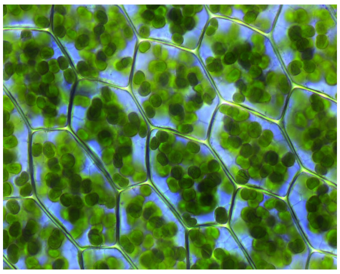

Figure 11: *Chloroplasts.*

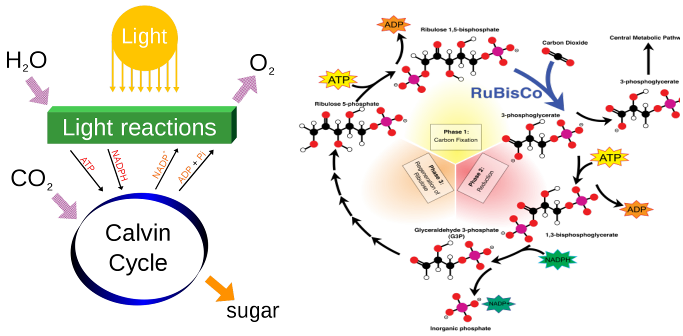

Figure 12: *Light-dependence and light-independence phases of photosynthesis.*

4.1 Photosynthesis model for one leaf unit

There are many ways to model the rate of the photosynthesis of the plants. In this framework, we will combine more than one model if necessary (see [Lom+75] for more details).

4.1.1 The diffusion of CO_2 into the leaves

The photosynthetic rate P per leaf unit can be seen as the rate at which CO_2 diffuses from the air into the leaf cells through stomata that are scattered on both sides of leaves as shown in Fig 13.

The diffusion process is represented by Fick's law given by

$$P = \frac{CO_{2\,Air} - CO_{2\,Stom}}{Res}. \tag{20}$$

The notation $CO_{2\,Stom}$ is the concentration of CO_2 in the stomata (μmol m^{-3}) and Res is the resistance-to-absorption coefficient (s m^{-1}). This coefficient of resistance depends on many factors including the speed of the wind blowing through the leaves.

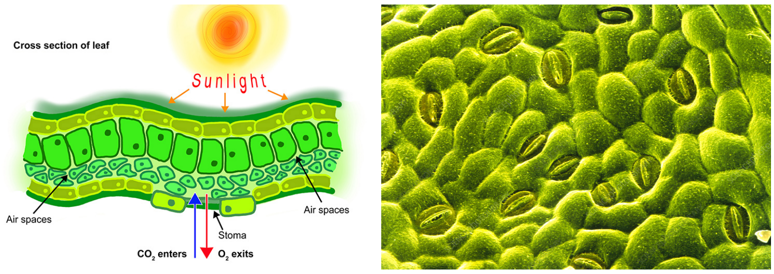

Figure 13: *Diffusion of CO_2 into the leaf cells (left) and stomata (right).*

4.1.2 Biochemistry in the dark phase

Michaelis–Menten kinetic models, named after the German biologist Leonor Michaelis and the Canadian physicist Maud Menten, can be used to represent the biochemical process in the dark phase of the photosynthesis. The process occurs in the chloroplasts when the amount of CO_2 absorbed into the stomata and the Rubisco enzyme present in the chloroplasts form an unstable complex. This, in turn, quickly degenerates into Rubisco and another substance. For example, in the first step, water will be separated into 4 cations H^+ and 4 free electrons with the accompanying product of O_2. In the second step, CO_2 in the substrate is combined with the free electrons and the H^+ cations to produce carbohydrates and water again.

$$2H_2O \to 4e^- + 4H^+ + O_2,$$
$$CO_2 + 4e^- + 4H^+ \to CH_2O + H_2O.$$

Let K_M be the substrate concentration in the case where the reaction rate is exactly 50% of the reaction rate at the saturation point. Then, for given substrate concentration, the reaction rate can be obtained by multiplying the rate at the saturation point (or the maximum rate) by the reciprocal of the sum of 1 and the ratio of K_M to the considered substrate concentration (see Figure 14).

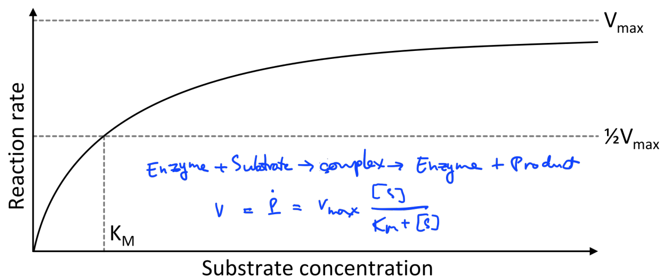

Figure 14: *Michaelis–Menten kinetic model.*

Then, the rate of the photosynthesis of the plants is given by

$$P = \frac{P_{Max} \cdot CO_{2\,Stom}}{CO_{2\,Stom} + CO_{2\,0.5}}, \tag{21}$$

where $CO_{2\,0.5}$ is the concentration of CO_2 in the substrate when $P = P_{Max}/2$ (μmol m^{-3}). Solve for $CO_{2\,Stom}$, from (20) and (21), the photosynthetic rate P satisfies the equation

$$ResP^2 - (CO_{2\,Air} + CO_{2\,0.5} + ResP_{Max})P + CO_{2\,Air}P_{Max} = 0. \tag{22}$$

For the quadratic equation above, the focus is in the solution P such that $P \to P_{Max}$ as $CO_{2\,Air} \to +\infty$. Noting that the photosynthetic rate P no longer depends on the concentration of CO_2 in the stomata but only on the concentration of CO_2 in the air, the resistance coefficient Res, and the maximum photosynthetic rate.

4.1.3 Maximum rate of photosynthesis

To solve the equation (22), the maximum rate of photosynthesis needs to be determined. For the model for the photosynthesis of one leaf unit, the maximum rate of photosynthesis is taken as a function of the leaf temperature, activation energy, and deactivation energy. Usually, that rate will be determined by the chemical reaction Arrhenius model

$$k(T) = k(T_0)e^{-\frac{H_a}{R}\left(\frac{1}{T}-\frac{1}{T_0}\right)}, \tag{23}$$

where $k(T)$ is the reaction rate at T (K), T_0 is the optimum temperature for which the reaction rate is known (K), H_a is the activation energy for the reaction (J mol^{-1}), and R is the ideal gas constant (J mol^{-1} K^{-1}).

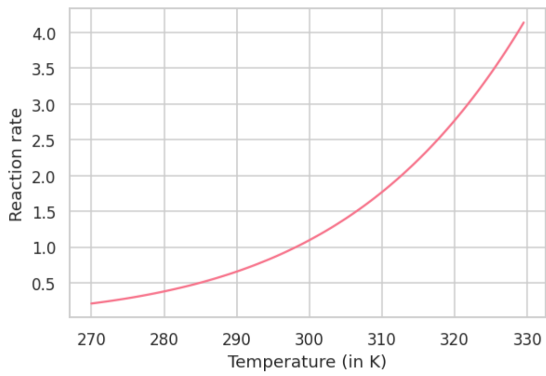

Figure 15: *Arrhenius model with* $T_0 = 298.15$, $k(T_0) = 1$, *and* $H_a = 37000$.

However, when the temperature is increasing, up to a certain threshold, the enzyme activity will be inhibited and the photosynthesis is slowed down and stop. Thus, the Arrhenius model is not sufficient to explain the inhibition of the enzyme and the following model is seen as the model for the activity of the Rubisco enzyme during photosynthesis and depends on the leaf temperature.

$$f(T) = \frac{1 + e^{-\frac{H_d}{R}\left(\frac{1}{T_0}-\frac{1}{\frac{H_d}{S}}\right)}}{1 + e^{-\frac{H_d}{R}\left(\frac{1}{T}-\frac{1}{\frac{H_d}{S}}\right)}}. \tag{24}$$

In the model (24), $f(T)$ represents the enzyme activity at T (K), H_d is the deactivation energy (J mol^{-1}), and S is the corresponding entropy quantity (J mol^{-1} K^{-1}).

By combining the model (23) and (24), the maximum rate of photosynthesis per leaf unit is given by the formula

$$P_{Max}(T) = k(T)f(T). \tag{25}$$

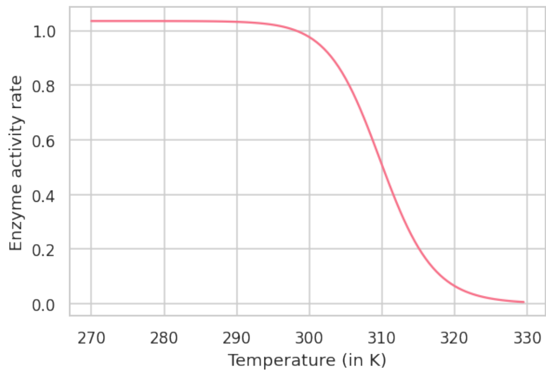

Figure 16: *Enzyme activity model with* $H_d = 220000$ *and* $S = 710$.

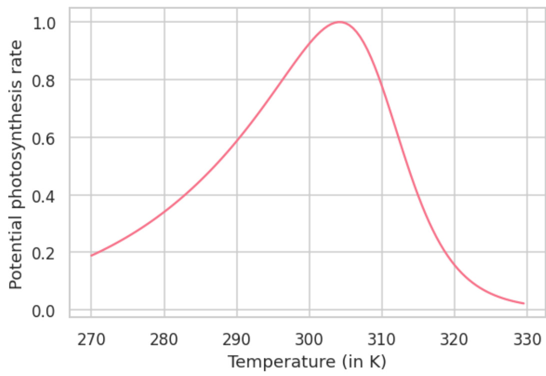

Figure 17: *Maximum photosynthetic rate* $P_{Max}(T) = k(T)f(T)$ *(normalized).*

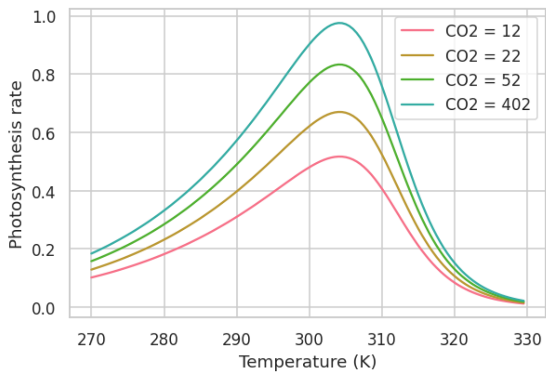

Figure 18: *Photosynthetic rate for different values of* CO_2 *concentration in the greenhouse air and resistance coefficient* $Res = 2.5$ *(normalized).*

4.2 Photosynthesis model for the whole canopy

In this section, we will develop a model for the whole foliage of the plants inside the greenhouse.

4.2.1 Leaf area index

First, we need to consider the concept of the leaf area index (LAI). The index LAI is calculated by the total leaf density per unit area of soil in a greenhouse. Then, the thicker the canopy is, the higher the index LAI is (see Figure 19). This index is very important for the canopy photosynthesis model because light absorbance is closely dependent on LAI. Due to Beer's law,

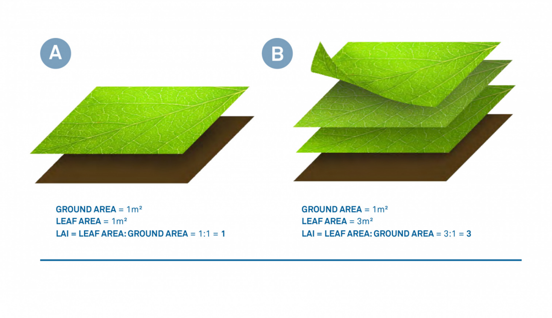

Figure 19: *Leaf area index*

if the light intensity before entering the canopy is I_0 (μ mol {photons} m^{-2} s^{-1}), the intensity of the transmitted beam I (μ mol {photons} m^{-2} s^{-1}) is equal to

$$I = \frac{I_0.K.e^{-K.LAI}}{1-m}, \tag{26}$$

If the leaves are horizontally stratified such as in the case of tomato, the dimensionless extinction coefficient K will be between 0.7 and 1.0. Meanwhile, if the leaves are sloping as in the case of wet rice, K will be between 0.3 and 0.5. m is the transmittance coefficient of the leaves which is set as 0.1. Hence, the amount of light absorbed by the canopy can be measured as the difference in the intensity of the light ray before entering the foliage and after passing through the foliage

$$L = L_0\left(1 - \frac{K.e^{-K.LAI}}{1-m}\right). \tag{27}$$

In this formula, L is luminous flux received by the leaves per unit area of the greenhouse floor (μ mol {photons} m^{-2} s^{-1}) and L_0 is the initial value of L before going through the canopy. Noting that the formula (27) does not take into account the light reflection factor and the absorption of radiation from greenhouse objects. A more complete formula can be found in [Van11].

4.2.2 The modified Arrhenius formula

To calculate the value of P_{Max}, which is the maximum photosynthetic rate of all leaves in the greenhouse, we consider the modified Arrhenius model (25) instead of (23).

$$k(T) = LAI \cdot k(T_0) \cdot e^{-\frac{H_a}{R}\left(\frac{1}{T}-\frac{1}{T_0}\right)}. \tag{28}$$

Here, $k(T)$ is the reaction rate for the whole canopy at T (K) and $k(T_0)$ is the reaction rate under the optimal condition T_0 (K) of one leaf unit, and H_a is also the activation energy for one leaf unit.

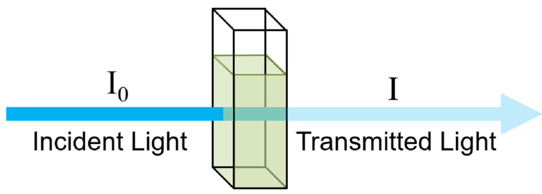

Figure 20: *The incident-ray intensity decreases after passing through a liquid.*

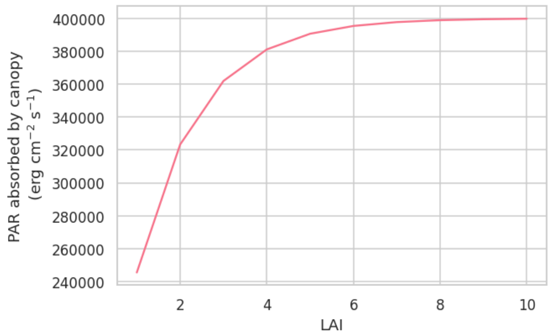

Figure 21: *Photosynthetically active radiation.*

4.2.3 Michaelis–Menten kinetic model for P_{Max}

Unlike the photosynthesis model for one leaf unit, the amount of light energy absorbed into the foliage in response to LAI needs to be added since it affects the maximum photosynthetic rate P_{Max}. Therefore, we consider the following formula of P_{Max}, which is a dependent function on L and T.

$$P_{Max}(L,T) = \frac{P_{MLT} \cdot P_{Max}(T) \cdot L}{L + L_{0.5}}. \tag{29}$$

In which, $L_{0.5}$ is light intensity when $P_{Max}(L,T) = P_{Max}(T)/2$ (μmol {photons} m^{-2} s^{-1}), $P_{Max}(T)$ is calculated by using the formula (25) with $k(T)$ as in (28), and P_{MLT} are the maximum photosynthetic rate at the point of light saturation and the optimal temperature T. Usually P_{MLT} is determined based on experiments and empirical works.

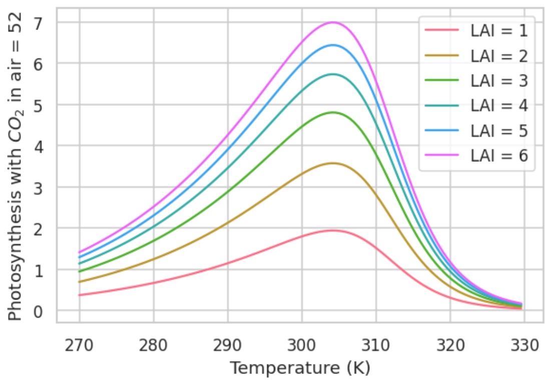

Figure 22: *Photosynthesis of the canopy at a fixed value of CO_2 concentration and different LAI.*

5 Data

In this topic, we will use the published greenhouse data at https://github.com/CEAOD/Data. Some other data regarding the coefficients of the model can be found from [Mar94; Van11; De 96] and related sources.

6 Instructions and Requirements

6.1 Instructions

Students have to follow the instructions and comply with the requirements below. *Teachers do not solve the cases arising due to the fact that students do not follow the instructions and comply with the requirements.*

Students can form groups among CC01-CC05 classes, it is not necessary that all the team members are in the same class, and each group has 4-5 people (5 is maximum). If a group has members from different classes, the class has to be specified in parentheses right after the student ID numbers of each member. In the member list, it is also necessary to indicate who is the "leader" of the group.

The assignment will not be presented by students directly in class, so all aspects related to this assignment will be quizzed (about 10 - 12 of about 25 multiple-choice questions) in the final exam of the subject. Therefore, team members should work together so that all of you understand all of the aspects of the assignment. The team leader should organize the work group so that this requirement will be met.

During the work, students should write emails to all of the advisors and the tutors "simultaneously" (their email addresses are on the cover of this assignment) for them to answer to all of the students of the 05 classes.

Regarding the background knowledge related to the topic, students have to read the information of the model in chapters 2, 3 or 8, 9 (the summary of chapters 2 and 3) in [Van11] and the citations therein carefully.

6.2 Requests

- Deadline for submission: January 20, 2020. Students have to answer each question in a clear and coherent way.
- Write a report in accordance with **the layout as in the template file** using LaTeX.
- Each group when submitting their report **need to submit also a log file (diary)** in which clearly state: **weekly work progress for all 06 weeks**, tasks, content of opinions exchanged of the members, ...

6.3 Submission

- Students must submit their group report via BK-eLearning system (to be opened in the coming weeks): compress all necessary files (.tex file, .py file, ...) into a single file named "*Assignment-CO2011-CSE201-The-student-ID-numbers.zip*" and submit it in the Assignment section on the BK-eLearning site.
- Note that for each group, **only the leader will submit the report, even for groups with members from different classes.**

7 Exercises

Students solve the following exercises.

Exercise 1 (*Compulsory*)**.** Write a report and break it into the following sections.

1. Background section:
    (a) Present the definition and classification according to different criteria, the general form of dynamical systems, and especially first-order differential equations systems with initial condition at time t_0, which are continuous dynamical systems used in this assignment.
    (b) Introduce a necessary and sufficient condition for the above systems of differential equations to exist and have unique solutions.
    (c) Give some examples of solvable first-order differential equations and their exact solutions.
    (d) Introduce and present the approximation steps of the Explicit Euler and Explicit Runge–Kutta of order 4 algorithms to solve general first-order differential equations.
    (e) Using Explicit Euler and Explicit Runge–Kutta, give approximate values of the exact solutions of the above examples at time t_0, $t_0 + h$, $t_0 + 2h$, ..., $t_0 + 5h$ with optional h.
2. Application section: Answer the questions 2, 3, 4, and 5.

Exercise 2 (*Compulsory*)**.** Do the following tasks.

a) Restate the model for the CO_2 concentration in greenhouses in detail as described in Sections 2, 3, and 4 of this assignment and write to the report.

b) Write programs that calculate the net CO_2 flux from one place to an other place using the formulas (3)-(7), (9)-(19). Each formula is a function with input parameters being the coefficients and variables involved in each respective formula. Then write a program that returns the right side of the system of (1) and (2) divided by $cap_{CO_2\,Air}$ and $cap_{CO_2\,Top}$ respectively and named this function **dx**. Present carefully in the report.

Exercise 3 (*Compulsory*)**.** Assuming constant temperature difference and constant air density difference, study from [Van11] and related citations to find specific and reasonable values for each input parameter of the function **dx** including the difference in temperature and in air density except for the variables $CO_{2\,Air}$ and $CO_{2\,Top}$. Noting that these values need to match the units considered in the report. Check if the programs work properly with the values found and specific $CO_{2\,Air}$ and $CO_{2\,Top}$ values. Present details in the report.

Exercise 4 (*Compulsory*)**.** Do the following tasks.

a) Study explicit Euler and Runge–Kutta of order 4 algorithms for solving first-order differential equations (see references [ESG93; EG96]). Write two programs performing the two algorithms and named them **euler** and **rk4** respectively with inputs: a callable function as **dx**, the values at time t of the two variables $CO_{2\,Air}$ and $CO_{2\,Top}$, the time step h. These solvers return the approximate values of $CO_{2\,Air}$ and $CO_{2\,Top}$ at time $t+h$. Present details in the report.

b) Select specific values of $CO_{2\,Air}$ and $CO_{2\,Top}$ at time t from the data set as initial values to run the solvers and find the approximate values of $CO_{2\,Air}$ and $CO_{2\,Top}$ in the next 5 minutes, 10 minutes, 20 minutes, ... and calculate the difference of the result from the actual data. Comment on the accuracy of the model and present details in the report.

Exercise 5 (*Compulsory*)**.** Do the same thing as in Ex. 2 - Ex. 4 for the vapor pressure VP_{Air} and VP_{Top} as described in chapters 2 and 8 in [Van11]. Present details in the report.

Exercise 6 (*Advanced and optional*)**.** Do the following tasks.

a) Read and learn how to solve differential equations using Deep Learning techniques as introduced in https://arxiv.org/pdf/1711.10561.pdf. Propose a similar (not too complicated) algorithm for the general ODE model

$$\begin{cases} \dot{x}(t) = f(t, x), \\ x(t_0) = x_0. \end{cases} \tag{30}$$

Here, f is a function depending on t and $x(t)$ (noting that x and f are vectors). Present details in the report.

b) Learn about using PyTorch or Tensorflow as well as some knowledge on how to build and train a basic Deep Learning model. Using the algorithm suggested in question a), build and train a simple Deep Learning model for at least one differential equation (30) for which the exact solution formula is known (training data is thus derived from the exact formula). Present details in the report.

c) Study how to use data on CO_2 concentration (see https://github.com/CEAOD/Data), proposed algorithm in question a), and the knowledge in b) to build and train a Deep Learning model to solve the system of differential equations related to $CO_{2\,Air}$ mentioned in the assignment. Compare with the result found by classical techniques as Euler and Runge–Kutta. Present details in the report.

8 Evaluation and Cheating

8.1 Evaluation

Each assignment will be evaluated as follows.

Content	**Score (%)**
- Analyze, answer coherently, systematically, focus on the goals of the questions and requests	30%
- The programs are neatly written and executable	30%
- Correct, clear, and intuitive graphs & diagrams	20%
- Background section is well written, correct and appropriate	15%
- Well written report and correct	5%

8.2 Cheating

The assignment must be done by the students (group). Students (groups) will be considered cheating if:

- There is an unusual similarity among the reports (especially the background section). In this case, ALL submissions that are similar are considered cheating. Therefore, the students (groups) must defend their own works.
- Students (groups) do not understand the works written by themselves. Students (groups) can consult from any source, but make sure they understand the meaning of everything they write.

If the article is found cheating, students will be judged according to the school's regulations.

References

[Bal89] LJ Balemans. *Assessment of criteria for energetic effectiveness of greenhouse screens.* 1989.

[BB95] Thierry Boulard and Alain Baille. "Modelling of air exchange rate in a greenhouse equipped with continuous roof vents". In: *Journal of Agricultural Engineering Research* 61.1 (1995), pp. 37–47.

[De 96] HF De Zwart. *Analyzing energy-saving options in greenhouse cultivation using a simulation model.* 1996.

[EG96] Hairer Ernst and Wanner Gerhard. *Solving Ordinary Differential Equations II: Stiff and Differential-Algebraic Problems.* Springer, 1996.

[ESG93] Hairer Ernst, P. Nørsett Syvert, and Wanner Gerhard. *Solving Ordinary Differential Equations I: Nonstiff Prolemns.* Springer, 1993.

[Kit+96] C Kittas et al. "Wind induced air exchange rates in a greenhouse tunnel with continuous side openings". In: *Journal of Agricultural Engineering Research* 65.1 (1996), pp. 37–49.

[Lom+75] Paul W Lommen et al. "Photosynthetic model". In: *Perspectives of Biophysical Ecology.* Springer, 1975, pp. 33–43.

[Mar94] LFM Marcelis. "A simulation model for dry matter partitioning in cucumber". In: *Annals of botany* 74.1 (1994), pp. 43–52.

[MG] N.J.van de Braak MIGUEL A.F. and 1995 G.P.ABot. "Mass flow through materials with pores and openings: II-natural convection". In: *submitted for publication in International Journal of Heat and Mass Transfer* ().

[Van11] Bram HE Vanthoor. *A model-based greenhouse design method.* 2011.

Nomenclatures

Some of the symbols used in this topic are given in the following table.

- Top/Air: the compartment above/below the thermal screen;
- Out/Ext: outside the greenhouse/external source;
- $Blow/Pad$: the direct air heater/pad and fan;
- Can: the canopy inside the greenhouse;
- MC_{AB}: the net CO_2 flux from A to B;
- $cap_{CO_2\,Top/Air}$: capacity of the compartment above/below the thermal screen to store CO_2 (m);
- $\dot{CO_2}_{Top/Air}$: the rate change of CO_2 concentration in the compartment above/below the thermal screen in time (mg m^{-3} s^{-1});
- MC_{AB}: the net CO_2 flux from A to B (mg m^{-2} s^{-1});

- η_{HeatCO_2}: the amount of CO_2 released when 1 Joule sensible energy is produced by the direct air heater (mg $\{CO_2\}$ J^{-1});
- $H_{BlowAir}$: the heat flux from the direct air heater to the greenhouse air (W m^{-2});
- U_{Blow}: the control valve of the direct air heater ranging in $[0, 1]$;
- P_{Blow}: the heat capacity of the direct air heater (W);
- A_{Flr}: the area of the greenhouse floor (m^2);
- U_{ExtCO_2}: the control valve of the external CO_2 source ranging in $[0, 1]$;
- ϕ_{ExtCO_2}: the capacity of the external CO_2 source (mg s^{-1});
- f_{Pad}: the ventilation flux due to the pad and fan system (m s^{-1});
- U_{Pad}: the control valve of the pad and fan system ranging in $[0, 1]$;
- ϕ_{Pad}: the capacity of the air flux through the pad (m^3 s^{-1});
- f_{ThScr}: the air flux through the thermal screen (m s^{-1});
- U_{ThScr}: the control of the thermal screen ranging in $[0, 1]$;
- K_{ThScr}: the screen flux coefficient determining the permeability of the screen (m K$^{-\frac{2}{3}}$ s^{-1});
- g: the gravitational acceleration (m s^{-2});
- $\rho_{Air/Top}$: the density of the greenhouse air below/above the thermal screen (kg m^{-3});
- ρ_{Air}^{Mean}: the mean density of the greenhouse air (kg m^{-3});
- $T_{Air/Top}$: the temperature below/above the thermal screen (K);
- $f_{VentSide}$: the rate for the sidewall ventilation system (m s^{-1});
- $f_{VentForced}$: the rate for the forced ventilation system (m s^{-1});
- $f_{VentRoofSide}$: the ventilation rate through both the roof and side vents (m s^{-1});
- $C_{d/w}$: discharge/global wind pressure coefficient depending on the greenhouse shape and the use of an outdoor thermal screen (-);
- $U_{Roof/Side}$: the control of the roof/side openings ranging in [0,1];
- $A_{Roof/Side}$: the roof/side opening area (m^2);
- $h_{SideRoof}$: the vertical distance between mid-points of side wall and roof ventilation openings (m);
- T_{Air}^{Mean}: the mean temperature between the indoor and outdoor temperatures (K); v_{Wind}: wind speed (m s^{-1});
- η_{InsScr}: reduction factor (-);
- ζ_{InsScr}: the screen porosity i.e. the area of holes per unit area of the insect screen (-);
- $f_{leakage}$: the leakage rate depending on wind speed (m s^{-1});

- $c_{leakage}$: the leakage coefficient depending on the greenhouse type (-);
- η_{Side}: the ratio between the side vents area and total ventilation area (-);
- η_{Side_Thr}: the threshold value above which no chimney effect is assumed to occur (-);
- $U_{VentForced}$: the control valve of the forced ventilation ranging in $[0, 1]$;
- $\phi_{VentForced}$: the air flow capacity of the forced ventilation system (m^3 s^{-1});
- h_{Vent}: the vertical dimension of a single ventilation opening (m);
- M_{CH_2O}: the molar mass of CH_2O (mg μmol^{-1});
- $h_{C_{Buf}}$: the inhibition of the photosynthetic rate by saturation of the leaves with carbohydrates (-);
- C_{Buf}/C_{Buf}^{Max}: the capacity/maximum capacity of carbohydrates storage in the canopy buffer (mg $\{CH_2O\}$ m^{-2});
- P/R: the photosynthesis/respiration rate of the canopy during the photosynthesis process (μmol $\{CO_2\}$ m^{-2} s^{-1});
- $CO_{2\,Stom}$: the amount of CO_2 in the chloroplasts (μmol m^{-3});
- Res: the resistance to CO_2 diffusion (s m^{-1});
- P_{Max}: the photosynthetic rate at saturating $CO_{2\,Chl}$ (μmol $\{CO_2\}$ m^{-2} s^{-1});
- $CO_{2\,0.5}$: the amount of $CO_{2\,Chl}$ such that $P = P_{Max}/2$ (μmol m^{-3});
- T: the temperature of the leaf (K);
- T_0: a specific temperature of the leaf that we know the reaction rate (K);
- $K(T)$: the reaction rate (-);
- H_a: the activation energy (J mol^{-1});
- R: the ideal gas constant (J mol^{-1} K^{-1});
- $f(T)$: the enzyme activity rate (-);
- H_d: the deactivation energy (J mol^{-1});
- S: the entropy term (J mol^{-1} K^{-1});
- K: the extinction coefficient in between 0.7-1.0 if the leaves are not inclined. Otherwise 0.3-0.5;
- I: the L measured at the ground surface (μmol {photons} m^{-2} s^{-1});
- I_0: the L measured above the canopy (μmol {photons} m^{-2} s^{-1});
- m: the transmittance of the leaves, which set as default 0.1;
- L: the photosynthetically active radiation absorbed by the canopy (μmol {photons} m^{-2} s^{-1});
- $L_{0.5}$: the photosynthetically active radiation at which $P_{Max}(L,T) = P_{MLT} \cdot P_{Max}(T)/2$ (μmol {photons} m^{-2} s^{-1});
- P_{MLT}: the value of P_{Max} at saturation L and optimum T (μmol $\{CO_2\}$ m^{-2} s^{-1}).