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Robotics FK & IK Report

KUKA KR 10 Robot

GitHub repo: https://github.com/phoenixfury/Kuka-kr-1000-FK---IK

I uploaded the new files again in the repo

1- Forward kinematics function
2- Inverse kinematics function
3- Test script
4- A live script

Robot Description:

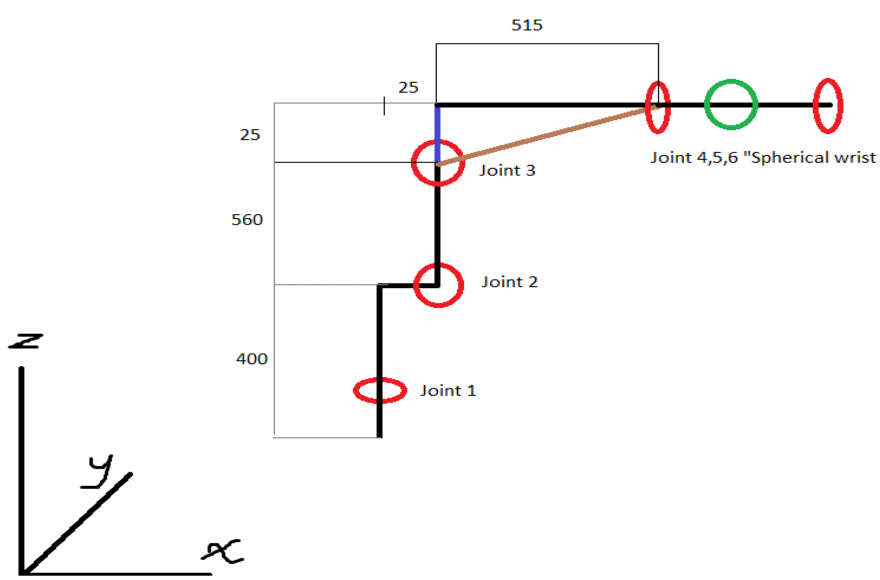
KR AGILUS-2:

The KR AGILUS-2 has six axes and is consistently rated for particularly high working speeds. At the same time, it offers extreme precision.

It has a spherical wrist for making the inverse kinematics problem easier.

The first 3 joints determine the position of the robot while the spherical wrist determines the orientation of the end effector.

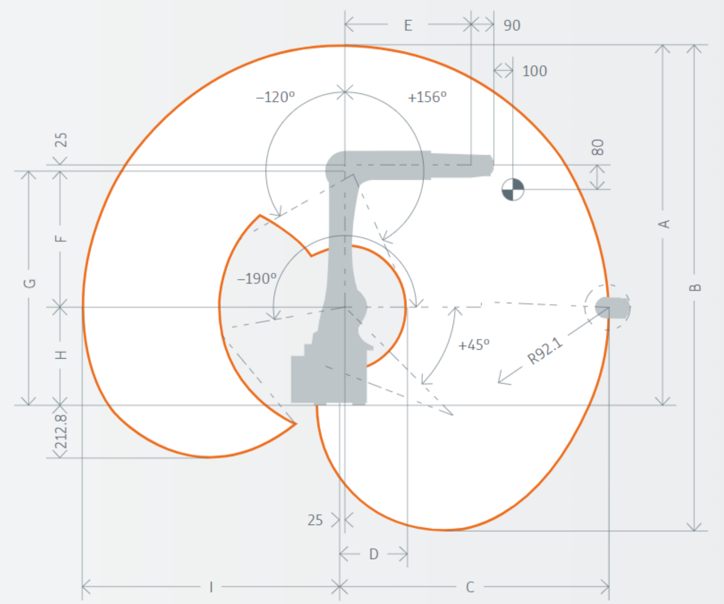
The kinematic Model

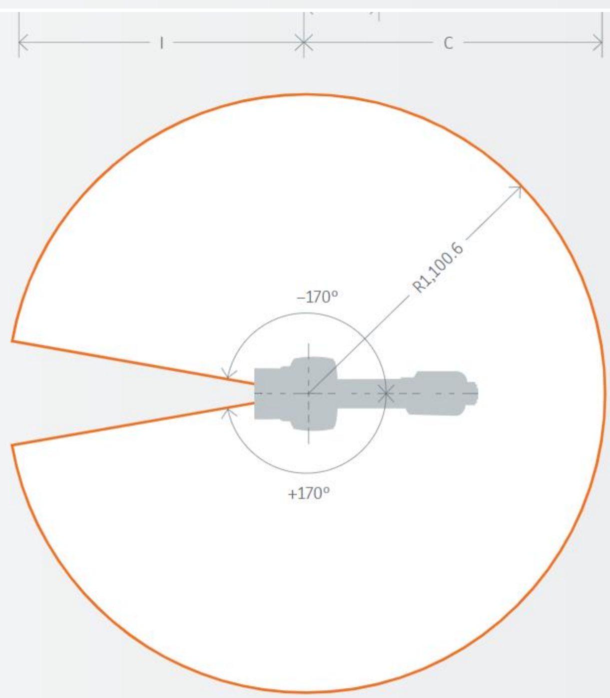


The home position of link 2 “between joint 2 and 3” is on the Z-axis and the home position of link 3 “between joint 3 and 4” is on the X-axis.

The parameters and dimensions:

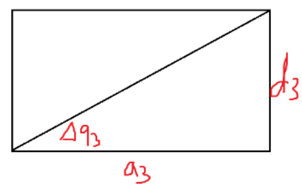
Work envelope [1]	Dimensions A	Dimensions B	Dimensions C	Dimensions D	Dimensions E	Dimensions F	Dimensions G	Dimensions H	Dimensions I
KR 10 R1100-2	1,475.6 mm	1,987.2 mm	1,100.6 mm	277.8 mm	515 mm	560 mm	960 mm	400 mm	1,050.6 mm





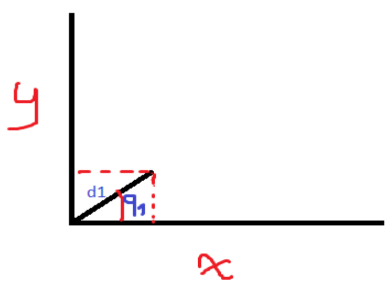
In order to simplify our inverse kinematics solution, 2 simplifications had to be made

The first one is this on joint 3



$$\Delta q_3 = Tan^{-1}\frac{d_3}{a_3}, \qquad d_3' = \sqrt{{d_3}^2 + {a_3}^2}$$

The 2nd one is on joint 1



$$dx = d_1 cos(q_1), dy = \ d_1 sin(q_1)$$

Now we are ready to define our forward kinematic model for this robot.

We will split this problem into 2 parts.

The 1st part is the forward kinematics for Translation

The 2nd part is the forward kinematics for Rotation of the spherical wrist

And then we will combine the 2 parts together to get the full forward kinematics.

The Forward kinematic model for Translation:

$$H_{transl} = R_z(q_1) * T_x(d_1) * T_z(a_1) * R_y(q_2) * T_z(a_2) * R_y(q_3 - \Delta q_3) * T_x(d_3') * R_z(\Delta q_3)$$

Here we added the simplification part of the Δq_3 and d_3' in order to simplify our inverse kinematics.

ans =

$$\begin{pmatrix} \frac{\cos(q_1)\,\sigma_3}{\sigma_1} & -\sin(q_1) & \frac{\cos(q_1)\,\sigma_4}{\sigma_1} & \cos(q_1)\,\sigma_5 \\ \frac{\sin(q_1)\,\sigma_3}{\sigma_1} & \cos(q_1) & \frac{\sin(q_1)\,\sigma_4}{\sigma_1} & \sin(q_1)\,\sigma_5 \\ -\frac{\sigma_4}{\sigma_2} & 0 & \frac{\sigma_3}{\sigma_2} & a_1 + a_2\cos(q_2) - \sin(\sigma_7)\,\sigma_6 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

The Forward kinematic model for Rotation:

$$H_{rot} = R_x(q_4) * R_y(q_5) * R_x(q_6)$$

ans =

$$\begin{pmatrix} \cos(q_5) & \sin(q_5)\sin(q_6) & \cos(q_6)\sin(q_5) & 0 \\ \sin(q_4)\sin(q_5) & \cos(q_4)\cos(q_6) - \cos(q_5)\sin(q_4)\sin(q_6) & -\cos(q_4)\sin(q_6) - \cos(q_5)\cos(q_6)\sin(q_4) & 0 \\ -\cos(q_4)\sin(q_5) & \cos(q_6)\sin(q_4) + \cos(q_4)\cos(q_5)\sin(q_6) & \cos(q_4)\cos(q_5)\cos(q_6) - \sin(q_4)\sin(q_6) & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

The full forward kinematic Model:

$$H = H_{transl} * H_{rot}$$

To test the home position of the robot set all the angles to zero

```
clear
d1 = 25;
a1 = 400;
a2 = 560;
d3 = 25;
a3 = 515;
q1 = 0; q2 = 0; q3 = 0; q4 = 0; q5 = 0; q6 = 0;
delta_q3 = atan2(d3,a3)
d3_dash = sqrt(d3^2 + a3^2)
H_transl = Rz(q1) * Tx(d1) * Tz(a1) * Ry(q2) * Tz(a2) * Ry(q3 - delta_q3) * Tx(d3_dash)* Ry(delta_q3);
H_rotation =  Rx(q4) * Ry(q5) * Rx(q6);
H = H_transl * H_rotation
```

```
delta_q3 = 0.0485
d3_dash = 515.6064
H = 4×4
     1     0     0   540
     0     1     0     0
     0     0     1   985
     0     0     0     1
```

The inverse kinematic Solution:

We will divide the problem into 2 parts, position and orientation.

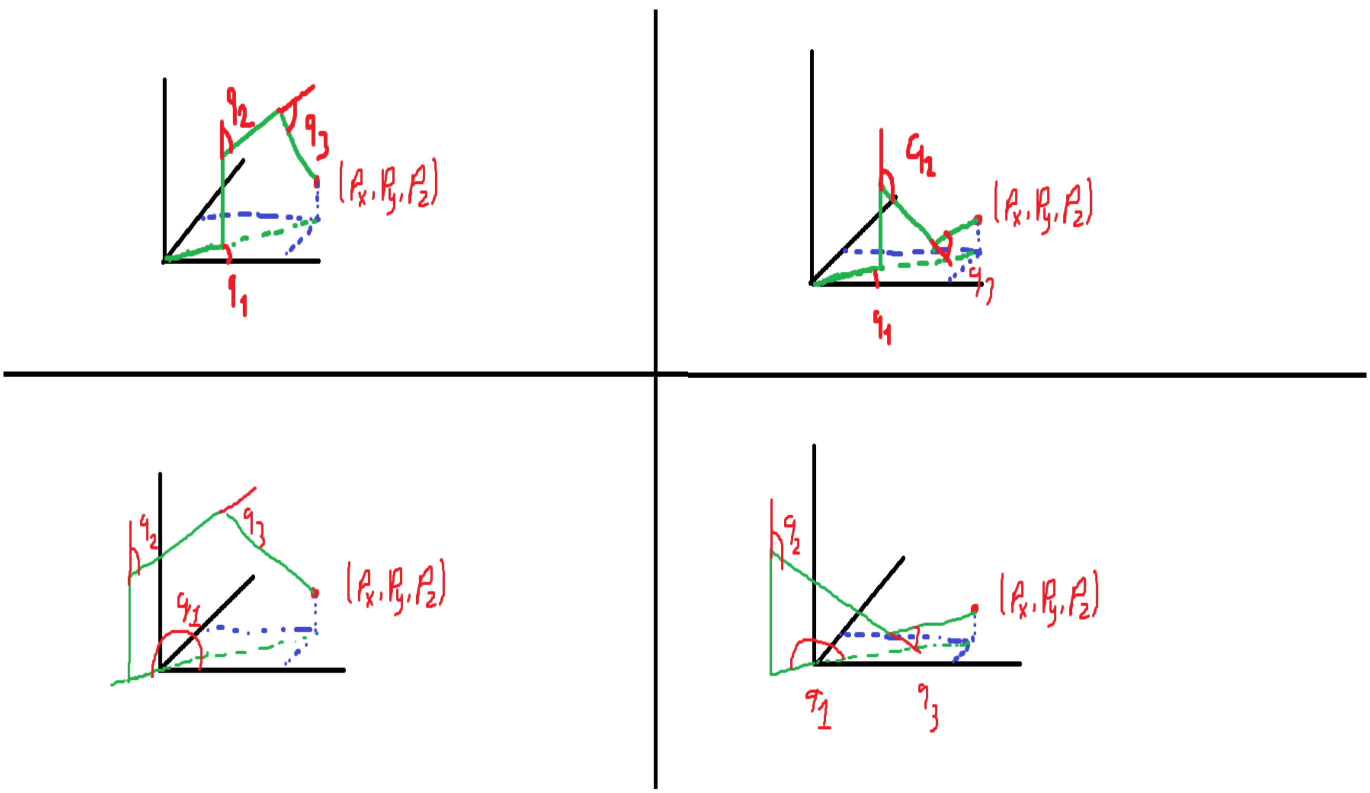
The inverse kinematic solution for the position will contain the 1st 3 joints of the robot.

The inverse kinematic solution for the rotation will contain the last 3 joints of the robot.

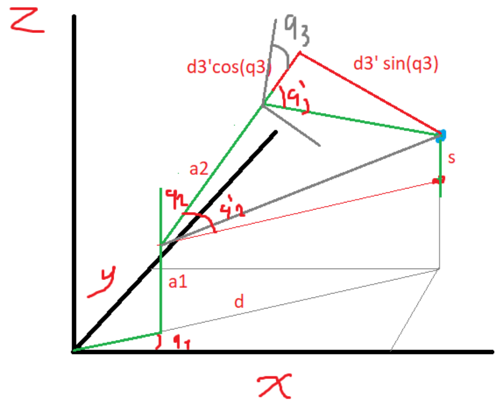
We have 4 solutions for the position of the robot and 2 solutions for the rotation, so we have 8 solutions for the whole robot.

IK for Position:

The 4 solutions:



How to get the 1st solution:



$s = z - a_1$

$\Delta q_3 = Tan^{-1}\frac{d_3}{a_3}$

$d_3' = \sqrt{{d_3}^2 + {a_3}^2}$

$d = \sqrt{x^2 + y^2} - d_1$

$q_1 = \operatorname{atan}\left(\frac{y}{x}\right)$

$D = \sqrt{s^2 + d^2}$

$q_3' = \operatorname{acos}\left(\frac{(D^2 - a_2^2 - d_3'^2)}{2a_2 d_3'}\right)$

$\alpha_1 = \operatorname{atan2}(s, d)$

$\alpha_2 = \operatorname{atan2}(d_3' \sin(q_3'), a_2 + d_3' \cos(q_3'))$

We Test q_3' to see if the current configuration is elbow up or down

If $q_3' > 0, q_2' = \alpha_1 + \alpha_2$

Else $q_2' = -\alpha_1 + \alpha_2$

Now we need the real q_2 and q_3 and we must counteract the effect of Δq_3

To get q_3: $q_3 = q_3' + \Delta q_3 - \frac{\pi}{2}$

To get q_2: $q_2 = \frac{\pi}{2} - q_2'$

The 2nd solution will be the opposite configuration of sol 1:

$q_{12} = \text{atan}\left(\frac{y}{x}\right)$

q_{32}: $q_{32} = -(q_3' - \Delta q_3 + \frac{\pi}{2})$

q_{22}: $q_{22} = \frac{\pi}{2} + q_2'$

The 3rd and 4th solutions will require q1 to be in the opposite direction and q2 and q3 will have 2 configurations "elbow up and down" and we must take into account the 25 mm shift in the x direction of the base joint

3rd Solution:

$q_{13} = \text{atan}\left(\frac{y}{x}\right) + \pi$

$dx = d_1 cos(q_{13}), dy = \ d_1 sin(q_{13})$

$d = \sqrt{(x - dx)^2 + (y - dy)^2}$

$D = \ \sqrt{s^2 + d^2}$

$q_3' = \text{acos}\left(\frac{(D^2 - a_2^2 - d_3'^2)}{2a_2 d_3'}\right)$

$\alpha_1 = \text{atan2}(s, d)$

$\alpha_2 = \text{atan2}(d_3' \sin{(q_3')}, a_2 + d_3' \ \cos{(q_3')})$

q_{33}: $q_{33} = -q_3' + \Delta q_3 - \frac{\pi}{2}$

q_{23}: $q_{23} = -(\frac{\pi}{2} - q_2')$

The 4th solution

$q_{14} = \text{atan}\left(\frac{y}{x}\right) + \pi$

q_{34}: $q_{34} = q_3' + \Delta q_3 - \frac{\pi}{2}$

q_{24}: $q_{24} = -(\frac{\pi}{2} + q_2')$

Now the 2nd part of the inverse kinematics problem:

The rotation part:

$H = H_{transl} * H_{rot}$

To get H_{rot} we must multiply H by the inverse of the forward kinematics of the translation

$H_{rot} = H_{transl}^{-1} * H$

And then extract the elements from the H_{rot} matrix.

We check the 1st element of the matrix to know if there is a singularity or not first.

```
ans =
```

$$\begin{pmatrix} \cos(q_5) & \sin(q_5)\sin(q_6) & \cos(q_6)\sin(q_5) & 0 \\ \sin(q_4)\sin(q_5) & \cos(q_4)\cos(q_6) - \cos(q_5)\sin(q_4)\sin(q_6) & -\cos(q_4)\sin(q_6) - \cos(q_5)\cos(q_6)\sin(q_4) & 0 \\ -\cos(q_4)\sin(q_5) & \cos(q_6)\sin(q_4) + \cos(q_4)\cos(q_5)\sin(q_6) & \cos(q_4)\cos(q_5)\cos(q_6) - \sin(q_4)\sin(q_6) & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

If there are no singularities, we will have 2 solutions.

The 1st solution:

$q_{41} = atan2\left(H_{rot}(2,1), -H_{rot}(3,1)\right)$

$q_{61} = atan2\left(H_{rot}(1,2), H_{rot}(1,3)\right)$

$q_{51} = atan2\left(\sqrt{H_{rot}(1,3)^2 + H_{rot}(1,2)^2}, H_{rot}(1,1)\right)$

The 2nd solution:

$q_{41} = atan2\left(-H_{rot}(2,1), H_{rot}(3,1)\right)$

$q_{61} = atan2\left(-H_{rot}(1,2), -H_{rot}(1,3)\right)$

$q_{51} = atan2\left(-\sqrt{H_{rot}(1,3)^2 + H_{rot}(1,2)^2}, H_{rot}(1,1)\right)$

Combining the 2-part solutions we have 8 solutions:

$$q = [\, q_1\; q_2\; q_3\; q_{41}\; q_{51}\; q_{61} \,]$$

$$q = [\, q_1\; q_2\; q_3\; q_{42}\; q_{52}\; q_{62} \,]$$

$$q = [\, q_{12}\; q_{22}\; q_{32}\; q_{41}\; q_{51}\; q_{61} \,]$$

$$q = [\, q_{12}\; q_{22}\; q_{32}\; q_{42}\; q_{52}\; q_{62} \,]$$

$$q = [\, q_{13}\; q_{23}\; q_{33}\; q_{41}\; q_{51}\; q_{61} \,]$$

$$q = [\, q_{13}\; q_{23}\; q_{33}\; q_{42}\; q_{52}\; q_{62} \,]$$

$$q = [\, q_{14}\; q_{24}\; q_{34}\; q_{41}\; q_{51}\; q_{61} \,]$$

$$q = [\, q_{14}\; q_{24}\; q_{34}\; q_{42}\; q_{52}\; q_{62} \,]$$

Checking for the workspace limits

```
Command Window
    0.7854    1.5708   -1.0472    0.7854   -0.6283    0.7854

>> fk = KukaFK(q)

fk =

    0.9363   -0.0022    0.3513  737.8681
    0.3485    0.1328   -0.9279  737.8681
   -0.0446    0.9911    0.1251  164.1506
         0         0         0    1.0000

>> w = KukaIK(fk)
Robot limit reached in 3rd sol
Robot limit reached in 4th sol
Warning: Matrix is singular, close to singular or badly scaled. Results may be inaccurate. RCOND = NaN.
> In KukaIK (line 120)

Warning: Matrix is singular, close to singular or badly scaled. Results may be inaccurate. RCOND = NaN.
> In KukaIK (line 121)


w =

    0.7854    1.5708   -1.0472   -2.3562    0.6283   -2.3562
    0.7854    1.5708   -1.0472    0.7854   -0.6283    0.7854
    0.7854    2.0259   -1.9974   -1.5260    0.4291    2.9871
    0.7854    2.0259   -1.9974    1.6156   -0.4291   -0.1545
       NaN       NaN       NaN       NaN       NaN       NaN
       NaN       NaN       NaN       NaN       NaN       NaN
       NaN       NaN       NaN       NaN       NaN       NaN
fx     NaN       NaN       NaN       NaN       NaN       NaN
```