Performance of Cooperative Spectrum Sensing in Hoyt Fading Channel under Hard Decision Fusion Rules

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Abstract— **Detection is compromised when a cognitive radio (CR) user experiences deep fading effect. To detect the primary user (PU) more accurately in fading channels, CR users need to cooperate by sharing their information. In this paper we investigate performance of cooperative spectrum sensing (CSS), using energy detection at each CR user, to improve the sensing performance in Hoyt/Nakagami-*q* fading channel. Hard decision combining fusion rule (*k*- out of-*N*) is performed at fusion center (FC) to make the final decision about PU. Comparison among fusion rules has been illustrated for a wide range of average signal-to-noise ratio (SNR) values. The performance of CSS is compared with single CR user based spectrum sensing under various network parameters such as number of CR users (*N*) and average SNRs. The performance of single CR user based spectrum sensing improves with increase in Hoyt fading parameter (*q*).**

***Keywords*— Cognitive radio, energy detection, fusion rules, missed detection probability**

I. Introduction

Cognitive radio (CR) technique has been proposed [1] to solve the conflicts between spectrum scarcity and spectrum under utilization. CR systems allow other CR users to share the spectrum with its primary users (PUs) through opportunistic access. However, a CR user can use the spectrum only when it does not create any intolerable interference to primary users. Spectrum sensing is an important feature of cognitive radio technology since it is necessary to detect the presence of PUs accurately and quickly in order to find availability of unused spectrum i.e. the spectrum holes. Accurate sensing of spectrum holes is a hard task because of shadowing, fading and time-varying nature of wireless channels [2]. Due to severe multipath fading, a cognitive radio may fail to detect the presence of the PU. Cooperative spectrum sensing (CSS) is a proven method for improving the detection performance. In CSS, all CR users sense the PU individually and send their sensing information in the form of 1-bit binary decisions (1 or 0) to fusion center (FC). The hard decision combining fusion rule (OR, AND, and Majority-logic fusion rule) is performed at fusion center (FC) using a counting rule to make the final decision regarding whether the PU is present or not [3]-[4].

In many wireless applications, it is of great interest to check the presence and availability of an active communication link when the signal is unknown. In such scenarios, one appropriate choice consists of using an energy detector which measures the energy in the received waveform over an observation time window [5]. The existing literature is focused on single CR user [6] and cooperative CR user [7] based spectrum sensing, using energy detection (ED) over popular fading models. e.g. Rayleigh and Nakagami-*m* fading channels. In this article, however, we extend performance analysis of CSS based on ED in Hoyt fading channel where we assume the links between PU and CR users are Hoyt faded. Hoyt distribution [8]-[10], also known as Nakagami-*q* distribution (*q* being the fading severity parameter), allows us to span the range of fading distribution from one-sided Gaussian (q=0) to Rayleigh fading (q=1), and is used extensively for modeling more severe than Rayleigh fading wireless links.

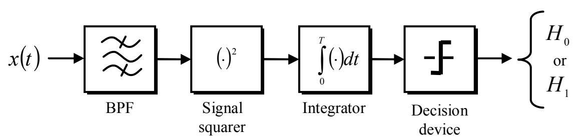

Figure 1. Block diagram of an energy detector.

Specifically, in this paper, we have investigated the impact of fading parameter (q) on single CR user's missed detection performance. Cooperative spectrum sensing is analyzed under several hard decision fusion rules such as AND, OR and Majority logic in presence of Hoyt fading in sensing channel. The performance comparison between single CR and cooperative CR user based spectrum sensing has been studied

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under different number of CR users and different sensing channel SNR values. Comparison among hard decision combining fusion rules for a wide range of SNR values has been illustrated and same is also highlighted through complementary receiver operating characteristics (ROC).

The rest of the paper is organized as follows. In Section II, the system model is described. Results and discussions are presented in Section III. Finally we conclude the paper in Section IV.

II. SYSTEM MODEL

We consider a network of N Cognitive radio (CR) users, one primary user (PU) and one fusion center (FC). A CR user, which is using an energy detector (ED) with a detection threshold (λ), makes hard binary decision (either binary bit '1' or binary bit '0') over Hoyt Fading sensing channel. We assume that all CR users are close to each other so that they form a cluster. All CR users use same threshold (λ). The distance between any two CR users is less than the distance between a PU and a CR user or the distance between a CR user and the FC. Each CR user is having one ED as shown in Fig. 1. An energy detector receives a signal $x(t)$ as defined below at input and gives a binary decision regarding the presence or absence of the PU. The received signal $x(t)$ at i-th CR user can be represented as:

$$x_i(t) = \begin{cases} n_i(t) & H_0 \\ h_i * s(t) + n_i(t) & H_1 \end{cases} \quad (1)$$

where h_i is the channel coefficient of Hoyt faded sensing channel. The noise $n_i(t)$ is modeled as a zero-mean white Gaussian random process, H_1 and H_0 are the two hypotheses associated with presence and absence of a PU. The noise energy at i-th CR user can be approximated over the time interval (0, T), as [6]:

$$\int_0^T n_i^2(t)dt = \frac{1}{2W}\sum_{j=1}^{2m} n_{i,j}^2, \quad (2)$$

where m is the time-bandwidth product and

$$n_{i,j} \sim N(0, N_{01}W), \text{ for all } j. \quad (3)$$

The decision statistic at i-th CR user, Y_i, can be written as:

$$Y_i = \sum_{j=1}^{2m} n'^2_{i,j} \quad (4)$$

where $n'_{i,j} = n_{i,j} / \sqrt{N_{01}W}$. Y_i can be viewed as the sum of the squares of $2m$ standard Gaussian variates with zero mean and unit variance. Therefore, Y_i follows a central chi-square (χ^2) distribution with $2m$ degrees of freedom. The same approach can be applied when the signal $s(t)$ is present by replacing each of $n_{i,j}$ with $n_{i,j} + s_j$ where $s_j = s(j/2W)$. The decision statistic Y_i in this case will have a non-central chi-square $\chi^2(2\gamma_i)$ distribution with $2m$ degrees of freedom and a non centrality parameter $2\gamma_i$ [6].

Non-fading environment (AWGN channel)

In non-fading environment i.e. when the sensing channels are corrupted by AWGN only, the probabilities of detection and false alarm at i-th CR user are given by the following formulas [3] - [7]:

$$P_{d,i} = P(Y_i > \lambda / H_1) = Q_m(\sqrt{2\gamma_i}, \quad \lambda) \quad (5)$$

$$P_{f,i} = P(Y_i > \lambda / H_0) = \Gamma(m, \lambda/2)/\Gamma(m) \quad (6)$$

where $\Gamma(.,.)$ is the incomplete gamma function [11] and

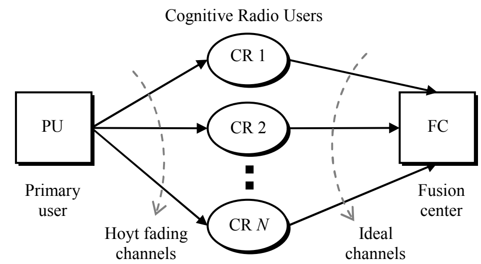

Figure 2. Block diagram of a cooperative spectrum sensing system.

$Q_m(.,.)$ is the generalized Marcum Q-function [12]. For simplicity, identical average SNRs $(\gamma_i = \bar{\gamma}; \forall i)$ has been assumed for all CR users. If the signal power is unknown, we can first set the false alarm probability (P_f) to a specific desired level. For a given desired level of P_f, the threshold (λ) of energy detector is set which determines the detection probability (P_d) along with instantaneous SNR (γ) of sensing channel as given in equation (5). As expected, P_f is independent of γ_i for all i since under H_0 there is no PU signal present.

When h is varying due to fading, equation (5) gives the probability of detection as a function of the instantaneous SNR (γ). In this case, the average probability of detection ($\bar{P}_d$) may be derived by averaging (5) over fading statistics [3],

$$\bar{P}_d = \int_x Q_m(\sqrt{2\gamma_i}, \quad \sqrt{\lambda}) f_\gamma(x)dx \quad (7)$$

where $f_\gamma(x)$ is the probability distribution function (PDF) of SNR under fading.

Hoyt or Nakagami-q fading in sensing Channel

We assume that the sensing channel between PU and CR user is Hoyt faded. Hoyt or Nakagami-q distribution is generally used to characterize the fading environments that are more severe than Rayleigh fading. The PDF of γ, i.e., $f_\gamma(\gamma)$, may be defined as [9]-[10]:

$$f_\gamma(\gamma) = \frac{1}{\sqrt{p}\bar{\gamma}} \exp\left(-\frac{\gamma}{p\bar{\gamma}}\right) I_0\left(\frac{\gamma\sqrt{1-p}}{p\bar{\gamma}}\right); \gamma \geq 0 \quad (8)$$

where $$p = \frac{4q^2}{(1+q^2)^2}; 0 \leq p \leq 1 \quad (9)$$

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where q is the fading severity parameter. The average P_d in this case, $\overline{P}_{d,Hoyt}$ can now be evaluated by substituting $f_\gamma(\gamma)$ from equation (8) in equation (7).

Hard decision fusion rules

Let N denote the number of CR users sensing the PU. Each CR user makes its own local decision regarding the presence or absence of PU (i.e. H_1 or H_0), and forwards the binary decision (1 or 0) to FC for data fusion as shown in Fig. 2. The PU is located far away from all CR users. All the CR users receive the PU signal with same local mean signal power, i.e. all CR users form a cluster with distance between any two CRs negligible compared to the distance from the PU to a CR. For simplicity we have assumed that the noise, fading statistics and average SNR in sensing channel are the same for each CR user. Further, the channels between CR users and FC are ideal channels (noiseless). Assuming independent decisions, the fusion problem where k-out of-N CR users are needed for decision, can be described by binomial distribution based on Bernoulli trials where each trial represents the decision of each CR user. The generalized formula for overall probability of detection, Q_d for the k-out of-N fusion rule is given by [5], [7]:

$$Q_d = \sum_{l=k}^{N} \binom{N}{l} \overline{P}_d^{\,l} \left(1 - \overline{P}_d\right)^{N-l} \tag{10}$$

where P_d is the probability of detection for each individual CR user as defined by equations (5) and (7).

The OR-logic fusion rule (i.e. 1-out of-N rule) can be evaluated by setting k=1 in equation (10):

$$Q_{d,OR} = \sum_{l=1}^{N} \binom{N}{l} \overline{P}_d^{\,l} \left(1-\overline{P}_d\right)^{N-l} = 1 - \binom{N}{l} \overline{P}_d^{\,l} \left(1-\overline{P}_d\right)^{N-l} \Bigg|_{l=0} = 1-(1-\overline{P}_d)^N \tag{11}$$

The AND-logic fusion rule (i.e. N-out of-N rule) can be evaluated by setting k=N in equation (10):

$$Q_{d,AND} = \sum_{l=N}^{N} \binom{N}{l} \overline{P}_d^{\,l} \left(1-\overline{P}_d\right)^{N-l} = (\overline{P}_d)^N \tag{12}$$

Finally, for the case of Majority-logic fusion rule (i.e. N/2-out of-N rule) the $Q_{d,Majo}$ is evaluated by setting $k = \lfloor N/2 \rfloor$ in equation (10).

Similarly, the overall probability of false alarm, Q_f for the case of OR, AND, and Majority-logic fusion rules can be evaluated by replacing P_d with P_f in equations (10), (11), and (12) respectively.

III. Results and Discussions

The simulation results are obtained using the following system parameters: Time-bandwidth product, $m = 5$, average SNR, $\overline{\gamma}$ =10 dB and Q_f=0.1. Fig. 3 shows complementary ROC (P_m vs. P_f) curves for the single CR user's spectrum sensing in Hoyt fading channel. Different values of Hoyt fading parameter, q=0.0, 0.3, 0.6, and 0.8 are considered. The performance with q=1 corresponds to Rayleigh fading sensing channel. Increase in Hoyt parameter q=0 to 1, significantly decrease the probability of missed detection [curves (i) to (iv)] at a fixed P_f. A plot for non-fading (pure AWGN) case is also provided for comparison. AWGN curve (v) matched with curve in [5] which validates our simulation framework.

Fig. 4 shows complementary ROC (Q_m vs. Q_f) curves for different cooperative CR users under Hoyt fading channel (q=0.5) in case of OR-logic fusion. Non- fading AWGN curve is also shown for comparison. We observe that fusing the decisions of different CR users cancels the effect of fading on the detection performance effectively. Moreover, with increase in N [curves (i) to (ii) & (iv) to v], cooperative spectrum sensing outperforms AWGN

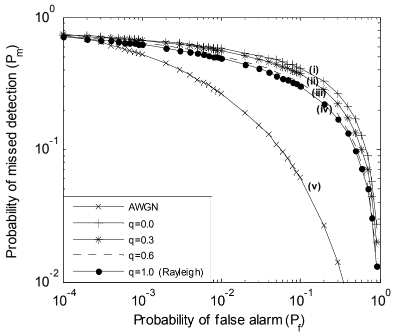

Figure .3. Effect of Hoyt fading parameter, q, on performance of single CR user based spectrum sensing.

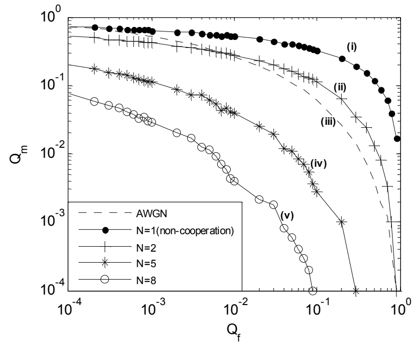

Figure 4. Q_m vs. Q_f under Hoyt fading channel (q=0.5) for different number of cooperative CR users ($\overline{\gamma}$ =10dB, m=5), OR rule.

local sensing [curve (iii)] and single CR user based sensing [curve (i)].

Fig. 5 shows the probability of detection (Q_d) vs. $\overline{\gamma}$ under Hoyt fading (with q=0.5) environment for different number of cooperative CR users under OR rule of fusion.

We have chosen Q_f as 0.1 and m=5 for each curve in this figure. We observe that there is an excellent improvement in performance of ED-CSS with increase in N and average SNR. In particular, for a probability of detection equal to 0.9, single CR based spectrum sensing requires $\overline{\gamma} \approx$ 17dB while cooperative sensing with N=8 only needs approximately 6 dB for individual CR users. Fig. 6 shows the performance of hard decision rules and their comparison based on Q_d vs. average SNR $\overline{\gamma}$ for 5 cooperative CR users under Hoyt fading channel (q=0.5), m=5 and Q_f=0.1.

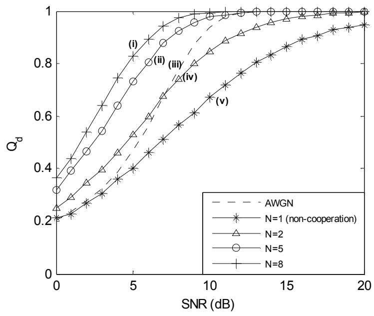

Figure 5. Q_d vs. $\overline{\gamma}$ under Hoyt fading channel (q=0.5) for different number of cooperative CR users (Q_f =0.1 , m=5) , OR rule.

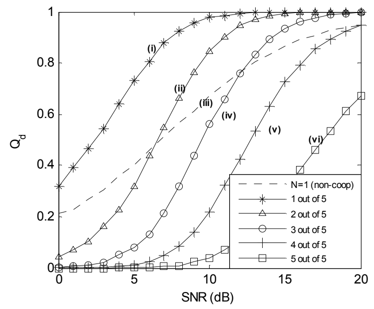

Figure 6. Performance of hard decision fusion rules via Q_d vs. $\overline{\gamma}$ under Hoyt fading channel (q=0.5) for N=5 CR users.

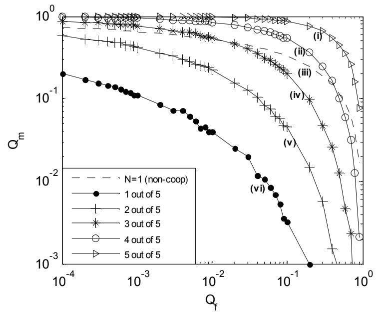

Figure 7. Performance of hard decision fusion rules via Q_m vs. Q_f under Hoyt fading channel (q=0.5) for N=5 CR users.

We observe that for a particular value of average SNR, of 6 dB, probability of detection is above 0.8, 0.08 and 0 for the OR-logic fusion (i.e. 1-out of- 5) [curve (i)], Majority-logic fusion (i.e.3-out of-5) [curve (iv)] and AND-logic fusion (i.e.5-out of-5) [curve (vi)] respectively. We can say that OR-logic fusion gives higher Q_d than Majority-logic fusion and AND-logic fusion. The curve (iii) for non-cooperation case (N=1) is provided for comparison purpose.

Fig. 7 shows the performance of several hard decision fusion rules and their comparison based on complementary ROC curves i.e. probability of missed detection (Q_m vs. Q_f) for 5 cooperative CR users. The Hoyt fading parameter is chosen as 0.5. We observe that for a particular value of Q_f=0.1, Q_m is 0.005, 0.5 and above 0.9 for OR-logic fusion [curve (vi)], Majority-logic fusion [curve (iv)] and AND-logic fusion [curve (i)] respectively. We observe that OR-logic fusion yields lower Q_m than Majority-logic fusion and AND-logic fusion. The curve (iii) for non-cooperation case is provided for comparison purpose.

IV. Conclusion

We have studied the performance of cooperative spectrum sensing (CSS) scheme using energy detection under Hoyt faded sensing channel. The performance of CSS also has been assessed in terms of probability of detection under several different average SNR values of the sensing channel. The impact of Hoyt fading parameter on single CR user's missed detection performance has also been investigated. Several fusion schemes such as AND, OR and Majority rules are studied and compared with each other through complementary ROC. We have observed that cooperative spectrum sensing, using energy detection performs better under OR-fusion rule compared to AND rule and Majority rules under same average SNR conditions. Performance of a CR user based spectrum sensing improves with increase in Hoyt parameter (q) i.e. with reduction in severity of fading.

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