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Energy Detection using Estimated Noise Variance for Spectrum Sensing in Cognitive Radio Networks

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Abstract- **In this paper, we analyze the performance of spectrum sensing based on energy detection. We do not assume the exact noise variance is known a priori. Instead, an estimated noise variance is used to calculate the threshold used in the spectrum sensing based on energy detection. We propose a new analytical model to evaluate the statistical performance of the energy detection. We claim some characteristics of this model, and analyze how these characteristics affect the performance of spectrum sensing. The analytical results are verified through numerical examples and simulations. Through these examples, we demonstrate the effectiveness of our analytical model: we show how it can be used to set the appropriate threshold such that more spectrum sharing can be facilitated, especially when combined with cooperative spectrum sensing method.**

I. INTRODUCTION

With the rapid development of wireless communications, the ever increasing demand for limited spectrum resources will eventually cause spectrum scarcity problem. However, a recent study conducted by Shared Spectrum shows that the average spectrum occupancy in the frequency band from 30 MHz to 3000 MHz over multiple locations is merely 5.2%. The maximum occupancy is about 13% in New York City [1]. It can be found that the spectrum scarcity is mostly caused by the fixed assignment to the wireless service operators, and there exist spectrum opportunities both spatially and temporally. Therefore the interest in allowing access to unutilized spectrum by unlicensed user (secondary user) has been growing in several regulatory bodies and standardization groups, e.g., FCC and 802.22 [2, 3].

In order to alleviate the problem of spectrum scarcity, a secondary user may be allowed to access the temporarily unused licensed bands of a primary user. The process of detecting primary user and then determining if the licensed spectrum is accessible is referred to as spectrum sensing. In this case, the secondary user adopts "detect and avoid" strategy to assure that no harmful interference is caused to the primary users' service. A cognitive radio network consists of secondary radio users that can perform spectrum sensing and then operate at the appropriate piece of unused spectrum.

In the aspect of spectrum sensing, the radio measures certain characteristics of the radio waveform, and then decides if a primary system is actively using that spectrum. In this paper, the energy detection method is chosen as the underlying detection scheme. The block diagram of a energy detector is shown in Fig. 1. The input band pass filter removes the out of band signals by selecting the center frequency f_c, and the bandwidth of interest W. We assume the secondary users have this information in order to perform spectrum sensing. After the signal is digitized by an analog to digital converter (ADC), a simple square and average device is used to estimate the received signal energy. Without loss of generality, we assume the input signal to the energy detector is real. The estimated energy, u, is then compared with a threshold, λ, to decide if a signal is present (H_1) or not (H_0).

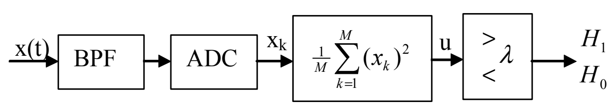

Fig. 1. Block diagram of a energy detector

The threshold can be calculated based on two principles: constant false alarm rate (CFAR) and constant detection rate (CDR) [4]. We will review CFAR and CDR principles in Section II. In both CFAR and CDR cases, the noise power[1] is needed to determine the threshold. In [4] and [5], it is assumed the exact noise variance is known and can be used to calculate the threshold. In practice, however, this information is rarely available. We can only estimate the noise power. A simple but reasonable method is to treat the estimate of noise power as the true noise power and calculate the threshold used in energy detection accordingly. However, to use this approach effectively, two questions need to be answered: 1) Does this simplified method result in the same detection performance? 2) If the result is different, how can we model the difference so that the threshold can be correctly set?

To the best of our knowledge, these questions have not been studied in any previous work. In [6], the authors use experimental method to set the threshold used in energy detection, after taking many measurements of the noise power. However, if the noise power fluctuates, which is very likely in practice, it is difficult to use this approach to set the appropriate threshold. In [5] and [7], the authors analyze and simulate the impact of noise uncertainty on the performance of spectrum sensing. They assume the dynamic range of the

[1] In this paper, if not specified, "noise power" and "noise variance" refer to the same physical measure.

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noise power is known. However, they did not show how the dynamic range can be derived or estimated. We believe our analysis is more applicable to the practical spectrum sensing problem: how can we set the threshold based on a real time estimation of the noise power, so that we can still guarantee the target probability of detection, or the probability of false alarm?

The rest of the paper is organized as follows. In Section II, we review the system model of energy detection using binary hypothesis testing, and the CFAR and CDR principles in spectrum sensing. In Section III, we present our model to analyze the expected performance of energy detection based on estimated noise variance. Section III also presents some characteristics of our analytical model and how they impact the performance of spectrum sensing. We give a simulation example in Section IV and conclude the paper in Section V with a summary.

II. SYSTEM MODEL

The goal of spectrum sensing is to determine if a licensed band is currently used by its primary owner or not. This can be formulated into a binary hypothesis testing problem [5]:

$$x(k) = \begin{cases} n(k), & H_0(vacant) \\ s(k)+n(k), & H_1(occupied) \end{cases}, \tag{1}$$

where the primary user's signal, the noise, and the received signal are denoted by *s(k)*, *n(k)*, and *x(k)* respectively. The noise is assumed to be iid random process of zero mean and variance of σ_n^2, whereas the signal is also assumed to be iid random process of zero mean and variance of σ_s^2. The signal to noise ratio is defined as the ratio of the signal variance to the noise variance

$$SNR = \sigma_s^2 / \sigma_n^2 . \tag{2}$$

The test statistic generated from the energy detector as shown in Fig. 1 is

$$u = \frac{1}{M}\sum_{k=1}^{M}(x_k)^2 \tag{3}$$

Under both hypotheses H_0 and H_1, the test statistic u is a random variable whose probability density function (PDF) is chi-square distributed. When M is sufficiently large, we can approximate the PDF using Gaussian distribution:

$$f_U(u) \sim \begin{cases} N(\sigma_n^2, 2\sigma_n^4/M), & under\ H_0 \\ N(\sigma_t^2, 2\sigma_t^4/M), & under\ H_1 \end{cases}, \tag{4}$$

where σ_t^2 is the total variance of signal plus noise, i.e.,

$$\sigma_t^2 = \sigma_n^2 + \sigma_s^2 = \sigma_n^2(1+SNR) . \tag{5}$$

For a given threshold λ, the probability of false alarm is given by the Q function:

$$P_{fa} = prob(u > \lambda \mid H_0) = Q\left(\frac{\lambda - \sigma_n^2}{\sigma_n^2/\sqrt{M/2}}\right) . \tag{6}$$

If the required probability of false alarm rate (P_{fa}) is predetermined, the threshold (λ_{fa}) can be set accordingly by

$$\lambda_{fa} = \sigma_n^2\left(1 + \frac{Q^{-1}(p_{fa})}{\sqrt{M/2}}\right) . \tag{7}$$

Similarly, under hypothesis H_1, we can derive the threshold in order to achieve a target probability of detection at the required signal level or SNR:

$$\lambda_d = \sigma_n^2(1+SNR)\left(1 + \frac{Q^{-1}(p_d)}{\sqrt{M/2}}\right) . \tag{8}$$

If the cognitive radio network is required to guarantee a reuse probability of the unused spectrum, the probability of false alarm is fixed to a small value (e.g., 5%) and the detection probability should be maximized as much as possible. This is referred to as constant false alarm rate (CFAR) principle [4]. On the other hand, if the cognitive radio is required to guarantee a non-interference probability to the incumbent systems, the probability of detection should be fixed to a high value (e.g., 95%) and the probability of false alarm should be minimized as much as possible. This requirement is referred to as the constant detection rate (CDR) principle [4]. It can be seen from (7) and (8) that the derivation of the threshold values for CFAR and CDR are similar, so the analytical results derived by assuming CFAR based detection can be applied to CDR based detection with minor modifications and vice versa.

III. EXPECTED ENERGY DETECTOR PERFORMANCE USING ESTIMATED NOISE VARIANCE

In Section II, we notice that the threshold values in (7) and (8) are derived from the exact noise variance σ_n^2. However, it is very difficult to assume the exact knowledge of noise variance will be available in practice. The total noise consists of thermal noise, receiver noise, and environmental noise, which can vary significantly from time to time. In this paper, instead, we assume only an estimated noise variance $\hat{\sigma}_n^2$ is available to calculate the thresholds used in energy detection.

The practicality of performing real time noise estimation can be justified with two examples. In one example, we can assume the spectrum regulators still wants to reserve certain channel for special applications, and secondary users are never allowed to access this channel. In addition, this special channel is rarely used and therefore can serve the purpose of noise estimation. In United States, for example, channel 37 (from 608 to 614 MHz) is reserved for radioastronomy and is used in very few occasions. In another example such as the detection of DTV signal in US, it is helpful to detect the DTV pilot signal which is a distinct narrow band spectral feature. After performing the power spectral density (PSD) estimation on the received signal, under low SNR assumptions, the noise variance can be estimated from some frequency bin not corresponding to the pilot frequency. In both examples, a threshold can be derived from the estimated noise variance based on CFAR or CDR requirement. Then the threshold is compared with the detected energy from the channel of interest or from the known pilot frequency bin, in order to determine whether a signal is present.

In this section, we want to find out the answers to these two important questions: 1) can we simply replace the exact noise variance σ_n^2 in (7) and (8) with the estimated noise variance $\hat{\sigma}_n^2$, i.e.,

$$\hat{\lambda}_{fa} = \hat{\sigma}_n^2 (1 + \frac{Q^{-1}(P_{fa})}{\sqrt{M/2}}), \; or \; \hat{\lambda}_d = \hat{\sigma}_n^2 (1 + SNR)(1 + \frac{Q^{-1}(P_d)}{\sqrt{M/2}}) \quad (9)$$

and still achieve, at least asymptotically, the target probability of false alarm or detection? 2) If not, what kind of performance in terms realistic detection probability and realistic false alarm probability can be expected, when the threshold is selected using (9)?

A. Asymptotic Analysis

Denote Y as the estimated noise variance from the reference channel (Ch_i) known to be vacant using energy detection method. Denote X as the energy detection result from the channel of interest (Ch_j). We assume the number of samples used to estimate noise variance is the same as the number of samples used to perform spectrum sensing, denoted by M. This assumption helps us to give concise expressions and insightful analysis in this paper. In practice, this assumption might not hold strictly. However, the results shown in this paper can be readily generalized to reflect the changes.

First, we consider CFAR based detection. Since we assume H_0 is true when deriving the probability of false alarm, X and Y should have the same PDF from (4) if M is sufficiently large:

$$X, Y \sim N(\sigma_n^2, 2\sigma_n^4 / M). \quad (10)$$

It can be seen that when M approaches infinity, both X and Y converges to the true noise variance σ_n^2. However, since we do not know the true mean of X (i.e., σ_n^2), we have to use Y (i.e., $\hat{\sigma}_n^2$) as the estimate of the mean of X instead. Therefore the variance of X should be calculated by

$$\begin{aligned} \mathrm{var}(X \mid \overline{X} = Y) &= E\{(X-Y)^2\} = E\{X^2 + Y^2 - 2XY\} \\ &= \sigma_x^2 + \overline{X}^2 + \sigma_y^2 + \overline{Y}^2 - 2\overline{XY} = 2\sigma_n^2 \approx 2Y^2 = 2\hat{\sigma}_n^2 \end{aligned} \quad (11)$$

So the PDF of X should actually be approximated by

$$X \sim N(\hat{\sigma}_n^2, 4\hat{\sigma}_n^4 / M), \quad (12)$$

instead of

$$X \sim N(\hat{\sigma}_n^2, 2\hat{\sigma}_n^4 / M). \quad (13)$$

When deriving (11), we include the condition of $\overline{X} = Y$ to reflect the fact that we are using an estimate of the real mean value. From (12) and (13), we can conclude that simply using (9) to calculate the estimated threshold, will never achieve the target probability of false alarm no matter how many samples are averaged. Asymptotically, since the real variance is approximately twice of the original variance (12), we have

$$\overline{P}_{fa}(\hat{\lambda}_{fa} \mid M \to \infty) = Q(\frac{Q^{-1}(P_{fa})}{\sqrt{2}}) . \quad (14)$$

The notations in (14), together with other notations to be used in the rest of the paper, are described in Table I. We only include the notations used in CFAR based detection to make this paper more concise. The notations used in the CDR based detection have *d* in the subscript and should have the similar definitions as their counterparts used in CFAR based detection.

The real probability of false alarm ($\overline{P}_{fa}$) is a function of estimated threshold, whereas the estimated threshold ($\hat{\lambda}_{fa}$) is determined by the number of samples being averaged (M). Therefore, (14) is written in a manner to emphasize these relationships.

TABLE I. SUMMARY OF NOTATIONS

Notation	Definition
P_{fa}	Target probability of false alarm
λ_{fa}	Threshold calculated using exact noise variance for CFAR detector: $\lambda_{fa} = \sigma_n^2(1+\frac{Q^{-1}(P_{fa})}{\sqrt{M/2}})$
$\hat{\lambda}_{fa}$	Threshold calculated using estimated noise variance for CFAR detector: $\hat{\lambda}_{fa} = \hat{\sigma}_n^2(1+\frac{Q^{-1}(P_{fa})}{\sqrt{M/2}})$
$\overline{P}_{fa}$	The expected probability of false alarm results from estimated threshold $\hat{\lambda}_{fa}$
Q_{fa}	The target system level probability of false alarm using OR rule based cooperative sensing
$\overline{Q}_{fa}$	The expected system level probability of false alarm using OR rule based cooperative sensing

Using the derivation above on CDR based detection, a similar formula can be given to asymptotically predict the real probability of detection ($\overline{P}_d$) when the threshold is calculated using (9):

$$\overline{P}_d(\hat{\lambda}_d \mid M \to \infty) = Q(\frac{Q^{-1}(P_d)}{\sqrt{2}}) \quad (15)$$

When M is not large enough, the asymptotic results in (14) and (15) cannot accurately predict the expected probabilities of the energy detection, with the threshold calculated from estimated noise variance by (9). Therefore, an analytical model is needed for the cases when M is moderate.

B. Analytical Model

Using the same notations of X and Y as in the asymptotic analysis, the estimated noise variance (Y) is a random variable itself, therefore the probability of false alarm or detection is conditioned on one observation of the random variable, e.g., y.

For example, if CFAR based detection is used, the probability of false alarm can be written as

$$prob\{X > \hat{\lambda}_{fa}\} = Q\left(\frac{y(1+Q^{-1}(P_{fa})/\sqrt{M/2})-\mu_x}{\sigma_x}\right), \quad (16)$$

where $\hat{\lambda}_{fa}$ is the threshold value calculated from (9), and μ_x and σ_x denote the mean and standard deviation of X. Since X and Y have the PDF as shown in (11), we can derive the expected probability of false alarm by integrating (16) over the PDF of Y. Once this integration is derived[2], the expected probability of false alarm is given by

$$\overline{P}_{fa} \approx \int_{-\infty}^{\infty} Q(x+t(1+x/\sqrt{M/2}))\frac{1}{\sqrt{2\pi}}e^{-t^2/2}dt \equiv Q_a(x,M). \quad (17)$$

We use $Q_a(x, M)$ to denote the integration to compute the expected probability, where $x = Q^{-1}(P_{fa})$ and M is the number of samples being averaged. In (17), t is defined by $t = (y-\mu_x)/\sigma_x$. The closed-form expression of $\overline{P}_{fa}$ is not available, however, numerical results can be obtained. In Fig. 2, we display $\overline{P}_{fa}$ as a function of M for $x = Q^{-1}(0.01)$. We can observe that when M goes to infinity, $\overline{P}_{fa}$ converges to $Q(x/\sqrt{2})$. This observation matches our asymptotic analysis in the previous subsection (14). In addition, when the exact noise variance is known, t becomes zero in (17). Evaluating (17) shows $\overline{P}_{fa}$ equals P_{fa} as expected. However, when t is not zero, no matter how many samples are averaged in the energy detection (M→∞), $\overline{P}_{fa}$ can never converge to P_{fa}.

If CDR based detection is used, the results shown above also apply except $\overline{P}_{fa}$ and P_{fa} are replaced by $\overline{P}_d$ and P_d respectively.

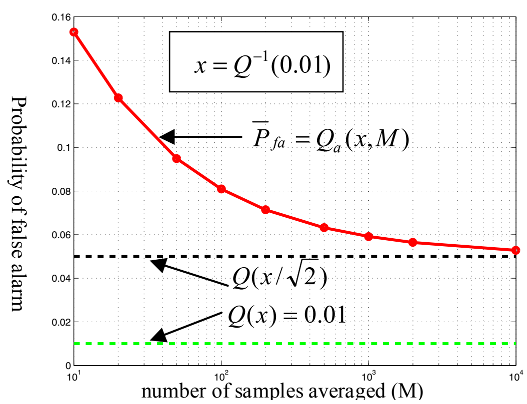

Fig. 2. Characterization of Q_a(x, M) as function of M

C. *Characteristics of the Analytical Model*

We study the characteristics of the analytical model, in particular, the function $Q_a(x, M)$, and then analyze how these characteristics can affect the performance of spectrum sensing.

CLAIM 1: Consider the generalized form of $Q_a(x, M)$, given by

$$Q_a(x,y) = \int_{-\infty}^{\infty} Q(x+t(1+y))\frac{1}{\sqrt{2\pi}}e^{-t^2/2}dt. \quad (18)$$

Then we claim:

1.1 when x>0, $Q_a(x,y) \geq Q_a(x,-1) = Q(x)$

1.2 when x<0, $Q_a(x,y) \leq Q_a(x,-1) = Q(x)$

1.3 when x=0, $Q_a(x,y) = Q(x) = 0.5$

PROOF: Take the derivative of $Q_a(x, y)$ with respect to y, and claims 1.1 and 1.2 can be proved by evaluating the derivative. Claim 1.3 can simply be proved by plugging in x=0 into (17).

CLAIM 2: Consider again $Q_a(x,y)$ as shown in (18), then,

2.1 when x>0, $Q_a(x, y)$ is monotonically decreasing with respect to y when y>0.

2.2 when x<0, $Q_a(x, y)$ is monotonically increasing with respect to y when -1<y<0.

PROOF: Claim 2 can be proved by evaluating the derivative of $Q_a(x, y)$ as in the proof of claim 1.

CLAIM 3: Consider $Q_a(x, M)$ as shown in (17), then,

3.1 when x>0, $Q_a(x, M) > Q_a(x, \infty) > Q(x)$

3.2 when x<0, $Q_a(x, \infty) < Q_a(x, M) < Q(x)$

3.3 given $M_1>M_2$, $Q_a(x, M_1) < Q_a(x, M_2)$

PROOF: We notice that $Q_a(x, M)$ is a special case of $Q_a(x, y)$ where $y = x/\sqrt{M/2}$. In this case, x and y should have the same sign, therefore claim 3 can be proved by using claim 1 and claim 2.

Fig. 3 displays two numerical examples of $Q_a(x, y)$, where x takes on a positive value and a negative value respectively. The first two claims are clearly illustrated in Fig. 3, whereas claim 3 can be observed from Fig. 2.

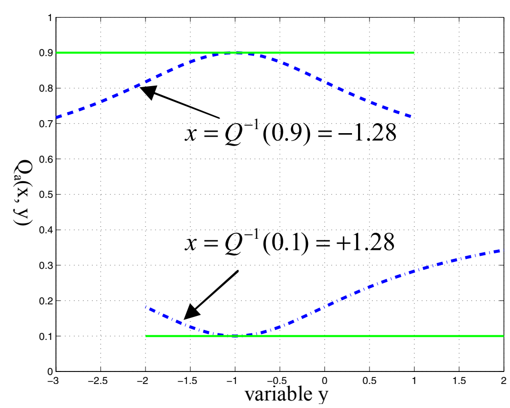

Fig. 3 Characterization of Q_a(x, y) as a function of y, using two different valued x

D. *Applications to Spectrum Sensing*

We consider first the non-cooperative spectrum sensing scenario, i.e., only one radio performs the energy detection. Generally, the probability of detection is required to be high

[2] We skip the derivation of (17) and the details in proving of the claims in Section C. Instead of showing algebraic details, we want to emphasize the numerical results and how they can be used in the spectrum sensing.

(e.g., P_d>90%), whereas the probability of false alarm needs to be low (e.g., P_{fa}<10%). When CFAR based detection is used, then $x = Q^{-1}(P_{fa}) > 0$ holds in this case. Therefore if the threshold is calculated using (9) with estimated noise variance, the expected probability of false alarm will be higher than the target false alarm rate (claim 1). It helps to increase the number of samples being averaged (M), however the target false alarm rate cannot be achieved. In order to achieve the target false alarm rate in practice, the threshold needs to be raised. On the other hand, the detection probability will decrease due to the raised threshold. Since we don't have closed-form expressions for (17) and (18), a numerical method is needed to calculate the correct threshold.

When CDR based detection is used, then $x = Q^{-1}(P_d) < 0$ holds in this case. Based on claim 1 again, $Q_a(x, M)$ equals Q(x) only when $x = -\sqrt{M}$ is satisfied. When M=10, the detection probability needs to be

$$P_d = Q(x) = Q(-\sqrt{10}) = 0.9992 . \quad (19)$$

When M is larger, the probability of detection is even greater than 0.9992. It is not likely an individual radio can achieve such high detection rate without experiencing high false alarm rate. Therefore, using (9) to set the threshold cannot achieve the target probability of detection in general. In addition, based on claim 3, the expected detection probability decreases when we increase M. Increasing M, by averaging more samples, can nonetheless help decrease the false alarm rate. We need to decrease the threshold calculated by (9) to meet the target detection probability, while keeping the false alarm rate at an acceptable level.

We then consider the cooperative spectrum sensing that uses OR fusion rule [6]. In cooperative sensing, the false alarm rate or detection probability is measured at the system level. In OR fusion rule, the system level false alarm rate (Q_{fa}) and the system level detection probability (Q_d) are determined by,

$$\begin{aligned} Q_{fa} &= 1-(1-P_{fa})^N \\ Q_d &= 1-(1-P_d)^N \end{aligned} , \quad (20)$$

where P_{fa} and P_d are, respectively, the radio level probability of false alarm and detection, and N is the number of radios participate in the cooperative sensing. In (20), the measurements taken by each radio are assumed to be independent from each other. Table II shows two cooperative sensing examples based on CFAR principle, using target false alarm rates of 10% and 1% respectively. The number of radios participate in the sensing operation is 10. In both cases, we calculate the threshold using (9) with the estimated noise variance, where P_{fa} is calculated based on (20) using the given Q_{fa}. Then we calculate the expected probabilities of false alarm at the radio level (M is assumed to be infinity) and the system level using (17) and (20). It can be seen that $\overline{Q}_{fa}$ and Q_{fa} can differ by a factor of 13. Therefore the precision of the threshold is very important in cooperative sensing using CFAR based detection.

TABLE II. EXPECTED FALSE ALARM RATE VS. TARGET FALSE ALARM RATE

Q_{fa}	P_{fa}	$\overline{P}_{fa}$	$\overline{Q}_{fa}$	$\overline{Q}_{fa}/Q_{fa}$
0.1	0.0105	0.0513	0.41	4.1
0.01	0.001	0.0145	0.136	13.6

If CDR based detection is used in cooperative spectrum sensing, we notice the radio level detection probability can be less than 0.5 when N is moderately large. For example, if Q_d is required to be 0.95, then P_d is only required to be 0.26 if 10 radios are used in the cooperative spectrum sensing using OR fusion rule. Then, different from the individual sensing scenario, we note $x = Q^{-1}(P_d) > 0$ holds instead. Based on claim 3, $\overline{P}_d$ is expected to be higher than P_d, and so is $\overline{Q}_d$ compared with Q_d. Since we are only required to achieve the detection probability of Q_d, the threshold calculated using (9) is too conservative. We can therefore raise the threshold, and the raised threshold can help minimize the probability of false alarm.

IV. NEW THRESHOLD AND SIMULATION EXAMPLE

We define a new variable u that satisfies

$$Q_a(u, M) = P_d . \quad (21)$$

Then the new threshold, given by,

$$\lambda_{new} = \hat{\sigma}_n^2 (1+SNR)(1+\frac{u}{\sqrt{M/2}}) , \quad (22)$$

can achieve the target probability of detection. We want to express the new threshold in a similar form as used in the original formula (9). To show the effectiveness of the new threshold, we give two simulation examples of cooperative sensing using CDR principle. In both simulations, we set Q_d to 95% when SNR is -6dB. We change the number of radios participate in the cooperative sensing between two simulations. We use N=10 radios in one simulation, and N=25 radios in the other. The simulation compares the cooperative sensing results using the old threshold resulted from (9) and the new threshold resulted from (20) and (21). The simulation results are displayed in Fig. 4. In both simulations, we can see Q_d is kept at the target 95% using the new threshold. As pointed in Section III, the old threshold is set too conservatively; therefore the resulted Q_d is in fact greater than 95%. In addition, the Q_{fa} resulted from the new threshold is lower than the Q_{fa} using the old threshold. Lower Q_{fa} indicates more spectrum sharing opportunities can be identified using the new threshold setting method.

When we compare the system false alarm rates Q_{fa} between the two simulations, we notice when 25 radios are used in the cooperative sensing, the resulted Q_{fa} is lower than the Q_{fa}

resulted from 10 radios cooperation. In addition, the difference between the Q_{fa} resulted from the old threshold and the Q_{fa} resulted from the new threshold is larger in the example of 25 radios cooperation than in the example of 10 radios cooperation. This is because when the number of radios increases, the required detection probability at the radio level (P_d) decreases if the system level detection probability is kept the same. To simplify the analysis, we assume M approaches infinity, then the expected probability of detection $\overline{P}_d$ should be $Q(Q^{-1}(P_d)/\sqrt{2})$, when the old threshold from (9) is used. The function $Q^{-1}(x)$ is monotonically decreasing with respect to x. This suggests when P_d decreases due to the increasing number of radios in cooperative sensing, the difference between $\overline{P}_d = Q(Q^{-1}(P_d)/\sqrt{2})$ and $P_d = Q(Q^{-1}(P_d))$ becomes more significant. Another example can be found in Table II, although P_{fa} is considered in that example. We have pointed out, in Section III, that the old threshold from (9) is set too conservatively, which is shown in both simulation examples. The additional analysis in this section shows the more radios participate in the cooperative sensing, the more conservative the threshold calculated from (9) will be. Therefore, using the new threshold setting method from (21) and (22), we can facilitate more spectrum sharing opportunities by minimizing the system false alarm probability.

V. CONCLUSION

In this paper, we present a new analytical model to evaluate the expected probability of false alarm or detection for spectrum sensing using energy detection method. More specifically, we assume only an estimated noise variance is used to set the threshold in energy detection, instead of the exact noise variance. By studying the characteristics of this analytical model, we show how they can affect the performance of spectrum sensing. The analytical results are verified using numerical and simulation examples. Based on the analytical model, we derive a method to determine the threshold that can achieve the desired probability of detection or false alarm using estimated noise variance. Through a CDR based spectrum sensing example, we show our threshold setting method can facilitate more spectrum sharing opportunities when combined with cooperative spectrum sensing, where the probability of false alarm is minimized and the desired probability of detection of the primary systems is guaranteed.

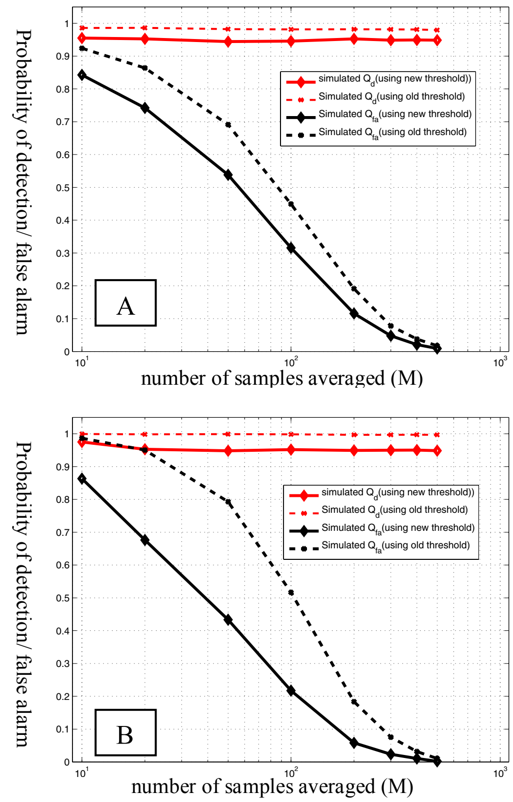

Fig. 4. Simulation results comparing two threshold setting methods using cooperative spectrum sensing among 10 radios (A) and 25 radios (B)

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