# Optimal Cooperative Sensing and Its Robustness to Decoding Errors

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Abstract—Based on the Neyman-Pearson theorem, the optimal cooperative sensing for distributed sensors with time independent signals is derived. It is shown that the optimal scheme is simply a linearly combined energy detection and the combining coefficient is a simple function of the signal to noise ratio (SNR). To reduce the required information at the fusion center and simplify the decision-making process and threshold setting, an approximated optimal cooperative sensing is proposed and compared with some other sub-optimal methods. Finally the impact of decoding error in the reported results is analyzed. Based on the closed-form expression for the performance, it is proved that the impact of decoding error is equivalent to the reduction of sensing time. Simulations are provided to support the results.

# I. INTRODUCTION

In recent years, cognitive radio is gradually moving from imagination to reality. Spectrum sensing in cognitive radio has a few special challenges, among others, as follows. (1) A cognitive radio may need to sense the primary signal at very low signal to noise ratio (SNR). (2) Propagation channel uncertainty makes the spectrum sensing difficult. The unknown time dispersive channel turns most coherent detections unreliable. (3) It is hard to synchronize the received signal with the primary signal in time and frequency. This will cause some methods like preamble/pilot based detections less effective. (4) The noise level may change with time and location, which yields the noise power uncertainty issue for detection [1], [2]. (5) The noise may not be white, which will affect many methods with white noise assumption.

Although there have been many methods (refer to [3], [4], [5], [6] and the references therein), most of them may not work well in such a hostile radio environment. It is extremely difficult for a single sensor to meet the sensing requirement in some cognitive radio applications. Cooperative sensing, i.e., multiple sensors sensing the common signal in a coordinated approach, is proved to be more reliable or have a better performance than single sensor sensing [7], [8], [9], [10], [11], [12], [13], [14], [15]. As a result, cooperative sensing for cognitive radio has received substantial concern in recent years.

In this paper, we first derive the optimal cooperative sensing for distributed sensors with time independent signals. It is proved that, if the sensors are distributed far apart and their signals are independent in time, the optimal scheme is simply a linearly combined energy detection and the combining coefficient is a simple function of the signal to noise ratio (SNR). The major difficulty of the optimal scheme is that its decision and threshold are related to the SNRs of the sensors. To avoid this difficulty we then consider sub-optimal cooperative methods. Especially a new method called approximated optimal cooperative sensing is proposed. Finally we analyze

the impact of decoding error in the reported results to the detection performance. Based on the closed-form expression of the performances we find the ultimate constraint of decoding errors. It is proved that the impact of decoding error is equivalent to the reduction of sensing time. Simulations are provided to support the results.

#### II. SYSTEM MODEL

We consider a cognitive radio network with  $M \geq 1$  secondary users, which share the same spectrum (one or multiple bands) with primary users. The network coordinates some or all secondary users to cooperatively sense the primary signals. It is assumed that a centralized unit is available for processing the signals from all the sensors. There are two other similar scenarios: (1) multi-antenna sensing, if we treat one antenna as a sensor; (2) multiple time slot sensing: the sensing is done in M equal length time slots, where each time slot is treated as a sensor. In the following, we consider a system model which can be applied to all the three scenarios.

There are two hypotheses:  $\mathcal{H}_0$ , signal absent; and  $\mathcal{H}_1$ , signal present. The received signal at sensor i is given by  $\mathcal{H}_0$ :  $x_i(n) = \eta_i(n)$  and  $\mathcal{H}_1$ :  $x_i(n) = s_i(n) + \eta_i(n)$ ,  $i = 1, \ldots, M$ , where  $\eta_i(n)$  is the noise. At hypothesis  $\mathcal{H}_1$ ,  $s_i(n)$  is the received source signal at antenna/receiver i. Note that  $s_i(n)$  is the transmitted primary signal after going through the fading and multipath propagation channel, and also time delay. That is,  $s_i(n)$  can be written as

$$s_i(n) = \sum_{k=1}^{N_p} \sum_{l=0}^{q_{ik}} h_{ik}(l) \tilde{s}_k(n - l - \tau_{ik}), \tag{1}$$

where  $N_p$  is the number of primary signals,  $\tilde{s}_k(n)$  stands for the transmitted primary signal from primary user or antenna k,  $h_{ik}(l)$  denotes the propagation channel coefficient from the kth primary user or antenna to the ith receiver/antenna,  $q_{ik}$  is the channel order for  $h_{ik}$ ,  $\tau_{ik}$  is the relative time delay from the primary user k to sensor i.

We form  $M \times 1$  vectors from the signals of M sensors as follows:  $\mathbf{x}(n) = \begin{bmatrix} x_1(n) & \cdots & x_M(n) \end{bmatrix}^T$ ,  $\mathbf{s}(n) = \begin{bmatrix} s_1(n) & \cdots & s_M(n) \end{bmatrix}^T$ ,  $\mathbf{\eta}(n) = \begin{bmatrix} \eta_1(n) & \cdots & \eta_M(n) \end{bmatrix}^T$ . The hypothesis testing problem based on N signal samples is then equivalent to

$$\mathcal{H}_0: \mathbf{x}(n) = \boldsymbol{\eta}(n)$$

$$\mathcal{H}_1: \mathbf{x}(n) = \mathbf{s}(n) + \boldsymbol{\eta}(n), \quad n = 0, \dots, N - 1.$$
 (2)

The probability of detection,  $P_d$ , and probability of false alarm,  $P_{fa}$ , are defined as follows:  $P_d = P(\mathcal{H}_1|\mathcal{H}_1)$  and  $P_{fa} = P(\mathcal{H}_1|\mathcal{H}_0)$ . In general a sensing algorithm is said to

be "optimal" if it achieves the highest  $P_d$  for a given  $P_{fa}$  with a fixed number of samples.

# III. OPTIMAL COOPERATIVE SENSING

Based on the Neyman-Pearson theorem [16], [17], for a given  $P_{fa}$ , the test statistic that maximizes the  $P_d$  is the likelihood ratio test (LRT). Hence the optimal cooperative sensing (OCS) has the test statistic as

$$T_{OCS}(\mathbf{x}) = p(\mathbf{x}|\mathcal{H}_1)/p(\mathbf{x}|\mathcal{H}_0)$$
 (3)

where  $p(\cdot)$  denotes the probability density function (PDF), and the vector  $\mathbf{x}$  represents the aggregation of  $\mathbf{x}(n)$ ,  $n=0,1,\cdots,N-1$ .

There are two major difficulties in using the OCS: (1) it needs the exact distribution of x, which is related to the source signal distribution, the wireless channels, and the noise distribution; (2) it may needs the raw data from all sensors, which is very expensive for practical applications.

In some situations, the signal samples are independent in time, that is,  $E(s_i(n)s_i(m)) = 0$ , for  $n \neq m$ . The PDFs in OCS can be decoupled as

$$p(\mathbf{x}|\mathcal{H}_1) = \prod_{n=0}^{N-1} p(\mathbf{x}(n)|\mathcal{H}_1), \ p(\mathbf{x}|\mathcal{H}_0) = \prod_{n=0}^{N-1} p(\mathbf{x}(n)|\mathcal{H}_0).$$
(4)

If we further assume that the noise and signal samples have Gaussian distribution, i.e.,  $\eta(n) \sim \mathcal{N}(\mathbf{0}, \mathbf{R}_{\eta})$  and  $\mathbf{s}(n) \sim \mathcal{N}(\mathbf{0}, \mathbf{R}_{s})$ , where

$$\mathbf{R}_{s} = \mathbf{E}(\mathbf{s}(n)\mathbf{s}^{T}(n)), \ \mathbf{R}_{n} = \mathbf{E}(\boldsymbol{\eta}(n)\boldsymbol{\eta}^{T}(n)),$$
 (5)

the OCS can be obtained explicitly as

$$\log T_{OCS} = \frac{1}{N} \sum_{n=0}^{N-1} \mathbf{x}^T(n) \mathbf{R}_{\eta}^{-1} \mathbf{R}_s (\mathbf{R}_s + \mathbf{R}_{\eta})^{-1} \mathbf{x}(n). \quad (6)$$

Note that in general the cross-correlations among the signals from different sensors are used in the detection here. It means that the fusion center needs the raw data from all sensors, if the signals from different sensors are correlated in space. The reporting of the raw data is very expensive for practical applications.

If the sensors are distributed at different locations and far apart, the primary signal will very likely arrive at different sensors at different times. That is, in (1)  $\tau_{ik}$  may be different for different i. In a large size network like a 802.22 cell (typically with radius 30km), the distance differences of different sensors to the primary user could be as large as several kilometers. Therefore, the relative time delays  $\tau_{ik}$  can be as large as 20 samples or more. If the delays are different, the signals at the sensors will be independent in space.

For distributed sensors, their noises are independent in space. If we aim for sensing at very low SNR, the received signal at a sensor will be dominated by noise. Hence even if the primary signals at different sensors may be weakly correlated, the whole signals (primary signals plus noises) can be treated approximately as independent in space at low SNR. So, in

the following, we further assume that  $E(s_i(n)s_j(n)) = 0$ , for  $i \neq j$ .

Under the assumptions we have

$$\mathbf{R}_{\eta} = \operatorname{diag}(\sigma_{\eta,1}^2, \cdots, \sigma_{\eta,M}^2), \ \mathbf{R}_s = \operatorname{diag}(\sigma_{s,1}^2, \cdots, \sigma_{s,M}^2)$$
 (7)

where  $\sigma_{\eta,i}^2 = \mathrm{E}(|\eta_i(n)|^2)$  and  $\sigma_{s,i}^2 = \mathrm{E}(|s_i(n)|^2)$ . Under the assumptions, we can express the OCS equivalently as

$$\log T_{OCS} = \frac{1}{N} \sum_{n=0}^{N-1} \sum_{i=1}^{M} \frac{\sigma_{s,i}^2}{\sigma_{\eta,i}^2(\sigma_{s,i}^2 + \sigma_{\eta,i}^2)} |x_i(n)|^2$$

$$= \sum_{i=1}^{M} \frac{\gamma_i}{1 + \gamma_i} T_{ED,i}$$
(8)

where 
$$\gamma_i = \sigma_{s,i}^2/\sigma_{\eta,i}^2$$
, and  $T_{ED,i} = \frac{1}{N\sigma_{\eta,i}^2} \sum_{n=0}^{N-1} |x_i(n)|^2$ .

Note that  $T_{ED,i}$  is the normalized energy at sensor i. The OCS is simply a linearly combined (LC) cooperative sensing  $^1$ . Thus there are three assertions for cooperative sensing by distributed sensors with time independent signals: (1) the optimal cooperative sensing is the linearly combined energy detection; (2) the combining coefficient is a simple function of the SNR at the sensor; (3) a sensor only needs to report its normalized energy and SNR to the fusion center, and no raw data transmission is necessary.

If the signals are time dependent, the derivation of the OCS becomes much more difficult. Furthermore, the information of correlation among the signal samples is required. There have been methods to exploit the time and space correlations of the signals in a multi-antenna system [3]. If the raw data from all sensors are sent to the fusion center, the sensor network may be treated as a single multi-antenna system (virtual multi-antenna system). If the fusion center does not have the raw data, how to fully use the time and space correlations is still an open question, though there have been some sub-optimal methods [14].

### IV. SUB-OPTIMAL METHODS AND COMPARISONS

A major difficulty in implementing the method is that the fusion center needs to know the SNR at each user. Also the decision and threshold are related to the SNR's, which means that the detection process changes dynamically with the signal strength and noise power.

If  $N_p=1$ , the propagation channels are flat-fading  $(q_{ik}=0,\forall i,k)$ , and  $\tau_{ik}=0,\forall i,k$ , the signal at different antennas can be coherently combined first and then the energy detection is used [12], [10]. The method is called maximum ratio combined (MRC) cooperative energy detection:  $T_{MRC}=\frac{1}{N}\sum_{n=0}^{N-1}|\sum_{i=1}^{M}h_ix_i(n)|^2$ . It is optimal if the noise powers at different sensors are equal. Note that the MRC needs the raw data from all sensors and also the channel information.

We have proved that the OCS is actually a LC scheme. It is natural to also consider other LC schemes. In general, a LC

 $<sup>^{1}</sup>$ By pre-assuming that each sensor uses the energy detection and M is even, Ma et al. proved in reference [10] that the optimal combination of energies from all sensors is equation (8). Here it is further proved that (8) is the optimal cooperative sensing in general sense.

scheme simply sums the weighted energy values to obtain the following test statistic

$$T_{LC} = \sum_{i=1}^{M} g_i T_{ED,i}$$
 (9)

where  $g_i$  is the combining coefficient with  $g_i \geq 0$ . If we allow the combining coefficients to depend on the SNRs of sensors, we know that the *optimal sensing* should choose  $g_i = \gamma_i/(1+\gamma_i)$ . So the problem is how we design a LC scheme that does not need the SNR information or only uses partially available SNR information, while its performance does not degrade much.

One such scheme is the equal gain combine (EGC) [9], [10], [12], [13], [3], i.e.,  $g_i = 1/M$  for all i:  $T_{EGC} = \frac{1}{M} \sum_{i=1}^{M} T_{ED,i}$ . EGC totally ignores the differences of sensors

We may estimate the optimal weights using the real-time signals. If N is large, we know that  $T_{ED,i}\approx 1+\gamma_i$  at hypothesis  $\mathcal{H}_1$ . Hence we have an estimation  $\gamma_i=T_{ED,i}-1$  at hypothesis  $\mathcal{H}_1$ . However, the estimation is definitely not valid at hypothesis  $\mathcal{H}_0$ . At hypothesis  $\mathcal{H}_0$ ,  $T_{ED,i}\approx 1$ . If our target is to obtain a reliable detection at SNR level  $\hat{\gamma}$ , a reasonable estimation is  $\gamma_i=\max(T_{ED,i}-1,\hat{\gamma})$ , which is better at  $\mathcal{H}_0$ . If we do have some information of the SNRs, we can replace  $\hat{\gamma}$  by another value constructed from that information, for example, the average SNR, in the above estimation. We call this method approximated OCS (AOCS), which has the test statistic:

$$T_{AOCS} = \sum_{i=1}^{M} \frac{\max(T_{ED,i} - 1, \hat{\gamma})}{1 + \max(T_{ED,i} - 1, \hat{\gamma})} T_{ED,i}.$$
 (10)

Please note that it is not in the LC class.

If the normalized signal energies at different sensors have large differences, a natural way is to choose the largest normalized energy for detection. We call this maximum normalized energy (MNE) cooperative sensing. The test statistic is:  $T_{MNE} = \max_{1 \leq i \leq M} T_{ED,i}$ . Note that this is different from the method that uses the known sensor with the largest normalized signal energies. The largest normalized energy may not always be at the same sensor due to the dynamic changes of wireless channels. The method is equivalent to the "OR decision rule" [7], [11].

When we evaluate a sensing scheme, we need to consider a couple of things. Among others we should consider the following: (1) the detection probabilities:  $P_d$  and  $P_{fa}$ ; (2) what information is required in the method? (3) how robust the method is to various impairments?

Before investigating the detection probabilities, let us first look at the required information for the methods.

 AOCS, EGC and MNE: the fusion center does not use any information of the signal and noise of the sensors; but each sensor needs to compute its normalized energy that requires the exact noise power of itself.

- MRC: the fusion center needs the raw data, the channel information and also the noise power of all sensors; but each sensor does not need any information.
- Optimal cooperative sensing: the fusion center needs the SNRs of all sensors, and each sensor needs to compute its normalized energy that requires the exact noise power of itself.

Obviously, MRC needs the most amount of information, especially the raw data. But we know that it's performance should not be better than the OCS at the case of distributed sensors with different time delays and noise powers. Hence in the following we will not consider this method. AOCS, EGC and MNE need the least amount of information.

In general, based on the central limit theorem the distribution of the  $T_{LC}$  can be well approximated by

$$\mathcal{H}_0: T_{LC} \sim \mathcal{N}\left(\sum_{i=1}^{M} g_i, \sum_{i=1}^{M} 2g_i^2/N\right)$$
 (11)

$$\mathcal{H}_1: T_{LC} \sim \mathcal{N}\left(\sum_{i=1}^{M} g_i(1+\gamma_i), \sum_{i=1}^{M} 2(1+\gamma_i)^2 g_i^2/N\right).$$
 (12)

For a given threshold  $\lambda$ , the detection probabilities are

$$P_{fa} = Q\left(\sqrt{\frac{N}{2}} \frac{\lambda - \sum_{i=1}^{M} g_i}{\sqrt{\sum_{i=1}^{M} g_i^2}}\right)$$
(13)

$$P_{d} = Q\left(\sqrt{\frac{N}{2}} \frac{\lambda - \sum_{i=1}^{M} g_{i}(1 + \gamma_{i})}{\sqrt{\sum_{i=1}^{M} (1 + \gamma_{i})^{2} g_{i}^{2}}}\right)$$
(14)

where  $Q(t) = \frac{1}{\sqrt{2\pi}} \int_t^{+\infty} e^{-u^2/2} \mathrm{d}u$ . For given  $P_d$  and  $P_{fa}$ , the number of samples required to achieve the required detection performance is given as

$$N_{LC} = 2 \frac{\left(Q^{-1}(P_d)\sqrt{\sum_{i=1}^{M}(1+\gamma_i)^2g_i^2} - Q^{-1}(P_{fa})\sqrt{\sum_{i=1}^{M}g_i^2}\right)^2}{\left(\sum_{i=1}^{M}g_i\gamma_i\right)^2}.$$
(15)

Based on this, we can get the required numbers of samples for OCS and EGC, respectively.

In general the distribution of  $T_{MNE}$  can be expressed as the production of multiple Q functions (defined above). It is difficult to find a closed-form expression for the distribution of  $T_{AOCS}$ .

#### V. IMPACT OF DECODING ERRORS

Here we only consider the case of sensor sending its normalized energy to the fusion center. Due to the decoding error, the fusion center actually receives the signal  $T_{ED,i}+w_i$  from sensor i, where  $w_i$  is the decoding error. In general,  $w_i$  can be modeled by a random variable with zero-mean Gaussian distribution. Let the variance of  $w_i$  be  $\sigma_0^2$ . Although it is possible to optimize the cooperative scheme by taking the decoding error into consideration like the optimization in [9], here we consider keeping the same cooperative scheme as if

there is no decoding error, because in practice it is hard to know the exact  $\sigma_0^2$ .

Here we take the LC scheme as an example. With decoding error, the actual test statistic for LC becomes

$$T_{LC} = \sum_{i=1}^{M} g_i (T_{ED,i} + w_i).$$
 (16)

The distribution of the  $T_{LC}$  turns to

$$\mathcal{H}_0: T_{LC} \sim \mathcal{N}\left(\sum_{i=1}^{M} g_i, \sum_{i=1}^{M} g_i^2(\frac{2}{N} + \sigma_0^2)\right)$$
 (17)

$$\mathcal{H}_1: T_{LC} \sim \mathcal{N}\left(\sum_{i=1}^{M} g_i(1+\gamma_i), \sum_{i=1}^{M} g_i^2(\frac{2(1+\gamma_i)^2}{N}+\sigma_0^2)\right).$$

For a given threshold  $\lambda$ , the detection probabilities are changed to

$$\hat{P}_{fa} = Q \left( \frac{\lambda - \sum_{i=1}^{M} g_i}{\sqrt{\sum_{i=1}^{M} g_i^2 (\frac{2}{N} + \sigma_0^2)}} \right)$$
(19)

$$\hat{P}_d = Q \left( \frac{\lambda - \sum_{i=1}^M g_i (1 + \gamma_i)}{\sqrt{\sum_{i=1}^M g_i^2 (\frac{2(1 + \gamma_i)^2}{N} + \sigma_0^2)}} \right). \tag{20}$$

Define  $\hat{N} = \frac{N}{1 + \sigma_0^2 N/2}$ . At low SNR case, it can be verified that

$$\hat{P}_{fa} = Q\left(\sqrt{\frac{\hat{N}}{2}} \frac{\lambda - \sum_{i=1}^{M} g_i}{\sqrt{\sum_{i=1}^{M} g_i^2}}\right)$$
(21)

$$\hat{P}_d \approx Q \left( \sqrt{\frac{\hat{N}}{2}} \frac{\lambda - \sum_{i=1}^M g_i (1 + \gamma_i)}{\sqrt{\sum_{i=1}^M g_i^2 (1 + \gamma_i)^2}} \right).$$
 (22)

Comparing (21) and (22) with (13) and (14), we see that the decoding error reduces the detection performance as if the sensing time is reduced from N to  $\hat{N}$ . That is, when there is a decoding error, the detector senses in time duration N but only gets the detection performance equal to that with sensing time  $\hat{N}$  when there is no decoding error. The assertion is also correct for the AOCS and MNE.

As a function of N,  $\hat{N}$  is an increasing function. Thus increasing the sensing time can in general improve the performance. But unfortunately there is a limit. In fact:  $\lim_{N\to\infty} \hat{N} = 2/\sigma_0^2$ .

When there is no decoding error, for given  $P_d$  and  $P_{fa}$ , the minimum sensing time is  $N_{LC}$ . Hence, when there is a decoding error, the sensing time must satisfy:  $\hat{N} \geq N_{LC}$ . It can only be satisfied if  $\sigma_0^2 < 2/N_{LC}$ . For given  $P_d$ ,  $P_{fa}$  and  $\sigma_0^2$ , the minimum number of samples required is therefore

$$\hat{N}_{LC} = \frac{N_{LC}}{\max(1 - \sigma_0^2 N_{LC}/2, 0)}.$$
 (23)

In summary we have two important assertions: (1) for given  $P_d$  and  $P_{fa}$ , the maximum allowed decoding error

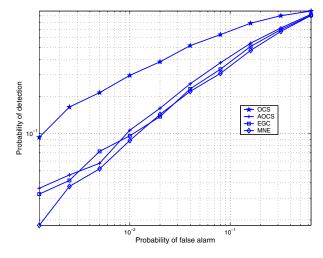


Fig. 1. ROC curve:  $M=10,\,N=2000,\,\bar{\gamma}=-20{\rm dB}$  and  $\beta=3{\rm dB}$ 

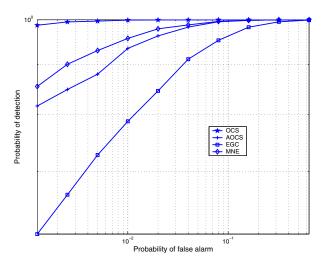


Fig. 2. ROC curve:  $M=10,\,N=2000,\,\bar{\gamma}=-15{\rm dB}$  and  $\beta=3{\rm dB}$ 

is  $2/N_{LC}$ ; (2) for given decoding error  $\sigma_0^2$ , the required sample size to meet the detection performances increases as shown in (23).

### VI. SIMULATIONS

We consider the case of one primary signal with real Gaussian distribution. The channels to each sensors are independent flat-fading and the channel coefficients are unknown but fixed in one sensing slot. The propagation delays of the primary signal to the sensors are random numbers in  $\{0,1,\cdots,M-1\}$ . For AOCS, we choose  $\hat{\gamma}=0.01$  (-20dB).

The relative performances of the methods are obviously affected by the distribution of the SNRs among the sensors. Here we use a simple model: the average SNR of all sensors is  $\bar{\gamma} = \frac{1}{M} \sum_{i=1}^{M} \gamma_i$  and the SNR of user i is  $\beta^{i-1} \gamma_{min}$  ( $i=1,\cdots,M$ ), where  $\gamma_{min}$  is the minimum SNR among those of all sensors.  $\beta$  is called the SNR gap among the users. For given average SNR  $\bar{\gamma}$  and SNR gap  $\beta$ , we can generate the SNRs of sensors, though the match of the SNRs to the sensors can be random. Note that the difference between the maximum

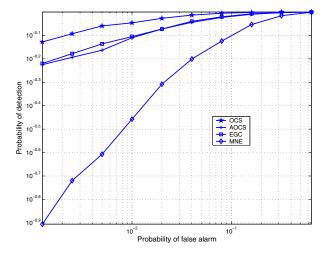


Fig. 3. ROC curve:  $M=10,\,N=2000,\,\bar{\gamma}=-15{\rm dB}$  and  $\beta=1{\rm dB}$ 

SNR and minimum SNR is  $(M-1)10\log_{10}(\beta)$  dB. All the simulations show that the theoretical results match well with simulated results. Hence we only show the results based on simulations. All simulation results are obtained from 10,000 Monte-Carlo tests.

When the SNR gap is  $\beta=3\text{dB}$ , the Receiver Operating Characteristics (ROC) curves are shown in Figure 1 and 2, respectively for  $\bar{\gamma}=-20\text{dB}$  and  $\bar{\gamma}=-15\text{dB}$ . It is no surprise that OCS is much better than AOCS, EGC and MNE. It is a little bit surprise that the relative performances of AOCS, EGC and MNE change with the average SNR level. In fact, at  $\bar{\gamma}=-15\text{dB}$  AOCS and MNE are much better than EGC, while at  $\bar{\gamma}=-20\text{dB}$  AOCS, EGC are slightly better than MNE. If we reduce the SNR gap to  $\beta=1\text{dB}$ , the ROC at  $\bar{\gamma}=-15\text{dB}$  is shown in Figure 3. AOCS and EGC is now approaching to OCS and MNE is much worse than AOCS and EGC.

Figure 4 shows the minimum number of samples required to achieve  $P_d=0.99$  and  $P_{fa}=0.01$  at different average SNR levels and different decoding errors, where M=10 and  $\beta=2\text{dB}$ . In some cases, the required number of samples is actually infinite due to the decoding errors. For better vision, we use  $10^{10}$  to represent the infinite. Simulations show that, at low SNR, the decoding error can greatly increase the required sensing time, while at high SNR, the detections can tolerate the decoding errors.

# VII. CONCLUSIONS

We have shown that, for distributed sensors with time independent signals, the optimal scheme is simply a linear combination of the normalized energies from all sensors. Furthermore, some sub-optimal schemes have been analyzed and compared. In general, the proposed AOCS has relatively good performance at all cases. If the SNR gap is not large, EGC achieves good performance. On the other hand, if the SNRs vary greatly among the sensors, it is better to choose the MNE. Finally it has been proved that the impact of decoding error is equivalent to the reduction of sensing time.

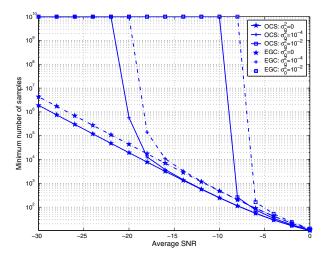


Fig. 4. Mini. sample size: M=10,  $\beta=2 dB$ ,  $P_d=0.99$  and  $P_{fa}=0.01$ 

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