NTUT_Kn1ghts ICPC Team Notebook

Contents

1 Advanced algorithms

1.1 2-SAT problem

```
// 2-SAT Problem demonstrated with 2018 ICPC Korea Regional - Problem K.
#include <bits/stdc++.h>

using namespace std;

#define LOCAL
#define blue(k) (k<<1)
#define red(k) (blue(k) + 1)
#define UNVISITED -1
#define neg(v) (v ^ 1) // [neg]ation of

typedef vector<int> vi;

int K, N;
int V;
vector<vi> AL;
bool possible = true;
vi sccNum;

int getVertex(pair<int, char> p)
{
    return p.second == ’B’ ? blue(p.first) : red(p.first);
}

pair<int, char> negation(pair<int, char> p)
{
    return make_pair(p.first, p.second == ’B’ ? ’R’ : ’B’);
}

void createEdge(pair<int, char> p, pair<int, char> q)
{
    int u, v;
    u = getVertex( negation(p) );
    v = getVertex( q );
//    printf("%d->%d\n", u, v);
    AL[u].push_back(v);

    u = getVertex( negation(q) );
    v = getVertex( p );
//    printf("%d->%d\n\n", u, v);
    AL[u].push_back(v);
}

vi dfs_num, dfs_low, visited;
int dfsNumberCounter, numSCC;
vi S;

void tarjanSCC(int u)
{
    dfs_low[u] = dfs_num[u] = dfsNumberCounter++;
    S.push_back(u);
    visited[u] = 1;
    for (int i = 0; i < (int)AL[u].size(); ++i)
    {
        int v = AL[u][i];
        if (dfs_num[v] == UNVISITED) tarjanSCC(v);
        if (visited[v]) dfs_low[u] = min( dfs_low[u], dfs_low[v] );
    }

    if (dfs_low[u] == dfs_num[u])
    {
        set<int> st;
        ++numSCC;
//        printf("SCC %d:", numSCC);
        while (true)
        {
            int v = S.back(); S.pop_back();
            visited[v] = 0;

            if ( st.find(neg(v)) != st.end() ) possible = false;
            st.insert(v);
            sccNum[v] = numSCC;    // Tarjan produces SCCs in reversed topo order
//            printf(" %d", v);
            if (u==v) break;
        }
//        printf("\n");
    }
}

void work()
{
    sccNum.assign(V, 0);
    dfs_num.assign(V, UNVISITED);
    dfs_low.assign(V, 0);
    visited.assign(V, 0);
    dfsNumberCounter = numSCC = 0;
    for (int u = 0; u < V; ++u)
        if (dfs_num[u] == UNVISITED)
            tarjanSCC(u);

    if (!possible)
    {
        printf("-1\n");
        return;
    }

    for (int i = 1; i <= K; ++i)                                    // 2-SAT assignment based
        printf("%c", sccNum[blue(i)] > sccNum[red(i)] ? ’R’ : ’B’); // on reversed topo order
    printf("\n");
}

int main()
{
    #ifdef LOCAL
    freopen("in.txt", "r", stdin);
    #endif // LOCAL
    scanf("%d %d", &K, &N);
    V = 2 * K + 2;
    AL.assign(V, vi());
    for (int i = 0; i < N; ++i)
    {
        pair<int, char> a[3];
        for (int j = 0; j < 3; ++j) scanf("%d %c", &a[j].first, &a[j].second);
//        for (int j = 0; j < 3; ++j) printf("%d %c ", a[j].first, a[j].second);
//        printf("\n");
        createEdge(a[0], a[1]);
        createEdge(a[0], a[2]);
        createEdge(a[1], a[2]);
    }

    work();

    return 0;
}
```

1.2 Closest pair problem

```
// UVa 10245 - The Closest Pair Problem solved in O(n log n).
#include <bits/stdc++.h>

using namespace std;

#define LOCAL
#define MAX_N 10050
#define INF 1000000000

typedef pair<double, double> dd;
typedef vector<dd> vdd;

dd a[MAX_N];

double dist(dd i, dd j)
{
    return sqrt( pow(i.first - j.first, 2.f) + pow(i.second - j.second, 2.f) );
}

double closest(int lo, int hi, vdd& y_sort)
{
    if (lo>hi) return INF;
    if (lo==hi)
    {
        y_sort.push_back(a[lo]);
        return INF;
    }

    // divide & conquer
    int mid = (lo+hi) / 2;
    vdd ys_o, ys_t;
    double d1 = closest(lo,    mid, ys_o);
    double d2 = closest(mid+1, hi,  ys_t);

    // merge sort
    int N_O = (int)ys_o.size();
    int N_T = (int)ys_t.size();
    int i = 0;
    int j = 0;
    while (true)
    {
        if (i >= N_O && j >= N_T) break;
        if (i >= N_O)
        {
            y_sort.push_back(ys_t[j++]);
            continue;
        }
        if (j >= N_T)
        {
            y_sort.push_back(ys_o[i++]);
            continue;
        }
        if ( ys_o[i].second < ys_t[j].second
          || (ys_o[i].second==ys_t[j].second && ys_o[i].first < ys_t[j].first) ) y_sort.push_back(ys_o
                [i++]);
          else y_sort.push_back(ys_t[j++]);
    }
//    for (int i = 0; i < (int)y_sort.size(); ++i)
//        printf("%lf %lf\n", y_sort[i].first, y_sort[i].second);
//    printf("\n");

    // retrieve d3 to combine
    if (lo + 1 == hi) return dist(a[lo], a[hi]);

    double d = min(d1, d2);
    double x_left = a[mid].first - d;
    double x_right = a[mid].first + d;
    vdd b;
    for (int i = 0; i < (int)y_sort.size(); ++i)
        if (x_left <= y_sort[i].first && y_sort[i].first <= x_right) b.push_back(y_sort[i]);

    double ret = d;
    for (int i = 1; i < (int)b.size(); ++i)
        for (int j = max(0, i-15); j < i; ++j)
            ret = min( ret, dist(b[i], b[j]) );

    return ret;
}

int main()
{
    #ifdef LOCAL
    freopen("in.txt", "r", stdin);
    #endif // LOCAL
    int N;
    while (scanf("%d", &N), N)
    {
        for (int i = 0; i < N; ++i) scanf("%lf %lf", &a[i].first, &a[i].second);
        sort(a, a+N);
//        for (int i = 0; i < N; ++i) printf("%lf %lf\n", a[i].first, a[i].second);
        vdd y_sort;
        double ret = closest(0, N-1, y_sort);
        if (ret<10000) printf("%.4lf\n", ret);
        else printf("INFINITY\n");
    }
    return 0;
}
```

1.3 Iterative deepening A* (IDA*)

```
// UVa 10181 - 15-Puzzle Problem solved with Iterative Deepening A* (IDA*).
#include <bits/stdc++.h>

using namespace std;

//#define LOCAL
#define N 4          // #rows/columns
#define B 15         // [B]lank tile id
#define PUZZLE (N*N)
#define MAX_STEPS 45 // given by the problem description
#define DIR 4        // 4 [DIR]ections

int dr[DIR] = {0, -1, 0, 1}; // must be right, up, left, down
int dc[DIR] = {1, 0, -1, 0}; // for the XOR operation to work
char dm[] = "RULD";          // [d]irection [m]ove

int p[PUZZLE];
int b_init_pos;      // [b]lank [init]ial [pos]ition
int lim;             // current [lim]it of the Iterative Deepening Search (IDS)
int pred[MAX_STEPS]; // [pre]viously used [d]irection to go to the current state

bool isViable()
{
    int sum;
    for (int i = 0; i < PUZZLE; ++i)
        for (int j = 0; j < i; ++j)
            if (p[j] > p[i]) ++sum;
    sum += b_init_pos / N + b_init_pos % N;
    sum -=          B / N +          B % N;
    return sum % 2 == 0;
}

int H()
{
    int h = 0;
    for (int pos = 0; pos < PUZZLE; ++pos)    // for all tile 'p[pos]'
    {                                         // compute Manhattan distance to goal state
        if (p[pos] == B) continue;
        h += abs( p[pos] / N - pos / N )      // position of 'p[pos]' in goal    state is 'p[pos]'
           + abs( p[pos] % N - pos % N );     // position of 'p[pos]' in current state is 'pos'
    }
    return h;
}

bool isValid(int r, int c)
{
    return 0 <= r && r < N && 0 <= c && c < N;
}

int Delta_H(int cur_r, int cur_c, int next_r, int next_c)
{
    int val = p[cur_r * N + cur_c]; // [val]ue of the tile being moved into the blank tile position
    int goal_r = val / N;           // position of 'val' in goal state is 'val'
    int goal_c = val % N;           // get row & column representation of the position
    return - ( abs(goal_r - cur_r ) + abs(goal_c - cur_c ) )
           + ( abs(goal_r - next_r) + abs(goal_c - next_c) );
}

bool dfs(int g, int h, int b_pos)
{
    if (g + h > lim) return false;
    if (h == 0) return true;       // found a solution!
    int r = b_pos / N;
    int c = b_pos % N;
    for (int d = 0; d < DIR; ++d)
    {
        if ( g != 0 && d == (pred[g] ^ 2) ) continue; // this direction gets us back to parent state
        int next_r = r + dr[d];
        int next_c = c + dc[d];
        if ( !isValid(next_r, next_c) ) continue;
        int next_h = h + Delta_H(next_r, next_c, r, c); // O(1)
        int b_next_pos = next_r * N + next_c;
        swap(p[b_pos], p[b_next_pos]);
        pred[g+1] = d;

        if ( dfs(g + 1, next_h, b_next_pos) ) return true;

        swap(p[b_pos], p[b_next_pos]);
    }
    return false;
}

int ida_star()
{
    int init_h = H();
    lim = init_h;
    while (lim <= MAX_STEPS)
    {
        if ( dfs(0, init_h, b_init_pos) ) return lim;
        ++lim;
    }
    return -1;
}

void output(int steps)
{
    for (int i = 1; i <= steps; ++i)
        printf("%c", dm[ pred[i] ]);
}

int main()
{
    #ifdef LOCAL
    freopen("in.txt", "r", stdin);
    #endif // LOCAL
    int T;
    scanf("%d", &T);
    while (T--)
    {
        for (int i = 0; i < N; ++i)
            for (int j = 0; j < N; ++j)
        {
            int pos = i * N + j;
            scanf("%d", &p[pos]);
            if (p[pos] == 0) p[pos] = B, b_init_pos = pos; // goal state 'p' is 0, 1, 2..14, 15
            else --p[pos];                                 // blank tile as 15
        }
        if ( !isViable() ) // must-consider condition otherwise TLE
        {
            printf("This puzzle is not solvable.\n");
            continue;
        }
        int ret = ida_star();
        if (ret == -1)
        {
            printf("This puzzle is not solvable.\n");
            continue;
        }
        output(ret), printf("\n");
    }
    return 0;
}
```

1.4 Lowest common ancestor (LCA)

```
// UVa 12238 - Ants Colony solved with Lowest Common Ancestor (LCA) using Range
// Minimum Query (RMQ) reduction and Spare Table data structure.
#include <bits/stdc++.h>

using namespace std;

#define LOCAL
#define root 0
#define MAX_N 100050
#define MAX_AN (2*MAX_N)
#define LOG_TWO_AN 20

typedef long long int ll;
typedef pair<int, ll> il;
typedef vector<il> vil;

int N;
vector< vil > CH; // [CH]ildren
ll dist[MAX_N];
int a[MAX_AN]; // RMQ [a]rray
int a_N;
int a2t[MAX_AN]; // RMQ [a]rray index -> [t]ree index
int t2a[MAX_N]; // [t]ree index -> RMQ [a]rray index

int _A[MAX_AN]; // RMQ member array (must be global otherwise MLE)
int SpT[MAX_AN][LOG_TWO_AN]; // [Sp]arse [T]able
class RMQ    // [R]ange [M]inimum [Q]uery
{
public:
    RMQ(int n, int A[])    // DP pre-process
    {
        for (int i = 0; i < n; ++i)
        {
            _A[i] = A[i];
            SpT[i][0] = i; // RMQ of sub-array starting at index [i] with length 2^[0] = 1
        }
        // the two nested loops below have overall time complexity O(n log n)
        for (int j = 1; (1<<j) <= n; ++j)    // for each [j] such that 2^j <= n. O(log n)
            for (int i = 0; i + (1<<j) - 1 < n; ++i)    // for each valid [i]. O(n)
                if (_A[ SpT[i][j-1] ] < _A[ SpT[ i + ( 1<<(j-1) ) ][j-1] ])    // RMQ
                    SpT[i][j] = SpT[i][j-1];    // start at index i          of length 2^(j-1)
                else                            // start at index i+2^(j-1) of length 2^(j-1)
                    SpT[i][j] = SpT[ i + ( 1<<(j-1) ) ][j-1];
    }

    int query(int i, int j)    // O(1)
    {
        int k = (int)floor( log( (double)j-i+1 ) / log(2.0) );    // find [k] such that 2^k <= (j-i+1)
        if (_A[ SpT[i][k] ] <= _A[ SpT[j - (1<<k) + 1][k] ]) return SpT[i][k];
        else return SpT[j - (1<<k) + 1][k];
    }
};

void build_dist(int u)
{
    for (int i = 0; i < (int)CH[u].size(); ++i)
    {
        int v = CH[u][i].first;
        dist[v] = dist[u] + CH[u][i].second;
        build_dist(v);
    }
}
```

```
void build_rmq(int u, int depth)
{
    a[a_N] = depth;
    a2t[a_N] = u;
    t2a[u] = a_N;
    ++a_N;
    for (int i = 0; i < (int)CH[u].size(); ++i)
    {
        int v = CH[u][i].first;
        build_rmq(v, depth + 1);
        a[a_N] = depth;
        a2t[a_N] = u;
        ++a_N;
    }
}

void preprocess()
{
//    for (int u = 0; u < N; ++u)
//        for (int i = 0; i < (int)CH[u].size(); ++i)
//            printf("%d->%d: %lld\n", u, CH[u][i].first, CH[u][i].second);
    dist[root] = 0;
    build_dist(root);
//    for (int u = 0; u < N; ++u) printf("%lld ", dist[u]);
//    printf("\n");
    a_N = 0;
    build_rmq(root, 0);
//    for (int i = 0; i < a_N; ++i) printf("%d ", a[i]);
//    printf("\n");
//    for (int i = 0; i < a_N; ++i) printf("%d ", a2t[i]);
//    printf("\n");
//    for (int u = 0; u < N; ++u) printf("node %d: %d\n", u, t2a[u]);
//    printf("\n");
}

int main()
{
#ifdef LOCAL
    freopen("in", "r", stdin);
#endif
    int Q;
    while (scanf("%d", &N), N)
    {
        CH.assign( N, vil() );
        for (int i = 1; i < N; ++i)
        {
            int parent;
            ll cost;
            scanf("%d %lld", &parent, &cost);
            CH[parent].push_back( make_pair(i, cost) );
        }
        preprocess();
        RMQ rmq(a_N, a);
        scanf("%d", &Q);
        for (int i = 0; i < Q; ++i)
        {
            if (i != 0) printf(" ");
            int s, t;
            scanf("%d %d", &s, &t);
            int l = min(t2a[s], t2a[t]);
            int r = max(t2a[s], t2a[t]);
            int lca = a2t[ rmq.query(l, r) ]; // [l]owest [c]ommon [a]ncestor
            printf("%lld", dist[s] + dist[t] - 2*dist[lca]);
        }
        printf("\n");
    }
    return 0;
}
```

1.5 Magic square construction

```
// UVa 1266 - Magic Square solved with Siamese method.
#include <bits/stdc++.h>

using namespace std;

int main()
{
    int n, i, j, first = 0;
    while (scanf("%d", &n) == 1)
    {
        if (first)
            puts("");
        first = 1;
        printf("n=%d, sum=%d\n", n, n * (n * n + 1) / 2);
        int map[15][15], x = 0, y = n / 2;
        memset(map, 0, sizeof(map));
        for (i = 1; i <= n * n; i++)
        {
            if (map[x][y])
            {
                x += 2, y--;
                if (x >= n)
                    x -= n;
                if (y < 0)
                    y += n;
                map[x][y] = i;
            }
            else
                map[x][y] = i;
            x--, y++;
            if (x < 0)
                x += n;
            if (y >= n)
                y -= n;
        }
        if (n * n <= 9)
        {
            for (i = 0; i < n; i++, puts(""))
                for (j = 0; j < n; j++)
                    printf("%2d", map[i][j]);
        }
        else if (n * n >= 10 && n * n <= 100)
        {
            for (i = 0; i < n; i++, puts(""))
                for (j = 0; j < n; j++)
                    printf("%3d", map[i][j]);
        }
        else
        {
            for (i = 0; i < n; i++, puts(""))
                for (j = 0; j < n; j++)
                    printf("%4d", map[i][j]);
        }
    }
    return 0;
}
```

1.6 Suffix automaton

```
#include <iostream>
#include <bits/stdc++.h>
#define LOCAl
#define N 10010
#define SAMN N*10
using namespace std;
int sz , last ;
struct state{
    int len , link ;
    map<char,int> next ;
}st[SAMN];

void sam_init(){
    sz = 0 ;
    st[0].len = 0 ;
    st[0].link = -1 ;
    st[0].next.clear();
    sz++ ;
    last = 0 ;
}

void sam_extend(char c ){
    int cur = sz++ ;
    st[cur].next.clear() ;
    st[cur].len = st[last].len+1 ;
    int p = last ;
    while(p != -1 && !st[p].next.count(c)){
        st[p].next[c] = cur ;
        p = st[p].link ;
    }
    if(p == -1){
        st[cur].link = 0 ;
    }
    else{
        int q = st[p].next[c] ;
        if(st[p].len + 1 == st[q].len){
            st[cur].link = q ;
        }
        else{
            int clone = sz++ ;
            st[clone].len = st[p].len + 1 ;
            st[clone].next = st[q].next ;
            st[clone].link = st[q].link ;
            while(p != -1 && st[p].next[c] == q){
                st[p].next[c] = clone ;
                p = st[p].link ;
            }
            st[q].link = st[cur].link = clone ;

        }
    }
    last = cur ;
}


int main()
{
#ifdef LOCAl
    freopen("in1.txt" , "r" , stdin );
#endif // LOCAl
    ios::sync_with_stdio(false);
    cin.tie(0);
    cout.tie(0);
    int n , len;
    string strA ;
    cin >> n ;
    while(n--){
        cin >> strA ;
        len = strA.length() ;
        strA += strA ;
        sam_init() ;
        for(int i = 0 ; i < strA.length() ; i++) sam_extend(strA[i]);

        int u = 0 , now = len  ;
        while(now--){
            for(auto it : st[u].next){
                u = it.second ;
                break ;
            }
        }
        cout << st[u].len - len + 1 << '\n' ;
    }
    return 0;
}
```

2 Dynamic programming algorithms

2.1 0-1 knapsack

```
#define W 1000 // Knapsack weight
#define N 100  // n item
int weight[N]; //item weight
int value[N];  //item value
int bag[W][2];

// 0/1 Knapsack
void ZeroOne(){
  memset(bag,0,sizeof(bag));
  for(int i = 0 ; i < N ; i++ ){
    for(int j = 0 ; j < W ; j++ )
      if( j >= weight[i] )
        bag[j][1] = max( bag[j][0] ,bag[j-weight[i]][0] + value[i] );

    for(int j = 0 ; j < W ; j++ )
      bag[j][0] = bag[j][1];
  }
}


// group knapsack
int group;    // hou much groups?
int how_many; // one group has many items?
int WEIGHT,VALUE;

void Grouping(){
  memset(bag,0,sizeof(bag));
  for(int i = 0 ; i < group ; i++ ){
    for(int j = 0 ; j < how_many ; j++ ){
      scanf("%d %d",&WEIGHT,&VALUE);

      for(int k = 0 ; k < W ; k++ ){
        if( j >= WEIGHT ){
          bag[j][1] = max( bag[j][1] , bag[j][0] );
          bag[j][1] = max( bag[j][1] ,bag[j-WEIGHT][0] + VALUE );
        }
      }
    }
    for(int j = 0 ; j < W ; j++ )
      bag[j][0] = bag[j][1];
  }
}

// mulipte knapsack
int limit[N]; // item limit
void Multiple() {
  for(int i = 0 ; i < N ; i++ ){
    int tmp = 1;
    while( tmp <= weight[i] ){
      for(int j = 0 ; j < W ; j++)
        if( j >= weight[i]*tmp )
          bag[j][1] = max( bag[j-weight[i]*tmp][0] + value[i]*tmp
                        , bag[j][0] );

      for(int j = 0 ; j < W ; j++ )
        bag[j][0] = bag[j][1];

      weight[i] = weight[i]-tmp;
      tmp = tmp*2;
    }
    if( weight[i] > 0 ){
      for(int j = 0 ; j < W ; j++)
        if( j >= weight[i]*tmp )
          bag[j][1] = max( bag[j-weight[i]*tmp][0] + value[i]*tmp , bag[j][0] );

      for(int j = 0 ; j < W ; j++ )
        bag[j][0] = bag[j][1];
    }
  }
}

// inf
void Unlimited(){
  memset(bag,0,sizeof(bag));
  for(int i = 0 ; i < N ; i++ ){
    for(int j = 0 ; j < W ; j++ )
      if( j >= weight[i] )
        bag[j][1] = max( bag[j][0] ,bag[j-weight[i]][1] + value[i] );

    for(int j = 0 ; j < W ; j++ )
      bag[j][0] = bag[j][1];
  }
}
```

2.2 Longest common subsequence (LCS)

```
#include <iostream>
#include <bits/stdc++.h>
#define LOCAL
using namespace std;


struct LCS{
    int step , max_len ;
}Dp[5000][5000];


int main()
{
#ifdef LOCAL
    freopen("in1.txt" , "r" , stdin );
#endif // LOCAL
    int intX , intY , Min_step , Max_len ;
    string strX , strY ;
    while(cin >> intX >> strX >> intY >> strY ){
        //init
        for(int i = 0 ; i <= intY ; i++){
            Dp[0][i].max_len = 0 ;
            Dp[0][i].step = i ;
        }
        for(int i = 0 ; i <= intX ; i++){
            Dp[i][0].max_len = 0 ;
            Dp[i][0].step = i ;
        }
        Max_len = 0 ;
        Min_step = 0 ;

        //lcs
        for(int i = 1 ; i <= intX ; i++){
            for(int j = 1 ; j <= intY ; j++){
                if(strX[i-1] == strY[j-1]){
                    Dp[i][j].max_len = Dp[i-1][j-1].max_len +1 ;
                    Dp[i][j].step = Dp[i-1][j-1].step ;

                    //debug
                    //cout << strX[i-1] << ' ' << strY[j-1] << ' ' << Dp[i][j].max_len << '\n' ;
                    //cout << strX[i-1] << ' ' << strY[j-1] << ' ' << Dp[i][j].step << '\n' ;
                }
                else{
                    Dp[i][j].max_len = max(Dp[i-1][j].max_len , Dp[i][j-1].max_len ) ;
                    Dp[i][j].step = min( min(Dp[i-1][j-1].step , Dp[i][j-1].step ) , Dp[i-1][j].step )
                           +1 ;
                }
            }
        }
        cout << Dp[intX][intY].step << '\n' ;
    }
    return 0;
}
```

2.3 Max 2D range sum

```
// Max 2D Range Sum - UVa 108 - solved with DP O(n^4).
// Abridged problem statement: Given an n x n square matrix of integers A where
// each integer ranges from [-127..127], find a sub-matrix of A with the maximum
// sum.
#include <bits/stdc++.h>
using namespace std;
int A[200][200];
int main() {
  int n; scanf("%d", &n);                        // square matrix size
  for (int i = 0; i < n; ++i)
    for (int j = 0; j < n; ++j) {
      scanf("%d", &A[i][j]);
      if (i > 0) A[i][j] += A[i-1][j];           // add from top
      if (j > 0) A[i][j] += A[i][j-1];           // add from left
      if (i > 0 && j > 0) A[i][j] -= A[i-1][j-1];// avoid double count
    }                                            // inclusion-exclusion
  int maxSubRect = -127*100*100;                 // the lowest possible val
  for (int i = 0; i < n; ++i)
    for (int j = 0; j < n; ++j)                  // start coordinate
      for (int k = i; k < n; ++k)
        for (int l = j; l < n; ++l) {            // end coord
          int subRect = A[k][l];                 // from (0, 0) to (k, l)
          if (i > 0) subRect -= A[i-1][l];       // O(1)
          if (j > 0) subRect -= A[k][j-1];       // O(1)
          if (i > 0 && j > 0) subRect += A[i-1][j-1]; // O(1)
          maxSubRect = max(maxSubRect, subRect); // the answer is here
        }
  printf("%d\n", maxSubRect);
  return 0;
}
```

2.4 Traveling salesman problem (TSP)

```
// This is a solution for UVa 10496 - Collecting Beepers. The problem is a
// variant of the Traveling Salesman Problem (TSP): Given n cities and their
// pairwise distances in the form of a matrix 'dist' of size n * n, compute the
// minimum cost of making a tour that starts from any city s, goes through all
// the other n - 1 cities exactly once, and finally returns to the city s. In
// this case, the salesman is Karel in a 2D world who can only move along the
// x and y axis. The cities are beepers whose coordinates are given, from which
// pairwise distances can be calculated. Algorithm takes time O(2^n * n^2).
// INPUT: The first line is the number of test cases. The first line of each
// test case is world's size (x-size and y-size). Next is the starting position
// of Karel. Next is the number of beepers. Next are the beepers' x- and y-
// coordinates.
// OUTPUT: For each test case, output the minimum distance to move from Karel's
// starting position to each of the beepers and back to the starting position.

#include <bits/stdc++.h>
using namespace std;

#define LSOne(S) ((S) & -(S))

const int MAX_n = 11;

int dist[MAX_n][MAX_n], memo[MAX_n][1<<(MAX_n-1)]; // Karel + max 10 beepers

int dp(int u, int mask) {                        // mask = free coordinates
  if (mask == 0) return dist[u][0];              // close the loop
  int &ans = memo[u][mask];
  if (ans != -1) return ans;                     // computed before
  ans = 2000000000;
  int m = mask;
  while (m) {                                    // up to O(n)
    int two_pow_v = LSOne(m);                    // but this is fast
    int v = __builtin_ctz(two_pow_v)+1;          // offset v by +1
    ans = min(ans, dist[u][v] + dp(v, mask^two_pow_v)); // keep the min
    m -= two_pow_v;
  }
  return ans;
}

int main() {
  int TC; scanf("%d", &TC);
  while (TC--) {
    int xsize, ysize; scanf("%d %d", &xsize, &ysize); // these two values are not used
    int x[MAX_n], y[MAX_n];
    scanf("%d %d", &x[0], &y[0]);
    int n; scanf("%d", &n); ++n;                 // include Karel
    for (int i = 1; i < n; ++i)                  // Karel is at index 0
      scanf("%d %d", &x[i], &y[i]);
    for (int i = 0; i < n; ++i)                  // build distance table
      for (int j = i; j < n; ++j)
        dist[i][j] = dist[j][i] = abs(x[i]-x[j]) + abs(y[i]-y[j]); // Manhattan distance
    memset(memo, -1, sizeof memo);
    printf("The shortest path has length %d\n", dp(0, (1<<(n-1))-1)); // DP-TSP
  }
  return 0;
}
```

3 Graph algorithms

3.1 All-pairs shortest paths (APSP)

```
// All-Pairs Shortest Paths (APSP) solved with Floyd Warshall O(V^3).
// inside int main()
    // Precondition: AdjMat[i][j] contains the weight of edge (i, j) or INF (1B)
    // if there is no such edge ('AdjMat' is a 32-bit signed integer array).
    // Let 'p' be 2D parent matrix, where p[i][j] is the last vertex before j on
    // a shortest path from i to j, i.e. i -> ... -> p[i][j] -> j.
    for (int i = 0; i < V; ++i)
        for (int j = 0; j < V; ++j)
            p[i][j] = i;    // initialize the parent matrix
    for (int k = 0; k < V; ++k)    // remember that loop order is k->i->j
        for (int i = 0; i < V; ++i)
            for (int j = 0; j < V; ++j)
                if (AdjMat[i][k] + AdjMat[k][j] < AdjMat[i][j])
                {
                    AdjMat[i][j] = AdjMat[i][k] + AdjMat[k][j];
                    p[i][j] = p[k][j];
                }
// print shortest paths
void printPath(int i, int j)
{
    if (i != j) printPath(i, p[i][j]);
    printf("%d ", j);
}
```

3.2 Bipartite matching BFS by David

```
#include <iostream>
#include <cstring>
#include <cstdio>
#include <vector>
#define LOCAL
using namespace std;

int fp[100010] ,fq[100010] ;
int vfp[100010] ,vfq[100010] ;
int turn = 0 ;
vector<int> cp[100010] , cq[100010];

int BFSBMfp(int n){
    vfp[n] = turn  ;
    for(int i = 0 ; i < cp[n].size() ; i++ ){
        if(vfq[cp[n][i]] != turn){
            vfq[cp[n][i]] = turn ;
            if(fq[cp[n][i]] == -1 || BFSBMfp(fq[cp[n][i]])){
                fp[n] = cp[n][i] ;
                fq[cp[n][i]] = n ;
                return 1 ;
            }
        }
    }
    return 0 ;
}
```

```
int main()
{
    ios::sync_with_stdio(false);
    cin.tie(0);
    cout.tie(0);

    int n ,p ,q ,k ,x, y ;
    cin >> n ;
    while(n--){
        cin >> p >> q >> k ;
        int MaxnPQ = max(p,q);
        for(int i = 1 ; i <= MaxnPQ ; i++){
            cp[i].clear();
            fp[i] = -1;
            cq[i].clear();
            fq[i] = -1;
        }
        int cnt = 0;
        for(int i = 0 ; i < k ; i++){
            cin >> x >> y ;
            cp[x].push_back(y) ;
            cq[y].push_back(x) ;
            if(fp[x] == -1 && fq[y] == -1 ){
                fp[x] = y ;
                fq[y] = x ;
                cnt++ ;
            }
        }
        for(int i = 1 ; i <= p ; i++){
            if(fp[i] == -1 ){
                turn++;
                if(BFSBMfp(i))
                   cnt++;
            }
        }
        cout << cnt << '\n';
    }
    return 0;
}
```

3.3 Centroid decomposition

```
#include<iostream>
#include<bits/stdc++.h>
#define LOCAL
#define MAXN 50005
using namespace std;

int n , k , a , b ;
int ans , cnt ;
int Max[MAXN] , sz[MAXN] , rt ;
int head[MAXN], dis[MAXN];
bool vis[MAXN] ;
struct node{
    int v , nx ;
}Edge[MAXN*2];

void init(int n ){
    Max[0] = n ;
    ans = cnt = 0 ;
    for(int i = 0 ; i <= n ; i++){
        head[i] = -1 ;
        vis[i] = 0 ;
    }
}

void add(int u , int v){
    Edge[cnt].v = v ;
    Edge[cnt].nx = head[u] ;
    head[u] = cnt++ ;
}

void get_rt(int u , int fa ){
    sz[u] = 1 ; Max[u] = 0 ;
    for(int i = head[u] ; ~i ; i=Edge[i].nx){
        int v = Edge[i].v ;
        if(vis[v] || v == fa )  continue ;
        get_rt(v,u);
        sz[u] += sz[v] ;
        Max[u] = max(Max[u] , sz[v]);
    }
    Max[u] = max(Max[u] , n - sz[u]);
    if(Max[rt] > Max[u])
        rt =  u ;
}

void get_dis(int u , int fa , int d){
    for(int i = head[u] ; ~i ; i= Edge[i].nx){
        int v = Edge[i].v ;
        if(vis[v] || v == fa ) continue ;
        dis[++cnt] = d + 1 ;
        get_dis(v,u,dis[cnt]);
    }
}


int get_ans(int u , int d ){
    dis[cnt=1] = d ;
    get_dis(u,0,d) ;
    sort(dis+1 , dis+cnt+1) ;
    int l = 1 , ans = 0 ;

    while(l < cnt && dis[l] + dis[cnt] < k ) l++ ;
    while(l < cnt && dis[l] <= k - dis[l]){
        ans += upper_bound(dis + l + 1 , dis + cnt + 1 , k - dis[l]) - lower_bound(dis+l+1 , dis+cnt+1
            , k-dis[l]);
        l++ ;
    }
    return ans ;
}

void dfs(int u ){
    vis[u] = 1 ;
    //cout << rt << ' ' << u << '\n' ;
    ans += get_ans(u , 0);
    for(int i = head[u] ; ~i ; i = Edge[i].nx){
        int v = Edge[i].v ;
        if(vis[v]) continue ;
        ans -= get_ans(v , 1) ;
        n = sz[v] , rt = 0  , get_rt(v,u);
        dfs(rt);
    }
}

int main(){
//#ifdef LOCAL
//    freopen("in1.txt" , "r" , stdin);
//#endif // LOCAL

    cin >> n >> k ;
    init(n);
    for(int i =1; i < n ; i++){
        cin >> a >> b ;
        add(a,b);
        add(b,a);
    }
    rt = 0 ; get_rt(1,0);
    dfs(rt);
    cout << ans << '\n' ;

}
```

3.4 Detect negative weight cycle

```
// Bellman Ford's O(VE)
    vi dist(V, INF); dist[s] = 0;
    for (int i = 0; i < V - 1; ++i)     // relax all E edges V - 1 times
        for (int u = 0; u < V; ++u)     // these two loops = O(E)
            for (int j = 0; j < (int)AL[u].size(); ++j) // [A]djacency [L]ist
            {
                ii vw = AL[u][j];
                dist[vw.first] = min( dist[vw.first], dist[u] + vw.second ); // relax
            }
```

3.5 DFS

```
#include <iostream>
#include <bits/stdc++.h>
#define LOCAL
using namespace std;
int m , n , flag=1;
int Maxn_city = 0 , Maxn_path = 0 ;
vector<int>tree[200020] ;
int city[200020] = {} ;
int visit[200020] = {} ;
vector<int> travel ;

void BFS_to_large_path(int root ){
    visit[root] = 1 ;
    travel.push_back(root);
    for(int i = 0 ; i < tree[root].size() ; i++){
        int node = tree[root][i] ;
        if(!visit[node]){
            BFS_to_large_path(node);
            travel.pop_back();
            visit[root] = 0 ;
        }
    }
    //debug to check large path
    //if (root == 1)
    //    cout << "1=" << travel.size() << ’ ’ << Maxn_path << ’ ’ << city[root] << ’\n’ ;

    if(city[root] && travel.size() > Maxn_path){
        Maxn_city = travel[travel.size()/2];
        Maxn_path = travel.size();
    }
}

void BFS_to_other_path(int root ,int path){
    visit[root] = 1 ;
    for(int i = 0 ; i < tree[root].size() ; i++){
        int node = tree[root][i] ;
        if(!visit[node]){
            BFS_to_other_path(node , path+1);
            visit[root] = 0 ;
        }
    }
    //debug
    if(root == 1 )
        cout << "city=" << root << " path= " << path << ’\n’ ;

    if(city[root] && path != Maxn_path)
        flag = 0 ;
}


int main(){
#ifdef LOCAL
    freopen("in1.txt" , "r" , stdin);
#endif // LOCAL
    cin >> n >> m ;
    int a , b ;
    for(int i = 0 ; i < n-1 ; i++){
        cin >> a >> b ;
        tree[a].push_back(b) ;
        tree[b].push_back(a) ;
    }

    for(int i = 0 ; i < m ; i++){
        cin >> a ;
        city[a] = 1 ;
    }
    BFS_to_large_path(a);
    //visit[a] = 0 ;
    BFS_to_other_path(Maxn_city , 1 );
    if(flag)
        cout << "YES\n" << Maxn_city ;
    else
        cout << "NO" ;

    //debug
    cout << "Maxn_path= " << Maxn_path << " Maxn_city= " << Maxn_city << ’\n’ ;
}
```

3.6 DFS ICPC 2019 Russia problem E

```
#include <iostream>
#include <bits/stdc++.h>
#define LOCAL
using namespace std;
int m , n , flag=1;
int Maxn_city = 0 , Maxn_path = 0 ;
vector<int>tree[200020] ;
int city[200020] = {} ;
int visit[200020] = {} ;
vector<int> travel ;

void BFS_to_large_path(int root ){
    visit[root] = 1 ;
    travel.push_back(root);
    for(int i = 0 ; i < tree[root].size() ; i++){
        int node = tree[root][i] ;
        if(!visit[node]){
            BFS_to_large_path(node);
            travel.pop_back();
            visit[root] = 0 ;
        }
    }
    //debug to check large path
    //if (root == 1)
    //    cout << "1=" << travel.size() << ’ ’ << Maxn_path << ’ ’ << city[root] << ’\n’ ;

    if(city[root] && travel.size() > Maxn_path){
        Maxn_city = travel[travel.size()/2];
        Maxn_path = travel.size();
    }
}

void BFS_to_other_path(int root ,int path){
    visit[root] = 1 ;
    for(int i = 0 ; i < tree[root].size() ; i++){
        int node = tree[root][i] ;
        if(!visit[node]){
            BFS_to_other_path(node , path+1);
            visit[root] = 0 ;
        }
    }
    //debug
    if(root == 1 )
        cout << "city=" << root << " path= " << path << ’\n’ ;

    if(city[root] && path != Maxn_path)
        flag = 0 ;
}


int main(){
#ifdef LOCAL
    freopen("in1.txt" , "r" , stdin);
#endif // LOCAL
    cin >> n >> m ;
    int a , b ;
    for(int i = 0 ; i < n-1 ; i++){
        cin >> a >> b ;
        tree[a].push_back(b) ;
        tree[b].push_back(a) ;
    }

    for(int i = 0 ; i < m ; i++){
        cin >> a ;
        city[a] = 1 ;
    }
    BFS_to_large_path(a);
    //visit[a] = 0 ;
    BFS_to_other_path(Maxn_city , 1 );
    if(flag)
        cout << "YES\n" << Maxn_city ;
    else
        cout << "NO" ;

    //debug
    cout << "Maxn_path= " << Maxn_path << " Maxn_city= " << Maxn_city << ’\n’ ;
}
```

3.7 Dijkstra

```
// Dijkstra implementation for negative weight edges O((V + E) log V)
    vi dist(V, INF); dist[s] = 0;
    priority_queue< ii, vii, greater<ii> > pq;
    pq.push( ii(0, s) );
    while (!pq.empty())
    {
        ii front = pq.top(); pq.pop();
        int d = front.first;
        int u = front.second;
        if (d > dist[u]) continue;
        for (int i = 0; i < (int)AL[u].size(); ++i) // [A]djacency [L]ist
        {
            ii vw = AL[u][i];
            int v = vw.first;
            int w = vw.second;
            if (dist[u] + w < dist[v])
            {
                dist[v] = dist[u] + w;    // relax operation
                pq.push( ii(dist[v], v) );
            }
        }
    } // this variant can cause duplicate items in the priority queue
```

3.8 Print Euler tour

```
// Given an Eulerian-tour graph - a connected undirected graph whose vertices a-
// ll have even degrees, produce its Euler tour. The graph is unweighted, stored
// in an adjacency list where the second attribute in edge info pair is a boole-
// an '1' (edge can still be used) or '0' (edge can no longer be used).
list<int> cyc;    // we need list for fast insertion in the middle

void EulerTour(list<int>::iterator i, int u)
{
    for (int j = 0; j < (int)AL[u].size(); ++j) // [A]djacency [L]ist
    {
        ii& vw = AL[u][j];
        int v = vw.first;
        if (vw.second)    // if this edge can still be used
        {
            vw.second = 0;    // remove this edge
            // remove bi-directional edge
            for (int k = 0; k < (int)AL[v].size(); ++k)
            {
                ii& uw = AL[v][k];
                if (uw.first == u && uw.second)
                {
                    uw.second = 0;
                    break;
                }
            }
            // continue the tour
            EulerTour(cyc.insert(i, u), v);
        }
    }
}

// inside int main()
    cyc.clear();
    EulerTour(cyc.end(), 0);    // 'cyc' contains an Euler tour starting at vertex '0'
    for (list<int>::iterator i = cyc.begin(); i != cyc.end(); ++i)
        printf("%d\n", *i);
```

3.9 Find articulation points and bridges for undirected graph

```
// Find articulation points & bridges for undirected graph solved with DFS O(V + E).
void articulationPointAndBridge(int u)
{
    dfs_low[u] = dfs_num[u] = dfsNumberCounter++;    // dfs_low[u] <= dfs_num[u]
    for (int i = 0; i < (int)AL[u].size(); ++i)    // [A]djacency [L]ist
    {
        int v = AL[u][i].first;
        if (dfs_num[v] == UNVISITED)    // a tree edge
        {
            dfs_parent[v] = u;
            if (u == dfsRoot) ++rootChildren;    // special case if 'u' is a root

            articulationPointAndBridge(v);

            if (dfs_low[v] >= dfs_num[u]) articulation_vertex[u] = true;
            if (dfs_low[v] > dfs_num[u]) printf("Edge (%d, %d) is a bridge\n", u, v);

            dfs_low[u] = min( dfs_low[u], dfs_low[v] );    // update dfs_low[u]
        }
        else if (v != dfs_parent[u]) dfs_low[u] = min( dfs_low[u], dfs_num[v] ); // update dfs_low[u]
    }
}

// inside int main()
    dfsNumberCounter = 0;
    dfs_num.assign(V, UNVISITED);
    dfs_low.assign(V, 0);
    dfs_parent.assign(V, 0);
    articulation_vertex.assign(V, 0);
    printf("Bridges:\n");
    for (int u = 0; u < V; ++u)
        if (dfs_num[u] == UNVISITED)
        {
            dfsRoot = u;
            rootChildren = 0;
            articulationPointAndBridge(u);
            articulation_vertex[dfsRoot] = (rootChildren > 1);    // special case
        }
    printf("Articulation Points:\n");
    for (int u = 0; u < V; ++u)
        if (articulation_vertex[u]) printf(" Vertex %d\n", u);
```

3.10 Floyd Warshall by David

```
#include <iostream>
#include <bits/stdc++.h>
#define LOCAL
using namespace std;
char before[520][520] = {} ;
int after[520][520] = {} ;

int main()
{
#ifdef LOCAL
    freopen("in1.txt" , "r" , stdin );
#endif // LOCAL

    int n ;
    cin >> n ;
    for(int i = 0 ; i < n ; i++){
        for(int j = 0 ; j < n ; j++)
            cin >> before[i][j] ;
    }

    for(int i = 0 ; i < n ; i++){
        for(int j = i+1 ; j < n ; j++){
            int sum = 0 ;
            for(int k = i + 1 ; k < j ; k++){
                if(after[i][k])
                    sum += before[k][j]-'0' ;
            }

            if( (sum +1) % 10 == before[i][j] - '0'){
                after[i][j] = 1 ;
            }

        }
    }

    for(int i = 0 ; i < n ; i++){
        for(int j = 0; j < n ; j++)
            cout << after[i][j] ;
        cout << '\n' ;
    }

    return 0;
}
```

3.11 Graph edges property check

```
// Graph Edges Property Check solved with DFS O(V + E).
void graphCheck(int u)    // DFS for checking graph edge properties
{
    dfs_num[u] = EXPLORED;
    for (int i = 0; i < (int)AL[u].size; ++i) // [A]djancency [L]ist
    {
        int v = AL[u][i].first;
        if (dfs_num[v] == UNVISITED)    // Tree Edge, EXPLORED->UNVISITED
        {
            dfs_parent[v] = u;    // parent of this child is me
            graphCheck(v);
        }
        else if (dfs_num[v] == EXPLORED)    // EXPLORED->EXPLORED
        {
            if (v == dfs_parent[u]) printf(" Two ways (%d, %d)-(%d, %d)\n", u, v, v, u);
            else printf(" Back Edge (%d, %d) (Cycle)\n", u, v); // can check if graph is cyclic
        }
        else if (dfs_num[v] == VISITED)    // EXPLORED->VISITED
            printf(" Forward/Cross Edge (%d, %d)\n", u, v);
    }
    dfs_num[u] = VISITED;
}

// inside int main()
    dfs_num.assign(V, UNVISITED);
    dfs_parent.assign(V, 0);
    for (int u = 0; u < V; ++u)
        if (dfs_num[u] == UNVISITED)
            printf("Component %d:\n", ++numComp), graphCheck(u);
```

3.12 Kruskal by David

```
#include <iostream>
#include <bits/stdc++.h>
#define LOCAL
#define ll long long
using namespace std;
int parent[1020] ;

struct edge{
    ll n1 , n2 , w ;
}node[25020];


int compare(edge A , edge B ){
    return A.w < B.w ;
}


int find_root(int a){
    if(a != parent[a] )
        return parent[a] = find_root(parent[a]) ;
    return a ;
}



int main()
{
#ifdef LOCAL
    freopen("in1.txt" , "r" , stdin );
    freopen("out.txt" , "w" , stdout );
#endif // LOCAL
    int n , m , p_n1 , p_n2 ; // parent_n1 , parent_n2
    vector<int> hce ; //heavy edge circle
    while(cin >> n >> m && n + m != 0 ){
        for(int i = 0 ; i < m ; i++ ){
            cin >> node[i].n1 >> node[i].n2 >> node[i].w ;
        }

        for(int i = 0 ; i < n ; i++)
            parent[i] = i ;
        sort(node , node + m , compare ) ;
        hce.clear() ;

        //kruskal
        for(int i = 0 ; i < m ; i++){
            p_n1 = find_root(node[i].n1) ;
            p_n2 = find_root(node[i].n2) ;
            if(p_n1 != p_n2 )
                parent[p_n2] = p_n1 ;
            else
                hce.push_back(node[i].w) ;

            //debug
            /**<
            for(int i = 0 ; i < n ; i++)
                cout << parent[i] << ' ' ;
            cout << '\n' ;
            */
        }
        sort(hce.begin() , hce.end()) ;
        if(hce.size()){
            for(int i = 0 ; i < hce.size()-1 ; i++)
                cout << hce[i] << ' ' ;
            cout << hce[hce.size()-1] ;
        }
        else
            cout << "forest" ;
        cout << '\n' ;
    }
    return 0;
}
```

3.13 Max flow

```
int res[MAX_V][MAX_V], mf, f, s, t;
vi p; // p stores the BFS spanning tree from s

void augment(int v, int minEdge)
{
    if (v == s) { f = minEdge; return; }
    else if ( p[v] != -1 )
    {
        augment( p[v], min(minEdge, res[ p[v] ][ v ]) );
        res[ p[v] ][ v ] -= f;
        res[ v ][ p[v] ] += f;
    }
}
```

```
// inside int main(): set up 'AL', 'res', 's', and 't' with appropriate values
// remember to add backward edges to 'AL'
    mf = 0;
    while (true)    // O(V * E^2) Edmonds Karp's algorithm
    {
        f = 0;
        vi dist(MAX_V, INF); dist[s] = 0;
        queue<int> q; q.push(s);
        p.assign(MAX_V, -1);
        while (!q.empty())
        {
            int u = q.front(); q.pop();
            if (u == t) break; // immediately stop BFS if we already reach sink t
            for (int i = 0; i < (int)AL[u].size(); ++i)
            {
                int v = AL[u][i];    // vector< vi > [A]djacency [L]ist
                if (res[u][v] > 0 && dist[v] == INF)
                    dist[v] = dist[u] + 1, q.push(v), p[v] = u;
            }
        }
        augment(t, INF);   // find the min edge weight 'f' in this path, if any
        if (f == 0) break; // we cannot send any more flow ('f' = 0), terminate
        mf += f;           // we can still send a flow, increase the max flow!
    }
    printf("%d\n", mf);
```

3.14 Max cardinality bipartite matching (MCBM)

```
// Max Cardinality Bipartite Matching (MCBM) solved with augmenting path algorithm O(VE).
vi match, vis;

int Aug(int l)    // return 1 if an augmenting path is found & 0 otherwise
{
    if (vis[l]) return 0;
    vis[l] = 1;
    for (int i = 0; i < (int)AL[l].size(); ++i) // [A]djacency [L]ist
    {
        int r = AL[l][i];    // edge weight not needed -> vector< vi > AL
        if ( match[r] == -1 || Aug(match[r]) )
        {
            match[r] = l;
            return 1;    // found 1 matching
        }
    }
    return 0;            // no matchings
}

// inside int main()
    // build unweighted bipartite graph with directed edge left->right set
    // left vertices [0..N-1], right vertices [N..V-1]
    int MCBM = 0;
    match.assign(V, -1); // V is the number of vertices in bipartite graph
    for (int l = 0; l < N; ++l)    // N = size of the left set
    {
        vis.assign(N, 0);    // reset before each recursion
        MCBM += Aug(l);
    }
    printf("Found %d matchings\n", MCBM);
```

3.15 Max cardinality matching (MCM)

```
// ACM Timus 1099 - Work Scheduling solved as Max Cardinality Matching (MCM) us-
// ing Edmonds Matching algorithm in O(V^2 * E).
#include <bits/stdc++.h>

using namespace std;

#define LOCAL
#define MAX_V 500

struct edge_t
{
    int v;
    edge_t* n; // [n]ext
};

typedef edge_t* edge_ptr;

int V;
edge_t pool[MAX_V*MAX_V*2];
edge_ptr p_top = pool;
edge_ptr adj[MAX_V];
```

```
int match[MAX_V];
int qh, qt;
int q[MAX_V];
int father[MAX_V], base[MAX_V];
bool inq[MAX_V], inb[MAX_V];

void add_edge(int u, int v)
{
    p_top->v = v, p_top->n = adj[u], adj[u] = p_top++;
    p_top->v = u, p_top->n = adj[v], adj[v] = p_top++;
}

int LCA(int root, int u, int v) // [L]owest [C]ommon [A]ncestor
{
    static bool inp[MAX_V];
    memset(inp, 0, sizeof inp);
    while (true)
    {
        inp[ u=base[u] ] = true;
        if (u==root) break;
        u = father[ match[u] ];
    }

    while (true)
    {
        if (inp[ v=base[v] ]) return v;
        else v = father[ match[v] ];
    }
}

void mark_blossom(int lca, int u)
{
    while (base[u] != lca)
    {
        int v = match[u];
        inb[ base[u] ] = inb[ base[v] ] = true;
        u = father[v];
        if (base[u] != lca) father[u] = v;
    }
}

void blossom_contraction(int s, int u, int v)
{
    int lca = LCA(s, u, v);
    memset(inb, 0, sizeof inb);
    mark_blossom(lca, u);
    mark_blossom(lca, v);
    if (base[u] != lca) father[u] = v;
    if (base[v] != lca) father[v] = u;
    for (int u = 0; u < V; ++u)
        if (inb[ base[u] ])
        {
            base[u] = lca;
            if (!inq[u]) inq[ q[++qt]=u ] = true;
        }
}

int find_augmenting_path(int s)
{
    memset(inq, 0, sizeof inq);
    memset(father, -1, sizeof father);
    for (int u = 0; u < V; ++u) base[u] = u;
    inq[ q[ qh=qt=0 ]=s ] = true;
    while (qh <= qt)
    {
        int u = q[qh++];
        for (edge_ptr p_e = adj[u]; p_e; p_e = p_e->n)
        {
            int v = p_e->v;
            if (base[u] == base[v] || match[u] == v) continue;
            if ( (v==s)
              || (match[v] != -1 && father[ match[v] ] != -1) )
            {
                blossom_contraction(s, u, v);
            }
            else if (father[v] == -1)
            {
                father[v] = u;
                if (match[v] == -1) return v;
                else if (!inq[ match[v] ]) inq[ q[++qt]=match[v] ] = true;
            }
        }
    }
    return -1;
}

bool augment_path(int s, int t)
{
    int v, w;
    int u = t;
    while (u != -1)
    {
        v = father[u];
        w = match[v];
        match[v] = u;
        match[u] = v;
        u = w;
    }
    return t != -1;
}

int edmonds()
{
    int match_N = 0;
    memset(match, -1, sizeof match);
    for (int u = 0; u < V; ++u)
        if (match[u] == -1) match_N += augment_path( u, find_augmenting_path(u) );
    return match_N;
}

int main()
{
#ifdef LOCAL
    freopen("in", "r", stdin);
#endif
    scanf("%d", &V);
    int u, v;
    while (scanf("%d %d", &u, &v) != EOF)
    {
        --u; --v; // 0-based index
        add_edge(u, v);
    }
    printf("%d\n", edmonds() * 2);
    bool visited[MAX_V];
    memset(visited, false, sizeof visited);
    for (int u = 0; u < V; ++u)
        if (!visited[u] && match[u] != -1)
        {
            visited[u] = visited[ match[u] ] = true;
            printf("%d %d\n", u+1, match[u]+1); // 1-based index
        }
    return 0;
}
```

3.16 Max weight perfect bipartite matching

```
// TopCoder ChessMatchup solved with Hungarian algorithm O(n^3).
#include <bits/stdc++.h>

using namespace std;

#define LOCAL
#define MAX_N 100
#define INF 1000000000

typedef vector<int> vi;

int N;
int cost[MAX_N][MAX_N];
int max_match;
int lx[MAX_N], ly[MAX_N]; // labels of X and Y parts
int xy[MAX_N]; // xy[x] - vertex that is matched with x
int yx[MAX_N]; // yx[y] - vertex that is matched with y
bool S[MAX_N], T[MAX_N]; // sets S and T in algorithm
int slack[MAX_N]; // as in the algorithm description
int slackx[MAX_N]; // slackx[y] such a vertex, that l(slackx[y]) + l(y) - w(slackx[y],y) = slack[y]
int pre[MAX_N]; // [pre]vious array for memorizing alternating paths

void init_labels()
{
    memset(lx, 0, sizeof lx);
    memset(ly, 0, sizeof ly);
    for (int x = 0; x < N; x++)
        for (int y = 0; y < N; y++) lx[x] = max(lx[x], cost[x][y]);
}

void update_labels()
{
    int x, y, delta = INF; // init delta as infinity
    for (y = 0; y < N; y++) // calculate delta using slack
        if ( !T[y] ) delta = min(delta, slack[y]);
    for (x = 0; x < N; x++) // update X labels
        if ( S[x] ) lx[x] -= delta;
    for (y = 0; y < N; y++) // update Y labels
        if ( T[y] ) ly[y] += delta;
    for (y = 0; y < N; y++) // update slack array
        if ( !T[y] ) slack[y] -= delta;
}
```

```
// x - current vertex,prevx - vertex from X before x in the alternating path, so
// we add edges (prevx, xy[x]), (xy[x], x)
void add_to_tree(int x, int prevx)
{
    S[x] = true; // add x to S
    pre[x] = prevx; // we need this when augmenting
    for (int y = 0; y < N; y++) // update slacks, because we add new vertex to S
        if (lx[x] + ly[y] - cost[x][y] < slack[y])
        {
            slack[y] = lx[x] + ly[y] - cost[x][y];
            slackx[y] = x;
        }
}

void augment() // main function of the algorithm
{
    if (max_match == N) return; // matching is perfect
    int x, y;
    int root = -1;
    int q[MAX_N], wr = 0, rd = 0; // q - queue for bfs, wr,rd - write and read pos in queue
    memset(S, false, sizeof S); // init set S
    memset(T, false, sizeof T); // init set T
    memset(pre, -1, sizeof pre); // init set prev - for the alternating tree
    for (x = 0; x < N; ++x) // finding root of the tree
        if (xy[x] == -1)
        {
            q[wr++] = root = x;
            pre[x] = -2;
            S[x] = true;
            break;
        }
    for (y = 0; y < N; ++y) // initializing slack array
    {
        slack[y] = lx[root] + ly[y] - cost[root][y];
        slackx[y] = root;
    }

    while (true) // main cycle
    {
        while (rd < wr) // building tree with bfs cycle
        {
            x = q[rd++]; // current vertex from X part
            for (y = 0; y < N; ++y) // iterate through all edges in equality graph
                if (cost[x][y] == lx[x] + ly[y] && !T[y])
                {
                    if (yx[y] == -1) break; // an exposed vertex in Y found, so augmenting path exists
                        !
                    T[y] = true;            // else just add y to T,
                    q[wr++] = yx[y];        // add vertex yx[y], which is matched with y, to the queue
                    add_to_tree( yx[y], x ); // add edges (x,y) and (y,yx[y]) to the tree
                }
            if (y < N) break; // augmenting path found!
        }
        if (y < N) break; // augmenting path found!
        update_labels(); // augmenting path not found, so improve labeling
        wr = rd = 0;
        // in this cycle we add edges that were added to the equality graph as a
        // result of improving the labeling, we add edge (slackx[y], y) to the
        // tree if and only if !T[y] && slack[y] == 0, also with this edge we a-
        // dd another one (y, yx[y]) or augment the matching, if y was exposed.
        for (y = 0; y < N; ++y)
            if ( !T[y] && slack[y] == 0 )
            {
                if (yx[y] == -1) // exposed vertex in Y found - augmenting path exists!
                {
                    x = slackx[y];
                    break;
                }
                // else just add y to T
                T[y] = true;
                if (!S[yx[y]])
                {
                    q[wr++] = yx[y]; // add vertex yx[y], which is matched with y, to the queue
                    add_to_tree( yx[y], slackx[y] ); // and add edges (x,y) and (y, yx[y]) to the tree
                }
            }
        if (y < N) break; // augmenting path found!
    } // end main cycle
    if (y < N) // we found augmenting path!
    {
        ++max_match; // increment matching in this cycle we inverse edges along augmenting path
        for (int cx = x, cy = y, ty;
                            cx != -2;
               cx = pre[cx], cy = ty)
        {
            ty = xy[cx];
            yx[cy] = cx;
            xy[cx] = cy;
        }
        augment(); // recall function, go to step 1 of the algorithm
    }
}
```

```
int max_weight_perfect_bipartite_matching()
{
    int ret = 0; // weight of the optimal matching
    max_match = 0; // number of vertices in current matching
    memset(xy, -1, sizeof xy);
    memset(yx, -1, sizeof yx);
    init_labels(); // step 0
    augment(); // steps 1-3
    for (int x = 0; x < N; ++x) ret += cost[ x ][ xy[x] ];
    return ret;
}

class ChessMatchup
{
public:
    static int maximumScore(vi us, vi them)
    {
        N = (int)us.size(); // for TopCoder submission
        for (int i = 0; i < N; ++i)
            for (int j = 0; j < N; ++j)
                if (us[i] > them[j]) cost[i][j] = 2;
                else if (us[i] == them[j]) cost[i][j] = 1;
                else cost[i][j] = 0;
        return max_weight_perfect_bipartite_matching();
    }
};

int main()
{
#ifdef LOCAL
    freopen("in2", "r", stdin);
#endif
    int us[MAX_N], them[MAX_N];
    scanf("%d", &N);
    for (int i = 0; i < N; ++i) scanf("%d", &us[i]);
    for (int i = 0; i < N; ++i) scanf("%d", &them[i]);
    printf( "%d\n", ChessMatchup::maximumScore( vi(us, us+N), vi(them, them+N) ) );
    return 0;
}
```

3.17 Min-cost flow (MCF)

```
// UVa 10594 - Data Flow solved as Min-Cost Flow (MCF) problem using Edmonds Ka-
// rp and Bellman Ford algorithms with total time O(V^2 * E^3).
#include <bits/stdc++.h>

using namespace std;

#define LOCAL
#define INF 1000000000000000 // 10^15
#define bwd 0 // [b]ack[w]ar[d] direction
#define fwd 1 // [f]or[w]ar[d] direction
#define MAX_V 200

typedef vector<int> vi;
typedef long long int ll;
typedef pair<ll, ll> ll2;
typedef vector<ll> vll;

int V;
vector<vi> AL;
ll res[MAX_V][MAX_V][2], cst[MAX_V][MAX_V][2];
ll mf, f, min_cost;
int s, t;
vector< pair<int, ll> > p;
ll FLOW, CAPACITY;

void augment(int v, ll minEdge)
{
    if (v == s) { f = minEdge; return; }
    else if ( p[v].first != -1 )
    {
        augment( p[v].first, min(minEdge, res[ p[v].first ][ v ][ p[v].second ]) );
        res[ p[v].first ][ v ][ p[v].second ] -= f;
        res[ v ][ p[v].first ][ p[v].second ] += f;
    }
}

void trace_cost(int v)
{
    if (p[v].first == -1) return;
    min_cost += cst[ p[v].first ][ v ][ p[v].second ] * f;
    trace_cost(p[v].first);
}

void min_cost_flow()
```

```
{
    min_cost = 0;
    mf = 0;
    while (true)
    {
        f = 0;
        p.assign(MAX_V, make_pair(-1, -1));
        vll dist(V, INF); dist[s] = 0;
        for (int i = 0; i < V - 1; ++i)
            for (int u = 0; u < V; ++u)
                for (int j = 0; j < (int)AL[u].size(); ++j)
                {
                    int v = AL[u][j];
                    for (int dir = 0; dir <= 1; ++dir)
                        if (res[u][v][dir] > 0 && dist[u] + cst[u][v][dir] < dist[v])
                        {
                            dist[v] = dist[u] + cst[u][v][dir];
                            p[v] = make_pair(u, dir);
                        }
                }
        augment(t, INF);
        if (f == 0) break;
        f = min(f, FLOW - mf);
        trace_cost(t);
        mf += f;
        if (mf == FLOW) break;
    }
    if (mf < FLOW) printf("Impossible.\n");
    else printf("%lld\n", min_cost);
}

int main()
{
#ifdef LOCAL
    freopen("in", "r", stdin);
#endif
    int E;
    while (scanf("%d %d", &V, &E) != EOF)
    {
        AL.assign(V, vi());
        memset(res, 0, sizeof res);
        memset(cst, 0, sizeof cst);
        for (int i = 0; i < E; ++i)
        {
            int u, v;
            ll w;
            scanf("%d %d %lld", &u, &v, &w);
            u--; v--; // 0-based index
            AL[u].push_back(v);
            AL[v].push_back(u);

            res[u][v][fwd] = res[v][u][bwd] = 1; // real edges
            cst[u][v][fwd] = cst[v][u][bwd] = w;

            res[u][v][bwd] = res[v][u][fwd] = 0; // additional reversed edges
            cst[u][v][bwd] = cst[v][u][fwd] = -w;
        }

        scanf("%lld %lld", &FLOW, &CAPACITY);
        for (int u = 0; u < V; ++u)
            for (int v = 0; v < V; ++v)
            {
                res[u][v][fwd] *= CAPACITY;
                res[v][u][bwd] *= CAPACITY;
            }

        s = 0;
        t = V-1;
        min_cost_flow();
    }
    return 0;
}
```

3.18 Minimum spanning tree (MST)

```
// Minimum Spanning Tree (MST) solved with Kruskal O(E log V)
// inside int main()
    vector< pair<int, ii> > EdgeList;    // (weight, two vertices) of the edge
    for (int i = 0; i < E; ++i)
    {
        scanf("%d %d %d", &u, &v, &w);
        EdgeList.push_back( make_pair( w, ii(u, v) ) );
    }
    sort(EdgeList.begin(), EdgeList.end()); // sort by edge weight O(E log E)
    int mst_cost = 0;
    UnionFind UF(V);    // all V are disjoint sets initially
    for (int i = 0; i < E; ++i)
    {
        pair<int, ii> front = EdgeList[i];
        if (!UF.isSameSet(front.second.first, front.second.second))
        {
            mst_cost += front.first;
            UF.unionSet(front.second.first, front.second.second);
        }
    }
    printf("MST cost = %d\n", mst_cost);
```

3.19 Strongly connected component (SCC)

```
// Tarjan O(V + E)
vi dfs_num, dfs_low, visited;
int dfsNumberCounter, numSCC;
vi S;

void tarjanSCC(int u)
{
    dfs_low[u] = dfs_num[u] = dfsNumberCounter++;    // dfs_low[u] <= dfs_num[u]
    S.push_back(u);    // stores 'u' in a vector baesd on order of visitation
    visited[u] = 1;
    for (int i = 0; i < (int)AL[u].size(); ++i) // [A]djacency [L]ist
    {
        int v = AL[u][i].first;
        if (dfs_num[v] == UNVISITED) tarjanSCC(v);
        if (visited[v]) dfs_low[u] = min( dfs_low[u], dfs_low[v] ); // condition for update
    }

    if (dfs_low[u] == dfs_num[u])    // if this is a root (start) of an SCC
    {                                // this part is done after recursion
        printf("SCC %d:", ++numSCC);
        while (true)
        {
            int v = S.back(); S.pop_back();
            visited[v] = 0;
            printf(" %d", v);
            if (u == v) break;
        }
        printf("\n");
    }
}

// inside int main()
    dfs_num.assign(V, UNVISITED);
    dfs_low.assign(V, 0);
    visited.assign(V, 0);
    dfsNumberCounter = numSCC = 0;
    for (int u = 0; u < V; ++u)
        if (dfs_num[u] == UNVISITED)
            tarjanSCC(u);
```

4 Greedy algorithms

4.1 Interval covering

```
// This is a solution for UVa 10382 - Watering Grass. The problem is a variant
// of Interval Covering problem, which is solved by O(n) Greedy algorithm.

#include <bits/stdc++.h>

#define pb push_back
#define not_set -1

using namespace std;

typedef pair<double, double> dd;
typedef vector<dd> vdd;
typedef enum { STOP = 0,
               CONTINUE } status;

int n, l, w;
vdd spinklers;
int answer;
double pivot;

struct sort_compare_t {
    bool operator()(dd a, dd b) const {
        return a.first < b.first || (a.first == b.first && a.second > b.second);
    }
} sort_compare;

void InputSpinklers() {
    for (int i = 0; i < n; i++) {
        double x, r;  // must be double otherwise WA.
        scanf("%lf %lf", &x, &r);
        if (w > 2 * r)  // ignore spinklers that cannot cover the width of the strip.
        {
            continue;
        }
        if (w == 2 * r)  // ignore spinklers that produce no intervals.
        {
            continue;
        }
        double dx = sqrt(r * r - w * w / 4.0);
        spinklers.pb(dd(x - dx, x + dx));
    }
}

status Check(int& j) {
    if (j == not_set)  // there is an interval after pivot that cannot be covered.
    {
        return STOP;
    }
    // record j.
    answer++;
    pivot = spinklers[j].second;
    if (pivot >= l)  // solution found!
    {
        return STOP;
    }
    j = not_set;
    return CONTINUE;
}

void SolveIntervalCovering() {
    sort(spinklers.begin(), spinklers.end(), sort_compare);
    answer = 0;
    pivot = 0.0;
    int j = not_set;
    int iter = 0;
    while (true) {
        if (iter == spinklers.size())  // iterated through all spinklers/intervals.
        {
            Check(j);
            break;
        }

        if (spinklers[iter].first <= pivot) {
            if (pivot < spinklers[iter].second)  // note the next candidate down!
            {
                if (j == not_set || spinklers[iter].second > spinklers[j].second)  // note down the
                      most right candidate.
                {
                    j = iter;
                }
                iter++;
            } else  // skip intervals that are completely covered by the previously selected ones.
            {
                iter++;
            }
        } else  // out bound.
        {
            if (Check(j) == STOP) {
                break;
            }
        }
    }

    if (pivot >= l) {
        printf("%d\n", answer);
    } else {
        printf("-1\n");
    }
}

int main() {
    while (scanf("%d %d %d", &n, &l, &w) != EOF) {
        spinklers.clear();
        InputSpinklers();
        SolveIntervalCovering();
    }
}
```

4.2 Longest increasing subsequence (LIS)

```
// UVa 481 - What Goes Up solved as Longest Increasing Subsequence (LIS) using
// greedy + divide & conquer algorithm in O(n log n).
#include <bits/stdc++.h>

using namespace std;

#define LOCAL

typedef vector<int> vi;

vi p;
vi a;

void print_lis(int i)
{
    if (p[i] == -1)
    {
        printf("%d\n", a[i]);
        return;
    }
    print_lis( p[i] );
    printf("%d\n", a[i]);
}

int main()
{
#ifdef LOCAL
    freopen("in", "r", stdin);
#endif
    int val;
    while (scanf("%d", &val) != EOF) a.push_back(val);
    int N = (int)a.size();
    int lis_length = 0;
    int lis_end = 0;
    vi L(N, 0);
    vi L_id(N, 0);
    p.assign(N, -1);
    for (int i = 0; i < N; ++i)
    {
        int pos = lower_bound( L.begin(), L.begin() + lis_length, a[i] ) - L.begin();
        L[pos] = a[i];                            // greedy - I am <= you (a[i] <= L[pos]). I am better.
        L_id[pos] = i;                            // which causes 'L' entry order to differ from 'a'.
        p[i] = (pos > 0) ? L_id[pos - 1] : -1;    // but we have [p]arent array on our back.
        if (pos == lis_length)                    // not found a guy to replace. we have a longer LIS!
        {
            lis_length = pos + 1;
            lis_end = i;
        }
        if (pos == lis_length - 1) lis_end = i; // take the LIS that occurs last.
//          printf("LIS ending at a[%d]=%d is of length %d: ", i, a[i], pos+1);
//          print_lis(i);
//          printf("\n");
    }
    printf("%d\n-\n", lis_length);
    print_lis(lis_end);
    return 0;
}
```

4.3 Max 1D range sum

```
// Max 1D Range Sum solved with Jay Kadane O(n).
// inside int main()
    int n = 9;
    int A[] = { 4, -5, 4, -3, 4, 4, -4, 4, -5 }; // a sample array A
    int sum = 0;
    int ans = 0; // important, 'ans' must be initialized to 0
    for (int i = 0; i < n; ++i)
    {
        sum += A[i];
        ans = max(ans, sum);
        if (sum < 0) sum = 0;
    }
    printf("Max 1D Range Sum = %d\n", ans);
```

5 Math algorithms

5.1 Chinese remainder theorem

```
#include <bits/stdc++.h>
#define qtr ios::sync_with_stdio(0); cin.tie(0);
#define endl '\n'
```

```
#define int long long
#define MOD 1000000
using namespace std;

int inv(int a, int m){
    int m0 = m, t, q;
    int x0 = 0, x1 = 1;
    if(m == 1){
        return 0;
    }
    while(a > 1){
        q = a/m;
        t = m;
        m = a%m, a = t;
        t = x0;
        x0 = x1 - q * x0;
        x1 = t;
    }
    if(x1 < 0){
        x1 += m0;
    }
    return x1;
}

int findMinX(vector<int> num, vector<int> rem, int k){
    int prod = 1;
    for(int i = 0; i < k; i++) prod *= num[i];
    int result = 0;
    for(int i = 0; i < k; i++){
        int pp = prod / num[i];
        result += rem[i] * inv(pp, num[i]) * pp;
    }
    return result % prod;
}

int32_t main() { //qtr
    int n = 3;
    vector<int> rem, factor;
    rem.resize(n);
    factor.resize(n);
    for(int i = 0; i < n; i++){
        cin >> factor[i];
    }
    for(int i = 0; i < n; i++){
        cin >> rem[i];
    }
    cout << findMinX(factor, rem, n) << endl;
}
```

5.2 Extended greatest common divisor (Ext-GCD)

```
// ax mod b = 1
// ax + by = 1,x=y=0
// a,b Relatively Prime
LL exgcd(LL a,LL b,LL &x,LL &y){
  if(b){
    LL tmd=exgcd(b,a%b,y,x);
    y-=a/b*x;
    return tmd;
  }
  x=1,y=0;
  return a;
}
```

5.3 Greatest common divisor (GCD) and least common multiple (LCM)

```
// or __gcd(a, b) in gcc
int gcd(int a, int b){
        return a%b?gcd(b,a%b):b;
}
int lcm(int a, int b){
        return a*b/gcd(a,b);
}
```

5.4 Generate list of prime numbers

```
// Generate list of prime numbers using Sieve of Eratosthenes.
ll _sieve_size;
bitset<10000010> bs; // [b]it [s]et 10^7 should be enough for most cases
vi primes;    // compact list of primes

void sieve(ll upperbound)    // create list of primes in [0..upperbound]
{
    _sieve_size = upperbound + 1; // add 1 to include upperbound
    bs.set();                     // set all bits to 1
    bs[0] = bs[1] = 0;            // exception index 0 and 1
    for (ll i = 2; i <= _sieve_size; ++i)
        if (bs[i])
        {   // cross out multiples of i starting from i * i!
            for (ll j = i * i; j <= _sieve_size; j += i) bs[j] = 0;
            primes.push_back( (int)i );
        }
}

bool isPrime(ll N)    // a good enough deterministic prime tester
{
    if (N <= _sieve_size) return bs[N];    // O(1) for small primes
    for (int i = 0; i < (int)primes.size(); ++i)
        if (N % primes[i] == 0) return false;
    return true;    // it takes longer if N is a large prime!
}                   // note: only work for N <= (last prime in vi 'primes')^2

// inside int main()
    sieve(10000000);    // can go up to 10^7 (need few seconds)
    printf("%d\n", isPrime(2147483647));     // 10-digit prime
    printf("%d\n", isPrime(136117223861LL)); // not a prime, 104729 * 1299709
```

5.5 N choose R combination (nCr)

```
#define MAXN 100
long long nCr[MAXN+5][MAXN+5];
// nCr[i][j] = \\(C_{n}^r\\)
void build_nCr() {
  for(int i = 1; i < MAXN+5; i++) {
    for(int j = 1; j < MAXN+5; j++) {
      if(i == j)
        nCr[i][j] = 1;
      else if(i > j)
        nCr[i][j] = nCr[i-1][j] * i / (i-j);
    }
  }
}
```

5.6 Stirling's approximation

```
double Stirling(int n){
        return (0.5*log(2.0*acos(-1.0)*n)+n*log(n+0.0)-n)/log(10.0);
}// n! Digits
```

6 String algorithms

6.1 Knuth Morris Pratt (KMP)

```
#include <iostream>
#include <bits/stdc++.h>
#define LOCAL
#define MAXN 100020
using namespace std;
string strA , strB ;
int b[MAXN] , p[MAXN] ;

void kmp_process(){
    int n = strB.length() ,i = 0 ,  j = -1 ;
    b[0] = -1 ;
    while(i < n ){
        while(j >= 0 && strB[i] != strB[j]) j = b[j] ;
        i++ ; j++ ;
        b[i] = j ;
    }

    //debug
//    for(int k = 0 ; k <= n ; k++)
//        cout << b[k] << ' ' ;
//    cout << '\n' ;
}

int kmp(){
    int n = strA.length() , m=strB.length() , i=0 , j=0  ;
    while(i < n ){
        while(j >= 0 && strA[i] != strB[j]) j = b[j] ;
        i++ ; j++ ;
    }
    return j ;
}


int main()
{
#ifdef LOCAL
    freopen("in1.txt" , "r" , stdin );
#endif // LOCAL

    while(cin >> strA){
        strB = strA;
        reverse(strB.begin() , strB.end());
        kmp_process();
        int n = kmp() ;
        cout << strA << strB.substr(n) << '\n' ;
    }
    return 0;
}
```

6.2 Longest palindromic substring

```
#include <iostream>
#include <bits/stdc++.h>
#define LOCAL
#define MAXN 1010
using namespace std;
int dp[MAXN][MAXN] = {} ;
string strA , strB ;
int n , m ;

int lcs(){
    n = strA.length();
    m = strB.length();
    for(int i = 0 ; i <= n ; i++) dp[i][0] = 0 ;
    for(int j = 0 ; j <= m ; j++) dp[j][0] = 0 ;
    for(int i = 1 ; i <= n ; i++){
        for(int j = 1 ; j <= m ; j++){
            if(strA[i-1] == strB[j-1]) dp[i][j] = dp[i-1][j-1]+1 ;
            else dp[i][j] = max(dp[i-1][j] , dp[i][j-1]);
        }
    }
    return dp[n][m] ;
}

int main()
{
#ifdef LOCAL
    freopen("in1.txt" , "r" , stdin );
#endif // LOCAL
    int t ;
    cin >> t ;
    cin.ignore();
    while(t--){
        getline(cin,strA);
        strB = strA ;
        reverse(strB.begin() , strB.end());
        cout << lcs() << '\n' ;
    }

    return 0;
}
```

6.3 Minimum edit distance

```
#include <iostream>
#include <bits/stdc++.h>
#define LOCAL
#define MAXN 100
using namespace std;
string strA , strB ;
int dis[MAXN][MAXN] , back_table[MAXN][MAXN] ;
int cnt , m , n  ;

void backtracking(int i , int j ){
    if(i==0 || j==0){
        while( i > 0 ){
            cout << cnt++ << " Delete " << i << '\n' ;
            i--;
        }
        while( j > 0){
            cout << cnt++ << " Insert " << i+1 << "," << strB[j-1] << '\n' ;
            j-- ;
        }
        return ;
    }

    if(strA[i-1] == strB[j-1])
        backtracking(i-1,j-1);
    else{
        if(dis[i][j] == dis[i-1][j-1]+1){
            cout << cnt++ << " Replace " << i << "," << strB[j-1] << '\n' ;
            backtracking(i-1,j-1);
        }
        else if(dis[i][j] == dis[i-1][j]+1){
            cout << cnt++ << " Delete " << i << '\n' ;
            backtracking(i-1,j) ;
        }
        else if(dis[i][j] == dis[i][j-1]+1){
            cout << cnt++ << " Insert " << i+1 << "," << strB[j-1] <<'\n'  ;
            backtracking(i,j-1);
        }

    }

}

void med(){ //Minimum Edit Distance
    for(int i = 0 ; i <= n ; i++) dis[i][0] = i ;
    for(int j = 0 ; j <= m ; j++) dis[0][j] = j ;
    for(int i = 1 ; i <= n ; i++){
        for(int j = 1 ; j <= m ; j++){
            if(strA[i-1] == strB[j-1]) dis[i][j] = dis[i-1][j-1] ;
            else dis[i][j] = min(dis[i-1][j-1], min(dis[i-1][j] , dis[i][j-1]))+1;
        }
    }
}


int main()
{
#ifdef LOCAL
    freopen("in1.txt" , "r" , stdin );
    freopen("out.txt" , "w" , stdout);
#endif // LOCAL
    cin.tie(0);
    cout.tie(0);
    ios::sync_with_stdio(false);
    int flag = 0 ;
    while(getline(cin ,strA) && getline(cin , strB)){
        n=strA.length() ;
        m=strB.length() ;
        cnt = 1 ;
        med();
        if(flag) cout << '\n' ;
        flag = 1 ;
        cout << dis[n][m] << '\n' ;
        backtracking(n,m);
    }
    return 0;
}
```

6.4 Z-algorithm

```
#include <iostream>
#include <bits/stdc++.h>
#define LOCAL
#define MAXN 1000020
using namespace std;
int z[MAXN] = {} ;
int x=0 , y=0 , maxn = 0;
string s ;


int main()
{
#ifdef LOCAL
    freopen("in1.txt","r",stdin);
#endif // LOCAL
```

```
    cin >> s ;
    for(int i = 1 ; i < s.length() ; i++ ){
        z[i] = max(0,min(z[i-x] , y - i + 1));
        while(i + z[i] < s.length() && s[z[i]] == s[i+z[i]] ){
            x = i ;
            y = i + z[i] ;
            z[i]++ ;
        }
    }

    for(int i = 0 ; i < s.length() ; i++)
        if(z[i] == s.length() - i && maxn >= s.length()-i ){
            cout << s.substr(0,z[i]);
            return 0 ;
        }
        maxn = max(maxn , z[i]);
    }
    cout << "Just a legend" ;
    return 0;
}
```

7 Data structures

7.1 Union-find disjoint sets (UFDS) by David

```
#include <iostream>
#include <bits/stdc++.h>
#define LOCAL

using namespace std;
int intSum[200080] , intParent[200080] , intSet[200080] ;


int find_root(int intA){
    if(intParent[intA] == intA)
        return intA ;
    intParent[intA] = find_root(intParent[intA]) ;
    return intParent[intA] ;
}


int each_debug(int n ){
    for(int i = 1 ; i <= n ; i++){
        cout << i << ’ ’ << intParent[i] << ’ ’  \
            << intSet[find_root(i)] << ’ ’ << intSum[find_root(i)] << ’\n’ ;
    }
    system("Pause") ;
}


int main()
{
#ifdef LOCAL
    freopen("in1.txt","r", stdin);
    freopen("out.txt" ,"w" , stdout) ;
#endif // LOCAL

    int n, m , operation , p , q  ;
    while(cin >> n >> m){
        for(int i = 1 ; i <= n ; i++){
            intParent[i] = i+n ;
            intParent[i+n] = i+n ;
            intSum[i+n] = i;
            intSet[i+n] = 1 ;
        }
        while(m--){
            cin >> operation ;
            if(operation == 1 ){
                cin >> p >> q ;
                int intRoot_p , intRoot_q ;
                intRoot_p = find_root(intParent[p]) ;
                intRoot_q = find_root(intParent[q]) ;
                if(intRoot_p != intRoot_q){
                    intParent[intRoot_q] = intRoot_p ;
                    intSum[intRoot_p] += intSum[intRoot_q] ;
                    intSet[intRoot_p] += intSet[intRoot_q] ;
                }
                //debug
                //each_debug(n) ;

            }
            else if (operation == 2 ){
                cin >> p >> q ;
                int intRoot_p , intRoot_q ;
                intRoot_p = find_root(intParent[p]) ;
                intRoot_q = find_root(intParent[q]) ;
                if(intRoot_p != intRoot_q){
                    intParent[p] = intRoot_q ;
                    intSum[intRoot_q] += p ;
                    intSum[intRoot_p] -= p ;
                    intSet[intRoot_q] ++ ;
                    intSet[intRoot_p] -- ;
                }
                //debug
                //each_debug(n) ;
            }
            else if (operation == 3){
                cin >> p ;
                cout << intSet[find_root(p)] << ’ ’ << intSum[find_root(p)] << ’\n’ ;
            }
        }
    }

    return 0;
}
```

7.2 Binary indexed/fenwick tree (BIT)

```
#include <iostream>
using namespace std;

#define LOGSZ 17

int tree[(1<<LOGSZ)+1];
int N = (1<<LOGSZ);

// add v to value at x
void set(int x, int v) {
  while(x <= N) {
    tree[x] += v;
    x += (x & -x);
  }
}

// get cumulative sum up to and including x
int get(int x) {
  int res = 0;
  while(x) {
    res += tree[x];
    x -= (x & -x);
  }
  return res;
}

// get largest value with cumulative sum less than or equal to x;
// for smallest, pass x-1 and add 1 to result
int getind(int x) {
  int idx = 0, mask = N;
  while(mask && idx < N) {
    int t = idx + mask;
    if(x >= tree[t]) {
      idx = t;
      x -= tree[t];
    }
    mask >>= 1;
  }
  return idx;
}
```

7.3 Rope

```
#include <iostream>
#include <bits/stdc++.h>
#include <ext/rope>
#define LOCAL
#define MAXN 50020
using namespace std;
using namespace __gnu_cxx ;

int main()
{
#ifdef LOCAL
    freopen("in1.txt" , "r" , stdin );
#endif // LOCAL
    int n , t , a , b , c , d=0 ;
    int v = 0 ;
    string strA ;
    rope<char> r[MAXN] , rtmp ;
    cin >> n ;
    while(n--){
        cin >> t ;

        if(t==1){
            cin >> a ;
            cin >> strA ;
            a -= d ;
            r[++v] = r[v] ;
            r[v].insert(a,strA.c_str());
            //debug
            //cout << r[v] << ’\n’ ;
        }
        else if(t==2){
            cin >> a >> b ;
            a -= d ; b -= d ;
            r[++v] = r[v] ;
            r[v].erase(a-1,b);
            //debug
            //cout << r[v] << ’ ’ << r[v-1] << ’\n’ ;
        }
        else if(t==3){
            cin >> a >> b >> c ;
            a -= d ; b -= d ; c -= d ;
            rtmp = r[a].substr(b-1,c) ;
            cout << rtmp << ’\n’ ;
            d += count(rtmp.begin() , rtmp.end() , ’c’ );
        }
    }
    return 0;
}
```

7.4 Segment tree

```
#include <iostream>
#include <bits/stdc++.h>
#include <string>
#define LOCAL
#define Lson(x) ((x << 1) +1)
#define Rson(x) ((x << 1) +2)
#define INF 99999999
using namespace std;
const int N = 100005 ;
int shift[35] , num[N] , len_shift ;
string strLine ;


struct Node{
    int left , right , Min_Value  ;
}node[4 * N ];


void build(int left , int right , int x = 0 ){
    node[x].left = left ;
    node[x].right = right ;
    if(left == right){
        node[x].Min_Value = num[left] ;
        return ;
    }
    int mid = (left + right ) / 2 ;

    //debug
    //cout << mid << ’\n’ ;
    //cout << x << ’ ’ << node[x].left << ’ ’ << node[x].right << ’ ’ << ’\n’ ;

    build(left , mid , Lson(x)) ;
    build(mid + 1 , right , Rson(x)) ;
    node[x].Min_Value = min(node[Lson(x)].Min_Value  , node[Rson(x)].Min_Value ) ;
}


void handle(){
    len_shift = 0 ;
    shift[len_shift] = 0;
    for(int i = 6 ; i < strLine.length() ; i++){
        if(strLine[i] >= ’0’ && strLine[i] <= ’9’ ){
            shift[len_shift] = shift[len_shift] * 10 + (int) (strLine[i] - ’0’ ) ;
        }
        else{
            shift[++len_shift ] = 0 ;
        }
    }
    //finaly char is ’)’ , so len_shift is right
    sort(shift , shift + len_shift ) ;

    //debug
    /**<
    for(int i = 0 ; i < len_shift ; i++)
        cout << shift[i] << ’ ’ ;
    cout << ’\n’ ;
    */
}


int query(int left , int right , int x = 0 ){
    if(node[x].left >= left && node[x].right <= right)
        return node[x].Min_Value ;
    int mid = (node[x].left + node[x].right ) / 2 ;
    int ans = INF ;

    //debug
    //cout << x << ’ ’ << node[x].left << ’ ’ << node[x].right << ’ ’ << node[x].Min_Value << ’\n’ ;

    if( left <= mid )
        ans = min(ans , query(left , right , Lson(x))) ;
    if(mid < right )
        ans = min(ans , query(left , right , Rson(x))) ;
    return ans ;
}


void set_num(int position , int value , int x = 0 ){
    if(node[x].left == position && node[x].right == position ){
       node[x].Min_Value = value ;
       return ;
    }
    int mid = (node[x].left + node[x].right ) / 2 ;
    if(position <= mid )
        set_num(position , value , Lson(x) );
    if(mid < position )
        set_num(position , value , Rson(x)) ;
    node[x].Min_Value = min(node[Lson(x)].Min_Value , node[Rson(x)].Min_Value );

}


int main()
{
    int n , q , intTemp ;
    ios::sync_with_stdio(0);
#ifdef LOCAL
    freopen("out.txt" , "w" , stdout ) ;
    freopen("in1.txt" , "r" , stdin ) ;
#endif // LOCAL
    cin >> n >> q ;
    for(int i = 1 ; i <= n ; i++)
        cin >> num[i] ;
    build(1,n);

    //debug
    /**<
    for(int i = 0 ; i < 13 ; i++){
        cout << node[i].left << ’ ’ << node[i].right << ’ ’ << node[i].Min_Value << ’\n’ ;
    }
    return 0 ;
    */

    while(q--){
        cin >> strLine ;
        if(strLine[0] == ’q’){
            handle();
            cout << query(shift[0] , shift[1] ) << ’\n’ ;
        }
        else if (strLine[0] == ’s’){
            handle();
            intTemp = num[shift[0]] ;

            for(int i = 1 ; i < len_shift ; i++){
                set_num(shift[i-1] , num[shift[i]]) ;
                num[shift[i-1]] = num[shift[i]] ;
            }
            num[shift[len_shift-1]] = intTemp ;
            set_num(shift[len_shift-1] , intTemp );

            //debug
            //cout << intTemp << ’ ’ << shift[len_shift-1] << ’\n’ ;
            //for(int i = 1 ; i <= n ; i++)
             //   cout << num[i] << ’ ’ ;
        }
    }
    return 0;
}
```

7.5 Suffix array

```
#include <iostream>
#include <bits/stdc++.h>
#define LOCAL
#define N 2000
using namespace std;
string strA="" , strB="" , strC="" ;
int sa[N] , rk[N<<1] , oldrk[N<<1] , id[N] , cnt[N] ;
int n , m , maxn , lenA , lenB , flag =0 ;

void build_sa(){
    int i , m , p , w ;
    n = strA.length()-1 ;
    m = max(n , 300 );
    memset(cnt,0,sizeof(cnt));
    memset(rk,0,sizeof(rk));
    for(i = 1 ; i <= n ; i++) ++cnt[rk[i] = (int)strA[i]] ;
    for(i = 1 ; i <= m ; i++) cnt[i] += cnt[i-1] ;
    for(i = n ; i >= 1 ; i--) sa[cnt[rk[i]]--] = i ;

    for(w = 1 ; w < n ; w <<= 1){
        memset(cnt,0,sizeof(cnt));
        for(i = 1 ; i <= n ; i++) id[i] = sa[i] ;
        for(i = 1 ; i <= n ; i++) ++cnt[rk[id[i]+w]] ;
        for(i = 1 ; i <= m ; i++) cnt[i] += cnt[i-1] ;
        for(i = n ; i >= 1 ; i--) sa[cnt[rk[id[i]+w]]--] = id[i] ;

        memset(cnt, 0 , sizeof(cnt));
        for(i = 1 ; i <= n ; i++) id[i] = sa[i] ;
        for(i = 1 ; i <= n ; i++) ++cnt[rk[id[i]]] ;
        for(i = 1 ; i <= m ; i++) cnt[i] += cnt[i-1] ;
        for(i = n ; i >= 1 ; i--) sa[cnt[rk[id[i]]]--] = id[i] ;

        memcpy(oldrk , rk , sizeof(rk));
        for(p = 0 , i = 1 ; i <= n ; i++){
            if(oldrk[sa[i]] == oldrk[sa[i-1]] &&
               oldrk[sa[i] + w] == oldrk[sa[i-1] + w])
                rk[sa[i]] = p ;
            else
                rk[sa[i]] = ++p ;
        }
    }

    //debug
//    cout << "Suffix Array is:\n" ;
//    for(int i = 1 ; i <= n ; i++){
//        cout << i << ' ' << strA.substr(sa[i]) << ' ' <<sa[i] << '\n' ;
//    }
}

void build_lcp(){
    int lcp[N] = {} ;
    int max_lcp = 0 ;
    for(int i = 1 , k = 0 ; i <= n ; i++){
        if(k) k-- ;
        while(strA[i+k] == strA[sa[rk[i]-1]+k]) ++k ;
        lcp[rk[i]] = k ;
    }

    for(int i = 1 ; i <= n ; i++){
        if((sa[i] < lenA && sa[i-1] < n+1 && sa[i-1] > lenA ) ||
            (sa[i] > lenA && sa[i-1] < n+1 && sa[i-1] < lenA))
            max_lcp = max(max_lcp , lcp[i]);
    }

    //debug
//    cout << "max_lcp is " << max_lcp << '\n' ;
//    for(int i = 0 ; i <= n ; i++)
//        cout << i << ' ' << lcp[i] << '\n' ;
//    cout << "lenA =" << lenA << "\nn is" << n << '\n' ;

    if(flag)
        cout << '\n' ;
    flag = 1 ;

    map<string,int> mp ;
    if(max_lcp == 0)
        {cout << "No common sequence.\n" ; return ;}
    for(int i = 1 ; i <= n ; i++){
        if((sa[i] < lenA && sa[i-1] < n+1 && sa[i-1] > lenA ) ||
           (sa[i] > lenA && sa[i-1] < n+1 && sa[i-1] < lenA))
           if(lcp[i] == max_lcp){
                string temp = strA.substr(sa[i] , max_lcp);
                if(mp[temp]) continue ;
                else mp[temp] = 1 ;
                cout << temp << '\n' ;
           }
    }
}

int main()
{
#ifdef LOCAL
    freopen("in1.txt" , "r" , stdin );
    //freopen("out.txt" , "w" , stdout );
#endif // LOCAL

    while(cin >> strA >> strB){
        lenA = strA.length()+1;
        lenB = strB.length();
        strA = ' ' + strA + '$' + strB + '#' ;
        //debug
        //cout << "strA is " <<strA << "\nstrA.length() is " << strA.length() << '\n' ;

        build_sa();
        build_lcp();
    }

    return 0;
}
```

7.6 Suffix tree

```
#include <iostream>
#include <bits/stdc++.h>
#define LOCAL
#define T 110
#define N 100100
using namespace std;
int root , cnt , pos , needSL , remainder_ , // note:remainder is cmath function
    active_node , active_e , active_len ;
string text ;
int oo ;
int max_lrs = 0 , lrs_index = 0 , lrs_repeat = 0 ;

struct node{
    int start , end ,slink ;
    map<char,int> next ;

    int edge_length(){
        return min(end , pos+1) - start ;
    }

    void init(int st , int ed = oo){
        start = st ;
        end = ed ;
        slink = 0 ;
        next.clear() ;
    }
}tree[2*N];

char active_edge(){
    return text[active_e] ;
}

void add_SL(int node){
    if(needSL > 0 ) tree[needSL].slink = node ;
    needSL = node ;
}

bool walkdown(int node){
    if(active_len >= tree[node].edge_length()){
        active_e += tree[node].edge_length() ;
        active_len -= tree[node].edge_length() ;
        active_node = node ;
        return true ;
    }
    return false ;
}

void st_init(){
    //tree root is 1 not zero
    needSL = remainder_ = 0 ;
    active_node = active_e = active_len = 0 ;
    pos = -1 ;

    cnt = root = 1 ;
    active_node = 1 ;
    tree[cnt++].init(-1,-1);
    return ;
}

void st_extend(char c){
    pos++;
    needSL = 0 ;
    remainder_++ ;
    while(remainder_ > 0){
        if(active_len == 0 ) active_e = pos ;
        if(tree[active_node].next[active_edge()] == 0){
            int leaf = cnt ;
            tree[cnt++].init(pos) ;
            tree[active_node].next[active_edge()] = leaf ;
            add_SL(active_node) ;
        }
        else{
            int nxt = tree[active_node].next[active_edge()] ;
            if(walkdown(nxt)) continue ;
            if(text[tree[nxt].start + active_len] == c){
                active_len++ ;
                add_SL(active_node) ;
                break ;
            }

            int split = cnt ;
            tree[cnt++].init(tree[nxt].start , tree[nxt].start + active_len) ;
            tree[active_node].next[active_edge()] = split ;
            int leaf = cnt ;
            tree[cnt++].init(pos) ;
            tree[split].next[c] = leaf ;
            tree[nxt].start += active_len ;
            tree[split].next[text[tree[nxt].start]] = nxt ;
            add_SL(split) ;
        }
        remainder_-- ;
        if(active_node == root && active_len > 0 ){
            active_len -- ;
            active_e = pos - remainder_ + 1 ;
        }
        else{
            active_node = tree[active_node].slink > 0 ? tree[active_node].slink : root ;
        }
    }
    return ;
}

void debug(){
        for(int i = 0 ; i < cnt ; i++){
        cout << i << ' ' << tree[i].start << ' ' << tree[i].end << ' ' << tree[i].slink << '\n' ;
        for(auto it : tree[i].next)
            cout << it.first << ' ' << it.second << '\n' ;
    }
    return ;
}

void lrs_dfs(int r , int len , int repeats ){ //dfs for suffix tree
    for(auto it : tree[r].next){
        lrs_dfs(it.second , len + tree[r].edge_length() , tree[r].next.size());
    }
    if(tree[r].slink == 0 && len > max_lrs){
        lrs_repeat = repeats ;
        max_lrs = len ;
        lrs_index = tree[r].start - len ;
    }
    return ;
}


int main()
{
#ifdef LOCAL
    freopen("in1.txt" , "r" , stdin );
    //freopen("out.txt" , "w" , stdout );
#endif // LOCAL
    int n ;
    cin >> n ;
    while(n--){
        cin >> text ;

        st_init() ;
        text += "$" ;
        oo = text.length() ;
        for(int i = 0 ; i < text.length() ; i++) st_extend(text[i]);

        max_lrs = 0 , lrs_index = 0 , lrs_repeat = 0 ;
        lrs_dfs(root , 0,0);
        if(max_lrs)
            cout << text.substr(lrs_index , max_lrs) << ' ' << lrs_repeat << '\n' ;
        else cout << "No repetitions found!\n" ;
    }
    return 0;
}
```

7.7 Trie

```
#include <iostream>
#include <bits/stdc++.h>
#define LOCAL
#define alp_MAXN 15
#define arr_MAXN 1000009
using namespace std;
int cnt = 0 , flag = 0 ;
string strA ;

struct node{
    bool isWord = false ;
    int next[alp_MAXN];

    void reset(){
        for(int i = 0 ; i < alp_MAXN ; i++)
            next[i] = -1 ;
        isWord = false ;
    }

}trie[arr_MAXN];

void insrt(){
    int c , cur = 0;
    for(int i = 0 ; i < strA.length() ; i++){
        c = strA[i] - '0' ;
        if(trie[cur].next[c] == -1 ){
            trie[cnt].reset();
            trie[cur].next[c] = cnt ;
            cur = cnt++ ;
        }
        else{
            cur = trie[cur].next[c];
            if(trie[cur].isWord || i == strA.length()-1 ) {flag = 1 ;return ;}
        }
        if(flag) break ;
    }
    trie[cur].isWord = true ;
}

int main()
{
#ifdef LOCAL
    freopen("in1.txt" , "r" , stdin );
    //freopen("out.txt" , "w" , stdout );
#endif // LOCAL

    int t , n ;
    cin >> t ;

    while(t--){
        cin >> n ;
        cnt = 1 ;
        flag = 0 ;
        trie[0].reset();
        for(int i = 0 ; i < n ; i++){
            cin >> strA ;
            insrt();
        }
        cout << (flag? "NO" : "YES") << '\n' ;

        //debug
//        for(int i = 0 ; i < 20 ; i++){
//            for(int j = 0 ; j < 10 ; j++)
//                cout << trie[i].next[j] << ' ' ;
//            cout << trie[i].isWord ;
//            cout << '\n' ;
//        }
    }

    return 0;
}
```

7.8 Union-find disjoint sets (UFDS) by Bill

```
class UnionFind
{
public:
    UnionFind(int N)
    {
        rank.assign(N, 0);
        p.assign(N, 0);
        for (int i = 0; i < N; ++i) p[i] = i;
    }
    int findSet(int i) { return (p[i] == i) ? i : ( p[i] = findSet(p[i]) ); }
    bool isSameSet(int i, int j) { return findSet(i) == findSet(j); }
    void unionSet(int i, int j)
    {
        if ( !isSameSet(i, j) )
        {
            int x = findSet(i);
            int y = findSet(j);
            if (rank[x] > rank[y]) p[y] = x;    // rank keeps the tree short
            else
            {
                p[x] = y;
                if (rank[x] == rank[y]) ++rank[y];
            }
        }
    }
private:
    vi p, rank;
};
```

8 Utilities

8.1 Bit manipulation

```
#define isOn(S, j) (S & (1<<j))
#define setBit(S, j) (S |= (1<<j))
#define clearBit(S, j) (S &= ~(1<<j))
#define toggleBit(S, j) (S ^= (1<<j))
#define lowBit(S) (S & (-S))
#define setAll(S, n) (S = (1<<n)-1)
```

8.2 C++ input output

```
#include <iostream>
#include <iomanip>

using namespace std;

int main()
{
    // Ouput a specific number of digits past the decimal point,
    // in this case 5
    cout.setf(ios::fixed); cout << setprecision(5);
    cout << 100.0/7.0 << endl;
    cout.unsetf(ios::fixed);

    // Output the decimal point and trailing zeros
    cout.setf(ios::showpoint);
    cout << 100.0 << endl;
    cout.unsetf(ios::showpoint);

    // Output a '+' before positive values
    cout.setf(ios::showpos);
    cout << 100 << " " << -100 << endl;
    cout.unsetf(ios::showpos);

    // Output numerical values in hexadecimal
    cout << hex << 100 << " " << 1000 << " " << 10000 << dec << endl;
}
```

8.3 C++ STL

```
// Example for using stringstreams and next_permutation

#include <algorithm>
#include <iostream>
#include <sstream>
#include <vector>

using namespace std;

int main(void){
  vector<int> v;

  v.push_back(1);
  v.push_back(2);
  v.push_back(3);
  v.push_back(4);

  // Expected output: 1 2 3 4
  //                  1 2 4 3
  //                  ...
  //                  4 3 2 1
  do {
    ostringstream oss;
    oss << v[0] << " " << v[1] << " " << v[2] << " " << v[3];

    // for input from a string s,
    //   istringstream iss(s);
    //   iss >> variable;

    cout << oss.str() << endl;
  } while (next_permutation (v.begin(), v.end()));

  v.clear();

  v.push_back(1);
  v.push_back(2);
  v.push_back(1);
  v.push_back(3);

  // To use unique, first sort numbers.  Then call
  // unique to place all the unique elements at the beginning
  // of the vector, and then use erase to remove the duplicate
  // elements.

  sort(v.begin(), v.end());
  v.erase(unique(v.begin(), v.end()), v.end());

  // Expected output: 1 2 3
  for (size_t i = 0; i < v.size(); i++)
    cout << v[i] << " ";
  cout << endl;
}
```

8.4 Dates

```
// Routines for performing computations on dates.  In these routines,
// months are expressed as integers from 1 to 12, days are expressed
// as integers from 1 to 31, and years are expressed as 4-digit
// integers.

#include <iostream>
#include <string>

using namespace std;

string dayOfWeek[] = {"Mon", "Tue", "Wed", "Thu", "Fri", "Sat", "Sun"};

// converts Gregorian date to integer (Julian day number)
int dateToInt (int m, int d, int y){
  return
    1461 * (y + 4800 + (m - 14) / 12) / 4 +
    367 * (m - 2 - (m - 14) / 12 * 12) / 12 -
    3 * ((y + 4900 + (m - 14) / 12) / 100) / 4 +
    d - 32075;
}

// converts integer (Julian day number) to Gregorian date: month/day/year
void intToDate (int jd, int &m, int &d, int &y){
  int x, n, i, j;

  x = jd + 68569;
  n = 4 * x / 146097;
  x -= (146097 * n + 3) / 4;
  i = (4000 * (x + 1)) / 1461001;
  x -= 1461 * i / 4 - 31;
  j = 80 * x / 2447;
  d = x - 2447 * j / 80;
  x = j / 11;
  m = j + 2 - 12 * x;
  y = 100 * (n - 49) + i + x;
}

// converts integer (Julian day number) to day of week
string intToDay (int jd){
  return dayOfWeek[jd % 7];
}

int main (int argc, char **argv){
  int jd = dateToInt (3, 24, 2004);
  int m, d, y;
  intToDate (jd, m, d, y);
  string day = intToDay (jd);

  // expected output:
  //    2453089
  //    3/24/2004
  //    Wed
  cout << jd << endl
    << m << "/" << d << "/" << y << endl
    << day << endl;
}
```

8.5 Prime numbers

```
// Primes less than 1000:
//      2     3     5     7    11    13    17    19    23    29    31    37
//     41    43    47    53    59    61    67    71    73    79    83    89
//     97   101   103   107   109   113   127   131   137   139   149   151
//    157   163   167   173   179   181   191   193   197   199   211   223
//    227   229   233   239   241   251   257   263   269   271   277   281
//    283   293   307   311   313   317   331   337   347   349   353   359
//    367   373   379   383   389   397   401   409   419   421   431   433
//    439   443   449   457   461   463   467   479   487   491   499   503
//    509   521   523   541   547   557   563   569   571   577   587   593
//    599   601   607   613   617   619   631   641   643   647   653   659
//    661   673   677   683   691   701   709   719   727   733   739   743
//    751   757   761   769   773   787   797   809   811   821   823   827
//    829   839   853   857   859   863   877   881   883   887   907   911
//    919   929   937   941   947   953   967   971   977   983   991   997

// Other primes:
//    The largest prime smaller than 10 is 7.
//    The largest prime smaller than 100 is 97.
//    The largest prime smaller than 1000 is 997.
//    The largest prime smaller than 10000 is 9973.
//    The largest prime smaller than 100000 is 99991.
//    The largest prime smaller than 1000000 is 999983.
//    The largest prime smaller than 10000000 is 9999991.
//    The largest prime smaller than 100000000 is 99999989.
//    The largest prime smaller than 1000000000 is 999999937.
//    The largest prime smaller than 10000000000 is 9999999967.
//    The largest prime smaller than 100000000000 is 99999999977.
//    The largest prime smaller than 1000000000000 is 999999999989.
//    The largest prime smaller than 10000000000000 is 9999999999971.
//    The largest prime smaller than 100000000000000 is 99999999999973.
//    The largest prime smaller than 1000000000000000 is 999999999999989.
//    The largest prime smaller than 10000000000000000 is 9999999999999937.
//    The largest prime smaller than 100000000000000000 is 99999999999999997.
//    The largest prime smaller than 1000000000000000000 is 999999999999999989.
```

8.6 Theorems

Euler path/tour theorems: An Euler path is a path that visits every edges exactly once. An Euler tour is an Euler path that starts and ends at the same vertex. A graph is an Eulerian-tour graph (i.e. it has an Euler tour) iff all of its vertices has even degrees. A graph is an Eulerian-path graph (i.e. it has an Euler path) iff all but 2 of its vertices has even degrees.

Euler's handshaking lemma: A graph does not have an Euler tour iff it has an even number of vertices of odd degrees.

Bipartite graph related theorems:
(1) Min vertex cover (MVC) = Max cardinality bipartite matching (MCBM).
(2) Max independent set (MIS) = V - MCBM.
(3) The number of spanning tree of a complete bipartite graph K(n,m) is m^(n-1) * n^(m-1).

Cayley's formula: There are n^(n-2) spanning trees of a complete graph with n labeled vertices.

Derangement: A permutation of the elements of a set such that none of the elements appear in their original position. The number of derangements 'der(n)' can be computed as follow: der(n) = (n-1) * (der(n-1) + der(n-2)) where der(0) = 1 and der(1) = 0.

Erdos Gallai's theorem: A necessary and sufficient condition for a finite sequence of natural numbers is the degree sequence of a simple graph. A sequence of non-negative integers d_1 >= d_2 >= ... >= d_n can be the degree sequence of a simple graph on n vertices iff
(1) sum[i: 1->n]{d_i} is even, and
(2) sum[i: 1->k]{d_i} <= k * (k-1) + sum[i: k+1->n]{min(d_i, k)} holds for 1 <= k <= n.