NTUT_Kn1ghts ICPC Team Notebook

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1 Dynamic programming algorithms

1.1 Longest common subsequence (LCS)

```
#include <iostream>
#include <bits/stdc++.h>
using namespace std;
struct LCS{
int step , max_len ;
}Dp[5000][5000];
int main()
    freopen("in1.txt" , "r" , stdin );
#endif // LOCAL
    int intX , intY , Min_step , Max_len ;
    while(cin >> intX >> strX >> intY >> strY ) {
        for(int i = 0 ; i <= intY ; i++) {</pre>
            Dp[0][i].max\_len = 0 ;
            Dp[0][i].step = i ;
        for (int i = 0 ; i <= intX ; i++) {</pre>
            Dp[i][0].max_len = 0;
            Dp[i][0].step = i ;
        Max_len = 0;
        Min_step = 0;
```

2 Graph algorithms

2.1 All-pairs shortest paths (APSP)

```
// All-Pairs Shortest Paths (APSP) solved with Floyd Warshall O(V^3).
// inside int main()
// precondition: AdjMat[i][j] contains the weight of edge (i, j)
// or INF (1B) if there is no such edge
// AdjMat is a 32-bit signed integer array
for (int k = 0; k < V; ++k)
    for (int i = 0; i < V; ++i)
    for (int j = 0; j < V; ++j)
        AdjMat[i][j] = min( AdjMat[i][j], AdjMat[i][k] + AdjMat[k][j] );</pre>
```

2.2 Centroid decomposition

```
#include<iostream>
#include <bits/stdc++.h>
#define LOCAL
#define MAXN 50005
using namespace std;
int n , k , a , b ;
int ans , cnt ;
int Max[MAXN] , sz[MAXN] , rt ;
int head[MAXN], dis[MAXN];
bool vis[MAXN];
struct node{
    int v , nx ;
}Edge[MAXN*2];
void init(int n ) {
    Max[0] = n;
    ans = cnt = 0 ;
for(int i = 0 ; i <= n ; i++) {</pre>
        head[i] = -1;
        vis[i] = 0 ;
void add(int u , int v) {
    Edge[cnt].v = v;
    Edge[cnt].nx = head[u] ;
    head[u] = cnt++ ;
void get_rt(int u , int fa ){
    sz[u] = 1 ; Max[u] = 0 ;
    for(int i = head[u] ; ~i ; i=Edge[i].nx) {
        int v = Edge[i].v ;
        if(vis[v] | v == fa ) continue;
        get_rt(v,u);
sz[u] += sz[v];
        Max[u] = max(Max[u], sz[v]);
    Max[u] = max(Max[u], n - sz[u]);
```

```
if(Max[rt] > Max[u])
       rt = u;
void get_dis(int u , int fa , int d) {
    for(int i = head[u] ; ~i ; i= Edge[i].nx){
       int v = Edge[i].v ;
        if(vis[v] \mid \mid v == fa) continue;
        dis[++cnt] = d + 1;
        get_dis(v,u,dis[cnt]);
int get_ans(int u , int d ){
    dis[cnt=1] = d;
    get_dis(u, 0, d);
    sort(dis+1 , dis+cnt+1) ;
    int 1 = 1 , ans = 0 ;
    while(1 < cnt && dis[1] + dis[cnt] < k ) 1++ ;
    while(1 < cnt && dis[1] <= k - dis[1]){
        ans += upper_bound(dis + 1 + 1 , dis + cnt + 1 , k - dis[1]) - lower_bound(dis+1+1 , dis+cnt+1
              , k-dis[1]);
       1++ •
    return ans :
void dfs(int u ) {
    vis[u] = 1 :
    //cout << rt << ' ' << u << '\n';
    ans += get_ans(u , 0);
    for (int i = head[u]; ~i; i = Edge[i].nx) {
       int v = Edge[i].v ;
        if(vis[v]) continue;
       ans -= get_ans(v , 1) ;
n = sz[v] , rt = 0 , get_rt(v,u);
        dfs(rt);
int main(){
//#ifdef LOCAL
     freopen("in1.txt", "r", stdin);
//#endif // LOCAL
    cin >> n >> k;
    init(n);
    for (int i =1; i < n ; i++) {</pre>
        cin >> a >> b;
       add(a,b);
       add(b, a);
   rt = 0 ; get_rt(1,0);
    dfs(rt):
    cout << ans << '\n' ;
```

2.3 Detect negative weight cycle

2.4 DFS

```
#include <iostream>
#include <bits/stdc++.h>
#define LOCAL
using namespace std;
int m , n , flag=1;
int Maxn_city = 0 , Maxn_path = 0 ;
vector<int>tree(200020] ;
int city[200020] = {};
int visit[200020] = {};
```

```
vector<int> travel ;
void BFS_to_large_path(int root ) {
    visit[root] = 1;
    travel.push_back(root);
    for(int i = 0 ; i < tree[root].size() ; i++){</pre>
        int node = tree[root][i] ;
        if(!visit[node]){
            BFS_to_large_path(node);
            travel.pop_back();
            visit[root] = 0 ;
    //debug to check large path
    //if (root == 1)
// cout << "1=" << travel.size() << ' ' << Maxn_path << ' ' << city[root] << '\n';
    if(city[root] && travel.size() > Maxn_path){
        Maxn_city = travel[travel.size()/2];
        Maxn_path = travel.size();
void BFS_to_other_path(int root ,int path) {
    visit[root] = 1 ;
for(int i = 0 ; i < tree[root].size() ; i++){</pre>
        int node = tree[root][i];
        if(!visit[node]){
            BFS_to_other_path(node , path+1);
visit[root] = 0 ;
    //debug
    if(root == 1 )
        cout << "city=" << root << " path= " << path << '\n' ;
    if(city[root] && path != Maxn_path)
        flag = 0;
int main() {
#ifdef LOCAL
    freopen("in1.txt" , "r" , stdin);
#endif // LOCAL
    cin >> n >> m;
    int a , b ;
    for (int i = 0; i < n-1; i++) {
        cin >> a >> b;
        tree[a].push_back(b) ;
        tree[b].push_back(a);
    for(int i = 0 ; i < m ; i++) {</pre>
        cin >> a :
        city[a] = 1;
    BFS_to_large_path(a);
    //visit[a] = 0 ;
    BFS_to_other_path(Maxn_city , 1 );
    if(flag)
        cout << "YES\n" << Maxn_city ;
    else
        cout << "NO" ;
    cout << "Maxn_path= " << Maxn_path << " Maxn_city= " << Maxn_city << '\n' ;</pre>
```

2.5 Dijkstra by Bill

```
// Dijkstra implementation for negative weight edges O((V + E) log V)
vi dist(V, INF); dist[s] = 0;
priority_queue< ii, vii, greater<ii>> pq;
pq.push( ii(0, s) );
while (!pq.empty())
{
    if front = pq.top(); pq.pop();
    int d = front.first;
    int u = front.second;
    if (d > dist[u]) continue;
    for (int i = 0; i < (int)AL[u].size(); ++i) // [A]djacency [L]ist
    {
        ii vw = AL[u][i];
        int v = vw.first;
        int w = vw.second;
        if (dist[u] + w < dist[v])</pre>
```

```
{
    dist[v] = dist[u] + w;  // relax operation
    pq.push( ii(dist[v], v) );
}
}// this variant can cause duplicate items in the priority queue
```

2.6 Dijkstra by David

```
#include <iostream>
#include <bits/stdc++.h>
#define LOCAL
#define INF 99999999
using namespace std;
int intMap[1010][1010] = {} , intValue[1010][1010] = {};
int m , n ;
struct Node {
    int x , y , v ;
    \label{eq:void_read} \mbox{ void read( int $\_x$ , int $\_y$ , int $\_v$)} \{
         x = _x ; y = _y ; v = _v ;
    bool operator < (const Node &a) const{
         return v > a.v ;
| nodNode:
void print_map() {
    for(int i = 1 ; i <= n ; i++) {</pre>
         for(int j = 1 ; j <= m ; j++) {
   if(intValue[i][j] == 99999999)
        cout << 'r' << ' ';</pre>
              else
                  cout << intValue[i][j] << ' ';
         cout << '\n' ;
    cout << '\n' ;
void bfs() {
    int x , y , intDirection[4][2] = {-1,0,0,1,1,0,0,-1};
    int intDx , intDy ;
    Node nodTemp ;
    priority_queue<Node> deqNode ;
    nodTemp.read(1,1,0);
    degNode.push (nodTemp);
    while(deqNode.size()){
         x = deqNode.top().x;
         y = deqNode.top().y;
         deqNode.pop() ;
         for (int i = 0; i < 4; i++) {
              intDx = intDirection[i][0] + x;
              intDy = intDirection[i][1] + y;
              //cout << intDx << ' ' << intDy << ' ' << intValue[x][y] + intMap[intDx][intDy] << ' ' <<
              if(intValue[x][y] + intMap[intDx][intDy] < intValue[intDx][intDy] ){</pre>
                  intValue[intDx][intDy] = intValue[x][y] + intMap[intDx][intDy];
nodTemp.read(intDx , intDy , intValue[intDx][intDy]);
                   degNode.push(nodTemp) :
         //print_map();
int main() {
#ifdef LOCAL
    freopen("in1.txt" , "r" , stdin );
freopen("out.txt" , "w" , stdout) ;
#endif
ios::sync_with_stdio(false);
    int intCase;
cin >> intCase;
    while (intCase --) {
         cin >> n >> m;
for(int i = 1; i <= n; i++) {
```

for (int j = 1; $j \le m$; j++) {

```
cin >> intMap[i][j];
    intValue[i][j] = INF;
}

for(int i = 1; i <= n; i++){
    intValue[i][0] = 0;
    intValue[i][m+1] = 0;
    intMap[i][0] = INF +1;
    intMap[i][m+1] = INF +1;
}
for(int i = 1; i <= m; i++){
    intValue[0][i] = 0;
    intValue[0][i] = 0;
    intValue[n+1][i] = 0;
    intMap[0][i] = INF +1;
    intMap[0][i] = INF +1;
    intMap[0][i] = INF +1;
}
intValue[1][i] = intMap[1][i];

//debug
//cout << intValue[1][i] << '\n';
bfs();
cout << intValue[n][m] << '\n';
}
return 0;</pre>
```

2.7 Euler tour

```
list<int> cyc; // we need list for fast insertion in the middle
void EulerTour(list<int>::iterator i, int u)
    for (int j = 0; j < (int)AL[u].size(); ++j) // [A]djacency [L]ist
       ii\& vw = AL[u][j];
       int v = vw.first;
       if (vw.second)
                        // if this edge can still be used
            vw.second = 0; // remove this edge
            // remove bi-directional edge
            for (int k = 0; k < (int)AL[v].size(); ++k)</pre>
                ii& uw = AL[v][k];
                if (uw.first == u && uw.second)
                    uw.second = 0;
                   break;
            // continue the tour
           EulerTour(cyc.insert(i, u), v);
// inside int main()
    EulerTour(cyc.end(), A); // 'cyc' contains an Euler tour starting at 'A'
    for (list<int>::iterator i = cyc.begin(); i != cyc.end(); ++i)
       printf("%d\n", *i);
```

2.8 Find articulation points and bridges

```
dfs_low[u] = min( dfs_low[u], dfs_low[v] );  // update dfs_low[u]
        else if (v != dfs_parent[u]) dfs_low[u] = min( dfs_low[u], dfs_num[v] ); // update dfs_low[u]
// inside int main()
    dfsNumberCounter = 0;
    dfs_num.assign(V, UNVISITED);
    dfs_low.assign(v, 0);
    dfs_parent.assign(V, 0);
    articulation_vertex.assign(V, 0);
    printf("Bridges:\n");
    for (int u = 0; u < V; ++u)
   if (dfs_num[u] == UNVISITED)</pre>
            dfsRoot = u;
            rootChildren = 0;
            articulationPointAndBridge(u);
            articulation_vertex[dfsRoot] = (rootChildren > 1);  // special case
    printf("Articulation Points:\n");
    for (int u = 0; u < V; ++u)
        if (articulation_vertex[u]) printf(" Vertex %d\n", u);
```

2.9 Floyd Warshall by David

```
#include <iostream>
#include <bits/stdc++.h>
#define LOCAL
using namespace std;
char before[5201[520] = {} ;
int after[520][520] = {};
int main()
    freopen("in1.txt" , "r" , stdin );
#endif // LOCAL
    int n ;
    cin >> n ;
    for(int i = 0 ; i < n ; i++) {</pre>
        for(int j = 0 ; j < n ; j++)
    cin >> before[i][j] ;
    for (int i = 0; i < n; i++) {
        for(int j = i+1; j < n; j++) {
             int sum = 0 ;
             for (int k = i + 1 ; k < j ; k++) {
                 if(after[i][k])
                    sum += before[k][j]-'0';
             if( (sum +1) % 10 == before[i][j] - '0'){
                 after[i][j] = 1;
    for (int i = 0 ; i < n ; i++) {</pre>
        for (int j = 0; j < n; j++)
            cout << after[i][j];</pre>
        cout << '\n' ;
    return 0;
```

2.10 Graph edges property check

2.11 Kruskal by David

```
#include <iostream>
#include <bits/stdc++.h>
#define LOCAL
#define || long long
using namespace std;
int parent[1020];
struct edge{
    ll n1 , n2 , w ;
}node[25020];
int compare(edge A , edge B ) {
    return A.w < B.w ;
int find_root(int a){
    if(a != parent[a] )
        return parent[a] = find_root(parent[a]);
    return a ;
int main()
#ifdef LOCAL
    freepen("in1.txt" , "r" , stdin );
freepen("out.txt" , "w" , stdout );
#endif // LOCAL
    int n , m , p_n1 , p_n2 ; // parent_n1 , parent_n2
vector<int> hce ; //heavy edge circle
    while (cin >> n >> m && n + m != 0 ) {
        for(int i = 0; i < m; i++) {
            cin >> node[i].nl >> node[i].n2 >> node[i].w ;
        for (int i = 0; i < n; i++)
           parent[i] = i ;
        sort(node , node + m , compare ) ;
        hce.clear();
         //kruskal
        for(int i = 0 : i < m : i++) {
            p_n1 = find_root(node[i].n1);
             p_n2 = find_root(node[i].n2);
            if (p_n1 != p_n2 )
                parent[p_n2] = p_n1;
            else
                hce.push_back(node[i].w);
             for(int i = 0 ; i < n ; i++)
                 cout << parent[i] << ' ';
             cout << '\n' ;
        sort(hce.begin() , hce.end());
        if(hce.size()){
            for(int i = 0 ; i < hce.size()-1 ; i++)
                cout << hce[i] << ' ';
            cout << hce[hce.size()-1];</pre>
```

```
}
else
    cout << "forest";
cout << '\n';
}
return 0;</pre>
```

2.12 Max flow

```
int res[MAX_V][MAX_V], mf, f, s, t;
vi p; // p stores the BFS spanning tree from s
void augment(int v. int minEdge)
    if (v == s) { f = minEdge; return; }
    else if (p[v] != -1)
         augment( p[v], min(minEdge, res[ p[v] ][ v ]) );
        res[ p[v] ][ v ] -= f;
        res[ v ][ p[v] ] += f;
// inside int main(): set up 'res', 's', and 't' with appropriate values
    while (true) // O(V^3 * E) Edmonds Karp s algorithm
        vi dist(MAX_V, INF); dist[s] = 0;
        queue<int> q; q.push(s);
        p.assign(MAX_V, -1);
         while (!q.empty())
             int u = q.front(); q.pop();
             if (u == t) break; // immediately stop BFS if we already reach sink t
             for (int v = 0; v < MAX_V; ++v)
   if (res[u][v] > 0 && dist[v] == INF)
                      dist[v] = dist[u] + 1, q.push(v), p[v] = u;
        augment(t, INF); // find the min edge weight f in this path, if any if (f == 0) break; // we cannot send any more flow (f == 0), terminate
                             // we can still send a flow, increase the max flow!
    printf("%d\n", mf);
```

2.13 Max cardinality bipartite matching (MCBM)

```
// Max Cardinality Bipartite Matching (MCBM) solved with augmenting path algorithm O(VE).
vi match, vis;
int Aug(int 1)
                // return 1 if an augmenting path is found & 0 otherwise
    if (vis[1]) return 0;
    vis[1] = 1;
    for (int i = 0; i < (int)AL[1].size(); ++i) // [A]djacency [L]ist</pre>
       int \ r = AL[1][i]; // edge weight not needed -> vector< vi > AL
       if ( match[r] == -1 || Aug(match[r]) )
            match[r] = 1;
            return 1; // found 1 matching
    return 0;
                        // no matchings
// inside int main()
    // build unweighted bipartite graph with directed edge left->right set
    match.assign(V, -1); // V is the number of vertices in bipartite graph
    for (int 1 = 0; i < N; ++1) // N = size of the left set
       vis.assign(N, 0);
                           // reset before each recursion
       MCBM += Aug(1);
    printf("Found %d matchings\n", MCBM);
```

2.14 Minimum Spanning Tree (MST)

```
// Minimum Spanning Tree (MST) solved with Kruskal O(E log V)
// inside int main()
    vector< pair<int, ii> > EdgeList; // (weight, two vertices) of the edge
   for (int i = 0; i < E; ++i)
       scanf("%d %d %d", &u, &v, &w);
       EdgeList.push_back( make_pair( w, ii(u, v) ) );
    sort(EdgeList.begin(), EdgeList.end()); // sort by edge weight O(E log E)
   int mst_cost = 0;
                       // all V are disjoint sets initially
   UnionFind UF(V);
   for (int i = 0; i < E; ++i)
       pair<int, ii> front = EdgeList[i];
       if (!UF.isSameSet(front.second.first, front.second.second))
            mst_cost += front.first;
           UF.unionSet(front.second.first, front.second.second);
   printf("MST cost = %d\n", mst_cost);
```

2.15 Strongly connected component (SCC)

```
// Tarjan O(V + E)
vi dfs_num, dfs_low, visited;
int dfsNumberCounter, numSCC;
vi S:
void tarjanSCC(int u)
    dfs_low[u] = dfs_num[u] = dfsNumberCounter++;  // dfs_low[u] <= dfs_num[u]</pre>
    S.push_back(u);
                      // stores 'u' in a vector baesd on order of visitation
    visited[u] = 1;
    for (int i = 0; i < (int)AL[u].size(); ++i) // [A]djacency [L]ist</pre>
        int v = AL[u][i].first;
        if (dfs_num[v] == UNVISITED) tarjanSCC(v);
        if (visited[v]) dfs_low[u] = min( dfs_low[u], dfs_low[v] ); // condition for update
                                     // if this is a root (start) of an SCC
    if (dfs low[u] == dfs num[u])
                                     // this part is done after recursion
        printf("SCC %d:", ++numSCC);
        while (true)
            int v = S.back(); S.pop_back();
            visited[v] = 0;
            if (u == v) break;
       printf("\n");
// inside int main()
    dfs_num.assign(V, UNVISITED);
    dfs low.assign(V, 0);
    visited.assign(V, 0);
    dfsNumberCounter = numSCC = 0;
    for (int u = 0; u < V; ++u)
        if (dfs_num[u] == UNVISITED)
            tarjanSCC(u);
```

3 String algorithms

3.1 Z-algorithm

```
#include <iostream>
#include <bits/stdc++.h>
#define LOCAL
#define MAXN 1000020
using namespace std;
int z[MAXN] = {};
int x=0 , y=0 , maxn = 0;
string s;
int main()
```

```
#iddef LOCAL
    freopen("in1.txt","r",stdin);
#endif // LOCAL

    cin >> s;
    for(int i = 1; i < s.length(); i++) {
        z[i] = max(0,min(z[i-x], y - i + 1));
        while(i + z[i] < s.length() && s[z[i]] == s[i+z[i]]) {
            x = i;
            y = i + z[i];
            z[i]++;
        }
}

for(int i = 0; i < s.length(); i++)
        if(z[i] == s.length() - i && maxn >= s.length()-i) {
            cout < s.substr(0,z[i]);
            return 0;
        }
        maxn = max(maxn, z[i]);
    }
    cout << "Just a legend";
    return 0;
}</pre>
```

4 Data structures

4.1 Rope

```
#include <iostream>
#include <bits/stdc++.h>
#include <ext/rope>
#define LOCAL
#define MAXN 50020
using namespace std;
using namespace __gnu_cxx ;
int main()
#ifdef LOCAL
    freopen("in1.txt" , "r" , stdin );
#endif // LOCAL
   int n , t , a , b , c , d=0 ;
    int v = 0;
    string strA ;
    rope<char> r[MAXN] , rtmp ;
    cin >> n;
       if(t==1){
           cin >> a ;
           cin >> strA ;
            a -= d;
           r[++v] = r[v] ;
           r[v].insert(a,strA.c_str());
            //debug
            //cout << r[v] << '\n';
       else if(t==2) {
           cin >> a >> b ;
            a -= d ; b -= d ;
           r[++v] = r[v] ;
           r[v].erase(a-1,b);
            //debug
            //cout << r[v] << ' ' << r[v-1] << '\n';
       else if(t==3){
           cin >> a >> b >> c;
           a = d; b = d; c = d;
           rtmp = r[a].substr(b-1,c);
           cout << rtmp << '\n' ;
           d += count(rtmp.begin() , rtmp.end() , 'c' );
    return 0:
```

4.2 Union-find disjoint sets (UFDS) by David

```
#include <iostream>
#include <bits/stdc++.h>
#define LOCAL
using namespace std;
int intSum[200080] , intParent[200080] , intSet[200080] ;
int find_root(int intA){
    if(intParent[intA] == intA)
       return intA ;
    intParent[intA] = find_root(intParent[intA]);
    return intParent[intA] ;
int each_debug(int n ) {
    system("Pause") ;
int main()
#ifdef LOCAL
    freopen("in1.txt","r", stdin);
freopen("out.txt","w", stdout);
#endif // LOCAL
    int n, m , operation , p , q ;
    while (cin >> n >> m) {
        for(int i = 1 ; i <= n ; i++) {
            intParent[i] = i+n ;
            intParent[i+n] = i+n ;
            intSum[i+n] = i;
            intSet[i+n] = 1;
        while (m--) {
            cin >> operation ;
            if(operation == 1){
                 cin >> p >> q;
                int intRoot_p , intRoot_q ;
intRoot_p = find_root(intParent[p]) ;
intRoot_q = find_root(intParent[q]) ;
if(intRoot_p != intRoot_q) {
                     intParent[intRoot_q] = intRoot_p;
                     intSum[intRoot_p] += intSum[intRoot_q];
                     intSet[intRoot_p] += intSet[intRoot_q] ;
                 //debug
                 //each_debug(n);
            else if (operation == 2 ) {
                cin >> p >> q;
                 int intRoot_p , intRoot_q ;
intRoot_p = find_root(intParent[p]) ;
                 intRoot_q = find_root(intParent[q]);
                 if(intRoot_p != intRoot_q){
                     intParent[p] = intRoot_q;
                     intSum[intRoot_q] += p ;
                     intSum[intRoot_p] -= p ;
                     intSet[intRoot_q] ++ ;
                     intSet[intRoot_p] -- ;
                 //debug
                 //each_debug(n) ;
            else if (operation == 3){
                 cout << intSet[find_root(p)] << ' ' << intSum[find_root(p)] << '\n';</pre>
    return 0;
```

4.3 Union-find disjoint sets (UFDS) by Bill

```
class UnionFind
{
public:
    UnionFind(int N)
```

5 Utilities

5.1 Bit manipulation

```
#define isOn(S, j) (S & (1<<j)) #define setBit(S, j) (S |= (1<<j)) #define clearBit(S, j) (S &= ^{\circ}(1<<j)) #define toggleBit(S, j) (S ^{\circ} = (1<<j)) #define lowBit(S) (S & (-S)) #define setAll(S, n) (S = (1<<n))
```

5.2 Prime numbers

```
// O(sqrt(x)) Exhaustive Primality Test
#include <cmath>
```

```
#define EPS 1e-7
typedef long long LL;
bool IsPrimeSlow (LL x)
  if(x<=1) return false;</pre>
  if(x<=3) return true;</pre>
  if (!(x%2) || !(x%3)) return false;
  LL s=(LL) (sqrt ((double)(x))+EPS);
  for(LL i=5;i<=s;i+=6)</pre>
    if (!(x%i) || !(x%(i+2))) return false;
  return true;
  Primes less than 1000:
                                 59
                                                                       83
                   103
                         107
                                109
                                      113
                                             127
                                                   131
                                                                139
                                                                      149
      157
            163
                  167
                         173
                               179
                                      181
                                            191
                                                   193
                                                         197
                                                               199
                                                                      211
                                                                            223
      227
            229
                  233
                         239
                               241
                                      251
                                            257
                                                   263
                                                         269
                                                                271
                                                                            281
                                      317
397
      283
            293
                  307
                         311
                                313
                                                         347
                                                                349
                                                                            359
                               389
      367
            373
                  379
                         383
                                            401
                                                   409
                                                         419
                                                                421
                                                                      431
                                                                            433
                                      463
557
                                                         487
571
      439
            443
                  449
                         457
                                461
                                            467
                                                   479
                                                               491
577
                                                                      499
                                                                            503
      509
            521
                  523
                         541
                               547
                                            563
                                                   569
                                                                      587
                                                                            593
                                                   641
719
      599
                         613
                               617
                                      619
                                            631
                                                         643
                                                                647
            601
                  607
                                                                      653
                                                                            659
      661
                  677
                         683
                                             709
                                                         727
                                                                733
                                                                      739
                                                                            743
            673
                               691
                                      787
                                                   809
      751
            757
                   761
                         769
                                773
                                             797
                                                         811
                                                                821
                                                                            827
                                                                      823
                                859
                                            877
      829
            839
                  853
                         857
                                      863
                                                   881
                                                         883
                                                               887
                                                                      907
                                                                            911
      919
            929
                  937
                         941
                               947
                                      953
                                            967
                                                   971
// Other primes:
      The largest prime smaller than 10 is 7.
      The largest prime smaller than 100 is 97.
      The largest prime smaller than 1000 is 997.
      The largest prime smaller than 10000 is 9973.
      The largest prime smaller than 100000 is 99991.
      The largest prime smaller than 1000000 is 999983.
The largest prime smaller than 1000000 is 9999983.
The largest prime smaller than 10000000 is 9999998.
The largest prime smaller than 100000000 is 99999989.
      The largest prime smaller than 10000000000 is 9999999967. The largest prime smaller than 100000000000 is 9999999977. The largest prime smaller than 1000000000000 is 9999999989.
      The largest prime smaller than 1000000000000 is 999999999971.
      The largest prime smaller than 1000000000000 is 9999999999973.
      The largest prime smaller than 100000000000000 is 99999999999937.
      The largest prime smaller than 100000000000000 is 999999999999997
```