NTUT_Kn1ghts ICPC Team Notebook

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1 Advanced algorithms

1.1 Iterative deepening A* (IDA*)

```
#define MAX_STEPS 45 // given by the problem description
#define DIR 4
                    // 4 [DIR]ections
int dr[DIR] = {0, -1, 0, 1}; // must be right, up, left, down
int dc[DIR] = \{1, 0, -1, 0\}; // for the XOR operation to work
char dm[] = "RULD";
                             // [d]irection [m]ove
int p[PUZZLE];
int b_init_pos;
                      // [b]lank [init]ial [pos]ition
                      // current [lim]it of the Iterative Deepening Search (IDS)
int lim;
int pred[MAX_STEPS]; // [pre]viously used [d]irection to go to the current state
    int sum:
    for (int i = 0; i < PUZZLE; ++i)</pre>
        for (int j = 0; j < i; ++j)
            if (p[j] > p[i]) ++sum;
    sum += b_init_pos / N + b_init_pos % N;
sum -= B / N + B % N;
    return sum % 2 == 0;
int H()
    int h = 0;
    for (int pos = 0; pos < PUZZLE; ++pos) // for all tile 'p[pos]'</pre>
                                                  // compute Manhattan distance to goal state
        if (p[pos] == B) continue;
        h += abs(p[pos] / N - pos / N)
+ abs(p[pos] % N - pos % N);
                                                // position of 'p[pos]' in goal state is 'p[pos]'
// position of 'p[pos]' in current state is 'pos'
    return h;
bool isValid(int r, int c)
    return 0 <= r && r < N && 0 <= c && c < N;
int Delta_H(int cur_r, int cur_c, int next_r, int next_c)
    int val = p[cur_r * N + cur_c]; // [val]ue of the tile being moved into the blank tile position
int goal_r = val / N; // position of 'val' in goal state is 'val'
    int goal_r = val / N;
int goal_c = val % N;
                                      // get row & column representation of the position
    return - ( abs(goal_r - cur_r ) + abs(goal_c - cur_c ) )
            + ( abs(goal_r - next_r) + abs(goal_c - next_c) );
bool dfs(int g, int h, int b_pos)
    if (g + h > lim) return false;
                                      // found a solution!
    if (h == 0) return true;
    int r = b_pos / N;
    int c = b_pos % N;
for (int d = 0; d < DIR; ++d)
        if ( q != 0 && d == (pred[q] ^ 2) ) continue; // this direction gets us back to parent state
        int next_r = r + dr[d];
        int next_c = c + dc[d];
        if (!isValid(next_r, next_c)) continue;
        int next_h = h + Delta_H(next_r, next_c, r, c); // O(1)
        int b_next_pos = next_r * N + next_c;
        swap(p[b_pos], p[b_next_pos]);
        pred[q+1] = d;
        if ( dfs(g + 1, next_h, b_next_pos) ) return true;
        swap(p[b_pos], p[b_next_pos]);
    return false:
int ida_star()
    int init_h = H();
    while (lim <= MAX_STEPS)
        if ( dfs(0, init_h, b_init_pos) ) return lim;
    return -1:
void output(int steps)
    for (int i = 1; i <= steps; ++i)</pre>
        printf("%c", dm[ pred[i] ]);
int main()
```

```
#ifdef LOCAL
freopen("in.txt", "r", stdin);
#endif // LOCAL
scanf("%d", &T);
while (T--)
    for (int i = 0; i < N; ++i)
        for (int j = 0; j < N; ++j)
        int pos = i * N + j;
       scanf("%d", &p[pos]);
        if (p[pos] == 0) p[pos] = B, b_init_pos = pos; // goal state 'p' is 0, 1, 2..14, 15
        else --p[pos];
                                                      // blank tile as 15
   if (!isViable()) // must-consider condition otherwise TLE
        printf("This puzzle is not solvable.\n");
        continue;
   if (ret == -1)
        printf("This puzzle is not solvable.\n");
        continue:
   output(ret), printf("\n");
return 0:
```

2 Dynamic programming algorithms

2.1 Longest common subsequence (LCS)

```
#include <iostream>
#include <bits/stdc++.h>
#define LOCAL
using namespace std;
struct LCS{
   int step , max_len ;
}Dp[5000][5000];
int main()
    freopen("in1.txt" , "r" , stdin );
#endif // LOCAL
   int intX , intY , Min_step , Max_len ;
    string strX , strY ;
    while(cin >> intX >> strX >> intY >> strY ) {
        for (int i = 0 ; i <= intY ; i++) {</pre>
            Dp[0][i].max\_len = 0;
            Dp[0][i].step = i ;
        for (int i = 0 ; i <= intX ; i++) {
            Dp[i][0].max\_len = 0 ;
            Dp[i][0].step = i ;
        Max len = 0:
        Min_step = 0;
        for(int i = 1 ; i <= intX ; i++) {</pre>
            for(int j = 1; j <= intY; j++) {
                if(strX[i-1] == strY[j-1]){
                    Dp[i][j].max_len = Dp[i-1][j-1].max_len +1;
                    Dp[i][j].step = Dp[i-1][j-1].step;
                    //cout << strX[i-1] << ' ' << strY[j-1] << ' ' << Dp[i][j].max_len << '\n'; //cout << strX[i-1] << ' ' << strY[j-1] << ' ' << Dp[i][j].step << '\n';
                    Dp[i][j].max_len = max(Dp[i-1][j].max_len , Dp[i][j-1].max_len );
```

```
}
cout << Dp[intX][intY].step << '\n';
}
return 0;</pre>
```

2.2 Max 2D range sum

```
// Max 2D Range Sum - UVa 108 - solved with DP O(n^4).
// Abridged problem statement: Given an n x n square matrix of integers A where
// each integer ranges from [-127..127], find a sub-matrix of A with the maximum
#include <hits/stdc++ h>
using namespace std:
int A[200][200];
int main() {
  int n; scanf("%d", &n);
                                                         // square matrix size
  for (int i = 0; i < n; ++i)
    for (int j = 0; j < n; ++j) {
      scanf("%d", &A[i][j]);
      if (i > 0) A[i][j] += A[i-1][j];
if (j > 0) A[i][j] += A[i][j-1];
                                                         // add from top
                                                          // add from left
      if (i > 0 && j > 0) A[i][j] -= A[i-1][j-1];// avoid double count
                                                          // inclusion-exclusion
  int maxSubRect = -127*100*100;
                                                          // the lowest possible val
  for (int i = 0; i < n; ++i)
for (int j = 0; j < n; ++j)</pre>
                                                         // start coordinate
      for (int k = i; k < n; ++k)
  for (int l = j; l < n; ++l) {
    int subRect = A[k][1];</pre>
                                                          // end coord
                                                         // from (0, 0) to (k, 1)
           if (i > 0) subRect -= A[i-1][1];
if (j > 0) subRect -= A[k][j-1];
           if (i > 0 && j > 0) subRect += A[i-1][j-1]; // O(1)
           maxSubRect = max(maxSubRect, subRect); // the answer is here
  printf("%d\n", maxSubRect);
  return 0;
```

2.3 Traveling salesman problem (TSP)

```
// This is a solution for UVa 10496 - Collecting Beepers. The problem is a
// variant of the Traveling Salesman Problem (TSP): Given n cities and their
// pairwise distances in the form of a matrix 'dist' of size n \, * \, n, compute the
// minimum cost of making a tour that starts from any city s, goes through all
// the other n - 1 cities exactly once, and finally returns to the city s. In
// this case, the salesman is Karel in a 2D world who can only move along the
// x and y axis. The cities are beepers whose coordinates are given, from which
// pairwise distances can be calculated. Algorithm takes time O(2^n \, \star \, n^2) .
// INPUT: The first line is the number of test cases. The first line of each
// test case is world's size (x-size and y-size). Next is the starting position
// of Karel. Next is the number of beepers. Next are the beepers' x- and y-
// coordinates.
// OUTPUT: For each test case, output the minimum distance to move from Karel's
// starting position to each of the beepers and back to the starting position.
#include <bits/stdc++.h>
using namespace std;
#define LSOne(S) ((S) & -(S))
const int MAX n = 11:
int dist[MAX_n] [MAX_n], memo[MAX_n] [1<<(MAX_n-1)]; // Karel + max 10 beepers</pre>
                                                  // mask = free coordinates
int dp(int u. int mask) {
  if (mask == 0) return dist[u][0];
                                                  // close the loop
  int &ans = memo[u][mask];
  if (ans != -1) return ans;
                                                  // computed before
  ans = 2000000000;
  int m = mask;
  while (m) {
                                                  // up to O(n)
   int two_pow_v = LSOne(m);
                                                  // but this is fast
    int v = __builtin_ctz(two_pow_v)+1;
                                                  // offset v by +1
    ans = min(ans, dist[u][v] + dp(v, mask^two_pow_v)); // keep the min
   m -= two_pow_v;
  return ans:
int main() {
  int TC; scanf("%d", &TC);
  while (TC--) {
```

3 Graph algorithms

3.1 All-pairs shortest paths (APSP)

```
// All-Pairs Shortest Paths (APSP) solved with Floyd Warshall O(V^3).
// inside int main()
   // precondition: AdjMat[i][j] contains the weight of edge (i, j)
   // or INF (1B) if there is no such edge
   // AdjMat is a 32-bit signed integer array
   // let p be a 2D parent matrix, where p[i][j] is the last vertex before j
    // on a shortest path from i to j, i.e. i \rightarrow ... \rightarrow p[i][j] \rightarrow j
   for (int i = 0; i < V; ++i)
   // remember that loop order is k->i->j
       for (int i = 0; i < V; ++i)
           for (int j = 0; j < V; ++j)
               if (AdjMat[i][k] + AdjMat[k][j] < AdjMat[i][j])</pre>
                   AdjMat[i][j] = AdjMat[i][k] + AdjMat[k][j];
                   p[i][j] = p[k][j];
// print shortest paths
void printPath(int i, int j)
   if (i != j) printPath(i, p[i][j]);
   printf(" %d", j);
```

3.2 Bipartite matching BFS by David

```
#include <iostream>
#include <cstring>
#include <cstdio>
#include <vector>
#define LOCAL
using namespace std;
int fp[100010] ,fq[100010];
int vfp[100010] ,vfq[100010];
vector<int> cp[100010] , cq[100010];
int BFSBMfp(int n) {
    for(int i = 0; i < cp[n].size(); i++){
   if(vfq[cp[n][i]]!= turn){</pre>
             vfq[cp[n][i]] = turn;
            if(fq[cp[n][i]] == -1 || BFSBMfp(fq[cp[n][i]])){
                 fp[n] = cp[n][i];
                 fq[cp[n][i]] = n;
                 return 1 ;
    return 0 ;
int main()
    ios::sync_with_stdio(false);
    cin.tie(0);
    cout.tie(0);
    int n ,p ,q ,k ,x, y ;
```

```
cin >> n ;
while (n--) {
    cin >> p >> q >> k;
    int MaxnPQ = max(p,q);
    for(int i = 1 ; i <= MaxnPQ ; i++) {</pre>
        cp[i].clear();
        fp[i] = -1;
        cq[i].clear();
        fq[i] = -1;
    for(int i = 0; i < k; i++) {
        cin >> x >> y;
        cp[x].push_back(y);
        cq[y].push_back(x);
if(fp[x] == -1 && fq[y] == -1){
            fp[x] = y;
            fq[y] = x;
    for(int i = 1; i <= p; i++) {
        if(fp[i] == -1){
            turn++;
            if(BFSBMfp(i))
               cnt++;
    cout << cnt << '\n';
return 0;
```

3.3 Centroid decomposition

```
#include<iostream>
#include <bits/stdc++.h>
#define LOCAL
#define MAXN 50005
using namespace std;
int n , k , a , b ;
int ans , cnt ;
int Max[MAXN] , sz[MAXN] , rt ;
int head[MAXN], dis[MAXN];
bool vis[MAXN];
struct node{
}Edge[MAXN*2];
void init(int n ) {
    Max[0] = n ;
    ans = cnt = 0;
    for (int i = 0; i <= n; i++) {
    head[i] = -1;</pre>
        vis[i] = 0;
void add(int u , int v) {
    Edge[cnt].v = v;
    Edge[cnt].nx = head[u] ;
    head[u] = cnt++;
void get_rt(int u , int fa ){
    sz[u] = 1 ; Max[u] = 0 ;
for(int i = head[u] ; ~i ; i=Edge[i].nx) {
        int v = Edge[i].v ;
        if(vis[v] || v == fa ) continue;
        get rt(v.u):
        sz[u] += sz[v];
        Max[u] = max(Max[u], sz[v]);
    Max[u] = max(Max[u], n - sz[u]);
    if(Max[rt] > Max[u])
void get_dis(int u , int fa , int d){
    for(int i = head[u] ; ~i ; i= Edge[i].nx){
        int v = Edge[i].v ;
        if(vis[v] | | v == fa ) continue ;
        dis[++cnt] = d + 1;
        get dis(v,u,dis[cnt]);
```

```
int get_ans(int u , int d ){
    dis[cnt=1] = d;
    get_dis(u,0,d);
    sort(dis+1 , dis+cnt+1) ;
    int 1 = 1 , ans = 0 ;
    while(1 < cnt && dis[1] + dis[cnt] < k ) 1++ ;
    while(1 < cnt && dis[1] <= k - dis[1]){</pre>
        ans += upper_bound(dis + 1 + 1 , dis + cnt + 1 , k - dis[1]) - lower_bound(dis+1+1 , dis+cnt+1
              , k-dis[1]);
    return ans :
void dfs(int u ) {
    vis[u] = 1;
    //cout << rt << ' ' << u << '\n' ;
    ans += get_ans(u , 0);
    for (int i = head[u]; ~i; i = Edge[i].nx) {
        int v = Edge[i].v ;
        if(vis[v]) continue;
       ans -= get_ans(v , 1) ;
n = sz[v] , rt = 0 , get_rt(v,u);
        dfs(rt);
int main(){
//#ifdef LOCAL
     freopen("in1.txt" , "r" , stdin);
    cin >> n >> k;
    init(n);
    for (int i =1; i < n; i++) {</pre>
        cin >> a >> b;
        add(a,b);
        add(b,a);
    rt = 0 ; get_rt(1,0);
    dfs(rt);
    cout << ans << '\n' ;
```

3.4 Detect negative weight cycle

3.5 DFS

```
#include <iostream>
#include <bits/stdc++.h>
#define LOCAL
using namespace std;
int m, n, flag=1;
int Maxn_city = 0 , Maxn_path = 0;
vector<int>tree[200020];
int city[200020] = {};
int visit[200020] = {};
vector<int> travel;

void BFS_to_large_path(int root) {
    visit[root] = 1;
    travel.push_back(root);
    for(int i = 0; i < tree[root].size(); i++){
        int node = tree[root][i];
        if(!visit[node]) {
            BFS_to_large_path(node);
            travel.pop_back();
            visit[root] = 0;
    }
}</pre>
```

```
//debug to check large path
    //if (root == 1)
    // cout << "1=" << travel.size() << ' ' << Maxn_path << ' ' << city[root] << '\n';
    if(city[root] && travel.size() > Maxn_path){
        Maxn_city = travel[travel.size()/2];
        Maxn_path = travel.size();
void BFS_to_other_path(int root ,int path) {
    visit[root] = 1 ;
    for(int i = 0 ; i < tree[root].size() ; i++) {
   int node = tree[root][i] ;</pre>
        if(!visit[node]){
            BFS_to_other_path(node , path+1);
            visit[root] = 0;
    //debug
    if(root == 1 )
        cout << "city=" << root << " path= " << path << '\n' ;
    if(city[root] && path != Maxn_path)
        flag = 0 ;
int main() {
#ifdef LOCAL
    freopen("in1.txt" , "r" , stdin);
#endif // LOCAL
    cin >> n >> m;
    int a , b ;
    for (int i = 0; i < n-1; i++) {
        cin >> a >> b ;
        tree[a].push_back(b);
        tree[b].push_back(a) ;
    for(int i = 0 ; i < m ; i++) {</pre>
        cin >> a :
        city[a] = 1;
    BFS_to_large_path(a);
    //visit[a] = 0 ;
    BFS_to_other_path(Maxn_city , 1 );
    if(flag)
        cout << "YES\n" << Maxn_city ;</pre>
    else
       cout << "NO" ;
    //dehua
    cout << "Maxn_path= " << Maxn_path << " Maxn_city= " << Maxn_city << '\n' ;
```

3.6 Dijkstra by Bill

```
// Dijkstra implementation for negative weight edges O((V + E) \log V)
   vi dist(V, INF); dist[s] = 0;
   priority_queue< ii, vii, greater<ii>> pq;
   pq.push( ii(0, s) );
    while (!pq.empty())
       ii front = pq.top(); pq.pop();
       int d = front.first;
       int u = front.second:
       if (d > dist[u]) continue;
       for (int i = 0; i < (int)AL[u].size(); ++i) // [A]djacency [L]ist</pre>
           ii vw = AL[u][i];
           int v = vw.first;
           int w = vw.second;
           if (dist[u] + w < dist[v])</pre>
                dist[v] = dist[u] + w;
                                          // relax operation
               pq.push( ii(dist[v], v) );
   } // this variant can cause duplicate items in the priority queue
```

B.7 Dijkstra by David

```
#include <iostream>
#include <bits/stdc++.h>
#define LOCAL
#define INF 99999999
using namespace std;
int intMap[1010][1010] = {} , intValue[1010][1010] = {};
struct Node {
    int x , y , v ;
    void read( int _x , int _y , int _v) {
        x = _x ; y = _y ; v = _v ;
    bool operator < (const Node &a) const(
        return v > a.v ;
void print_map(){
    for (int i = 1; i <= n; i++) {
        for(int j = 1 ; j <= m ; j++) {
   if(intValue[i][j] == 99999999)</pre>
                 cout << 'r' << ' ';
             else
                 cout << intValue[i][j] << ' ';
        cout << '\n' ;
    cout << '\n' ;
void bfs() {
    int x , y , intDirection[4][2] = \{-1,0,0,1,1,0,0,-1\};
    int intDx , intDy ;
    Node nodTemp ;
    priority_queue<Node> deqNode ;
    nodTemp.read(1,1,0);
    degNode.push (nodTemp);
    while (deqNode.size()) {
        x = deqNode.top().x;
        y = deqNode.top().y;
        deqNode.pop() ;
        for (int i = 0; i < 4; i++) {
             intDx = intDirection[i][0] + x ;
             intDy = intDirection[i][1] + y;
             //cout << intDx << ' ' << intDy << ' ' << intValue[x][y] + intMap[intDx][intDy] << ' ' <<
                  i << '\n';
            if(intValue[x][y] + intMap[intDx][intDy] < intValue[intDx][intDy] ) {
   intValue[intDx][intDy] = intValue[x][y] + intMap[intDx][intDy];</pre>
                 nodTemp.read(intDx , intDy , intValue[intDx][intDy]);
                 deqNode push (nodTemp) ;
         //print_map();
int main() {
#ifdef LOCAL
    freopen("in1.txt" , "r" , stdin );
freopen("out.txt" , "w" , stdout) ;
#endif
ios::sync_with_stdio(false);
    int intCase ;
    cin >> intCase ;
    while(intCase --) {
        cin >> n >> m ;
        for (int i = 1; i \le n; i++) {
             for (int j = 1; j \le m; j++) {
                  cin >> intMap[i][j];
                 intValue[i][j] = INF ;
        for(int i = 1 ; i <= n ; i++) {</pre>
             intValue[i][0] = 0;
             intValue[i][m+1] = 0;
             intMap[i][0] = INF +1;
             intMap[i][m+1] = INF +1;
        for(int i = 1 ; i <= m ; i++) {
             intValue[0][i] = 0;
             intValue[n+1][i] = 0;
```

intMap[0][i] = INF +1;

```
intMap[n+1][i] = INF +1;
} intValue[1][1] = intMap[1][1];

//debug
//cout << intValue[1][1] << '\n';

bfs();
cout << intValue[n][m] << '\n';
}
return 0;</pre>
```

3.8 Euler tour

```
list<int> cyc; // we need list for fast insertion in the middle
void EulerTour(list<int>::iterator i, int u)
    for (int j = 0; j < (int)AL[u].size(); ++j) // [A]djacency [L]ist</pre>
        ii\& vw = AL[u][j];
        int v = vw.first;
       if (vw.second)
                         // if this edge can still be used
            vw.second = 0; // remove this edge
            // remove bi-directional edge
            for (int k = 0; k < (int)AL[v].size(); ++k)</pre>
                ii\& uw = AL[v][k];
                if (uw.first == u && uw.second)
                    uw.second = 0:
                    break;
            // continue the tour
           EulerTour(cyc.insert(i, u), v);
// inside int main()
    cyc.clear();
    EulerTour(cyc.end(), A); // 'cyc' contains an Euler tour starting at 'A'
    for (list<int>::iterator i = cyc.begin(); i != cyc.end(); ++i)
        printf("%d\n", *i);
```

3.9 Find articulation points and bridges

```
// Find articulation points & bridges solved with DFS O(V + E).
void articulationPointAndBridge(int u)
    dfs_low[u] = dfs_num[u] = dfsNumberCounter++; // dfs_low[u] <= dfs_num[u]
    for (int i = 0; i < (int)AL[u].size(); ++i) // [A]djacency [L]ist</pre>
        int v = AL[u][i].first;
        if (dfs_num[v] == UNVISITED)
                                        // a tree edge
            dfs_parent[v] = u;
if (u == dfsRoot) ++rootChildren; // special case if 'u' is a root
            articulationPointAndBridge(v):
            if (dfs_low[v] >= dfs_num[u]) articulation_vertex[u] = true;
            if (dfs_low[v] > dfs_num[u]) printf("Edge (%d, %d) is a bridge\n", u, v);
            dfs_low[u] = min( dfs_low[u], dfs_low[v] );  // update dfs_low[u]
        else if (v != dfs_parent[u]) dfs_low[u] = min( dfs_low[u], dfs_num[v]); // update dfs_low[u]
// inside int main()
    dfsNumberCounter = 0:
    dfs_num.assign(V, UNVISITED);
    dfs_low.assign(v, 0);
    dfs_parent.assign(V, 0);
articulation vertex.assign(V, 0);
    printf("Bridges:\n");
    for (int u = 0; u < V; ++u)
        if (dfs_num[u] == UNVISITED)
```

```
{
    dfsRoot = u;
    rootChildren = 0;
    articulationPointAndBridge(u);
    articulation_vertex[dfsRoot] = (rootChildren > 1);  // special case
}
printf("Articulation Points:\n");
for (int u = 0; u < V; ++u)
    if (articulation_vertex[u]) printf(" Vertex %d\n", u);</pre>
```

3.10 Floyd Warshall by David

```
#include <iostream>
#include <bits/stdc++.h>
#define LOCAL
using namespace std;
char before[520][520] = {};
int after[520][520] = {};
int main()
#ifdef LOCAL
    freopen("in1.txt" , "r" , stdin );
#endif // LOCAL
    int n ;
    cin >> n ;
   for(int i = 0 ; i < n ; i++) {
       for(int j = 0 ; j < n ; j++)
    cin >> before[i][j] ;
    for (int i = 0; i < n; i++) {
        for(int j = i+1; j < n; j++) {
            int sum = 0 ;
            for (int k = i + 1; k < j; k++) {
                if(after[i][k])
                   sum += before[k][j]-'0';
            if( (sum +1) % 10 == before[i][j] - '0'){
                after[i][j] = 1;
    for (int i = 0; i < n; i++) {
        for(int j = 0; j < n; j++)
           cout << after[i][j];
        cout << '\n' ;
    return 0;
```

3.11 Graph edges property check

```
dfs_parent.assign(V, 0);
for (int u = 0; u < V; ++u)
   if (dfs_num[u] == UNVISITED)
        printf("Component %d:\n", ++numComp), graphCheck(u);</pre>
```

3.12 Kruskal by David

```
#include <iostream>
#include <bits/stdc++.h>
#define LOCAL
#define 11 long long
using namespace std;
int parent[1020];
    ll n1 , n2 , w ;
}node[25020];
int compare(edge A , edge B ) {
    return A.w < B.w ;
int find_root(int a) {
    if(a != parent[a] )
       return parent[a] = find_root(parent[a]);
    return a ;
int main()
#ifdef LOCAL
    freopen("in1.txt" , "r" , stdin );
freopen("out.txt" , "w" , stdout );
#endif // LOCAL
    int n , m , p_n1 , p_n2 ; // parent_n1 , parent_n2
    vector<int> hce ; //heavy edge circle
    while (cin >> n >> m && n + m != 0 ) {
        for(int i = 0; i < m; i++) {
           cin >> node[i].n1 >> node[i].n2 >> node[i].w ;
        for (int i = 0; i < n; i++)
           parent[i] = i ;
        sort(node , node + m , compare ) ;
        hce.clear();
        //kruskal
        for(int i = 0 ; i < m ; i++) {
            p_n1 = find_root(node[i].n1);
            p_n2 = find_root(node[i].n2);
            if (p n1 != p n2 )
                parent[p_n2] = p_n1 ;
                hce.push_back(node[i].w) ;
            //debug
            for(int i = 0 ; i < n ; i++)
            cout << parent[i] << ' ';
cout << '\n';</pre>
        sort(hce.begin() , hce.end());
        if(hce.size()){
            for(int i = 0 ; i < hce.size()-1 ; i++)
                cout << hce[i] << ' ';
            cout << hce[hce.size()-1];</pre>
           cout << "forest";
        cout << '\n' ;
    return 0:
```

3.13 Max flow

```
int res[MAX_V][MAX_V], mf, f, s, t;
vi p; // p stores the BFS spanning tree from s
```

```
void augment(int v, int minEdge)
    if (v == s) { f = minEdge; return; }
    else if (p[v] != -1)
        augment(\ p[v],\ min(minEdge,\ res[\ p[v]\ ][\ v\ ])\ );
        res[ p[v] ][ v ] -= f;
        res[ v ][ p[v] ] += f;
// inside int main(): set up 'res', 's', and 't' with appropriate values
                    // O(V^3 * E) Edmonds Karp's algorithm
    while (true)
        vi dist(MAX_V, INF); dist[s] = 0;
        queue<int> q; q.push(s);
         o.assign(MAX_V, -1);
        while (!q.empty())
            int u = q.front(); q.pop();
if (u == t) break; // immediately stop BFS if we already reach sink t
            for (int v = 0; v < MAX_V; ++v)
   if (res[u][v] > 0 && dist[v] == INF)
                     dist[v] = dist[u] + 1, q.push(v), p[v] = u;
        augment(t, INF); // find the min edge weight 'f' in this path, if any
        if (f == 0) break; // we cannot send any more flow ('f' = 0), terminate
        mf += f;
                            // we can still send a flow, increase the max flow!
    printf("%d\n", mf);
```

3.14 Max cardinality bipartite matching (MCBM)

```
// Max Cardinality Bipartite Matching (MCBM) solved with augmenting path algorithm O(VE).
vi match, vis:
               // return 1 if an augmenting path is found & 0 otherwise
int Aug(int 1)
    if (vis[1]) return 0;
    vis[1] = 1;
    for (int i = 0; i < (int)AL[1].size(); ++i) // [A]djacency [L]ist</pre>
       int r = AL[1][i];  // edge weight not needed -> vector< vi > AL
       if ( match[r] == -1 || Aug(match[r]) )
            match[r] = 1;
            return 1; // found 1 matching
    return 0:
                        // no matchings
// inside int main()
    // build unweighted bipartite graph with directed edge left->right set
    // left vertices [0..N-1], right vertices [N..V-1]
    int MCBM = 0;
    match.assign(V, -1); // V is the number of vertices in bipartite graph
    for (int 1 = 0; 1 < N; ++1) // N = size of the left set
       vis.assign(N, 0); // reset before each recursion
       MCBM += Aug(1);
    printf("Found %d matchings\n", MCBM);
```

3.15 Minimum spanning tree (MST)

```
pair<int, ii> front = EdgeList[i];
if (!UF.isSameSet(front.second.first, front.second.second))
{
    mst_cost += front.first;
    UF.unionSet(front.second.first, front.second.second);
}
}
printf("MST cost = %d\n", mst_cost);
```

3.16 Strongly connected component (SCC)

```
// Tarjan O(V + E)
vi dfs_num, dfs_low, visited;
int dfsNumberCounter, numSCC;
vi S:
void tarjanSCC(int u)
   S.push_back(u);
                     // stores 'u' in a vector baesd on order of visitation
    visited[u] = 1;
   for (int i = 0; i < (int)AL[u].size(); ++i) // [A]djacency [L]ist</pre>
       int v = AL[u][i].first;
       if (dfs_num[v] == UNVISITED) tarjanSCC(v);
       if (visited[v]) dfs_low[u] = min( dfs_low[u], dfs_low[v] ); // condition for update
                                  // if this is a root (start) of an SCC
   if (dfs low[u] == dfs num[u])
                                   // this part is done after recursion
       printf("SCC %d:", ++numSCC);
       while (true)
           int v = S.back(); S.pop_back();
           visited[v] = 0;
printf(" %d", v);
           if (u == v) break;
       printf("\n");
// inside int main()
   dfs num.assign(V, UNVISITED);
   dfs_low.assign(V, 0);
   visited.assign(V, 0);
    dfsNumberCounter = numSCC = 0;
    for (int u = 0; u < V; ++u)
       if (dfs_num[u] == UNVISITED)
           tarjanSCC(u);
```

4 Greedy algorithms

4.1 Interval covering

```
// This is a solution for UVa 10382 - Watering Grass. The problem is a variant
// of Interval Covering problem, which is solved by O(n) Greedy algorithm.
#include <hits/stdc++ h>
#define pb push back
#define not set -1
using namespace std;
typedef pair<double, double> dd;
typedef vector<dd> vdd;
typedef enum { STOP = 0,
               CONTINUE } status;
int n, 1, w;
vdd spinklers;
int answer:
double pivot;
struct sort compare t {
    bool operator()(dd a, dd b) const {
        return a.first < b.first || (a.first == b.first && a.second > b.second);
} sort_compare;
```

```
void InputSpinklers() {
   for (int i = 0; i < n; i++) {
       double x, r; // must be double otherwise WA.
        scanf("%lf %lf", &x, &r);
        if (w > 2 * r) // ignore spinklers that cannot cover the width of the strip.
       if (w == 2 * r) // ignore spinklers that produce no intervals.
           continue:
       double dx = sqrt(r * r - w * w / 4.0);
       spinklers.pb(dd(x - dx, x + dx));
status Check(int& j) {
   if (j == not_set) // there is an interval after pivot that cannot be covered.
        return STOP:
    // record j.
   answer++;
    pivot = spinklers[j].second;
   if (pivot >= 1) // solution found!
       return STOP:
     = not set:
   return CONTINUE;
void SolveIntervalCovering() {
   sort(spinklers.begin(), spinklers.end(), sort_compare);
   answer = 0;
   pivot = 0.0;
   int j = not_set;
   int iter = 0:
   while (true) {
       if (iter == spinklers.size()) // iterated through all spinklers/intervals.
           Check(j);
           break;
       if (spinklers[iter].first <= pivot) {</pre>
           if (pivot < spinklers[iter].second) // note the next candidate down!
               if (j == not_set || spinklers[iter].second > spinklers[j].second) // note down the
                      most right candidate.
                   j = iter;
               iter++:
            } else // skip intervals that are completely covered by the previously selected ones.
               iter++:
        else // out bound.
           if (Check(j) == STOP) {
   if (pivot >= 1) {
       printf("%d\n", answer);
   else (
       printf("-1\n");
   while (scanf("%d %d %d", &n, &1, &w) != EOF) {
       spinklers.clear();
        InputSpinklers();
        SolveIntervalCovering();
```

4.2 Longest increasing subsequence (LIS)

```
void print_array(const char *s, vi &L, int n) {
 for (int i = 0; i < n; ++i) {
  if (i) printf(", ");</pre>
    else printf("%s: [", s);
    printf("%d", L[i]);
 printf("]\n");
vi p;
                                                  // predecessor array
void print_LIS(int i) {
                                                  // backtracking routine
 if (p[i] == -1) { printf("%d", A[i]); return; }// base case
  print_LIS(p[i]);
 printf(" %d", A[i]);
int memo[10010];
                                                  // old limit: up to 10^4
                                                  // O(n^2) overall
int LIS(int i) {
 if (i == 0) return 1:
  int &ans = memo[i];
  if (ans != -1) return ans;
                                                  // was computed before
                                                  // LIS can start anywhere
  ans = 1:
 for (int j = 0; j < i; ++j)
  if (A[j] < A[i])</pre>
                                                  // O(n) here
                                                  // increasing condition
      ans = \max(ans, LIS(j)+1);
                                                  // pick the max
int main() {
  // note: A[n-1] must be set as the largest value ("INF")  
  // so that all LIS (that can start anywhere) will end at n-1
  srand(time(NULL)):
  int n = 10 + rand() %11;
                                                  // [10..201
  A.assign(n, 0);
  A[n-1] = 99;
                                                  // set A[n-1] = INF
  for (int i = 0; i < n-1; ++i)
   A[i] = rand() %101-50;
                                                  // [-50..501
  vi sample({-7, 10, 9, 2, 3, 8, 8, 1, 2, 3, 4, 99});
  printf("n = %d:", n);
  for (int i = 0; i < n; ++i)
   printf(" %d", A[i]);
  printf("\n");
  // early 2000 problems usually accept O(n^2) solution
  memset (memo, -1, sizeof memo);
  printf("LIS length is %d\n\n", LIS(n-1));
                                                 // with O(n^2) DP
  // 2020s problems will likely only accept O(n log k) solution
  // new limit: n can be up to 200K
  int k = 0, lis_end = 0;
  vi L(n, 0), L_id(n, 0);
  p.assign(n, -1);
  for (int i = 0; i < n; ++i) {
    int pos = lower_bound(L.begin(), L.begin()+k, A[i]) - L.begin();
    L[pos] = A[i];
                                                  // greedily overwrite this
    L_id[pos] = i;
                                                  // remember the index too
    p[i] = pos ? L_id[pos-1] : -1;
                                                  // predecessor info
    if (pos == k) {
                                                  // can extend LTS?
     k = pos+1;
                                                  // k = longer LIS by +1
      lis end = i:
                                                  // keep best ending i
    printf("Considering element A[%d] = %d\n", i, A[i]);
    printf("LIS ending at A[%d] is of length %d: ", i, pos+1);
    print_LIS(i);
    printf("]\n")
    print_array("L is now", L, k);
    printf("\n");
  printf("Final LIS is of length %d: ", k);
  print_LIS(lis_end); printf("\n");
  assert (LIS (n-1) == k);
                                                  // both must be identical
  return 0;
```

typedef vector<int> vi;

4.3 Max 1D range sum

```
// Max 1D Range Sum solved with Jay Kadane O(n).
/ inside int main()
int n = 9;
int A[] = { 4, -5, 4, -3, 4, 4, -4, 4, -5 }; // a sample array A
int sum = 0;
int ans = 0; // important, 'ans' must be initialized to 0
for (int i = 0; i < n; ++i)
{
    sum += A[i];
    ans = max(ans, sum);
    if (sum < 0) sum = 0;
}
printf("Max 1D Range Sum = %d\n", ans);</pre>
```

5 String algorithms

5.1 Z-algorithm

```
#include <iostream>
#include <bits/stdc++.h>
#define LOCAL
#define MAXN 1000020
using namespace std;
int z [MAXN] = { } ;
int x=0 , y=0 , maxn = 0;
string s ;
int main()
#ifdef LOCAL
    freopen("in1.txt", "r", stdin);
#endif // LOCAL
    cin >> s ;
    for (int i = 1 ; i < s.length() ; i++ ) {
   z[i] = max(0,min(z[i-x] , y - i + 1));</pre>
         while(i + z[i] < s.length() && s[z[i]] == s[i+z[i]]){
              y = i + z[i];
              z[i]++;
    for(int i = 0 ; i < s.length() ; i++)
   if(z[i] == s.length() - i && maxn >= s.length()-i ){
              cout << s.substr(0,z[i]);</pre>
              return 0 :
         maxn = max(maxn , z[i]);
    cout << "Just a legend" ;
    return 0;
```

6 Data structures

6.1 Union-find disjoint sets (UFDS) by David

```
#include <iostream>
#include <bits/stdc++.h>
#define LOCAL

using namespace std;
int intSum[200080] , intParent[200080] , intSet[200080] ;

int find_root(int intA) {
    if(intParent[intA] == intA)
        return intA ;
    intParent[intA] = find_root(intParent[intA]) ;
    return intParent[intA] ;
```

```
int each_debug(int n ){
    system("Pause");
int main()
#ifdef LOCAL
    freopen("in1.txt","r", stdin);
freopen("out.txt","w", stdout);
#endif // LOCAL
    int n, m , operation , p , q ;
    while(cin >> n >> m) {
        for(int i = 1; i <= n; i++) {
             intParent[i] = i+n ;
             intParent[i+n] = i+n ;
             intSum[i+n] = i;
             intSet[i+n] = 1;
        while (m--) {
             cin >> operation :
             if(operation == 1){
                 cin >> p >> q;
int intRoot_p, intRoot_q;
intRoot_p = find_root(intParent[p]);
                  intRoot_q = find_root(intParent[q]);
                  if(intRoot_p != intRoot_q){
                      intParent[intRoot_q] = intRoot_p;
intSum[intRoot_p] += intSum[intRoot_q];
intSet[intRoot_p] += intSet[intRoot_q];
                  //debug
                 //each_debug(n);
             else if (operation == 2 ) {
                 cin >> p >> q;
int intRoot_p , intRoot_q;
intRoot_p = find_root(intParent[p]);
                  intRoot_q = find_root(intParent[q]);
                  if(intRoot_p != intRoot_q){
                      intParent[p] = intRoot_q;
                      intSum[intRoot_q] += p ;
                      intSum[intRoot_p] -= p ;
                      intSet[intRoot_q] ++ ;
                      intSet[intRoot_p] -- ;
                  //debua
                 //each_debug(n);
             else if (operation == 3) {
                  cout << intSet[find_root(p)] << ' ' << intSum[find_root(p)] << '\n';</pre>
    return 0;
```

6.2 Binary indexed/fenwick tree (BIT)

```
#include <iostream>
using namespace std;

#define LOGSZ 17

int tree[(1<<LOGSZ)+1];
int N = (1<<LOGSZ);

// add v to value at x
void set(int x, int v) {
   while(x <= N) {
        tree[x] += v;
        x += (x & -x);
   }
}

// get cumulative sum up to and including x</pre>
```

```
int get(int x) {
 int res = 0;
  while(x) {
   res += tree[x];
    x = (x & -x);
  return res;
// get largest value with cumulative sum less than or equal to x;
// for smallest, pass x-1 and add 1 to result
int getind(int x) {
 int idx = 0, mask = N;
while (mask && idx < N) {</pre>
   int t = idx + mask;
    if(x >= tree[t]) {
     idx = t;
     x -= tree[t];
    mask >>= 1;
  return idx;
```

6.3 Rope

```
#include <iostream>
#include <bits/stdc++.h>
#include <ext/rope>
#define LOCAL
#define MAXN 50020
using namespace std:
using namespace __gnu_cxx ;
int main()
#ifdef LOCAL
    freopen("in1.txt" , "r" , stdin );
#endif // LOCAL
   int n , t , a , b , c , d=0 ;
    int v = 0;
    string strA ;
    rope<char> r[MAXN] , rtmp ;
    cin >> n ;
    while (n--) {
       cin >> t :
       if(t==1){
           cin >> a ;
            cin >> strA ;
            a -= d;
            r[++v] = r[v];
            r[v].insert(a,strA.c_str());
            //debug
            //cout << r[v] << '\n' ;
        else if(t==2){
           cin >> a >> b;
            a -= d : b -= d :
            r[++v] = r[v] ;
            r[v].erase(a-1,b);
            //debua
            //cout << r[v] << ' ' << r[v-1] << '\n';
        else if(t==3) {
           cin >> a >> b >> c;
            a -= d; b -= d; c -= d;
            rtmp = r[a].substr(b-1,c);
            cout << rtmp << '\n' ;
           d += count(rtmp.begin() , rtmp.end() , 'c' );
       }
    return 0:
```

6.4 Segment tree

```
#include <iostream>
#include <bits/stdc++.h>
#include <string>
#define LOCAL
#define LSOn(x) ((x << 1) +1)
#define Rson(x) ((x << 1) +2)</pre>
```

```
#define INF 99999999
using namespace std;
const int N = 100005;
int shift[35] , num[N] , len_shift;
string strLine ;
struct Node {
    int left , right , Min_Value ;
}node[4 * N ];
void build(int left , int right , int x = 0 ){
    node[x].left = left ;
    node[x].right = right;
    if(left == right){
        node[x].Min_Value = num[left] ;
        return :
    int mid = (left + right ) / 2;
    //debug
    //cout << mid << '\n' ;
    //cout << x << ' ' << node[x].left << ' ' << node[x].right << ' ' << '\n';
    build(left , mid , Lson(x));
    build(mid + 1 , right , Rson(x)) ;
node[x].Min_Value = min(node[Lson(x)].Min_Value , node[Rson(x)].Min_Value ) ;
void handle(){
    len_shift = 0 ;
     shift[len_shift] = 0;
    else{
             shift[++len_shift] = 0;
    //finaly char is ')' , so len_shift is right
sort(shift , shift + len shift );
    //debug
    for(int i = 0 ; i < len_shift ; i++)
    cout << shift[i] << ' ';</pre>
    cout << '\n' ;
int query(int left , int right , int x = 0 ){
   if(node[x].left >= left && node[x].right <= right)</pre>
       return node[x].Min_Value;
    int mid = (node[x].left + node[x].right ) / 2;
    int ans = INF ;
    //cout << x << ' ' << node[x].left << ' ' << node[x].right << ' ' << node[x].Min_Value << '\n';
    if( left <= mid )</pre>
        ans = min(ans , query(left , right , Lson(x))) ;
    if(mid < right )</pre>
        ans = min(ans , query(left , right , Rson(x))) ;
    return ans ;
void set_num(int position , int value , int x = 0 ){
    if(node[x].left == position && node[x].right == position ) {
       node[x].Min_Value = value;
       return ;
    int mid = (node[x].left + node[x].right ) / 2;
    if(position <= mid )</pre>
        set_num(position , value , Lson(x) );
    if(mid < position )</pre>
        set_num(position , value , Rson(x)) ;
    node[x].Min_Value = min(node[Lson(x)].Min_Value , node[Rson(x)].Min_Value );
int main()
    int n , q , intTemp ;
    ios::sync_with_stdio(0);
#ifdef LOCAL
    freopen("out.txt" , "w" , stdout );
```

```
freopen("in1.txt" , "r" , stdin ) ;
#endif // LOCAL
    cin >> n >> q;
    for (int i = 1; i \le n; i++)
        cin >> num[i] ;
    build(1,n);
    for(int i = 0 ; i < 13 ; i++){
    cout << node[i].left << ' ' << node[i].right << ' ' << node[i].Min_Value << '\n';</pre>
    return 0 ;
    while (q--) {
         cin >> strLine ;
         if(strLine[0] == 'q'){
             handle();
             cout << query(shift[0] , shift[1] ) << '\n';
        else if (strLine[0] == 's'){
             handle();
             intTemp = num[shift[0]];
             for(int i = 1 ; i < len_shift ; i++) {</pre>
                 set_num(shift[i-1] , num[shift[i]]);
num[shift[i-1]] = num[shift[i]];
             num[shift[len_shift-1]] = intTemp;
             set_num(shift[len_shift-1] , intTemp );
             //cout << intTemp << ' ' << shift[len_shift-1] << '\n';
             //for(int i = 1; i <= n; i++)
// cout << num[i] << ' ';
    return 0:
```

6.5 Union-find disjoint sets (UFDS) by Bill

7 Utilities

7.1 Bit manipulation

```
#define isOn(S, j) (S & (1<<j)) #define setBit(S, j) (S = (1<<j)) #define clearBit(S, j) (S &= (1<<j)) #define toggleBit(S, j) (S = (1<<j))
```

```
#define lowBit(S) (S & (-S))
#define setAll(S, n) (S = (1<<n)-1)</pre>
```

7.2 C++ input output

```
#include <iostream>
#include <iomanip>
using namespace std;
int main()
    // Ouput a specific number of digits past the decimal point,
    // in this case 5
    cout.setf(ios::fixed); cout << setprecision(5);</pre>
    cout << 100.0/7.0 << endl;
    cout.unsetf(ios::fixed);
    // Output the decimal point and trailing zeros \,
    cout.setf(ios::showpoint);
    cout << 100.0 << endl;
    cout.unsetf(ios::showpoint);
    // Output a '+' before positive values
    cout.setf(ios::showpos);
    cout << 100 << " " << -100 << endl;
    cout.unsetf(ios::showpos);
    // Output numerical values in hexadecimal
    cout << hex << 100 << " " << 1000 << " " << 10000 << dec << endl;
```

7.3 C++ STL

```
// Example for using stringstreams and next_permutation
#include <algorithm>
#include <iostream>
#include <sstream>
#include <vector>
using namespace std;
int main (void) {
  vector<int> v;
  v.push back(1);
  v.push_back(2);
  v.push back(3);
  v.push_back(4);
  // Expected output: 1 2 3 4
   ostringstream oss;
oss << v[0] << " " << v[1] << " " << v[2] << " " << v[3];
    // for input from a string s,
    // istringstream iss(s);
// iss >> variable;
    cout << oss.str() << endl:
  } while (next_permutation (v.begin(), v.end()));
  v.clear();
  v.push_back(1);
  v.push_back(2);
  v.push_back(1);
  // To use unique, first sort numbers. Then call
  // unique to place all the unique elements at the beginning
  // of the vector, and then use erase to remove the duplicate
  sort(v.begin(), v.end());
  v.erase(unique(v.begin(), v.end()), v.end());
  // Expected output: 1 2 3
  for (size_t i = 0; i < v.size(); i++)</pre>
```

```
cout << v[i] << " ";
cout << endl;</pre>
```

7.4 Dates

```
// Routines for performing computations on dates. In these routines,
// months are expressed as integers from 1 to 12, days are expressed
// as integers from 1 to 31, and years are expressed as 4-digit
#include <iostream>
#include <string>
using namespace std:
string dayOfWeek[] = {"Mon", "Tue", "Wed", "Thu", "Fri", "Sat", "Sun"};
// converts Gregorian date to integer (Julian day number)
int dateToInt (int m, int d, int y) {
  return
    1461 * (y + 4800 + (m - 14) / 12) / 4 +
    367 * (m - 2 - (m - 14) / 12 * 12) / 12 -
    3 * ((y + 4900 + (m - 14) / 12) / 100) / 4 +
// converts integer (Julian day number) to Gregorian date: month/day/year
void intToDate (int jd, int &m, int &d, int &y) {
 int x, n, i, j;
 x = jd + 68569;

n = 4 * x / 146097;
  x -= (146097 * n + 3) / 4;
  i = (4000 * (x + 1)) / 1461001;
  x -= 1461 * i / 4 - 31;
  j = 80 * x / 2447;
  d = x - 2447 * j / 80;
  x = j / 11;
  m = j + 2 - 12 * x;
  y = 100 * (n - 49) + i + x;
// converts integer (Julian day number) to day of week
string intToDay (int jd) {
 return dayOfWeek[jd % 7];
int main (int arge, char **argv) {
  int jd = dateToInt (3, 24, 2004);
  int m, d, y;
intToDate (jd, m, d, y);
  string day = intToDay (jd);
  // expected output:
       2453089
       3/24/2004
      Wed
```

7.5 Prime numbers

<< day << endl;

```
// O(sqrt(x)) Exhaustive Primality Test
#include <cmath>
#define EPS 1e-7
typedef long long LL;
bool IsPrimeSlow (LL x)
  if(x<=1) return false;</pre>
 if(x<=3) return true;</pre>
  if (!(x%2) || !(x%3)) return false;
  LL s=(LL) (sqrt ((double)(x))+EPS);
  for (LL i=5; i<=s; i+=6)
   if (!(x%i) || !(x%(i+2))) return false;
  return true:
// Primes less than 1000:
                                   13
                                         17
                                               19
                              11
                              59
                                   61
                                               71
                                                           79
            43
                                         67
                                                                 83
           101
                 103
                       107
                             109
                                                               149
     157
                             179
                                   181
                                        191
                                              193
                                                    197
                                                          199
     227
           229
                 233
                       239
                             241
                                   251
                                        257
                                              263
                                                    269
                                                          271
     283
           293
                 307
                       311
                             313
                                   317
                                         331
                                              337
                                                    347
                                                          349
                                                                353
                                                                      359
     367
           373
                 379
                       383
                             389
                                  397
                                         401
                                              409
                                                    419
                                                          421
     439
           443
                 449
                       457
                             461
                                   463
                                        467
                                              479
                                                    487
                                                          491
                                                                499
                                                                      503
                                   557
                                              569
                                                    571
     509
           521
                 523
                       541
                             547
                                        563
                                                          577
                                                                587
                                                   643
727
811
     599
                                   619
                                                          647
           601
                 607
                       613
                             617
                                        631
                                              641
                                                                653
                                                                      659
     661
751
           673
757
                 677
                            691
773
                                  701
787
                                        709
797
                                              719
809
                                                                     743
                       683
                                                                7.39
                                                          821
                 761
                       769
                                                               823
                                                                     827
                       857
                            859
                                  863
                                        877
                                              881
           839
     829
                 853
                                                    883
                                                          887
                                                               907
                                                                     911
           929
                 937
                       941
                            947
                                  953
                                        967
                                              971
     919
// Other primes:
     The largest prime smaller than 10 is 7.
     The largest prime smaller than 100 is 97.
     The largest prime smaller than 1000 is 997.
     The largest prime smaller than 10000 is 9973.
     The largest prime smaller than 100000 is 99991.
     The largest prime smaller than 1000000 is 999983.
     The largest prime smaller than 10000000 is 99999991.
     The largest prime smaller than 100000000 is 99999989.
     The largest prime smaller than 1000000000 is 999999937.
     The largest prime smaller than 10000000000 is 9999999967.
     The largest prime smaller than 10000000000 is 99999999977.
     The largest prime smaller than 100000000000 is 999999999999.
     The largest prime smaller than 1000000000000 is 999999999971.
     The largest prime smaller than 1000000000000 is 9999999999973.
     The largest prime smaller than 10000000000000 is 9999999999999999.
     The largest prime smaller than 100000000000000 is 9999999999997.
     The largest prime smaller than 1000000000000000 is 99999999999997.
```