

NTUT_Knights ICPC Team Notebook

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1 Dynamic programming algorithms

1.1 Longest common subsequence

```
/*
Calculates the length of the longest common subsequence of two vectors.
Backtracks to find a single subsequence or all subsequences. Runs in
O(m*n) time except for finding all longest common subsequences, which
may be slow depending on how many there are.
*/
```

```
#include <iostream>
#include <vector>
#include <set>
#include <algorithm>

using namespace std;
```

```
typedef int T;
typedef vector<T> VT;
typedef vector<VT> VVT;

typedef vector<int> VI;
typedef vector<VI> VVI;

void backtrack(VVI& dp, VT& res, VT& A, VT& B, int i, int j)
{
    if(!i || !j) return;
    if(A[i-1] == B[j-1]) { res.push_back(A[i-1]); backtrack(dp, res, A, B, i-1, j-1); }
    else
    {
        if(dp[i][j-1] >= dp[i-1][j]) backtrack(dp, res, A, B, i, j-1);
        else backtrack(dp, res, A, B, i-1, j);
    }
}

void backtrackall(VVI& dp, set<VT>& res, VT& A, VT& B, int i, int j)
{
    if(!i || !j) { res.insert(VI()); return; }
    if(A[i-1] == B[j-1])
    {
        set<VT> tempres;
        backtrackall(dp, tempres, A, B, i-1, j-1);
        for(set<VT>::iterator it=tempres.begin(); it!=tempres.end(); it++)
        {
            VT temp = *it;
            temp.push_back(A[i-1]);
            res.insert(temp);
        }
    }
    else
    {
        if(dp[i][j-1] >= dp[i-1][j]) backtrackall(dp, res, A, B, i, j-1);
        if(dp[i][j-1] <= dp[i-1][j]) backtrackall(dp, res, A, B, i-1, j);
    }
}

VT LCS(VT& A, VT& B)
{
    VVI dp;
    int n = A.size(), m = B.size();
    dp.resize(n+1);
    for(int i=0; i<=n; i++) dp[i].resize(m+1, 0);

    for(int i=1; i<=n; i++)
        for(int j=1; j<=m; j++)
        {
            if(A[i-1] == B[j-1]) dp[i][j] = dp[i-1][j-1]+1;
            else dp[i][j] = max(dp[i-1][j], dp[i][j-1]);
        }

    VT res;
    backtrack(dp, res, A, B, n, m);
    reverse(res.begin(), res.end());
    return res;
}

set<VT> LCSall(VT& A, VT& B)
{
    VVI dp;
    int n = A.size(), m = B.size();
    dp.resize(n+1);
    for(int i=0; i<=n; i++) dp[i].resize(m+1, 0);
    for(int i=1; i<=n; i++)
        for(int j=1; j<=m; j++)
        {
            if(A[i-1] == B[j-1]) dp[i][j] = dp[i-1][j-1]+1;
            else dp[i][j] = max(dp[i-1][j], dp[i][j-1]);
        }

    set<VT> res;
    backtrackall(dp, res, A, B, n, m);
    return res;
}

int main()
{
    int a[] = { 0, 5, 5, 2, 1, 4, 2, 3 }, b[] = { 5, 2, 4, 3, 2, 1, 2, 1, 3 };
    VI A = VI(a, a+8), B = VI(b, b+9);
    VI C = LCS(A, B);

    for(int i=0; i<C.size(); i++) cout << C[i] << " ";
    cout << endl << endl;

    set<VI> D = LCSall(A, B);
    for(set<VI>::iterator it = D.begin(); it != D.end(); it++)
    {
        for(int i=0; i<(*it).size(); i++) cout << (*it)[i] << " ";
        cout << endl;
    }
}
```

1.2 Max 2D range sum

```
// This is a solution for UVa 108 - Maximum Sum. In this problem, we are given a
// square matrix of integers and are asked to find a sub-matrix with the maximum
// sum. This 1D DP + greedy (Kadane's) solution takes time  $O(n^3)$ .
```

```
#include <bits/stdc++.h>
using namespace std;

#define MAX_n 110

int A[MAX_n][MAX_n];

int main() {
    int n; scanf("%d", &n);           // square matrix size
    for (int i = 0; i < n; ++i)
        for (int j = 0; j < n; ++j) {
            scanf("%d", &A[i][j]);
            if (j > 0) A[i][j] += A[i][j-1]; // pre-processing
        }
    int maxSubRect = -127*100*100;     // lowest possible val
    for (int l = 0; l < n; ++l)
        for (int r = l; r < n; ++r) {
            int subRect = 0;
            for (int row = 0; row < n; ++row) {
                // Max 1D Range Sum on columns of this row
                if (l > 0) subRect += A[row][r] - A[row][l-1];
                else subRect += A[row][r];
                // Kadane's algorithm on rows
                if (subRect < 0) subRect = 0; // restart if negative
                maxSubRect = max(maxSubRect, subRect);
            }
        }
    printf("%d\n", maxSubRect);
    return 0;
}
```

1.3 0-1 Knapsack

```
// This is a solution for UVa 10130 - SuperSale. This is a 0-1 Knapsack variant
// solved using top-down DP  $O(n*W)$ .
```

```
#include <bits/stdc++.h>
using namespace std;

const int MAX_N = 1010;
const int MAX_W = 40;

int N, V[MAX_N], W[MAX_N], memo[MAX_N][MAX_W];

int dp(int id, int remW) {
    if ((id == N) || (remW == 0)) return 0; // two base cases
    int &ans = memo[id][remW];
    if (ans != -1) return ans; // computed before
    if (W[id] > remW) return ans = dp(id+1, remW); // no choice, skip
    return ans = max(dp(id+1, remW), // has choice, skip
                    V[id]+dp(id+1, remW-W[id])); // or take
}

int main() {
    int T; scanf("%d", &T);
    while (T--) {
        memset(memo, -1, sizeof memo);
        scanf("%d", &N);
        for (int i = 0; i < N; ++i)
            scanf("%d %d", &V[i], &W[i]);
        int ans = 0;
        int G; scanf("%d", &G);
        while (G--) {
            int MW; scanf("%d", &MW);
            ans += dp(0, MW);
        }
        printf("%d\n", ans);
    }
    return 0;
}
```

1.4 Traveling salesman

```
// This is a solution for UVa 10496 - Collecting Beepers. The problem is a
// variant of the Traveling Salesman Problem (TSP): Given  $n$  cities and their
// pairwise distances in the form of a matrix 'dist' of size  $n \times n$ , compute the
// minimum cost of making a tour that starts from any city  $s$ , goes through all
// the other  $n - 1$  cities exactly once, and finally returns to the city  $s$ . In
// this case, the salesman is Karel in a 2D world who can only move along the
//  $x$  and  $y$  axis. The cities are beepers whose coordinates are given, from which
// pairwise distances can be calculated. Algorithm takes time  $O(2^n \times n^2)$ .
// INPUT: The first line is the number of test cases. The first line of each
// test case is world's size ( $x$ -size and  $y$ -size). Next is the starting position
// of Karel. Next is the number of beepers. Next are the beepers'  $x$ - and  $y$ -
// coordinates.
// OUTPUT: For each test case, output the minimum distance to move from Karel's
// starting position to each of the beepers and back to the starting position.
```

```
#include <bits/stdc++.h>
using namespace std;

#define LSONe(S) ((S) & -(S))

const int MAX_n = 11;

int dist[MAX_n][MAX_n], memo[MAX_n][1<<(MAX_n-1)]; // Karel + max 10 beepers

int dp(int u, int mask) {
    if (mask == 0) return dist[u][0]; // mask = free coordinates
    int &ans = memo[u][mask]; // close the loop
    if (ans != -1) return ans; // computed before
    ans = 2000000000;
    int m = mask;
    while (m) { // up to  $O(n)$ 
        int two_pow_v = LSONe(m); // but this is fast
        int v = _builtin_ctz(two_pow_v)+1; // offset v by +1
        ans = min(ans, dist[u][v] + dp(v, mask^two_pow_v)); // keep the min
        m -= two_pow_v;
    }
    return ans;
}

int main() {
    int TC; scanf("%d", &TC);
    while (TC--) {
        int xsize, ysize; scanf("%d %d", &xsize, &ysize); // these two values are not used
        int x[MAX_n], y[MAX_n];
        scanf("%d %d", &x[0], &y[0]);
        int n; scanf("%d", &n); ++n; // include Karel
        for (int i = 1; i < n; ++i) // Karel is at index 0
            scanf("%d %d", &x[i], &y[i]);
        for (int i = 0; i < n; ++i) // build distance table
            for (int j = i; j < n; ++j)
                dist[i][j] = dist[j][i] = abs(x[i]-x[j]) + abs(y[i]-y[j]); // Manhattan distance
        memset(memo, -1, sizeof memo);
        printf("The shortest path has length %d\n", dp(0, (1<<(n-1))-1)); // DP-TSP
    }
    return 0;
}
```

2 Geometry

2.1 Convex hull

```
// Compute the 2D convex hull of a set of points using the monotone chain
// algorithm. Eliminate redundant points from the hull if REMOVE_REDUNDANT is
// #defined.
// Running time:  $O(n \log n)$ 
// INPUT: a vector of input points, unordered.
// OUTPUT: a vector of points in the convex hull, counterclockwise, starting
// with bottommost/leftmost point

#include <cstdio>
#include <cassert>
#include <vector>
#include <algorithm>
#include <cmath>
// BEGIN CUT
#include <map>
// END CUT
```

2.2 Miscellaneous geometry

```
using namespace std;

#define REMOVE_REDUNDANT

typedef double T;
const T EPS = 1e-7;
struct PT {
    T x, y;
    PT() {}
    PT(T x, T y) : x(x), y(y) {}
    bool operator<(const PT &rhs) const { return make_pair(y,x) < make_pair(rhs.y,rhs.x); }
    bool operator==(const PT &rhs) const { return make_pair(y,x) == make_pair(rhs.y,rhs.x); }
};

T cross(PT p, PT q) { return p.x*q.y-p.y*q.x; }
T area2(PT a, PT b, PT c) { return cross(a,b) + cross(b,c) + cross(c,a); }

#ifdef REMOVE_REDUNDANT
bool between(const PT &a, const PT &b, const PT &c) {
    return (fabs(area2(a,b,c)) < EPS && (a.x-b.x)*(c.x-b.x) <= 0 && (a.y-b.y)*(c.y-b.y) <= 0);
}
#endif

void ConvexHull(vector<PT> &pts) {
    sort(pts.begin(), pts.end());
    pts.erase(unique(pts.begin(), pts.end(), pts.end()));
    vector<PT> up, dn;
    for (int i = 0; i < pts.size(); i++) {
        while (up.size() > 1 && area2(up[up.size()-2], up.back(), pts[i]) >= 0) up.pop_back();
        while (dn.size() > 1 && area2(dn[dn.size()-2], dn.back(), pts[i]) <= 0) dn.pop_back();
        up.push_back(pts[i]);
        dn.push_back(pts[i]);
    }
    pts = dn;
    for (int i = (int) up.size() - 2; i >= 1; i--) pts.push_back(up[i]);

#ifdef REMOVE_REDUNDANT
    if (pts.size() <= 2) return;
    dn.clear();
    dn.push_back(pts[0]);
    dn.push_back(pts[1]);
    for (int i = 2; i < pts.size(); i++) {
        if (between(dn[dn.size()-2], dn[dn.size()-1], pts[i])) dn.pop_back();
        dn.push_back(pts[i]);
    }
    if (dn.size() >= 3 && between(dn.back(), dn[0], dn[1])) {
        dn[0] = dn.back();
        dn.pop_back();
    }
    pts = dn;
#endif
}

// BEGIN CUT
// The following code solves SPOJ problem #26: Build the Fence (BSHEEP)

int main() {
    int t;
    scanf("%d", &t);
    for (int caseno = 0; caseno < t; caseno++) {
        int n;
        scanf("%d", &n);
        vector<PT> v(n);
        for (int i = 0; i < n; i++) scanf("%lf%lf", &v[i].x, &v[i].y);
        vector<PT> h(v);
        map<PT,int> index;
        for (int i = n-1; i >= 0; i--) index[v[i]] = i+1;
        ConvexHull(h);

        double len = 0;
        for (int i = 0; i < h.size(); i++) {
            double dx = h[i].x - h[(i+1)%h.size()].x;
            double dy = h[i].y - h[(i+1)%h.size()].y;
            len += sqrt(dx*dx+dy*dy);
        }

        if (caseno > 0) printf("\n");
        printf("%.2f\n", len);
        for (int i = 0; i < h.size(); i++) {
            if (i > 0) printf(" ");
            printf("%d", index[h[i]]);
        }
        printf("\n");
    }
}

// END CUT
```

// C++ routines for computational geometry.

```
#include <iostream>
#include <vector>
#include <cmath>
#include <cassert>

using namespace std;

double INF = 1e100;
double EPS = 1e-12;

struct PT {
    double x, y;
    PT() {}
    PT(double x, double y) : x(x), y(y) {}
    PT(const PT &p) : x(p.x), y(p.y) {}
    PT operator + (const PT &p) const { return PT(x+p.x, y+p.y); }
    PT operator - (const PT &p) const { return PT(x-p.x, y-p.y); }
    PT operator * (double c) const { return PT(x*c, y*c); }
    PT operator / (double c) const { return PT(x/c, y/c); }
};

double dot(PT p, PT q) { return p.x*q.x+p.y*q.y; }
double dist2(PT p, PT q) { return dot(p-q,p-q); }
double cross(PT p, PT q) { return p.x*q.y-p.y*q.x; }
ostream &operator<<(ostream &os, const PT &p) {
    return os << "(" << p.x << ", " << p.y << ")";
}

// rotate a point CCW or CW around the origin
PT RotateCCW90(PT p) { return PT(-p.y,p.x); }
PT RotateCW90(PT p) { return PT(p.y,-p.x); }
PT RotateCCW(PT p, double t) {
    return PT(p.x*cos(t)-p.y*sin(t), p.x*sin(t)+p.y*cos(t));
}

// project point c onto line through a and b
// assuming a != b
PT ProjectPointLine(PT a, PT b, PT c) {
    return a + (b-a)*dot(c-a, b-a)/dot(b-a, b-a);
}

// project point c onto line segment through a and b
PT ProjectPointSegment(PT a, PT b, PT c) {
    double r = dot(b-a,b-a);
    if (fabs(r) < EPS) return a;
    r = dot(c-a, b-a)/r;
    if (r < 0) return a;
    if (r > 1) return b;
    return a + (b-a)*r;
}

// compute distance from c to segment between a and b
double DistancePointSegment(PT a, PT b, PT c) {
    return sqrt(dist2(c, ProjectPointSegment(a, b, c)));
}

// compute distance between point (x,y,z) and plane ax+by+cz=d
double DistancePointPlane(double x, double y, double z,
    double a, double b, double c, double d)
{
    return fabs(a*x+b*y+c*z-d)/sqrt(a*a+b*b+c*c);
}

// determine if lines from a to b and c to d are parallel or collinear
bool LinesParallel(PT a, PT b, PT c, PT d) {
    return fabs(cross(b-a, c-d)) < EPS;
}

bool LinesCollinear(PT a, PT b, PT c, PT d) {
    return LinesParallel(a, b, c, d)
        && fabs(cross(a-b, a-c)) < EPS
        && fabs(cross(c-d, c-a)) < EPS;
}

// determine if line segment from a to b intersects with
// line segment from c to d
bool SegmentsIntersect(PT a, PT b, PT c, PT d) {
    if (LinesCollinear(a, b, c, d)) {
        if (dist2(a, c) < EPS || dist2(a, d) < EPS ||
            dist2(b, c) < EPS || dist2(b, d) < EPS) return true;
        if (dot(c-a, c-b) > 0 && dot(d-a, d-b) > 0 && dot(c-b, d-b) > 0)
            return false;
        return true;
    }
}
```

```

    if (cross(d-a, b-a) * cross(c-a, b-a) > 0) return false;
    if (cross(a-c, d-c) * cross(b-c, d-c) > 0) return false;
    return true;
}

// compute intersection of line passing through a and b
// with line passing through c and d, assuming that unique
// intersection exists; for segment intersection, check if
// segments intersect first
PT ComputelineIntersection(PT a, PT b, PT c, PT d) {
    b=b-a; d=d-c; c=c-a;
    assert(dot(b, b) > EPS && dot(d, d) > EPS);
    return a + b*cross(c, d)/cross(b, d);
}

// compute center of circle given three points
PT ComputeCircleCenter(PT a, PT b, PT c) {
    b=(a+b)/2;
    c=(a+c)/2;
    return ComputelineIntersection(b, b+RotateCW90(a-b), c, c+RotateCW90(a-c));
}

// determine if point is in a possibly non-convex polygon (by William
// Randolph Franklin); returns 1 for strictly interior points, 0 for
// strictly exterior points, and 0 or 1 for the remaining points.
// Note that it is possible to convert this into an *exact* test using
// integer arithmetic by taking care of the division appropriately
// (making sure to deal with signs properly) and then by writing exact
// tests for checking point on polygon boundary
bool PointInPolygon(const vector<PT> &p, PT q) {
    bool c = 0;
    for (int i = 0; i < p.size(); i++){
        int j = (i+1)%p.size();
        if ((p[i].y <= q.y && q.y < p[j].y ||
            p[j].y <= q.y && q.y < p[i].y) &&
            q.x < p[i].x + (p[j].x - p[i].x) * (q.y - p[i].y) / (p[j].y - p[i].y))
            c = !c;
    }
    return c;
}

// determine if point is on the boundary of a polygon
bool PointOnPolygon(const vector<PT> &p, PT q) {
    for (int i = 0; i < p.size(); i++)
        if (dist2(ProjectPointSegment(p[i], p[(i+1)%p.size()], q), q) < EPS)
            return true;
    return false;
}

// compute intersection of line through points a and b with
// circle centered at c with radius r > 0
vector<PT> CircleLineIntersection(PT a, PT b, PT c, double r) {
    vector<PT> ret;
    b = b-a;
    a = a-c;
    double A = dot(b, b);
    double B = dot(a, b);
    double C = dot(a, a) - r*r;
    double D = B*B - A*C;
    if (D < -EPS) return ret;
    ret.push_back(c+a+b*(-B+sqrt(D+EPS))/A);
    if (D > EPS)
        ret.push_back(c+a+b*(-B-sqrt(D))/A);
    return ret;
}

// compute intersection of circle centered at a with radius r
// with circle centered at b with radius R
vector<PT> CircleCircleIntersection(PT a, PT b, double r, double R) {
    vector<PT> ret;
    double d = sqrt(dist2(a, b));
    if (d > r+R || d+min(r, R) < max(r, R)) return ret;
    double x = (d*d-R*R+r*r)/(2*d);
    double y = sqrt(r*r-x*x);
    PT v = (b-a)/d;
    ret.push_back(a+v*x + RotateCCW90(v)*y);
    if (y > 0)
        ret.push_back(a+v*x - RotateCCW90(v)*y);
    return ret;
}

// This code computes the area or centroid of a (possibly nonconvex)
// polygon, assuming that the coordinates are listed in a clockwise or
// counterclockwise fashion. Note that the centroid is often known as
// the "center of gravity" or "center of mass".
double ComputeSignedArea(const vector<PT> &p) {
    double area = 0;
    for(int i = 0; i < p.size(); i++) {
        int j = (i+1) % p.size();
        area += p[i].x*p[j].y - p[j].x*p[i].y;
    }
    return area / 2.0;
}

}

double ComputeArea(const vector<PT> &p) {
    return fabs(ComputeSignedArea(p));
}

PT ComputeCentroid(const vector<PT> &p) {
    PT c(0,0);
    double scale = 6.0 * ComputeSignedArea(p);
    for (int i = 0; i < p.size(); i++){
        int j = (i+1) % p.size();
        c = c + (p[i]*p[j])*(p[i].x*p[j].y - p[j].x*p[i].y);
    }
    return c / scale;
}

// tests whether or not a given polygon (in CW or CCW order) is simple
bool IsSimple(const vector<PT> &p) {
    for (int i = 0; i < p.size(); i++) {
        for (int k = i+1; k < p.size(); k++) {
            int j = (i+1) % p.size();
            int l = (k+1) % p.size();
            if (i == 1 || j == k) continue;
            if (SegmentsIntersect(p[i], p[j], p[k], p[l]))
                return false;
        }
    }
    return true;
}

int main() {
    // expected: (-5,2)
    cerr << RotateCCW90(PT(2,5)) << endl;

    // expected: (5,-2)
    cerr << RotateCW90(PT(2,5)) << endl;

    // expected: (-5,2)
    cerr << RotateCCW(PT(2,5), M_PI/2) << endl;

    // expected: (5,2)
    cerr << ProjectPointLine(PT(-5,-2), PT(10,4), PT(3,7)) << endl;

    // expected: (5,2) (7.5,3) (2.5,1)
    cerr << ProjectPointSegment(PT(-5,-2), PT(10,4), PT(3,7)) << " "
        << ProjectPointSegment(PT(7.5,3), PT(10,4), PT(3,7)) << " "
        << ProjectPointSegment(PT(-5,-2), PT(2.5,1), PT(3,7)) << endl;

    // expected: 6.78903
    cerr << DistancePointPlane(4,-4,3,2,-2,5,-8) << endl;

    // expected: 1 0 1
    cerr << LinesParallel(PT(1,1), PT(3,5), PT(2,1), PT(4,5)) << " "
        << LinesParallel(PT(1,1), PT(3,5), PT(2,0), PT(4,5)) << " "
        << LinesParallel(PT(1,1), PT(3,5), PT(5,9), PT(7,13)) << endl;

    // expected: 0 0 1
    cerr << LinesCollinear(PT(1,1), PT(3,5), PT(2,1), PT(4,5)) << " "
        << LinesCollinear(PT(1,1), PT(3,5), PT(2,0), PT(4,5)) << " "
        << LinesCollinear(PT(1,1), PT(3,5), PT(5,9), PT(7,13)) << endl;

    // expected: 1 1 1 0
    cerr << SegmentsIntersect(PT(0,0), PT(2,4), PT(3,1), PT(-1,3)) << " "
        << SegmentsIntersect(PT(0,0), PT(2,4), PT(4,3), PT(0,5)) << " "
        << SegmentsIntersect(PT(0,0), PT(2,4), PT(2,-1), PT(-2,1)) << " "
        << SegmentsIntersect(PT(0,0), PT(2,4), PT(5,5), PT(1,7)) << endl;

    // expected: (1,2)
    cerr << ComputelineIntersection(PT(0,0), PT(2,4), PT(3,1), PT(-1,3)) << endl;

    // expected: (1,1)
    cerr << ComputeCircleCenter(PT(-3,4), PT(6,1), PT(4,5)) << endl;

    vector<PT> v;
    v.push_back(PT(0,0));
    v.push_back(PT(5,0));
    v.push_back(PT(5,5));
    v.push_back(PT(0,5));

    // expected: 1 1 1 0 0
    cerr << PointInPolygon(v, PT(2,2)) << " "
        << PointInPolygon(v, PT(2,0)) << " "
        << PointInPolygon(v, PT(0,2)) << " "
        << PointInPolygon(v, PT(5,2)) << " "
        << PointInPolygon(v, PT(2,5)) << endl;

    // expected: 0 1 1 1 1
    cerr << PointOnPolygon(v, PT(2,2)) << " "
        << PointOnPolygon(v, PT(2,0)) << " "
        << PointOnPolygon(v, PT(0,2)) << " "
        << PointOnPolygon(v, PT(5,2)) << " "
        << PointOnPolygon(v, PT(2,5)) << " "
}

```

```

    << PointOnPolygon(v, PT(2,5)) << endl;

// expected: (1,6)
// (5,4) (4,5)
// blank line
// (4,5) (5,4)
// blank line
// (4,5) (5,4)
vector<PT> u = CircleLineIntersection(PT(0,6), PT(2,6), PT(1,1), 5);
for (int i = 0; i < u.size(); i++) cerr << u[i] << " "; cerr << endl;
u = CircleLineIntersection(PT(0,9), PT(9,0), PT(1,1), 5);
for (int i = 0; i < u.size(); i++) cerr << u[i] << " "; cerr << endl;
u = CircleCircleIntersection(PT(1,1), PT(10,10), 5, 5);
for (int i = 0; i < u.size(); i++) cerr << u[i] << " "; cerr << endl;
u = CircleCircleIntersection(PT(1,1), PT(8,8), 5, 5);
for (int i = 0; i < u.size(); i++) cerr << u[i] << " "; cerr << endl;
u = CircleCircleIntersection(PT(1,1), PT(4.5,4.5), 10, sqrt(2.0)/2.0);
for (int i = 0; i < u.size(); i++) cerr << u[i] << " "; cerr << endl;
u = CircleCircleIntersection(PT(1,1), PT(4.5,4.5), 5, sqrt(2.0)/2.0);
for (int i = 0; i < u.size(); i++) cerr << u[i] << " "; cerr << endl;

// area should be 5.0
// centroid should be (1.1666666, 1.1666666)
PT pa[] = { PT(0,0), PT(5,0), PT(1,1), PT(0,5) };
vector<PT> p(pa, pa+4);
PT c = ComputeCentroid(p);
cerr << "Area: " << ComputeArea(p) << endl;
cerr << "Centroid: " << c << endl;

return 0;
}

```

3 Numerical algorithms

3.1 Number theory (modular, Chinese remainder, linear Diophantine)

```

// This is a collection of useful code for solving problems that
// involve modular linear equations. Note that all of the
// algorithms described here work on nonnegative integers.

```

```

#include <iostream>
#include <vector>
#include <algorithm>

using namespace std;

typedef vector<int> VI;
typedef pair<int, int> PII;

// return a % b (positive value)
int mod(int a, int b) {
    return ((a%b) + b) % b;
}

// computes gcd(a,b)
int gcd(int a, int b) {
    while (b) { int t = a%b; a = b; b = t; }
    return a;
}

// computes lcm(a,b)
int lcm(int a, int b) {
    return a / gcd(a, b)*b;
}

// (a^b) mod m via successive squaring
int powermod(int a, int b, int m)
{
    int ret = 1;
    while (b)
    {
        if (b & 1) ret = mod(ret*a, m);
        a = mod(a*a, m);
        b >>= 1;
    }
    return ret;
}

// returns g = gcd(a, b); finds x, y such that d = ax + by
int extended_euclid(int a, int b, int &x, int &y) {
    int xx = y = 0;
    int yy = x = 1;

```

```

    while (b) {
        int q = a / b;
        int t = b; b = a%b; a = t;
        t = xx; xx = x - q*xx; x = t;
        t = yy; yy = y - q*yy; y = t;
    }
    return a;
}

// finds all solutions to ax = b (mod n)
VI modular_linear_equation_solver(int a, int b, int n) {
    int x, y;
    VI ret;
    int g = extended_euclid(a, n, x, y);
    if (!(b%g)) {
        x = mod(x*(b / g), n);
        for (int i = 0; i < g; i++)
            ret.push_back(mod(x + i*(n / g), n));
    }
    return ret;
}

// computes b such that ab = 1 (mod n), returns -1 on failure
int mod_inverse(int a, int n) {
    int x, y;
    int g = extended_euclid(a, n, x, y);
    if (g > 1) return -1;
    return mod(x, n);
}

// Chinese remainder theorem (special case): find z such that
// z % m1 = r1, z % m2 = r2. Here, z is unique modulo M = lcm(m1, m2).
// Return (z, M). On failure, M = -1.
PII chinese_remainder_theorem(int m1, int r1, int m2, int r2) {
    int s, t;
    int g = extended_euclid(m1, m2, s, t);
    int g = extended_euclid(m1, m2, s, t);
    if (r1%g != r2%g) return make_pair(0, -1);
    return make_pair(mod(s*r2*m1 + t*r1*m2, m1*m2) / g, m1*m2 / g);
}

// Chinese remainder theorem: find z such that
// z % m[i] = r[i] for all i. Note that the solution is
// unique modulo M = lcm_i (m[i]). Return (z, M). On
// failure, M = -1. Note that we do not require the a[i]'s
// to be relatively prime.
PII chinese_remainder_theorem(const VI &m, const VI &r) {
    PII ret = make_pair(r[0], m[0]);
    for (int i = 1; i < m.size(); i++) {
        ret = chinese_remainder_theorem(ret.second, ret.first, m[i], r[i]);
        if (ret.second == -1) break;
    }
    return ret;
}

// computes x and y such that ax + by = c
// returns whether the solution exists
bool linear_diophantine(int a, int b, int c, int &x, int &y) {
    if (!a && !b)
    {
        if (c) return false;
        x = 0; y = 0;
        return true;
    }
    if (!a)
    {
        if (c % b) return false;
        x = 0; y = c / b;
        return true;
    }
    if (!b)
    {
        if (c % a) return false;
        x = c / a; y = 0;
        return true;
    }
    int g = gcd(a, b);
    if (c % g) return false;
    x = c / g + mod_inverse(a / g, b / g);
    y = (c - a*x) / b;
    return true;
}

int main() {
    // expected: 2
    cout << gcd(14, 30) << endl;

    // expected: 2 -2 1
    int x, y;
    int g = extended_euclid(14, 30, x, y);
    cout << g << " " << x << " " << y << endl;

    // expected: 95 451

```

```

VI sols = modular_linear_equation_solver(14, 30, 100);
for (int i = 0; i < sols.size(); i++) cout << sols[i] << " ";
cout << endl;

// expected: 8
cout << mod_inverse(8, 9) << endl;

// expected: 23 105
//          11 12
PII ret = chinese_remainder_theorem(VI({ 3, 5, 7 }), VI({ 2, 3, 2 }));
cout << ret.first << " " << ret.second << endl;
ret = chinese_remainder_theorem(VI({ 4, 6 }), VI({ 3, 5 }));
cout << ret.first << " " << ret.second << endl;

// expected: 5 -15
if (!linear_diophantine(7, 2, 5, x, y)) cout << "ERROR" << endl;
cout << x << " " << y << endl;
return 0;
}

```

3.2 Systems of linear equations, matrix inverse, determinant

```

// Gauss-Jordan elimination with full pivoting.
//
// Uses:
// (1) solving systems of linear equations (AX=B)
// (2) inverting matrices (AX=I)
// (3) computing determinants of square matrices
//
// Running time: O(n^3)
//
// INPUT:   a[][] = an nxn matrix
//          b[][] = an nxm matrix
//
// OUTPUT:  X      = an nxm matrix (stored in b[][])
//          A^-1    = an nxn matrix (stored in a[][])
//          returns determinant of a[][]

#include <iostream>
#include <vector>
#include <cmath>

using namespace std;

const double EPS = 1e-10;

typedef vector<int> VI;
typedef double T;
typedef vector<T> VT;
typedef vector<VT> VVT;

T GaussJordan(VVT &a, VVT &b) {
    const int n = a.size();
    const int m = b[0].size();
    VI irow(n), icol(n), ipiv(n);
    T det = 1;

    for (int i = 0; i < n; i++) {
        int pj = -1, pk = -1;
        for (int j = 0; j < n; j++) if (!ipiv[j])
            for (int k = 0; k < n; k++) if (!ipiv[k])
                if (pj == -1 || fabs(a[j][k]) > fabs(a[pj][pk])) { pj = j; pk = k; }
        if (fabs(a[pj][pk]) < EPS) { cerr << "Matrix is singular." << endl; exit(0); }
        ipiv[pj]++;
        swap(a[pj], a[pk]);
        swap(b[pj], b[pk]);
        if (pj != pk) det *= -1;
        irow[i] = pj;
        icol[i] = pk;

        T c = 1.0 / a[pj][pk];
        det *= a[pj][pk];
        a[pj][pk] = 1.0;
        for (int p = 0; p < n; p++) a[pk][p] *= c;
        for (int p = 0; p < m; p++) b[pk][p] *= c;
        for (int p = 0; p < n; p++) if (p != pk) {
            c = a[p][pk];
            a[p][pk] = 0;
            for (int q = 0; q < n; q++) a[p][q] -= a[pk][q] * c;
            for (int q = 0; q < m; q++) b[p][q] -= b[pk][q] * c;
        }
    }

    for (int p = n-1; p >= 0; p--) if (irow[p] != icol[p]) {
        for (int k = 0; k < n; k++) swap(a[k][irow[p]], a[k][icol[p]]);
    }
}

```

```

}

return det;
}

int main() {
    const int n = 4;
    const int m = 2;
    double A[n][n] = { { 1,2,3,4 }, { 1,0,1,0 }, { 5,3,2,4 }, { 6,1,4,6 } };
    double B[n][m] = { { 1,2 }, { 4,3 }, { 5,6 }, { 8,7 } };
    VVT a(n), b(n);
    for (int i = 0; i < n; i++) {
        a[i] = VT(A[i], A[i] + n);
        b[i] = VT(B[i], B[i] + m);
    }

    double det = GaussJordan(a, b);

    // expected: 60
    cout << "Determinant: " << det << endl;

    // expected: -0.233333 0.166667 0.133333 0.066667
    //          0.166667 0.166667 0.333333 -0.333333
    //          0.233333 0.833333 -0.133333 -0.066667
    //          0.05 -0.75 -0.1 0.2
    cout << "Inverse: " << endl;
    for (int i = 0; i < n; i++) {
        for (int j = 0; j < n; j++)
            cout << a[i][j] << ' ';
        cout << endl;
    }

    // expected: 1.63333 1.3
    //          -0.166667 0.5
    //          2.36667 1.7
    //          -1.85 -1.35
    cout << "Solution: " << endl;
    for (int i = 0; i < n; i++) {
        for (int j = 0; j < m; j++)
            cout << b[i][j] << ' ';
        cout << endl;
    }
}

```

3.3 Reduced row echelon form, matrix rank

```

// Reduced row echelon form via Gauss-Jordan elimination
// with partial pivoting. This can be used for computing
// the rank of a matrix.
//
// Running time: O(n^3)
//
// INPUT:   a[][] = an nxm matrix
//
// OUTPUT:  rref[][] = an nxm matrix (stored in a[][])
//          returns rank of a[][]

#include <iostream>
#include <vector>
#include <cmath>

using namespace std;

const double EPSILON = 1e-10;

typedef double T;
typedef vector<T> VT;
typedef vector<VT> VVT;

int rref(VVT &a) {
    int n = a.size();
    int m = a[0].size();
    int r = 0;
    for (int c = 0; c < m && r < n; c++) {
        int j = r;
        for (int i = r + 1; i < n; i++)
            if (fabs(a[i][c]) > fabs(a[j][c])) j = i;
        if (fabs(a[j][c]) < EPSILON) continue;
        swap(a[j], a[r]);

        T s = 1.0 / a[r][c];
        for (int j = 0; j < m; j++) a[r][j] *= s;
        for (int i = 0; i < n; i++) if (i != r) {
            T t = a[i][c];
            for (int j = 0; j < m; j++) a[i][j] -= t * a[r][j];
        }
        r++;
    }
}

```

```

    }
    return r;
}

int main() {
    const int n = 5, m = 4;
    double A[n][m] = {
        {16, 2, 3, 13},
        {5, 11, 10, 8},
        {9, 7, 6, 12},
        {4, 14, 15, 1},
        {13, 21, 21, 13}};
    VVT a(n);
    for (int i = 0; i < n; i++)
        a[i] = VT(A[i], A[i] + m);

    int rank = rref(a);

    // expected: 3
    cout << "Rank: " << rank << endl;

    // expected: 1 0 0 1
    //           0 1 0 3
    //           0 0 1 -3
    //           0 0 0 3.10862e-15
    //           0 0 0 2.22045e-15
    cout << "rref: " << endl;
    for (int i = 0; i < 5; i++) {
        for (int j = 0; j < 4; j++)
            cout << a[i][j] << ' ';
        cout << endl;
    }
}

```

3.4 Fast Fourier transform

```

#include <cassert>
#include <cstdio>
#include <cmath>

struct cpx
{
    cpx() {}
    cpx(double aa):a(aa),b(0) {}
    cpx(double aa, double bb):a(aa),b(bb) {}
    double a;
    double b;
    double modsq(void) const
    {
        return a * a + b * b;
    }
    cpx bar(void) const
    {
        return cpx(a, -b);
    }
};

cpx operator +(cpx a, cpx b)
{
    return cpx(a.a + b.a, a.b + b.b);
}

cpx operator *(cpx a, cpx b)
{
    return cpx(a.a * b.a - a.b * b.b, a.a * b.b + a.b * b.a);
}

cpx operator /(cpx a, cpx b)
{
    cpx r = a * b.bar();
    return cpx(r.a / b.modsq(), r.b / b.modsq());
}

cpx EXP(double theta)
{
    return cpx(cos(theta), sin(theta));
}

const double two_pi = 4 * acos(0);

// in:    input array
// out:   output array
// step:  {SET TO 1} (used internally)
// size:  length of the input/output {MUST BE A POWER OF 2}
// dir:   either plus or minus one (direction of the FFT)
// RESULT: out[k] = \sum_{j=0}^{size-1} in[j] * exp(dir * 2pi * i * j * k / size)
void FFT(cpx *in, cpx *out, int step, int size, int dir)
{

```

```

    if(size < 1) return;
    if(size == 1)
    {
        out[0] = in[0];
        return;
    }
    FFT(in, out, step * 2, size / 2, dir);
    FFT(in + step, out + size / 2, step * 2, size / 2, dir);
    for(int i = 0; i < size / 2; i++)
    {
        cpx even = out[i];
        cpx odd = out[i + size / 2];
        out[i] = even + EXP(dir * two_pi * i / size) * odd;
        out[i + size / 2] = even + EXP(dir * two_pi * (i + size / 2) / size) * odd;
    }

    // Usage:
    // f[0...N-1] and g[0...N-1] are numbers
    // Want to compute the convolution h, defined by
    // h[n] = sum of f[k]g[n-k] (k = 0, ..., N-1).
    // Here, the index is cyclic; f[-1] = f[N-1], f[-2] = f[N-2], etc.
    // Let F[0...N-1] be FFT(f), and similarly, define G and H.
    // The convolution theorem says H[n] = F[n]G[n] (element-wise product).
    // To compute h[] in O(N log N) time, do the following:
    // 1. Compute F and G (pass dir = 1 as the argument).
    // 2. Get H by element-wise multiplying F and G.
    // 3. Get h by taking the inverse FFT (use dir = -1 as the argument)
    // and *dividing by N*. DO NOT FORGET THIS SCALING FACTOR.

    int main(void)
    {
        printf("If rows come in identical pairs, then everything works.\n");

        cpx a[8] = {0, 1, cpx(1,3), cpx(0,5), 1, 0, 2, 0};
        cpx b[8] = {1, cpx(0,-2), cpx(0,1), 3, -1, -3, 1, -2};
        cpx A[8];
        cpx B[8];
        FFT(a, A, 1, 8, 1);
        FFT(b, B, 1, 8, 1);

        for(int i = 0; i < 8; i++)
        {
            printf("%7.21f%7.21f", A[i].a, A[i].b);
        }
        printf("\n");
        for(int i = 0; i < 8; i++)
        {
            cpx Ai(0,0);
            for(int j = 0; j < 8; j++)
            {
                Ai = Ai + a[j] * EXP(j * i * two_pi / 8);
            }
            printf("%7.21f%7.21f", Ai.a, Ai.b);
        }
        printf("\n");

        cpx AB[8];
        for(int i = 0; i < 8; i++)
            AB[i] = A[i] * B[i];
        cpx aconvb[8];
        FFT(AB, aconvb, 1, 8, -1);
        for(int i = 0; i < 8; i++)
            aconvb[i] = aconvb[i] / 8;
        for(int i = 0; i < 8; i++)
        {
            printf("%7.21f%7.21f", aconvb[i].a, aconvb[i].b);
        }
        printf("\n");
        for(int i = 0; i < 8; i++)
        {
            cpx aconvbi(0,0);
            for(int j = 0; j < 8; j++)
            {
                aconvbi = aconvbi + a[j] * b[(8 + i - j) % 8];
            }
            printf("%7.21f%7.21f", aconvbi.a, aconvbi.b);
        }
        printf("\n");
        return 0;
    }
}

```

3.5 Simplex algorithm

```

// Two-phase simplex algorithm for solving linear programs of the form
//

```

```
// maximize    c^T x
// subject to  Ax <= b
//            x >= 0
//
// INPUT: A -- an m x n matrix
//        b -- an m-dimensional vector
//        c -- an n-dimensional vector
//        x -- a vector where the optimal solution will be stored
//
// OUTPUT: value of the optimal solution (infinity if unbounded
//        above, nan if infeasible)
//
// To use this code, create an LPSolver object with A, b, and c as
// arguments. Then, call Solve(x).

#include <iostream>
#include <iomanip>
#include <vector>
#include <cmath>
#include <limits>

using namespace std;

typedef long double DOUBLE;
typedef vector<DOUBLE> VD;
typedef vector<VD> VVD;
typedef vector<int> VI;

const DOUBLE EPS = 1e-9;

struct LPSolver {
    int m, n;
    VI B, N;
    VVD D;

    LPSolver(const VVD &A, const VD &b, const VD &c) :
        m(b.size()), n(c.size()), N(n + 1), B(m), D(m + 2, VD(n + 2)) {
        for (int i = 0; i < m; i++) for (int j = 0; j < n; j++) D[i][j] = A[i][j];
        for (int i = 0; i < m; i++) { B[i] = n + 1; D[i][n] = -1; D[i][n + 1] = b[i]; }
        for (int j = 0; j < n; j++) { N[j] = j; D[m][j] = -c[j]; }
        N[n] = -1; D[m + 1][n] = 1;
    }

    void Pivot(int r, int s) {
        double inv = 1.0 / D[r][s];
        for (int i = 0; i < m + 2; i++) if (i != r)
            for (int j = 0; j < n + 2; j++) if (j != s)
                D[i][j] -= D[r][j] * D[i][s] * inv;
        for (int j = 0; j < n + 2; j++) if (j != s) D[r][j] *= inv;
        for (int i = 0; i < m + 2; i++) if (i != r) D[i][s] *= -inv;
        D[r][s] = inv;
        swap(B[r], N[s]);
    }

    bool Simplex(int phase) {
        int x = phase == 1 ? m + 1 : m;
        while (true) {
            int s = -1;
            for (int j = 0; j <= n; j++) {
                if (phase == 2 && N[j] == -1) continue;
                if (s == -1 || D[x][j] < D[x][s] || D[x][j] == D[x][s] && N[j] < N[s]) s = j;
            }
            if (D[x][s] > -EPS) return true;
            int r = -1;
            for (int i = 0; i < m; i++) {
                if (D[i][s] < EPS) continue;
                if (r == -1 || D[i][n + 1] / D[i][s] < D[r][n + 1] / D[r][s] ||
                    (D[i][n + 1] / D[i][s] == (D[r][n + 1] / D[r][s]) && B[i] < B[r]) r = i;
            }
            if (r == -1) return false;
            Pivot(r, s);
        }
    }

    DOUBLE Solve(VD &x) {
        int r = 0;
        for (int i = 1; i < m; i++) if (D[i][n + 1] < D[r][n + 1]) r = i;
        if (D[r][n + 1] < -EPS) {
            Pivot(r, n);
            if (!Simplex(1) || D[m + 1][n + 1] < -EPS) return numeric_limits<DOUBLE>::infinity();
            for (int i = 0; i < m; i++) if (B[i] == -1) {
                int s = -1;
                for (int j = 0; j <= n; j++)
                    if (s == -1 || D[i][j] < D[i][s] || D[i][j] == D[i][s] && N[j] < N[s]) s = j;
                Pivot(i, s);
            }
        }
        if (!Simplex(2)) return numeric_limits<DOUBLE>::infinity();
        x = VD(n);
        for (int i = 0; i < m; i++) if (B[i] < n) x[B[i]] = D[i][n + 1];
        return D[m][n + 1];
    }
}
```

```
};

int main() {
    const int m = 4;
    const int n = 3;
    DOUBLE _A[m][n] = {
        { 6, -1, 0 },
        { -1, -5, 0 },
        { 1, 5, 1 },
        { -1, -5, -1 }
    };
    DOUBLE _b[m] = { 10, -4, 5, -5 };
    DOUBLE _c[n] = { 1, -1, 0 };

    VVD A(m);
    VD b(_b, _b + m);
    VD c(_c, _c + n);
    for (int i = 0; i < m; i++) A[i] = VD(_A[i], _A[i] + n);

    LPSolver solver(A, b, c);
    VD x;
    DOUBLE value = solver.Solve(x);

    cerr << "VALUE: " << value << endl; // VALUE: 1.29032
    cerr << "SOLUTION:"; // SOLUTION: 1.74194 0.451613 1
    for (size_t i = 0; i < x.size(); i++) cerr << " " << x[i];
    cerr << endl;
    return 0;
}
```

4 Graph algorithms

4.1 Bellman-Ford shortest paths with negative edge weights

```
// This function runs the Bellman-Ford algorithm for single source
// shortest paths with negative edge weights. The function returns
// false if a negative weight cycle is detected. Otherwise, the
// function returns true and dist[i] is the length of the shortest
// path from start to i.
//
// Running time: O(|V|^3)
//
// INPUT: start, w[i][j] = cost of edge from i to j
// OUTPUT: dist[i] = min weight path from start to i
//         prev[i] = previous node on the best path from the
//         start node
```

```
#include <iostream>
#include <queue>
#include <cmath>
#include <vector>

using namespace std;

typedef double T;
typedef vector<T> VT;
typedef vector<VT> VVT;

typedef vector<int> VI;
typedef vector<VI> VVI;

bool BellmanFord(const VVT &w, VT &dist, VI &prev, int start) {
    int n = w.size();
    prev = VI(n, -1);
    dist = VT(n, 1000000000);
    dist[start] = 0;

    for (int k = 0; k < n; k++) {
        for (int i = 0; i < n; i++) {
            for (int j = 0; j < n; j++) {
                if (dist[j] > dist[i] + w[i][j]) {
                    if (k == n-1) return false;
                    dist[j] = dist[i] + w[i][j];
                    prev[j] = i;
                }
            }
        }
    }

    return true;
}
```


4.2 Fast Dijkstra's algorithm

```
// Implementation of Dijkstra's algorithm using adjacency lists
// and priority queue for efficiency.
//
// Running time:  $O(|E| \log |V|)$ 

#include <queue>
#include <cstdio>

using namespace std;
const int INF = 2000000000;
typedef pair<int, int> PII;

int main() {
    int N, s, t;
    scanf("%d%d%d", &N, &s, &t);
    vector<vector<PII>> edges(N);
    for (int i = 0; i < N; i++) {
        int M;
        scanf("%d", &M);
        for (int j = 0; j < M; j++) {
            int vertex, dist;
            scanf("%d%d", &vertex, &dist);
            edges[i].push_back(make_pair(dist, vertex)); // note order of arguments here
        }
    }

    // use priority queue in which top element has the "smallest" priority
    priority_queue<PII, vector<PII>, greater<PII>> Q;
    vector<int> dist(N, INF), dad(N, -1);
    Q.push(make_pair(0, s));
    dist[s] = 0;
    while (!Q.empty()) {
        PII p = Q.top();
        Q.pop();
        int here = p.second;
        if (here == t) break;
        if (dist[here] != p.first) continue;

        for (vector<PII>::iterator it = edges[here].begin(); it != edges[here].end(); it++) {
            if (dist[here] + it->first < dist[it->second]) {
                dist[it->second] = dist[here] + it->first;
                dad[it->second] = here;
                Q.push(make_pair(dist[it->second], it->second));
            }
        }
    }

    printf("%d\n", dist[t]);
    if (dist[t] < INF)
        for (int i = t; i != -1; i = dad[i])
            printf("%d%c", i, (i == s ? '\n' : ' '));

    return 0;
}

/*
Sample input:
5 0 4
2 1 2 3 1
2 2 4 4 5
3 1 4 3 3 4 1
2 0 1 2 3
2 1 5 2 1

Expected:
5
4 2 3 0
*/
```

4.3 Strongly connected components

```
#include <memory.h>
struct edge{int e, nxt;};
int V, E;
edge e[MAXE], er[MAXE];
int sp[MAXV], spr[MAXV];
int group_cnt, group_num[MAXV];
bool v[MAXV];
int stk[MAXV];
void fill_forward(int x)
{
    int i;
```

```
v[x]=true;
for(i=sp[x];i;i=e[i].nxt) if(!v[e[i].e]) fill_forward(e[i].e);
stk[++stk[0]]=x;
}
void fill_backward(int x)
{
    int i;
    v[x]=false;
    group_num[x]=group_cnt;
    for(i=spr[x];i;i=er[i].nxt) if(v[er[i].e]) fill_backward(er[i].e);
}
void add_edge(int v1, int v2) //add edge v1->v2
{
    e[++E].e=v2; e[E].nxt=sp[v1]; sp[v1]=E;
    er[E].e=v1; er[E].nxt=spr[v2]; spr[v2]=E;
}
void SCC()
{
    int i;
    stk[0]=0;
    memset(v, false, sizeof(v));
    for(i=1;i<=V;i++) if(!v[i]) fill_forward(i);
    group_cnt=0;
    for(i=stk[0];i>=1;i--) if(v[stk[i]]){group_cnt++; fill_backward(stk[i]);}
```

4.4 Eulerian path

```
struct Edge;
typedef list<Edge>::iterator iter;

struct Edge
{
    int next_vertex;
    iter reverse_edge;

    Edge(int next_vertex)
        :next_vertex(next_vertex)
    { }
};

const int max_vertices = ;
int num_vertices;
list<Edge> adj[max_vertices]; // adjacency list

vector<int> path;

void find_path(int v)
{
    while(adj[v].size() > 0)
    {
        int vn = adj[v].front().next_vertex;
        adj[vn].erase(adj[v].front().reverse_edge);
        adj[v].pop_front();
        find_path(vn);
    }
    path.push_back(v);
}

void add_edge(int a, int b)
{
    adj[a].push_front(Edge(b));
    iter ita = adj[a].begin();
    adj[b].push_front(Edge(a));
    iter itb = adj[b].begin();
    ita->reverse_edge = itb;
    itb->reverse_edge = ita;
}
```

4.5 Kruskal's algorithm

```
// This is a solution for UVa 11747 - Heavy Cycle Edges. Abridged problem
// description: Undirected graph. Generate Minimum Spanning Tree (MST).

#include <bits/stdc++.h>

#define LOCAL
#define ll long long

using namespace std;

int parent[1020];
```

```

struct edge {
    ll n1, n2, w;
} node[25020];

int compare(edge A, edge B) {
    return A.w < B.w;
}

int find_root(int a) {
    if (a != parent[a])
        return parent[a] = find_root(parent[a]);
    return a;
}

int main() {
#ifdef LOCAL
    freopen("in1.txt", "r", stdin);
    freopen("out.txt", "w", stdout);
#endif
    int n, m, p_n1, p_n2; // LOCAL
    vector<int> hce; //heavy edge circle
    while (cin >> n >> m && n + m != 0) {
        for (int i = 0; i < m; i++) {
            cin >> node[i].n1 >> node[i].n2 >> node[i].w;
        }

        for (int i = 0; i < n; i++)
            parent[i] = i;
        sort(node, node + m, compare);
        hce.clear();

        //kruskal
        for (int i = 0; i < m; i++) {
            p_n1 = find_root(node[i].n1);
            p_n2 = find_root(node[i].n2);
            if (p_n1 != p_n2)
                parent[p_n2] = p_n1;
            else
                hce.push_back(node[i].w);

            //debug
            /*
            for(int i = 0; i < n; i++)
                cout << parent[i] << ' ';
            cout << '\n';
            */
        }
        sort(hce.begin(), hce.end());
        if (hce.size()) {
            for (int i = 0; i < hce.size() - 1; i++)
                cout << hce[i] << ' ';
            cout << hce[hce.size() - 1];
        } else
            cout << "forest";
        cout << '\n';
        return 0;
    }
}

```

```

typedef vector<VI> VVI;
typedef pair<int,int> PII;
typedef vector<PII> VPPII;

T Prim (const VVT &w, VPPII &edges) {
    int n = w.size();
    VI found (n);
    VI prev (n, -1);
    VT dist (n, 1000000000);
    int here = 0;
    dist[here] = 0;

    while (here != -1) {
        found[here] = true;
        int best = -1;
        for (int k = 0; k < n; k++) if (!found[k]) {
            if (w[here][k] != -1 && dist[k] > w[here][k]) {
                dist[k] = w[here][k];
                prev[k] = here;
            }
            if (best == -1 || dist[k] < dist[best]) best = k;
        }
        here = best;
    }

    T tot_weight = 0;
    for (int i = 0; i < n; i++) if (prev[i] != -1) {
        edges.push_back(make_pair(prev[i], i));
        tot_weight += w[prev[i]][i];
    }
    return tot_weight;
}

int main() {
    int ww[5][5] = {
        {0, 400, 400, 300, 600},
        {400, 0, 3, -1, 7},
        {400, 3, 0, 2, 0},
        {300, -1, 2, 0, 5},
        {600, 7, 0, 5, 0}
    };
    VVT w(5, VT(5));
    for (int i = 0; i < 5; i++)
        for (int j = 0; j < 5; j++)
            w[i][j] = ww[i][j];

    // expected: 305
    // 2 1
    // 3 2
    // 0 3
    // 2 4

    VPPII edges;
    cout << Prim (w, edges) << endl;
    for (int i = 0; i < edges.size(); i++)
        cout << edges[i].first << " " << edges[i].second << endl;
}

```

4.6 Minimum spanning trees

```

// This function runs Prim's algorithm for constructing minimum
// weight spanning trees.
//
// Running time: O(|V|^2)
//
// INPUT:   w[i][j] = cost of edge from i to j
//
// NOTE: Make sure that w[i][j] is nonnegative and
//        symmetric. Missing edges should be given -1
//        weight.
//
// OUTPUT:  edges = list of pair<int,int> in minimum spanning tree
//           return total weight of tree

#include <iostream>
#include <queue>
#include <cmath>
#include <vector>

using namespace std;

typedef double T;
typedef vector<T> VT;
typedef vector<VI> VVT;
typedef vector<int> VI;

```

5 Data structures

5.1 Suffix array

```

// Suffix array construction in O(L log^2 L) time. Routine for
// computing the length of the longest common prefix of any two
// suffixes in O(log L) time.
//
// INPUT:   string s
//
// OUTPUT:  array suffix[] such that suffix[i] = index (from 0 to L-1)
//           of substring s[i...L-1] in the list of sorted suffixes.
//           That is, if we take the inverse of the permutation suffix[],
//           we get the actual suffix array.

#include <vector>
#include <iostream>
#include <string>

using namespace std;

struct SuffixArray {
    const int L;
    string s;
    vector<vector<int>> > P;
    vector<pair<pair<int,int>,int>> > M;
}

```

```

SuffixArray(const string &s) : L(s.length()), s(s), P(1, vector<int>(L, 0)), M(L) {
    for (int i = 0; i < L; i++) P[0][i] = int(s[i]);
    for (int skip = 1, level = 1; skip < L; skip *= 2, level++) {
        P.push_back(vector<int>(L, 0));
        for (int i = 0; i < L; i++)
            M[i] = make_pair(make_pair(P[level-1][i], i + skip < L ? P[level-1][i + skip] : -1000), i);
        sort(M.begin(), M.end());
        for (int i = 0; i < L; i++)
            P[level][M[i].second] = (i > 0 && M[i].first == M[i-1].first) ? P[level][M[i-1].second] : i;
    }

    vector<int> GetSuffixArray() { return P.back(); }

    // returns the length of the longest common prefix of s[i...L-1] and s[j...L-1]
    int LongestCommonPrefix(int i, int j) {
        int len = 0;
        if (i == j) return L - i;
        for (int k = P.size() - 1; k >= 0 && i < L && j < L; k--) {
            if (P[k][i] == P[k][j]) {
                i += 1 << k;
                j += 1 << k;
                len += 1 << k;
            }
        }
        return len;
    }
};

// BEGIN CUT
// The following code solves UVA problem 11512: GATTACA.
#define TESTING
#ifdef TESTING
int main() {
    int T;
    cin >> T;
    for (int caseno = 0; caseno < T; caseno++) {
        string s;
        cin >> s;
        SuffixArray array(s);
        vector<int> v = array.GetSuffixArray();
        int bestlen = -1, bestpos = -1, bestcount = 0;
        for (int i = 0; i < s.length(); i++) {
            int len = 0, count = 0;
            for (int j = i+1; j < s.length(); j++) {
                int l = array.LongestCommonPrefix(i, j);
                if (l >= len) {
                    if (l > len) count = 2; else count++;
                    len = l;
                }
            }
            if (len > bestlen || len == bestlen && s.substr(bestpos, bestlen) > s.substr(i, len)) {
                bestlen = len;
                bestcount = count;
                bestpos = i;
            }
        }
        if (bestlen == 0) {
            cout << "No repetitions found!" << endl;
        } else {
            cout << s.substr(bestpos, bestlen) << " " << bestcount << endl;
        }
    }
}

#else
// END CUT
int main() {

    // bobocel is the 0'th suffix
    // obocel is the 5'th suffix
    // bocel is the 1'st suffix
    // ocel is the 6'th suffix
    // cel is the 2'nd suffix
    // el is the 3'rd suffix
    // l is the 4'th suffix
    SuffixArray suffix("bobocel");
    vector<int> v = suffix.GetSuffixArray();

    // Expected output: 0 5 1 6 2 3 4
    //
    for (int i = 0; i < v.size(); i++) cout << v[i] << " ";
    cout << endl;
    cout << suffix.LongestCommonPrefix(0, 2) << endl;
}

// BEGIN CUT
#endif
// END CUT

```

5.2 Binary Indexed (Fenwick) Tree

```

#include <iostream>
using namespace std;

#define LOGSZ 17

int tree[(1<<LOGSZ)+1];
int N = (1<<LOGSZ);

// add v to value at x
void set(int x, int v) {
    while(x <= N) {
        tree[x] += v;
        x += (x & -x);
    }
}

// get cumulative sum up to and including x
int get(int x) {
    int res = 0;
    while(x) {
        res += tree[x];
        x -= (x & -x);
    }
    return res;
}

// get largest value with cumulative sum less than or equal to x;
// for smallest, pass x-1 and add 1 to result
int getind(int x) {
    int idx = 0, mask = N;
    while(mask && idx < N) {
        int t = idx + mask;
        if(x >= tree[t]) {
            idx = t;
            x -= tree[t];
        }
        mask >>= 1;
    }
    return idx;
}

```

5.3 Union-find disjoint sets

```

#include <iostream>
#include <vector>
using namespace std;
struct UnionFind {
    vector<int> C;
    UnionFind(int n) : C(n) { for (int i = 0; i < n; i++) C[i] = i; }
    int find(int x) { return (C[x] == x) ? x : C[x] = find(C[x]); }
    void merge(int x, int y) { C[find(x)] = find(y); }
};

int main()
{
    int n = 5;
    UnionFind uf(n);
    uf.merge(0, 2);
    uf.merge(1, 0);
    uf.merge(3, 4);
    for (int i = 0; i < n; i++) cout << i << " " << uf.find(i) << endl;
    return 0;
}

```

5.4 KD-tree

```

// -----
// A straightforward, but probably sub-optimal KD-tree implementation
// that's probably good enough for most things (current it's a
// 2D-tree)
//
// - constructs from n points in O(n lg^2 n) time
// - handles nearest-neighbor query in O(lg n) if points are well
//   distributed
// - worst case for nearest-neighbor may be linear in pathological
//   case
//
// Sonny Chan, Stanford University, April 2009

```

```
// -----
#include <iostream>
#include <vector>
#include <limits>
#include <cstdlib>

using namespace std;

// number type for coordinates, and its maximum value
typedef long long ntype;
const ntype sentry = numeric_limits<ntype>::max();

// point structure for 2D-tree, can be extended to 3D
struct point {
    ntype x, y;
    point(ntype xx = 0, ntype yy = 0) : x(xx), y(yy) {}
};

bool operator==(const point &a, const point &b)
{
    return a.x == b.x && a.y == b.y;
}

// sorts points on x-coordinate
bool on_x(const point &a, const point &b)
{
    return a.x < b.x;
}

// sorts points on y-coordinate
bool on_y(const point &a, const point &b)
{
    return a.y < b.y;
}

// squared distance between points
ntype pdist2(const point &a, const point &b)
{
    ntype dx = a.x-b.x, dy = a.y-b.y;
    return dx*dx + dy*dy;
}

// bounding box for a set of points
struct bbox
{
    ntype x0, x1, y0, y1;

    bbox() : x0(sentry), x1(-sentry), y0(sentry), y1(-sentry) {}

    // computes bounding box from a bunch of points
    void compute(const vector<point> &v) {
        for (int i = 0; i < v.size(); ++i) {
            x0 = min(x0, v[i].x);    x1 = max(x1, v[i].x);
            y0 = min(y0, v[i].y);    y1 = max(y1, v[i].y);
        }
    }

    // squared distance between a point and this bbox, 0 if inside
    ntype distance(const point &p) {
        if (p.x < x0) {
            if (p.y < y0) return pdist2(point(x0, y0), p);
            else if (p.y > y1) return pdist2(point(x0, y1), p);
            else return pdist2(point(x0, p.y), p);
        }
        else if (p.x > x1) {
            if (p.y < y0) return pdist2(point(x1, y0), p);
            else if (p.y > y1) return pdist2(point(x1, y1), p);
            else return pdist2(point(x1, p.y), p);
        }
        else {
            if (p.y < y0) return pdist2(point(p.x, y0), p);
            else if (p.y > y1) return pdist2(point(p.x, y1), p);
            else return 0;
        }
    }
};

// stores a single node of the kd-tree, either internal or leaf
struct kndnode
{
    bool leaf; // true if this is a leaf node (has one point)
    point pt; // the single point of this is a leaf
    bbox bound; // bounding box for set of points in children

    kndnode *first, *second; // two children of this kd-node

    kndnode() : leaf(false), first(0), second(0) {}
    ~kndnode() { if (first) delete first; if (second) delete second; }

    // intersect a point with this node (returns squared distance)
    ntype intersect(const point &p) {
```

```
        return bound.distance(p);
    }
};

// recursively builds a kd-tree from a given cloud of points
void construct(vector<point> &vp)
{
    // compute bounding box for points at this node
    bound.compute(vp);

    // if we're down to one point, then we're a leaf node
    if (vp.size() == 1) {
        leaf = true;
        pt = vp[0];
    }
    else {
        // split on x if the bbox is wider than high (not best heuristic...)
        if (bound.x1-bound.x0 >= bound.y1-bound.y0)
            sort(vp.begin(), vp.end(), on_x);
        // otherwise split on y-coordinate
        else
            sort(vp.begin(), vp.end(), on_y);

        // divide by taking half the array for each child
        // (not best performance if many duplicates in the middle)
        int half = vp.size()/2;
        vector<point> vl(vp.begin(), vp.begin()+half);
        vector<point> vr(vp.begin()+half, vp.end());
        first = new kndnode(); first->construct(vl);
        second = new kndnode(); second->construct(vr);
    }
}

// simple kd-tree class to hold the tree and handle queries
struct kdtree
{
    kndnode *root;

    // constructs a kd-tree from a points (copied here, as it sorts them)
    kdtree(const vector<point> &vp) {
        vector<point> v(vp.begin(), vp.end());
        root = new kndnode();
        root->construct(v);
    }
    ~kdtree() { delete root; }

    // recursive search method returns squared distance to nearest point
    ntype search(kndnode *node, const point &p)
    {
        if (node->leaf) {
            // commented special case tells a point not to find itself
            // if (p == node->pt) return sentry;
            // else
            return pdist2(p, node->pt);
        }

        ntype bfirst = node->first->intersect(p);
        ntype bsecond = node->second->intersect(p);

        // choose the side with the closest bounding box to search first
        // (note that the other side is also searched if needed)
        if (bfirst < bsecond) {
            ntype best = search(node->first, p);
            if (bsecond < best)
                best = min(best, search(node->second, p));
            return best;
        }
        else {
            ntype best = search(node->second, p);
            if (bfirst < best)
                best = min(best, search(node->first, p));
            return best;
        }
    }

    // squared distance to the nearest
    ntype nearest(const point &p) {
        return search(root, p);
    }
};

// -----
// some basic test code here

int main()
{
    // generate some random points for a kd-tree
    vector<point> vp;
    for (int i = 0; i < 100000; ++i) {
        vp.push_back(point(rand()%100000, rand()%100000));
    }
    kdtree tree(vp);
```

```
// query some points
for (int i = 0; i < 10; ++i) {
    point q(rand()%100000, rand()%100000);
    cout << "Closest squared distance to " << q.x << ", " << q.y << " "
    << " is " << tree.nearest(q) << endl;
}

return 0;
}

// -----
```

5.5 Splay tree

```
#include <cstdio>
#include <algorithm>
using namespace std;

const int N_MAX = 130010;
const int oo = 0x3f3f3f3f;
struct Node
{
    Node *ch[2], *pre;
    int val, size;
    bool isTurned;
} nodePool[N_MAX], *null, *root;

Node *allocNode(int val)
{
    static int freePos = 0;
    Node *x = &nodePool[freePos++];
    x->val = val, x->isTurned = false;
    x->ch[0] = x->ch[1] = x->pre = null;
    x->size = 1;
    return x;
}

inline void update(Node *x)
{
    x->size = x->ch[0]->size + x->ch[1]->size + 1;
}

inline void makeTurned(Node *x)
{
    if (x == null)
        return;
    swap(x->ch[0], x->ch[1]);
    x->isTurned ^= 1;
}

inline void pushDown(Node *x)
{
    if (x->isTurned)
    {
        makeTurned(x->ch[0]);
        makeTurned(x->ch[1]);
        x->isTurned ^= 1;
    }
}

inline void rotate(Node *x, int c)
{
    Node *y = x->pre;
    x->pre = y->pre;
    if (y->pre != null)
        y->pre->ch[y == y->pre->ch[1]] = x;
    y->ch[!c] = x->ch[c];
    if (x->ch[c] != null)
        x->ch[c]->pre = y;
    x->ch[!c] = y, y->pre = x;
    update(y);
    if (y == root)
        root = x;
}

void splay(Node *x, Node *p)
{
    while (x->pre != p)
    {
        if (x->pre->pre == p)
            rotate(x, x == x->pre->ch[0]);
        else
        {
            Node *y = x->pre, *z = y->pre;
            if (y == z->ch[0])
            {

```

```
                if (x == y->ch[0])
                    rotate(y, 1), rotate(x, 1);
                else
                    rotate(x, 0), rotate(x, 1);
            }
        }
        else
        {
            if (x == y->ch[1])
                rotate(y, 0), rotate(x, 0);
            else
                rotate(x, 1), rotate(x, 0);
        }
    }
    update(x);
}

void select(int k, Node *fa)
{
    Node *now = root;
    while (1)
    {
        pushDown(now);
        int tmp = now->ch[0]->size + 1;
        if (tmp == k)
            break;
        else if (tmp < k)
            now = now->ch[1], k -= tmp;
        else
            now = now->ch[0];
    }
    splay(now, fa);
}

Node *makeTree(Node *p, int l, int r)
{
    if (l > r)
        return null;
    int mid = (l + r) / 2;
    Node *x = allocNode(mid);
    x->pre = p;
    x->ch[0] = makeTree(x, l, mid - 1);
    x->ch[1] = makeTree(x, mid + 1, r);
    update(x);
    return x;
}

int main()
{
    int n, m;
    null = allocNode(0);
    null->size = 0;
    root = allocNode(0);
    root->ch[1] = allocNode(oo);
    root->ch[1]->pre = root;
    update(root);

    scanf("%d%d", &n, &m);
    root->ch[1]->ch[0] = makeTree(root->ch[1], 1, n);
    splay(root->ch[1]->ch[0], null);

    while (m --)
    {
        int a, b;
        scanf("%d%d", &a, &b);
        a ++, b ++;
        select(a - 1, null);
        select(b + 1, root);
        makeTurned(root->ch[1]->ch[0]);
    }

    for (int i = 1; i <= n; i++)
    {
        select(i + 1, null);
        printf("%d ", root->val);
    }
}
```

5.6 Segment tree

```
// This is a solution for UVA 12299 - RMQ with Shifts. In this problem, we are
// given an array of integers A and a list of queries of 2 types:
// 1. query(L, R) returns the minimum value in A[L...R].
// 2. shift(i_1, i_2, ..., i_k) performs a left "circular shift" of A[i_1],
// A[i_2], ..., A[i_k]. For example, if A = {6, 2, 4, 8, 5, 1, 4}, then
// shift(2, 4, 5, 7) yields 6, 8, 4, 5, 4, 1, 2. After that, shift(1, 2)
// yields 8, 6, 4, 5, 4, 1, 2.
```

```

#include <bits/stdc++.h>

#define LOCAL
#define Lson(x) ((x << 1) + 1)
#define Rson(x) ((x << 1) + 2)
#define INF 99999999

using namespace std;

const int N = 100005;
int shift[35], num[N], len_shift;
string strLine;

struct Node {
    int left, right, Min_Value;
} node[4 * N];

void build(int left, int right, int x = 0) {
    node[x].left = left;
    node[x].right = right;
    if (left == right) {
        node[x].Min_Value = num[left];
        return;
    }
    int mid = (left + right) / 2;

    //debug
    //cout << mid << '\n' ;
    //cout << x << ' ' << node[x].left << ' ' << node[x].right << ' ' << '\n' ;

    build(left, mid, Lson(x));
    build(mid + 1, right, Rson(x));
    node[x].Min_Value = min(node[Lson(x)].Min_Value, node[Rson(x)].Min_Value);
}

void handle() {
    len_shift = 0;
    shift[len_shift] = 0;
    for (int i = 6; i < strLine.length(); i++) {
        if (strLine[i] >= '0' && strLine[i] <= '9') {
            shift[len_shift] = shift[len_shift] * 10 + (int)(strLine[i] - '0');
        } else {
            shift[++len_shift] = 0;
        }
    }
    //finally char is ')', so len_shift is right
    sort(shift, shift + len_shift);

    //debug
    //cout << shift[i] << ' ' ;
    //cout << '\n' ;
    //
}

int query(int left, int right, int x = 0) {
    if (node[x].left >= left && node[x].right <= right)
        return node[x].Min_Value;
    int mid = (node[x].left + node[x].right) / 2;
    int ans = INF;

    //debug
    //cout << x << ' ' << node[x].left << ' ' << node[x].right << ' ' << node[x].Min_Value << '\n' ;

    if (left <= mid)
        ans = min(ans, query(left, right, Lson(x)));
    if (mid < right)
        ans = min(ans, query(left, right, Rson(x)));
    return ans;
}

void set_num(int position, int value, int x = 0) {
    if (node[x].left == position && node[x].right == position) {
        node[x].Min_Value = value;
        return;
    }
    int mid = (node[x].left + node[x].right) / 2;
    if (position <= mid)
        set_num(position, value, Lson(x));
    if (mid < position)
        set_num(position, value, Rson(x));
    node[x].Min_Value = min(node[Lson(x)].Min_Value, node[Rson(x)].Min_Value);
}

int main() {
    int n, q, intTemp;
    ios::sync_with_stdio(0);
    #ifdef LOCAL
        freopen("out.txt", "w", stdout);
        freopen("in1.txt", "r", stdin);
    #endif

```

```

#endif // LOCAL
    cin >> n >> q;
    for (int i = 1; i <= n; i++)
        cin >> num[i];
    build(1, n);

    //debug
    //cout << node[i].left << ' ' << node[i].right << ' ' << node[i].Min_Value << '\n' ;
    //
    return 0;
}

while (q--) {
    cin >> strLine;
    if (strLine[0] == 'q') {
        handle();
        cout << query(shift[0], shift[1]) << '\n';
    } else if (strLine[0] == 's') {
        handle();
        intTemp = num[shift[0]];

        for (int i = 1; i < len_shift; i++) {
            set_num(shift[i - 1], num[shift[i]]);
            num[shift[i - 1]] = num[shift[i]];
        }
        num[shift[len_shift - 1]] = intTemp;
        set_num(shift[len_shift - 1], intTemp);

        //debug
        //cout << intTemp << ' ' << shift[len_shift-1] << '\n' ;
        //for(int i = 1 ; i <= n ; i++)
        //    cout << num[i] << ' ' ;
    }
}
return 0;
}

```

5.7 Lowest common ancestor

```

const int max_nodes, log_max_nodes;
int num_nodes, log_num_nodes, root;

vector<int> children[max_nodes]; // children[i] contains the children of node i
int A[max_nodes][log_max_nodes+1]; // A[i][j] is the 2^j-th ancestor of node i, or -1 if that
// ancestor does not exist
int L[max_nodes]; // L[i] is the distance between node i and the root

// floor of the binary logarithm of n
int lb(unsigned int n)
{
    if(n==0)
        return -1;
    int p = 0;
    if (n >= 1<<16) { n >>= 16; p += 16; }
    if (n >= 1<< 8) { n >>= 8; p += 8; }
    if (n >= 1<< 4) { n >>= 4; p += 4; }
    if (n >= 1<< 2) { n >>= 2; p += 2; }
    if (n >= 1<< 1) { p += 1; }
    return p;
}

void DFS(int i, int l)
{
    L[i] = l;
    for(int j = 0; j < children[i].size(); j++)
        DFS(children[i][j], l+1);
}

int LCA(int p, int q)
{
    // ensure node p is at least as deep as node q
    if(L[p] < L[q])
        swap(p, q);

    // "binary search" for the ancestor of node p situated on the same level as q
    for(int i = log_num_nodes; i >= 0; i--)
        if (L[p] - (1<<i) >= L[q])
            p = A[p][i];

    if(p == q)
        return p;

    // "binary search" for the LCA
    for(int i = log_num_nodes; i >= 0; i--)
        if (A[p][i] != -1 && A[p][i] != A[q][i])
            continue;
    return A[p][i];
}

```

```

        p = A[p][i];
        q = A[q][i];
    }
    return A[p][0];
}

int main(int argc, char* argv[])
{
    // read num_nodes, the total number of nodes
    log_num_nodes = lb(num_nodes);

    for(int i = 0; i < num_nodes; i++)
    {
        int p;
        // read p, the parent of node i or -1 if node i is the root
        A[i][0] = p;
        if(p != -1)
            children[p].push_back(i);
        else
            root = i;
    }

    // precompute A using dynamic programming
    for(int j = 1; j <= log_num_nodes; j++)
        for(int i = 0; i < num_nodes; i++)
            if(A[i][j-1] != -1)
                A[i][j] = A[A[i][j-1]][j-1];
            else
                A[i][j] = -1;

    // precompute L
    DFS(root, 0);

    return 0;
}

```

6 Miscellaneous

6.1 Dijkstra's algorithm using struct

```

// This is a solution for UVa 929 - Number Maze. Abridged problem statement:
// What is the shortest path from the maze's top-left to bottom-right corner?

#include <bits/stdc++.h>

#define LOCAL
#define INF 999999999

using namespace std;
int intMap[1010][1010] = {}, intValue[1010][1010] = {};
int m, n;

struct Node {
    int x, y, v;
    void read(int _x, int _y, int _v) {
        x = _x;
        y = _y;
        v = _v;
    }
    bool operator<(const Node &a) const {
        return v > a.v;
    }
} nodNode;

void print_map() {
    for (int i = 1; i <= n; i++) {
        for (int j = 1; j <= m; j++) {
            if (intValue[i][j] == 999999999)
                cout << 'x' << ' ';
            else
                cout << intValue[i][j] << ' ';
        }
        cout << '\n';
    }
    cout << '\n';
}

void bfs() {
    int x, y, intDirection[4][2] = {-1, 0, 0, 1, 1, 0, 0, -1};
    int intDx, intDy;
    Node nodTemp;
    priority_queue<Node> deqNode;

```

```

    nodTemp.read(1, 1, 0);
    deqNode.push(nodTemp);
    while (deqNode.size()) {
        x = deqNode.top().x;
        y = deqNode.top().y;
        deqNode.pop();

        for (int i = 0; i < 4; i++) {
            intDx = intDirection[i][0] + x;
            intDy = intDirection[i][1] + y;

            //debug
            //cout << intDx << ' ' << intDy << ' ' << intValue[x][y] + intMap[intDx][intDy] << ' ' <<
                i << '\n';

            if (intValue[x][y] + intMap[intDx][intDy] < intValue[intDx][intDy]) {
                intValue[intDx][intDy] = intValue[x][y] + intMap[intDx][intDy];
                nodTemp.read(intDx, intDy, intValue[intDx][intDy]);
                deqNode.push(nodTemp);
            }
        }
        //print_map();
    }
}

int main() {
    #ifndef LOCAL
        freopen("in1.txt", "r", stdin);
        freopen("out.txt", "w", stdout);
    #endif
    ios::sync_with_stdio(false);
    int intCase;
    cin >> intCase;
    while (intCase--) {
        cin >> n >> m;
        for (int i = 1; i <= n; i++) {
            for (int j = 1; j <= m; j++) {
                cin >> intMap[i][j];
                intValue[i][j] = INF;
            }
        }

        for (int i = 1; i <= n; i++) {
            intValue[i][0] = 0;
            intValue[i][m + 1] = 0;
            intMap[i][0] = INF + 1;
            intMap[i][m + 1] = INF + 1;
        }

        for (int i = 1; i <= m; i++) {
            intValue[0][i] = 0;
            intValue[n + 1][i] = 0;
            intMap[0][i] = INF + 1;
            intMap[n + 1][i] = INF + 1;
        }
        intValue[1][1] = intMap[1][1];

        //debug
        //cout << intValue[1][1] << '\n';

        bfs();
        cout << intValue[n][m] << '\n';
    }
    return 0;
}

```

6.2 Longest increasing subsequence

```

// Given a list of numbers of length n, this routine extracts a
// longest increasing subsequence.
//
// Running time: O(n log n)
//
// INPUT: a vector of integers
// OUTPUT: a vector containing the longest increasing subsequence

#include <iostream>
#include <vector>
#include <algorithm>

using namespace std;

typedef vector<int> VI;
typedef pair<int, int> PII;
typedef vector<PII> VPPII;

#define STRICTLY_INCREASNG

```

```

VI LongestIncreasingSubsequence(VI v) {
    VPII best;
    VI dad(v.size(), -1);

    for (int i = 0; i < v.size(); i++) {
#ifdef STRICTLY_INCREASNG
        PII item = make_pair(v[i], 0);
        VPII::iterator it = lower_bound(best.begin(), best.end(), item);
        item.second = i;
    #else
        PII item = make_pair(v[i], i);
        VPII::iterator it = upper_bound(best.begin(), best.end(), item);
    #endif
        if (it == best.end()) {
            dad[i] = (best.size() == 0 ? -1 : best.back().second);
            best.push_back(item);
        } else {
            dad[i] = it == best.begin() ? -1 : prev(it)->second;
            *it = item;
        }
    }

    VI ret;
    for (int i = best.back().second; i >= 0; i = dad[i])
        ret.push_back(v[i]);
    reverse(ret.begin(), ret.end());
    return ret;
}

```

6.3 Knuth-Morris-Pratt

```

/*
Finds all occurrences of the pattern string p within the
text string t. Running time is O(n + m), where n and m
are the lengths of p and t, respectively.
*/

#include <iostream>
#include <string>
#include <vector>

using namespace std;

typedef vector<int> VI;

void buildPi(string& p, VI& pi)
{
    pi = VI(p.length());
    int k = -2;
    for(int i = 0; i < p.length(); i++) {
        while(k >= -1 && p[k+1] != p[i])
            k = (k == -1) ? -2 : pi[k];
        pi[i] = ++k;
    }
}

int KMP(string& t, string& p)
{
    VI pi;
    buildPi(p, pi);
    int k = -1;
    for(int i = 0; i < t.length(); i++) {
        while(k >= -1 && p[k+1] != t[i])
            k = (k == -1) ? -2 : pi[k];
        k++;
        if(k == p.length() - 1) {
            // p matches t[i-m+1, ..., i]
            cout << "matched at index " << i-k << ": ";
            cout << t.substr(i-k, p.length()) << endl;
            k = (k == -1) ? -2 : pi[k];
        }
    }
    return 0;
}

int main()
{
    string a = "AABAACAADAABAABA", b = "AABA";
    KMP(a, b); // expected matches at: 0, 9, 12
    return 0;
}

```

6.4 Dates

```

// Routines for performing computations on dates. In these routines,
// months are expressed as integers from 1 to 12, days are expressed
// as integers from 1 to 31, and years are expressed as 4-digit
// integers.

#include <iostream>
#include <string>

using namespace std;

string dayOfWeek[] = {"Mon", "Tue", "Wed", "Thu", "Fri", "Sat", "Sun"};

// converts Gregorian date to integer (Julian day number)
int dateToInt (int m, int d, int y){
    return
        1461 * (y + 4800 + (m - 14) / 12) / 4 +
        367 * (m - 2 - (m - 14) / 12 * 12) / 12 -
        3 * ((y + 4900 + (m - 14) / 12) / 100) / 4 +
        d - 32075;
}

// converts integer (Julian day number) to Gregorian date: month/day/year
void intToDate (int jd, int &m, int &d, int &y){
    int x, n, i, j;

    x = jd + 68569;
    n = 4 * x / 146097;
    x -= (146097 * n + 3) / 4;
    i = (4000 * (x + 1)) / 1461001;
    x -= 1461 * i / 4 - 31;
    j = 80 * x / 2447;
    d = x - 2447 * j / 80;
    x = j / 11;
    m = j + 2 - 12 * x;
    y = 100 * (n - 49) + i + x;
}

// converts integer (Julian day number) to day of week
string intToDay (int jd){
    return dayOfWeek[jd % 7];
}

int main (int argc, char **argv){
    int jd = dateToInt (3, 24, 2004);
    int m, d, y;
    intToDate (jd, m, d, y);
    string day = intToDay (jd);

    // expected output:
    // 2453089
    // 3/24/2004
    // Wed
    cout << jd << endl
         << m << "/" << d << "/" << y << endl
         << day << endl;
}

```

6.5 Prime numbers

```

// O(sqrt(x)) Exhaustive Primality Test
#include <cmath>
#define EPS 1e-7
typedef long long LL;
bool isPrimeSlow (LL x)
{
    if(x<=1) return false;
    if(x<=3) return true;
    if (!(x%2) || !(x%3)) return false;
    LL s = (LL)(sqrt((double)(x))+EPS);
    for(LL i=5; i<=s; i+=6)
    {
        if (!(x%i) || !(x%(i+2))) return false;
    }
    return true;
}

// Primes less than 1000:
// 2 3 5 7 11 13 17 19 23 29 31 37
// 41 43 47 53 59 61 67 71 73 79 83 89
// 97 101 103 107 109 113 127 131 137 139 149 151
// 157 163 167 173 179 181 191 193 197 199 211 223
// 227 229 233 239 241 251 257 263 269 271 277 281
// 283 293 307 311 313 317 331 337 347 349 353 359
// 367 373 379 383 389 397 401 409 419 421 431 433
// 439 443 449 457 461 463 467 479 487 491 499 503
// 509 521 523 541 547 557 563 569 571 577 587 593
// 599 601 607 613 617 619 631 641 643 647 653 659
// 661 673 677 683 691 701 709 719 727 733 739 743
// 751 757 761 769 773 787 797 809 811 821 823 827

```



```
//      829   839   853   857   859   863   877   881   883   887   907   911
//      919   929   937   941   947   953   967   971   977   983   991   997

// Other primes:
// The largest prime smaller than 10 is 7.
// The largest prime smaller than 100 is 97.
// The largest prime smaller than 1000 is 997.
// The largest prime smaller than 10000 is 9973.
// The largest prime smaller than 100000 is 99991.
// The largest prime smaller than 1000000 is 999983.
// The largest prime smaller than 10000000 is 9999991.
// The largest prime smaller than 100000000 is 99999989.
// The largest prime smaller than 1000000000 is 999999937.
// The largest prime smaller than 10000000000 is 9999999967.
// The largest prime smaller than 100000000000 is 99999999977.
// The largest prime smaller than 1000000000000 is 99999999989.
// The largest prime smaller than 10000000000000 is 99999999971.
// The largest prime smaller than 100000000000000 is 99999999973.
// The largest prime smaller than 1000000000000000 is 99999999989.
// The largest prime smaller than 10000000000000000 is 999999999937.
// The largest prime smaller than 100000000000000000 is 999999999997.
// The largest prime smaller than 1000000000000000000 is 9999999999989.
```

6.6 Latitude/longitude

```
/*
Converts from rectangular coordinates to latitude/longitude and vice
versa. Uses degrees (not radians).
*/

#include <iostream>
#include <cmath>

using namespace std;

struct ll
{
    double r, lat, lon;
};

struct rect
{
    double x, y, z;
};

ll convert(rect& P)
{
    ll Q;
    Q.r = sqrt(P.x*P.x+P.y*P.y+P.z*P.z);
    Q.lat = 180/M_PI*asin(P.z/Q.r);
    Q.lon = 180/M_PI*acos(P.x/sqrt(P.x*P.x+P.y*P.y));

    return Q;
}

rect convert(ll& Q)
{
    rect P;
    P.x = Q.r*cos(Q.lon*M_PI/180)*cos(Q.lat*M_PI/180);
    P.y = Q.r*sin(Q.lon*M_PI/180)*cos(Q.lat*M_PI/180);
    P.z = Q.r*sin(Q.lat*M_PI/180);

    return P;
}

int main()
{
    rect A;
    ll B;

    A.x = -1.0; A.y = 2.0; A.z = -3.0;

    B = convert(A);
    cout << B.r << " " << B.lat << " " << B.lon << endl;

    A = convert(B);
    cout << A.x << " " << A.y << " " << A.z << endl;
}
```

6.7 C++ input/output

```
#include <iostream>
```

```
#include <iomanip>

using namespace std;

int main()
{
    // Output a specific number of digits past the decimal point,
    // in this case 5
    cout.setf(ios::fixed); cout << setprecision(5);
    cout << 100.0/7.0 << endl;
    cout.unsetf(ios::fixed);

    // Output the decimal point and trailing zeros
    cout.setf(ios::showpoint);
    cout << 100.0 << endl;
    cout.unsetf(ios::showpoint);

    // Output a '+' before positive values
    cout.setf(ios::showpos);
    cout << 100 << " " << -100 << endl;
    cout.unsetf(ios::showpos);

    // Output numerical values in hexadecimal
    cout << hex << 100 << " " << 1000 << " " << 10000 << dec << endl;
}
```

6.8 Random STL stuff

```
// Example for using stringstream and next_permutation

#include <algorithm>
#include <iostream>
#include <sstream>
#include <vector>

using namespace std;

int main(void) {
    vector<int> v;

    v.push_back(1);
    v.push_back(2);
    v.push_back(3);
    v.push_back(4);

    // Expected output: 1 2 3 4
    //                  1 2 4 3
    //                  ...
    //                  4 3 2 1
    do {
        stringstream oss;
        oss << v[0] << " " << v[1] << " " << v[2] << " " << v[3];

        // for input from a string s,
        // istream iss(s);
        // iss >> variable;

        cout << oss.str() << endl;
    } while (next_permutation(v.begin(), v.end()));

    v.clear();

    v.push_back(1);
    v.push_back(2);
    v.push_back(1);
    v.push_back(3);

    // To use unique, first sort numbers. Then call
    // unique to place all the unique elements at the beginning
    // of the vector, and then use erase to remove the duplicate
    // elements.

    sort(v.begin(), v.end());
    v.erase(unique(v.begin(), v.end()), v.end());

    // Expected output: 1 2 3
    for (size_t i = 0; i < v.size(); i++)
        cout << v[i] << " ";
    cout << endl;
}
```

6.9 Regular expressions (Java)

```
// Code which demonstrates the use of Java's regular expression libraries.
// This is a solution for
```

```
//
// Loglan: a logical language
// http://acm.uva.es/p/v1/134.html
//
```

```
// In this problem, we are given a regular language, whose rules can be
// inferred directly from the code. For each sentence in the input, we must
// determine whether the sentence matches the regular expression or not. The
// code consists of (1) building the regular expression (which is fairly
// complex) and (2) using the regex to match sentences.
```

```
import java.util.*;
import java.util.regex.*;
```

```
public class LogLan {
```

```
    public static String BuildRegex () {
        String space = " ";
```

```
        String A = "[aeiou]";
        String C = "[a-z&&[^aeiou]]";
        String MOD = "(g" + A + ")";
        String BA = "(b" + A + ")";
        String DA = "(d" + A + ")";
        String LA = "(l" + A + ")";
        String NAM = "([a-z]*" + C + ")";
        String PRED = "((" + C + C + A + C + A + "|" + C + A + C + C + A + C + ")";
```

```
        String predstring = "(" + PRED + "(" + space + PRED + ")*)";
        String predname = "(" + LA + space + predstring + "|" + NAM + ")";
        String preds = "(" + predstring + "(" + space + A + space + predstring + ")*)";
        String predclaim = "(" + predname + space + BA + space + preds + "|" + DA + space +
            preds + ")";
        String verbpred = "(" + MOD + space + predstring + ")";
        String statement = "(" + predname + space + verbpred + space + predname + "|" +
            predname + space + verbpred + ")";
        String sentence = "(" + statement + "|" + predclaim + ")";
```

```
        return "^" + sentence + "$";
```

```
    }

    public static void main (String args[]) {
```

```
        String regex = BuildRegex();
        Pattern pattern = Pattern.compile (regex);
```

```
Scanner s = new Scanner(System.in);
```

```
while (true) {
```

```
    // In this problem, each sentence consists of multiple lines, where the last
    // line is terminated by a period. The code below reads lines until
    // encountering a line whose final character is a '.'. Note the use of
```

```
    //
    // s.length() to get length of string
    // s.charAt() to extract characters from a Java string
    // s.trim() to remove whitespace from the beginning and end of Java string
    //
```

```
    // Other useful String manipulation methods include
```

```
    //
    // s.compareTo(t) < 0 if s < t, lexicographically
    // s.indexOf("apple") returns index of first occurrence of "apple" in s
    // s.lastIndexOf("apple") returns index of last occurrence of "apple" in s
    // s.replace(c,d) replaces occurrences of character c with d
    // s.startsWith("apple") returns (s.indexOf("apple") == 0)
    // s.toLowerCase() / s.toUpperCase() returns a new lower/uppercased string
    //
```

```
    // Integer.parseInt(s) converts s to an integer (32-bit)
    // Long.parseLong(s) converts s to a long (64-bit)
    // Double.parseDouble(s) converts s to a double
    //
```

```
String sentence = "";
```

```
while (true) {
    sentence = (sentence + " " + s.nextLine()).trim();
    if (sentence.equals(".")) return;
    if (sentence.charAt(sentence.length()-1) == '.' break;
}
```

```
// now, we remove the period, and match the regular expression
```

```
String removed_period = sentence.substring(0, sentence.length()-1).trim();
if (pattern.matcher (removed_period).find()) {
    System.out.println ("Good");
} else {
    System.out.println ("Bad!");
}
```

```
    }
}
```