NTUT_Kn1ghts ICPC Team Notebook

Contents

1 Advanced algorithms

1.1 2-SAT problem

```cpp
// 2-SAT Problem demonstrated with 2018 ICPC Korea Regional - Problem K.
#include <bits/stdc++.h>

using namespace std;

#define LOCAL
#define blue(k) (k<<1)
#define red(k) (blue(k) + 1)
#define UNVISITED -1
#define neg(v) (v ^ 1) // [neg]ation of

typedef vector<int> vi;

int K, N;
int V;
vector<vi> AL;
bool possible = true;
vi sccNum;

int getVertex(pair<int, char> p)
{
    return p.second == 'B' ? blue(p.first) : red(p.first);
}

pair<int, char> negation(pair<int, char> p)
{
    return make_pair(p.first, p.second == 'B' ? 'R' : 'B');
}

void createEdge(pair<int, char> p, pair<int, char> q)
{
    int u, v;
    u = getVertex( negation(p) );
    v = getVertex( q );
//    printf("%d->%d\n", u, v);
    AL[u].push_back(v);

    u = getVertex( negation(q) );
    v = getVertex( p );
//    printf("%d->%d\n\n", u, v);
    AL[u].push_back(v);
}

vi dfs_num, dfs_low, visited;
int dfsNumberCounter, numSCC;
vi S;

void tarjanSCC(int u)
{
    dfs_low[u] = dfs_num[u] = dfsNumberCounter++;
    S.push_back(u);
    visited[u] = 1;
    for (int i = 0; i < (int)AL[u].size(); ++i)
    {
        int v = AL[u][i];
        if (dfs_num[v] == UNVISITED) tarjanSCC(v);
        if (visited[v]) dfs_low[u] = min( dfs_low[u], dfs_low[v] );
    }

    if (dfs_low[u] == dfs_num[u])
    {
        set<int> st;
        ++numSCC;
//        printf("SCC %d:", numSCC);
        while (true)
        {
            int v = S.back(); S.pop_back();
            visited[v] = 0;

            if ( st.find(neg(v)) != st.end() ) possible = false;
            st.insert(v);

            sccNum[v] = numSCC;    // Tarjan produces SCCs in reversed topo order
//            printf(" %d", v);
            if (u==v) break;
        }
//        printf("\n");
    }
}
```

```
void work()
{
    sccNum.assign(V, 0);
    dfs_num.assign(V, UNVISITED);
    dfs_low.assign(V, 0);
    visited.assign(V, 0);
    dfsNumberCounter = numSCC = 0;
    for (int u = 0; u < V; ++u)
        if (dfs_num[u] == UNVISITED)
            tarjanSCC(u);

    if (!possible)
    {
        printf("-1\n");
        return;
    }

    for (int i = 1; i <= K; ++i)                                    // 2-SAT assignment based
        printf("%c", sccNum[blue(i)] > sccNum[red(i)] ? 'R' : 'B'); // on reversed topo order
    printf("\n");
}

int main()
{
    #ifdef LOCAL
    freopen("in.txt", "r", stdin);
    #endif // LOCAL
    scanf("%d %d", &K, &N);
    V = 2 * K + 2;
    AL.assign(V, vi());
    for (int i = 0; i < N; ++i)
    {
        pair<int, char> a[3];
        for (int j = 0; j < 3; ++j) scanf("%d %c", &a[j].first, &a[j].second);
//        for (int j = 0; j < 3; ++j) printf("%d %c ", a[j].first, a[j].second);
//        printf("\n");
        createEdge(a[0], a[1]);
        createEdge(a[0], a[2]);
        createEdge(a[1], a[2]);
    }

    work();

    return 0;
}
```

1.2 Iterative deepening A* (IDA*)

```
// UVa 10181 - 15-Puzzle Problem solved with Iterative Deepening A* (IDA*).
#include <bits/stdc++.h>

using namespace std;

//#define LOCAL
#define N 4           // #rows/columns
#define B 15          // [B]lank tile id
#define PUZZLE (N*N)
#define MAX_STEPS 45 // given by the problem description
#define DIR 4         // 4 [DIR]ections

int dr[DIR] = {0, -1, 0, 1}; // must be right, up, left, down
int dc[DIR] = {1, 0, -1, 0}; // for the XOR operation to work
char dm[] = "RULD";          // [d]irection [m]ove

int p[PUZZLE];
int b_init_pos;       // [b]lank [init]ial [pos]ition
int lim;              // current [lim]it of the Iterative Deepening Search (IDS)
int pred[MAX_STEPS]; // [pre]viously used [d]irection to go to the current state

bool isViable()
{
    int sum;
    for (int i = 0; i < PUZZLE; ++i)
        for (int j = 0; j < i; ++j)
            if (p[j] > p[i]) ++sum;
    sum += b_init_pos / N + b_init_pos % N;
    sum -=          B / N +          B % N;
    return sum % 2 == 0;
}

int H()
{
    int h = 0;
    for (int pos = 0; pos < PUZZLE; ++pos)    // for all tile 'p[pos]'
    {                                         // compute Manhattan distance to goal state
        if (p[pos] == B) continue;
        h += abs( p[pos] / N - pos / N )      // position of 'p[pos]' in goal    state is 'p[pos]'
           + abs( p[pos] % N - pos % N );     // position of 'p[pos]' in current state is 'pos'
    }
    return h;
}

bool isValid(int r, int c)
{
    return 0 <= r && r < N && 0 <= c && c < N;
}

int Delta_H(int cur_r, int cur_c, int next_r, int next_c)
{
    int val = p[cur_r * N + cur_c]; // [val]ue of the tile being moved into the blank tile position
    int goal_r = val / N;           // position of 'val' in goal state is 'val'
    int goal_c = val % N;           // get row & column representation of the position
    return - ( abs(goal_r - cur_r ) + abs(goal_c - cur_c ) )
           + ( abs(goal_r - next_r) + abs(goal_c - next_c) );
}

bool dfs(int g, int h, int b_pos)
{
    if (g + h > lim) return false;
    if (h == 0) return true;       // found a solution!
    int r = b_pos / N;
    int c = b_pos % N;
    for (int d = 0; d < DIR; ++d)
    {
        if ( g != 0 && d == (pred[g] ^ 2) ) continue; // this direction gets us back to parent state
        int next_r = r + dr[d];
        int next_c = c + dc[d];
        if ( !isValid(next_r, next_c) ) continue;
        int next_h = h + Delta_H(next_r, next_c, r, c); // O(1)
        int b_next_pos = next_r * N + next_c;
        swap(p[b_pos], p[b_next_pos]);
        pred[g+1] = d;

        if ( dfs(g + 1, next_h, b_next_pos) ) return true;

        swap(p[b_pos], p[b_next_pos]);
    }
    return false;
}

int ida_star()
{
    int init_h = H();
    lim = init_h;
    while (lim <= MAX_STEPS)
    {
        if ( dfs(0, init_h, b_init_pos) ) return lim;
        ++lim;
    }
    return -1;
}

void output(int steps)
{
    for (int i = 1; i <= steps; ++i)
        printf("%c", dm[ pred[i] ]);
}

int main()
{
    #ifdef LOCAL
    freopen("in.txt", "r", stdin);
    #endif // LOCAL
    int T;
    scanf("%d", &T);
    while (T--)
    {
        for (int i = 0; i < N; ++i)
            for (int j = 0; j < N; ++j)
        {
            int pos = i * N + j;
            scanf("%d", &p[pos]);
            if (p[pos] == 0) p[pos] = B, b_init_pos = pos; // goal state 'p' is 0, 1, 2..14, 15
            else --p[pos];                                 // blank tile as 15
        }
        if ( !isViable() ) // must-consider condition otherwise TLE
        {
            printf("This puzzle is not solvable.\n");
            continue;
        }
        int ret = ida_star();
        if (ret == -1)
        {
            printf("This puzzle is not solvable.\n");
            continue;
        }
        output(ret), printf("\n");
    }
    return 0;
}
```

2 Dynamic programming algorithms

2.1 0-1 knapsack

```
#define W 1000 // Knapsack weight
#define N 100  // n item
int weight[N]; //item weight
int value[N];  //item value
int bag[W][2];

// 0/1 Knapsack
void ZeroOne(){
  memset(bag,0,sizeof(bag));
  for(int i = 0 ; i < N ; i++ ){
    for(int j = 0 ; j < W ; j++ )
      if( j >= weight[i] )
        bag[j][1] = max( bag[j][0] ,bag[j-weight[i]][0] + value[i] );

    for(int j = 0 ; j < W ; j++ )
      bag[j][0] = bag[j][1];
  }
}


// group knapsack
int group;    // hou much groups?
int how_many; // one group has many items?
int WEIGHT,VALUE;

void Grouping(){
  memset(bag,0,sizeof(bag));
  for(int i = 0 ; i < group ; i++ ){
    for(int j = 0 ; j < how_many ; j++ ){
      scanf("%d %d",&WEIGHT,&VALUE);

      for(int k = 0 ; k < W ; k++ ){
        if( j >= WEIGHT ){
          bag[j][1] = max( bag[j][1] , bag[j][0] );
          bag[j][1] = max( bag[j][1] ,bag[j-WEIGHT][0] + VALUE );
        }
      }
    }

    for(int j = 0 ; j < W ; j++ )
      bag[j][0] = bag[j][1];
  }
}

// mulipte knapsack
int limit[N]; // item limit
void Multiple() {
  for(int i = 0 ; i < N ; i++ ){
    int tmp = 1;
    while( tmp <= weight[i] ){
      for(int j = 0 ; j < W ; j++)
        if( j >= weight[i]*tmp )
          bag[j][1] = max( bag[j-weight[i]*tmp][0] + value[i]*tmp
                      , bag[j][0] );

      for(int j = 0 ; j < W ; j++ )
        bag[j][0] = bag[j][1];

      weight[i] = weight[i]-tmp;
      tmp = tmp*2;
    }
    if( weight[i] > 0 ){
      for(int j = 0 ; j < W ; j++)
        if( j >= weight[i]*tmp )
          bag[j][1] = max( bag[j-weight[i]*tmp][0] + value[i]*tmp , bag[j][0] );

      for(int j = 0 ; j < W ; j++ )
        bag[j][0] = bag[j][1];
    }
  }
}

// inf
void Unlimited(){
  memset(bag,0,sizeof(bag));
  for(int i = 0 ; i < N ; i++ ){
    for(int j = 0 ; j < W ; j++ )
      if( j >= weight[i] )
        bag[j][1] = max( bag[j][0] ,bag[j-weight[i]][1] + value[i] );
```

```
    for(int j = 0 ; j < W ; j++ )
      bag[j][0] = bag[j][1];
  }
}
```

2.2 Longest common subsequence (LCS)

```
#include <iostream>
#include <bits/stdc++.h>
#define LOCAL
using namespace std;


struct LCS{
    int step , max_len ;
}Dp[5000][5000];


int main()
{
#ifdef LOCAL
    freopen("in1.txt" , "r" , stdin );
#endif // LOCAL
    int intX , intY , Min_step , Max_len ;
    string strX , strY ;
    while(cin >> intX >> strX >> intY >> strY ){
        //init
        for(int i = 0 ; i <= intY ; i++){
            Dp[0][i].max_len = 0 ;
            Dp[0][i].step = i ;
        }
        for(int i = 0 ; i <= intX ; i++){
            Dp[i][0].max_len = 0 ;
            Dp[i][0].step = i ;
        }
        Max_len = 0 ;
        Min_step = 0 ;

        //lcs
        for(int i = 1 ; i <= intX ; i++){
            for(int j = 1 ; j <= intY ; j++){
                if(strX[i-1] == strY[j-1]){
                    Dp[i][j].max_len = Dp[i-1][j-1].max_len +1 ;
                    Dp[i][j].step = Dp[i-1][j-1].step ;

                    //debug
                    //cout << strX[i-1] << ' ' << strY[j-1] << ' ' << Dp[i][j].max_len << '\n' ;
                    //cout << strX[i-1] << ' ' << strY[j-1] << ' ' << Dp[i][j].step << '\n' ;
                }
                else{
                    Dp[i][j].max_len = max(Dp[i-1][j].max_len , Dp[i][j-1].max_len ) ;
                    Dp[i][j].step = min( min(Dp[i-1][j-1].step , Dp[i][j-1].step ) , Dp[i-1][j].step )
                           +1 ;
                }
            }
        }
        cout << Dp[intX][intY].step << '\n' ;
    }
    return 0;
}
```

2.3 Max 2D range sum

```
// Max 2D Range Sum - UVa 108 - solved with DP O(n^4).
// Abridged problem statement: Given an n x n square matrix of integers A where
// each integer ranges from [-127..127], find a sub-matrix of A with the maximum
// sum.
#include <bits/stdc++.h>
using namespace std;
int A[200][200];
int main() {
  int n; scanf("%d", &n);                      // square matrix size
  for (int i = 0; i < n; ++i)
    for (int j = 0; j < n; ++j) {
      scanf("%d", &A[i][j]);
      if (i > 0) A[i][j] += A[i-1][j];           // add from top
      if (j > 0) A[i][j] += A[i][j-1];           // add from left
      if (i > 0 && j > 0) A[i][j] -= A[i-1][j-1];// avoid double count
    }                                            // inclusion-exclusion
  int maxSubRect = -127*100*100;               // the lowest possible val
  for (int i = 0; i < n; ++i)
    for (int j = 0; j < n; ++j)                // start coordinate
```

```
      for (int k = i; k < n; ++k)
        for (int l = j; l < n; ++l) {            // end coord
          int subRect = A[k][l];                 // from (0, 0) to (k, l)
          if (i > 0) subRect -= A[i-1][l];       // O(1)
          if (j > 0) subRect -= A[k][j-1];       // O(1)
          if (i > 0 && j > 0) subRect += A[i-1][j-1]; // O(1)
          maxSubRect = max(maxSubRect, subRect); // the answer is here
        }
  printf("%d\n", maxSubRect);
  return 0;
}
```

2.4 Traveling salesman problem (TSP)

```
// This is a solution for UVa 10496 - Collecting Beepers. The problem is a
// variant of the Traveling Salesman Problem (TSP): Given n cities and their
// pairwise distances in the form of a matrix 'dist' of size n * n, compute the
// minimum cost of making a tour that starts from any city s, goes through all
// the other n - 1 cities exactly once, and finally returns to the city s. In
// this case, the salesman is Karel in a 2D world who can only move along the
// x and y axis. The cities are beepers whose coordinates are given, from which
// pairwise distances can be calculated. Algorithm takes time O(2^n * n^2).
// INPUT: The first line is the number of test cases. The first line of each
// test case is world's size (x-size and y-size). Next is the starting position
// of Karel. Next is the number of beepers. Next are the beepers' x- and y-
// coordinates.
// OUTPUT: For each test case, output the minimum distance to move from Karel's
// starting position to each of the beepers and back to the starting position.

#include <bits/stdc++.h>
using namespace std;

#define LSOne(S) ((S) & -(S))

const int MAX_n = 11;

int dist[MAX_n][MAX_n], memo[MAX_n][1<<(MAX_n-1)]; // Karel + max 10 beepers

int dp(int u, int mask) {                        // mask = free coordinates
  if (mask == 0) return dist[u][0];              // close the loop
  int &ans = memo[u][mask];
  if (ans != -1) return ans;                     // computed before
  ans = 2000000000;
  int m = mask;
  while (m) {                                    // up to O(n)
    int two_pow_v = LSOne(m);                    // but this is fast
    int v = __builtin_ctz(two_pow_v)+1;          // offset v by +1
    ans = min(ans, dist[u][v] + dp(v, mask^two_pow_v)); // keep the min
    m -= two_pow_v;
  }
  return ans;
}

int main() {
  int TC; scanf("%d", &TC);
  while (TC--) {
    int xsize, ysize; scanf("%d %d", &xsize, &ysize); // these two values are not used
    int x[MAX_n], y[MAX_n];
    scanf("%d %d", &x[0], &y[0]);
    int n; scanf("%d", &n); ++n;                 // include Karel
    for (int i = 1; i < n; ++i)                  // Karel is at index 0
      scanf("%d %d", &x[i], &y[i]);
    for (int i = 0; i < n; ++i)                  // build distance table
      for (int j = i; j < n; ++j)
        dist[i][j] = dist[j][i] = abs(x[i]-x[j]) + abs(y[i]-y[j]); // Manhattan distance
    memset(memo, -1, sizeof memo);
    printf("The shortest path has length %d\n", dp(0, (1<<(n-1))-1)); // DP-TSP
  }
  return 0;
}
```

3 Graph algorithms

3.1 All-pairs shortest paths (APSP)

```
// All-Pairs Shortest Paths (APSP) solved with Floyd Warshall O(V^3).
// inside int main()
    // Precondition: AdjMat[i][j] contains the weight of edge (i, j) or INF (1B)
    // if there is no such edge ('AdjMat' is a 32-bit signed integer array).
    // Let 'p' be 2D parent matrix, where p[i][j] is the last vertex before j on
    // a shortest path from i to j, i.e. i -> ... -> p[i][j] -> j.
    for (int i = 0; i < V; ++i)
        for (int j = 0; j < V; ++j)
            p[i][j] = i;    // initialize the parent matrix
    for (int k = 0; k < V; ++k)    // remember that loop order is k->i->j
        for (int i = 0; i < V; ++i)
            for (int j = 0; j < V; ++j)
                if (AdjMat[i][k] + AdjMat[k][j] < AdjMat[i][j])
                {
                    AdjMat[i][j] = AdjMat[i][k] + AdjMat[k][j];
                    p[i][j] = p[k][j];
                }
// print shortest paths
void printPath(int i, int j)
{
    if (i != j) printPath(i, p[i][j]);
    printf("%d ", j);
}
```

3.2 Bipartite matching BFS by David

```
#include <iostream>
#include <cstring>
#include <cstdio>
#include <vector>
#define LOCAL
using namespace std;

int fp[100010] ,fq[100010] ;
int vfp[100010] ,vfq[100010] ;
int turn = 0 ;
vector<int> cp[100010] , cq[100010];

int BFSBMfp(int n){
    vfp[n] = turn  ;
    for(int i = 0 ; i < cp[n].size() ; i++ ){
        if(vfq[cp[n][i]] != turn){
            vfq[cp[n][i]] = turn ;
            if(fq[cp[n][i]] == -1 || BFSBMfp(fq[cp[n][i]])){
                fp[n] = cp[n][i] ;
                fq[cp[n][i]] = n ;
                return 1 ;
            }
        }
    }
    return 0 ;
}

int main()
{
    ios::sync_with_stdio(false);
    cin.tie(0);
    cout.tie(0);

    int n ,p ,q ,k ,x, y ;
    cin >> n ;
    while(n--){
        cin >> p >> q >> k ;
        int MaxnPQ = max(p,q);
        for(int i = 1 ; i <= MaxnPQ ; i++){
            cp[i].clear();
            fp[i] = -1;
            cq[i].clear();
            fq[i] = -1;
        }
        int cnt = 0;
        for(int i = 0 ; i < k ; i++){
            cin >> x >> y ;
            cp[x].push_back(y) ;
            cq[y].push_back(x) ;
            if(fp[x] == -1 && fq[y] == -1 ){
                fp[x] = y ;
                fq[y] = x ;
                cnt++ ;
            }
        }
        for(int i = 1 ; i <= p ; i++){
            if(fp[i] == -1 ){
                turn++;
                if(BFSBMfp(i))
                   cnt++;
            }
        }
        cout << cnt << '\n';
    }
    return 0;
}
```

3.3 Centroid decomposition

```cpp
#include<iostream>
#include<bits/stdc++.h>
#define LOCAL
#define MAXN 50005
using namespace std;

int n , k , a , b ;
int ans , cnt ;
int Max[MAXN] , sz[MAXN] , rt ;
int head[MAXN], dis[MAXN];
bool vis[MAXN] ;
struct node{
    int v , nx ;
}Edge[MAXN*2];

void init(int n ){
    Max[0] = n ;
    ans = cnt = 0 ;
    for(int i = 0 ; i <= n ; i++){
        head[i] = -1 ;
        vis[i] = 0 ;
    }
}

void add(int u , int v){
    Edge[cnt].v = v ;
    Edge[cnt].nx = head[u] ;
    head[u] = cnt++ ;
}

void get_rt(int u , int fa ){
    sz[u] = 1 ; Max[u] = 0 ;
    for(int i = head[u] ; ~i ; i=Edge[i].nx){
        int v = Edge[i].v ;
        if(vis[v] || v == fa )  continue ;
        get_rt(v,u);
        sz[u] += sz[v] ;
        Max[u] = max(Max[u] , sz[v]);
    }
    Max[u] = max(Max[u] , n - sz[u]);
    if(Max[rt] > Max[u])
        rt =  u ;
}

void get_dis(int u , int fa , int d){
    for(int i = head[u] ; ~i ; i= Edge[i].nx){
        int v = Edge[i].v ;
        if(vis[v] || v == fa ) continue ;
        dis[++cnt] = d + 1 ;
        get_dis(v,u,dis[cnt]);
    }
}


int get_ans(int u , int d ){
    dis[cnt=1] = d ;
    get_dis(u,0,d) ;
    sort(dis+1 , dis+cnt+1) ;
    int l = 1 , ans = 0 ;

    while(l < cnt && dis[l] + dis[cnt] < k ) l++ ;
    while(l < cnt && dis[l] <= k - dis[l]){
        ans += upper_bound(dis + l + 1 , dis + cnt + 1 , k - dis[l]) - lower_bound(dis+l+1 , dis+cnt+1
            , k-dis[l]);
        l++ ;
    }
    return ans ;
}

void dfs(int u ){
    vis[u] = 1 ;
    //cout << rt << ' ' << u << '\n' ;
    ans += get_ans(u , 0);
    for(int i = head[u] ; ~i ; i = Edge[i].nx){
        int v = Edge[i].v ;
        if(vis[v]) continue ;
        ans -= get_ans(v , 1) ;
        n = sz[v] , rt = 0  , get_rt(v,u);
        dfs(rt);
    }
}

int main(){
//#ifdef LOCAL
//    freopen("in1.txt" , "r" , stdin);
//#endif // LOCAL
    cin >> n >> k ;
    init(n);
    for(int i =1; i < n ; i++){
        cin >> a >> b ;
        add(a,b);
        add(b,a);
    }
    rt = 0 ; get_rt(1,0);
    dfs(rt);
    cout << ans << '\n' ;

}
```

3.4 Detect negative weight cycle

```cpp
// Bellman Ford's O(VE)
    vi dist(V, INF); dist[s] = 0;
    for (int i = 0; i < V - 1; ++i)    // relax all E edges V - 1 times
        for (int u = 0; u < V; ++u)    // these two loops = O(E)
            for (int j = 0; j < (int)AL[u].size(); ++j) // [A]djacency [L]ist
            {
                ii vw = AL[u][j];
                dist[vw.first] = min( dist[vw.first], dist[u] + vw.second ); // relax
            }
```

3.5 DFS

```cpp
#include <iostream>
#include <bits/stdc++.h>
#define LOCAL
using namespace std;
int m , n , flag=1;
int Maxn_city = 0 , Maxn_path = 0 ;
vector<int>tree[200020] ;
int city[200020] = {} ;
int visit[200020] = {} ;
vector<int> travel ;

void BFS_to_large_path(int root ){
    visit[root] = 1 ;
    travel.push_back(root);
    for(int i = 0 ; i < tree[root].size() ; i++){
        int node = tree[root][i] ;
        if(!visit[node]){
            BFS_to_large_path(node);
            travel.pop_back();
            visit[root] = 0 ;
        }
    }
    //debug to check large path
    //if (root == 1)
    //    cout << "1=" << travel.size() << ' ' << Maxn_path << ' ' << city[root] << '\n' ;

    if(city[root] && travel.size() > Maxn_path){
        Maxn_city = travel[travel.size()/2];
        Maxn_path = travel.size();
    }
}

void BFS_to_other_path(int root ,int path){
    visit[root] = 1 ;
    for(int i = 0 ; i < tree[root].size() ; i++){
        int node = tree[root][i] ;
        if(!visit[node]){
            BFS_to_other_path(node , path+1);
            visit[root] = 0 ;
        }
    }
    //debug
    if(root == 1 )
        cout << "city=" << root << " path= " << path << '\n' ;

    if(city[root] && path != Maxn_path)
        flag = 0 ;
}


int main(){
#ifdef LOCAL
    freopen("in1.txt" , "r" , stdin);
#endif // LOCAL
    cin >> n >> m ;
    int a , b ;
    for(int i = 0 ; i < n-1 ; i++){
        cin >> a >> b ;
        tree[a].push_back(b) ;
        tree[b].push_back(a) ;
    }

    for(int i = 0 ; i < m ; i++){
        cin >> a ;
        city[a] = 1 ;
    }
    BFS_to_large_path(a);
    //visit[a] = 0 ;
    BFS_to_other_path(Maxn_city , 1 );
    if(flag)
        cout << "YES\n" << Maxn_city ;
    else
        cout << "NO" ;

    //debug
    cout << "Maxn_path= " << Maxn_path << " Maxn_city= " << Maxn_city << '\n' ;
}
```

3.6 DFS ICPC 2019 Russia problem E

```
#include <iostream>
#include <bits/stdc++.h>
#define LOCAL
using namespace std;
int m , n , flag=1;
int Maxn_city = 0 , Maxn_path = 0 ;
vector<int>tree[200020] ;
int city[200020] = {} ;
int visit[200020] = {} ;
vector<int> travel ;

void BFS_to_large_path(int root ){
    visit[root] = 1 ;
    travel.push_back(root);
    for(int i = 0 ; i < tree[root].size() ; i++){
        int node = tree[root][i] ;
        if(!visit[node]){
            BFS_to_large_path(node);
            travel.pop_back();
            visit[root] = 0 ;
        }
    }
    //debug to check large path
    //if (root == 1)
    //    cout << "1=" << travel.size() << ' ' << Maxn_path << ' ' << city[root] << '\n' ;

    if(city[root] && travel.size() > Maxn_path){
        Maxn_city = travel[travel.size()/2];
        Maxn_path = travel.size();
    }
}

void BFS_to_other_path(int root ,int path){
    visit[root] = 1 ;
    for(int i = 0 ; i < tree[root].size() ; i++){
        int node = tree[root][i] ;
        if(!visit[node]){
            BFS_to_other_path(node , path+1);
            visit[root] = 0 ;
        }
    }
    //debug
    if(root == 1 )
        cout << "city=" << root << " path= " << path << '\n' ;

    if(city[root] && path != Maxn_path)
        flag = 0 ;
}


int main(){
#ifdef LOCAL
    freopen("in1.txt" , "r" , stdin);
#endif // LOCAL
    cin >> n >> m ;
    int a , b ;
    for(int i = 0 ; i < n-1 ; i++){
        cin >> a >> b ;
        tree[a].push_back(b) ;
        tree[b].push_back(a) ;
    }
    for(int i = 0 ; i < m ; i++){
        cin >> a ;
        city[a] = 1 ;
    }
    BFS_to_large_path(a);
    //visit[a] = 0 ;
    BFS_to_other_path(Maxn_city , 1 );
    if(flag)
        cout << "YES\n" << Maxn_city ;
    else
        cout << "NO" ;

    //debug
    cout << "Maxn_path= " << Maxn_path << " Maxn_city= " << Maxn_city << '\n' ;
}
```

3.7 Dijkstra by Bill

```
// Dijkstra implementation for negative weight edges O((V + E) log V)
    vi dist(V, INF); dist[s] = 0;
    priority_queue< ii, vii, greater<ii> > pq;
    pq.push( ii(0, s) );
    while (!pq.empty())
    {
        ii front = pq.top(); pq.pop();
        int d = front.first;
        int u = front.second;
        if (d > dist[u]) continue;
        for (int i = 0; i < (int)AL[u].size(); ++i) // [A]djacency [L]ist
        {
            ii vw = AL[u][i];
            int v = vw.first;
            int w = vw.second;
            if (dist[u] + w < dist[v])
            {
                dist[v] = dist[u] + w;    // relax operation
                pq.push( ii(dist[v], v) );
            }
        }
    } // this variant can cause duplicate items in the priority queue
```

3.8 Dijkstra by David

```
#include <iostream>
#include <bits/stdc++.h>
#define LOCAL
#define INF 99999999
using namespace std;
int intMap[1010][1010] = {} , intValue[1010][1010] = {};
int m , n ;


struct Node{
    int x , y , v ;
    void read( int _x , int _y , int _v){
        x = _x ; y = _y ; v = _v ;
    }
    bool operator < (const Node &a) const{
        return v > a.v ;
    }
}nodNode;


void print_map(){
    for(int i = 1 ; i <= n ; i++){
        for(int j = 1 ; j <= m ; j++){
            if(intValue[i][j] == 99999999)
                cout << 'r' << ' ' ;
            else
                cout << intValue[i][j] << ' ' ;
        }
        cout << '\n' ;
    }
    cout << '\n' ;
}


void bfs(){
    int x , y , intDirection[4][2] = {-1,0 ,0,1 ,1,0 ,0,-1};
    int intDx , intDy ;
    Node nodTemp ;
    priority_queue<Node> deqNode ;
    nodTemp.read(1,1,0);
    deqNode.push(nodTemp);
    while(deqNode.size()){
        x = deqNode.top().x ;
        y = deqNode.top().y ;
        deqNode.pop() ;

        for(int i = 0 ; i < 4 ; i++){
            intDx = intDirection[i][0] + x ;
            intDy = intDirection[i][1] + y ;

            //debug
            //cout << intDx << ' ' << intDy << ' ' << intValue[x][y] + intMap[intDx][intDy] << ' ' <<
                  i <<  '\n' ;

            if(intValue[x][y] + intMap[intDx][intDy] < intValue[intDx][intDy]  ){
                intValue[intDx][intDy] = intValue[x][y] + intMap[intDx][intDy] ;
                nodTemp.read(intDx , intDy , intValue[intDx][intDy]);
                deqNode.push(nodTemp) ;
            }
        }
        //print_map() ;
    }
}


int main() {
#ifdef LOCAL
    freopen("in1.txt" , "r" , stdin );
    freopen("out.txt" , "w" , stdout) ;
#endif
ios::sync_with_stdio(false);
    int intCase ;
    cin >> intCase ;
    while(intCase --){
        cin >> n >> m ;
        for(int i = 1 ; i <= n ; i++){
            for(int j = 1 ; j <= m ; j++){
                 cin >> intMap[i][j] ;
                intValue[i][j] = INF ;
            }
        }

        for(int i = 1 ; i <= n ; i++){
            intValue[i][0] = 0 ;
            intValue[i][m+1] = 0 ;
            intMap[i][0] = INF +1 ;
            intMap[i][m+1] = INF +1 ;
        }
        for(int i = 1 ; i <= m ; i++){
            intValue[0][i] = 0 ;
            intValue[n+1][i] = 0 ;
            intMap[0][i] = INF +1 ;
            intMap[n+1][i] = INF +1 ;
        }
        intValue[1][1] = intMap[1][1] ;

        //debug
        //cout << intValue[1][1] << '\n' ;

        bfs();
        cout << intValue[n][m] << '\n' ;
    }
        return 0;

}
```

3.9 Print Euler tour

```
// Given an Eulerian-tour graph - a connected undirected graph whose vertices a-
// ll have even degrees, produce its Euler tour. The graph is unweighted, stored
// in an adjacency list where the second attribute in edge info pair is a boole-
// an '1' (edge can still be used) or '0' (edge can no longer be used).
list<int> cyc;    // we need list for fast insertion in the middle

void EulerTour(list<int>::iterator i, int u)
{
    for (int j = 0; j < (int)AL[u].size(); ++j) // [A]djacency [L]ist
    {
        ii& vw = AL[u][j];
        int v = vw.first;
        if (vw.second)    // if this edge can still be used
        {
            vw.second = 0;    // remove this edge
            // remove bi-directional edge
            for (int k = 0; k < (int)AL[v].size(); ++k)
            {
                ii& uw = AL[v][k];
                if (uw.first == u && uw.second)
                {
                    uw.second = 0;
                    break;
                }
            }
            // continue the tour
            EulerTour(cyc.insert(i, u), v);
        }
    }
}

// inside int main()
    cyc.clear();
    EulerTour(cyc.end(), 0);    // 'cyc' contains an Euler tour starting at vertex '0'
    for (list<int>::iterator i = cyc.begin(); i != cyc.end(); ++i)
        printf("%d\n", *i);
```

3.10 Find articulation points and bridges for undirected graph

```
// Find articulation points & bridges for undirected graph solved with DFS O(V + E).
void articulationPointAndBridge(int u)
{
    dfs_low[u] = dfs_num[u] = dfsNumberCounter++;    // dfs_low[u] <= dfs_num[u]
    for (int i = 0; i < (int)AL[u].size(); ++i)    // [A]djacency [L]ist
    {
        int v = AL[u][i].first;
        if (dfs_num[v] == UNVISITED)    // a tree edge
        {
            dfs_parent[v] = u;
            if (u == dfsRoot) ++rootChildren;    // special case if 'u' is a root

            articulationPointAndBridge(v);

            if (dfs_low[v] >= dfs_num[u]) articulation_vertex[u] = true;
            if (dfs_low[v] > dfs_num[u]) printf("Edge (%d, %d) is a bridge\n", u, v);

            dfs_low[u] = min( dfs_low[u], dfs_low[v] );    // update dfs_low[u]
        }
        else if (v != dfs_parent[u]) dfs_low[u] = min( dfs_low[u], dfs_num[v] ); // update dfs_low[u]
    }
}

// inside int main()
    dfsNumberCounter = 0;
    dfs_num.assign(V, UNVISITED);
    dfs_low.assign(V, 0);
    dfs_parent.assign(V, 0);
    articulation_vertex.assign(V, 0);
    printf("Bridges:\n");
    for (int u = 0; u < V; ++u)
        if (dfs_num[u] == UNVISITED)
        {
            dfsRoot = u;
            rootChildren = 0;
            articulationPointAndBridge(u);
            articulation_vertex[dfsRoot] = (rootChildren > 1);    // special case
        }
    printf("Articulation Points:\n");
    for (int u = 0; u < V; ++u)
        if (articulation_vertex[u]) printf(" Vertex %d\n", u);
```

3.11 Floyd Warshall by David

```
#include <iostream>
#include <bits/stdc++.h>
#define LOCAL
using namespace std;
char before[520][520] = {} ;
int after[520][520] = {} ;

int main()
{
#ifdef LOCAL
    freopen("in1.txt" , "r" , stdin );
#endif // LOCAL

    int n ;
    cin >> n ;
    for(int i = 0 ; i < n ; i++){
        for(int j = 0 ; j < n ; j++)
```

```
            cin >> before[i][j] ;
    }

    for(int i = 0 ; i < n ; i++){
        for(int j = i+1 ; j < n ; j++){
            int sum = 0 ;
            for(int k = i + 1 ; k < j ; k++){
                if(after[i][k])
                    sum += before[k][j]-’0’ ;
            }

            if( (sum +1) % 10 == before[i][j] - ’0’){
                after[i][j] = 1 ;
            }

        }
    }

    for(int i = 0 ; i < n ; i++){
        for(int j = 0; j < n ; j++)
            cout << after[i][j] ;
        cout << ’\n’ ;
    }

    return 0;
}
```

3.12 Graph edges property check

```
// Graph Edges Property Check solved with DFS O(V + E).
void graphCheck(int u)    // DFS for checking graph edge properties
{
    dfs_num[u] = EXPLORED;
    for (int i = 0; i < (int)AL[u].size; ++i) // [A]djancency [L]ist
    {
        int v = AL[u][i].first;
        if (dfs_num[v] == UNVISITED)    // Tree Edge, EXPLORED->UNVISITED
        {
            dfs_parent[v] = u;    // parent of this child is me
            graphCheck(v);
        }
        else if (dfs_num[v] == EXPLORED)    // EXPLORED->EXPLORED
        {
            if (v == dfs_parent[u]) printf(" Two ways (%d, %d)-(%d, %d)\n", u, v, v, u);
            else printf(" Back Edge (%d, %d) (Cycle)\n", u, v); // can check if graph is cyclic
        }
        else if (dfs_num[v] == VISITED)    // EXPLORED->VISITED
            printf(" Forward/Cross Edge (%d, %d)\n", u, v);
    }
    dfs_num[u] = VISITED;
}

// inside int main()
    dfs_num.assign(V, UNVISITED);
    dfs_parent.assign(V, 0);
    for (int u = 0; u < V; ++u)
        if (dfs_num[u] == UNVISITED)
            printf("Component %d:\n", ++numComp), graphCheck(u);
```

3.13 Kruskal by David

```
#include <iostream>
#include <bits/stdc++.h>
#define LOCAL
#define ll long long
using namespace std;
int parent[1020] ;

struct edge{
    ll n1 , n2 , w ;
}node[25020];


int compare(edge A , edge B ){
    return A.w < B.w ;
}


int find_root(int a){
    if(a != parent[a] )
        return parent[a] = find_root(parent[a]) ;
    return a ;
}

int main()
{
#ifdef LOCAL
    freopen("in1.txt" , "r" , stdin );
    freopen("out.txt" , "w" , stdout );
#endif // LOCAL
    int n , m , p_n1 , p_n2 ; // parent_n1 , parent_n2
    vector<int> hce ; //heavy edge circle
    while(cin >> n >> m && n + m != 0 ){
        for(int i = 0 ; i < m ; i++ ){
            cin >> node[i].n1 >> node[i].n2 >> node[i].w ;
        }

        for(int i = 0 ; i < n ; i++)
            parent[i] = i ;
        sort(node , node + m , compare ) ;
        hce.clear() ;

        //kruskal
        for(int i = 0 ; i < m ; i++){
            p_n1 = find_root(node[i].n1) ;
            p_n2 = find_root(node[i].n2) ;
            if(p_n1 != p_n2 )
                parent[p_n2] = p_n1 ;
            else
                hce.push_back(node[i].w) ;

            //debug
            /**<
            for(int i = 0 ; i < n ; i++)
                cout << parent[i] << ’ ’ ;
            cout << ’\n’ ;
            */
        }
        sort(hce.begin() , hce.end()) ;
        if(hce.size()){
            for(int i = 0 ; i < hce.size()-1 ; i++)
                cout << hce[i] << ’ ’ ;
            cout << hce[hce.size()-1] ;
        }
        else
            cout << "forest" ;
        cout << ’\n’ ;
    }
    return 0;
}
```

3.14 Max flow

```
int res[MAX_V][MAX_V], mf, f, s, t;
vi p; // p stores the BFS spanning tree from s

void augment(int v, int minEdge)
{
    if (v == s) { f = minEdge; return; }
    else if ( p[v] != -1 )
    {
        augment( p[v], min(minEdge, res[ p[v] ][ v ]) );
        res[ p[v] ][ v ] -= f;
        res[ v ][ p[v] ] += f;
    }
}

// inside int main(): set up ’AL’, ’res’, ’s’, and ’t’ with appropriate values
// remember to add backward edges to ’AL’
    mf = 0;
    while (true)    // O(V * E^2) Edmonds Karp’s algorithm
    {
        f = 0;
        vi dist(MAX_V, INF); dist[s] = 0;
        queue<int> q; q.push(s);
        p.assign(MAX_V, -1);
        while (!q.empty())
        {
            int u = q.front(); q.pop();
            if (u == t) break; // immediately stop BFS if we already reach sink t
            for (int i = 0; i < (int)AL[u].size(); ++i)
            {
                int v = AL[u][i];    // vector< vi > [A]djacency [L]ist
                if (res[u][v] > 0 && dist[v] == INF)
                    dist[v] = dist[u] + 1, q.push(v), p[v] = u;
            }
        }
        augment(t, INF);   // find the min edge weight ’f’ in this path, if any
        if (f == 0) break; // we cannot send any more flow (’f’ = 0), terminate
        mf += f;           // we can still send a flow, increase the max flow!
    }
    printf("%d\n", mf);
```

3.15 Max cardinality bipartite matching (MCBM)

```
// Max Cardinality Bipartite Matching (MCBM) solved with augmenting path algorithm O(VE).
vi match, vis;

int Aug(int l)    // return 1 if an augmenting path is found & 0 otherwise
{
    if (vis[l]) return 0;
    vis[l] = 1;
    for (int i = 0; i < (int)AL[l].size(); ++i) // [A]djacency [L]ist
    {
        int r = AL[l][i];    // edge weight not needed -> vector< vi > AL
        if ( match[r] == -1 || Aug(match[r]) )
        {
            match[r] = l;
            return 1;    // found 1 matching
        }
    }
    return 0;            // no matchings
}

// inside int main()
    // build unweighted bipartite graph with directed edge left->right set
    // left vertices [0..N-1], right vertices [N..V-1]
    int MCBM = 0;
    match.assign(V, -1); // V is the number of vertices in bipartite graph
    for (int l = 0; l < N; ++l)    // N = size of the left set
    {
        vis.assign(N, 0);    // reset before each recursion
        MCBM += Aug(l);
    }
    printf("Found %d matchings\n", MCBM);
```

3.16 Min-cost flow (MCF)

```
// UVa 10594 - Data Flow solved as Min-Cost Flow (MCF) problem using Edmonds Ka-
// rp and Bellman Ford algorithms with total time O(V^2 * E^3).
#include <bits/stdc++.h>

using namespace std;

#define LOCAL
#define INF 1000000000000000 // 10^15
#define bwd 0 // [b]ack[w]ar[d] direction
#define fwd 1 // [f]or[w]ar[d] direction
#define MAX_V 200

typedef vector<int> vi;
typedef long long int ll;
typedef pair<ll, ll> ll2;
typedef vector<ll> vll;

int V;
vector<vi> AL;
ll res[MAX_V][MAX_V][2], cst[MAX_V][MAX_V][2];
ll mf, f, min_cost;
int s, t;
vector< pair<int, ll> > p;
ll FLOW, CAPACITY;

void augment(int v, ll minEdge)
{
    if (v == s) { f = minEdge; return; }
    else if ( p[v].first != -1 )
    {
        augment( p[v].first, min(minEdge, res[ p[v].first ][ v ][ p[v].second ]) );
        res[ p[v].first ][ v ][ p[v].second ] -= f;
        res[ v ][ p[v].first ][ p[v].second ] += f;
    }
}

void trace_cost(int v)
{
    if (p[v].first == -1) return;
    min_cost += cst[ p[v].first ][ v ][ p[v].second ] * f;
    trace_cost(p[v].first);
}

void min_cost_flow()
{
    min_cost = 0;
    mf = 0;
    while (true)
    {
        f = 0;
        p.assign(MAX_V, make_pair(-1, -1));
        vll dist(V, INF); dist[s] = 0;
        for (int i = 0; i < V - 1; ++i)
            for (int u = 0; u < V; ++u)
                for (int j = 0; j < (int)AL[u].size(); ++j)
                {
                    int v = AL[u][j];
                    for (int dir = 0; dir <= 1; ++dir)
                        if (res[u][v][dir] > 0 && dist[u] + cst[u][v][dir] < dist[v])
                        {
                            dist[v] = dist[u] + cst[u][v][dir];
                            p[v] = make_pair(u, dir);
                        }
                }
        augment(t, INF);
        if (f == 0) break;
        f = min(f, FLOW - mf);
        trace_cost(t);
        mf += f;
        if (mf == FLOW) break;
    }
    if (mf < FLOW) printf("Impossible.\n");
    else printf("%lld\n", min_cost);
}

int main()
{
#ifdef LOCAL
    freopen("in", "r", stdin);
#endif
    int E;
    while (scanf("%d %d", &V, &E) != EOF)
    {
        AL.assign(V, vi());
        memset(res, 0, sizeof res);
        memset(cst, 0, sizeof cst);
        for (int i = 0; i < E; ++i)
        {
            int u, v;
            ll w;
            scanf("%d %d %lld", &u, &v, &w);
            u--; v--; // 0-based index
            AL[u].push_back(v);
            AL[v].push_back(u);

            res[u][v][fwd] = res[v][u][bwd] = 1; // real edges
            cst[u][v][fwd] = cst[v][u][bwd] = w;

            res[u][v][bwd] = res[v][u][fwd] = 0; // additional reversed edges
            cst[u][v][bwd] = cst[v][u][fwd] = -w;
        }

        scanf("%lld %lld", &FLOW, &CAPACITY);
        for (int u = 0; u < V; ++u)
            for (int v = 0; v < V; ++v)
            {
                res[u][v][fwd] *= CAPACITY;
                res[v][u][bwd] *= CAPACITY;
            }

        s = 0;
        t = V-1;
        min_cost_flow();
    }
    return 0;
}
```

3.17 Minimum spanning tree (MST)

```
// Minimum Spanning Tree (MST) solved with Kruskal O(E log V)
// inside int main()
    vector< pair<int, ii> > EdgeList;    // (weight, two vertices) of the edge
    for (int i = 0; i < E; ++i)
    {
        scanf("%d %d %d", &u, &v, &w);
        EdgeList.push_back( make_pair( w, ii(u, v) ) );
    }
    sort(EdgeList.begin(), EdgeList.end()); // sort by edge weight O(E log E)
    int mst_cost = 0;
    UnionFind UF(V);    // all V are disjoint sets initially
    for (int i = 0; i < E; ++i)
    {
        pair<int, ii> front = EdgeList[i];
        if (!UF.isSameSet(front.second.first, front.second.second))
        {
            mst_cost += front.first;
            UF.unionSet(front.second.first, front.second.second);
        }
    }
    printf("MST cost = %d\n", mst_cost);
```

3.18 Strongly connected component (SCC)

```
// Tarjan O(V + E)
vi dfs_num, dfs_low, visited;
int dfsNumberCounter, numSCC;
vi S;

void tarjanSCC(int u)
{
    dfs_low[u] = dfs_num[u] = dfsNumberCounter++;    // dfs_low[u] <= dfs_num[u]
    S.push_back(u);    // stores 'u' in a vector baesd on order of visitation
    visited[u] = 1;
    for (int i = 0; i < (int)AL[u].size(); ++i) // [A]djacency [L]ist
    {
        int v = AL[u][i].first;
        if (dfs_num[v] == UNVISITED) tarjanSCC(v);
        if (visited[v]) dfs_low[u] = min( dfs_low[u], dfs_low[v] ); // condition for update
    }

    if (dfs_low[u] == dfs_num[u])    // if this is a root (start) of an SCC
    {                                // this part is done after recursion
        printf("SCC %d:", ++numSCC);
        while (true)
        {
            int v = S.back(); S.pop_back();
            visited[v] = 0;
            printf(" %d", v);
            if (u == v) break;
        }
        printf("\n");
    }
}

// inside int main()
    dfs_num.assign(V, UNVISITED);
    dfs_low.assign(V, 0);
    visited.assign(V, 0);
    dfsNumberCounter = numSCC = 0;
    for (int u = 0; u < V; ++u)
        if (dfs_num[u] == UNVISITED)
            tarjanSCC(u);
```

4 Greedy algorithms

4.1 Interval covering

```
// This is a solution for UVa 10382 - Watering Grass. The problem is a variant
// of Interval Covering problem, which is solved by O(n) Greedy algorithm.

#include <bits/stdc++.h>

#define pb push_back
#define not_set -1

using namespace std;

typedef pair<double, double> dd;
typedef vector<dd> vdd;
typedef enum { STOP = 0,
               CONTINUE } status;

int n, l, w;
vdd spinklers;
int answer;
double pivot;

struct sort_compare_t {
    bool operator()(dd a, dd b) const {
        return a.first < b.first || (a.first == b.first && a.second > b.second);
    }
} sort_compare;

void InputSpinklers() {
    for (int i = 0; i < n; i++) {
        double x, r;  // must be double otherwise WA.
        scanf("%lf %lf", &x, &r);
        if (w > 2 * r)  // ignore spinklers that cannot cover the width of the strip.
        {
            continue;
        }
        if (w == 2 * r)  // ignore spinklers that produce no intervals.
        {
            continue;
        }
        double dx = sqrt(r * r - w * w / 4.0);
        spinklers.pb(dd(x - dx, x + dx));
    }
}

status Check(int& j) {
    if (j == not_set)  // there is an interval after pivot that cannot be covered.
    {
        return STOP;
    }
    // record j.
    answer++;
    pivot = spinklers[j].second;
    if (pivot >= l)  // solution found!
    {
        return STOP;
    }
    j = not_set;
    return CONTINUE;
}

void SolveIntervalCovering() {
    sort(spinklers.begin(), spinklers.end(), sort_compare);
    answer = 0;
    pivot = 0.0;
    int j = not_set;
    int iter = 0;
    while (true) {
        if (iter == spinklers.size())  // iterated through all spinklers/intervals.
        {
            Check(j);
            break;
        }

        if (spinklers[iter].first <= pivot) {
            if (pivot < spinklers[iter].second)  // note the next candidate down!
            {
                if (j == not_set || spinklers[iter].second > spinklers[j].second)  // note down the
                      most right candidate.
                {
                    j = iter;
                }
                iter++;
            } else  // skip intervals that are completely covered by the previously selected ones.
            {
                iter++;
            }
        } else  // out bound.
        {
            if (Check(j) == STOP) {
                break;
            }
        }
    }

    if (pivot >= l) {
        printf("%d\n", answer);
    } else {
        printf("-1\n");
    }
}

int main() {
    while (scanf("%d %d %d", &n, &l, &w) != EOF) {
        spinklers.clear();
        InputSpinklers();
        SolveIntervalCovering();
    }
}
```

4.2 Longest increasing subsequence (LIS)

```
#include <bits/stdc++.h>
using namespace std;

typedef vector<int> vi;

int n;
vi A;

void print_array(const char *s, vi &L, int n) {
  for (int i = 0; i < n; ++i) {
    if (i) printf(", ");
    else printf("%s: [", s);
    printf("%d", L[i]);
  }
  printf("]\n");
}

vi p;                                            // predecessor array

void print_LIS(int i) {                          // backtracking routine
  if (p[i] == -1) { printf("%d", A[i]); return; }// base case
  print_LIS(p[i]);                               // backtrack
  printf(" %d", A[i]);
}

int memo[10010];                                 // old limit: up to 10^4

int LIS(int i) {                                 // O(n^2) overall
  if (i == 0) return 1;
  int &ans = memo[i];
  if (ans != -1) return ans;                     // was computed before
  ans = 1;                                       // LIS can start anywhere
  for (int j = 0; j < i; ++j)                    // O(n) here
    if (A[j] < A[i])                             // increasing condition
      ans = max(ans, LIS(j)+1);                  // pick the max
  return ans;
}

int main() {
  // note: A[n-1] must be set as the largest value ("INF")
  // so that all LIS (that can start anywhere) will end at n-1
  srand(time(NULL));
  int n = 10+rand()%11;                          // [10..20]
  A.assign(n, 0);
  A[n-1] = 99;                                   // set A[n-1] = INF
  for (int i = 0; i < n-1; ++i)
    A[i] = rand()%101-50;                        // [-50..50]

  n = 12;
  vi sample({-7, 10, 9, 2, 3, 8, 8, 1, 2, 3, 4, 99});
  A = sample;

  printf("n = %d:", n);
  for (int i = 0; i < n; ++i)
    printf(" %d", A[i]);
  printf("\n");

  // early 2000 problems usually accept O(n^2) solution
  memset(memo, -1, sizeof memo);
  printf("LIS length is %d\n\n", LIS(n-1));      // with O(n^2) DP

  // 2020s problems will likely only accept O(n log k) solution
  // new limit: n can be up to 200K
  int k = 0, lis_end = 0;
  vi L(n, 0), L_id(n, 0);
  p.assign(n, -1);

  for (int i = 0; i < n; ++i) {                  // O(n)
    int pos = lower_bound(L.begin(), L.begin()+k, A[i]) - L.begin();
    L[pos] = A[i];                               // greedily overwrite this
    L_id[pos] = i;                               // remember the index too
    p[i] = pos ? L_id[pos-1] : -1;               // predecessor info
    if (pos == k) {                              // can extend LIS?
      k = pos+1;                                 // k = longer LIS by +1
      lis_end = i;                               // keep best ending i
    }

    printf("Considering element A[%d] = %d\n", i, A[i]);
    printf("LIS ending at A[%d] is of length %d: ", i, pos+1);
    printf("[");
    print_LIS(i);
    printf("]\n");
    print_array("L is now", L, k);
    printf("\n");
  }

  printf("Final LIS is of length %d: ", k);
  print_LIS(lis_end); printf("\n");

  assert(LIS(n-1) == k);                         // both must be identical
  return 0;
}
```

4.3 Max 1D range sum

```
// Max 1D Range Sum solved with Jay Kadane O(n).
// inside int main()
    int n = 9;
    int A[] = { 4, -5, 4, -3, 4, 4, -4, 4, -5 }; // a sample array A
    int sum = 0;
    int ans = 0; // important, 'ans' must be initialized to 0
    for (int i = 0; i < n; ++i)
    {
        sum += A[i];
        ans = max(ans, sum);
        if (sum < 0) sum = 0;
    }
    printf("Max 1D Range Sum = %d\n", ans);
```

5 Math algorithms

5.1 Chinese remainder theorem

```
#include <bits/stdc++.h>
#define qtr ios::sync_with_stdio(0); cin.tie(0);
#define endl '\n'
#define int long long
#define MOD 1000000
using namespace std;

int inv(int a, int m){
    int m0 = m, t, q;
    int x0 = 0, x1 = 1;
    if(m == 1){
        return 0;
    }
    while(a > 1){
        q = a/m;
        t = m;
        m = a%m, a = t;
        t = x0;
        x0 = x1 - q * x0;
        x1 = t;
    }
    if(x1 < 0){
        x1 += m0;
    }
    return x1;
}

int findMinX(vector<int> num, vector<int> rem, int k){
    int prod = 1;
    for(int i = 0; i < k; i++) prod *= num[i];
    int result = 0;
    for(int i = 0; i < k; i++){
        int pp = prod / num[i];
        result += rem[i] * inv(pp, num[i]) * pp;
    }
    return result % prod;
}

int32_t main() { //qtr
    int n = 3;
    vector<int> rem, factor;
    rem.resize(n);
    factor.resize(n);
    for(int i = 0; i < n; i++){
        cin >> factor[i];
    }
    for(int i = 0; i < n; i++){
        cin >> rem[i];
    }
    cout << findMinX(factor, rem, n) << endl;
}
```

5.2 Extended greatest common divisor (Ext-GCD)

```
// ax mod b = 1
// ax + by = 1,x=y=0
// a,b Relatively Prime
```

```
LL exgcd(LL a,LL b,LL &x,LL &y){
  if(b){
    LL tmd=exgcd(b,a%b,y,x);
    y-=a/b*x;
    return tmd;
  }
  x=1,y=0;
  return a;
}
```

5.3 Greatest common divisor (GCD) and least common multiple (LCM)

```
// or __gcd(a, b) in gcc
int gcd(int a, int b){
        return a%b?gcd(b,a%b):b;
}
int lcm(int a, int b){
        return a*b/gcd(a,b);
}
```

5.4 Generate list of prime numbers

```
// Generate list of prime numbers using Sieve of Eratosthenes.
ll _sieve_size;
bitset<10000010> bs; // [b]it [s]et 10^7 should be enough for most cases
vi primes;    // compact list of primes

void sieve(ll upperbound)    // create list of primes in [0..upperbound]
{
    _sieve_size = upperbound + 1; // add 1 to include upperbound
    bs.set();                     // set all bits to 1
    bs[0] = bs[1] = 0;            // exception index 0 and 1
    for (ll i = 2; i <= _sieve_size; ++i)
        if (bs[i])
        {   // cross out multiples of i starting from i * i!
            for (ll j = i * i; j <= _sieve_size; j += i) bs[j] = 0;
            primes.push_back( (int)i );
        }
}

bool isPrime(ll N)    // a good enough deterministic prime tester
{
    if (N <= _sieve_size) return bs[N];    // O(1) for small primes
    for (int i = 0; i < (int)primes.size(); ++i)
        if (N % primes[i] == 0) return false;
    return true;    // it takes longer if N is a large prime!
}                   // note: only work for N <= (last prime in vi 'primes')^2

// inside int main()
    sieve(10000000);    // can go up to 10^7 (need few seconds)
    printf("%d\n", isPrime(2147483647));     // 10-digit prime
    printf("%d\n", isPrime(136117223861LL)); // not a prime, 104729 * 1299709
```

5.5 N choose R combination (nCr)

```
#define MAXN 100
long long nCr[MAXN+5][MAXN+5];
// nCr[i][j] = \\(C_{n}^r\\)
void build_nCr() {
  for(int i = 1; i < MAXN+5; i++) {
    for(int j = 1; j < MAXN+5; j++) {
      if(i == j)
        nCr[i][j] = 1;
      else if(i > j)
        nCr[i][j] = nCr[i-1][j] * i / (i-j);
    }
  }
}
```

5.6 Stirling's approximation

```
double Stirling(int n){
        return (0.5*log(2.0*acos(-1.0)*n)+n*log(n+0.0)-n)/log(10.0);
}// n! Digits
```

6 String algorithms

6.1 KnuthMorrisPratt algorithm

```
#include <iostream>
#include <bits/stdc++.h>
#define LOCAL
#define MAXN 100020
using namespace std;
string strA , strB ;
int b[MAXN] , p[MAXN] ;

void kmp_process(){
    int n = strB.length() ,i = 0 ,  j = -1 ;
    b[0] = -1 ;
    while(i < n ){
        while(j >= 0 && strB[i] != strB[j]) j = b[j] ;
        i++ ; j++ ;
        b[i] = j ;
    }

    //debug
//    for(int k = 0 ; k <= n ; k++)
//        cout << b[k] << ' ' ;
//    cout << '\n' ;
}

int kmp(){
    int n = strA.length() , m=strB.length() , i=0 , j=0  ;
    while(i < n ){
        while(j >= 0 && strA[i] != strB[j]) j = b[j] ;
        i++ ; j++ ;
    }
    return j ;
}


int main()
{
#ifdef LOCAL
    freopen("in1.txt" , "r" , stdin );
#endif // LOCAL

    while(cin >> strA){
        strB = strA;
        reverse(strB.begin() , strB.end());
        kmp_process();
        int n = kmp() ;
        cout << strA << strB.substr(n) << '\n' ;
    }
    return 0;
}
```

6.2 Longest palindromic substring

```
#include <iostream>
#include <bits/stdc++.h>
#define LOCAL
#define MAXN 1010
using namespace std;
int dp[MAXN][MAXN] = {} ;
string strA , strB ;
int n , m ;

int lcs(){
    n = strA.length();
    m = strB.length();
    for(int i = 0 ; i <= n ; i++) dp[i][0] = 0 ;
    for(int j = 0 ; j <= m ; j++) dp[j][0] = 0 ;
    for(int i = 1 ; i <= n ; i++){
        for(int j = 1 ; j <= m ; j++){
            if(strA[i-1] == strB[j-1]) dp[i][j] = dp[i-1][j-1]+1 ;
            else dp[i][j] = max(dp[i-1][j] , dp[i][j-1]);
        }
    }
    return dp[n][m] ;
}

int main()
{
#ifdef LOCAL
    freopen("in1.txt" , "r" , stdin );
#endif // LOCAL
    int t ;
    cin >> t ;
    cin.ignore();
    while(t--){
        getline(cin,strA);
        strB = strA ;
        reverse(strB.begin() , strB.end());
        cout << lcs() << ’\n’ ;
    }

    return 0;
}
```

6.3 Minimum edit distance

```
#include <iostream>
#include <bits/stdc++.h>
#define LOCAL
#define MAXN 100
using namespace std;
string strA , strB ;
int dis[MAXN][MAXN] , back_table[MAXN][MAXN] ;
int cnt , m , n  ;

void backtracking(int i , int j ){
    if(i==0 || j==0){
        while( i > 0 ){
            cout << cnt++ << " Delete " << i << ’\n’ ;
            i--;
        }
        while( j > 0){
            cout << cnt++ << " Insert " << i+1 << "," << strB[j-1] << ’\n’ ;
            j-- ;
        }
        return ;
    }

    if(strA[i-1] == strB[j-1])
        backtracking(i-1,j-1);
    else{
        if(dis[i][j] == dis[i-1][j-1]+1){
            cout << cnt++ << " Replace " << i << "," << strB[j-1] << ’\n’ ;
            backtracking(i-1,j-1);
        }
        else if(dis[i][j] == dis[i-1][j]+1){
            cout << cnt++ << " Delete " << i << ’\n’ ;
            backtracking(i-1,j) ;
        }
        else if(dis[i][j] == dis[i][j-1]+1){
            cout << cnt++ << " Insert " << i+1 << "," << strB[j-1] <<’\n’  ;
            backtracking(i,j-1);
        }

    }

}

void med(){ //Minimum Edit Distance
    for(int i = 0 ; i <= n ; i++) dis[i][0] = i ;
    for(int j = 0 ; j <= m ; j++) dis[0][j] = j ;
    for(int i = 1 ; i <= n ; i++){
        for(int j = 1 ; j <= m ; j++){
            if(strA[i-1] == strB[j-1]) dis[i][j] = dis[i-1][j-1] ;
            else dis[i][j] = min(dis[i-1][j-1], min(dis[i-1][j] , dis[i][j-1]))+1;
        }
    }
}


int main()
{
#ifdef LOCAL
    freopen("in1.txt" , "r" , stdin );
    freopen("out.txt" , "w" , stdout);
#endif // LOCAL
    cin.tie(0);
    cout.tie(0);
    ios::sync_with_stdio(false);
    int flag = 0 ;
    while(getline(cin ,strA) && getline(cin , strB)){
        n=strA.length() ;
        m=strB.length() ;
        cnt = 1 ;
        med();
        if(flag) cout << ’\n’ ;
        flag = 1 ;
        cout << dis[n][m] << ’\n’ ;
        backtracking(n,m);
    }
    return 0;
}
```

6.4 Z-algorithm

```
#include <iostream>
#include <bits/stdc++.h>
#define LOCAL
#define MAXN 1000020
using namespace std;
int z[MAXN] = {} ;
int x=0 , y=0 , maxn = 0;
string s ;


int main()
{
#ifdef LOCAL
    freopen("in1.txt","r",stdin);
#endif // LOCAL

    cin >> s ;
    for(int i = 1 ; i < s.length() ; i++ ){
        z[i] = max(0,min(z[i-x] , y - i + 1));
        while(i + z[i] < s.length() && s[z[i]] == s[i+z[i]] ){
            x = i ;
            y = i + z[i] ;
            z[i]++ ;
        }
    }

    for(int i = 0 ; i < s.length() ; i++)
        if(z[i] == s.length() - i && maxn >= s.length()-i ){
            cout << s.substr(0,z[i]);
            return 0 ;
        }
        maxn = max(maxn , z[i]);
    }
    cout << "Just a legend" ;
    return 0;
}
```

7 Data structures

7.1 Union-find disjoint sets (UFDS) by David

```
#include <iostream>
#include <bits/stdc++.h>
#define LOCAL

using namespace std;
int intSum[200080] , intParent[200080] , intSet[200080] ;


int find_root(int intA){
    if(intParent[intA] == intA)
        return intA ;
    intParent[intA] = find_root(intParent[intA]) ;
    return intParent[intA] ;
}


int each_debug(int n ){
    for(int i = 1 ; i <= n ; i++){
        cout << i << ’ ’ << intParent[i] << ’ ’  \
            << intSet[find_root(i)] << ’ ’ << intSum[find_root(i)] << ’\n’ ;
    }
    system("Pause") ;
}


int main()
```

```
{
#ifdef LOCAL
    freopen("in1.txt","r", stdin);
    freopen("out.txt" ,"w" , stdout) ;
#endif // LOCAL

    int n, m , operation , p , q  ;
    while(cin >> n >> m){
        for(int i = 1 ; i <= n ; i++){
            intParent[i] = i+n ;
            intParent[i+n] = i+n ;
            intSum[i+n] = i;
            intSet[i+n] = 1 ;
        }
        while(m--){
            cin >> operation ;
            if(operation == 1 ){
                cin >> p >> q ;
                int intRoot_p , intRoot_q ;
                intRoot_p = find_root(intParent[p]) ;
                intRoot_q = find_root(intParent[q]) ;
                if(intRoot_p != intRoot_q){
                    intParent[intRoot_q] = intRoot_p ;
                    intSum[intRoot_p] += intSum[intRoot_q] ;
                    intSet[intRoot_p] += intSet[intRoot_q] ;
                }
                //debug
                //each_debug(n) ;

            }
            else if (operation == 2 ){
                cin >> p >> q ;
                int intRoot_p , intRoot_q ;
                intRoot_p = find_root(intParent[p]) ;
                intRoot_q = find_root(intParent[q]) ;
                if(intRoot_p != intRoot_q){
                    intParent[p] = intRoot_q ;
                    intSum[intRoot_q] += p ;
                    intSum[intRoot_p] -= p ;
                    intSet[intRoot_q] ++ ;
                    intSet[intRoot_p] -- ;
                }
                //debug
                //each_debug(n) ;
            }
            else if (operation == 3){
                cin >> p ;
                cout << intSet[find_root(p)] << ’ ’ << intSum[find_root(p)] << ’\n’ ;
            }
        }
    }

    return 0;
}
```

7.2 Binary indexed/fenwick tree (BIT)

```
#include <iostream>
using namespace std;

#define LOGSZ 17

int tree[(1<<LOGSZ)+1];
int N = (1<<LOGSZ);

// add v to value at x
void set(int x, int v) {
  while(x <= N) {
    tree[x] += v;
    x += (x & -x);
  }
}

// get cumulative sum up to and including x
int get(int x) {
  int res = 0;
  while(x) {
    res += tree[x];
    x -= (x & -x);
  }
  return res;
}

// get largest value with cumulative sum less than or equal to x;
// for smallest, pass x-1 and add 1 to result
int getind(int x) {
  int idx = 0, mask = N;
  while(mask && idx < N) {
    int t = idx + mask;
    if(x >= tree[t]) {
      idx = t;
      x -= tree[t];
    }
    mask >>= 1;
  }
  return idx;
}
```

7.3 Rope

```
#include <iostream>
#include <bits/stdc++.h>
#include <ext/rope>
#define LOCAL
#define MAXN 50020
using namespace std;
using namespace __gnu_cxx ;

int main()
{
#ifdef LOCAL
    freopen("in1.txt" , "r" , stdin );
#endif // LOCAL
    int n , t , a , b , c , d=0 ;
    int v = 0 ;
    string strA ;
    rope<char> r[MAXN] , rtmp ;
    cin >> n ;
    while(n--){
        cin >> t ;

        if(t==1){
            cin >> a ;
            cin >> strA ;
            a -= d ;
            r[++v] = r[v] ;
            r[v].insert(a,strA.c_str());
            //debug
            //cout << r[v] << ’\n’ ;
        }
        else if(t==2){
            cin >> a >> b ;
            a -= d ; b -= d ;
            r[++v] = r[v] ;
            r[v].erase(a-1,b);
            //debug
            //cout << r[v] << ’ ’ << r[v-1] << ’\n’ ;
        }
        else if(t==3){
            cin >> a >> b >> c ;
            a -= d ; b -= d ; c -= d ;
            rtmp = r[a].substr(b-1,c) ;
            cout << rtmp << ’\n’ ;
            d += count(rtmp.begin() , rtmp.end() , ’c’ );
        }
    }
    return 0;
}
```

7.4 Segment tree

```
#include <iostream>
#include <bits/stdc++.h>
#include <string>
#define LOCAL
#define Lson(x) ((x << 1) +1)
#define Rson(x) ((x << 1) +2)
#define INF 99999999
using namespace std;
const int N = 100005 ;
int shift[35] , num[N] , len_shift ;
string strLine ;


struct Node{
    int left , right , Min_Value  ;
}node[4 * N ];


void build(int left , int right , int x = 0 ){
    node[x].left = left ;
    node[x].right = right ;
    if(left == right){
        node[x].Min_Value = num[left] ;
        return ;
    }
    int mid = (left + right ) / 2 ;

    //debug
    //cout << mid << '\n' ;
    //cout << x << ' ' << node[x].left << ' ' << node[x].right << ' ' << '\n' ;

    build(left , mid , Lson(x)) ;
    build(mid + 1 , right , Rson(x)) ;
    node[x].Min_Value = min(node[Lson(x)].Min_Value  , node[Rson(x)].Min_Value ) ;
}


void handle(){
    len_shift = 0 ;
    shift[len_shift] = 0;
    for(int i = 6 ; i < strLine.length() ; i++){
        if(strLine[i] >= '0' && strLine[i] <= '9' ){
            shift[len_shift] = shift[len_shift] * 10 + (int) (strLine[i] - '0' ) ;
        }
        else{
            shift[++len_shift ] = 0 ;
        }
    }
    //finaly char is ')' , so len_shift is right
    sort(shift , shift + len_shift ) ;

    //debug
    /**<
    for(int i = 0 ; i < len_shift ; i++)
        cout << shift[i] << ' ' ;
    cout << '\n' ;
    */
}


int query(int left , int right , int x = 0 ){
    if(node[x].left >= left && node[x].right <= right)
        return node[x].Min_Value ;
    int mid = (node[x].left + node[x].right ) / 2 ;
    int ans = INF ;

    //debug
    //cout << x << ' ' << node[x].left << ' ' << node[x].right << ' ' << node[x].Min_Value << '\n' ;

    if( left <= mid )
        ans = min(ans , query(left , right , Lson(x))) ;
    if(mid < right )
        ans = min(ans , query(left , right , Rson(x))) ;
    return ans ;
}


void set_num(int position , int value , int x = 0 ){
    if(node[x].left == position && node[x].right == position ){
       node[x].Min_Value = value ;
       return ;
    }
    int mid = (node[x].left + node[x].right ) / 2 ;
    if(position <= mid )
        set_num(position , value , Lson(x) );
    if(mid < position )
        set_num(position , value , Rson(x)) ;
    node[x].Min_Value = min(node[Lson(x)].Min_Value , node[Rson(x)].Min_Value );

}


int main()
{
    int n , q , intTemp ;
    ios::sync_with_stdio(0);
#ifdef LOCAL
    freopen("out.txt" , "w" , stdout ) ;
    freopen("in1.txt" , "r" , stdin ) ;
#endif // LOCAL
    cin >> n >> q ;
    for(int i = 1 ; i <= n ; i++)
        cin >> num[i] ;
    build(1,n);

    //debug
    /**<
    for(int i = 0 ; i < 13 ; i++){
        cout << node[i].left << ' ' << node[i].right << ' ' << node[i].Min_Value << '\n' ;
    }
    return 0 ;
    */

    while(q--){
        cin >> strLine ;
        if(strLine[0] == 'q'){
            handle();
            cout << query(shift[0] , shift[1] ) << '\n' ;
        }
        else if (strLine[0] == 's'){
            handle();
            intTemp = num[shift[0]] ;

            for(int i = 1 ; i < len_shift ; i++){
                set_num(shift[i-1] , num[shift[i]]) ;
                num[shift[i-1]] = num[shift[i]] ;
            }
            num[shift[len_shift-1]] = intTemp ;
            set_num(shift[len_shift-1] , intTemp );

            //debug
            //cout << intTemp << ' ' << shift[len_shift-1] << '\n' ;
            //for(int i = 1 ; i <= n ; i++)
             //   cout << num[i] << ' ' ;
        }
    }
    return 0;
}
```

7.5 Union-find disjoint sets (UFDS) by Bill

```
class UnionFind
{
public:
    UnionFind(int N)
    {
        rank.assign(N, 0);
        p.assign(N, 0);
        for (int i = 0; i < N; ++i) p[i] = i;
    }
    int findSet(int i) { return (p[i] == i) ? i : ( p[i] = findSet(p[i]) ); }
    bool isSameSet(int i, int j) { return findSet(i) == findSet(j); }
    void unionSet(int i, int j)
    {
        if ( !isSameSet(i, j) )
        {
            int x = findSet(i);
            int y = findSet(j);
            if (rank[x] > rank[y]) p[y] = x;    // rank keeps the tree short
            else
            {
                p[x] = y;
                if (rank[x] == rank[y]) ++rank[y];
            }
        }
    }
private:
    vi p, rank;
};
```

8 Utilities

8.1 Bit manipulation

```
#define isOn(S, j) (S & (1<<j))
#define setBit(S, j) (S |= (1<<j))
#define clearBit(S, j) (S &= ~(1<<j))
#define toggleBit(S, j) (S ^= (1<<j))
#define lowBit(S) (S & (-S))
#define setAll(S, n) (S = (1<<n)-1)
```

8.2 C++ input output

```
#include <iostream>
#include <iomanip>

using namespace std;
```

```
int main()
{
    // Ouput a specific number of digits past the decimal point,
    // in this case 5
    cout.setf(ios::fixed); cout << setprecision(5);
    cout << 100.0/7.0 << endl;
    cout.unsetf(ios::fixed);

    // Output the decimal point and trailing zeros
    cout.setf(ios::showpoint);
    cout << 100.0 << endl;
    cout.unsetf(ios::showpoint);

    // Output a '+' before positive values
    cout.setf(ios::showpos);
    cout << 100 << " " << -100 << endl;
    cout.unsetf(ios::showpos);

    // Output numerical values in hexadecimal
    cout << hex << 100 << " " << 1000 << " " << 10000 << dec << endl;
}
```

8.3 C++ STL

```
// Example for using stringstreams and next_permutation

#include <algorithm>
#include <iostream>
#include <sstream>
#include <vector>

using namespace std;

int main(void){
  vector<int> v;

  v.push_back(1);
  v.push_back(2);
  v.push_back(3);
  v.push_back(4);

  // Expected output: 1 2 3 4
  //                  1 2 4 3
  //                  ...
  //                  4 3 2 1
  do {
    ostringstream oss;
    oss << v[0] << " " << v[1] << " " << v[2] << " " << v[3];

    // for input from a string s,
    //   istringstream iss(s);
    //   iss >> variable;

    cout << oss.str() << endl;
  } while (next_permutation (v.begin(), v.end()));

  v.clear();

  v.push_back(1);
  v.push_back(2);
  v.push_back(1);
  v.push_back(3);

  // To use unique, first sort numbers.  Then call
  // unique to place all the unique elements at the beginning
  // of the vector, and then use erase to remove the duplicate
  // elements.

  sort(v.begin(), v.end());
  v.erase(unique(v.begin(), v.end()), v.end());

  // Expected output: 1 2 3
  for (size_t i = 0; i < v.size(); i++)
    cout << v[i] << " ";
  cout << endl;
}
```

8.4 Dates

```
// Routines for performing computations on dates.  In these routines,
// months are expressed as integers from 1 to 12, days are expressed
// as integers from 1 to 31, and years are expressed as 4-digit
// integers.
```

```
#include <iostream>
#include <string>

using namespace std;

string dayOfWeek[] = {"Mon", "Tue", "Wed", "Thu", "Fri", "Sat", "Sun"};

// converts Gregorian date to integer (Julian day number)
int dateToInt (int m, int d, int y){
  return
    1461 * (y + 4800 + (m - 14) / 12) / 4 +
    367 * (m - 2 - (m - 14) / 12 * 12) / 12 -
    3 * ((y + 4900 + (m - 14) / 12) / 100) / 4 +
    d - 32075;
}

// converts integer (Julian day number) to Gregorian date: month/day/year
void intToDate (int jd, int &m, int &d, int &y){
  int x, n, i, j;

  x = jd + 68569;
  n = 4 * x / 146097;
  x -= (146097 * n + 3) / 4;
  i = (4000 * (x + 1)) / 1461001;
  x -= 1461 * i / 4 - 31;
  j = 80 * x / 2447;
  d = x - 2447 * j / 80;
  x = j / 11;
  m = j + 2 - 12 * x;
  y = 100 * (n - 49) + i + x;
}

// converts integer (Julian day number) to day of week
string intToDay (int jd){
  return dayOfWeek[jd % 7];
}

int main (int argc, char **argv){
  int jd = dateToInt (3, 24, 2004);
  int m, d, y;
  intToDate (jd, m, d, y);
  string day = intToDay (jd);

  // expected output:
  //    2453089
  //    3/24/2004
  //    Wed
  cout << jd << endl
    << m << "/" << d << "/" << y << endl
    << day << endl;
}
```

8.5 Prime numbers

```
// O(sqrt(x)) Exhaustive Primality Test
#include <cmath>
#define EPS 1e-7
typedef long long LL;
bool IsPrimeSlow (LL x)
{
  if(x<=1) return false;
  if(x<=3) return true;
  if (!(x%2) || !(x%3)) return false;
  LL s=(LL)(sqrt((double)(x))+EPS);
  for(LL i=5;i<=s;i+=6)
  {
    if (!(x%i) || !(x%(i+2))) return false;
  }
  return true;
}
// Primes less than 1000:
//      2     3     5     7    11    13    17    19    23    29    31    37
//     41    43    47    53    59    61    67    71    73    79    83    89
//     97   101   103   107   109   113   127   131   137   139   149   151
//    157   163   167   173   179   181   191   193   197   199   211   223
//    227   229   233   239   241   251   257   263   269   271   277   281
//    283   293   307   311   313   317   331   337   347   349   353   359
//    367   373   379   383   389   397   401   409   419   421   431   433
//    439   443   449   457   461   463   467   479   487   491   499   503
//    509   521   523   541   547   557   563   569   571   577   587   593
//    599   601   607   613   617   619   631   641   643   647   653   659
//    661   673   677   683   691   701   709   719   727   733   739   743
//    751   757   761   769   773   787   797   809   811   821   823   827
//    829   839   853   857   859   863   877   881   883   887   907   911
//    919   929   937   941   947   953   967   971   977   983   991   997

// Other primes:
```

```
//    The largest prime smaller than 10 is 7.
//    The largest prime smaller than 100 is 97.
//    The largest prime smaller than 1000 is 997.
//    The largest prime smaller than 10000 is 9973.
//    The largest prime smaller than 100000 is 99991.
//    The largest prime smaller than 1000000 is 999983.
//    The largest prime smaller than 10000000 is 9999991.
//    The largest prime smaller than 100000000 is 99999989.
//    The largest prime smaller than 1000000000 is 999999937.
//    The largest prime smaller than 10000000000 is 9999999967.
//    The largest prime smaller than 100000000000 is 99999999977.
//    The largest prime smaller than 1000000000000 is 999999999989.
//    The largest prime smaller than 10000000000000 is 9999999999971.
//    The largest prime smaller than 100000000000000 is 99999999999973.
//    The largest prime smaller than 1000000000000000 is 999999999999989.
//    The largest prime smaller than 10000000000000000 is 9999999999999937.
//    The largest prime smaller than 100000000000000000 is 99999999999999997.
//    The largest prime smaller than 1000000000000000000 is 999999999999999989.
```

8.6 Theorems

Euler path/tour theorems: An Euler path is a path that visits every edges exactly once. An Euler tour is an Euler path that starts and ends at the same vertex. A graph is an Eulerian-tour graph (i.e. it has an Euler tour) iff all of its vertices has even degrees. A graph is an Eulerian-path graph (i.e. it has an Euler path) iff all but 2 of its vertices has even degrees.

Euler’s handshaking lemma: A graph does not have an Euler tour iff it has an even number of vertices of odd degrees.

Bipartite graph related theorems:
(1) Min vertex cover (MVC) = Max cardinality bipartite matching (MCBM).
(2) Max independent set (MIS) = V - MCBM.
(3) The number of spanning tree of a complete bipartite graph K(n,m) is m^(n-1) * n^(m-1).

Cayley’s formula: There are n^(n-2) spanning trees of a complete graph with n labeled vertices.

Derangement: A permutation of the elements of a set such that none of the elements appear in their original position. The number of derangements ’der(n)’ can be computed as follow: der(n) = (n-1) * (der(n-1) + der(n-2)) where der(0) = 1 and der(1) = 0.

Erdos Gallai’s theorem: A necessary and sufficient condition for a finite sequence of natural numbers is the degree sequence of a simple graph. A sequence of non-negative integers d_1 >= d_2 >= ... >= d_n can be the degree sequence of a simple graph on n vertices iff
(1) sum[i: 1->n]{d_i} is even, and
(2) sum[i: 1->k]{d_i} <= k * (k-1) + sum[i: k+1->n]{min(d_i, k)} holds for 1 <= k <= n.