

NTUT_Knights ICPC Team Notebook

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1 Advanced algorithms

1.1 2-SAT problem

```
// 2-SAT Problem demonstrated with 2018 ICPC Korea Regional - Problem K.
#include <bits/stdc++.h>

using namespace std;

#define LOCAL
#define blue(k) (k<<1)
#define red(k) (blue(k) + 1)
#define UNVISITED -1
#define neg(v) (v ^ 1) // [negation of

typedef vector<int> vi;

int K, N;
int V;
vector<vi> AL;
bool possible = true;
vi sccNum;

int getVertex(pair<int, char> p)
{
    return p.second == 'B' ? blue(p.first) : red(p.first);
}

pair<int, char> negation(pair<int, char> p)
{
    return make_pair(p.first, p.second == 'B' ? 'R' : 'B');
}

void createEdge(pair<int, char> p, pair<int, char> q)
{
    int u, v;
    u = getVertex( negation(p) );
    v = getVertex( q );
    // printf("%d->%d\n", u, v);
    AL[u].push_back(v);

    u = getVertex( negation(q) );
    v = getVertex( p );
    // printf("%d->%d\n", u, v);
    AL[u].push_back(v);
}

vi dfs_num, dfs_low, visited;
int dfsNumberCounter, numSCC;
vi S;

void tarjanSCC(int u)
{
    dfs_low[u] = dfs_num[u] = dfsNumberCounter++;
    S.push_back(u);
    visited[u] = 1;
    for (int i = 0; i < (int)AL[u].size(); ++i)
    {
        int v = AL[u][i];
        if (dfs_num[v] == UNVISITED) tarjanSCC(v);
        if (visited[v]) dfs_low[u] = min( dfs_low[u], dfs_low[v] );
    }

    if (dfs_low[u] == dfs_num[u])
    {
        set<int> st;
        ++numSCC;
        // printf("SCC %d:", numSCC);
        while (true)
        {
            int v = S.back(); S.pop_back();
            visited[v] = 0;
            if ( st.find(neg(v)) != st.end() ) possible = false;
            st.insert(v);
        }

        sccNum[v] = numSCC; // Tarjan produces SCCs in reversed topo order
        // printf(" %d", v);
        if (u==v) break;
    }
    // printf("\n");
}
```

```

    }
}

void work()
{
    sccNum.assign(V, 0);
    dfs_num.assign(V, UNVISITED);
    dfs_low.assign(V, 0);
    visited.assign(V, 0);
    dfsNumberCounter = numSCC = 0;
    for (int u = 0; u < V; ++u)
        if (dfs_num[u] == UNVISITED)
            tarjanSCC(u);

    if (!possible)
    {
        printf("-1\n");
        return;
    }

    for (int i = 1; i <= K; ++i)
        printf("%c", sccNum[blue(i)] > sccNum[red(i)] ? 'R' : 'B'); // 2-SAT assignment based
    printf("\n"); // on reversed topo order
}

int main()
{
    #ifdef LOCAL
    freopen("in.txt", "r", stdin);
    #endif // LOCAL
    scanf("%d %d", &K, &N);
    V = 2 * K + 2;
    AL.assign(V, vi());
    for (int i = 0; i < N; ++i)
    {
        pair<int, char> a[3];
        for (int j = 0; j < 3; ++j) scanf("%d %c", &a[j].first, &a[j].second);
        for (int j = 0; j < 3; ++j) printf("%d %c ", a[j].first, a[j].second);
        printf("\n");
        createEdge(a[0], a[1]);
        createEdge(a[0], a[2]);
        createEdge(a[1], a[2]);
    }

    work();

    return 0;
}

```

1.2 Closest pair problem

```

// UVa 10245 - The Closest Pair Problem solved in O(n log n).
#include <bits/stdc++.h>

using namespace std;

#define LOCAL
#define MAX_N 10050
#define INF 1000000000

typedef pair<double, double> dd;
typedef vector<dd> vdd;

dd a[MAX_N];

double dist(dd i, dd j)
{
    return sqrt( pow(i.first - j.first, 2.f) + pow(i.second - j.second, 2.f) );
}

double closest(int lo, int hi, vdd& y_sort)
{
    if (lo > hi) return INF;
    if (lo == hi)
    {
        y_sort.push_back(a[lo]);
        return INF;
    }

    // divide & conquer
    int mid = (lo + hi) / 2;
    vdd ys_o, ys_t;
    double d1 = closest(lo, mid, ys_o);
    double d2 = closest(mid + 1, hi, ys_t);

    // merge sort
    int N_O = (int)ys_o.size();

```

```

    int N_T = (int)ys_t.size();
    int i = 0;
    int j = 0;
    while (true)
    {
        if (i >= N_O && j >= N_T) break;
        if (i >= N_O)
        {
            y_sort.push_back(ys_t[j++]);
            continue;
        }
        if (j >= N_T)
        {
            y_sort.push_back(ys_o[i++]);
            continue;
        }
        if ( ys_o[i].second < ys_t[j].second
            || (ys_o[i].second == ys_t[j].second && ys_o[i].first < ys_t[j].first) ) y_sort.push_back(ys_o[i++]);
        else y_sort.push_back(ys_t[j++]);
    }
    for (int i = 0; i < (int)y_sort.size(); ++i)
        printf("%lf %lf\n", y_sort[i].first, y_sort[i].second);
    printf("\n");

    // retrieve d3 to combine
    if (lo + 1 == hi) return dist(a[lo], a[hi]);

    double d = min(d1, d2);
    double x_left = a[mid].first - d;
    double x_right = a[mid].first + d;
    vdd b;
    for (int i = 0; i < (int)y_sort.size(); ++i)
        if (x_left <= y_sort[i].first && y_sort[i].first <= x_right) b.push_back(y_sort[i]);

    double ret = d;
    for (int i = 1; i < (int)b.size(); ++i)
        for (int j = max(0, i - 15); j < i; ++j)
            ret = min( ret, dist(b[i], b[j]) );

    return ret;
}

int main()
{
    #ifdef LOCAL
    freopen("in.txt", "r", stdin);
    #endif // LOCAL
    int N;
    while (scanf("%d", &N), N)
    {
        for (int i = 0; i < N; ++i) scanf("%lf %lf", &a[i].first, &a[i].second);
        sort(a, a + N);
        for (int i = 0; i < N; ++i) printf("%lf %lf\n", a[i].first, a[i].second);
        double ret = closest(0, N - 1, y_sort);
        if (ret < 10000) printf("%.4lf\n", ret);
        else printf("INFINITY\n");
    }
    return 0;
}

```

1.3 Iterative deepening A* (IDA*)

```

// UVa 10181 - 15-Puzzle Problem solved with Iterative Deepening A* (IDA*).
#include <bits/stdc++.h>

using namespace std;

// #define LOCAL
#define N 4 // #rows/columns
#define B 15 // [B]lank tile id
#define PUZZLE (N*N)
#define MAX_STEPS 45 // given by the problem description
#define DIR 4 // 4 [DIR]ections

int dr[DIR] = {0, -1, 0, 1}; // must be right, up, left, down
int dc[DIR] = {1, 0, -1, 0}; // for the XOR operation to work
char dm[] = "RULD"; // [d]irection [m]ove

int p[PUZZLE];
int b_init_pos; // [b]lank [init]ial [pos]ition
int lim; // current [lim]it of the Iterative Deepening Search (IDS)
int pred[MAX_STEPS]; // [pre]viously used [d]irection to go to the current state

bool isViable()
{

```

```

int sum;
for (int i = 0; i < PUZZLE; ++i)
    for (int j = 0; j < i; ++j)
        if (p[j] > p[i]) ++sum;
sum += b_init_pos / N + b_init_pos % N;
sum -= B / N + B % N;
return sum % 2 == 0;
}

int H()
{
    int h = 0;
    for (int pos = 0; pos < PUZZLE; ++pos) // for all tile 'p[pos]'
    { // compute Manhattan distance to goal state
        if (p[pos] == B) continue;
        h += abs( p[pos] / N - pos / N ) // position of 'p[pos]' in goal state is 'p[pos]'
            + abs( p[pos] % N - pos % N ); // position of 'p[pos]' in current state is 'pos'
    }
    return h;
}

bool isValid(int r, int c)
{
    return 0 <= r && r < N && 0 <= c && c < N;
}

int Delta_H(int cur_r, int cur_c, int next_r, int next_c)
{
    int val = p[cur_r * N + cur_c]; // [val]ue of the tile being moved into the blank tile position
    int goal_r = val / N;           // position of 'val' in goal state is 'val'
    int goal_c = val % N;           // get row & column representation of the position
    return - ( abs(goal_r - cur_r ) + abs(goal_c - cur_c ) )
        + ( abs(goal_r - next_r ) + abs(goal_c - next_c ) );
}

bool dfs(int g, int h, int b_pos)
{
    if (g + h > lim) return false;
    if (h == 0) return true; // found a solution!
    int r = b_pos / N;
    int c = b_pos % N;
    for (int d = 0; d < DIR; ++d)
    {
        if ( g != 0 && d == (pred[g] ^ 2) ) continue; // this direction gets us back to parent state
        int next_r = r + dr[d];
        int next_c = c + dc[d];
        if ( !isValid(next_r, next_c) ) continue;
        int next_h = h + Delta_H(next_r, next_c, r, c); // O(1)
        int b_next_pos = next_r * N + next_c;
        swap(p[b_pos], p[b_next_pos]);
        pred[g+1] = d;

        if ( dfs(g + 1, next_h, b_next_pos) ) return true;

        swap(p[b_pos], p[b_next_pos]);
    }
    return false;
}

int ida_star()
{
    int init_h = H();
    lim = init_h;
    while (lim <= MAX_STEPS)
    {
        if ( dfs(0, init_h, b_init_pos) ) return lim;
        ++lim;
    }
    return -1;
}

void output(int steps)
{
    for (int i = 1; i <= steps; ++i)
        printf("%c", dm[ pred[i] ]);
}

int main()
{
#ifdef LOCAL
    freopen("in.txt", "r", stdin);
#endif // LOCAL
    int T;
    scanf("%d", &T);
    while (T--)
    {
        for (int i = 0; i < N; ++i)
            for (int j = 0; j < N; ++j)
            {
                int pos = i * N + j;
                scanf("%d", &p[pos]);
                if (p[pos] == 0) p[pos] = B, b_init_pos = pos; // goal state 'p' is 0, 1, 2..14, 15
            }
    }
}

```

```

        else --p[pos]; // blank tile as 15
    }
    if ( !isViable() ) // must-consider condition otherwise TLE
    {
        printf("This puzzle is not solvable.\n");
        continue;
    }
    int ret = ida_star();
    if (ret == -1)
    {
        printf("This puzzle is not solvable.\n");
        continue;
    }
    output(ret), printf("\n");
}
return 0;
}

```

1.4 Lowest common ancestor (LCA)

```

// UVa 12238 - Ants Colony solved with Lowest Common Ancestor (LCA) using Range
// Minimum Query (RMQ) reduction and Spare Table data structure.
#include <bits/stdc++.h>

using namespace std;

#define LOCAL
#define root 0
#define MAX_N 100050
#define MAX_AN (2*MAX_N)
#define LOG_TWO_AN 20

typedef long long int ll;
typedef pair<int, ll> il;
typedef vector<ll> vil;

int N;
vector<vil > CH; // [CH]ildren
ll dist[MAX_N];
int a[MAX_AN]; // RMQ [a]rray
int a_N;
int a2t[MAX_AN]; // RMQ [a]rray index -> [t]ree index
int t2a[MAX_N]; // [t]ree index -> RMQ [a]rray index

int _A[MAX_AN]; // RMQ member array (must be global otherwise MLE)
int SpT[MAX_AN][LOG_TWO_AN]; // [S]parse [T]able
class RMQ // [R]ange [M]inimum [Q]uery
{
public:
    RMQ(int n, int A[]) // DP pre-process
    {
        for (int i = 0; i < n; ++i)
        {
            _A[i] = A[i];
            SpT[i][0] = i; // RMQ of sub-array starting at index [i] with length 2^0 = 1
        }
        // the two nested loops below have overall time complexity O(n log n)
        for (int j = 1; (1<<j) <= n; ++j) // for each [j] such that 2^j <= n. O(log n)
            for (int i = 0; i + (1<<j) - 1 < n; ++i) // for each valid [i]. O(n)
                if (_A[ SpT[i][j-1] ] < _A[ SpT[ i + ( 1<<(j-1) ) ][j-1] ] ) // RMQ
                    SpT[i][j] = SpT[i][j-1]; // start at index i of length 2^(j-1)
                else
                    SpT[i][j] = SpT[ i + ( 1<<(j-1) ) ][j-1]; // start at index i+2^(j-1) of length 2^(j-1)
    }

    int query(int i, int j) // O(1)
    {
        int k = (int)floor( log( (double)j-i+1 ) / log(2.0) ); // find [k] such that 2^k <= (j-i+1)
        if (_A[ SpT[i][k] ] <= _A[ SpT[j - (1<<k) + 1][k] ] ) return SpT[i][k];
        else return SpT[j - (1<<k) + 1][k];
    }
};

void build_dist(int u)
{
    for (int i = 0; i < (int)CH[u].size(); ++i)
    {
        int v = CH[u][i].first;
        dist[v] = dist[u] + CH[u][i].second;
        build_dist(v);
    }
}

void build_rmq(int u, int depth)
{
    a[a_N] = depth;
    a2t[a_N] = u;
}

```

```

t2a[u] = a_N;
++a_N;
for (int i = 0; i < (int)CH[u].size(); ++i)
{
    int v = CH[u][i].first;
    build_rmq(v, depth + 1);
    a[a_N] = depth;
    a2t[a_N] = u;
    ++a_N;
}

void preprocess()
{
    // for (int u = 0; u < N; ++u)
    // for (int i = 0; i < (int)CH[u].size(); ++i)
    // printf("%d->%d: %lld\n", u, CH[u][i].first, CH[u][i].second);
    dist[root] = 0;
    build_dist(root);
    // for (int u = 0; u < N; ++u) printf("%lld ", dist[u]);
    // printf("\n");
    a_N = 0;
    build_rmq(root, 0);
    // for (int i = 0; i < a_N; ++i) printf("%d ", a[i]);
    // printf("\n");
    // for (int i = 0; i < a_N; ++i) printf("%d ", a2t[i]);
    // printf("\n");
    // for (int u = 0; u < N; ++u) printf("node %d: %d\n", u, t2a[u]);
    // printf("\n");
}

int main()
{
#ifdef LOCAL
    freopen("in", "r", stdin);
#endif
    int Q;
    while (scanf("%d", &N), N)
    {
        CH.assign( N, vil() );
        for (int i = 1; i < N; ++i)
        {
            int parent;
            ll cost;
            scanf("%d %lld", &parent, &cost);
            CH[parent].push_back( make_pair(i, cost) );
        }
        preprocess();
        RMQ rmq(a_N, a);
        scanf("%d", &Q);
        for (int i = 0; i < Q; ++i)
        {
            if (i != 0) printf(" ");
            int s, t;
            scanf("%d %d", &s, &t);
            int l = min(t2a[s], t2a[t]);
            int r = max(t2a[s], t2a[t]);
            int lca = a2t[ rmq.query(l, r) ]; // [l]owest [c]ommon [a]ncestor
            printf("%lld", dist[s] + dist[t] - 2*dist[lca]);
            printf("\n");
        }
        return 0;
    }
}

```

2 Dynamic programming algorithms

2.1 0-1 knapsack

```

#define W 1000 // Knapsack weight
#define N 100 // n item
int weight[N]; //item weight
int value[N]; //item value
int bag[W][2];

// 0/1 Knapsack
void ZeroOne() {
    memset(bag, 0, sizeof(bag));
    for (int i = 0; i < N; i++) {
        for (int j = 0; j < W; j++)
            if (j >= weight[i])
                bag[j][1] = max( bag[j][0], bag[j-weight[i]][0] + value[i] );

        for (int j = 0; j < W; j++)
            bag[j][0] = bag[j][1];
    }
}

```

```

}

// group knapsack
int group; // hou much groups?
int how_many; // one group has many items?
int WEIGHT, VALUE;

void Grouping() {
    memset(bag, 0, sizeof(bag));
    for (int i = 0; i < group; i++) {
        for (int j = 0; j < how_many; j++) {
            scanf("%d %d", &WEIGHT, &VALUE);

            for (int k = 0; k < W; k++) {
                if (j >= WEIGHT) {
                    bag[j][1] = max( bag[j][1], bag[j][0] );
                    bag[j][1] = max( bag[j][1], bag[j-WEIGHT][0] + VALUE );
                }
            }
        }

        for (int j = 0; j < W; j++)
            bag[j][0] = bag[j][1];
    }
}

// mulipte knapsack
int limit[N]; // item limit
void Multiple() {
    for (int i = 0; i < N; i++) {
        int tmp = 1;
        while (tmp <= weight[i]) {
            for (int j = 0; j < W; j++)
                if (j >= weight[i]*tmp)
                    bag[j][1] = max( bag[j-weight[i]*tmp][0] + value[i]*tmp,
                                     bag[j][0] );

            for (int j = 0; j < W; j++)
                bag[j][0] = bag[j][1];

            weight[i] = weight[i]*tmp;
            tmp = tmp*2;
        }
        if (weight[i] > 0) {
            for (int j = 0; j < W; j++)
                if (j >= weight[i]*tmp)
                    bag[j][1] = max( bag[j-weight[i]*tmp][0] + value[i]*tmp, bag[j][0] );

            for (int j = 0; j < W; j++)
                bag[j][0] = bag[j][1];
        }
    }
}

// inf
void Unlimited() {
    memset(bag, 0, sizeof(bag));
    for (int i = 0; i < N; i++) {
        for (int j = 0; j < W; j++)
            if (j >= weight[i])
                bag[j][1] = max( bag[j][0], bag[j-weight[i]][1] + value[i] );

        for (int j = 0; j < W; j++)
            bag[j][0] = bag[j][1];
    }
}
}

```

2.2 Longest common subsequence (LCS)

```

#include <iostream>
#include <bits/stdc++.h>
#define LOCAL
using namespace std;

struct LCS {
    int step, max_len;
} Dp[5000][5000];

int main()
{
#ifdef LOCAL
    freopen("in1.txt", "r", stdin);
#endif // LOCAL
}

```

```

int intX, intY, Min_step, Max_len;
string strX, strY;
while(cin >> intX >> strX >> intY >> strY){
    //init
    for(int i = 0; i <= intY; i++){
        Dp[0][i].max_len = 0;
        Dp[0][i].step = i;
    }
    for(int i = 0; i <= intX; i++){
        Dp[i][0].max_len = 0;
        Dp[i][0].step = i;
    }
    Max_len = 0;
    Min_step = 0;

    //lcs
    for(int i = 1; i <= intX; i++){
        for(int j = 1; j <= intY; j++){
            if(strX[i-1] == strY[j-1]){
                Dp[i][j].max_len = Dp[i-1][j-1].max_len + 1;
                Dp[i][j].step = Dp[i-1][j-1].step;
            }
            //debug
            //cout << strX[i-1] << ' ' << strY[j-1] << ' ' << Dp[i][j].max_len << '\n';
            //cout << strX[i-1] << ' ' << strY[j-1] << ' ' << Dp[i][j].step << '\n';
            else{
                Dp[i][j].max_len = max(Dp[i-1][j].max_len, Dp[i][j-1].max_len);
                Dp[i][j].step = min(min(Dp[i-1][j-1].step, Dp[i][j-1].step), Dp[i-1][j].step)
                    + 1;
            }
        }
        cout << Dp[intX][intY].step << '\n';
    }
    return 0;
}

```

2.3 Max 2D range sum

```

// Max 2D Range Sum - UVa 108 - solved with DP O(n^4).
// Abridged problem statement: Given an n x n square matrix of integers A where
// each integer ranges from [-127..127], find a sub-matrix of A with the maximum
// sum.
#include <bits/stdc++.h>
using namespace std;
int A[200][200];
int main() {
    int n; scanf("%d", &n); // square matrix size
    for (int i = 0; i < n; ++i)
        for (int j = 0; j < n; ++j) {
            scanf("%d", &A[i][j]);
            if (i > 0) A[i][j] += A[i-1][j]; // add from top
            if (j > 0) A[i][j] += A[i][j-1]; // add from left
            if (i > 0 && j > 0) A[i][j] -= A[i-1][j-1]; // avoid double count
        } // inclusion-exclusion
    int maxSubRect = -127*100*100; // the lowest possible val
    for (int i = 0; i < n; ++i)
        for (int j = 0; j < n; ++j) // start coordinate
            for (int k = i; k < n; ++k)
                for (int l = j; l < n; ++l) { // end coord
                    int subRect = A[k][l]; // from (0, 0) to (k, l)
                    if (i > 0) subRect -= A[i-1][l]; // O(1)
                    if (j > 0) subRect -= A[k][j-1]; // O(1)
                    if (i > 0 && j > 0) subRect += A[i-1][j-1]; // O(1)
                    maxSubRect = max(maxSubRect, subRect); // the answer is here
                }
    printf("%d\n", maxSubRect);
    return 0;
}

```

2.4 Traveling salesman problem (TSP)

```

// This is a solution for UVa 10496 - Collecting Beepers. The problem is a
// variant of the Traveling Salesman Problem (TSP): Given n cities and their
// pairwise distances in the form of a matrix 'dist' of size n * n, compute the
// minimum cost of making a tour that starts from any city s, goes through all
// the other n - 1 cities exactly once, and finally returns to the city s. In
// this case, the salesman is Karel in a 2D world who can only move along the
// x and y axis. The cities are beepers whose coordinates are given, from which
// pairwise distances can be calculated. Algorithm takes time O(2^n * n^2).
// INPUT: The first line is the number of test cases. The first line of each
// test case is world's size (x-size and y-size). Next is the starting position

```

```

// of Karel. Next is the number of beepers. Next are the beepers' x- and y-
// coordinates.
// OUTPUT: For each test case, output the minimum distance to move from Karel's
// starting position to each of the beepers and back to the starting position.

```

```

#include <bits/stdc++.h>
using namespace std;

#define LSONe(S) ((S) & -(S))

const int MAX_n = 11;

int dist[MAX_n][MAX_n], memo[MAX_n][1<<(MAX_n-1)]; // Karel + max 10 beepers

int dp(int u, int mask) {
    if (mask == 0) return dist[u][0]; // mask = free coordinates
    int &ans = memo[u][mask]; // close the loop
    if (ans != -1) return ans; // computed before
    ans = 2000000000;
    int m = mask;
    while (m) { // up to O(n)
        int two_pow_v = LSONe(m); // but this is fast
        int v = __builtin_ctz(two_pow_v)+1; // offset v by +1
        ans = min(ans, dist[u][v] + dp(v, mask^two_pow_v)); // keep the min
        m -= two_pow_v;
    }
    return ans;
}

int main() {
    int TC; scanf("%d", &TC);
    while (TC--) {
        int xsize, ysize; scanf("%d %d", &xsize, &ysize); // these two values are not used
        int x[MAX_n], y[MAX_n];
        scanf("%d", &x[0]);
        int n; scanf("%d", &n); ++n; // include Karel
        for (int i = 1; i < n; ++i) // Karel is at index 0
            scanf("%d", &x[i], &y[i]);
        for (int i = 0; i < n; ++i) // build distance table
            for (int j = i; j < n; ++j)
                dist[i][j] = dist[j][i] = abs(x[i]-x[j]) + abs(y[i]-y[j]); // Manhattan distance
        memset(memo, -1, sizeof memo);
        printf("The shortest path has length %d\n", dp(0, (1<<(n-1))-1)); // DP-TSP
    }
    return 0;
}

```

3 Graph algorithms

3.1 All-pairs shortest paths (APSP)

```

// All-Pairs Shortest Paths (APSP) solved with Floyd Warshall O(V^3).
// inside int main()
// Precondition: AdjMat[i][j] contains the weight of edge (i, j) or INF (1B)
// if there is no such edge ('AdjMat' is a 32-bit signed integer array).
// Let 'p' be 2D parent matrix, where p[i][j] is the last vertex before j on
// a shortest path from i to j, i.e. i -> ... -> p[i][j] -> j.
for (int i = 0; i < V; ++i)
    for (int j = 0; j < V; ++j)
        p[i][j] = i; // initialize the parent matrix
for (int k = 0; k < V; ++k) // remember that loop order is k->i->j
    for (int i = 0; i < V; ++i)
        for (int j = 0; j < V; ++j)
            if (AdjMat[i][k] + AdjMat[k][j] < AdjMat[i][j])
                AdjMat[i][j] = AdjMat[i][k] + AdjMat[k][j];
                p[i][j] = p[k][j];
}

// print shortest paths
void printPath(int i, int j)
{
    if (i != j) printPath(i, p[i][j]);
    printf("%d ", j);
}

```

3.2 Bipartite matching BFS by David

```

#include <iostream>
#include <string>
#include <cstdio>

```

```

#include <vector>
#define LOCAL
using namespace std;

int fp[100010],fq[100010];
int vfp[100010],vfq[100010];
int turn = 0;
vector<int> cp[100010], cq[100010];

int BFSBMfp(int n){
    vfp[n] = turn;
    for(int i = 0; i < cp[n].size(); i++){
        if(vfp[cp[n][i]] != turn){
            vfp[cp[n][i]] = turn;
            if(fq[cp[n][i]] == -1 || BFSBMfp(fq[cp[n][i]])){
                fp[n] = cp[n][i];
                fq[cp[n][i]] = n;
                return 1;
            }
        }
    }
    return 0;
}

int main()
{
    ios::sync_with_stdio(false);
    cin.tie(0);
    cout.tie(0);

    int n, p, q, k, x, y;
    cin >> n;
    while(n--){
        cin >> p >> q >> k;
        int MaxnPQ = max(p,q);
        for(int i = 1; i <= MaxnPQ; i++){
            cp[i].clear();
            fp[i] = -1;
            cq[i].clear();
            fq[i] = -1;
        }
        int cnt = 0;
        for(int i = 0; i < k; i++){
            cin >> x >> y;
            cp[x].push_back(y);
            cq[y].push_back(x);
            if(fp[x] == -1 && fq[y] == -1){
                fp[x] = y;
                fq[y] = x;
                cnt++;
            }
        }
        for(int i = 1; i <= p; i++){
            if(fp[i] == -1){
                turn++;
                if(BFSBMfp(i))
                    cnt++;
            }
        }
        cout << cnt << '\n';
    }
    return 0;
}

```

3.3 Centroid decomposition

```

#include<iostream>
#include<bits/stdc++.h>
#define LOCAL
#define MAXN 50005
using namespace std;

int n, k, a, b;
int ans, cnt;
int Max[MAXN], sz[MAXN], rt;
int head[MAXN], dis[MAXN];
bool vis[MAXN];
struct node{
    int v, nx;
}Edge[MAXN*2];

void init(int n){
    Max[0] = n;
    ans = cnt = 0;
    for(int i = 0; i <= n; i++){
        head[i] = -1;
        vis[i] = 0;
    }
}

```

```

    }

void add(int u, int v){
    Edge[cnt].v = v;
    Edge[cnt].nx = head[u];
    head[u] = cnt++;
}

void get_rt(int u, int fa){
    sz[u] = 1; Max[u] = 0;
    for(int i = head[u]; ~i; i = Edge[i].nx){
        int v = Edge[i].v;
        if(vis[v] || v == fa) continue;
        get_rt(v,u);
        sz[u] += sz[v];
        Max[u] = max(Max[u], sz[v]);
    }
    Max[u] = max(Max[u], n - sz[u]);
    if(Max[rt] > Max[u])
        rt = u;
}

void get_dis(int u, int fa, int d){
    for(int i = head[u]; ~i; i = Edge[i].nx){
        int v = Edge[i].v;
        if(vis[v] || v == fa) continue;
        dis[++cnt] = d + 1;
        get_dis(v,u,dis[cnt]);
    }
}

int get_ans(int u, int d){
    dis[cnt=1] = d;
    get_dis(u,0,d);
    sort(dis+1, dis+cnt+1);
    int l = 1, ans = 0;

    while(l < cnt && dis[l] + dis[cnt] < k) l++;
    while(l < cnt && dis[l] <= k - dis[l]){
        ans += upper_bound(dis + l + 1, dis + cnt + 1, k - dis[l]) - lower_bound(dis+1+1, dis+cnt+1, k-dis[l]);
        l++;
    }
    return ans;
}

void dfs(int u){
    vis[u] = 1;
    //cout << rt << ' ' << u << '\n';
    ans += get_ans(u, 0);
    for(int i = head[u]; ~i; i = Edge[i].nx){
        int v = Edge[i].v;
        if(vis[v]) continue;
        ans -= get_ans(v, 1);
        n = sz[v], rt = 0, get_rt(v,u);
        dfs(rt);
    }
}

int main(){
    // #ifdef LOCAL
    //     freopen("in1.txt", "r", stdin);
    // #endif // LOCAL

    cin >> n >> k;
    init(n);
    for(int i = 1; i < n; i++){
        cin >> a >> b;
        add(a,b);
        add(b,a);
    }
    rt = 0; get_rt(1,0);
    dfs(rt);
    cout << ans << '\n';
}

```

3.4 Detect negative weight cycle

```

// Bellman Ford's O(VE)
vi dist(V, INF); dist[s] = 0;
for (int i = 0; i < V - 1; ++i) // relax all E edges V - 1 times
    for (int u = 0; u < V; ++u) // these two loops = O(E)
        for (int j = 0; j < (int)AL[u].size(); ++j) // [A]dacency [L]ist
            {

```

3.5 DFS

```

    ii vw = AL[u][j];
    dist[vw.first] = min( dist[vw.first], dist[u] + vw.second ); // relax
}

#include <iostream>
#include <bits/stdc++.h>
#define LOCAL
using namespace std;
int m, n, flag=1;
int Maxn_city = 0, Maxn_path = 0 ;
vector<int> tree[200020] ;
int city[200020] = {} ;
int visit[200020] = {} ;
vector<int> travel ;

void BFS_to_large_path(int root ){
    visit[root] = 1 ;
    travel.push_back(root);
    for(int i = 0 ; i < tree[root].size() ; i++){
        int node = tree[root][i] ;
        if(!visit[node]){
            BFS_to_large_path(node);
            travel.pop_back();
            visit[root] = 0 ;
        }
    }
    //debug to check large path
    //if (root == 1)
    //    cout << "1=" << travel.size() << ' ' << Maxn_path << ' ' << city[root] << '\n' ;

    if(city[root] && travel.size() > Maxn_path){
        Maxn_city = travel[travel.size()/2];
        Maxn_path = travel.size();
    }
}

void BFS_to_other_path(int root ,int path){
    visit[root] = 1 ;
    for(int i = 0 ; i < tree[root].size() ; i++){
        int node = tree[root][i] ;
        if(!visit[node]){
            BFS_to_other_path(node , path+1);
            visit[root] = 0 ;
        }
    }
    //debug
    if(root == 1 )
        cout << "city=" << root << " path=" << path << '\n' ;

    if(city[root] && path != Maxn_path)
        flag = 0 ;
}

int main(){
#ifdef LOCAL
    freopen("in1.txt" , "r" , stdin);
#endif // LOCAL
    cin >> n >> m ;
    int a, b ;
    for(int i = 0 ; i < n-1 ; i++){
        cin >> a >> b ;
        tree[a].push_back(b) ;
        tree[b].push_back(a) ;
    }

    for(int i = 0 ; i < m ; i++){
        cin >> a ;
        city[a] = 1 ;
    }
    BFS_to_large_path(a);
    //visit[a] = 0 ;
    BFS_to_other_path(Maxn_city , 1 );
    if(flag)
        cout << "YES\n" << Maxn_city ;
    else
        cout << "NO" ;

    //debug
    cout << "Maxn_path=" << Maxn_path << " Maxn_city=" << Maxn_city << '\n' ;
}

```

3.6 DFS ICPC 2019 Russia problem E

```

#include <iostream>
#include <bits/stdc++.h>
#define LOCAL
using namespace std;
int m, n, flag=1;
int Maxn_city = 0, Maxn_path = 0 ;
vector<int> tree[200020] ;
int city[200020] = {} ;
int visit[200020] = {} ;
vector<int> travel ;

void BFS_to_large_path(int root ){
    visit[root] = 1 ;
    travel.push_back(root);
    for(int i = 0 ; i < tree[root].size() ; i++){
        int node = tree[root][i] ;
        if(!visit[node]){
            BFS_to_large_path(node);
            travel.pop_back();
            visit[root] = 0 ;
        }
    }
    //debug to check large path
    //if (root == 1)
    //    cout << "1=" << travel.size() << ' ' << Maxn_path << ' ' << city[root] << '\n' ;

    if(city[root] && travel.size() > Maxn_path){
        Maxn_city = travel[travel.size()/2];
        Maxn_path = travel.size();
    }
}

void BFS_to_other_path(int root ,int path){
    visit[root] = 1 ;
    for(int i = 0 ; i < tree[root].size() ; i++){
        int node = tree[root][i] ;
        if(!visit[node]){
            BFS_to_other_path(node , path+1);
            visit[root] = 0 ;
        }
    }
    //debug
    if(root == 1 )
        cout << "city=" << root << " path=" << path << '\n' ;

    if(city[root] && path != Maxn_path)
        flag = 0 ;
}

int main(){
#ifdef LOCAL
    freopen("in1.txt" , "r" , stdin);
#endif // LOCAL
    cin >> n >> m ;
    int a, b ;
    for(int i = 0 ; i < n-1 ; i++){
        cin >> a >> b ;
        tree[a].push_back(b) ;
        tree[b].push_back(a) ;
    }

    for(int i = 0 ; i < m ; i++){
        cin >> a ;
        city[a] = 1 ;
    }
    BFS_to_large_path(a);
    //visit[a] = 0 ;
    BFS_to_other_path(Maxn_city , 1 );
    if(flag)
        cout << "YES\n" << Maxn_city ;
    else
        cout << "NO" ;

    //debug
    cout << "Maxn_path=" << Maxn_path << " Maxn_city=" << Maxn_city << '\n' ;
}

```

3.7 Dijkstra by Bill

```

// Dijkstra implementation for negative weight edges O((V + E) log V)
vi dist(V, INF); dist[s] = 0;

```

```

priority_queue< ii, vii, greater<ii> > pq;
pq.push( ii(0, s) );
while (!pq.empty())
{
    ii front = pq.top(); pq.pop();
    int d = front.first;
    int u = front.second;
    if (d > dist[u]) continue;
    for (int i = 0; i < (int)AL[u].size(); ++i) // [A]djacency [L]ist
    {
        ii vw = AL[u][i];
        int v = vw.first;
        int w = vw.second;
        if (dist[u] + w < dist[v])
        {
            dist[v] = dist[u] + w;    // relax operation
            pq.push( ii(dist[v], v) );
        }
    }
}
} // this variant can cause duplicate items in the priority queue

```

3.8 Dijkstra by David

```

#include <iostream>
#include <bits/stdc++.h>
#define LOCAL
#define INF 999999999
using namespace std;
int intMap[1010][1010] = {} , intValue[1010][1010] = {};
int m , n ;

struct Node{
    int x , y , v ;
    void read( int _x , int _y , int _v ){
        x = _x ; y = _y ; v = _v ;
    }
    bool operator < (const Node &a) const{
        return v > a.v ;
    }
} nodNode;

void print_map(){
    for(int i = 1 ; i <= n ; i++){
        for(int j = 1 ; j <= m ; j++){
            if(intValue[i][j] == 99999999)
                cout << 'x' << ' ' ;
            else
                cout << intValue[i][j] << ' ' ;
        }
        cout << '\n' ;
    }
    cout << '\n' ;
}

void bfs(){
    int x , y , intDirection[4][2] = {-1,0 , 0,1 , 1,0 , 0,-1};
    int intDx , intDy ;
    Node nodTemp ;
    priority_queue<Node> deqNode ;
    nodTemp.read(1,1,0);
    deqNode.push(nodTemp);
    while(deqNode.size()){
        x = deqNode.top().x ;
        y = deqNode.top().y ;
        deqNode.pop() ;

        for(int i = 0 ; i < 4 ; i++){
            intDx = intDirection[i][0] + x ;
            intDy = intDirection[i][1] + y ;

            //debug
            //cout << intDx << ' ' << intDy << ' ' << intValue[x][y] + intMap[intDx][intDy] << ' ' <<
            //      i << '\n' ;

            if(intValue[x][y] + intMap[intDx][intDy] < intValue[intDx][intDy] ){
                intValue[intDx][intDy] = intValue[x][y] + intMap[intDx][intDy] ;
                nodTemp.read(intDx , intDy , intValue[intDx][intDy]);
                deqNode.push(nodTemp) ;
            }
        }
    }
    //print_map() ;
}

```

```

int main() {
#ifdef LOCAL
    freopen("in1.txt" , "r" , stdin );
    freopen("out.txt" , "w" , stdout) ;
#endif
    ios::sync_with_stdio(false);
    int intCase ;
    cin >> intCase ;
    while(intCase --){
        cin >> n >> m ;
        for(int i = 1 ; i <= n ; i++){
            for(int j = 1 ; j <= m ; j++){
                cin >> intMap[i][j] ;
                intValue[i][j] = INF ;
            }
        }

        for(int i = 1 ; i <= n ; i++){
            intValue[i][0] = 0 ;
            intValue[i][m+1] = 0 ;
            intMap[i][0] = INF +1 ;
            intMap[i][m+1] = INF +1 ;
        }

        for(int i = 1 ; i <= m ; i++){
            intValue[0][i] = 0 ;
            intValue[n+1][i] = 0 ;
            intMap[0][i] = INF +1 ;
            intMap[n+1][i] = INF +1 ;
        }
        intValue[1][1] = intMap[1][1] ;

        //debug
        //cout << intValue[1][1] << '\n' ;

        bfs();
        cout << intValue[n][m] << '\n' ;

        return 0;
    }
}

```

3.9 Print Euler tour

```

// Given an Eulerian-tour graph - a connected undirected graph whose vertices a-
// ll have even degrees, produce its Euler tour. The graph is unweighted, stored
// in an adjacency list where the second attribute in edge info pair is a boole-
// an '1' (edge can still be used) or '0' (edge can no longer be used).
list<int> cyc;    // we need list for fast insertion in the middle

void EulerTour(list<int>::iterator i, int u)
{
    for (int j = 0; j < (int)AL[u].size(); ++j) // [A]djacency [L]ist
    {
        ii& vw = AL[u][j];
        int v = vw.first;
        if (vw.second)    // if this edge can still be used
        {
            vw.second = 0;    // remove this edge
            // remove bi-directional edge
            for (int k = 0; k < (int)AL[v].size(); ++k)
            {
                ii& uw = AL[v][k];
                if (uw.first == u && uw.second)
                {
                    uw.second = 0;
                    break;
                }
            }
            // continue the tour
            EulerTour(cyc.insert(i, u), v);
        }
    }
}

// inside int main()
cyc.clear();
EulerTour(cyc.end(), 0);    // 'cyc' contains an Euler tour starting at vertex '0'
for (list<int>::iterator i = cyc.begin(); i != cyc.end(); ++i)
    printf("%d\n", *i);

```


3.10 Find articulation points and bridges for undirected graph

```
// Find articulation points & bridges for undirected graph solved with DFS O(V + E).
void articulationPointAndBridge(int u)
{
    dfs_low[u] = dfs_num[u] = dfsNumberCounter++; // dfs_low[u] <= dfs_num[u]
    for (int i = 0; i < (int)AL[u].size(); ++i) // [A]dacency [L]ist
    {
        int v = AL[u][i].first;
        if (dfs_num[v] == UNVISITED) // a tree edge
        {
            dfs_parent[v] = u;
            if (u == dfsRoot) ++rootChildren; // special case if 'u' is a root

            articulationPointAndBridge(v);

            if (dfs_low[v] >= dfs_num[u]) articulation_vertex[u] = true;
            if (dfs_low[v] > dfs_num[u]) printf("Edge (%d, %d) is a bridge\n", u, v);

            dfs_low[u] = min(dfs_low[u], dfs_low[v]); // update dfs_low[u]
        }
        else if (v != dfs_parent[u]) dfs_low[u] = min(dfs_low[u], dfs_num[v]); // update dfs_low[u]
    }
}

// inside int main()
dfsNumberCounter = 0;
dfs_num.assign(V, UNVISITED);
dfs_low.assign(V, 0);
dfs_parent.assign(V, 0);
articulation_vertex.assign(V, 0);
printf("Bridges:\n");
for (int u = 0; u < V; ++u)
    if (dfs_num[u] == UNVISITED)
    {
        dfsRoot = u;
        rootChildren = 0;
        articulationPointAndBridge(u);
        articulation_vertex[dfsRoot] = (rootChildren > 1); // special case
    }
printf("Articulation Points:\n");
for (int u = 0; u < V; ++u)
    if (articulation_vertex[u]) printf("Vertex %d\n", u);
```

3.11 Floyd Warshall by David

```
#include <iostream>
#include <bits/stdc++.h>
#define LOCAL
using namespace std;
char before[520][520] = {};
int after[520][520] = {};

int main()
{
    #ifdef LOCAL
        freopen("in1.txt", "r", stdin);
    #endif // LOCAL

    int n;
    cin >> n;
    for (int i = 0; i < n; i++)
        for (int j = 0; j < n; j++)
            cin >> before[i][j];

    for (int i = 0; i < n; i++)
        for (int j = i+1; j < n; j++)
        {
            int sum = 0;
            for (int k = i+1; k < j; k++)
                if (after[i][k])
                    sum += before[k][j] - '0';
        }

    if ((sum + 1) % 10 == before[i][j] - '0')
        after[i][j] = 1;
}
```

```
for (int i = 0; i < n; i++)
    for (int j = 0; j < n; j++)
        cout << after[i][j] << '\n';
}

return 0;
```

3.12 Graph edges property check

```
// Graph Edges Property Check solved with DFS O(V + E).
void graphCheck(int u) // DFS for checking graph edge properties
{
    dfs_num[u] = EXPLORED;
    for (int i = 0; i < (int)AL[u].size(); ++i) // [A]dacency [L]ist
    {
        int v = AL[u][i].first;
        if (dfs_num[v] == UNVISITED) // Tree Edge, EXPLORED->UNVISITED
        {
            dfs_parent[v] = u; // parent of this child is me
            graphCheck(v);
        }
        else if (dfs_num[v] == EXPLORED) // EXPLORED->EXPLORED
        {
            if (v == dfs_parent[u]) printf("Two ways (%d, %d)-(%d, %d)\n", u, v, v, u);
            else printf("Back Edge (%d, %d) (Cycle)\n", u, v); // can check if graph is cyclic
        }
        else if (dfs_num[v] == VISITED) // EXPLORED->VISITED
            printf("Forward/Cross Edge (%d, %d)\n", u, v);
    }
    dfs_num[u] = VISITED;
}

// inside int main()
dfs_num.assign(V, UNVISITED);
dfs_parent.assign(V, 0);
for (int u = 0; u < V; ++u)
    if (dfs_num[u] == UNVISITED)
        printf("Component %d:\n", ++numComp, graphCheck(u));
```

3.13 Kruskal by David

```
#include <iostream>
#include <bits/stdc++.h>
#define LOCAL
using namespace std;
int parent[1020];

struct edge{
    int n1, n2, w;
}node[25020];

int compare(edge A, edge B){
    return A.w < B.w;
}

int find_root(int a){
    if (a != parent[a])
        return parent[a] = find_root(parent[a]);
    return a;
}

int main()
{
    #ifdef LOCAL
        freopen("in1.txt", "r", stdin);
        freopen("out.txt", "w", stdout);
    #endif // LOCAL

    int n, m, p_n1, p_n2; // parent_n1, parent_n2
    vector<int> hce; // heavy edge circle
    while (cin >> n >> m && n+m != 0)
        for (int i = 0; i < m; i++)
            cin >> node[i].n1 >> node[i].n2 >> node[i].w;

    for (int i = 0; i < n; i++)
        parent[i] = i;
```

```

sort(node , node + m , compare ) ;
hce.clear() ;

//kruskal
for(int i = 0 ; i < m ; i++){
    p_n1 = find_root(node[i].n1) ;
    p_n2 = find_root(node[i].n2) ;
    if(p_n1 != p_n2 )
        parent[p_n2] = p_n1 ;
    else
        hce.push_back(node[i].w) ;

    //debug
    /**<
    for(int i = 0 ; i < n ; i++)
        cout << parent[i] << ' ' ;
    cout << '\n' ;
    */
}
sort(hce.begin() , hce.end()) ;
if(hce.size()){
    for(int i = 0 ; i < hce.size()-1 ; i++)
        cout << hce[i] << ' ' ;
    cout << hce[hce.size()-1] ;
}
else
    cout << "forest" ;
cout << '\n' ;
}
return 0;
}

```

3.14 Max flow

```

int res[MAX_V][MAX_V], mf, f, s, t;
vi p; // p stores the BFS spanning tree from s

void augment(int v, int minEdge)
{
    if (v == s) { f = minEdge; return; }
    else if ( p[v] != -1 )
    {
        augment( p[v], min(minEdge, res[ p[v] ][ v ] ) );
        res[ p[v] ][ v ] -= f;
        res[ v ][ p[v] ] += f;
    }
}

// inside int main(): set up 'AL', 'res', 's', and 't' with appropriate values
// remember to add backward edges to 'AL'
mf = 0;
while (true) // O(V * E^2) Edmonds Karp's algorithm
{
    f = 0;
    vi dist(MAX_V, INF); dist[s] = 0;
    queue<int> q; q.push(s);
    p.assign(MAX_V, -1);
    while (!q.empty())
    {
        int u = q.front(); q.pop();
        if (u == t) break; // immediately stop BFS if we already reach sink t
        for (int i = 0; i < (int)AL[u].size(); ++i)
        {
            int v = AL[u][i]; // vector<vi> [A]dacency [L]ist
            if (res[u][v] > 0 && dist[v] == INF)
                dist[v] = dist[u] + 1, q.push(v), p[v] = u;
        }
    }
    augment(t, INF); // find the min edge weight 'f' in this path, if any
    if (f == 0) break; // we cannot send any more flow ('f' = 0), terminate
    mf += f; // we can still send a flow, increase the max flow!
}
printf("%d\n", mf);

```

3.15 Max cardinality bipartite matching (MCBM)

```

// Max Cardinality Bipartite Matching (MCBM) solved with augmenting path algorithm O(VE).
vi match, vis;

int Aug(int l) // return 1 if an augmenting path is found & 0 otherwise
{
    if (vis[l]) return 0;
    vis[l] = 1;

```

```

for (int i = 0; i < (int)AL[l].size(); ++i) // [A]dacency [L]ist
{
    int r = AL[l][i]; // edge weight not needed -> vector<vi> > AL
    if ( match[r] == -1 || Aug(match[r]) )
    {
        match[r] = l;
        return 1; // found 1 matching
    }
}
return 0; // no matchings
}

// inside int main()
// build unweighted bipartite graph with directed edge left->right set
// left vertices [0..N-1], right vertices [N..V-1]
int MCBM = 0;
match.assign(V, -1); // V is the number of vertices in bipartite graph
for (int l = 0; l < N; ++l) // N = size of the left set
{
    vis.assign(N, 0); // reset before each recursion
    MCBM += Aug(l);
}
printf("Found %d matchings\n", MCBM);

```

3.16 Min-cost flow (MCF)

```

// Uva 10594 - Data Flow solved as Min-Cost Flow (MCF) problem using Edmonds Ka-
// rp and Bellman Ford algorithms with total time O(V^2 * E^3).
#include <bits/stdc++.h>

using namespace std;

#define LOCAL
#define INF 1000000000000000 // 10^15
#define bwd 0 // [b]ack[w]ar[d] direction
#define fwd 1 // [f]or[w]ar[d] direction
#define MAX_V 200

typedef vector<int> vi;
typedef long long int ll;
typedef pair<ll, ll> ll2;
typedef vector<ll> vll;

int V;
vector<vi> AL;
ll res[MAX_V][MAX_V][2], cst[MAX_V][MAX_V][2];
ll mf, f, min_cost;
int s, t;
vector< pair<int, ll> > p;
ll FLOW, CAPACITY;

void augment(int v, ll minEdge)
{
    if (v == s) { f = minEdge; return; }
    else if ( p[v].first != -1 )
    {
        augment( p[v].first, min(minEdge, res[ p[v].first ][ v ][ p[v].second ] ) );
        res[ p[v].first ][ v ][ p[v].second ] -= f;
        res[ v ][ p[v].first ][ p[v].second ] += f;
    }
}

void trace_cost(int v)
{
    if (p[v].first == -1) return;
    min_cost += cst[ p[v].first ][ v ][ p[v].second ] * f;
    trace_cost(p[v].first);
}

void min_cost_flow()
{
    min_cost = 0;
    mf = 0;
    while (true)
    {
        f = 0;
        p.assign(MAX_V, make_pair(-1, -1));
        vll dist(V, INF); dist[s] = 0;
        for (int i = 0; i < V - 1; ++i)
            for (int u = 0; u < V; ++u)
                for (int j = 0; j < (int)AL[u].size(); ++j)
                {
                    int v = AL[u][j];
                    for (int dir = 0; dir <= 1; ++dir)
                        if (res[u][v][dir] > 0 && dist[u] + cst[u][v][dir] < dist[v])
                        {
                            dist[v] = dist[u] + cst[u][v][dir];

```

```

        p[v] = make_pair(u, dir);
    }
    }
    augment(t, INF);
    if (f == 0) break;
    f = min(f, FLOW - mf);
    trace_cost(t);
    mf += f;
    if (mf == FLOW) break;
}
if (mf < FLOW) printf("Impossible.\n");
else printf("%lld\n", min_cost);
}

int main()
{
    #ifdef LOCAL
        freopen("in", "r", stdin);
    #endif
    int E;
    while (scanf("%d %d", &V, &E) != EOF)
    {
        AL.assign(V, vi());
        memset(res, 0, sizeof res);
        memset(cst, 0, sizeof cst);
        for (int i = 0; i < E; ++i)
        {
            int u, v;
            ll w;
            scanf("%d %d %lld", &u, &v, &w);
            u--; v--; // 0-based index
            AL[u].push_back(v);
            AL[v].push_back(u);

            res[u][v][fwd] = res[v][u][bwd] = 1; // real edges
            cst[u][v][fwd] = cst[v][u][bwd] = w;

            res[u][v][bwd] = res[v][u][fwd] = 0; // additional reversed edges
            cst[u][v][bwd] = cst[v][u][fwd] = -w;
        }

        scanf("%lld %lld", &FLOW, &CAPACITY);
        for (int u = 0; u < V; ++u)
            for (int v = 0; v < V; ++v)
            {
                res[u][v][fwd] += CAPACITY;
                res[v][u][bwd] += CAPACITY;
            }

        s = 0;
        t = V-1;
        min_cost_flow();
    }
    return 0;
}

```

3.17 Minimum spanning tree (MST)

```

// Minimum Spanning Tree (MST) solved with Kruskal O(E log V)
// inside int main()
vector< pair<int, ii> > EdgeList; // (weight, two vertices) of the edge
for (int i = 0; i < E; ++i)
{
    scanf("%d %d %d", &u, &v, &w);
    EdgeList.push_back( make_pair( w, ii(u, v) ) );
}
sort(EdgeList.begin(), EdgeList.end()); // sort by edge weight O(E log E)
int mst_cost = 0;
UnionFind UF(V); // all V are disjoint sets initially
for (int i = 0; i < E; ++i)
{
    pair<int, ii> front = EdgeList[i];
    if (!UF.isSameSet(front.second.first, front.second.second))
    {
        mst_cost += front.first;
        UF.unionSet(front.second.first, front.second.second);
    }
}
printf("MST cost = %d\n", mst_cost);

```

3.18 Strongly connected component (SCC)

```

// Tarjan O(V + E)
vi dfs_num, dfs_low, visited;
int dfsNumberCounter, numSCC;
vi S;

void tarjanSCC(int u)
{
    dfs_low[u] = dfs_num[u] = dfsNumberCounter++; // dfs_low[u] <= dfs_num[u]
    S.push_back(u); // stores 'u' in a vector based on order of visitation
    visited[u] = 1;
    for (int i = 0; i < (int)AL[u].size(); ++i) // [A]dacency [L]ist
    {
        int v = AL[u][i].first;
        if (dfs_num[v] == UNVISITED) tarjanSCC(v);
        if (visited[v]) dfs_low[u] = min( dfs_low[u], dfs_low[v] ); // condition for update
    }

    if (dfs_low[u] == dfs_num[u]) // if this is a root (start) of an SCC
    {
        printf("SCC %d:", ++numSCC); // this part is done after recursion
        while (true)
        {
            int v = S.back(); S.pop_back();
            visited[v] = 0;
            printf(" %d", v);
            if (u == v) break;
        }
        printf("\n");
    }
}

// inside int main()
dfs_num.assign(V, UNVISITED);
dfs_low.assign(V, 0);
visited.assign(V, 0);
dfsNumberCounter = numSCC = 0;
for (int u = 0; u < V; ++u)
    if (dfs_num[u] == UNVISITED)
        tarjanSCC(u);

```

4 Greedy algorithms

4.1 Interval covering

```

// This is a solution for UVa 10382 - Watering Grass. The problem is a variant
// of Interval Covering problem, which is solved by O(n) Greedy algorithm.

#include <bits/stdc++.h>

#define pb push_back
#define not_set -1

using namespace std;

typedef pair<double, double> dd;
typedef vector<dd> vdd;
typedef enum { STOP = 0,
              CONTINUE } status;

int n, l, w;
vdd spinklers;
int answer;
double pivot;

struct sort_compare_t {
    bool operator()(dd a, dd b) const {
        return a.first < b.first || (a.first == b.first && a.second > b.second);
    }
} sort_compare;

void InputSpinklers() {
    for (int i = 0; i < n; i++) {
        double x, r; // must be double otherwise WA.
        scanf("%lf %lf", &x, &r);
        if (w > 2 * r) // ignore spinklers that cannot cover the width of the strip.
        {
            continue;
        }
        if (w == 2 * r) // ignore spinklers that produce no intervals.
        {
            continue;
        }
        double dx = sqrt(r * r - w * w / 4.0);
        spinklers.pb(dd(x - dx, x + dx));
    }
}

```

```

}

status Check(int& j) {
    if (j == not_set) // there is an interval after pivot that cannot be covered.
    {
        return STOP;
    }
    // record j.
    answer++;
    pivot = spinklers[j].second;
    if (pivot >= 1) // solution found!
    {
        return STOP;
    }
    j = not_set;
    return CONTINUE;
}

void SolveIntervalCovering() {
    sort(spinklers.begin(), spinklers.end(), sort_compare);
    answer = 0;
    pivot = 0.0;
    int j = not_set;
    int iter = 0;
    while (true) {
        if (iter == spinklers.size()) // iterated through all spinklers/intervals.
        {
            Check(j);
            break;
        }

        if (spinklers[iter].first <= pivot) {
            if (pivot < spinklers[iter].second) // note the next candidate down!
            {
                if (j == not_set || spinklers[iter].second > spinklers[j].second) // note down the
                    most right candidate.
                {
                    j = iter;
                }
                iter++;
            }
            else // skip intervals that are completely covered by the previously selected ones.
            {
                iter++;
            }
        }
        else // out bound.
        {
            if (Check(j) == STOP) {
                break;
            }
        }
    }

    if (pivot >= 1) {
        printf("%d\n", answer);
    }
    else {
        printf("-1\n");
    }
}

int main() {
    while (scanf("%d %d %d", &n, &l, &w) != EOF) {
        spinklers.clear();
        InputSpinklers();
        SolveIntervalCovering();
    }
}

```

4.2 Longest increasing subsequence (LIS)

```

#include <bits/stdc++.h>
using namespace std;

typedef vector<int> vi;

int n;
vi A;

void print_array(const char *s, vi &L, int n) {
    for (int i = 0; i < n; ++i) {
        if (i) printf(" ");
        else printf("%s: [", s);
        printf("%d", L[i]);
    }
    printf("]\n");
}

vi p; // predecessor array

```

```

void print_LIS(int i) {
    if (p[i] == -1) { printf("%d", A[i]); return; } // base case
    print_LIS(p[i]);
    printf(" %d", A[i]);
}

int memo[10010]; // old limit: up to 10^4

int LIS(int i) {
    if (i == 0) return 1;
    int &ans = memo[i];
    if (ans != -1) return ans; // was computed before
    ans = 1; // LIS can start anywhere
    for (int j = 0; j < i; ++j) // O(n) here
        if (A[j] < A[i]) // increasing condition
            ans = max(ans, LIS(j)+1); // pick the max
    return ans;
}

int main() {
    // note: A[n-1] must be set as the largest value ("INF")
    // so that all LIS (that can start anywhere) will end at n-1
    srand(time(NULL));
    int n = 10+rand()%11; // [10..20]
    A.assign(n, 0);
    A[n-1] = 99; // set A[n-1] = INF
    for (int i = 0; i < n-1; ++i)
        A[i] = rand()%101-50; // [-50..50]

    n = 12;
    vi sample({-7, 10, 9, 2, 3, 8, 8, 1, 2, 3, 4, 99});
    A = sample;

    printf("n = %d:", n);
    for (int i = 0; i < n; ++i)
        printf(" %d", A[i]);
    printf("\n");

    // early 2000 problems usually accept O(n^2) solution
    memset(memo, -1, sizeof memo);
    printf("LIS length is %d\n", LIS(n-1)); // with O(n^2) DP

    // 2020s problems will likely only accept O(n log k) solution
    // new limit: n can be up to 200K
    int k = 0, lis_end = 0;
    vi L(n, 0), L_id(n, 0);
    p.assign(n, -1);

    for (int i = 0; i < n; ++i) {
        int pos = lower_bound(L.begin(), L.begin()+k, A[i]) - L.begin();
        L[pos] = A[i];
        L_id[pos] = i; // greedily overwrite this
        p[i] = pos ? L_id[pos-1] : -1; // remember the index too
        // predecessor info
        if (pos == k) { // can extend LIS?
            k = pos+1; // k = longer LIS by +1
            lis_end = i; // keep best ending i
        }

        printf("Considering element A[%d] = %d\n", i, A[i]);
        printf("LIS ending at A[%d] is of length %d: ", i, pos+1);
        printf("[");
        print_LIS(i);
        printf("]\n");
        print_array("L is now", L, k);
        printf("\n");
    }

    printf("Final LIS is of length %d: ", k);
    print_LIS(lis_end); printf("\n");

    assert(LIS(n-1) == k); // both must be identical
    return 0;
}

```

4.3 Max 1D range sum

```

// Max 1D Range Sum solved with Jay Kadane O(n).
// inside int main()
int n = 9;
int A[] = { 4, -5, 4, -3, 4, 4, -4, 4, -5 }; // a sample array A
int sum = 0;
int ans = 0; // important, 'ans' must be initialized to 0
for (int i = 0; i < n; ++i)
{
    sum += A[i];
    ans = max(ans, sum);
}

```

```

    if (sum < 0) sum = 0;
}
printf("Max 1D Range Sum = %d\n", ans);

```

5 Math algorithms

5.1 Chinese remainder theorem

```

#include <bits/stdc++.h>
#define qtr ios::sync_with_stdio(0); cin.tie(0);
#define endl '\n'
#define int long long
#define MOD 1000000
using namespace std;

int inv(int a, int m){
    int m0 = m, t, q;
    int x0 = 0, x1 = 1;
    if(m == 1){
        return 0;
    }
    while(a > 1){
        q = a/m;
        t = m;
        m = a%m, a = t;
        t = x0;
        x0 = x1 - q * x0;
        x1 = t;
    }
    if(x1 < 0){
        x1 += m0;
    }
    return x1;
}

int findMinX(vector<int> num, vector<int> rem, int k){
    int prod = 1;
    for(int i = 0; i < k; i++) prod *= num[i];
    int result = 0;
    for(int i = 0; i < k; i++){
        int pp = prod / num[i];
        result += rem[i] * inv(pp, num[i]) * pp;
    }
    return result % prod;
}

int32_t main() { //qtr
    int n = 3;
    vector<int> rem, factor;
    rem.resize(n);
    factor.resize(n);
    for(int i = 0; i < n; i++){
        cin >> factor[i];
    }
    for(int i = 0; i < n; i++){
        cin >> rem[i];
    }
    cout << findMinX(factor, rem, n) << endl;
}

```

5.2 Extended greatest common divisor (Ext-GCD)

```

// ax mod b = 1
// ax + by = 1, x=y=0
// a, b Relatively Prime
LL exgcd(LL a, LL b, LL &x, LL &y){
    if(b){
        LL tmd=exgcd(b, a%b, y, x);
        y-=a/b*x;
        return tmd;
    }
    x=1, y=0;
    return a;
}

```

5.3 Greatest common divisor (GCD) and least common multiple (LCM)

```

// or __gcd(a, b) in gcc
int gcd(int a, int b){
    return a%b?gcd(b, a%b):b;
}
int lcm(int a, int b){
    return a*b/gcd(a, b);
}

```

5.4 Generate list of prime numbers

```

// Generate list of prime numbers using Sieve of Eratosthenes.
ll _sieve_size;
bitset<10000010> bs; // [b]it [s]et 10^7 should be enough for most cases
vi primes; // compact list of primes

void sieve(ll upperbound) // create list of primes in [0..upperbound]
{
    _sieve_size = upperbound + 1; // add 1 to include upperbound
    bs.set(); // set all bits to 1
    bs[0] = bs[1] = 0; // exception index 0 and 1
    for (ll i = 2; i <= _sieve_size; ++i)
        if (bs[i])
        {
            // cross out multiples of i starting from i * i!
            for (ll j = i * i; j <= _sieve_size; j += i) bs[j] = 0;
            primes.push_back( (int)i );
        }
}

bool isPrime(ll N) // a good enough deterministic prime tester
{
    if (N <= _sieve_size) return bs[N]; // O(1) for small primes
    for (int i = 0; i < (int)primes.size(); ++i)
        if (N % primes[i] == 0) return false;
    return true; // it takes longer if N is a large prime!
} // note: only work for N <= (last prime in vi 'primes')^2

// inside int main()
sieve(10000000); // can go up to 10^7 (need few seconds)
printf("%d\n", isPrime(2147483647)); // 10-digit prime
printf("%d\n", isPrime(136117223861LL)); // not a prime, 104729 * 1299709

```

5.5 N choose R combination (nCr)

```

#define MAXN 100
long long nCr[MAXN+5][MAXN+5];
// nCr[i][j] = C_n(i)^r(j)
void build_nCr() {
    for(int i = 1; i < MAXN+5; i++) {
        for(int j = 1; j < MAXN+5; j++) {
            if(i == j)
                nCr[i][j] = 1;
            else if(i > j)
                nCr[i][j] = nCr[i-1][j] * i / (i-j);
        }
    }
}

```

5.6 Stirling's approximation

```

double Stirling(int n){
    return (0.5*log(2.0*acos(-1.0)*n)+n*log(n+0.0)-n)/log(10.0);
} // n! Digits

```

6 String algorithms

6.1 Knuth Morris Pratt (KMP)

```
#include <iostream>
#include <bits/stdc++.h>
#define LOCAL
#define MAXN 100020
using namespace std;
string strA , strB ;
int b[MAXN] , p[MAXN] ;

void kmp_process(){
    int n = strB.length() , i = 0 , j = -1 ;
    b[0] = -1 ;
    while(i < n){
        while(j >= 0 && strB[i] != strB[j]) j = b[j] ;
        i++ ; j++ ;
        b[i] = j ;
    }

    //debug
    // for(int k = 0 ; k <= n ; k++)
    //     cout << b[k] << ' ' ;
    // cout << '\n' ;
}

int kmp(){
    int n = strA.length() , m = strB.length() , i = 0 , j = 0 ;
    while(i < n){
        while(j >= 0 && strA[i] != strB[j]) j = b[j] ;
        i++ ; j++ ;
    }
    return j ;
}

int main()
{
    #ifdef LOCAL
        freopen("in1.txt" , "r" , stdin) ;
    #endif // LOCAL

    while(cin >> strA){
        strB = strA;
        reverse(strB.begin() , strB.end());
        kmp_process();
        int n = kmp() ;
        cout << strA << strB.substr(n) << '\n' ;
    }
    return 0;
}
```

6.2 Longest palindromic substring

```
#include <iostream>
#include <bits/stdc++.h>
#define LOCAL
#define MAXN 1010
using namespace std;
int dp[MAXN][MAXN] = {} ;
string strA , strB ;
int n , m ;

int lcs(){
    n = strA.length();
    m = strB.length();
    for(int i = 0 ; i <= n ; i++) dp[i][0] = 0 ;
    for(int j = 0 ; j <= m ; j++) dp[0][j] = 0 ;
    for(int i = 1 ; i <= n ; i++){
        for(int j = 1 ; j <= m ; j++){
            if(strA[i-1] == strB[j-1]) dp[i][j] = dp[i-1][j-1]+1 ;
            else dp[i][j] = max(dp[i-1][j] , dp[i][j-1]);
        }
    }
    return dp[n][m] ;
}

int main()
{
}
```

```
#ifdef LOCAL
    freopen("in1.txt" , "r" , stdin) ;
#endif // LOCAL
int t ;
cin >> t ;
cin.ignore();
while(t--){
    getline(cin, strA);
    strB = strA ;
    reverse(strB.begin() , strB.end());
    cout << lcs() << '\n' ;
}

return 0;
}
```

6.3 Minimum edit distance

```
#include <iostream>
#include <bits/stdc++.h>
#define LOCAL
#define MAXN 100
using namespace std;
string strA , strB ;
int dis[MAXN][MAXN] , back_table[MAXN][MAXN] ;
int cnt , m , n ;

void backtracking(int i , int j){
    if(i==0 || j==0){
        while(i > 0){
            cout << cnt++ << " Delete " << i << '\n' ;
            i--;
        }
        while(j > 0){
            cout << cnt++ << " Insert " << i+1 << " , " << strB[j-1] << '\n' ;
            j-- ;
        }
        return ;
    }

    if(strA[i-1] == strB[j-1])
        backtracking(i-1, j-1);
    else{
        if(dis[i][j] == dis[i-1][j-1]+1){
            cout << cnt++ << " Replace " << i << " , " << strB[j-1] << '\n' ;
            backtracking(i-1, j-1);
        }
        else if(dis[i][j] == dis[i-1][j]+1){
            cout << cnt++ << " Delete " << i << '\n' ;
            backtracking(i-1, j) ;
        }
        else if(dis[i][j] == dis[i][j-1]+1){
            cout << cnt++ << " Insert " << i+1 << " , " << strB[j-1] << '\n' ;
            backtracking(i, j-1);
        }
    }
}

void med(){ //Minimum Edit Distance
    for(int i = 0 ; i <= n ; i++) dis[i][0] = i ;
    for(int j = 0 ; j <= m ; j++) dis[0][j] = j ;
    for(int i = 1 ; i <= n ; i++){
        for(int j = 1 ; j <= m ; j++){
            if(strA[i-1] == strB[j-1]) dis[i][j] = dis[i-1][j-1] ;
            else dis[i][j] = min(dis[i-1][j-1] , min(dis[i-1][j] , dis[i][j-1]))+1;
        }
    }
}

int main()
{
    #ifdef LOCAL
        freopen("in1.txt" , "r" , stdin) ;
        freopen("out.txt" , "w" , stdout);
    #endif // LOCAL
    cin.tie(0);
    cout.tie(0);
    ios::sync_with_stdio(false);
    int flag = 0 ;
    while(getline(cin, strA) && getline(cin , strB)){
        n = strA.length() ;
        m = strB.length() ;
        cnt = 1 ;
        med();
        if(flag) cout << '\n' ;
    }
}
```

```

        flag = 1 ;
        cout << dis[n][m] << '\n' ;
        backtracking(n,m);
    }
    return 0;
}

```

6.4 Z-algorithm

```

#include <iostream>
#include <bits/stdc++.h>
#define LOCAL
#define MAXN 1000020
using namespace std;
int z[MAXN] = {} ;
int x=0 , y=0 , maxn = 0;
string s ;

int main()
{
    #ifdef LOCAL
        freopen("in1.txt","r",stdin);
    #endif // LOCAL

    cin >> s ;
    for(int i = 1 ; i < s.length() ; i++) {
        z[i] = max(0,min(z[i-x] , y - i + 1));
        while(i + z[i] < s.length() && s[z[i]] == s[i+z[i]] ) {
            x = i ;
            y = i + z[i] ;
            z[i]++;
        }
    }

    for(int i = 0 ; i < s.length() ; i++)
        if(z[i] == s.length() - i && maxn >= s.length()-i ) {
            cout << s.substr(0,z[i]);
            return 0 ;
        }
    maxn = max(maxn , z[i]);
    cout << "Just a legend" ;
    return 0;
}

```

7 Data structures

7.1 Union-find disjoint sets (UFDS) by David

```

#include <iostream>
#include <bits/stdc++.h>
#define LOCAL

using namespace std;
int intSum[200080] , intParent[200080] , intSet[200080] ;

int find_root(int intA) {
    if(intParent[intA] == intA)
        return intA ;
    intParent[intA] = find_root(intParent[intA]) ;
    return intParent[intA] ;
}

int each_debug(int n ) {
    for(int i = 1 ; i <= n ; i++) {
        cout << i << ' ' << intParent[i] << ' ' \
            << intSet[find_root(i)] << ' ' << intSum[find_root(i)] << '\n' ;
    }
    system("Pause") ;
}

int main()
{
    #ifdef LOCAL
        freopen("in1.txt","r", stdin);
        freopen("out.txt","w" , stdout) ;
    #endif
}

```

```

#endif // LOCAL

int n, m , operation , p , q ;
while(cin >> n >> m) {
    for(int i = 1 ; i <= n ; i++) {
        intParent[i] = i+n ;
        intParent[i+n] = i+n ;
        intSum[i+n] = i;
        intSet[i+n] = 1 ;
    }
    while(m--) {
        cin >> operation ;
        if(operation == 1 ) {
            cin >> p >> q ;
            int intRoot_p , intRoot_q ;
            intRoot_p = find_root(intParent[p]) ;
            intRoot_q = find_root(intParent[q]) ;
            if(intRoot_p != intRoot_q) {
                intParent[intRoot_q] = intRoot_p ;
                intSum[intRoot_p] += intSum[intRoot_q] ;
                intSet[intRoot_p] += intSet[intRoot_q] ;
            }
            //debug
            //each_debug(n) ;
        }
        else if (operation == 2 ) {
            cin >> p >> q ;
            int intRoot_p , intRoot_q ;
            intRoot_p = find_root(intParent[p]) ;
            intRoot_q = find_root(intParent[q]) ;
            if(intRoot_p != intRoot_q) {
                intParent[p] = intRoot_q ;
                intSum[intRoot_q] += p ;
                intSum[intRoot_p] -= p ;
                intSet[intRoot_q] ++ ;
                intSet[intRoot_p] -- ;
            }
            //debug
            //each_debug(n) ;
        }
        else if (operation == 3) {
            cin >> p ;
            cout << intSet[find_root(p)] << ' ' << intSum[find_root(p)] << '\n' ;
        }
    }
    return 0;
}

```

7.2 Binary indexed/fenwick tree (BIT)

```

#include <iostream>
using namespace std;

#define LOGSZ 17

int tree[(1<<LOGSZ)+1];
int N = (1<<LOGSZ);

// add v to value at x
void set(int x, int v) {
    while(x <= N) {
        tree[x] += v;
        x += (x & -x);
    }
}

// get cumulative sum up to and including x
int get(int x) {
    int res = 0;
    while(x) {
        res += tree[x];
        x -= (x & -x);
    }
    return res;
}

// get largest value with cumulative sum less than or equal to x;
// for smallest, pass x-1 and add 1 to result
int getind(int x) {
    int idx = 0, mask = N;
    while(mask && idx < N) {
        int t = idx + mask;
        if(x >= tree[t]) {
            idx = t;
        }
    }
}

```

```

    x -= tree[t];
}
mask >= 1;
}
return idx;
}

```

7.3 Rope

```

#include <iostream>
#include <bits/stdc++.h>
#include <ext/rope>
#define LOCAL
#define MAXN 50020
using namespace std;
using namespace __gnu_cxx;

int main()
{
#ifdef LOCAL
    freopen("in1.txt", "r", stdin);
#endif // LOCAL
    int n, t, a, b, c, d=0;
    int v = 0;
    string strA;
    rope<char> r[MAXN], rtmp;
    cin >> n;
    while(n--){
        cin >> t;

        if(t==1){
            cin >> a;
            cin >> strA;
            a -= d;
            r[++v] = r[v];
            r[v].insert(a, strA.c_str());
            //debug
            //cout << r[v] << '\n';
        }
        else if(t==2){
            cin >> a >> b;
            a -= d; b -= d;
            r[++v] = r[v];
            r[v].erase(a-1, b);
            //debug
            //cout << r[v] << ' ' << r[v-1] << '\n';
        }
        else if(t==3){
            cin >> a >> b >> c;
            a -= d; b -= d; c -= d;
            rtmp = r[a].substr(b-1, c);
            cout << rtmp << '\n';
            d += count(rtmp.begin(), rtmp.end(), 'c');
        }
    }
    return 0;
}

```

7.4 Segment tree

```

#include <iostream>
#include <bits/stdc++.h>
#include <string>
#define LOCAL
#define Lson(x) ((x <= 1) + 1)
#define Rson(x) ((x <= 1) + 2)
#define INF 999999999
using namespace std;
const int N = 100005;
int shift[35], num[N], len_shift;
string strLine;

struct Node{
    int left, right, Min_Value;
}node[4 * N];

void build(int left, int right, int x = 0){
    node[x].left = left;
    node[x].right = right;
    if(left == right){
        node[x].Min_Value = num[left];
    }
}

```

```

        return;
    }
    int mid = (left + right) / 2;

    //debug
    //cout << mid << '\n';
    //cout << x << ' ' << node[x].left << ' ' << node[x].right << ' ' << '\n';

    build(left, mid, Lson(x));
    build(mid + 1, right, Rson(x));
    node[x].Min_Value = min(node[Lson(x)].Min_Value, node[Rson(x)].Min_Value);
}

void handle(){
    len_shift = 0;
    shift[len_shift] = 0;
    for(int i = 6; i < strLine.length(); i++){
        if(strLine[i] >= '0' && strLine[i] <= '9'){
            shift[len_shift] = shift[len_shift] * 10 + (int)(strLine[i] - '0');
        }
        else{
            shift[++len_shift] = 0;
        }
    }
    //finally char is ')', so len_shift is right
    sort(shift, shift + len_shift);

    //debug
    /**<
    for(int i = 0; i < len_shift; i++)
        cout << shift[i] << ' ';
    cout << '\n';
    */
}

int query(int left, int right, int x = 0){
    if(node[x].left >= left && node[x].right <= right)
        return node[x].Min_Value;
    int mid = (node[x].left + node[x].right) / 2;
    int ans = INF;

    //debug
    //cout << x << ' ' << node[x].left << ' ' << node[x].right << ' ' << node[x].Min_Value << '\n';

    if(left <= mid)
        ans = min(ans, query(left, right, Lson(x)));
    if(mid < right)
        ans = min(ans, query(left, right, Rson(x)));
    return ans;
}

void set_num(int position, int value, int x = 0){
    if(node[x].left == position && node[x].right == position){
        node[x].Min_Value = value;
        return;
    }
    int mid = (node[x].left + node[x].right) / 2;
    if(position <= mid)
        set_num(position, value, Lson(x));
    if(mid < position)
        set_num(position, value, Rson(x));
    node[x].Min_Value = min(node[Lson(x)].Min_Value, node[Rson(x)].Min_Value);
}

int main()
{
    int n, q, intTemp;
    ios::sync_with_stdio(0);
#ifdef LOCAL
    freopen("out.txt", "w", stdout);
    freopen("in1.txt", "r", stdin);
#endif // LOCAL
    cin >> n >> q;
    for(int i = 1; i <= n; i++)
        cin >> num[i];
    build(1, n);

    //debug
    /**<
    for(int i = 0; i < 13; i++){
        cout << node[i].left << ' ' << node[i].right << ' ' << node[i].Min_Value << '\n';
    }
    return 0;
    */

    while(q--){
        cin >> strLine;
    }
}

```



```

    if(strLine[0] == 'q'){
        handle();
        cout << query(shift[0] , shift[1] ) << '\n' ;
    }
    else if (strLine[0] == 's'){
        handle();
        intTemp = num[shift[0]] ;

        for(int i = 1 ; i < len_shift ; i++){
            set_num(shift[i-1] , num[shift[i]] ) ;
            num[shift[i-1]] = num[shift[i]] ;
        }
        num[shift[len_shift-1]] = intTemp ;
        set_num(shift[len_shift-1] , intTemp );

        //debug
        //cout << intTemp << ' ' << shift[len_shift-1] << '\n' ;
        //for(int i = 1 ; i <= n ; i++)
        //    cout << num[i] << ' ' ;
    }
}
return 0;
}

```

7.5 Union-find disjoint sets (UFDS) by Bill

```

class UnionFind
{
public:
    UnionFind(int N)
    {
        rank.assign(N, 0);
        p.assign(N, 0);
        for (int i = 0; i < N; ++i) p[i] = i;
    }
    int findSet(int i) { return (p[i] == i) ? i : ( p[i] = findSet(p[i]) ); }
    bool isSameSet(int i, int j) { return findSet(i) == findSet(j); }
    void unionSet(int i, int j)
    {
        if ( !isSameSet(i, j) )
        {
            int x = findSet(i);
            int y = findSet(j);
            if (rank[x] > rank[y]) p[y] = x;    // rank keeps the tree short
            else
            {
                p[x] = y;
                if (rank[x] == rank[y]) ++rank[y];
            }
        }
    }
private:
    vi p, rank;
};

```

8 Utilities

8.1 Bit manipulation

```

#define isOn(S, j) (S & (1<<j))
#define setBit(S, j) (S |= (1<<j))
#define clearBit(S, j) (S &= ~(1<<j))
#define toggleBit(S, j) (S ^= (1<<j))
#define lowBit(S) (S & (-S))
#define setAll(S, n) (S = (1<<n)-1)

```

8.2 C++ input output

```

#include <iostream>
#include <iomanip>

using namespace std;

int main()
{

```

```

// Output a specific number of digits past the decimal point,
// in this case 5
cout.setf(ios::fixed); cout << setprecision(5);
cout << 100.0/7.0 << endl;
cout.unsetf(ios::fixed);

// Output the decimal point and trailing zeros
cout.setf(ios::showpoint);
cout << 100.0 << endl;
cout.unsetf(ios::showpoint);

// Output a '+' before positive values
cout.setf(ios::showpos);
cout << 100 << " " << -100 << endl;
cout.unsetf(ios::showpos);

// Output numerical values in hexadecimal
cout << hex << 100 << " " << 1000 << " " << 10000 << dec << endl;
}

```

8.3 C++ STL

```

// Example for using stringstream and next_permutation

#include <algorithm>
#include <iostream>
#include <sstream>
#include <vector>

using namespace std;

int main(void){
    vector<int> v;

    v.push_back(1);
    v.push_back(2);
    v.push_back(3);
    v.push_back(4);

    // Expected output: 1 2 3 4
    //                  1 2 4 3
    //                  ...
    //                  4 3 2 1
    do {
        stringstream oss;
        oss << v[0] << " " << v[1] << " " << v[2] << " " << v[3];

        // for input from a string s,
        //   stringstream iss(s);
        //   iss >> variable;

        cout << oss.str() << endl;
    } while (next_permutation (v.begin(), v.end()));

    v.clear();

    v.push_back(1);
    v.push_back(2);
    v.push_back(1);
    v.push_back(3);

    // To use unique, first sort numbers. Then call
    // unique to place all the unique elements at the beginning
    // of the vector, and then use erase to remove the duplicate
    // elements.

    sort(v.begin(), v.end());
    v.erase(unique(v.begin(), v.end()), v.end());

    // Expected output: 1 2 3
    for (size_t i = 0; i < v.size(); i++)
        cout << v[i] << " ";
    cout << endl;
}

```

8.4 Dates

```

// Routines for performing computations on dates. In these routines,
// months are expressed as integers from 1 to 12, days are expressed
// as integers from 1 to 31, and years are expressed as 4-digit
// integers.

#include <iostream>

```

```

#include <string>

using namespace std;

string dayOfWeek[] = {"Mon", "Tue", "Wed", "Thu", "Fri", "Sat", "Sun"};

// converts Gregorian date to integer (Julian day number)
int dateToInt (int m, int d, int y){
    return
        1461 * (y + 4800 + (m - 14) / 12) / 4 +
        367 * (m - 2 - (m - 14) / 12 * 12) / 12 -
        3 * ((y + 4900 + (m - 14) / 12) / 100) / 4 +
        d - 32075;
}

// converts integer (Julian day number) to Gregorian date: month/day/year
void intToDate (int jd, int &m, int &d, int &y){
    int x, n, i, j;

    x = jd + 68569;
    n = 4 * x / 146097;
    x -= (146097 * n + 3) / 4;
    i = (4000 * (x + 1)) / 1461001;
    x -= 1461 * i / 4 - 31;
    j = 80 * x / 2447;
    d = x - 2447 * j / 80;
    x = j / 11;
    m = j + 2 - 12 * x;
    y = 100 * (n - 49) + i + x;
}

// converts integer (Julian day number) to day of week
string intToDay (int jd){
    return dayOfWeek[jd % 7];
}

int main (int argc, char **argv){
    int jd = dateToInt (3, 24, 2004);
    int m, d, y;
    intToDate (jd, m, d, y);
    string day = intToDay (jd);

    // expected output:
    //      2453089
    //      3/24/2004
    //      Wed
    cout << jd << endl
         << m << "/" << d << "/" << y << endl
         << day << endl;
}

```

8.5 Prime numbers

```

// O(sqrt(x)) Exhaustive Primality Test
#include <cmath>
#define EPS 1e-7
typedef long long LL;
bool isPrimeSlow (LL x)
{
    if (x<=1) return false;
    if (x<=3) return true;
    if (!(x%2) || !(x%3)) return false;
    LL s=(LL)(sqrt((double)(x))+EPS);
    for (LL i=5; i<=s; i+=6)
    {
        if (!(x%i) || !(x%(i+2))) return false;
    }
    return true;
}

// Primes less than 1000:

```

```

//      2      3      5      7      11      13      17      19      23      29      31      37
//      41      43      47      53      59      61      67      71      73      79      83      89
//      97      101      103      107      109      113      127      131      137      139      149      151
//      157      163      167      173      179      181      191      193      197      199      211      223
//      227      229      233      239      241      251      257      263      269      271      277      281
//      283      293      307      311      313      317      331      337      347      349      353      359
//      367      373      379      383      389      397      401      409      419      421      431      433
//      439      443      449      457      461      463      467      479      487      491      499      503
//      509      521      523      541      547      557      563      569      571      577      587      593
//      599      601      607      613      617      619      631      641      643      647      653      659
//      661      673      677      683      691      701      709      719      727      733      739      743
//      751      757      761      769      773      787      797      809      811      821      823      827
//      829      839      853      857      859      863      877      881      883      887      907      911
//      919      929      937      941      947      953      967      971      977      983      991      997

// Other primes:
//      The largest prime smaller than 10 is 7.
//      The largest prime smaller than 100 is 97.
//      The largest prime smaller than 1000 is 997.
//      The largest prime smaller than 10000 is 9973.
//      The largest prime smaller than 100000 is 99991.
//      The largest prime smaller than 1000000 is 999983.
//      The largest prime smaller than 10000000 is 9999991.
//      The largest prime smaller than 100000000 is 99999989.
//      The largest prime smaller than 1000000000 is 999999937.
//      The largest prime smaller than 10000000000 is 999999997.
//      The largest prime smaller than 100000000000 is 9999999997.
//      The largest prime smaller than 1000000000000 is 99999999997.
//      The largest prime smaller than 10000000000000 is 999999999997.
//      The largest prime smaller than 100000000000000 is 9999999999997.
//      The largest prime smaller than 1000000000000000 is 99999999999997.
//      The largest prime smaller than 10000000000000000 is 999999999999997.
//      The largest prime smaller than 100000000000000000 is 9999999999999997.
//      The largest prime smaller than 1000000000000000000 is 99999999999999997.

```

8.6 Theorems

Euler path/tour theorems: An Euler path is a path that visits every edges exactly once. An Euler tour is an Euler path that starts and ends at the same vertex. A graph is an Eulerian-tour graph (i.e. it has an Euler tour) iff all of its vertices has even degrees. A graph is an Eulerian-path graph (i.e. it has an Euler path) iff all but 2 of its vertices has even degrees.

Euler's handshaking lemma: A graph does not have an Euler tour iff it has an even number of vertices of odd degrees.

Bipartite graph related theorems:

- (1) Min vertex cover (MVC) = Max cardinality bipartite matching (MCBM).
- (2) Max independent set (MIS) = $V - \text{MCBM}$.
- (3) The number of spanning tree of a complete bipartite graph $K(n,m)$ is $m^{n-1} * n^{m-1}$.

Cayley's formula: There are n^{n-2} spanning trees of a complete graph with n labeled vertices.

Derangement: A permutation of the elements of a set such that none of the elements appear in their original position. The number of derangements 'der(n)' can be computed as follow: $\text{der}(n) = (n-1) * (\text{der}(n-1) + \text{der}(n-2))$ where $\text{der}(0) = 1$ and $\text{der}(1) = 0$.

Erds Gallai's theorem: A necessary and sufficient condition for a finite sequence of natural numbers is the degree sequence of a simple graph. A sequence of non-negative integers $d_1 \geq d_2 \geq \dots \geq d_n$ can be the degree sequence of a simple graph on n vertices iff

- (1) $\sum_{i=1}^n d_i$ is even, and
- (2) $\sum_{i=1}^k d_i \leq k * (k-1) + \sum_{i=k+1}^n \min(d_i, k)$ holds for $1 \leq k \leq n$.