**Exercises -** Section 1: Lecture 1 – Fixed Costs, Linking to Continuous Variables - Questions

For each of the following situations, create binary variables to model the yes/no decisions, and use those binary variables to model the specified expressions (you might also need to define some notation for data). The first one has been done for you as an example.

1. A law office has been offered eight different cases, and needs to choose which ones among the eight to take on. Each case has an expected profit and an expected amount of person-time required.

| English | Math | gurobipy |
|---|---|---|
| a. Create binary variables to model whether or not the office takes each case. | Variables<br>$x_i \in \{0,1\}$ for all cases $i$ (where $x_i = 1$ denotes that the office takes case $i$, and $x_i = 0$ denotes that the office does not take case $i$ | `#DATA`<br>`# read N from file`<br>`# N is number of cases`<br><br>`#VARIABLES`<br>`x = m.addVars(N, vtype = GRB.BINARY)` |
| b. Write an expression to model the total expected profit from the cases taken. | Data<br>$p_i$: profit from case $i$<br><br>Total profit<br>$\sum_i p_i x_i$ | `#DATA`<br>`# read p[] from file`<br>`# p[i] is profit from case i`<br><br>`#EXPRESSION`<br>`sum(p[i]*x[i] for i in range(N))`<br>*or*<br>`x.prod(p)` |
| c. Write an expression to model total expected person-time required for the cases taken. | Data<br>$t_i$: person-time for case $i$<br><br>Total person-time<br>$\sum_i t_i x_i$ | `#DATA`<br>`# read t[] from file`<br>`# t[i] is profit from case i`<br><br>`#EXPRESSION`<br>`sum(t[i]*x[i] for i in range(N))`<br>*or*<br>`x.prod(t)` |
| d. Let $z_i$ be a variable denoting the amount of office paper that will be purchased for each case $i$ (up to a maximum of 10 reams). Write a constraint for each case to | Variables<br>$z_i \geq 0$ for all cases $i$<br><br>Constraint<br>$z_i \leq 10x_i$ | `#VARIABLES`<br>`z = m.addVars(N, vtype = GRB.CONTINUOUS)`<br><br>`#CONSTRAINT` |

| | | |
|---|---|---|
| specify that no paper will be purchased for the case unless the office takes the case. | | `m.addConstrs(z[i] <= 10*x[i] for i in range(N))` |

2. A manufacturer wants to rent warehouses in which to store raw materials before use. There are six different locations available for rent, each with a different annual rental cost and a different capacity.

| **English** | **Math** | **gurobipy** |
|---|---|---|
| a. Create binary variables to model whether or not the manufacturer rents each warehouse. | | |
| b. Write an expression to model the total annual rental cost for the warehouses that are rented. | | |
| c. Write a constraint to specify that the total capacity of the rented warehouses must be at least 100,000 square feet. | | |
| d. Let $L_i$ be a variable denoting the number of square feet of each warehouse $i$ that will be devoted to storing large items. Write a constraint for each warehouse to specify that no space can be devoted to large items unless the warehouse is rented. | | |

3. A supermarket chain is considering expanding by placing new stores in up to 100 communities next year. For each community, the chain has built a model to predict annual store profit.

| **English** | **Math** | **gurobipy** |
|---|---|---|
| a. Create binary variables to model whether or not the chain puts a store in each community. | | |
| b. Write an expression for the store's overall expected profit from the new stores. | | |
| c. Let $f_i$ be a variable denoting the fraction of each new supermarket $i$ that will hold frozen items. Write a constraint for each community to specify that no frozen items can be stocked in that community's store unless the store is built. | | |

4. A professional sports team is considering which of 50 players to offer a contract to. Contract offers can be from 1 to 10 years  For each player, the team has built a model to predict the player's value to the team in each of the next 10 years.

| **English** | **Math** | **gurobipy** |
|---|---|---|
| a. Create binary variables to model whether or not the team offers a contract to each of the players.<br><br>b. Create a new binary variable to model whether or not the team offers a contract to each player for each number of years.<br><br>c. Write a constraint to link the variables you created in parts a. and b., so that if a player is offered a contract then the player must be offered a contract of 1 through 10 years, and vice versa.<br><br>d. Assuming every player offered a contract accepts it, write an expression for the overall value the team would get in each year. | | |

| | | |
|---|---|---|
| e. Let $m_i$ be a variable denoting the money spent (out of a total marketing budget $M$) marketing each new player $i$ to season ticket holders. Write a constraint for each player to specify that no money will be spent marketing the player unless the player is offered (and, we assume, accepts) a contract. | | |

5. A nation's military has identified 16 potential research projects that it might fund this year. For each project, the military has estimated the total cost of research.

| **English** | **Math** | **gurobipy** |
|---|---|---|
| a. Create binary variables to model whether or not the military funds each project. | | |
| b. Write a constraint to specify that the total cost of funded projects cannot exceed the total annual budget for new research projects. | | |
| c. Let $s_i$ be a variable denoting the amount of hours each project $i$ requires on a specific supercomputer per day on average. Write a constraint for each project to specify that no supercomputer time will be used unless the project is funded. | | |

**NOTES:**