Exercises - Section 3: Lecture 10 – Dealing with Infeasibility - Questions

1. [This question is based on an exercise from an old optimization textbook.[1]] A family owns a 45-acre farm on which they plant wheat and corn. Each acre of wheat planted requires 3 labor hours per week and 2 tons of fertilizer, and each acre of corn planted requires 2 labor hours per week and 4 tons of fertilizer. Total, the family has 100 hours of labor available per week, and 120 tons of fertilizer available.

   The family would like to find the best amount of wheat and corn to plant in order to maximize its profit for the year. The profit per acre of wheat planted is $2000 and the profit per acre of corn planted is $3000. Acres planted do not have to be integer (so, for example, a solution with 22 7/8 acres of wheat and 22 1/8 acres of corn is allowable).

   a. Assuming that all 45 acres must be planted, create a mathematical model and a gurobipy model that the family can optimize to maximize its profit, and solve the gurobipy model.

   b. If you did part a. correctly, you found that the model was infeasible. Now, suppose that not all acres need to be planted. Change your mathematical and gurobipy models, and solve the new gurobipy model. (The optimal solution should specify planting 20 acres of wheat and 20 acres of corn, which leaves 5 acres unplanted.)

2. A high-capacity print shop has three different machines that each print, staple, and/or bind booklets. The machines are not identical; they take different amounts of time for the same job, and they have different ink costs. There are 10 different jobs that need to be completed today in the 13 hours (7am to 8pm) that the print shop is open. The table below shows how long each job will take on each machine, and the cost of ink for each job on each machine.

JOB	TIME (HOURS)			INK COST		
	Machine 1	Machine 2	Machine 3	Machine 1	Machine 2	Machine 3
1	2	3	4	400	200	300
2	3	3	3	350	300	350
3	4	6	6	400	300	350
4	4	6	7	200	350	250
5	2	3	6	300	300	300
6	5	5	7	300	250	400
7	3	4	7	250	200	300
8	4	4	5	300	300	350
9	5	5	7	400	300	500
10	3	5	6	250	250	300

[1] Winston, Wayne L. *Introduction to Mathematical Programming*, 2nd ed. (1995). p. 60, question #6.

a. Create a mathematical model and a gurobipy model that the print shop manager can optimize to determine which machine to run each job on, in order to minimize the total ink cost while printing all ten jobs.

b. If you did part a. correctly, you found that the model was infeasible; there isn't enough time to complete all of the jobs in one day. Change your mathematical and gurobipy models to maximize the number of jobs that can be done today, regardless of cost.

c. Instead of your model in part b. that leaves some jobs undone, suppose the print shop manager will stay late in order to get all jobs completed. Change your original model of part a. to minimize the number of hours between 7am and when all jobs are finished. [Hint: Consider adding a variable for the number of hours between 7am and when all of the jobs are finished, and require that the its value be greater than or equal to the total time required on each machine.]

d. Instead of your models in parts b. and c., suppose the print shop manager can split some jobs between machines (e.g., if there are many copies of a booklet to be printed, some fraction of them can be printed on each machine). Change your original model of part a. to allow fractional printing, and minimize the total ink cost while printing all jobs.

e. Your models in b., c, and d. are three different ways of dealing with the infeasibility you discovered in a. Take a minute to think about the differences between them, and which you might suggest using in different situations.
   - Compromising on the number of jobs completed as done in (b) might be considered when working overtime is not an option or the cost of opening the shop for one hour is much more than the cost of losing a customer. This option can be only considered if the owner is not contractually obligated to complete all the jobs on the same day.
   - Keeping the shop open for more hours as done in (c) might be considered when customer-satisfaction is key for the manager or there is contractual obligation.
   - Splitting the jobs among different machines as done in (d) might be a good solution if it does not affect the quality of the finished products.

NOTES: