Exercises - Section 2: Lecture 12 – Assignment and Generalized Assignment Archetypes - Questions

For each of the following network pictures of assignment models, write the optimization model mathematically and in gurobipy, and solve the gurobipy model.

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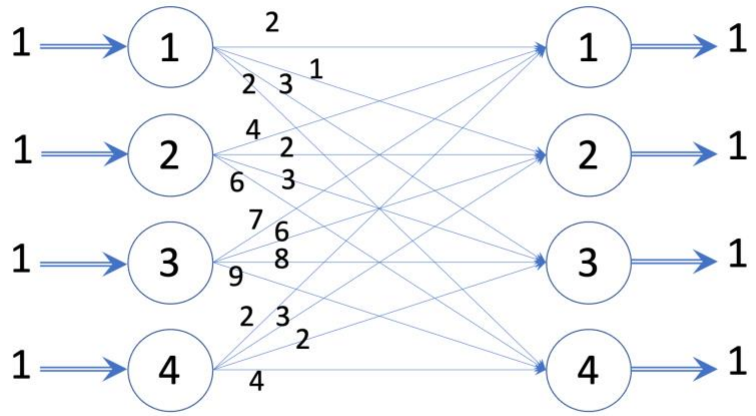

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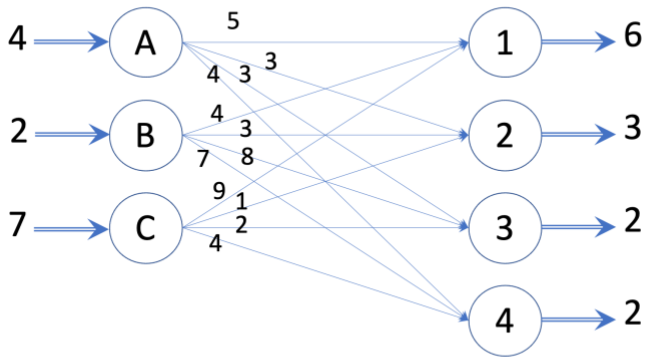


For each of the following situations, draw the appropriate assignment model network picture.

3. The department chair of a small college math department needs to decide which of four professors will teach each of the department's four calculus courses: Calculus 1 (Differentiation), Calculus 2 (Integration), Multivariable Calculus, and Survey of Calculus. The chair has data from student ratings of each professor teaching each course:

	Calculus 1	Calculus 2	Multivariable	Survey
Professor Smith	4.4	4.3	4.3	4.1
Professor Johnson	4.1	4.1	4.2	4.2
Professor Williams	3.8	3.2	3.4	4.1
Professor Plum	4.8	4.9	4.7	5.0

The department chair wants to create a course assignment where each professor teaches one course (and each course is taught by one professor), in order to maximize the average student rating in the four courses. Draw this scenario as an assignment network problem.

4. A Canadian refrigerator manufacturer operates in the provinces of Saskatchewan, Manitoba, and Alberta, as well as U.S. states Montana and North Dakota. The company's three manufacturing plants supply customers throughout the region. Refrigerators are shipped from the plants to appliance superstores by truck.

The refrigerator manufacturer has three plants: Saskatoon (Saskatchewan), Portage la Prairie (Manitoba), and Medicine Hat (Alberta). Its goods are sold in appliance superstores in five cities: Moose Jaw (Saskatchewan), Winnipeg (Manitoba), Great Falls (Montana), Bismarck (North Dakota), and Prince Albert (Saskatchewan).

The trucking company employed by the refrigerator manufacturer has quoted shipping costs as shown below in Table 1.

	Moose Jaw, SK	**Winnipeg, MB**	**Great Falls, MT**	**Bismarck, ND**	**Prince Albert, SK**	**Storage**
Saskatoon, SK	$5750	$7500	$7625	$7750	$5500	$1000
Portage la Prairie, MB	$6750	$5250	$9750	$6875	$7750	$800
Medicine Hat, AB	$6250	$8125	$6500	$8250	$7000	$1300

Table 1. Approximate truckload costs between plants and superstores (Canadian dollars).

This month, the company estimates that the demand for refrigerators at each superstore is as shown in Table 2.

Moose Jaw	2
Winnipeg	4
Great Falls	8
Bismarck	4
Prince Albert	1

Table 2. Refrigerator demand by location (# of truckloads of refrigerators)

The refrigerator manufacturer's main plant is the Saskatoon factory, where it produces 9 truckloads of refrigerators each month. The smaller plants in Portage la Prairie and Medicine Hat produce 6 truckloads of refrigerators each month. Total demand is low this month, so the excess (unshipped) refrigerators are kept in storage. Because of each Canadian province's different tax laws, the storage/inventory costs differ from plant to plant. Each truckload of refrigerators costs $1000 to keep in inventory in Saskatoon, $800 in Portage la Prairie, and $1300 in Medicine Hat.

The manufacturer wants to know how many truckloads of refrigerators to send from each plant to each superstore, and how many truckloads of refrigerators at each plant to keep in storage. Draw this scenario as an assignment network problem.

5. Two nights ago, an astronomer took a picture of a small portion of the sky containing 10,000 objects (stars, planets, asteroids, etc. Last night, the astronomer took a picture of the same portion of sky (containing the same 10,000 objects, but in slightly different locations). The objects have all moved, and by measuring the distance they have moved the astronomer can estimate their speed relative to Earth.

   Unfortunately, it's not quite that easy. Because the astronomer does not know the speed or direction of motion of each object beforehand, there is no way for the astronomer to be sure which dot in the second picture is the same object as a certain dot in the first picture.

   Although the astronomer can't be sure which dot is which, based on the size and brightness of each dot, it's possible to estimate the probability p_{ij} that object i in the first picture is the same as object j in the second picture. (Assume that all probabilities are independent.)

   Using these probabilities, it is possible to use an assignment model to guess which object is which. Draw this scenario as an assignment model (you can just draw a small part of it; you don't need all 10,000 objects!) including the objective function value on each arc. (Hint: The joint probability of all assignments is the product of each assignment's probability; remember (from many maximum likelihood-based models in data science) that one way to maximize the product of probabilities is to maximize the sum of their logs.)

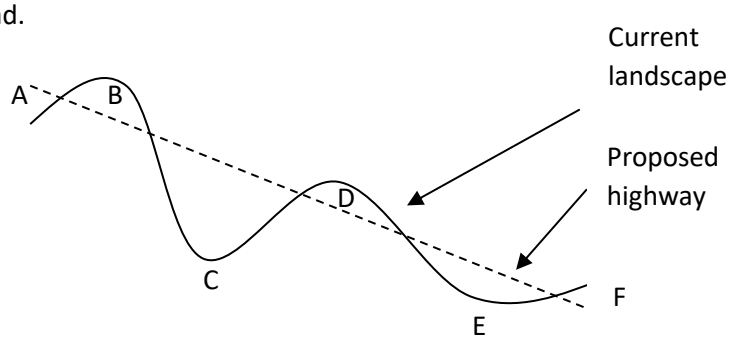
For each of the following situations, draw the appropriate assignment model network picture, write the optimization model mathematically and in gurobipy, and solve the gurobipy model.

6. A youth basketball coach has a team of 6 players. Based on the players' skills, the coach wants to assign primary positions to each player. The team needs to have a center, power forward, small forward, shooting guard, point guard, and sixth player (who doesn't start the game but comes in later off the bench). The table below shows the coach's estimate of how valuable each player would be in each position.

	Center	Power forward	Small forward	Shooting guard	Point guard	Sixth player
Player A	5	4	2	3	3	2
Player B	8	6	4	2	2	4
Player C	8	5	2	1	1	4
Player D	4	4	5	8	7	4
Player E	1	2	4	10	8	5
Player F	1	1	2	3	6	2

Draw the assignment network picture, write the model mathematically and in gurobipy, and solve the gurobipy model to see which player should play each position in order to maximize the total value to the team.

7. The Department of Transportation (DOT) is building a new highway. The highway goes through a hilly region and instead of having the road go up and down repeatedly, the DOT would like to have the road ascend steadily. The figure below shows the current landscape and the proposed road.



In order to give the landscape a constant slope, dirt must be removed from areas B, D, and F and dirt must be added to areas A, C, and E. Dirt can be moved from area to area, and dirt can also be brought in from outside, if necessary. The dirt requirements are given in the tables below.

A. Area	B. Dirt Needed
A	20 tons
C	80 tons
E	40 tons

C. Area	D. Extra Dirt
B	15 tons
D	10 tons
F	90 tons

The table below shows the cost per ton of shipping dirt between areas:

To / From	A	C	E
B	$500	$400	$1000
D	$1000	$600	$500
F	$1600	$1200	$700
Outside	$2000	$2200	$2000

Draw the assignment network picture, write the model mathematically and in gurobipy, and solve the gurobipy model to find the dirt transportation plan that minimizes the total costs.

8. In a game of chess, one player is "White" and the other is "Black". The results of the first round of a 1963 tournament between eight of the best chessplayers in the world are shown below.

White	Black	Winner
Paul Keres	Tigran Petrosian	None (Draw)
Miguel Najdorf	Oscar Panno	Najdorf
Fredrik Olafsson	Svetozar Gligoric	None (Draw)
Samuel Reshevsky	Pal Benko	Reshevsky

Players receive one point for winning a game, zero points for losing a game, and ½ of a point each if nobody wins (a "draw"). Therefore, the standings after the first round were the following:

Players	E. Points
Reshevsky, Najdorf	1
Keres, Petrosian, Olafsson, Gligoric	½
Panno, Benko	0

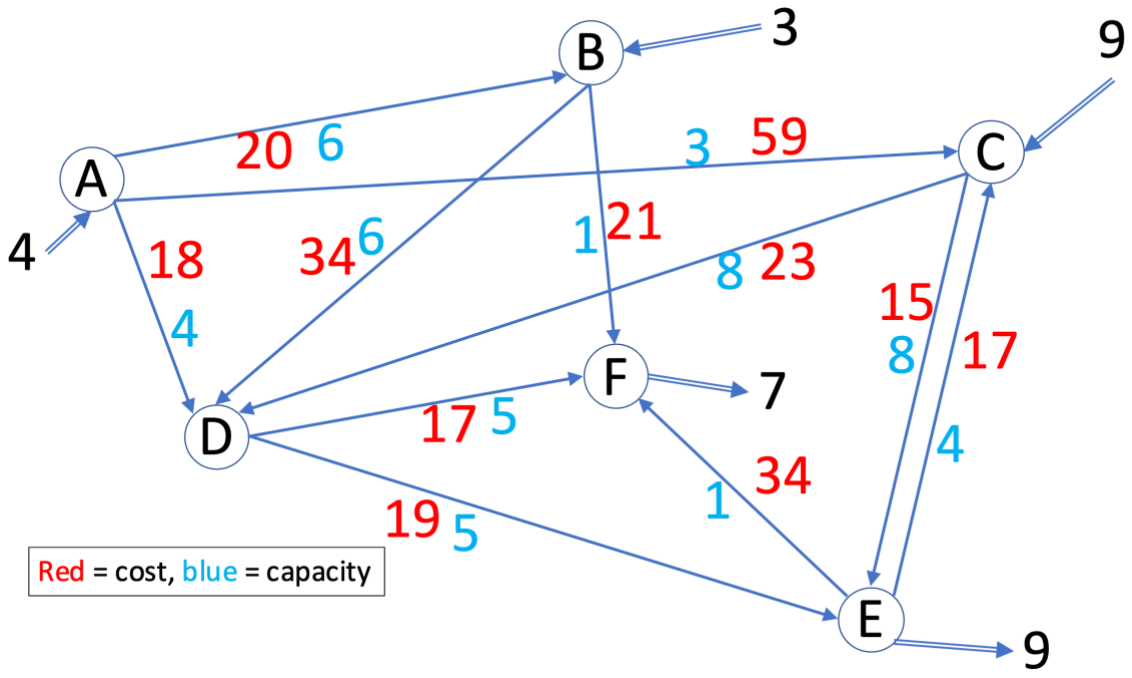
After seeing the results of round 1, the tournament director would like to determine who should play each other in round 2. Players who were White in round one, must be Black in round 2, and players who were Black in round 1 must be White in round 2. If two players were opponents in the first round, they cannot play each other again in the second round. Each player must play one game in round 2.

To get "interesting" games in round 2, the tournament director has decided to use the following formula: the "value" of a game is equal to the square of the sum of the scores of the two players. For example, a game between Reshevsky and Gligoric would have a value of $\left(1+\frac{1}{2}\right)^2 = \frac{9}{4}$. The tournament director would like to maximize the total value of the four games that are played in round 2.

Draw a network picture to show how the tournament director's question fits the assignment archetype.

For each of the following network pictures, identify which type of network optimization archetype (shortest path, assignment or neither) it is.

9.



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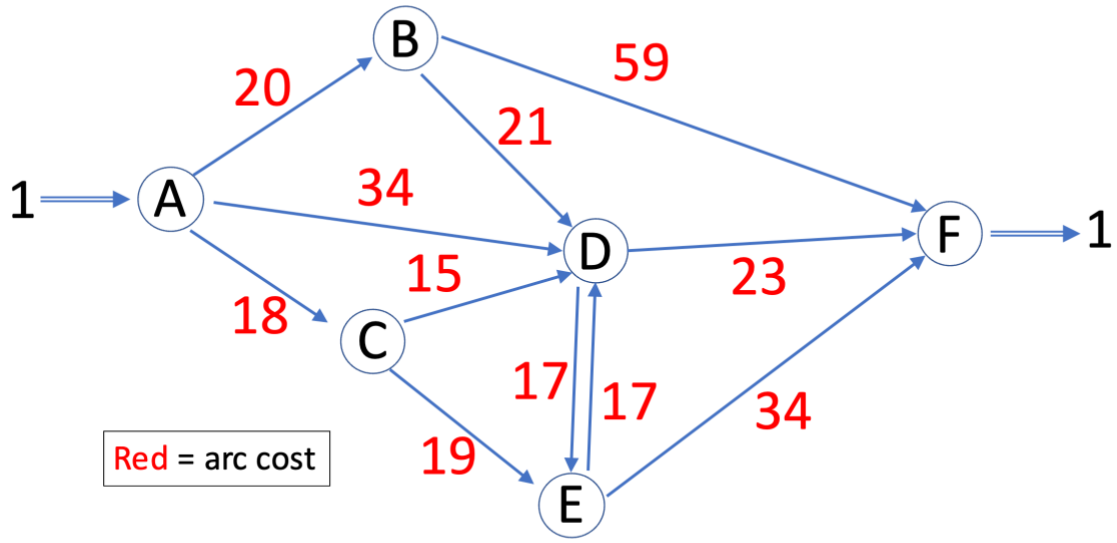
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Red = arc cost
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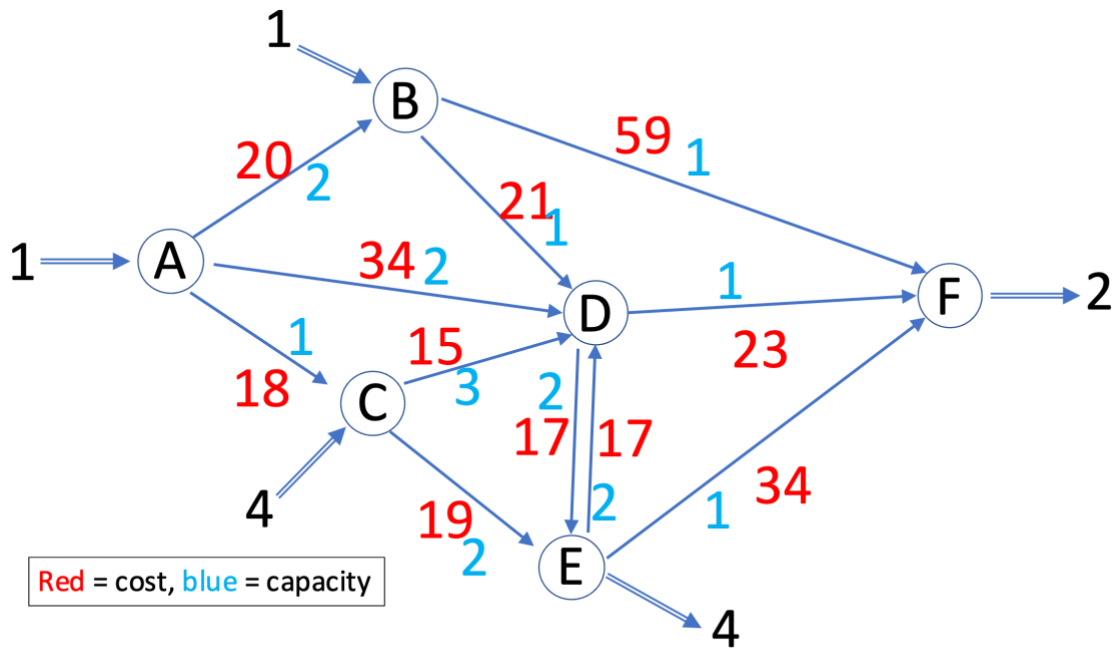
11.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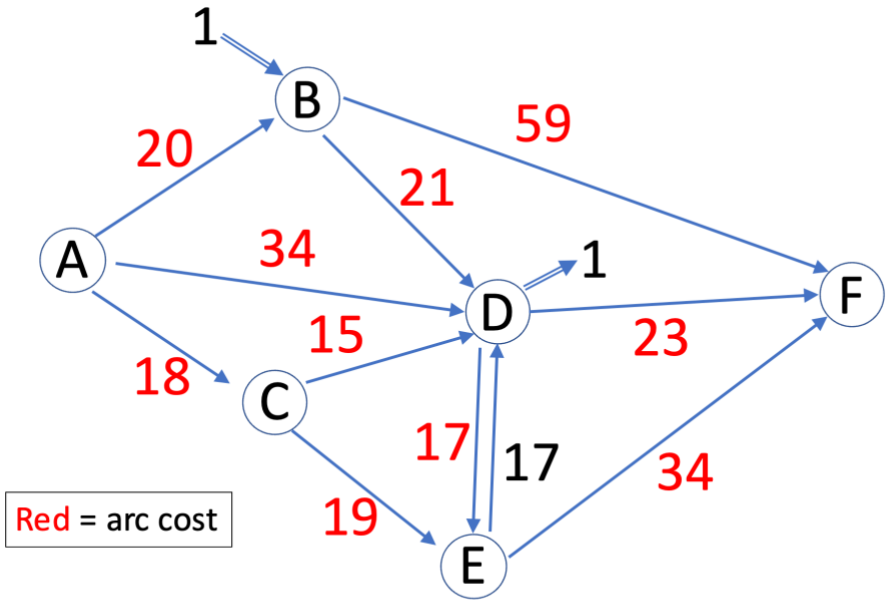
12.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Red = arc cost

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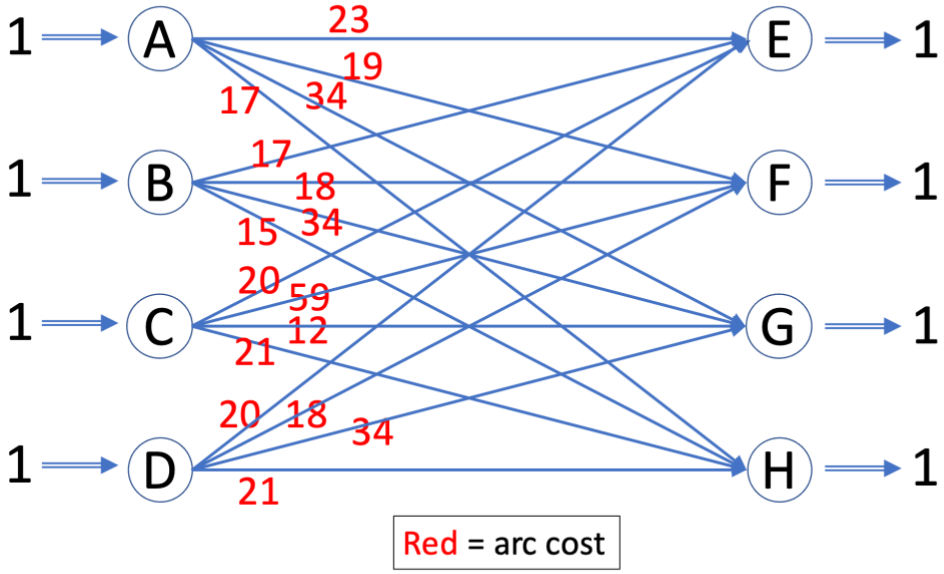
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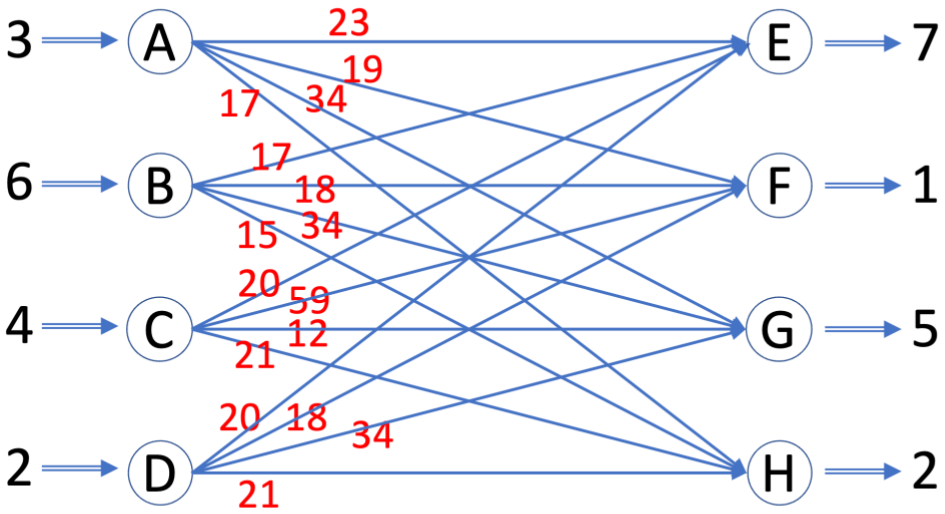
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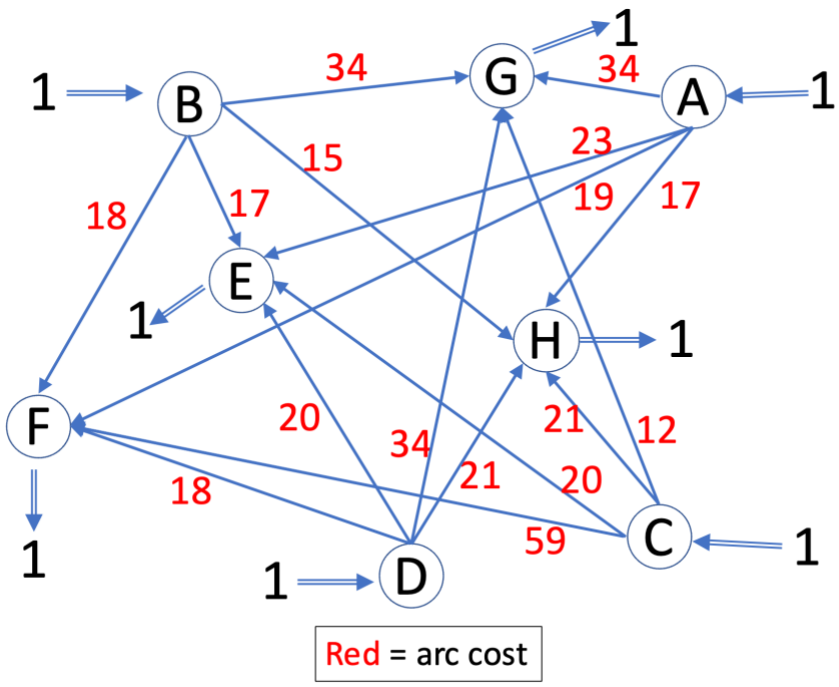
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Red = arc cost

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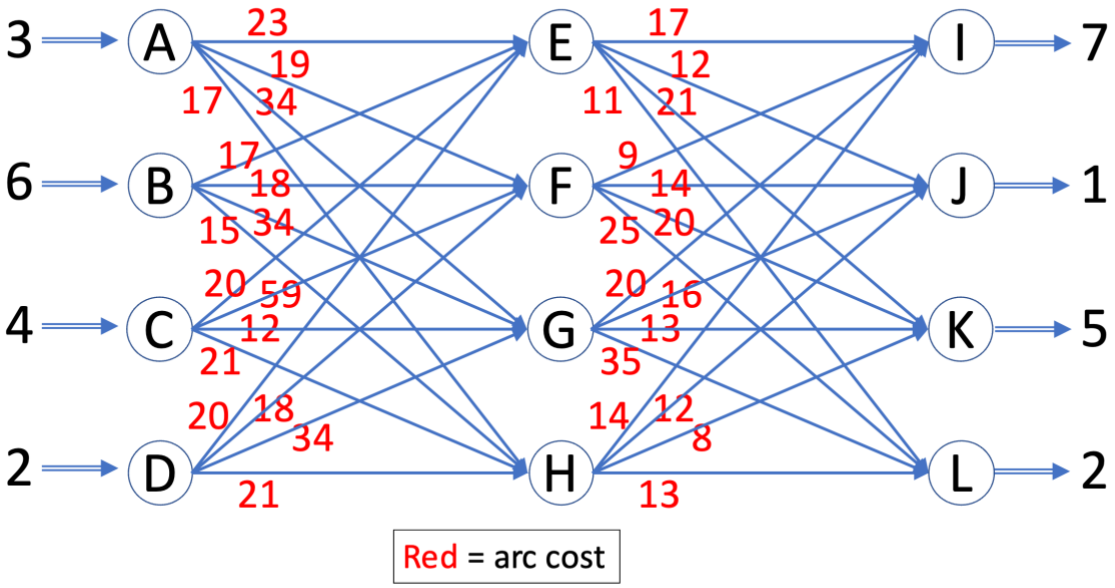
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Red = arc cost

NOTES: