Exercises - Section 1: Lecture 6 – Integer Programming Modeling Examples - Solutions

1. A small city has a significant budget surplus that the mayor wants to spend on special projects. Each member of the city council has suggested one or more projects, and the mayor would like to select a set of projects so that (1) every council member has at least one suggestion selected, and (2) the total number of person-benefits is highest (if a person benefits from k different projects, count that person as having k person-benefits). Of course, the mayor also needs to stay within budget: the total surplus is $20 million, so the selected projects can't cost more than that total.

   (a) Create mathematical and gurobipy models that the mayor could use to choose a set of projects according to the guidelines above. Solve the gurobipy model.

Project	Council member(s) who suggested project	People who would primarily benefit from project	Project cost
Large new city park	A,C,E	20,000	$ 6,000,000
Pothole repairs	A,B,D,F	200,000	$ 12,000,000
Renovate city hall	E	500	$ 2,400,000
Install electric vehicle chargers	D,E	16,000	$ 4,000,000
Tax incentive to attract investment	A,F,G	10,000	$ 3,000,000
Property tax rebate	B,G	100,000	$ 18,000,000
Increase police force	B,C,G	25,000	$ 1,000,000

   (b) After conferring with political advisors, the mayor has realized that it is important to fund projects that help people in different areas of the city and people with different interests, and has therefore come up with some additional guidelines for choosing projects:

   - If there is a property tax rebate, then the police force should be increased.
   - If either the new park is built or the electric vehicle chargers are installed, then there should be a tax incentive to attract investment.
   - No more than one tax program (incentive to attract investment, property tax rebate) can be chosen.
   - City hall can be renovated only if potholes are repaired.
   - The new park and the increased police force must either both be chosen or neither be chosen.

   Add these guidelines to your mathematical and gurobipy models, and re-solve the gurobipy model. Have the advisors been helpful?

2. The terms of a will require the executor to divide the deceased person's most valuable items among three heirs in a way that the total value given to each heir is as close as possible, without selling any of the items and without dividing any of the items (so, for example, the stamp collection must remain together rather than being split up between two or more heirs). A solution where each heir receives a value of $234,000 (1/3 of the total value) would be ideal, but it might not be feasible. An additional restriction is that all of the music and art items (painting, sculpture, and bass clarinet) must be given to different people.

   Create mathematical and gurobipy models that the executor could use to find the division of items into three sets that minimizes the sum of each set's squared deviation from the ideal of $234,000. (I.e., if one set of items has value $260,000, then that set's squared deviation would be ($260,000 – $234,000)^2.) Solve the gurobipy model.

Item Number	Description	A. Value
1	Painting	$145,000
2	Sculpture	$120,000
3	Stamp Collection	$115,000
4	Diamond Necklace	$135,000
5	Pickup Truck	$32,000
6	Stock Portfolio	$123,000
7	Professional Model Bass Clarinet	$15,000
8	Baseball Memorabilia Collection	$17,000

3. Map makers need to ensure that countries adjacent on a map are shown in different colors. Given the map of South America shown below, create mathematical and gurobipy optimization models that can determine what color to use for each country in a way that minimizes the number of ink colors required. Solve the gurobipy model.

Image taken from https://en.wikipedia.org/wiki/File:South_America_(orthographic_projection).svg under Creative Commons Attribution 3.0 Unported license. Country names were added on top of image and color was adjusted.

Hint: One approach is for one set of variables to be x_{ij} that are 1 if country i is colored with color j and 0 if not, and for another set of variables to be y_j that are 1 if color j is used and 0 if not.

4. A expert network firm is putting together a focus group for a software company, and they would like to be able to achieve sufficient diversity in the dimensions that are important to their client. The table below shows all of the firm's experts who have expressed interest in participating in the focus group, along with their attributes and their hourly consulting rate. It also shows the minimum and maximum participants required with each attribute. Because experts 4 and 8 do not get along with expert 10, the firm can either choose expert 10 without 4 and 8; or one or both of experts 4 and 8 without expert 10; or none of experts 4, 8, and 10.

   Create mathematical and gurobipy models that the expert network firm could use to determine which participants to use in the focus group. Solve the gurobipy model.

Expert	Familiarity with company's software	Years of experience in industry	Level of employment	Country of residence	Cost per hour
1	Expert user	12	Director	USA	$ 1,000
2	Occasional user	30	VP	USA	$ 3,000
3	None	7	Senior Analyst	India	$ 300
4	Occasional user	22	Director	UK	$ 1,000
5	Expert user	14	Director	USA	$ 1,800
6	Expert user	9	Senior Analyst	India	$ 400
7	None	25	VP	UK	$ 1,500
8	Expert user	3	Analyst	UK	$ 300
9	Occasional user	12	Senior Analyst	USA	$ 1,000
10	Expert user	10	Director	India	$ 800
Total of 4-6 people	2-3 experts, 1-2 occasional users, 1 non-user	at least one < 10 at least one > 20 at least one 10-20	At least one of each level (VP, Director, Senior Analyst, Analyst	At least one from each country (USA, UK, India)	As low as possible

5. A certain company is planning to build assembly facilities from which it will transport finished products to each of the 48 contiguous United States, plus the District of Columbia. The company needs to decide where to build the assembly facilities, and it has identified one candidate location in each state.

The cost of a facility (operating cost plus prorated building cost) depends on conditions in each state. The company wants to satisfy all of its demand, and to do so it pays to transport its products. The shipping charge is 10 cents per unit per mile, so shipping 5 units a distance of 200 miles each incurs a total cost of 5*10*200 = 10,000 cents (or $100). State-to-state distances are approximated by using the distances between state capitals. All data (demand, facility cost, and distance) are given in the accompanying Excel file.

(a) Create mathematical and gurobipy models that the company could use to determine where to build assembly facilities, and how much of each state's demand will be shipped from each assembly facility, assuming that facilities can assemble as much finished product as needed.

(b) The model in part (a) is great if you have access to a full version of Gurobi's software, either because your employer has a license for it or because you're an academic (faculty or student) and have access to Gurobi's always-free-for-academics license.

But if not, you're limited to 2000 variables and 2000 constraints, which is less that what's in the model above, so the model size needs to be decreased.

The constraints are easy to deal with; there are only 49 of the first constraint, and if you chose the first option for the second constraint (one for each pair of states, or 49x49=2401) then you can switch to the second option instead, which only is 49. The binary-variable constraints aren't needed in gurobipy at all because they're part of the variable declaration.

The variables are harder to deal with, though. There are 49 x-variables and 2401 y-variables, for a total of 2450 – which means 450 need to be eliminated. As a heuristic, one might assume that no product will be shipped more than 1600 miles. Conveniently, there are 450 state-to-state distances that are farther than 1600 miles – so the gurobipy model can be rewritten to avoid declaring or using any of those variables (changes shown in **bold red** in the solutions – it's more-advanced than what has been covered so far, so if you're not sure how to do it, go ahead and look at the solutions):

Then, whether with the full model or the reduced-size model, solve the gurobipy model.

(c) Suppose no facility can assemble more than 250,000 units of product per year. Modify the models in part (a) and re-solve the gurobipy model.

(d) To try to ensure that the company's supply chain doesn't get completely disrupted by weather events, the company would like to build at least three facilities must be built in each of the following areas of the country:

i. WA, OR, CA, MT, ID, WY, NV, UT, CO, AZ, NM
ii. ND, SD, NE, KS, MO, IA, MN, WI, IL, IN, MI, OH
iii. TX, OK, AR, LA, FL, GA, AL, MS, TN, SC, NC, VA, KY, WV, MD, DE, DC
iv. NY, NJ, PA, CT, RI, MA, NH, VT, ME

The company also requires that if there is a facility in California, then there must be one in either Arizona or Nevada (or both).

Finally, to balance between growing and shrinking states, if there is a facility in at least two of California, New York, and Illinois, then the must be a facility in at least one of Texas, Florida, Georgia, and North Carolina.

Add these constraints to the mathematical and gurobipy models, and re-solve the gurobipy model.

NOTES: