Exercises - Section 2: Lecture 6 - **Variable Substitution** - Questions

1. A high-end musical instrument manufacturer projects that it can sell up to 100 trumpets, 40 flugelhorns, 40 baritones, and 20 contrabasses in the next quarter-year. Creating pistons for these instruments requires three precision machining steps: broaching, brazing, and grinding. The table below shows how many minutes are required in each process for each type of musical instrument.

	Trumpet	**Flugelhorn**	**Baritone**	**Contrabass**
Broaching	10	8	12	15
Brazing	20	20	20	20
Grinding	10	12	15	20

Table. Minutes required per instrument on each type of machine

To allow for maintenance and downtime, the company does not want to its machines beyond a certain limit. The total time available on the machines is 2000 hours for broaching, 3000 for brazing, and 2000 hours for grinding.

Once instruments are manufactured, they go to a tester, who is a professional musician; the musician is contracted to test exactly 150 instruments per quarter, no more and no less. Therefore, the company must manufacture exactly 150 instruments this quarter.

The company's profit is approximately $600 per trumpet, $700 per flugelhorn, $1000 per baritone, and $1500 per contrabass.

a. Create a mathematical model and a gurobipy model that the manufacturer can optimize to determine how many instruments of each type to manufacture in order to maximize its quarterly profit. Include an equality constraint to say that the tester can test exactly 150 instruments per quarter (so exactly 150 must be manufactured). Solve the gurobipy model.

b. Instead of the equality constraint to say that exactly 150 instruments should be manufactured, use that constraint to substitute out the variable for flugelhorn production. Write the new mathematical and gurobipy models, and solve the gurobipy model.

c. If your solution to model b. produces more than 150 instruments, what do you think went wrong? (Hint: If you plug the solution's recommended number of trumpets, baritones, and contrabasses into the equality constraint for the tester in model a., how many flugelhorns would be manufactured?)

d. On the other hand, if your solution to model b. did produce exactly 150 instruments, you probably remembered to substitute the equality constraint into the nonnegativity and upper-bound constraints for the variable for manufacturing flugelhorns (the variable should be greater than or equal to zero, and less than or equal to 40). If so, remove those

constraints to see what wrong solution you might've received if you had forgotten to substitute into the bound.

2. A marketing firm is purchasing data sets to help train forecasting models. The firm would like to purchase exactly eight out of twelve available data sets. The table below lists the data sets, their cost, and the type of data they contain.

Data Set	Cost	Type of Data
1	$1.1M	Retail, USA
2	$0.5M	Retail, Europe
3	$0.2M	Housing, Canada
4	$0.7M	Credit, Europe
5	$1.1M	Credit, USA
6	$0.9M	Credit, Japan
7	$1.0M	Retail, Japan
8	$1.6M	Credit and Retail, USA
9	$0.8M	Online retail, Canada
10	$0.5M	Housing, USA
11	$0.7M	Online retail, Europe
12	$1.0M	Online retail, USA

To train its models, the company needs at least three data sets that contain retail data (including at least one online retail data set and at least one general retail data set), at least one housing data set, and at least two credit data sets. The company needs at least two data sets from the USA, at least two from Europe, and at least one each from Japan and Canada.

a. Create a mathematical model and a gurobipy model that the company can optimize to determine which data sets to purchase in order to minimize its spend. Include an equality constraint to say that the company must purchase exactly eight of the data sets. Solve the gurobipy model.

b. Instead of the equality constraint to say that exactly eight data sets should be purchased, use that constraint to substitute out the variable for purchasing data set 2. Write the new mathematical and gurobipy models, and solve the gurobipy model.

c. If your solution to model b. purchases fewer than seven data sets, what do you think went wrong? (Hint: check your solution against all of the original constraints and objective. How many duplicates of data set 2 does the model seem to be purchasing?)

d. On the other hand, if your solution to model b. did purchase exactly eight data sets, you probably remembered to substitute the equality constraint into the implied bound constraints for the variable for purchasing data set 2 (the variable should be greater than or

equal to zero, and the variable should be less than or equal to one). If so, remove those constraints to see what wrong solution you might've received if you had forgotten to substitute into the bounds.

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