

Introduction to Optimization Through the Lens of Data Science Course Exercises

Exercises - Section 3: Lecture 11 – Sensitivity - Questions

This set of exercises builds on the ones used in the infeasibility lesson (7.2). Your initial models will be similar to your models in that lesson's exercises.

1. [This question is based on an exercise from an old optimization textbook.¹] A family owns a 45-acre farm on which they plant wheat and corn. Each acre of wheat planted requires 3 labor hours per week and 2 tons of fertilizer, and each acre of corn planted requires 2 labor hours per week and 4 tons of fertilizer. Total, the family has 100 hours of labor available per week, and 120 tons of fertilizer available.

The family would like to find the best amount of wheat and corn to plant in order to maximize its profit for the year. The profit per acre of wheat planted is \$2000 and the profit per acre of corn planted is \$3000. Acres planted do not have to be integer (so, for example, a solution with 22 7/8 acres of wheat and 22 1/8 acres of corn is allowable). Not every acre needs to be planted.

- a. Create a mathematical model and a gurobipy model that the family can optimize to
 maximize its profit, and solve the gurobipy model. [The optimal solution should be to plant
 20 acres of wheat and 20 acres of corn, and leave 5 acres unplanted.]
- b. Suppose the number of weekly labor hours available increases to 150 when another family member comes to live and work at the farm. What is the new optimal solution?
- c. In addition to the changes in b., suppose a drought in the wheat-producing areas of Europe and Asia increases the profit per acre to \$4000 per acre of wheat and \$3200 per acre of corn. What is the new optimal solution?
- d. In response to the changes in c., many corn farmers are considering switching over to wheat. To ensure sufficient corn supplies, the US government will give a subsidy equivalent to \$300 per acre of corn grown. What is the new optimal solution? [Modeling hint: the subsidy will increase the profit per acre of corn by the amount of the subsidy.]
- e. Oops, the subsidy in e. isn't enough for this farmer's optimal solution to include planting some corn! Try to determine (by changing the profit per acre of corn) how much of a subsidy is required for this farmer's optimal solution to include planting both wheat and corn.

¹ Winston, Wayne L. *Introduction to Mathematical Programming*, 2nd ed. (1995). p. 60, question #6.

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2. A high-capacity print shop has three different machines that each print, staple, and/or bind booklets. The machines are not identical; they take different amounts of time for the same job, and they have different ink costs. There are 10 different jobs that need to be completed today in the 14 hours (6am to 9pm) that the print shop is open. [Note that compared to the question in the infeasibility exercises, the print shop will now be open for one additional hour per day.] The table below shows how long each job will take on each machine, and the cost of ink for each job on each machine. Jobs can <u>not</u> be split between machines; each job must be run on a single machine.

	TIME (HOURS)			INK COST		
JOB	Machine 1	Machine 2	Machine 3	Machine 1	Machine 2	Machine 3
1	2	3	4	40	20	30
2	3	3	3	35	30	35
3	4	6	6	40	30	35
4	4	6	7	20	35	25
5	2	3	6	30	30	30
6	5	5	7	30	25	40
7	3	4	7	25	20	30
8	4	4	5	30	30	35
9	5	5	7	40	30	50
10	3	5	6	25	25	30

- a. Create a mathematical model and a gurobipy model that the print shop manager can optimize to determine which machine to run each job on, in order to minimize the total ink cost while printing all ten jobs.
- b. The print shop manager is considering keeping the shop open 15 hours today rather than 14. In return for the extra expense of keeping the shop open an extra hour (you don't need to calculate that number), how much less expensive would ink be if the shop was open 15 hours on this day?
- c. The manager has decided to keep the print shop open 14 hours today, as in part a. A customer calls just before the shop opens, asking if it is possible to do a short rush job today. The job would take just 2 hours on any of the three machines, with ink cost \$150, \$100, and \$150 on the three machines. Can the print shop fit the new job into its 14-hour schedule?

NOTES:		

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