Exercises - Section 4: Lecture 14 – Building a Model - Questions

1. A real estate investment company has an investment budget that they want to spend in a bunch of cities, in a way that maximizes the predicted increase in value. To diversify, they want to limit their spending in each region, and they also have set minimum and maximum spending limits in each city.

    A dataset for this question is given in file `question_one_data.json`. The mathematical and gurobipy models are described in the lessons. Create a file with the gurobipy model, and solve it.

2. A metal alloy manufacturer has two processes that it can use to produce alloy. Operating Process 1 costs $28 per ton of alloy created, and operating Process 2 costs $53 per ton of alloy created. The plant would like to produce its yearly requirement of 3400 tons of alloy at minimum cost, subject to the requirement that at most two thirds of its production can be from Process 1. However, the law limits the amount of pollution the plant can put out to at most 200,000 pounds per year. If the plant goes beyond that limit, it will be shut down by the government. Operating Process 1 produces 90 pounds of pollution per ton of alloy created, and operating Process 2 produces 50 pounds of pollution per ton of alloy created.

    Create mathematical and gurobipy models that the manufacturer can use to find the minimum-cost way of producing its annual requirement of alloy without going over the pollution limit. Solve the gurobipy model.

3. A large new furniture store wants to decide how much of its floor space will be devoted to beds, bookcases, chairs (dining room, living room, and office), coffee tables, couches, desks, dining tables, and dressers.

    In order to not be known as a specialized store, it needs to devote at least 20% of its floor space to bedroom furniture, at least 20% to living room furniture, at least 15% to dining room furniture, and at least 10% to office furniture.

    At least 10% of the floor space must be devoted to beds, at least 50% to seating (chairs, couches), at least 10% to tables (coffee, dining), and at least 5% to storage and work (bookcases, desks, dressers).

    Because dining room tables and chairs are often purchased together (and tables take 1.5 times the space of chairs on display), the amount of floor space devoted to dining room tables must be 1.5 times the floor space devoted to dining room chairs.

    Every type of furniture must get at least 5% of the floor space.

The table below shows how each type of furniture is categorized, and the marginal profit of increasing the size of space devoted to each type of furniture.

Index	Furniture type	Categories	Marginal annual profit per extra percent of floor space (scaled)
1	Beds	Bedroom	10
2	Bookcases	Office, Storage/Work	7
3	Chairs, dining room	Dining Room, Seating	12
4	Chairs, living room	Living Room, Seating	9
5	Chairs, office	Office, Seating	3
6	Coffee tables	Living Room, Tables	5
7	Couches	Living Room, Seating	8
8	Desks	Office, Storage/Work	6
9	Dining tables	Dining Room, Tables	13
10	Dressers	Bedroom, Storage/Work	4

Create mathematical and gurobipy models that the manufacturer can use to find the most profitable way to divide its floor space. Solve the gurobipy model.

4. A regional recycling center collects used paper, glass, and plastic. Every month, it sells the paper, glass, and plastic to large recycling companies (and those companies recycle the materials). Each large recycling company purchases different combinations of materials. The table below shows the weight ratio that each company purchases materials to recycle, and the per-pound cost they pay (for example, Company A purchases paper, glass, and plastic in a ratio of 5 pounds of paper to 3 pounds of glass to 1 pound of plastic, and pays $2 for each 9 pounds of total materials).

Recycling Company	Pounds of paper per bundle	Pounds of glass per bundle	Pounds of plastic per bundle	Purchase price for bundle of paper, glass, and plastic
A	5 lbs	3 lbs	1 lb	$2 per 9 lbs bundle
B	7 lbs	3 lbs	2 lbs	$3 per 10 lbs bundle
C	2 lbs	1 lb	1 lb	$1 per 4 lbs bundle
D	3 lbs	1 lbs	4 lbs	$2 per 9 lbs bundle

In addition to bundles, the recycling center can also sell paper, glass, and plastic to small recyclers for $0.05 per pound of paper, $0.03 per pound of glass, and $0.02 per pound of plastic.

To maintain their business relationship, the recycling center must sell at least 100 full bundles to each large recycler. Beyond that minimum, either full or partial bundles may be sold.

(a) This month, the recycling center has collected 15,000 pounds of paper, 7,000 pounds of glass, and 14,000 pounds of plastic. Create mathematical and gurobipy models that the recycling center can use to maximize its revenue for selling all that it collected this month. Solve the gurobipy model.

(b) Next month, the recycling center collects 18,000 pounds of paper, 5,000 pounds of glass, and 10,000 pounds of plastic. Solve your gurobipy model from part (a) with this new data, to determine how the recycling center can maximize its revenue this month.

(c) Suppose the recycling center was able store two months of recyclables, so it could sell both months' recyclables at once. Solve your gurobipy model from part (a) with the combined two-month data, to determine how much additional revenue the recycling center could get if it had two months of storage capacity. (Note that the need to sell 100 bundles/month to each large recycler means that the recycling center needs to sell at least 200 bundles to each large recycler over this two-month time period.)

5.   The environmental regulators of a certain country (with very strong central government control) would like to make sure that the air over a city does not get too polluted, while at the same time making sure that businesses are not forced into bankruptcy by environmental regulations.

To control the air pollution, the regulators have identified C companies to regulate. Each company j currently emits p_j tons of pollutants each year. To reduce the amount of pollution, they estimate that it will cost them d_j dollars per year per ton decrease. Each company j currently has operating expenses of c_j dollars per year; in order to maintain a reasonable level of profit, they must keep their expenses below m_j per year.

Create mathematical and gurobipy models that the regulators can use to assign allowable pollution levels to each company that minimize the total expense of the companies while getting the total pollution to be no greater that T tons per year and while allowing each company to maintain its reasonable level of profit.

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