Exercises - Section 2: Lecture 8 – Quadratic Modeling - Questions

The following four pieces of mathematical models require quadratic terms. In each example, x, y, and z denote variables and all other letters denote data. For each one, write mathematical expressions and gurobipy code. Use auxiliary variables when helpful.

a. maximize $(x + y + z)^2$

b. minimize $\sum_{i=1}^{N} \sum_{j=1}^{N} a_{ij} x_i x_j$

c. $(x + y + z)^2 \leq R$

d. $\sum_{i=1}^{N} \sum_{j=1}^{N} a_{ij} x_i x_j \leq R$

2. An investment portfolio manager wants to determine how much money to invest in each of 1000 stocks (the S&P 500 plus 500 smaller stocks). The manager's data science team has built models to predict the expected return (per dollar invested) relative to the market of each stock i (denoted by α_i) and they have calculated the historical covariance β_{ij} for each pair of stocks i and j. The covariances are used as a proxy for investment risk; if x_i is the amount invested in stock i, then the total risk can be written as $\sum_i \sum_j \beta_{ij} x_i x_j$. The portfolio has a total of B dollars available for investment.

   a. Create mathematical and gurobipy models that the manager can use to determine how much money to invest in each stock in order to maximize the total expected return, while adhering to the budget and keeping the risk below a specific tolerance T. Use auxiliary variables for the quadratic terms if needed.

   b. Create mathematical and gurobipy models that the manager can use to determine how much money to invest in each stock in order to minimize the risk, while adhering to the budget and having an expected return of at least R. Use auxiliary variables for the quadratic terms if needed.

3. A data scientist would like to run a regression with special restrictions: the regression constant (a_0) must be zero, the sum of all regression coefficients $a_1, \ldots, a_m$ must be zero, and no coefficient a_j can be greater than 1 or less than -1. The data scientist has n data points, each consisting of a response y_i and predictors x_{ij}. Create mathematical and gurobipy models that

the data scientist can use to find the constrained regression solution. Use auxiliary variables for the quadratic terms if it would be helpful.

4. An appliance manufacturer is getting ready to launch production of three new models of oven. The three oven models do the same thing (cook food), but they have different features. Oven A has the most features (e.g., convection, self-cleaning, Sabbath mode, on/off timer), Oven B has only a subset of those features (e.g., self-cleaning, Sabbath mode, on/off timer), and Oven C has even fewer of the features (e.g., self-cleaning, Sabbath mode). The table below shows the cost to produce each model of oven.

Model	Production cost
A	$600
B	$400
C	$375

The manufacturer would like to maximize its total profit (the sum over all ovens sold of selling price minus production cost) on the new ovens, and the prices they charge will affect the number of ovens they sell. The manufacturer's marketing experts and data scientists believe that the number of ovens sold of each model will depend on the price of its model, and how much different that price is from the prices of the next model up and/or down. They have collaborated to come up with the following estimates. If P_i is the selling price of oven model i and Q_i is the number of ovens of model i they can sell, then:

$$Q_A = 3{,}000 - 5P_A + (7{,}000 - 25(P_A - P_B))$$

$$Q_B = 20{,}000 - 25P_B - \big(7{,}000 - 25(P_A - P_B)\big) - 40(P_B - P_C)$$

$$Q_C = 10{,}000 - 20P_C + 40(P_B - P_C)$$

Create mathematical and gurobipy models that the manufacturer can use to maximize its profit. [Remember to use auxiliary variables for quadratic terms if they're helpful, and don't forget obvious restrictions like the prices and quantities sold can't be negative.] Solve the gurobipy model.

Note: Because this is a nonconvex model, in the gurobipy code you'll need to include the statement `m.setParam("NonConvex",2)` before optimizing.

NOTES: