1. Find the following limit. (If the limit does not exist or has an infinite limit, you should point it out. In addition, also remember the definition of definite integral) (15%)

(a) $\lim_{n\to\infty} \ln(\frac{1}{n}\times\frac{2}{n}\ldots\times\frac{n}{n})^{\frac{1}{n}}$

(b) $\lim_{x\to 0}\frac{1}{x}\int_0^x \cos(4t)dt$

(c) $\lim_{x\to 0}\cot(x)-\frac{1}{x}$

(d) $\lim_{x\to 0}\frac{\arctan(\sin(2x))}{\tan(\arcsin(3x))}$

(e) $\lim_{x\to\infty}(1+\frac{1}{x})^{4x}$

Notice $\lim_{x\to 0}\frac{sinx}{x}=1$

Ans:

Theorem 5.17 (L'Hôpital's Rule)

Let f and g be functions that are differentiable on an open interval (a, b) containing c, except possibly at c itself. Assume that $g'(x)\neq 0$ for all x in (a, b), except possibly at c itself. If the limit of $f(x)/g(x)$ as x approaches c produces the indeterminate form $0/0$, then

$$\lim_{x\to c}\frac{f(x)}{g(x)}=\lim_{x\to c}\frac{f'(x)}{g'(x)}$$

provided the limit on the right exists (or is infinite). This result also applies if the limit of $f(x)/g(x)$ as x approaches c produces anyone of the indeterminate forms ∞/∞,$(-\infty)/\infty$, $\infty/(-\infty)$ or $(-\infty)/(-\infty)$.

(a) $$\lim_{n\to\infty}\ln\left(\frac{1}{n}\times\frac{2}{n}\ldots\times\frac{n}{n}\right)^{\frac{1}{n}}=\lim_{n\to\infty}\frac{1}{n}\Sigma_{i=1}^{n}\ln\left(\frac{i}{n}\right)=\int_0^1\ln x\,dx$$

$$=\lim_{b\to 0^+}\int_b^1\ln x\,dx$$

$$=\lim_{b\to 0^+}\left(x\ln x\Big|_b^1-\int_b^1 dx\right)=\lim_{b\to 0^+}\left(-b\ln b-(1-b)\right)$$

$$=-1-\lim_{b\to 0^+}\frac{\ln b}{\frac{1}{b}}=-1-\lim_{b\to 0^+}\frac{\frac{1}{b}}{\frac{-1}{b^2}}=-1$$

Definition 4.5 (Definite integral)

If f is defined on the closed interval $[a, b]$ and the limit of Riemann sums over partitions Δ

$$\lim_{\|\Delta\|\to 0} \sum_{i=1}^{n} f(c_i)\Delta x_i$$

exists (as described above), then f is said to be integrable on $[a, b]$ and the limit is denoted by

$$\lim_{\|\Delta\|\to 0} \sum_{i=1}^{n} f(c_i)\Delta x_i = \int_a^b f(x)\,\mathrm{d}x.$$

The limit is called the definite integral of f from a to b. The number a is the lower limit of integration, and the number b is the upper limit of integration.

(b) $\lim\limits_{x\to 0} \frac{1}{x}\int_0^x \cos(4t)dt = \lim\limits_{x\to 0} \frac{\cos(4x)}{1} = 1$

Theorem 3.4 (The Mean Value Theorem)

If f is continuous on the closed interval $[a, b]$ and differentiable on the open interval (a, b), then there exists a number c in (a, b) such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}.$$

Theorem 4.11 (The Second Fundamental Theorem of Calculus)

If f is continuous on an open interval I containing a, then, for every x in the interval,

$$\frac{\mathrm{d}}{\mathrm{d}x}\left[\int_a^x f(t)\,\mathrm{d}t\right] = f(x).$$

(c) $\lim\limits_{x\to 0} \cot(x) - \frac{1}{x} = \lim\limits_{x\to 0} \frac{\cos(x)}{\sin(x)} - \frac{1}{x} = \lim\limits_{x\to 0} \frac{\mathrm{x}\cos(x) - \sin(x)}{\mathrm{x}\sin(x)} = \lim\limits_{x\to 0} \frac{cosx - xsinx - cosx}{\mathrm{sinx+xcosx}} =$

$\lim\limits_{x\to 0} \frac{-xcosx - sinx}{\mathrm{cosx+cosx-xsinx}} = 0$ (Indeterminate form $\infty - \infty$)

(d) $\lim\limits_{x\to 0} \frac{\arctan(\sin(2x))}{\tan(\arcsin(3x))} = \lim\limits_{x\to 0} \frac{\frac{1}{1+sin^2(2x)}\cos(2x)2}{sec^2(\arcsin(3x))\frac{1}{\sqrt{1-(3x)^2}}3} = \frac{2}{3}$

(e) $\lim\limits_{x\to\infty} (1+\frac{1}{x})^{4x} = \lim\limits_{x\to\infty} \left[(1+\frac{1}{x})^x\right]^4 = e^4$ (Indeterminate form 1^∞, can take log)

Theorem 5.15 (A limit involving e)

$$\lim_{x\to\infty} \left(1 + \frac{1}{x}\right)^x = \lim_{x\to\infty} \left(\frac{x+1}{x}\right)^x = e$$

2. Find the point $a > 0$ satisfying (8%)

$$\int_1^2 \frac{2}{t} dt = \int_a^{\frac{1}{8}} \frac{1}{t} dt$$

Ans:

Applying the definition of the function ln x to the equation shows that

$$2(\ln a - \ln 1) = \ln \frac{1}{8} - \ln a$$

$$3 \ln a = \ln \frac{1}{8}$$

$$a^3 = \frac{1}{8} \rightarrow a = \frac{1}{2}$$

Definition 5.1 (The natural logarithmic function)

The natural logarithmic function is defined by

$$\ln x = \int_1^x \frac{1}{t} \mathrm{d}t, \quad x > 0.$$

The domain of the natural logarithmic function is the set of all positive real numbers.

3. Assume the inverse function of $f(x) = x^5 + 2x^3 + x - 2$ is $g(x)$, find $g'(f(1))$ (5%)

Ans:

$$g\big(f(x)\big) = x$$

$$g'\big(f(x)\big)f'(x) = 1$$

$$g'\big(f(x)\big) = \frac{1}{f'(x)} = \frac{1}{5x^2 + 6x^2 + 1}$$

$$g'\big(f(1)\big) = \frac{1}{12}$$

Theorem 5.7 (The existence of an inverse function)

1. *A function has an inverse function if and only if it is one-to-one.*
2. *If f is strictly monotonic on its entire domain, then it is one-to-one and therefore has an inverse function.*

Definition 5.3 (Inverse function)

A function g is the inverse function of the function f if $f(g(x)) = x$ for each x in the domain of g and $g(f(x)) = x$ for each x in the domain of f. The function g is denoted by f^{-1} (read "f inverse").

4. Evaluate. (12%)

(a) $\int_{\sqrt{3}}^{3} \frac{1}{x\sqrt{4x^2-9}} dx$?

(b) $\int \frac{2}{\sqrt{-x^2+4x}} dx$

(c) $\int \frac{5^{2x}}{1+5^{2x}} dx$

(d) $\int_{\frac{1}{4}}^{\frac{1}{2}} \frac{arcsin(\sqrt{x})}{\sqrt{x(1-x)}} dx$

Ans: (a) $\int_{\sqrt{3}}^{3} \frac{1}{x\sqrt{4x^2-9}} dx = \left[\frac{1}{3} arcsec \frac{|2x|}{3}\right] \begin{matrix} 3 \\ \sqrt{3} \end{matrix} = \frac{1}{3}\text{arcsec}(2) - \frac{1}{3} arcsec \frac{2\sqrt{3}}{3} = \frac{1}{3}\left(\frac{\pi}{3}\right) - \frac{1}{3}\left(\frac{\pi}{6}\right) = \frac{\pi}{18}$

(b) $\int \frac{2}{\sqrt{-x^2+4x}} dx = \int \frac{2}{\sqrt{4-(x^2-4x+4)}} dx = \int \frac{2}{\sqrt{4-(x-2)^2}} dx = 2\arcsin\left(\frac{x-2}{2}\right) + C$

Table 1: Integrals of the six basic trigonometric functions

$\int \sin u \, du = -\cos u + C$	$\int \cos u \, du = \sin u + C$
$\int \tan u \, du = -\ln\lvert\cos u\rvert + C$	$\int \cot u \, du = \ln\lvert\sin u\rvert + C$
$\int \sec u \, du = \ln\lvert\sec u + \tan u\rvert + C$	$\int \csc u \, du = -\ln\lvert\csc u + \cot u\rvert + C$

Theorem 5.19 (Integrals involving inverse trigonometric functions)

Let u be a differentiable function of x, and let $a > 0$.
1. $\int \frac{du}{\sqrt{a^2-u^2}} = \arcsin\frac{u}{a} + C$ **2.** $\int \frac{du}{a^2+u^2} = \frac{1}{a}\arctan\frac{u}{a} + C$ **3.** $\int \frac{du}{u\sqrt{u^2-a^2}} = \frac{1}{a}\text{arcsec}\frac{|u|}{a} + C$

(c) Let $u = 1 + 5^{2x}, du = 2(\ln 5)5^{2x}dx$

$$\int \frac{5^{2x}}{1+5^{2x}} dx = \frac{1}{2\ln 5}\int \frac{1}{u} dx = \frac{1}{2\ln 5}\ln u + C = \frac{1}{2\ln 5}\ln(1+5^{2x}) + C$$

Theorem 5.5 (Log Rule for integration)

Let u be a differentiable function of x.
1. $\int \frac{1}{x} dx = \ln|x| + C$ **2.** $\int \frac{1}{u} du = \ln|u| + C$

(d) Let $u = arcsin(\sqrt{x}), \frac{du}{dx} = \frac{1}{\sqrt{1-x}}\frac{1}{2\sqrt{x}}$

$$\int_{\frac{1}{4}}^{\frac{1}{2}} \frac{arcsin(\sqrt{x})}{\sqrt{x(1-x)}} dx = \int_{\frac{\pi}{6}}^{\frac{\pi}{4}} 2u du = u^2\Big|_{\frac{\pi}{6}}^{\frac{\pi}{4}} = \frac{5\pi^2}{144}$$

5. Find the equation of the tangent line $y = \log_{10}(3x)$ at $x = 5$. (5%)

Ans:

$$y = \log_{10}(3x) = \log_{10} 3 + \frac{\ln x}{\ln 10}$$

$y' = \frac{1}{x \ln 10}$ and the slope of the tangent line is $y'(5) = \frac{1}{5 \ln 10}$

Therefore, the tangent line is $y = \frac{1}{5 \ln 10}(x - 5) + \log_{10} 15$

Theorem 5.13 (Derivatives for bases other than e)

Let a be a positive real number ($a \neq 1$) and let u be a differentiable function of x.

1. $\frac{d}{dx}[a^x] = (\ln a)a^x$
2. $\frac{d}{dx}[a^u] = (\ln a)a^u \frac{du}{dx}$
3. $\frac{d}{dx}[\log_a x] = \frac{1}{(\ln a)x}$
4. $\frac{d}{dx}[\log_a u] = \frac{1}{(\ln a)u} \frac{du}{dx}$

6. Evaluate (12%)

(a) $\int (\ln x)^3 dx$

(b) $\int e^x cosx \ \ dx$

(c) $\int_{-\pi}^{\pi} \sin(mx)cos(nx)dx$, where m and n are positive integers

(d) $\int_0^{\ln 4} \frac{e^x}{\sqrt{e^{2x}+9}} dx$

Ans:

Theorem 8.1 (Integration by Parts)

If u and v are functions of x and have continuous derivatives, then

$$\int u\,dv = uv - \int v\,du = uv - \int vu'\,dx.$$

(a)

Let $u = (\ln x)^3, dv = dx \ \rightarrow du = 3(\ln x)^2 \frac{1}{x} dx,\ v = x$

$$\int (\ln x)^3 dx = x(\ln x)^3 - 3\int (\ln x)^2 dx$$

Let $u = (\ln x)^2, dv = dx \ \rightarrow du = 2\ln x \frac{1}{x} dx,\ v = x$

$$\int (\ln x)^2 dx = x(\ln x)^2 - 2\int \ln x \, dx$$

Let $u = \ln x$, $dv = dx \rightarrow du = \frac{1}{x}dx,\ v = x$

$$\int \ln x = x\ln x - \int dx = x\ln x - x + C$$

$$\int (\ln x)^3 dx = x(\ln x)^3 - 3\int (\ln x)^2 dx$$

$$= x(\ln x)^3 - 3\left(x(\ln x)^2 - 2\int \ln x\,dx\right)$$

$$= x(\ln x)^3 - 3(x(\ln x)^2 - 2(x\ln x - x + C))$$

$$= x(\ln x)^3 - 3x(\ln x)^2 + 6x\ln x - 6x + C'$$

(b)

Let $u = \cos x$, $dv = e^x dx \rightarrow du = -\sin x\ dx,\ v = e^x$

$$\int e^x \cos x\ dx = e^x \cos x + \int e^x \sin x\ dx$$

Let $u = \sin x$, $dv = e^x dx \rightarrow du = \cos x\ dx,\ v = e^x$

$$\int e^x \cos x\ dx = e^x \cos x + \int e^x \sin x\ dx = e^x \cos x + \left(e^x \sin x - \int e^x \cos x\ dx\right)$$

$$\int e^x \cos x\ dx = \frac{1}{2}(e^x \cos x + e^x \sin x) + C$$

(c)

$\int_{-\pi}^{\pi} \sin(mx)cos(nx)dx$, where m and n are positive integers

If $m \neq n$, $\int_{-\pi}^{\pi} \sin(mx)cos(nx)dx = \frac{1}{2}\int_{-\pi}^{\pi} \sin\big((m+n)x\big) + sin((m-n)x)dx = 0$ since $\sin x$ is an odd function

If $m = n$, $\int_{-\pi}^{\pi} \sin(mx)cos(nx)dx = \frac{1}{2}\int_{-\pi}^{\pi} \sin(2mx)\,dx = 0$

$$\sin mx \cos nx = \frac{1}{2}(\sin[(m-n)x] + \sin[(m+n)x])$$

$$\sin^2 x = \frac{1-\cos 2x}{2} \quad \text{and} \quad \cos^2 x = \frac{1+\cos 2x}{2}$$

(d) $\int_0^{\ln 4} \frac{e^x}{\sqrt{e^{2x}+9}} dx$

(Let $u = e^x, du = e^x \ dx$)

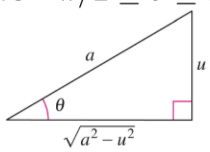
$$= \int_1^4 \frac{du}{\sqrt{u^2+9}}$$

(Let $u = 3\tan\theta,\ du = 3sec^2\theta d\theta$)

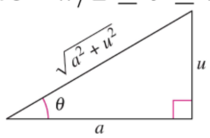
$$= \int_{\arctan(\frac{1}{3})}^{\arctan(\frac{4}{3})} sec\theta d\theta = \ln|sec\theta + tan\theta|\Big|_{\arctan\left(\frac{1}{3}\right)}^{\arctan\left(\frac{4}{3}\right)} = \ln\left(\frac{5}{3}+\frac{4}{3}\right) - \ln\left(\frac{\sqrt{10}+1}{3}\right)$$

$$= 2\ln 3 - \ln(\sqrt{10}+1)$$

For integrals involving $\sqrt{a^2 - u^2}$, let $u = a\sin\theta$. Then $\sqrt{a^2-u^2} = a\cos\theta$, where $-\pi/2 \le \theta \le \pi/2$.

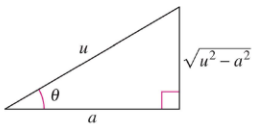


For integrals involving $\sqrt{a^2 + u^2}$, let $u = a\tan\theta$. Then $\sqrt{a^2+u^2} = a\sec\theta$, where $-\pi/2 \le \theta \le \pi/2$.



For integrals involving $\sqrt{u^2 - a^2}$, let $u = a\sec\theta$.

Then $\sqrt{u^2-a^2} = \begin{cases} a\tan\theta & \text{if } u > a, \text{ where } 0 \le \theta < \pi/2 \\ -a\tan\theta, & \text{if } u < -a, \text{ where } \pi/2 < \theta \le \pi. \end{cases}$



7. Evaluate (20%)

(a) $\int \frac{sec^2(x)}{(tanx)(tanx+1)}$dx

(b) $\int \frac{x^4-2x^2+4x+1}{x^3-x^2-x+1}$dx

(c) $\int_{-\infty}^{0} xe^x \, dx$

(d) $\int_0^3 \frac{1}{\sqrt[3]{x-1}} dx$

Ans:

(a) Let $u = tanx, du = sec^2(x)dx$

$$\int \frac{sec^2(x)}{(tanx)(tanx+1)} \mathrm{dx} = \int \frac{1}{(u)(u+1)} \mathrm{du}$$

$$\frac{1}{(u)(u+1)} = \frac{A}{u} + \frac{B}{u+1},$$

$$1 = A(u+1) + Bu$$

When $u = 0, A = 1$

When $u = 1, B = -1$

$$\int \frac{sec^2(x)}{(tanx)(tanx+1)} \mathrm{dx} = \int \frac{1}{(u)(u+1)} \mathrm{du} = \int \frac{1}{u} - \frac{1}{u+1} \mathrm{du}$$

$$= \ln|u| - \ln|u+1| + C = \ln\left|\frac{u}{u+1}\right| + C$$

$$= \ln\left|\frac{tanx}{tanx+1}\right| + C$$

(b)

$$\frac{x^4 - 2x^2 + 4x + 1}{x^3 - x^2 - x + 1} = \mathrm{x} + 1 + \frac{4x}{x^3 - x^2 - x + 1}$$

$$\frac{4x}{x^3 - x^2 - x + 1} = \frac{4x}{(x-1)^2(x+1)} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{(x+1)}$$

$$4x = A(x-1)(x+1) + B(x+1) + C(x-1)^2$$

$$\mathrm{A} = 1, \mathrm{B} = 2, \mathrm{C} = -1$$

$$\int \frac{x^4 - 2x^2 + 4x + 1}{x^3 - x^2 - x + 1} \mathrm{dx} = \int (\mathrm{x} + 1 + \frac{1}{x-1} + \frac{2}{(x-1)^2} + \frac{-1}{(x+1)}) \mathrm{dx}$$

$$= \frac{1}{2}x^2 + x + \ln|x-1| - \frac{2}{x-1} - \ln|x+1| + C$$

3. Linear factors: For each factor of the form $(px+q)^m$, the partial fraction decomposition must include the following sum of m fractions.

$$\frac{A_1}{(px+q)} + \frac{A_2}{(px+q)^2} + \cdots + \frac{A_m}{(px+q)^m}$$

4. Quadratic factors: For each factor of the form $(ax^2+bx+c)^n$, the partial fraction decomposition must include the following sum of n fractions.

$$\frac{B_1x + C_1}{ax^2+bx+c} + \frac{B_2x + C_2}{(ax^2+bx+c)^2} + \cdots + \frac{B_nx + C_n}{(ax^2+bx+c)^n}$$

(c)

$$\int_{-\infty}^{0} xe^x\,dx = \lim_{a\to-\infty}\int_a^0 xe^x dx = \lim_{a\to-\infty}\left((xe^x)|_a^0 - \int_a^0 e^x dx\right)$$

$$= \lim_{a\to-\infty}(-ae^a - 1 + e^a) = -\lim_{a\to-\infty}\left(\frac{a}{e^{-a}}\right) - 1$$

$$= -\lim_{a\to-\infty}\left(\frac{1}{-e^{-a}}\right) - 1 = \lim_{a\to-\infty}(e^a) - 1 = -1$$

(d)

$$\int_0^3 \frac{1}{\sqrt[3]{x-1}}dx = \int_0^1 \frac{1}{\sqrt[3]{x-1}}dx + \int_1^3 \frac{1}{\sqrt[3]{x-1}}dx$$

$$= \lim_{b\to1^-}\int_0^b \frac{1}{\sqrt[3]{x-1}}dx + \lim_{c\to1^+}\int_c^3 \frac{1}{\sqrt[3]{x-1}}dx$$

$$= \lim_{b\to1^-}\frac{3}{2}(x-1)^{\frac{2}{3}}|_0^b + \lim_{c\to1^+}\frac{3}{2}(x-1)^{\frac{2}{3}}|_c^3 = \frac{-3}{2} + \frac{3\sqrt[3]{4}}{2}$$

Definition 8.1 (Improper integrals with infinite integration limits)

1. If f is continuous on the interval $[a,\infty)$, then

$$\int_a^{\infty} f(x)\,\mathrm{d}x = \lim_{b\to\infty}\int_a^b f(x)\,\mathrm{d}x.$$

2. If f is continuous on the interval $(-\infty,b]$, then

$$\int_{-\infty}^{b} f(x)\,\mathrm{d}x = \lim_{a\to-\infty}\int_a^b f(x)\,\mathrm{d}x.$$

3. If f is continuous on the interval $(-\infty,\infty)$, then

$$\int_{-\infty}^{\infty} f(x)\,\mathrm{d}x = \int_{-\infty}^{c} f(x)\,\mathrm{d}x + \int_c^{\infty} f(x)\,\mathrm{d}x$$

where c is any real number.

Definition 8.2 (Improper integrals with infinite discontinuities)

1. If f is continuous on the interval $[a,b)$ and has an infinite discontinuity at b, then

$$\int_a^b f(x)\,\mathrm{d}x = \lim_{c\to b^-}\int_a^c f(x)\,\mathrm{d}x.$$

2. If f is continuous on the interval $(a,b]$ and has an infinite discontinuity at a, then

$$\int_a^b f(x)\,\mathrm{d}x = \lim_{c\to a^+}\int_c^b f(x)\,\mathrm{d}x.$$

3. If f is continuous on the interval $[a,b]$, except for some c in (a,b) at which f has an infinite discontinuity, then

$$\int_a^b f(x)\,\mathrm{d}x = \int_a^c f(x)\,\mathrm{d}x + \int_c^b f(x)\,\mathrm{d}x.$$

8. Determine whether the following integral diverges or converges (9%)

(a) $\int_0^1 \frac{1}{\sqrt[7]{x}} dx$

(b) $\int_1^\infty \frac{sin^2x}{x^2} dx$

(c) $\int_1^\infty \frac{1}{\sqrt{x}} dx$

Ans: (a) If $p = 1, \int_0^1 \frac{1}{x^p} dx = \int_0^1 \frac{1}{x} dx = \lim_{a \to 0^+} \ln x \,|_a^1 = \infty$ diverges

If $p \neq 1, \int_0^1 \frac{1}{x^p} dx = \lim_{a \to 0^+} \frac{x^{1-p}}{1-p} |_a^1 = \lim_{a \to 0^+} \left(\frac{1}{1-p} - \frac{a^{1-p}}{1-p}\right) = \frac{1}{1-p} - \frac{1}{1-p} \lim_{a \to 0^+} a^{1-p}$

Which converges to $\frac{1}{1-p}$ if $1 - p > 0 \leftrightarrow p < 1$

Therefore, $\int_0^1 \frac{1}{\sqrt[7]{x}} dx$ converges

(b) Since $\frac{1}{x^2} \geq \frac{sin^2x}{x^2} \geq 0$ on $[1, \infty)$ and $\int_1^\infty \frac{1}{x^2}$dx is convergent.

$\int_1^\infty \frac{sin^2x}{x^2} dx$ is convergent by the comparison test

Theorem 8.7 (A special type of improper integral)

$$\int_1^\infty \frac{dx}{x^p} = \begin{cases} \frac{1}{p-1}, & \text{if } p > 1 \\ \text{diverges}, & \text{if } p \leq 1 \end{cases}$$

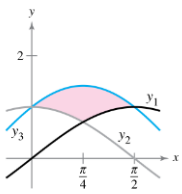
(c) $\int_1^\infty \frac{1}{\sqrt{x}} dx$ is divergent

Theorem 8.8 (Comparison Test for Improper Integrals)

Suppose the function f and g are continuous and $0 \leq g(x) \leq f(x)$ on the interval $[a, \infty)$. It can be shown that if $\int_a^\infty f(x)\,dx$ converges, then $\int_a^\infty g(x)\,dx$ also converges, and if $\int_a^\infty g(x)\,dx$ diverges, then $\int_a^\infty f(x)\,dx$ also diverges.

9. Find the area of the given region bounded by the graph y_1, y_2 and y_3 (8%)

$$y_1 = sinx, y_2 = cosx, y_3 = cosx + sinx$$



Ans:

$$A = \int_0^{\frac{\pi}{4}} (y_3 - y_2)dx + \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} (y_3 - y_1)dx = \int_0^{\frac{\pi}{4}} sinx\ \ dx + \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} cosx\ \ dx$$

$$= -cosx\Big|_0^{\frac{\pi}{4}} + \text{sin}x\Big|_{\frac{\pi}{4}}^{\frac{\pi}{2}} = 2 - \sqrt{2}$$

Area of a region between two curves

If f and g are continuous on $[a, b]$ and $g(x) \le f(x)$ for all x in $[a, b]$, then the area of the region bounded by the graphs of f and g and the vertical lines $x = a$ and $x = b$ is

$$A = \int_a^b [f(x) - g(x)]\,\mathrm{d}x.$$

10. Find the volume of the solid generated by revolving the region bounded by the graphs of $\text{f(x)} = e^{-x}$ $(0 \le \text{x} \le \ln 2)$ about the line $\text{y} = -1$ (6%)

Ans:

$$\text{V} = \pi \int_0^{\ln 2} (e^{-x} + 1)^2 dx = \pi \int_0^{\ln 2} e^{-2x} + 2e^{-x} + 1\ \ dx$$

$$= \pi\left(\frac{e^{-2x}}{-2} - 2e^{-x} + x\right)\Big|_0^{\ln 2} = (\ln 2 + \frac{11}{8})\pi$$

The Disk Method

To find the volume of a solid of revolution with the Disk Method, use one of the following, as shown in Figure 12.

Horizontal axis of revolution	Vertical axis of revolution
Volume $= V = \pi \int_a^b [R(x)]^2\,\mathrm{d}x$	Volume $= V = \pi \int_c^d [R(y)]^2\,\mathrm{d}y$

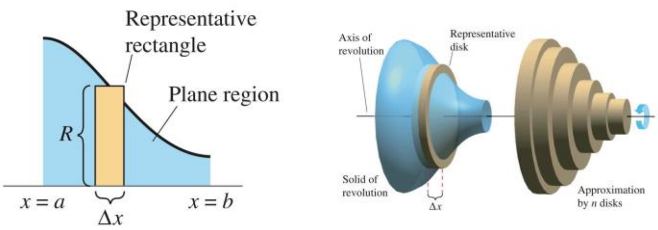


Figure 11: Disk Method.