1. Find the following limit. (If the limit does not exist or has an infinite limit, you should point it out. In addition, also remember the definition of definite integral). (15%)

   (a) $\lim_{n\to\infty} \ln(\frac{1}{n} \times \frac{2}{n} \dots \times \frac{n}{n})^{\frac{1}{n}}$

   (b) $\lim_{x\to 0} \frac{1}{x} \int_0^x \cos(4t) dt$

   (c) $\lim_{x\to 0} \cot(x) - \frac{1}{x}$

   (d) $\lim_{x\to 0} \frac{\arctan(\sin(2x))}{\tan(\arcsin(3x))}$

   (e) $\lim_{x\to\infty} (1 + \frac{1}{x})^{4x}$

2. Find the point $a > 0$ satisfying (8%)

$$\int_1^a \frac{2}{t} dt = \int_a^{\frac{1}{8}} \frac{1}{t} dt$$

3. Assume the inverse function of $f(x) = x^5 + 2x^3 + x - 2$ is $g(x)$, find $g'(f(1))$ (5%).

4. Evaluate the following integral. (12%)

   (a) $\int_{\sqrt{3}}^{3} \frac{1}{x\sqrt{4x^2-9}} dx$?

   (b) $\int \frac{2}{\sqrt{-x^2+4x}} dx$

   (c) $\int \frac{5^{2x}}{1+5^{2x}} dx$

   (d) $\int_{\frac{1}{4}}^{\frac{1}{2}} \frac{\arcsin(\sqrt{x})}{\sqrt{x(1-x)}} dx$

5. Find the equation of the tangent line $y = \log_{10}(3x)$ at $x = 5$. (5%)

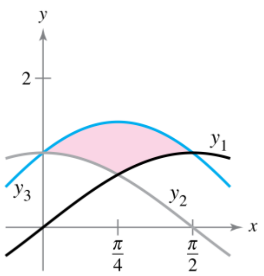
6. Evaluate the following integral. (16%)
   (a) $\int(\ln x)^3 dx$
   (b) $\int e^x cosx \ \ dx$
   (c) $\int_{-\pi}^{\pi} \sin(mx)cos(nx)dx,$ where m and n are positive integers
   (d) $\int_0^{\ln 4} \frac{e^x}{\sqrt{e^{2x}+9}}dx$

7. Evaluate the following integral. (16%)
   (a) $\int \frac{sec^2(x)}{(tanx)(tanx+1)}$ dx
   (b) $\int \frac{x^4-2x^2+4x+1}{x^3-x^2-x+1}$ dx
   (c) $\int_{-\infty}^{0} xe^x \ dx$
   (d) $\int_0^3 \frac{1}{\sqrt[3]{x-1}}dx$

8. Determine whether the following integral diverges or converges. (9%)
   (a) $\int_0^1 \frac{1}{\sqrt[7]{x}}dx$
   (b) $\int_1^{\infty} \frac{sin^2x}{x^2}dx$
   (c) $\int_1^{\infty} \frac{1}{\sqrt{x}}dx$

9. Find the area of the given region bounded by the graphs y_1, y_2 and y_3 (The pink region in the following figure). (6%)

$$y_1 = sinx, y_2 = cosx, y_3 = cosx + sinx$$



10. Find the volume of the solid generated by revolving the region bounded by the graphs of f(x) $= e^{-x}$ (where 0≤ x ≤ ln 2) about the line y $= -1$. (8%)