If the limit does not exist or has an infinite limit, you should point it out. In addition, do not use the L'Hôpital's rule to solve the limit problem.

1. (20%) Determine the following limit.

   (a) $\lim_{x\to 4} \frac{x^2-16}{|x-4|}$

   (b) $\lim_{x\to -\infty} \sqrt{4x^2-2x}+2x$

   (c) $\lim_{x\to 0} x\left(\cos 2x + \cos\frac{1}{2x}\right)$

   (d) $\lim_{x\to 1} \frac{x^2+5x-6}{x^2-1}$

2. (8%) Assume $f(x)=\begin{cases} x^2-a & \text{for } x\geq 2 \\ bx+6 & \text{for } x<2 \end{cases}$ is a differentiable function

What is the value of a and b?

3. (8%) Verify that $f(x)=x^5+x+\frac{3}{2}$ satisfies the hypotheses of Intermediate Value Theorem and the Mean Value Theorem on any closed interval $[a,b]$ of the real numbers. Then, proof that $f(x)=x^5+x+\frac{3}{2}$ has exactly one real root.

4. (20%) Remember that you can solve the derivative using the definition or the differentiation rule for the following question.

(a) Find the following limit. $\lim_{x\to 1} \frac{\frac{x}{\sqrt{x^2+1}}-\frac{1}{\sqrt{2}}}{x-1}$

(b) Use chain rule to find the derivative of $f(x)=x^3\sec\left(\frac{1}{x^2}\right)$

(c) Use implicit differentiation to find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ of the expression $3xy+\sin(x)=2$

(d) Find the equation of the tangent line to the graph of $f(x)=x^3-\sqrt{x}$ at the point (1,0)

5. (24%) Let $f(x) = \frac{(x+1)^2}{x^2+1}$

(a) Find the critical numbers and the possible points of inflection of $f(x)$

(b) Find the open intervals on which f is increasing or decreasing

(c) Find the open intervals of concavity

(d) Find all the asymptotes (Vertical/horizontal/Slant)

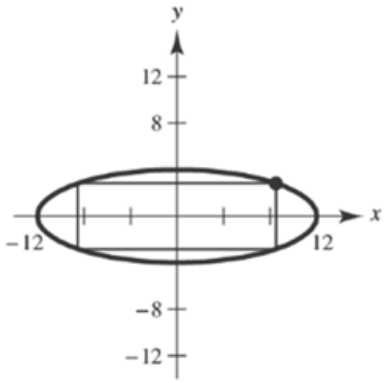
(e) Sketch the graph of $f(x)$ (Label any intercepts, relative extrema, points of inflection,and asymptotes)

(f) What is the domain and range of $f(x)$?

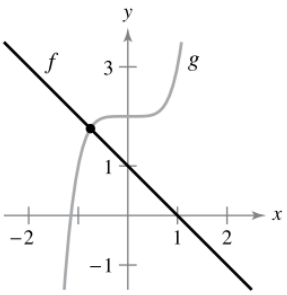
6. (8%) Find the width and height of a rectangle with maximum area that can be inscribed in the ellipse

$$\frac{x^2}{144} + \frac{y^2}{16} = 1$$

where the rectangle's sides are parallel to the coordinate axes.



7. (6%) Apply Newton's method to approximate the x-value of the indicated points of the two graphs defined by $f(x) = 1 - x, g(x) = x^5 + 2$. Continue the iterations until two successive approxmiations differ by less than 0.01. (Hint: let $h(x) = f(x) - g(x)$)



8. (6%) Use differential to approximat $\sqrt{63.9}$