

1. (20%) Find the following limit. (If the limit does not exist or has an infinite limit, you should point it out)

(a) (5%)  $\lim_{x \rightarrow 1^+} \frac{2x}{x-1} - \frac{2}{\ln x}$

(b) (5%)  $\lim_{x \rightarrow 0} \cos(x)^{\frac{1}{x^2}}$

(c) (5%)  $\lim_{x \rightarrow \infty} \frac{e^x + e^{-x}}{e^x - e^{-x}}$

(d) (5%)  $\lim_{x \rightarrow \infty} \frac{e^x}{x}$

2. (15%) Solve the following problems

(a) Evaluate  $\int_{-2}^2 x e^{-x^4} dx$

(b) Find  $\lim_{n \rightarrow \infty} \sum_{i=1}^n \frac{1}{n} \left[ \frac{1}{1 + \frac{i^2}{n^2}} \right]$

(c)  $\int_0^5 (5 - |x - 5|) dx$

3. (9%) Let  $f(x) = \int_1^x \sqrt{1+t^3} dt$

(a) (3%) Show that  $f(x)$  has an inverse function

(b) (3%) What is the value of  $f^{-1}(x)$  when  $x = 0$

(c) (3%) What is the value of  $(f^{-1})'(x)$  when  $x = 0$

4. (10%) Compute the derivative.

(a) (5%)  $f(x) = \log_7 \frac{x\sqrt{x-1}}{2}$

(b) (5%)  $f(x) = e^{\sin(x)} + \cos(e^{2x}) - 2^{\sqrt{x}}$

5. (25%) Evaluate the following integral.

(a) (5%)  $\int_0^1 \frac{dx}{2\sqrt{3-x}\sqrt{x+1}}$

(b) (5%)  $\int \frac{\cos(1-\ln \theta)d\theta}{\theta}$

(c) (5%)  $\int_0^1 3^{x^2+2x}(x+1)dx$

(d) (5%)  $\int \frac{4\tan^{-1}\left(\frac{x}{2}\right)}{4+x^2} dx$

(e) (5%)  $\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{\cos(x)}{\sin(x)\sqrt{\sin^2(x)-\frac{1}{4}}} dx$

6. (7%) Find the area of the given region bounded by the graphs of  $x = \cos(y)$  and

$x = \frac{1}{2}$  on the interval  $\frac{\pi}{3} \leq y \leq \frac{7\pi}{3}$ .

7. (7%) Find the volume of the solid formed by revolving the region bounded by the

graphs of the equations  $y = \frac{1}{x^2}, y = 0, x = 2, x = 5$  about the y-axis.

8. (7%) Find the area of the surface generated by revolving the curve  $y = \frac{x^3}{18}$  on

the interval  $3 \leq x \leq 6$  about the  $x$ -axis.