1. (a)

$$f'(c) = \frac{f(b)-f(a)}{b-a} = \frac{2-0}{2-1} = 2$$

$f'(x) = 2x^3 - 5x + 2 \Rightarrow 2x^3 - 5x + 2 = 2$

$\Rightarrow 2x^3 - 5x = 0 \Rightarrow x = \pm\sqrt{\frac{5}{2}}$ or 0

Only $\sqrt{\frac{5}{2}}$ in the interval $[1, 2]$

$\therefore c = \sqrt{\frac{5}{2}}$ and $f'(\sqrt{\frac{5}{2}}) = 2$ #

(b)

$$f'(c) = \frac{f(b)-f(a)}{b-a} = \frac{-7-(-7)}{4} = 0$$

$f'(x) = 2x - 8 \Rightarrow 2x - 8 = 0$

$\Rightarrow 2x = 8 \Rightarrow x = 4$

Ans $c = 4$ and $f'(4) = 0$ #

2. $x^3 + 3x^2 + 5$

(1)

$f'(x) = 3x^2 + 6x$

令 $3x^2 + 6x = 0 \Rightarrow x(3x+6) = 0$

$x = 0$ or -2

Interval	$(-\infty, -2)$	$[-2, 0]$	$(0, \infty)$
$f'(x)$	+	−	+
	increasing	decreasing	increasing

Ans: Increasing on $(-\infty, -2)$ and $(0, \infty)$

Decreasing on $[-2, 0]$

(2)

$f''(x) = 6x + 6$

令 $6x + 6 = 0 \Rightarrow x = -1$

inflection point $(-1, 7)$

Interval	$(-\infty, -1)$	$(-1, \infty)$
$f''(x)$	−	+
	downward	upward

Ans: Concave upward on $(-1, \infty)$

Concave downward on $(-\infty, -1)$

(3)

point	$(0, 5)$	$(-2, 9)$
$f''(x)$	+	−
	minimum	maximum

Ans: relative maximum $(-2, 9)$

relative minimum $(0, 5)$

3 (a)

$$\lim_{x \to \infty} \frac{-4x^2+2x-5}{x} = \lim_{x \to \infty} \frac{-4x+2-\frac{5}{x}}{1} = -\infty \ \#$$

(b) $$\lim_{x \to \infty} \frac{-4x^2-2x-5}{x^2} = \lim_{x \to \infty} \frac{-4-\frac{2}{x}-\frac{5}{x^2}}{1} = -4 \ \#$$

(c) $$\lim_{x \to \infty} \frac{-4x^2-2x-5}{x^3} = \lim_{x \to \infty} \frac{\frac{-4}{x}-\frac{2}{x^2}-\frac{5}{x^3}}{1} = 0 \ \#$$