1. (20%) Find the following limit. (If the limit does not exist or has an infinite limit, you should point it out.)

   (a) $\lim_{(x,y)\to(0,0)} arccos(\frac{x^3+y^3}{x^2+y^2})$

   (b) $\lim_{(x,y)\to(0,0)} \frac{xy^3}{x^2+y^6}$

   (c) $\lim_{(x,y,z)\to(0,0,0)} \frac{e^{xyz}-1}{x^2+y^2+z^2}$

   (d) $\lim_{(x,y)\to(0,0)} \frac{xy^2}{x^2+y^2}$

2. (15%)

(a) Let $f(x,y) = \begin{cases} \frac{xy}{x^2+y^2} & \text{when } (x,y) \neq (0,0) \\ 0 & \text{when } (x,y) = (0,0) \end{cases}$, evaluate $f_x(0,0)$ and $f_{xy}(0,0)$

(b) Given the equation $w - \sqrt{x-y} - \sqrt{y-z} = 0$, differentiate implicitly to find the three first partial derivatives $\frac{\partial w}{\partial x}, \frac{\partial w}{\partial y}$ and $\frac{\partial w}{\partial z}$

(c) Find a set of parametric equations for the tangent line to the curve of intersection of the surface $x^2 + y^2 + z^2 = 4$ and $(x-1)^2 + y^2 = 1$ at the point $(1, 1, \sqrt{2})$.

3. (10%) Given $f(x,y) = y^2 + \sin(xy)$. Find the directions at the point (1,1) where the directional derivative of $f(x,y)$ in that direction is 1. Express your result as unit vector.

4. (15%) Let $f(x,y) = x^4 - 2x^2 - 2xy^2 - y^2$
   (a) Find the critical points of $f(x,y)$
   (b) Determine whether they are local maximum, local minimum or saddle points

5. (15%) Evaluate the following expression

(a) $\int_0^1 \int_{\sqrt{x}}^1 e^{y^3}\, dydx$

(b) $\int_1^3 \int_0^x \frac{1}{\sqrt{x^2+y^2}} dydx$

(c) $\int_0^{\frac{\pi}{4}} \int_0^6 \int_0^{6-r} rz\ \ dzdrd\theta$

6. (10%) Find the area of the surface given by $z = f(x, y) = xy$ that lies above the region R where $R = \{(x, y): x^2 + y^2 \leq 9\}$

7. (15%) Evaluate the triple integral $\int\int\int_Q\ x^2 + y^2 dV$ where $Q = \{-1 \leq \mathrm{x} \leq 1, -\sqrt{1-x^2} \leq \mathrm{y} \leq \sqrt{1-x^2}, \sqrt{x^2+y^2} \leq \mathrm{z} \leq 1\}$