

Homework 7

May 2, 2025

1. $\lim_{(x,y) \rightarrow (0,0)} \left[x^2 \sin\left(\frac{1}{y}\right) + y^2 \cos\left(\frac{1}{x}\right) \right] = ?$

2. $\lim_{y \rightarrow 0} \left(\lim_{x \rightarrow 0} \frac{\sin(x^2) - y^2}{x^2 + y^2} \right) = ?$

3. $\lim_{x \rightarrow 0} \left(\lim_{y \rightarrow 0} \frac{\sin(x^2) - y^2}{x^2 + y^2} \right) = ?$

4. $\lim_{(x,y) \rightarrow (1,-1)} \frac{xy + 1}{x^2 - y^2} = ?$

Hint: Use the change of variables $u = x - 1$, $v = y + 1$ to rewrite the limit around the origin.

5. Let $z = f(x, y) = x^2 y^3 + e^{xy}$, find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y} = ?$