Algorithm Analysis

1. What Is Algorithm Analysis?

2. Big O Notation

3. An Anagram Detection Example

4. Performance of `Python` Data Structures: `Lists/Dictionaries`

2.1~2.2 What Is Algorithm Analysis?

An interesting question often arises. When two programs solve the same problem but look different, is one program better than the other?

An interesting question often arises. When two programs solve the same problem but look different, is one program better than the other?

In order to answer this question, we need to remember that there is an important difference between a program and the underlying algorithm that the program is representing!

An interesting question often arises. When two programs solve the same problem but look different, is one program better than the other?

In order to answer this question, we need to remember that there is an important difference between a program and the underlying algorithm that the program is representing!

There may be many programs for the same algorithm, depending on the programmer and the programming language being used!

To explore this difference further, consider the function that computes the sum of the first integers. The algorithm uses the idea of an accumulator variable that is initialized to 0. The solution then iterates through the integers, adding each to the accumulator.

To explore this difference further, consider the function that computes the sum of the first integers. The algorithm uses the idea of an accumulator variable that is initialized to 0. The solution then iterates through the integers, adding each to the accumulator.

In [1]:
```
def sum_of_n(n):
    the_sum = 0
    for i in range(1, n + 1):
        the_sum = the_sum + i
    return the_sum

print(sum_of_n(10))
```

```
55
```

Now look at the function below:

Now look at the function below:

In [2]:
```
def foo(tom):
    fred = 0
    for bill in range(1, tom + 1):
        barney = bill
        fred = fred + barney
    return fred

print(foo(10))
```

```
55
```

Now look at the function below:

In [2]:
```
def foo(tom):
    fred = 0
    for bill in range(1, tom + 1):
        barney = bill
        fred = fred + barney
    return fred

print(foo(10))
```

```
55
```

At first glance it may look strange, but this function is essentially doing the same thing as the previous one. Here, we did not use good identifiers for readability, and we used an extra assignment statement that was not really necessary!

The function `sum_of_n()` is certainly better than the function `foo()` if you are concerned with readability. Easy to read and easy to understand is important for beginner. In this course, however, we are also interested in characterizing the algorithm itself.

The function `sum_of_n()` is certainly better than the function `foo()` if you are concerned with readability. Easy to read and easy to understand is important for beginner. In this course, however, we are also interested in characterizing the algorithm itself.

Algorithm analysis is concerned with comparing algorithms based upon the **amount of computing resources that each algorithm uses**.

The function `sum_of_n()` is certainly better than the function `foo()` if you are concerned with readability. Easy to read and easy to understand is important for beginner. In this course, however, we are also interested in characterizing the algorithm itself.

Algorithm analysis is concerned with comparing algorithms based upon the **amount of computing resources that each algorithm uses**.

We want to be able to consider two algorithms and say that one is better than the other because it is more efficient in its use of those resources or perhaps because it simply uses fewer.

There are two different ways to look at this. One way is to consider the **amount of space or memory** an algorithm requires to solve the problem.

There are two different ways to look at this. One way is to consider the **amount of space or memory** an algorithm requires to solve the problem.

As an alternative to space requirements, we can analyze and compare algorithms based on the **amount of time** they require to execute.

There are two different ways to look at this. One way is to consider the **amount of space or memory** an algorithm requires to solve the problem.

As an alternative to space requirements, we can analyze and compare algorithms based on the **amount of time** they require to execute.

One way is that we can measure the execution time for the function `sum_of_n()` to do a benchmark analysis. In `Python`, we can benchmark a function by noting the starting time and ending time within the system we are using.

In [3]:
```
import time

def sum_of_n_2(n):
    start = time.time()
    the_sum = 0
    for i in range(1, n + 1):
        the_sum = the_sum + i
    end = time.time()
    return the_sum, end - start
```

In [3]:
```
import time

def sum_of_n_2(n):
    start = time.time()
    the_sum = 0
    for i in range(1, n + 1):
        the_sum = the_sum + i
    end = time.time()
    return the_sum, end - start
```

In [4]:
```
for i in range(5):
    print("Sum is %d required %10.7f seconds" % sum_of_n_2(100_000))
```

```
Sum is 5000050000 required  0.0040002 seconds
Sum is 5000050000 required  0.0030003 seconds
Sum is 5000050000 required  0.0040002 seconds
Sum is 5000050000 required  0.0029995 seconds
Sum is 5000050000 required  0.0039990 seconds
```

We discover that the time is fairly consistent. What if we run the function adding the first 1,000,000 integers?

We discover that the time is fairly consistent. What if we run the function adding the first 1,000,000 integers?

In [5]:
```
for i in range(5):
    print("Sum is %d required %10.7f seconds" % sum_of_n_2(1_000_000))
```

```
Sum is 500000500000 required  0.0390027 seconds
Sum is 500000500000 required  0.0360374 seconds
Sum is 500000500000 required  0.0359602 seconds
Sum is 500000500000 required  0.0359993 seconds
Sum is 500000500000 required  0.0360005 seconds
```

Now consider the following function, which shows a different means of solving the summation problem. This function takes advantage of a closed equation to compute the sum of the first integers without iterating.

Now consider the following function, which shows a different means of solving the summation problem. This function takes advantage of a closed equation to compute the sum of the first integers without iterating.

In [6]:
```
def sum_of_n_3(n):
    start = time.time()
    the_sum = (n * (n + 1)) / 2
    end = time.time()
    return the_sum, end - start
print(sum_of_n_3(10)[0])
```

```
55.0
```

If we do the same benchmark measurement for `sum_of_n_3()`, using four different values for (100,000, 1,000,000, 10,000,000, and 100,000,000), we get the following results:

If we do the same benchmark measurement for `sum_of_n_3()`, using four different values for (100,000, 1,000,000, 10,000,000, and 100,000,000), we get the following results:

In [7]:
```
print("Sum is %d required %10.7f seconds" % sum_of_n_3(100_000))
print("Sum is %d required %10.7f seconds" % sum_of_n_3(1_000_000))
print("Sum is %d required %10.7f seconds" % sum_of_n_3(10_000_000))
print("Sum is %d required %10.7f seconds" % sum_of_n_3(100_000_000))
```

```
Sum is 5000050000 required  0.0000000 seconds
Sum is 500000500000 required  0.0000000 seconds
Sum is 50000005000000 required  0.0000000 seconds
Sum is 5000000050000000 required  0.0000000 seconds
```

If we do the same benchmark measurement for `sum_of_n_3()`, using four different values for (100,000, 1,000,000, 10,000,000, and 100,000,000), we get the following results:

In [7]:
```
print("Sum is %d required %10.7f seconds" % sum_of_n_3(100_000))
print("Sum is %d required %10.7f seconds" % sum_of_n_3(1_000_000))
print("Sum is %d required %10.7f seconds" % sum_of_n_3(10_000_000))
print("Sum is %d required %10.7f seconds" % sum_of_n_3(100_000_000))
```

```
Sum is 5000050000 required  0.0000000 seconds
Sum is 500000500000 required  0.0000000 seconds
Sum is 50000005000000 required  0.0000000 seconds
Sum is 5000000050000000 required  0.0000000 seconds
```

First, the times recorded above are shorter than any of the previous examples. Second, they are very consistent no matter what the value of . It appears that `sum_of_n_3()` is hardly impacted by the number of integers being added.

Intuitively, we can see that the iterative solutions seem to be doing more work since some program steps are being repeated. Also, the time required for the iterative solution seems to increase as we increase the value of .

Intuitively, we can see that the iterative solutions seem to be doing more work since some program steps are being repeated. Also, the time required for the iterative solution seems to increase as we increase the value of .

However, if we ran the same function on a different computer or used a different programming language, we would likely get different results. It could take even longer to perform `sum_of_n_3()` if the computer were older.

We need a better way to characterize these algorithms with respect to execution time. The benchmark does not really provide us with a useful measurement because it is dependent on a **particular machine, program, time of day, compiler, and programming language.**

We need a better way to characterize these algorithms with respect to execution time. The benchmark does not really provide us with a useful measurement because it is dependent on a **particular machine, program, time of day, compiler, and programming language.**

we would like to have a characterization that is independent of the program or computer being used. This measure would then be useful for judging the algorithm alone and could be used to compare algorithms **across implementations**!

2.3 Big O Notation

If each of these steps is considered to be a **basic unit** of computation, then the execution time for an algorithm can be expressed as the number of steps required to solve the problem!

If each of these steps is considered to be a **basic unit** of computation, then the execution time for an algorithm can be expressed as the number of steps required to solve the problem!

Deciding on an appropriate basic unit of computation can be a complicated problem and will depend on how the algorithm is implemented.

If each of these steps is considered to be a **basic unit** of computation, then the execution time for an algorithm can be expressed as the number of steps required to solve the problem!

Deciding on an appropriate basic unit of computation can be a complicated problem and will depend on how the algorithm is implemented.

A good basic unit of computation for comparing the summation algorithms might be the number of assignment statements performed to compute the sum.

In the function `sum_of_n()`, the number of assignment statements is 1 (`the_sum = 0`) plus the value of (the number of times we perform `the_sum = the_sum + 1`). We can denote this by a function, call it , where .

In the function `sum_of_n()`, the number of assignment statements is 1 (`the_sum = 0`) plus the value of (the number of times we perform `the_sum = the_sum + 1`). We can denote this by a function, call it , where .

The parameter is often referred to as the **size of the problem,** and we can read this as **is the time it takes to solve a problem of size , namely steps.**

In the function `sum_of_n()`, the number of assignment statements is 1 (`the_sum = 0`) plus the value of (the number of times we perform `the_sum = the_sum + 1`). We can denote this by a function, call it , where .

The parameter is often referred to as the **size of the problem,** and we can read this as **is the time it takes to solve a problem of size , namely steps.**

We can then say that the sum of the first 100,000 integers is a bigger instance of the summation problem than the sum of the first 1,000. Because of this, it might seem reasonable that the time required to solve the larger case would be greater than for the smaller case.

Our goal then is to show how the algorithm's execution time changes with respect to the size of the problem.

Our goal then is to show how the algorithm's execution time changes with respect to the size of the problem.

It turns out that the exact number of operations is not as important as determining the most dominant part of the function. In other words, as the problem gets larger, some portion of the function tends to overpower the rest.

Our goal then is to show how the algorithm's execution time changes with respect to the size of the problem.

It turns out that the exact number of operations is not as important as determining the most dominant part of the function. In other words, as the problem gets larger, some portion of the function tends to overpower the rest.

The **order of th emagnitude of the function** describes the part of that increases the fastest as the value of increases. Order of magnitude is often called Big O notation (for order) and written as .

It provides a useful approximation of the actual number of steps in the computation. The function provides a simple representation of the dominant part of the original .

It provides a useful approximation of the actual number of steps in the computation. The function provides a simple representation of the dominant part of the original .

In the above example, . As gets larger, the constant 1 will become less and less significant to the final result. If we are looking for an approximation for , then we can drop the 1 and simply say that the running time is .

It provides a useful approximation of the actual number of steps in the computation. The function provides a simple representation of the dominant part of the original .

In the above example, . As gets larger, the constant 1 will become less and less significant to the final result. If we are looking for an approximation for , then we can drop the 1 and simply say that the running time is .

It is important to note that the 1 is certainly significant for . However, as gets large, our approximation will be just as accurate without it.

As another example, suppose that for some algorithm, the exact number of steps is . When is small, say 1 or 2, the constant 1005 seems to be the dominant part of the function. However, as gets larger, the term becomes the most important! .

As another example, suppose that for some algorithm, the exact number of steps is . When is small, say 1 or 2, the constant 1005 seems to be the dominant part of the function. However, as gets larger, the term becomes the most important! .

In fact, when is really large, the other two terms become insignificant for the final result. Again, to approximate as gets large, we can ignore the other terms and focus on .

As another example, suppose that for some algorithm, the exact number of steps is . When is small, say 1 or 2, the constant 1005 seems to be the dominant part of the function. However, as gets larger, the term becomes the most important! .

In fact, when is really large, the other two terms become insignificant for the final result. Again, to approximate as gets large, we can ignore the other terms and focus on .

In addition, the coefficient becomes insignificant as gets large. We would say then that the function has an order of magnitude , or simply that it is .

Sometimes the performance of an algorithm depends on the **exact values of the data rather than simply the size of the problem.** For these kinds of algorithms we need to characterize their performance in terms of best-case, worst-case, or average-case performance.

Sometimes the performance of an algorithm depends on the **exact values of the data rather than simply the size of the problem.** For these kinds of algorithms we need to characterize their performance in terms of best-case, worst-case, or average-case performance.

The worst-case performance refers to a particular data set where the algorithm performs especially poorly, whereas a different data set for the exact same algorithm might have extraordinarily good (best-case) performance.

Sometimes the performance of an algorithm depends on the **exact values of the data rather than simply the size of the problem.** For these kinds of algorithms we need to characterize their performance in terms of best-case, worst-case, or average-case performance.

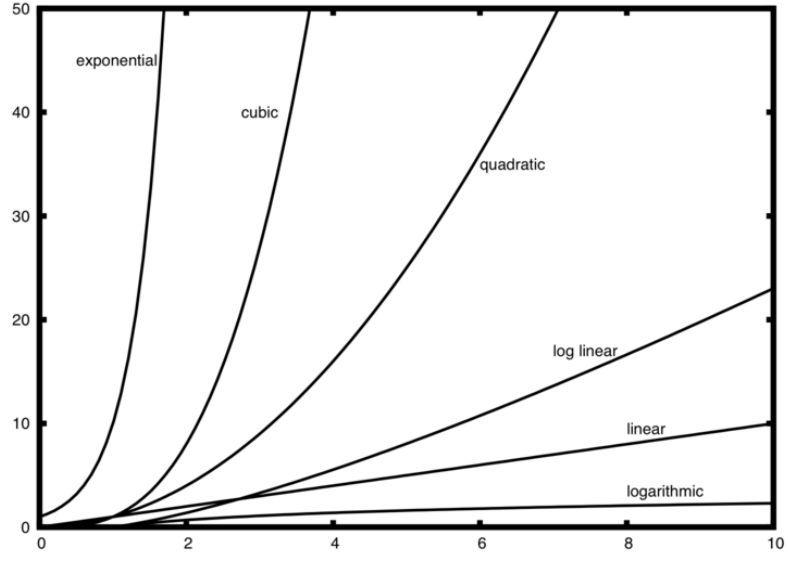
The worst-case performance refers to a particular data set where the algorithm performs especially poorly, whereas a different data set for the exact same algorithm might have extraordinarily good (best-case) performance.

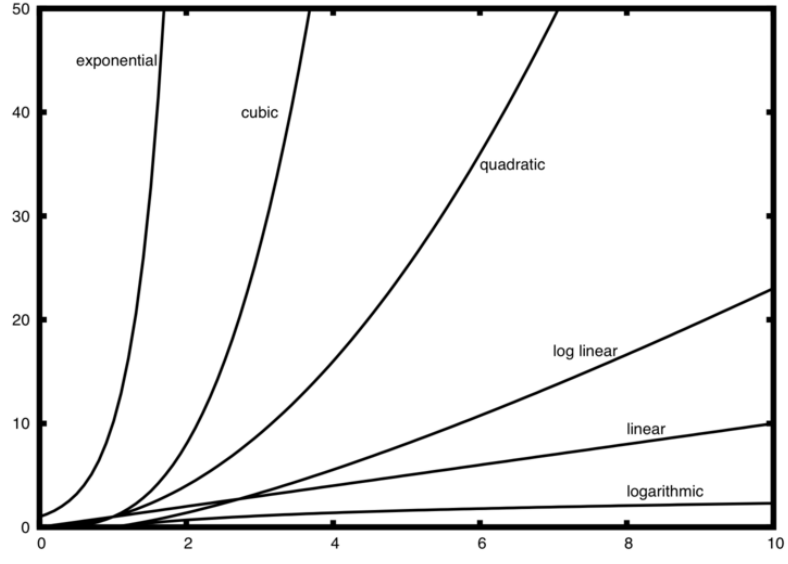
However, in most cases the algorithm performs somewhere in between these two extremes (average-case performance).

A number of very common order of magnitude functions will come up over and over as you study algorithms:

A number of very common order of magnitude functions will come up over and over as you study algorithms:

f(n)	Name
1	Constant
log(n)	Logarithmic
n	Linear
n log(n)	Log linear
n^2	Quadratic
n^3	Cubic
2^n	Exponential


50
40
30
20
10
0
0
2
4
6
8
10
exponential
cubic
quadratic
log linear
linear
logarithmic



Notice that when is small, the functions are not very well defined with respect to one another. It is hard to tell which is dominant.

As a final example, suppose that we have the fragment of `Python` code:

As a final example, suppose that we have the fragment of `Python` code:

In [8]:
```
n = 100
# Start of the code
a = 5
b = 6
c = 10
for i in range(n):
    for j in range(n):
        x = i * i
        y = j * j
        z = i * j
for k in range(n):
    w = a * k + 45
    v = b * b
d = 33
```

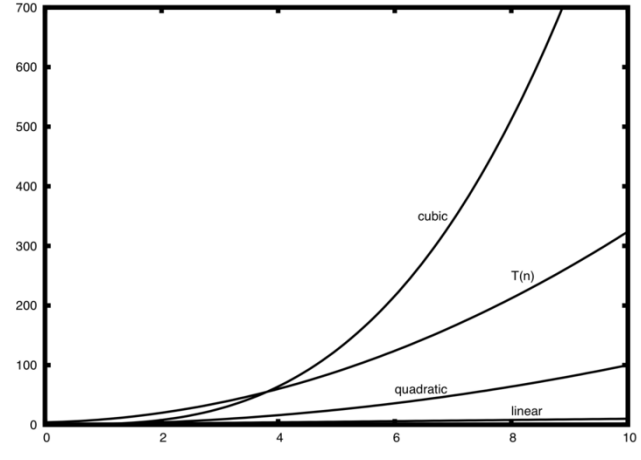
As a final example, suppose that we have the fragment of `Python` code:

In [8]:
```
n = 100
# Start of the code
a = 5
b = 6
c = 10
for i in range(n):
    for j in range(n):
        x = i * i
        y = j * j
        z = i * j
for k in range(n):
    w = a * k + 45
    v = b * b
d = 33
```

- The first part is the constant 3, representing the three assignment statements at the start of the fragment.
- The second part is due to the nested iteration.
- The third part is and the fourth part is the constant 1, representing the final assignment statement.

This gives us . We can see that the term will be dominant and therefore this code is . All of the other terms as well as the coefficient on the dominant term can be ignored as grows larger!

This gives us . We can see that the term will be dominant and therefore this code is . All of the other terms as well as the coefficient on the dominant term can be ignored as grows larger!



Exercise: Write two `Python` functions to find the minimum number in a list. The first function should compare each number to every other number on the list. . The second function should be linear .

Exercise: Write two `Python` functions to find the minimum number in a list. The first function should compare each number to every other number on the list. . The second function should be linear .

In [ ]:
```
## Your code here
```

2.4 An Anagram Detection Example

One string is an **anagram** of another if the second is simply a rearrangement of the first. For example, "heart" and "earth" are anagrams. The strings "python" and "typhon" are anagrams as well!

One string is an **anagram** of another if the second is simply a rearrangement of the first. For example, "heart" and "earth" are anagrams. The strings "python" and "typhon" are anagrams as well!

For the sake of simplicity, we will assume that the two strings in question are of **equal length** and that they are made up of symbols from the set of 26 lowercase alphabetic characters.

One string is an **anagram** of another if the second is simply a rearrangement of the first. For example, "heart" and "earth" are anagrams. The strings "python" and "typhon" are anagrams as well!

For the sake of simplicity, we will assume that the two strings in question are of **equal length** and that they are made up of symbols from the set of 26 lowercase alphabetic characters.

Our goal is to write a boolean function that will take two strings and return whether they are anagrams.

2.4.1 Solution 1: Anagram Detection Checking Off

Our first solution to the anagram problem will:

1. Check the lengths of the strings

Our first solution to the anagram problem will:

1. Check the lengths of the strings
2. Check to see that each character in the first string actually occurs in the second. If it is possible to check off each character, then the two strings must be anagrams.

Our first solution to the anagram problem will:

1. Check the lengths of the strings
2. Check to see that each character in the first string actually occurs in the second. If it is possible to check off each character, then the two strings must be anagrams.

Checking off a character will be accomplished by replacing it with the special value `None`. However, since strings in `Python` are immutable, the first step will be to convert the second string to a `list`. Each character from the first string can be checked against the characters in the list and if found, checked off by `None`.

In [9]:

```
def anagram_solution_1(s1, s2):
    still_ok = True
    if len(s1) != len(s2): # Step1
        still_ok = False

    a_list = list(s2)
    pos_1 = 0

    while pos_1 < len(s1) and still_ok: # Step2
        pos_2 = 0
        found = False
        while pos_2 < len(a_list) and not found:
            if s1[pos_1] == a_list[pos_2]:
                found = True
            else:
                pos_2 = pos_2 + 1
        if found:
            a_list[pos_2] = None
        else:
            still_ok = False
        pos_1 = pos_1 + 1

    return still_ok
```

In [ ]:

```
print(anagram_solution_1("apple", "pleap"))  # expected: True
print(anagram_solution_1("abcd", "dcba"))  # expected: True
print(anagram_solution_1("abcd", "dcda"))  # expected: False
```

In [ ]:
```
print(anagram_solution_1("apple", "pleap"))  # expected: True
print(anagram_solution_1("abcd", "dcba"))  # expected: True
print(anagram_solution_1("abcd", "dcda"))  # expected: False
```

Each of the characters in `s1` will cause an iteration through up to characters in the list from `s2`. Each of the positions in the list will be visited once to match a character from s1. The number of visits then becomes the sum of the integers from 1 to . Therfore, !

In [ ]:
```
print(anagram_solution_1("apple", "pleap"))  # expected: True
print(anagram_solution_1("abcd", "dcba"))  # expected: True
print(anagram_solution_1("abcd", "dcda"))  # expected: False
```

Each of the characters in `s1` will cause an iteration through up to characters in the list from `s2`. Each of the positions in the list will be visited once to match a character from s1. The number of visits then becomes the sum of the integers from 1 to . Therfore, !

As gets large, the term will dominate. Therefore, this solution is .

2.4.2 Solution 2: Sort and Compare

Another solution to the anagram problem will make use of the fact that even though `s1` and `s2` are different, they are anagrams only if they consist of exactly the same characters.

Another solution to the anagram problem will make use of the fact that even though `s1` and `s2` are different, they are anagrams only if they consist of exactly the same characters.

So if we begin by sorting each string alphabetically from a to z, we will end up with the same string if the original two strings are anagrams!

In [10]:

```
def anagram_solution_2(s1, s2):
    a_list_1 = list(s1)
    a_list_2 = list(s2)

    a_list_1.sort()
    a_list_2.sort()

    pos = 0
    matches = True

    while pos < len(s1) and matches:
        if a_list_1[pos] == a_list_2[pos]:
            pos = pos + 1
        else:
            matches = False

    return matches
```

In [10]:
```
def anagram_solution_2(s1, s2):
    a_list_1 = list(s1)
    a_list_2 = list(s2)

    a_list_1.sort()
    a_list_2.sort()

    pos = 0
    matches = True

    while pos < len(s1) and matches:
        if a_list_1[pos] == a_list_2[pos]:
            pos = pos + 1
        else:
            matches = False

    return matches
```

In [11]:
```
print(anagram_solution_2("apple", "pleap"))  # expected: True
print(anagram_solution_2("abcd", "dcba"))  # expected: True
print(anagram_solution_2("abcd", "dcda"))  # expected: False
```

```
True
True
False
```

At first glance you may be tempted to think that this algorithm is $O(n)$, since there is one simple iteration to compare the n characters after the sorting process. However, the two calls to the `Python sort()` method are not without their own cost. As we will see in Chapter 5, sorting is typically either $O(n^2)$ or $O(n \log n)$, so the sorting operations dominate the iteration.

2.4.3 Solution 3: Brute Force

A brute force technique for solving a problem tries to exhaust all possibilities. For the anagram detection problem, we can simply generate a list of all possible strings using the characters from `s1` and then see if `s2` occurs.

A brute force technique for solving a problem tries to exhaust all possibilities. For the anagram detection problem, we can simply generate a list of all possible strings using the characters from `s1` and then see if `s2` occurs.

However, when generating all possible strings from `s1`, there are possible first characters, possible characters for the second position, and so on. The total number of candidate strings is . Although some of the strings may be duplicates, the program cannot know this ahead of time!

It turns out that $n!$ grows even faster than 2^n as n gets large. In fact, if `s1` were 20 characters long, there would be $20! = 2,432,902,008,176,640,000$ possible candidate strings. If we processed one possibility every second, it would still take us $77,146,816,596$ years to go through the entire list!

2.4.4 Solution 4: Count and Compare

Our final solution to the anagram problem takes advantage of the fact that any two anagrams will have the same number of a's, the same number of b's, the same number of c's, and so on.

Our final solution to the anagram problem takes advantage of the fact that any two anagrams will have the same number of a's, the same number of b's, the same number of c's, and so on.

In order to decide whether two strings are anagrams

1. First count the number of times each character occurs. Since there are 26 possible characters, we can use a list of 26 counters, one for each possible character. Each time we see a particular character, we will increment the counter at that position.

Our final solution to the anagram problem takes advantage of the fact that any two anagrams will have the same number of a's, the same number of b's, the same number of c's, and so on.

In order to decide whether two strings are anagrams

1. First count the number of times each character occurs. Since there are 26 possible characters, we can use a list of 26 counters, one for each possible character. Each time we see a particular character, we will increment the counter at that position.

2. In the end, if the two lists of counters are identical, the strings must be anagrams!

In [12]:

```
def anagram_solution_4(s1, s2):
    c1 = [0] * 26 # Step 1, use ASCII code
    c2 = [0] * 26

    for i in range(len(s1)):
        pos = ord(s1[i]) - ord("a")
        c1[pos] = c1[pos] + 1

    for i in range(len(s2)):
        pos = ord(s2[i]) - ord("a")
        c2[pos] = c2[pos] + 1

    j = 0
    still_ok = True # Step 2
    while j < 26 and still_ok:
        if c1[j] == c2[j]:
            j = j + 1
        else:
            still_ok = False

    return still_ok
```

In [12]:
```
def anagram_solution_4(s1, s2):
    c1 = [0] * 26 # Step 1, use ASCII code
    c2 = [0] * 26

    for i in range(len(s1)):
        pos = ord(s1[i]) - ord("a")
        c1[pos] = c1[pos] + 1

    for i in range(len(s2)):
        pos = ord(s2[i]) - ord("a")
        c2[pos] = c2[pos] + 1

    j = 0
    still_ok = True # Step 2
    while j < 26 and still_ok:
        if c1[j] == c2[j]:
            j = j + 1
        else:
            still_ok = False

    return still_ok
```

In [13]:
```
print(anagram_solution_4("apple", "pleap"))  # expected: True
print(anagram_solution_4("abcd", "dcba"))  # expected: True
print(anagram_solution_4("abcd", "dcda"))  # expected: False
```

```
True
True
False
```

Unlike the first solution, none of the loops are nested. The first two iterations used to count the characters are both based on . The third iteration, comparing the two lists of counts, always takes 26 steps since there are 26 possible characters in the strings. Adding it all up gives us steps. That is . We have found a linear order of magnitude algorithm for solving this problem!

Unlike the first solution, none of the loops are nested. The first two iterations used to count the characters are both based on . The third iteration, comparing the two lists of counts, always takes 26 steps since there are 26 possible characters in the strings. Adding it all up gives us steps. That is . We have found a linear order of magnitude algorithm for solving this problem!

Before leaving this example, we need to say something about **space requirements**. Although the last solution was able to run in linear time, it could only do so by using additional storage to keep the two lists of character counts. **In other words, this algorithm sacrificed space in order to gain time.**

Unlike the first solution, none of the loops are nested. The first two iterations used to count the characters are both based on . The third iteration, comparing the two lists of counts, always takes 26 steps since there are 26 possible characters in the strings. Adding it all up gives us steps. That is . We have found a linear order of magnitude algorithm for solving this problem!

Before leaving this example, we need to say something about **space requirements**. Although the last solution was able to run in linear time, it could only do so by using additional storage to keep the two lists of character counts. **In other words, this algorithm sacrificed space in order to gain time.**

On many occasions you will need to make decisions between time and space trade-offs. In this case, the amount of extra space is not significant. However, if the underlying alphabet had millions of characters, there would be more concern.

2.6 Performance of `Python` Data Structures: Lists

`Python` had many choices to make when they implemented the `list` data structure. To help them make the right choices they looked at the ways that people would most commonly use the `list`, and they optimized their implementation of a `list` so that the most common operations were very fast!

`Python` had many choices to make when they implemented the `list` data structure. To help them make the right choices they looked at the ways that people would most commonly use the `list`, and they optimized their implementation of a `list` so that the most common operations were very fast!

Of course they also tried to make the less common operations fast, but when a trade-off had to be made the performance of a less common operation was often sacrificed in favor of the more common operation.

`Python` had many choices to make when they implemented the `list` data structure. To help them make the right choices they looked at the ways that people would most commonly use the `list`, and they optimized their implementation of a `list` so that the most common operations were very fast!

Of course they also tried to make the less common operations fast, but when a trade-off had to be made the performance of a less common operation was often sacrificed in favor of the more common operation.

Two common operations are indexing and assigning to an index position. Both of these operations take the same amount of time no matter how large the list becomes. When an operation like this is independent of the size of the list, it is .

Another very common programming task is to grow a list. You can use the `append()` method or the concatenation operator. The `append()` method is . However, the concatenation operator is , where is the size of the list that is being concatenated. This is important because it can help you make your own programs more efficient by choosing the right tool for the job.

Another very common programming task is to grow a list. You can use the `append()` method or the concatenation operator. The `append()` method is . However, the concatenation operator is , where is the size of the list that is being concatenated. This is important because it can help you make your own programs more efficient by choosing the right tool for the job.

Let's look at four different ways we might generate a list of numbers starting with 0.

1. First we'll try a `for` loop and create the `list` by concatenation
2. We'll use `append()` rather than concatenation.

Another very common programming task is to grow a list. You can use the `append()` method or the concatenation operator. The `append()` method is . However, the concatenation operator is , where is the size of the list that is being concatenated. This is important because it can help you make your own programs more efficient by choosing the right tool for the job.

Let's look at four different ways we might generate a list of numbers starting with 0.

1. First we'll try a `for` loop and create the `list` by concatenation
2. We'll use `append()` rather than concatenation.

3. Next, we'll try creating the `list` using list comprehension
4. using the `range()` function wrapped by a call to the `list` constructor.

In [14]:

```
def test1():
    l = []
    for i in range(1000):
        l = l + [i]

def test2():
    l = []
    for i in range(1000):
        l.append(i)

def test3():
    l = [i for i in range(1000)]

def test4():
    l = list(range(1000))
```

In [14]:
```
def test1():
    l = []
    for i in range(1000):
        l = l + [i]

def test2():
    l = []
    for i in range(1000):
        l.append(i)

def test3():
    l = [i for i in range(1000)]

def test4():
    l = list(range(1000))
```

We will use `Python`'s `timeit` module. The module is designed to allow developers to make cross-platform timing measurements by running functions in a consistent environment and using timing mechanisms that are as similar as possible across operating systems!

To use `timeit` you create a `Timer` object whose parameters are two `Python` statements.

1. The first parameter is a `Python` statement that you want to time
2. The second parameter is a statement that will run once to set up the test.

To use `timeit` you create a `Timer` object whose parameters are two `Python` statements.

1. The first parameter is a `Python` statement that you want to time
2. The second parameter is a statement that will run once to set up the test.

By default, `timeit` will try to run the statement one million times. When it's done it returns the time as a floating-point value representing the total number of seconds.

To use `timeit` you create a `Timer` object whose parameters are two `Python` statements.

1. The first parameter is a `Python` statement that you want to time
2. The second parameter is a statement that will run once to set up the test.

By default, `timeit` will try to run the statement one million times. When it's done it returns the time as a floating-point value representing the total number of seconds.

You can also pass `timeit` a named parameter called `number` that allows you to specify how many times the test statement is executed.

In [15]:
```
from timeit import Timer

t1 = Timer("test1()", "from __main__ import test1")
print(f"concatenation: {t1.timeit(number=1000):15.2f} milliseconds")
t2 = Timer("test2()", "from __main__ import test2")
print(f"appending: {t2.timeit(number=1000):19.2f} milliseconds")
t3 = Timer("test3()", "from __main__ import test3")
print(f"list comprehension: {t3.timeit(number=1000):10.2f} milliseconds")
t4 = Timer("test4()", "from __main__ import test4")
print(f"list range: {t4.timeit(number=1000):18.2f} milliseconds")
```

```
concatenation:            1.09 milliseconds
appending:                0.04 milliseconds
list comprehension:       0.02 milliseconds
list range:               0.01 milliseconds
```

In [15]:
```
from timeit import Timer

t1 = Timer("test1()", "from __main__ import test1")
print(f"concatenation: {t1.timeit(number=1000):15.2f} milliseconds")
t2 = Timer("test2()", "from __main__ import test2")
print(f"appending: {t2.timeit(number=1000):19.2f} milliseconds")
t3 = Timer("test3()", "from __main__ import test3")
print(f"list comprehension: {t3.timeit(number=1000):10.2f} milliseconds")
t4 = Timer("test4()", "from __main__ import test4")
print(f"list range: {t4.timeit(number=1000):18.2f} milliseconds")
```

```
concatenation:            1.09 milliseconds
appending:                0.04 milliseconds
list comprehension:       0.02 milliseconds
list range:               0.01 milliseconds
```

In the experiment above the statement that we are timing is the function call to `test1()`, `test2()`, and so on. You are probably very familiar with the `from... import` statement, but this is usually used at the beginning of a `Python` program file.

In this case the statement `from __main__ import test1` imports the function `test1` from the `__main__` namespace into the namespace that `timeit` sets up experiment. The `timeit` module wants to run the timing tests in an environment that is uncluttered by any variables you may have created that may interfere with your function's performance in some way!

In this case the statement `from __main__ import test1` imports the function `test1` from the `__main__` namespace into the namespace that `timeit` sets up experiment. The `timeit` module wants to run the timing tests in an environment that is uncluttered by any variables you may have created that may interfere with your function's performance in some way!

From the experiment above it is clear that the `append()` operation is much faster than concatenation. It is interesting to note that the list comprehension is twice as fast as a `for` loop with an `append()` operation.

In this case the statement `from __main__ import test1` imports the function `test1` from the `__main__` namespace into the namespace that `timeit` sets up experiment. The `timeit` module wants to run the timing tests in an environment that is uncluttered by any variables you may have created that may interfere with your function's performance in some way!

From the experiment above it is clear that the `append()` operation is much faster than concatenation. It is interesting to note that the list comprehension is twice as fast as a `for` loop with an `append()` operation.

One final observation is that all of the times that you see above include some overhead for actually calling the test function, but we can assume that the function call overhead is identical in all four cases so we still get a meaningful comparison of the operations. As an exercise you could test the time it takes to call an empty function and subtract that from the numbers above!

You can look at Table below to see the Big O efficiency of all the basic list operations.

You can look at Table below to see the Big O efficiency of all the basic list operations.

Operation	Big O Efficiency
index `[]`	O(1)
index assignment	O(1)
`append()`	O(1)
`pop()`	O(1)
`pop(i)`	O(n)
`insert(i, item)`	O(n)
`del` operator	O(n)
iteration	O(n)
contains (`in`)	O(n)
get slice `[x:y]`	O(k)
del slice	O(n)
set slice	O(n+k)
`reverse()`	O(n)
concatenate	O(k)
`sort()`	O(n log n)
multiply	O(nk)

You may be wondering about the two different times for `pop()`. When pop is called on the end of the `list` it takes , but when pop is called on the first element in the `list` — or anywhere in the middle — it is .

You may be wondering about the two different times for `pop()`. When pop is called on the end of the `list` it takes , but when pop is called on the first element in the `list` — or anywhere in the middle — it is .

When an item is taken from the front of the `list`, all the other elements in the `list` are shifted one position closer to the beginning. This may seem silly to you now, but if you look at Table you will see that this implementation also allows the index operation to be . This is a tradeoff that the Python designers thought was a good one!

Let's do another experiment using the `timeit` module. Our goal is to be able to verify the performance of the `pop()` operation on a `list` of a known size when the program pops from the end of the list, and again when the program pops from the beginning of the list.

Let's do another experiment using the `timeit` module. Our goal is to be able to verify the performance of the `pop()` operation on a `list` of a known size when the program pops from the end of the list, and again when the program pops from the beginning of the list.

In [16]:
```
# we do want to be able to use the list object x in our test
# This approach allows us to time just the single pop statement
# and get the most accurate measure of the time for that single operation
pop_zero = Timer("x.pop(0)", "from __main__ import x")
pop_end = Timer("x.pop()", "from __main__ import x")

x = list(range(2000000))
print(f"pop(0): {pop_zero.timeit(number=1000):10.5f} milliseconds")

x = list(range(2000000))
print(f"pop(): {pop_end.timeit(number=1000):11.5f} milliseconds")
```

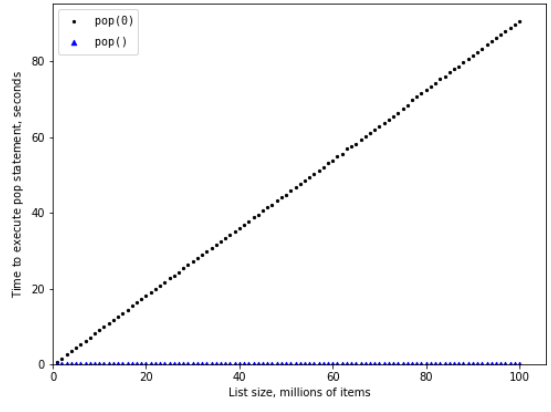
```
pop(0):    0.72283 milliseconds
pop():     0.00004 milliseconds
```

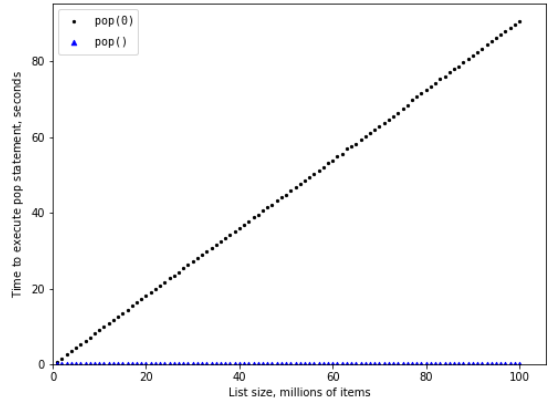
The above shows one attempt to measure the difference between the two uses of `pop()`. Popping from the end is much faster than popping from the beginning. However, this does not validate the claim that `pop(0)` is while `pop()` is . To validate that claim we need to look at the performance of both calls over a range of `list` sizes:

The above shows one attempt to measure the difference between the two uses of `pop()`. Popping from the end is much faster than popping from the beginning. However, this does not validate the claim that `pop(0)` is while `pop()` is . To validate that claim we need to look at the performance of both calls over a range of `list` sizes:

In [17]:
```
pop_zero = Timer("x.pop(0)", "from __main__ import x")
pop_end = Timer("x.pop()", "from __main__ import x")
print(f"{'n':10s}{'pop(0)':>15s}{'pop()':>15s}")
for i in range(1_000_000, 10_000_001, 1_000_000):
    x = list(range(i))
    pop_zero_t = pop_zero.timeit(number=1000)
    x = list(range(i))
    pop_end_t = pop_end.timeit(number=1000)
    print(f"{i:<10d}{pop_zero_t:>15.5f}{pop_end_t:>15.5f}")
```

```
n                   pop(0)          pop()
1000000            0.20570        0.00004
2000000            0.77414        0.00004
3000000            1.56184        0.00004
4000000            1.85473        0.00004
5000000            2.43492        0.00004
6000000            2.95523        0.00004
7000000            3.55318        0.00004
8000000            4.09937        0.00004
9000000            4.67260        0.00004
10000000           5.17886        0.00004
```


pop(0)
pop()
Time to execute pop statement, seconds
List size, millions of items
0
20
40
60
80
100



You can see that as the list gets longer and longer the time it takes to `pop(0)` also increases while the time for `pop()` stays very flat. This is exactly what we would expect to see for an and algorithm!

Exercise: Devise an experiment to verify that the list index operator is .

Exercise: Devise an experiment to verify that the list index operator is .

In [ ]:
```
## Your code here
```

2.7 Dictionaries

The second major `Python` data structure is the `dictionary`. As you probably recall, `dictionaries` differ from `lists` in that you can access items in a `dictionary` by a **key** rather than a position. Later in this book you will see that there are many ways to implement a dictionary!

The second major `Python` data structure is the `dictionary`. As you probably recall, `dictionaries` differ from `lists` in that you can access items in a `dictionary` by a **key** rather than a position. Later in this book you will see that there are many ways to implement a dictionary!

The thing that is most important to notice right now is that the get item and set item operations on a dictionary are . Another important dictionary operation is the `contains` operation. Checking to see whether a key is in the dictionary or not is also . The efficiency of all dictionary operations is summarized in Table below:

Operation	Big O Efficiency
copy	O(n)
get item	O(1)
set item	O(1)
delete item	O(1)
contains (in)	O(1)
iteration	O(n)

Operation	Big O Efficiency
copy	O(n)
get item	O(1)
set item	O(1)
delete item	O(1)
contains (in)	O(1)
iteration	O(n)

One important side note on dictionary performance is that the efficiencies we provide in the table are for **average performance**. In some rare cases the contains, get item, and set item operations can degenerate into performance, you can refer to Chapter 8 for more information.

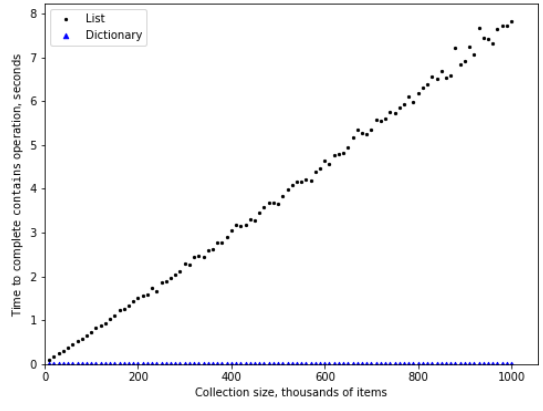
For our last performance experiment we will compare the performance of the `contains` operation between lists and dictionaries. In the process we will confirm that the contains operator for lists is and the contains operator for dictionaries is .

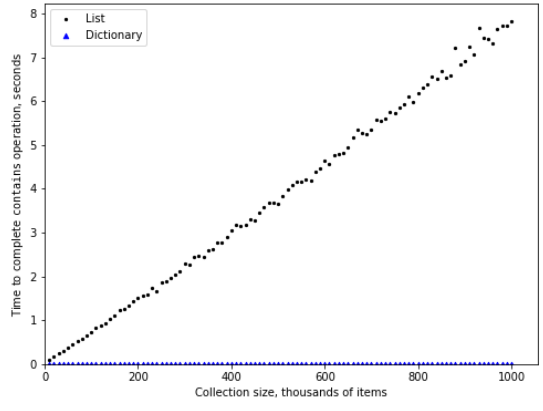
For our last performance experiment we will compare the performance of the `contains` operation between lists and dictionaries. In the process we will confirm that the contains operator for lists is and the contains operator for dictionaries is .

In [18]:
```
import timeit
import random

print(f"{'n':10s}{'list':>10s}{'dict':>10s}")
for i in range(10_000, 1_000_001, 100_000):
    t = timeit.Timer(f"random.randrange({i}) in x",
    "from __main__ import random, x")
    x = list(range(i))
    lst_time = t.timeit(number=1000)
    x = {j: None for j in range(i)}
    dict_time = t.timeit(number=1000)
    print(f"{i:<10,}{lst_time:>10.3f}{dict_time:>10.3f}")
```

```
n                 list      dict
10,000           0.040     0.000
110,000          0.438     0.001
210,000          0.833     0.001
310,000          1.242     0.001
410,000          1.729     0.001
510,000          2.091     0.001
610,000          2.596     0.001
710,000          2.988     0.001
810,000          3.377     0.001
910,000          3.856     0.001
```


List
Dictionary
Time to complete contains operation, seconds
Collection size, thousands of items
0
1
2
3
4
5
6
7
8
0
200
400
600
800
1000



You can see that the `dictionary` is consistently faster. You can also see that the time it takes for the `contains` operator on the `list` grows linearly with the size of the `list`. This verifies the assertion that the `contains` operator on a `list` is . It can also be seen that the time for the `contains` operator on a dictionary is constant even as the dictionary size grows.

References

1. Textbook CH2