A Reference Material

Selection of CMOS imaging sensors (Section 1.7.1)

R1

C: charge saturation capacity in electrons, FR: frame rate in s^{-1}, PC: pixel clock in MHz, QE: peak quantum efficiency

Chip	Format H × V	FR	PC	Pixel size H × V, μm	Comments
Linear response					
Micron[3] MT9V403	656×491	200	66	9.9×9.9	QE 0.32 @ 520 nm
Fillfactory[2] IBIS54-1300	1280×1024	30	40	6.7×6.7	QE 0.30–0.35 @ 600 nm, C 60k
Fillfactory[2] IBIS4-4000	2496×1692	4.5		11.4×11.4	C 150k
Fast frame rate linear response					
Fillfactory[2] LUPA1300	1280×1024	450	40	12.0×12.0	16 parallel ports
Micron[3] MV40	2352×1728	240	80	7.0×7.0	16 parallel 10-bit ports
Micron[3,5] MT9M413	1280×1024	600	80	12.0×12.0	QE 0.27 @ 520 nm, C 63k, 10 parallel 10-bit ports
Micron [4] MV02	512×512	4000	80	16.0×16.0	16 parallel 10-bit ports
Logarithmic response					
IMS HDRC VGA [4]	640×480	25	8	12×12	
PhotonFocus[1]	1024×1024	150	80	10.6×10.6	QE 0.29 @ 600 nm, C 200k, linear response at low light levels with adjustable transition to logarithmic response

Sources:

[1] `http://www.photonfocus.com`

[2] `http://www.fillfactory.com`

[3] `http://www.photobit.com`

[4] `http://www.ims-chips.de`

[5] `http://www.pco.de`

ISBN 3-540-24035-7

R2 Selection of CCD imaging sensors (Section 1.7.1)

C: charge saturation capacity in electrons, eNIR: enhanced NIR sensitivity, FR: frame rate in s^{-1}, ID: image diagonal in mm, QE: peak quantum efficiency, Sony (ICX...) and Kodak (KAI...) sensors

Chip	Format H × V	FR	ID	Pixel size H × V, μm	Comments
Interlaced EIA video					
ICX278AL 1/4"	768 × 494	30	4.56	4.75 × 5.55	eNIR
ICX258AL 1/3"	768 × 494	30	6.09	6.35 × 7.4	eNIR
ICX248AL 1/2"	768 × 494	30	8.07	8.4 × 9.8	eNIR
ICX422AL 2/3"	768 × 494	30	11.1	11.6 × 13.5	
Interlaced CCIR video					
ICX279AL 1/4"	752 × 582	25	4.54	4.85 × 4.65	eNIR
ICX259AL 1/3"	752 × 582	25	6.09	6.5 × 6.25	eNIR
ICX249AL 1/2"	752 × 582	25	8.07	8.6 × 8.3	eNIR
ICX423AL 2/3"	752 × 582	25	10.9	11.6 × 11.2	
Progressive scanning interline					
ICX098AL 1/4"	659 × 494	30	4.61	5.6 × 5.6	
ICX424AL 1/3"	659 × 494	30	6.09	7.4 × 7.4	
ICX074AL 1/2"	659 × 494	40	8.15	9.9 × 9.9	C 32k, QE 0.43 @ 340 nm
ICX414AL 1/2"	659 × 494	50	8.15	9.9 × 9.9	C 30k, QE 0.40 @ 500 nm
ICX075AL 1/2"	782 × 582	30	8.09	8.3 × 8.3	
ICX204AL 1/3"	1024 × 768	15	5.95	4.65 × 4.65	
ICX205AL 1/2"	1360 × 1024	9.5	7.72	4.65 × 4.65	C 13 ke
ICX285AL 2/3"	1360 × 1024	10	11.0	6.45 × 6.45	C 18k, QE 0.65 @ 500 nm
ICX085AL 2/3"	1300 × 1030	12.5	11.1	6.7 × 6.7	C 20k, QE 0.54 @ 380 nm
ICX274AL 1/1.8"	1628 × 1236	12	8.99	4.4 × 4.4	
KAI-0340DM 1/3"	640 × 480	200	5.92	7.4 × 7.4	C 20k, QE 0.55 @ 500 nm
KAI-1010M	1008 × 1018	30	12.9	9.0 × 9.0	QE 0.37 @ 500 nm
KAI-1020M	1000 × 1000	49	10.5	7.4 × 7.4	C 42k, QE 0.45 @ 490 nm
KAI-2001M	1600 × 1200	30	14.8	7.4 × 7.4	C 40k, QE 0.55 @ 480 nm
KAI-4020M	2048 × 2048	15	21.4	7.4 × 7.4	C 40k, QE 0.55 @ 480 nm
KAI-10000M	4008 × 2672	3	43.3	9.0 × 9.0	C 60k, QE 0.50 @ 500 nm

Sources:
`http://www.framos.de`
`http://www.kodak.com/global/en/digital/ccd/`
`http://www.pco.de`

Imaging sensors for the infrared (IR, Section 1.7.1)

C: full well capacity in millions of electrons [Me], IT: integration time, NETD: noise equivalent temperature difference, QE: peak quantum efficiency

Chip	Format H × V	FR	PC	Pixel size H × V, μm	Comments
Near infrared (NIR)					
Indigo[1] InGaAs	320 × 256	345		30 × 30	0.9–1.68 μm, C 3.5 Me
Mid wave infrared (MWIR)					
AIM[2] PtSi	640 × 486	50	12	24 × 24	3.0–5.0 μm, NETD < 75 mK @ 33 ms IT
Indigo[1] InSb	320 × 256	345		30 × 30	2.0–5.0 μm, C 18 Me
Indigo[1] InSb	640 × 512	100		25 × 25	2.0–5.0 μm, C 11 Me
AIM[2] HgCdTe	384 × 288	120	20	24 × 24	3.0–5.0 μm, NETD < 20 mK @ 2 ms IT
AIM[2]/IaF FhG[3] QWIP	640 × 512	30	18	24 × 24	3.0–5.0 μm, NETD < 15 mK @ 20 ms IT
Long wave infrared (LWIR)					
AIM[2] HgCdTe	256 × 256	200	16	40 × 40	8–10 μm, NETD < 20 mK @ 0.35 ms IT
Indigo[1] QWIP	320 × 256	345		30 × 30	8.0–9.2 μm, C 18 Me, NETD < 30 mK
AIM[2]/IaF FhG[3] QWIP	256 × 256	200	16	40 × 40	8.0–9.2 μm, NETD < 8 mK @ 20 ms IT
AIM[2]/IaF FhG[3] QWIP	640 × 512	30	18	24 × 24	8.0–9.2 μm, NETD < 10 mK @ 30 ms IT
Uncooled sensors					
Indigo[1] Microbolometer	320 × 240	60		30 × 30	7.0–14.0 μm, NETD < 120 mK

Sources:

[1] `http://www.indigosystems.com`

[2] `http://www.aim-ir.de`

[3] `http://www.iaf.fhg.de/tpqw/frames_d.htm`

R4 **Properties of the W-dimensional Fourier transform (Section 2.3.4)**

$g(\boldsymbol{x}) \circ\!\!-\!\!\bullet\ \hat{g}(\boldsymbol{k})$ and $h(\boldsymbol{x}) \circ\!\!-\!\!\bullet\ \hat{h}(\boldsymbol{k})$ are Fourier transform pairs: $\mathbb{R}^W \mapsto \mathbb{C}$:

$$\hat{g}(\boldsymbol{k}) = \int\limits_{-\infty}^{\infty} g(\boldsymbol{x}) \exp\left(-2\pi \mathrm{i} \boldsymbol{k}^T \boldsymbol{x}\right) \mathrm{d}^W x = \left\langle \exp\left(2\pi \mathrm{i} \boldsymbol{k}^T \boldsymbol{x}\right) \middle| g(\boldsymbol{x}) \right\rangle;$$

s is a real, nonzero number, a and b are complex constants; $\boldsymbol{A}$ is a W×W matrix, $\boldsymbol{R}$ is an orthogonal rotation matrix ($\boldsymbol{R}^{-1} = \boldsymbol{R}^T$, $\det R = 1$)

Property	Spatial domain	Fourier domain
Linearity	$a g(\boldsymbol{x}) + b h(\boldsymbol{x})$	$a\hat{g}(\boldsymbol{k}) + b\hat{h}(\boldsymbol{k})$
Similarity	$g(s\boldsymbol{x})$	$\hat{g}(\boldsymbol{k}/s)/\lvert s\rvert^W$
Generalized similarity	$g(\boldsymbol{A}\boldsymbol{x})$	$\hat{g}\left((\boldsymbol{A}^{-1})^T \boldsymbol{k}\right) / \det \boldsymbol{A}$
Rotation	$g(\boldsymbol{R}\boldsymbol{x})$	$\hat{g}(\boldsymbol{R}\boldsymbol{k})$
Separability	$\prod\limits_{w=1}^{W} g_w(x_w)$	$\prod\limits_{w=1}^{W} \hat{g}_w(k_w)$
Shift in x space	$g(\boldsymbol{x} - \boldsymbol{x}_0)$	$\exp(-2\pi \mathrm{i} \boldsymbol{k}^T \boldsymbol{x}_0)\hat{g}(\boldsymbol{k})$
Finite difference	$g(\boldsymbol{x} + \boldsymbol{x}_0/2) - g(\boldsymbol{x} - \boldsymbol{x}_0/2)$	$2\mathrm{i}\sin(\pi \boldsymbol{x}_0^T \boldsymbol{k})\hat{g}(\boldsymbol{k})$
Shift in k space	$\exp(2\pi \mathrm{i} \boldsymbol{k}_0^T \boldsymbol{x}) g(\boldsymbol{x})$	$\hat{g}(\boldsymbol{k} - \boldsymbol{k}_0)$
Modulation	$\cos(2\pi \boldsymbol{k}_0^T \boldsymbol{x}) g(\boldsymbol{x})$	$(\hat{g}(\boldsymbol{k} - \boldsymbol{k}_0) + \hat{g}(\boldsymbol{k} + \boldsymbol{k}_0))/2$
Differentiation in x space	$\dfrac{\partial g(\boldsymbol{x})}{\partial x_p}$	$2\pi \mathrm{i} k_p \hat{g}(\boldsymbol{k})$
Differentiation in k space	$-2\pi \mathrm{i} x_p g(\boldsymbol{x})$	$\dfrac{\partial \hat{g}(\boldsymbol{k})}{\partial k_p}$
Definite integral, mean	$\int\limits_{-\infty}^{\infty} g(\boldsymbol{x}')\mathrm{d}^W x'$	$\hat{g}(\boldsymbol{0})$
Moments	$\int\limits_{-\infty}^{\infty} x_p^m x_q^n g(\boldsymbol{x})\mathrm{d}^W x$	$\left(\dfrac{\mathrm{i}}{2\pi}\right)^{m+n} \left(\dfrac{\partial^{m+n}\hat{g}(\boldsymbol{k})}{\partial k_p^m \partial k_q^n}\right)\Bigg\rvert_{\boldsymbol{0}}$
Convolution	$\int\limits_{-\infty}^{\infty} h(\boldsymbol{x}')g(\boldsymbol{x} - \boldsymbol{x}')\mathrm{d}^W x'$	$\hat{h}(\boldsymbol{k})\hat{g}(\boldsymbol{k})$
Spatial correlation	$\int\limits_{-\infty}^{\infty} h(\boldsymbol{x}')g(\boldsymbol{x}' + \boldsymbol{x})\mathrm{d}^W x'$	$\hat{g}^*(\boldsymbol{k})\,\hat{h}(\boldsymbol{k})$
Multiplication	$h(\boldsymbol{x})g(\boldsymbol{x})$	$\int\limits_{-\infty}^{\infty} \hat{h}(\boldsymbol{k}')\hat{g}(\boldsymbol{k} - \boldsymbol{k}')\mathrm{d}^W k'$
Inner product	$\int\limits_{-\infty}^{\infty} g^*(\boldsymbol{x})\,h(\boldsymbol{x})\mathrm{d}^W x$	$\int\limits_{-\infty}^{\infty} \hat{g}^*(\boldsymbol{k})\hat{h}(\boldsymbol{k})\mathrm{d}^W k$

Elementary transform pairs for the continuous Fourier transform

R5

2-D and 3-D functions are marked by † and ‡, respectively.

Space domain	Fourier domain
Delta, $\delta(x)$	const., 1
const., 1	Delta, $\delta(k)$
$\cos(k_0 x)$	$\frac{1}{2}\left(\delta(k-k_0)+\delta(k+k_0)\right)$
$\sin(k_0 x)$	$\frac{\mathrm{i}}{2}\left(\delta(k-k_0)-\delta(k+k_0)\right)$
$\operatorname{sgn}(x) = \begin{cases} 1 & x \geq 0 \\ -1 & x < 0 \end{cases}$	$\frac{-\mathrm{i}}{\pi k}$
Box, $\Pi(x) = \begin{cases} 1 & \lvert x\rvert < 1/2 \\ 0 & \lvert x\rvert \geq 1/2 \end{cases}$	$\operatorname{sinc}(k) = \frac{\sin(\pi k)}{\pi k}$
Disk, † $\frac{1}{\pi r^2}\Pi\left(\frac{\lvert\boldsymbol{x}\rvert}{2r}\right)$	Bessel, $\frac{J_1(2\pi r\lvert\boldsymbol{k}\rvert)}{\pi r\lvert\boldsymbol{k}\rvert}$
Ball, ‡ $\Pi\left(\frac{\lvert\boldsymbol{x}\rvert}{2}\right)$	$\frac{\sin(\lvert\boldsymbol{k}\rvert)-\lvert\boldsymbol{k}\rvert\cos(\lvert\boldsymbol{k}\rvert)}{\lvert\boldsymbol{k}\rvert^3/(4\pi)}$
Bessel, $\frac{J_1(2\pi x)}{x}$	$2(1-k)^{1/2}\Pi\left(\frac{k}{2}\right)$
$\exp(-\lvert x\rvert)$, $\exp(-\lvert\boldsymbol{x}\rvert)^\dagger$	$\frac{2}{1+(2\pi k)^2}$, $\frac{2\pi}{(1+(2\pi\lvert\boldsymbol{k}\rvert)^2)^{3/2}}{}^\dagger$

Functions invariant under the Fourier transform

R6

Space domain	Fourier domain
Gaussian, $\exp\left(-\pi\boldsymbol{x}^T\boldsymbol{x}\right)$	Gaussian, $\exp\left(-\pi\boldsymbol{k}^T\boldsymbol{k}\right)$
$x_p\exp\left(-\pi\boldsymbol{x}^T\boldsymbol{x}\right)$	$-\mathrm{i}k_p\exp\left(-\pi\boldsymbol{k}^T\boldsymbol{k}\right)$
$\operatorname{sech}(\pi x) = \frac{1}{\exp(\pi x)+\exp(-\pi x)}$	$\operatorname{sech}(\pi k) = \frac{1}{\exp(\pi k)+\exp(-\pi k)}$
Hyperbola, $\lvert\boldsymbol{x}\rvert^{-W/2}$	$\lvert\boldsymbol{k}\rvert^{-W/2}$
1-D δ comb, $Ш(x) = \sum\limits_{n=-\infty}^{\infty}\delta(x-n)$	$Ш(k) = \sum\limits_{v=-\infty}^{\infty}\delta(k-v)$

R7 Properties of the 2-D DFT (Section 2.3.4)

$\boldsymbol{G}$ and $\boldsymbol{H}$ are complex-valued M×N matrices, $\hat{\boldsymbol{G}}$ and $\hat{\boldsymbol{H}}$ their Fourier transforms,

$$\hat{g}_{u,v} = \frac{1}{MN}\sum_{m=0}^{M-1}\sum_{n=0}^{N-1} g_{m,n} \mathrm{w}_M^{-mu} \mathrm{w}_N^{-nv}, \quad \mathrm{w}_N = \exp(2\pi \mathrm{i}/N)$$
$$g_{m,n} = \sum_{u=0}^{M-1}\sum_{v=0}^{N-1} \hat{g}_{u,v} \mathrm{w}_M^{mu} \mathrm{w}_N^{nv},$$

and a and b complex-valued constants. Stretching and replication by factors $K, L \in \mathbb{N}$ yields KM×LN matrices. For proofs see Cooley and Tukey [25], Poularikas [156].

Property	Space domain	Wave-number domain
Mean	$\frac{1}{MN}\sum_{m=0}^{M-1}\sum_{n=0}^{N-1} G_{mn}$	$\hat{g}_{0,0}$
Linearity	$a\boldsymbol{G} + b\boldsymbol{H}$	$a\hat{\boldsymbol{G}} + b\hat{\boldsymbol{H}}$
Spatial stretching (up-sampling)	$g_{Km,Ln}$	$\hat{g}_{uv}/(KL)$ $(\hat{g}_{kM+u,lN+v} = \hat{g}_{u,v})$
Replication (frequency stretching)	$g_{m,n}$ $(g_{kM+m,lN+n} = g_{m,n})$	$\hat{g}_{Ku,Lv}$
Shifting	$g_{m-m',n-n'}$	$\mathrm{w}_M^{-m'u}\mathrm{w}_N^{-n'v}\hat{g}_{uv}$
Modulation	$\mathrm{w}_M^{u'm}\mathrm{w}_N^{v'n} g_{m,n}$	$\hat{g}_{u-u',v-v'}$
Finite differences	$(g_{m+1,n} - g_{m-1,n})/2$ $(g_{m,n+1} - g_{m,n-1})/2$	$\mathrm{i}\sin(2\pi u/M)\hat{g}_{uv}$ $\mathrm{i}\sin(2\pi v/N)\hat{g}_{uv}$
Convolution	$\sum_{m'=0}^{M-1}\sum_{n'=0}^{N-1} h_{m'n'} g_{m-m',n-n'}$	$MN\hat{h}_{uv}\hat{g}_{uv}$
Spatial correlation	$\sum_{m'=0}^{M-1}\sum_{n'=0}^{N-1} h_{m'n'} g_{m+m',n+n'}$	$MN\hat{h}_{uv}\hat{g}^*_{uv}$
Multiplication	$g_{mn}h_{mn}$	$\sum_{u'=0}^{M-1}\sum_{v'=0}^{N-1} h_{u'v'} g_{u-u',v-v'}$
Inner product	$\sum_{m=0}^{M-1}\sum_{n=0}^{N-1} g^*_{mn} h_{mn}$	$\sum_{u=0}^{M-1}\sum_{v=0}^{N-1} \hat{g}^*_{uv}\hat{h}_{uv}$
Norm	$\sum_{m=0}^{M-1}\sum_{n=0}^{N-1} \lvert g_{mn}\rvert^2$	$\sum_{u=0}^{M-1}\sum_{v=0}^{N-1} \lvert\hat{g}_{uv}\rvert^2$

Properties of the continuous 1-D Hartley transform (Section 2.4.2) **R8**

$g(x) \circ\!\!-\!\!\bullet\ \hat{g}(k)$ and $h(x) \circ\!\!-\!\!\bullet\ \hat{h}(k)$ are Hartley transform pairs: $\mathbb{R} \mapsto \mathbb{R}$,

$$ {}^{h}\hat{g}(k) = \int_{-\infty}^{\infty} g(x)\,\mathrm{cas}(2\pi kx)\mathrm{d}x \quad \circ\!\!-\!\!\bullet \quad g(x) = \int_{-\infty}^{\infty} {}^{h}\hat{g}(k)\,\mathrm{cas}(2\pi kx)\mathrm{d}k $$

with

$$ \mathrm{cas}\,2\pi kx = \cos(2\pi kx) + \sin(2\pi kx). $$

s is a real, nonzero number, a and b are real constants.

Property	Spatial domain	Fourier domain
Linearity	$ag(x) + bh(x)$	$a\hat{g}(k) + b\hat{h}(k)$
Similarity	$g(sx)$	$\hat{g}(k/s)/\lvert s\rvert$
Shift in x space	$g(x - x_0)$	$\cos(2\pi kx_0)\hat{g}(k) - \sin(2\pi kx_0)\hat{g}(-k)$
Modulation	$\cos(2\pi k_0 x)g(x)$	$(\hat{g}(k-k_0) + \hat{g}(k+k_0))/2$
Differentiation in x space	$\dfrac{\partial g(\boldsymbol{x})}{\partial x_p}$	$-2\pi k_p \hat{g}(-k)$
Definite integral, mean	$\int_{-\infty}^{\infty} g(\boldsymbol{x}')\mathrm{d}x'$	$\hat{g}(0)$
Convolution	$\int_{-\infty}^{\infty} h(x')g(x-x')\mathrm{d}x'$	$[\hat{g}(k)\hat{h}(k) + \hat{g}(k)\hat{h}(-k)$ $+\hat{g}(-k)\hat{h}(k) - \hat{g}(-k)\hat{h}(-k)]/2$
Multiplication	$h(x)g(x)$	$[\hat{g}(k) * \hat{h}(k) + \hat{g}(k) * \hat{h}(-k)$ $+\hat{g}(-k) * \hat{h}(k) - \hat{g}(-k) * \hat{h}(-k)]/2$
Autocorrelation	$\int_{-\infty}^{\infty} g(x')g(x'+x)\mathrm{d}x'$	$[\hat{g}^2(k) + \hat{g}^2(-k)]/2$

1. Fourier transform expressed in terms of the Hartley transform

$$ \hat{g}(k) = \frac{1}{2}\left({}^{h}\hat{g}(k) +{}^{h}\hat{g}(-k)\right) - \frac{\mathrm{i}}{2}\left({}^{h}\hat{g}(k) -{}^{h}\hat{g}(-k)\right) $$

2. Hartley transform expressed in terms of the Fourier transform

$$ {}^{h}\hat{g}(k) = \Re[\hat{g}(k)] - \Im[\hat{g}(k)] = \frac{1}{2}\left(\hat{g}(k) + \hat{g}^*(k)\right) + \frac{\mathrm{i}}{2}\left(\hat{g}(k) - \hat{g}^*(k)\right) $$

R9 **Probability density functions (PDFs, Section 3.4).**

Definition, mean, and variance of some PDFs

Name	Definition	Mean	Variance
Discrete PDFs f_n			
Poisson $P(\mu)$	$\exp(-\mu)\dfrac{\mu^n}{n!},\ n \geq 0$	μ	μ
Binomial $B(Q,p)$	$\dfrac{Q!}{n!\,(Q-n)!}p^n(1-p)^{Q-n}, 0 \leq n < Q$	Qp	$Qp(1-p)$
Continuous PDFs $f(x)$			
Uniform $U(a,b)$	$\dfrac{1}{b-a}$	$\dfrac{a+b}{2}$	$\dfrac{(b-a)^2}{12}$
Normal $N(\mu,\sigma)$	$\dfrac{1}{\sqrt{2\pi}\sigma}\exp\left(-\dfrac{(x-\mu)^2}{2\sigma^2}\right)$	μ	σ^2
Rayleigh $R(\sigma)$	$\dfrac{x}{\sigma^2}\exp\left(-\dfrac{x^2}{2\sigma^2}\right),\ x > 0$	$\sigma\sqrt{\pi/2}$	$\sigma^2(4-\pi)/2$
Chi-square $\chi^2(Q,\sigma)$	$\dfrac{x^{Q/2-1}}{2^{Q/2}\Gamma(Q/2)\sigma^Q}\exp\left(-\dfrac{x}{2\sigma^2}\right),\ x > 0$	$Q\,\sigma^2$	$2Q\,\sigma^4$

Addition theorems for independent random variables g_1 and g_2

PDF	g_1	g_2	$g_1 + g_2$
Binomial	$B(Q_1,p)$	$B(Q_2,p)$	$B(Q_1+Q_2,p)$
Poisson	$P(\mu_1)$	$P(\mu_2)$	$P(\mu_1+\mu_2)$
Normal	$N(\mu_1,\sigma_1)$	$N(\mu_2,\sigma_2)$	$N(\mu_1+\mu_2,(\sigma_1^2+\sigma_2^2)^{1/2})$
Chi-square	$\chi^2(Q_1,\sigma)$	$\chi^2(Q_2,\sigma)$	$\chi^2(Q_1+Q_2,\sigma)$

PDFs of functions of independent random variables g_n

PDF of variable	Function	PDF of function
g_n: $N(0,\sigma)$	$(g_1^2+g_2^2)^{1/2}$	$R(\sigma)$
g_n: $N(0,\sigma)$	$\arctan(g_2^2/g_1^2)$	$U(0,2\pi)$
g_n: $N(0,\sigma)$	$\sum_{n=1}^{Q} g_n^2$	$\chi^2(Q,\sigma)$

Error propagation (Sections 3.2.3, 3.3.3, and 4.2.8) **R10**

f_g is the PDF of the random variable (RV) g, a, and b are constants, $g' = p(g)$ a differentiable monotonic function with the derivative $\mathrm{d}p/\mathrm{d}g$ and the inverse function $g = p^{-1}(g')$.
Let $\boldsymbol{g}$ be a vector with P RVs with the covariance matrix $\mathrm{cov}(\boldsymbol{g})$, $\boldsymbol{g}'$ a vector with Q RVs and with the covariance matrix $\mathrm{cov}(\boldsymbol{g}')$, $\boldsymbol{M}$ a $Q \times P$ matrix, and $\boldsymbol{a}$ a column vector with Q elements.

1. PDF, mean, and variance of a linear function $g' = ag + b$

$$f_{g'}(g') = \frac{f_g((g'-a)/b)}{|a|}, \quad \mu_{g'} = a\mu_g + b, \quad \sigma^2_{g'} = a^2\sigma^2_g$$

2. PDF of monotonous differentiable nonlinear function $g' = p(g)$

$$f_{g'}(g') = \frac{f_g(p^{-1}(g'))}{|\mathrm{d}p(p^{-1}(g'))/\mathrm{d}g|},$$

3. Mean and variance of differentiable nonlinear function $g' = p(g)$

$$\mu_{g'} \approx p(\mu_g) + \frac{\sigma^2_g}{2}\frac{\mathrm{d}^2 p(\mu_g)}{\mathrm{d}g^2}, \quad \sigma^2_{g'} \approx \left|\frac{\mathrm{d}p(\mu_g)}{\mathrm{d}g}\right|^2 \sigma^2_g$$

4. Covariance matrix of a linear combination of RVs, $\boldsymbol{g}' = \boldsymbol{M}\boldsymbol{g} + \boldsymbol{a}$

$$\mathrm{cov}(\boldsymbol{g}') = \boldsymbol{M}\,\mathrm{cov}(\boldsymbol{g})\boldsymbol{M}^T$$

5. Covariance matrix of a nonlinear combination of RVs, $\boldsymbol{g}' = \boldsymbol{p}(\boldsymbol{g})$

$$\mathrm{cov}(\boldsymbol{g}') \approx \boldsymbol{J}\,\mathrm{cov}(\boldsymbol{g})\boldsymbol{J}^T \quad \text{with the Jacobian matrix } \boldsymbol{J}, \quad j_{q,p} = \frac{\partial p_q}{\partial g_p}.$$

6. Homogeneous stochastic field: convolution of a random vector by the filter $\boldsymbol{h}$ $\boldsymbol{g}' = \boldsymbol{h} * \boldsymbol{g}$ (Section 4.2.8)
   (a) With the autocovariance vector $\boldsymbol{c}$

$$\boldsymbol{c}' = \boldsymbol{c} \star (\boldsymbol{h} \star \boldsymbol{h}) \quad \circ\!\!-\!\!\bullet \quad \hat{\boldsymbol{c}}'(k) = \hat{\boldsymbol{c}}(k)\left|\hat{\boldsymbol{h}}(k)\right|^2.$$

   (b) With the autocovariance vector $\boldsymbol{c} = \sigma^2\delta_n$ (uncorrelated elements)

$$\boldsymbol{c}' = \sigma^2(\boldsymbol{h} \star \boldsymbol{h}) \quad \circ\!\!-\!\!\bullet \quad \hat{\boldsymbol{c}}'(k) = \sigma^2\left|\hat{\boldsymbol{h}}(k)\right|^2.$$

R11 1-D LSI filters (Sections 4.2.6, 11.2, and 12.3)

1. Transfer function of a 1-D filter with an odd number of coefficients ($2R+1$, $[h_{-R},\ldots,h_{-1},h_0,h_1,\ldots,h_R]$)

   (a) General

$$\hat{h}(\tilde{k}) = \sum_{v'=-R}^{R} h_{v'} \exp(-\pi \mathrm{i} v' \tilde{k})$$

   (b) Even symmetry ($h_{-v} = h_v$)

$$\hat{h}_v = h_0 + 2\sum_{v'=1}^{R} h_{v'} \cos(\pi v' \tilde{k})$$

   (c) Odd symmetry ($h_{-v} = -h_v$)

$$\hat{h}_v = -2\mathrm{i}\sum_{v'=1}^{R} h_{v'} \sin(\pi v' \tilde{k})$$

2. Transfer function of a 1-D filter with an even number of coefficients ($2R$, $[h_{-R},\ldots,h_{-1},h_1,\ldots,h_R]$, convolution results put on intermediate grid)

   (a) Even symmetry ($h_{-v} = h_v$)

$$\hat{h}_v = 2\sum_{v'=1}^{R} h_{v'} \cos(\pi(v'-1/2)\tilde{k})$$

   (b) Odd symmetry ($h_{-v} = -h_v$)

$$\hat{h}_v = -2\mathrm{i}\sum_{v'=1}^{R} h_{v'} \sin(\pi(v'-1/2)\tilde{k})$$

3. Transfer function of the two elementary filters

   (a) Averaging of two neighboring points

$$\boldsymbol{B} = [1 \quad 1]/2 \quad \circ\!\!-\!\!\bullet \quad \hat{b}(\tilde{k}) = \cos(\pi\tilde{k}/2)$$

   (b) Difference of two neighboring points

$$\boldsymbol{D}_1 = [1 \quad -1] \quad \circ\!\!-\!\!\bullet \quad \hat{d}_1(\tilde{k}) = 2\mathrm{i}\sin(\pi\tilde{k}/2)$$

1-D recursive filters (Section 4.5). **R12**

1. General filter equation

$$g'_n = -\sum_{n''=1}^{S} a_{n''} g'_{n-n''} + \sum_{n'=-R}^{R} h_{n'} g_{n-n'}$$

2. General transfer function

$$\hat{h}(\tilde{k}) = \frac{\displaystyle\sum_{n'=-R}^{R} h_{n'} \exp(-\pi \mathrm{i} n' \tilde{k})}{\displaystyle\sum_{n''=0}^{S} a_{n''} \exp(-\pi \mathrm{i} n'' \tilde{k})}$$

3. Factorization of the transfer function using the z transform and the fundamental law of algebra

$$\hat{h}(z) = h_{-R} z^R \frac{\displaystyle\prod_{n'=1}^{2R} (1 - c_{n'} z^{-1})}{\displaystyle\prod_{n''=1}^{S} (1 - d_{n''} z^{-1})}$$

4. Relaxation filter
   (a) Filter equation ($|\alpha| < 1$)

$$g'_n = \alpha g'_{n\mp 1} + (1 - \alpha) g_n$$

   (b) Point spread function

$${}^{\pm}r_{\pm n} = \begin{cases} (1-\alpha)\alpha^n & n \geq 0 \\ 0 & \text{else} \end{cases}$$

   (c) Transfer function of the symmetric filter (running filter successively in positive and negative direction)

$$\hat{r}(\tilde{k}) = \frac{1}{1 + \beta - \beta \cos \pi \tilde{k}}, \quad \left(\hat{r}(0) = 1, \hat{r}(1) = \frac{1}{1 + 2\beta}\right)$$

   with

$$\beta = \frac{2\alpha}{(1-\alpha)^2}, \ \alpha = \frac{1 + \beta - \sqrt{1 + 2\beta}}{\beta}, \ \beta \in]-1/2, \infty[$$

5. Resonance filter with unit response at resonance wave number $\tilde{k}_0$ in the limit of low damping $1 - r \ll 1$

   (a) Filter equation (damping coefficient $r \in [0,1[$, resonance wave number $\tilde{k}_0 \in [0,1]$)

$$g'_n = (1 - r^2)\sin(\pi\tilde{k}_0)g_n + 2r\cos(\pi\tilde{k}_0)g'_{n\mp 1} - r^2 g'_{n\mp 2}$$

   (b) Point spread function

$$h_{\pm n} = \begin{cases} (1 - r^2)r^n \sin[(n+1)\pi\tilde{k}_0] & n \geq 0 \\ 0 & n < 0 \end{cases}$$

   (c) Transfer function of the symmetric filter (running filter successively in positive and negative direction)

$$\hat{s}(\tilde{k}) = \frac{\sin^2(\pi\tilde{k}_0)(1 - r^2)^2}{\left(1 - 2r\cos[\pi(\tilde{k} - \tilde{k}_0)] + r^2\right)\left(1 - 2r\cos[\pi(\tilde{k} + \tilde{k}_0)] + r^2\right)}$$

   (d) For low damping, the transfer function can be approximated by

$$\hat{s}(\tilde{k}) \approx \frac{1}{1 + (\tilde{k} - \tilde{k}_0)^2 \Big/ \frac{(1-r^2)^2}{4r^2\pi^2}} \quad \text{for} \quad 1 - r \ll 1$$

   (e) Halfwidth Δk, defined by $\hat{s}(\tilde{k}_0 + \Delta k) = 1/2$

$$\Delta k \approx (1 - r)/\pi$$

R13 Gaussian and Laplacian pyramids (Section 5.2)

1. Construction of the *Gaussian pyramid* $\boldsymbol{G}^{(0)}, \boldsymbol{G}^{(1)}, \ldots, \boldsymbol{G}^{(P)}$ with $P + 1$ planes by iterative smoothing and subsampling by a factor of two in all directions

$$\boldsymbol{G}^{(0)} = \boldsymbol{G}, \quad \boldsymbol{G}^{(p+1)} = \mathcal{B}_{\downarrow 2}\boldsymbol{G}^{(p)}$$

2. Condition for smoothing filter to avoid aliasing

$$\hat{B}(\tilde{\boldsymbol{k}}) = 0 \quad \forall \tilde{k}_p \geq \frac{1}{2}$$

3. Construction of the *Laplacian pyramid* $\boldsymbol{L}^{(0)}, \boldsymbol{L}^{(1)}, \ldots, \boldsymbol{L}^{(P)}$ with $P + 1$ planes from the Gaussian pyramid

$$\boldsymbol{L}^{(p)} = \boldsymbol{G}^{(p)} - \uparrow_2 \boldsymbol{G}^{(p+1)}, \quad \boldsymbol{L}^{(P)} = \boldsymbol{G}^{(P)}$$

   The last plane of the Laplacian pyramid is the last plane of the Gaussian pyramid.

4. Interpolation filters for upsampling operation $\uparrow_2$ (≻ R22)
5. Iterative reconstruction of the original image from the Laplacian pyramid. Compute

$$G^{(p-1)} = L^{(p-1)} + \uparrow_2 \; G^{(p)}$$

   starting with the highest plane ($p = P$). When the same upsampling operator is used as for the construction of the Laplacian pyramid, the reconstruction is perfect except for rounding errors.
6. Directio-pyramidal decomposition in two directional components

$$\begin{aligned}
G^{(p+1)} &= \downarrow_2 \mathcal{B}_x \mathcal{B}_y G^{(p)} \\
L^{(p)} &= G^{(p)} - \uparrow_2 G^{(p+1)} \\
L_x^{(p)} &= 1/2(L^{(p)} - (\mathcal{B}_x - \mathcal{B}_y) G^{(p)}) \\
L_y^{(p)} &= 1/2(L^{(p)} + (\mathcal{B}_x - \mathcal{B}_y) G^{(p)})
\end{aligned}$$

Basic properties of electromagnetic waves (Section 6.3) R14

1. The *frequency* ν (cycles per unit time) and *wavelength* λ (length of a period) are related by the *phase speed* c (in vacuum *speed of light* $c = 2.9979 \times 10^8\,\mathrm{m\,s^{-1}}$):

$$\lambda \nu = c$$

2. Classification of the ultraviolet, visible and infrared part of the electromagnetic spectrum (see also Fig. 6.6)

Name	Wavelength range	Comment
VUV (vacuum UV)	30–180 nm	Strongly absorbed by air; requires evacuated equipment
UV-C	100–280 nm	CIE standard definition
UV-B	280–315 nm	CIE standard definition
UV-A	315–400 nm	CIE standard definition
Visible (light)	400–700 nm	Visible by the human eye
VNIR (very near IR)	0.7–1.0 μm	IR wavelength range to which standard silicon image sensors respond
NIR (near IR)	0.7–3.0 μm	
TIR (thermal IR)	3.0–14.0 μm	Range of largest emission at environmental temperatures
MIR (middle IR)	3–100 μm	
FIR (far IR)	100–1000 μm	

3. Energy and momentum of particulate radiation such as β radiation (electrons), α radiation (helium nuclei), neutrons, and photons (electromagnetic radiation):

$$\begin{aligned} \nu &= E/h \quad \text{Bohr frequency condition,} \\ \lambda &= h/p \quad \text{de Broglie wavelength relation.} \end{aligned}$$

R15 Radiometric and photometric terms (Section 6.2)

dA_0 is an element of area in the surface, θ the angle of incidence, Ω the solid angle. For energy-, photon-, and photometry-related terms, often the indices e, p, and ν, respectively, are used.

Term	Energy-related	Photon-related	Photometric quantity
Energy	Radiant energy Q [Ws]	Photon number [1]	Luminous energy [lm s]
Energy flux (power)	Radiant flux $\Phi = \frac{dQ}{dt}$ [W]	Photon flux [s^{-1}]	Luminous flux [lumen (lm)]
Incident energy flux density	Irradiance $E = \frac{d\Phi}{dA_0}$ [$W\,m^{-2}$]	Photon irradiance [$m^{-2}s^{-1}$]	Illuminance [lm/m^2 = lux [(lx)]
Excitant energy flux density	Radiant excitance (emittance) $M = \frac{d\Phi}{dA_0}$ [$W\,m^{-2}$]	Photon flux density [$m^{-2}s^{-1}$]	Luminous excitance [lm/m^2]
Energy flux per solid angle	Radiant intensity $I = \frac{d\Phi}{d\Omega}$ [Wsr^{-1}]	Photon intensity [$s^{-1}sr^{-1}$]	Luminous intensity [lm/sr = candela (cd)]
Energy flux density per solid angle	Radiance $L = \frac{d^2\Phi}{d\Omega dA_0 \cos\theta}$ [$W\,m^{-2}\,sr^{-1}$]	Photon radiance [$m^{-2}s^{-1}sr^{-1}$]	Luminance [$cd\,m^{-2}$]
Energy/area	Energy density [$W\,s\,m^2$]	Photon density [m^{-2}]	Exposure [$lm\,s\,m^{-2}$ = lx s]

Computation of luminous quantities from the corresponding radiometric quantity by the *spectral luminous efficacy* $V(\lambda)$ for daylight (photopic) vision:

$$Q_v = 683 \frac{\mathrm{lm}}{\mathrm{W}} \int_{380\,\mathrm{nm}}^{780\,\mathrm{nm}} Q(\lambda) V(\lambda)\, \mathrm{d}\lambda$$

Table with the 1980 CIE values of the spectral luminous efficacy $V(\lambda)$ for photopic vision

λ [nm]	$V(\lambda)$	λ [nm]	$V(\lambda)$	λ [nm]	$V(\lambda)$
380	0.00004	520	0.710	660	0.061
390	0.00012	530	0.862	670	0.032
400	0.0004	540	0.954	680	0.017
410	0.0012	550	0.995	690	0.0082
420	0.0040	560	0.995	700	0.0041
430	0.0116	570	0.952	710	0.0021
440	0.023	580	0.870	720	0.00105
450	0.038	590	0.757	730	0.00052
460	0.060	600	0.631	740	0.00025
470	0.091	610	0.503	750	0.00012
480	0.139	620	0.381	760	0.00006
490	0.208	630	0.265	770	0.00003
500	0.323	640	0.175	780	0.000015
510	0.503	650	0.107		

Color systems (Section 6.2.4) R16

1. Human color vision based on three types of cones with maximal sensitivities at 445 nm, 535 nm, and 575 nm (Fig. 6.4b).
2. RGB color system; additive color system with the three primary colors red, green, and blue. This could either be monochromatic colors with wavelengths 700 nm, 646.1 nm, and 435.8 nm or red, green, and blue phosphor as used in RGB monitors (e. g., according to the European EBU norm). Not all colors can be represented by the RGB color system (see Fig. 6.5a).
3. Chromaticity diagram: reduction of the 3-D color space to a 2-D color plane normalized by the intensity:

$$r = \frac{R}{R+G+B}, \quad g = \frac{G}{R+G+B}, \quad b = \frac{B}{R+G+B}.$$

   It is sufficient to use the two components r and g: $b = 1 - r - g$.
4. XYZ color system (Fig. 6.5c): additive color system with three virtual primaries X, Y, and Z that can represent all possible colors and is given by the following linear transform from the EBU RGB color

system:

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} 0.490 & 0.310 & 0.200 \\ 0.177 & 0.812 & 0.011 \\ 0.000 & 0.010 & 0.990 \end{bmatrix} \begin{bmatrix} R \\ G \\ B \end{bmatrix}.$$

5. Color difference or YUV system: color system with an origin at the white point (Fig. 6.5b).
6. Hue-saturation (HSI) color system: color system using polar coordinates in a color difference system. The saturation is given by the radius and the hue by the angle.

R17 Thermal emission (Section 6.4.1)

1. Spectral emittance (law of Planck)

$$M_e(\lambda, T) = \frac{2\pi h c^2}{\lambda^5} \frac{1}{\exp\left(\frac{hc}{k_B T \lambda}\right) - 1}$$

with

$h = 6.6262 \times 10^{-34}\,\mathrm{J\,s}$	Planck constant,
$k_B = 1.3806 \times 10^{-23}\,\mathrm{J K^{-1}}$	Boltzmann constant, and
$c = 2.9979 \times 10^{8}\,\mathrm{m\,s^{-1}}$	speed of light in vacuum.

2. Total emittance (law of Stefan and Boltzmann)

$$M_e = \frac{2}{15} \frac{k_B^4 \pi^5}{c^2 h^3} T^4 = \sigma T^4 \quad \text{with} \quad \sigma \approx 5.67 \cdot 10^{-8} \mathrm{W\,m^{-2} K^{-4}}$$

3. Wavelength of maximum emittance (Wien's law)

$$\lambda_m \approx \frac{2898 \mathrm{K}\,\mu\mathrm{m}}{T}$$

R18 Interaction of radiation with matter (Section 6.4)

1. *Snell's law* of *refraction* at the boundary of two optical media with the indices of refraction n_1 and n_2

$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{n_2}{n_1}$$

θ_1 and θ_2 are the angles of incidence and refraction, respectively.

2. *Reflectivity* ρ: ratio of the reflected radiant flux to the incident flux at the surface. *Fresnel's equations* give the reflectivity for parallel polarized light

$$\rho_\parallel = \frac{\tan^2(\theta_1 - \theta_2)}{\tan^2(\theta_1 + \theta_2)},$$

for perpendicular polarized light

$$\rho_\perp = \frac{\sin^2(\theta_1 - \theta_2)}{\sin^2(\theta_1 + \theta_2)},$$

and for unpolarized light

$$\rho = \frac{\rho_\parallel + \rho_\perp}{2}.$$

3. Reflectivity at normal incidence ($\theta_1 = 0$) for all polarization states

$$\rho = \frac{(n_1 - n_2)^2}{(n_1 + n_2)^2} = \frac{(n-1)^2}{(n+1)^2} \quad \text{with} \quad n = n_1/n_2$$

4. *Total reflection.* When a ray enters into a medium with lower refractive index, beyond the critical angle θ_c all light is reflected and none enters the optically thinner medium:

$$\theta_c = \arcsin\frac{n_1}{n_2} \quad \text{with} \quad n_1 < n_2$$

Optical imaging

R19

1. Perspective projection with *pinhole camera* model

$$x_1 = -\frac{d'X_1}{X_3}, \quad x_2 = -\frac{d'X_2}{X_3}$$

Pinhole located at origin of world coordinate system $[X_1, X_2, X_3]^T$, d' is distance of image plane to projection center, X_3 axis aligned perpendicular to image plane.

2. Image equation (Newtonian and Gaussian form)

$$dd' = f^2 \quad \text{or} \quad \frac{1}{d'+f} + \frac{1}{d+f} = \frac{1}{f}$$

d and d' are the distances of the object and image to the front and back focal points of the optical system, respectively (see Fig. 7.7).

3. Lateral magnification

$$m_l = \frac{x_1}{X_1} = \frac{f}{d} = \frac{d'}{f}$$

4. Axial magnification

$$m_a \approx \frac{d'}{d} = \frac{f^2}{d^2} = \frac{d'^2}{f^2} = m_l^2$$

5. The f-number n_f of an optical system is the ratio of the focal length and diameter of lens aperture

$$n_f = \frac{f}{2r}$$

6. Depth of focus (image space)

$$\Delta x_3 = 2n_f \left(1 + \frac{d'}{f}\right)\epsilon = 2n_f(1 + m_l)\epsilon$$

7. Depth of field (object space)

$$\begin{array}{llcl} \text{Distant objects } (\Delta X_3 \ll d) & \Delta X_3 & \approx & 2n_f \cdot \dfrac{1+m_l}{m_l^2}\epsilon \\ d_{\min} \text{ for range including infinity} & d_{\min} & \approx & \dfrac{f^2}{4n_f\epsilon} \\ \text{Microscopy } (m_l \gg 1) & \Delta X_3 & \approx & \dfrac{2n_f\epsilon}{m_l} \end{array}$$

8. Resolution of a diffraction-limited optical system: angular resolution

$$\begin{array}{llcl} \text{Angular resolution} & \Delta\theta_0 & = & 0.61\dfrac{\lambda}{r} \\ \text{Lateral resolution at image plane} & \Delta x & = & 0.61\dfrac{\lambda}{n'_a} \\ \text{Lateral resolution at object plane} & \Delta X & = & 0.61\dfrac{\lambda}{n_a} \end{array}$$

The resolution is given by the Rayleigh criterion (see Fig. 7.15b); n_a and $n_{a'}$ are the object-sided and image sided numerical aperture of the light cone entering the optical system:

$$n_a = n \sin\theta_0 = \frac{2n}{n_f} = \frac{nr}{f};$$

n is the index of refraction.

9. Relation of the irradiance at image plane E' to the object radiance L (see Fig. 7.10)

$$E' = t\pi \left(\frac{r}{f + d'}\right)^2 \cos^4\theta\, L \approx t\pi \frac{\cos^4\theta}{n_f^2} L \quad \text{for} \quad d \gg f$$

Homogeneous point operation (Section 10.2) R20

Point operation that is independent of the position of the pixel

$$G'_{mn} = P(G_{mn})$$

1. Negative
$$P_N(q) = Q - 1 - q$$
2. Detection of underflow and overflow by a pseudocolor $[r, g, b]$ mapping
$$P_{uo}(q) = \begin{cases} [0, 0, Q-1] & \text{(blue)} \quad q = 0 \\ [q, q, q] & \text{(gray)} \quad q \in [1, Q-2] \\ [Q-1, 0, 0] & \text{(red)} \quad q = Q-1 \end{cases}$$
3. Contrast stretching of range $[q_1, q_2]$
$$P_{cs}(q) = \begin{cases} 0 & q < q_1 \\ \dfrac{(q-q_1)(Q-1)}{q_2 - q_1} & q \in [q_1, q_2] \\ Q-1 & q > q_2 \end{cases}$$

Calibration procedures R21

1. Noise equalization (Section 10.2.3)
If the variance of the noise depends on the image intensity, it can be equalized by a nonlinear grayscale transformation
$$h(g) = \sigma_h \int_0^g \frac{\mathrm{d}g'}{\sqrt{\sigma^2(g')}} + C$$
with the two free parameters σ_h and C. With a linear variance function (Section 3.4.5)
$$\sigma_g^2(g) = \sigma_0^2 + \alpha g$$
the transformation becomes for $g \in [0, g_{\max}] \mapsto h \in [0, \gamma g_{\max}]$
$$h(g) = \gamma g_{\max} \frac{\sqrt{\sigma_0^2 + Kg} - \sigma_0}{\sqrt{\sigma_0^2 + Kg_{\max}} - \sigma_0}, \quad \sigma_h = \frac{\gamma K g_{\max}/2}{\sqrt{\sigma_0^2 + Kg_{\max}} - \sigma_0}.$$
2. Linear photometric two-point calibration (Section 10.3.3)
Two calibration images are taken, a dark image $\boldsymbol{B}$ without any illumination and a reference image $\boldsymbol{R}$ with an object of constant radiance. A normalized image corrected for both the fixed pattern noise and inhomogeneous sensitivity is given by
$$\boldsymbol{G}' = c\,\frac{\boldsymbol{G} - \boldsymbol{B}}{\boldsymbol{R} - \boldsymbol{B}}.$$

R22 Interpolation (Section 10.5)

1. Interpolation of continuous function from sampled points at distances Δx_w is an convolution operation:

$$g_r(\boldsymbol{x}) = \sum_{\boldsymbol{n}} g(\boldsymbol{x_n}) h(\boldsymbol{x} - \boldsymbol{x_n}).$$

Reproduction of the grid points results in the *interpolation condition*

$$h(\boldsymbol{x_n}) = \begin{cases} 1 & \boldsymbol{n} = \boldsymbol{0} \\ 0 & \text{otherwise.} \end{cases}$$

2. Ideal interpolation function

$$h(\boldsymbol{x}) = \prod_{w=1}^{W} \operatorname{sinc}(x_w/\Delta x_w) \quad \circ\!\!-\!\!\bullet \quad \hat{h}(\boldsymbol{k}) = \prod_{w=1}^{W} \Pi(\tilde{k}_w/2)$$

3. Discrete 1-D interpolation filters for interpolation of intermediate grid points halfway between the existing points

Type	Mask	Transfer function
Linear	$\begin{bmatrix} 1 & 1 \end{bmatrix}/2$	$\cos(\pi\tilde{k}/2)$
Cubic	$\begin{bmatrix} -1 & 9 & 9 & -1 \end{bmatrix}/16$	$\dfrac{9\cos(\pi\tilde{k}/2) - \cos(3\pi\tilde{k}/2)}{8}$
Cubic B-spline	$\begin{bmatrix} 1 & 23 & 23 & 1 \end{bmatrix}/48$ $\begin{bmatrix} 3-\sqrt{3}, & \sqrt{3}-2 \end{bmatrix}^{\dagger}$	$\dfrac{23\cos(\pi\tilde{k}/2) + \cos(3\pi\tilde{k}/2)}{16 + 8\cos(\pi\tilde{k})}$

†Recursive filter applied in forward and backward direction, see Section 10.6.1

Averaging convolution filters (Chapter 11)

1. Summary of general constraints for averaging convolution filters

Property	Space domain	Wave-number domain
Preservation of mean	$\sum_n h_n = 1$	$\hat{h}(\mathbf{0}) = 1$
Zero shift, even symmetry	$h_{-\boldsymbol{n}} = h_{\boldsymbol{n}}$	$\Im\left(\hat{h}(\boldsymbol{k})\right) = 0$
Monotonic decrease from one to zero	—	$\hat{h}(\tilde{k}_2) \leq \hat{h}(\tilde{k}_1)$ if $\tilde{k}_2 > \tilde{k}_1$, $\hat{h}(\boldsymbol{k}) \in [0, 1]$
Isotropy	$h(\boldsymbol{x}) = h(\lvert\boldsymbol{x}\rvert)$	$\hat{h}(\boldsymbol{k}) = \hat{h}(\lvert\boldsymbol{k}\rvert)$

2. 1-D smoothing box filters

Mask	Transfer function	Noise suppression†
${}^3\boldsymbol{R} = [1\ 1\ 1]/3$	$\dfrac{1}{3} + \dfrac{2}{3}\cos(\pi\tilde{k})$	$\dfrac{1}{\sqrt{3}} \approx 0.577$
${}^4\boldsymbol{R} = [1\ 1\ 1\ 1]/4$	$\cos(\pi\tilde{k})\cos(\pi\tilde{k}/2)$	$1/2 = 0.5$
${}^R\boldsymbol{R} = \underbrace{[1\ \ldots\ 1]}_{R\text{ times}}/R$	$\dfrac{\sin(\pi R\tilde{k}/2)}{R\sin(\pi\tilde{k}/2)}$	$\dfrac{1}{\sqrt{R}}$

†For white noise

3. 1-D smoothing binomial filters

Mask	TF	Noise suppression†
$\boldsymbol{B}^2 = [1\ 2\ 1]/4$	$\cos^2(\pi\tilde{k}/2)$	$\sqrt{\dfrac{3}{8}} \approx 0.612$
$\boldsymbol{B}^4 = [1\ 4\ 6\ 4\ 1]/16$	$\cos^4(\pi\tilde{k}/2)$	$\sqrt{\dfrac{35}{128}} \approx 0.523$
$\boldsymbol{B}^{2R}$	$\cos^{2R}(\pi\tilde{k}/2)$	$\left(\dfrac{\Gamma(R+1/2)}{\sqrt{\pi}\,\Gamma(R+1)}\right)^{1/2} \approx \left(\dfrac{1}{R\pi}\right)^{1/4}\left(1 - \dfrac{1}{16R}\right)$

†For white noise

R24 First-order derivative convolution filters (Chapter 12)

1. Summary of general constraints for a first-order derivative filter into the direction x_w for W-dimensional signals; w' denotes any of the possible directions and $\boldsymbol{n}$ vector indexing (Section 4.2.1)

Property	Space domain	Wave-number domain
Zero mean	$\sum_{\boldsymbol{n}} h_{\boldsymbol{n}} = 0$	$\hat{h}(\tilde{\boldsymbol{k}})\Big\vert_{\tilde{\boldsymbol{k}}=\boldsymbol{0}} = 0$
Zero shift, odd symmetry	$h_{n_1,\ldots,-n_w,\ldots,n_W} = -h_{n_1,\ldots,n_w,\ldots,n_W}$	$\Re\left(\hat{H}(\boldsymbol{k})\right) = 0$
First-order derivative	$\sum_{\boldsymbol{n}} n_{w'} h_{\boldsymbol{n}} = \delta_{w'-w}$	$\left.\dfrac{\partial \hat{h}(\tilde{\boldsymbol{k}})}{\partial \tilde{k}_w}\right\vert_{\tilde{\boldsymbol{k}}=\boldsymbol{0}} = \pi \mathrm{i} \delta_{w'-w}$
Isotropy		$\hat{h}(\tilde{\boldsymbol{k}}) = \pi \mathrm{i} \tilde{k}_w \hat{b}(\vert\tilde{\boldsymbol{k}}\vert)$ with $\hat{b}(0) = 1$, $\nabla_k \hat{b}(\vert\tilde{\boldsymbol{k}}\vert) = \boldsymbol{0}$

2. First-order discrete difference filters

Name	Mask	Transfer function
$\mathcal{D}_x$	$\begin{bmatrix} 1 & -1 \end{bmatrix}$	$2\mathrm{i}\sin(\pi\tilde{k}_x/2)$
Symmetric difference, $\mathcal{D}_{2x}$	$\begin{bmatrix} 1 & 0 & -1 \end{bmatrix}/2$	$\mathrm{i}\sin(\pi\tilde{k}_x)$
Cubic B-spline $\mathcal{D}_{2x}\,{}^{\pm}\mathcal{R}$	$\begin{bmatrix} 1 & 0 & -1 \end{bmatrix}/2$, $\begin{bmatrix} 3-\sqrt{3}, & \sqrt{3}-2 \end{bmatrix}^{\dagger}$	$\mathrm{i}\dfrac{\sin(\pi\tilde{k}_x)}{2/3 + 1/3\cos(\pi\tilde{k}_x)}$

†Recursive filter applied in forward and backward direction, see Section 10.6.1

3. Regularized first-order discrete difference filters

Name	Mask	Transfer function
2×2, $\mathcal{D}_x\mathcal{B}_y$	$\frac{1}{2}\begin{bmatrix} 1 & -1 \\ 1 & -1 \end{bmatrix}$	$2\mathrm{i}\sin(\pi\tilde{k}_x/2)\cos(\pi\tilde{k}_y/2)$
Sobel, $\mathcal{D}_{2x}\mathcal{B}_y^2$	$\frac{1}{8}\begin{bmatrix} 1 & 0 & -1 \\ 2 & 0 & -2 \\ 1 & 0 & -1 \end{bmatrix}$	$\mathrm{i}\sin(\pi\tilde{k}_x)\cos^2(\pi\tilde{k}_y/2)$
Optimized Sobel $\mathcal{D}_{2x}(3\mathcal{B}_y^2 + \mathcal{I})/4$	$\frac{1}{32}\begin{bmatrix} 3 & 0 & -3 \\ 10 & 0 & -10 \\ 3 & 0 & -3 \end{bmatrix}$	$\mathrm{i}\sin(\pi\tilde{k}_x)(3\cos^2(\pi\tilde{k}_y/2) + 1)/4$

4. Performance characteristics of edge detectors: angle error, magnitude error, and noise suppression for white noise. The three values in the two error columns give the errors for a wave number range of 0–0.25, 0.25–0.5, and 0.5–0.75, respectively.

Name	Angle error [°]	Magnitude error	Noise factor
$\mathcal{D}_x$			$\sqrt{2} \approx 1.414$
$\mathcal{D}_{2x}$	1.36 4.90 12.66	0.026 0.151 0.398	$1/\sqrt{2} \approx 0.707$
$\mathcal{D}_{2x}\,{}^{\pm}\mathcal{R}$	0.02 0.33 2.26	0.001 0.023 0.220	$\sqrt{3\ln 3/\pi} \approx 1.024$
$\mathcal{D}_x\mathcal{B}_y$	0.67 2.27 5.10	0.013 0.079 0.221	1
$\mathcal{D}_{2x}\mathcal{B}_y^2$	0.67 2.27 5.10	0.012 0.053 0.070	$\sqrt{3}/4 \approx 0.433$
$\mathcal{D}_{2x}(3\mathcal{B}_y^2 + \mathcal{I})/4$	0.15 0.32 0.72	0.003 0.005 0.047	$\sqrt{59}/16 \approx 0.480$

R25 Second-order derivative convolution filters (Chapter 12)

1. Summary of general constraints for a second-order derivative filter into the direction x_w for W-dimensional signals; w' denotes any of the possible directions and $\boldsymbol{n}$ vector indexing (Section 4.2.1)

Property	Space domain	Wave-number domain
Zero mean	$\sum\limits_{\boldsymbol{n}} h_{\boldsymbol{n}} = 0$	$\hat{h}(\tilde{\boldsymbol{k}})\Big\vert_{\tilde{\boldsymbol{k}}=\boldsymbol{0}} = 0$
Zero slope	$\sum\limits_{\boldsymbol{n}} n_{w'} h_{\boldsymbol{n}} = 0$	$\left.\dfrac{\partial \hat{h}(\tilde{\boldsymbol{k}})}{\partial \tilde{k}_{w'}}\right\vert_{\tilde{\boldsymbol{k}}=\boldsymbol{0}} = 0$
Zero shift, even symmetry	$h_{-\boldsymbol{n}} = h_{\boldsymbol{n}}$	$\Im\left(\hat{H}(\boldsymbol{k})\right) = 0$
2nd-order derivative	$\sum\limits_{\boldsymbol{n}} n_{w'}^2 h_{\boldsymbol{n}} = 2\delta_{w'-w}$	$\left.\dfrac{\partial^2 \hat{h}(\tilde{\boldsymbol{k}})}{\partial \tilde{k}_{w'}^2}\right\vert_{\tilde{\boldsymbol{k}}=\boldsymbol{0}} = -2\pi^2\delta_{w'-w}$
Isotropy		$\hat{h}(\tilde{\boldsymbol{k}}) = -(\pi\tilde{k}_w)^2\hat{b}(\vert\tilde{\boldsymbol{k}}\vert)$ with $\hat{b}(0) = 1$, $\nabla_k \hat{b}(\vert\tilde{\boldsymbol{k}}\vert) = \boldsymbol{0}$

2. Second-order discrete difference filters

Name	Mask	Transfer function
1-D Laplace $\mathcal{D}_x^2$	$\begin{bmatrix} 1 & -2 & 1 \end{bmatrix}$	$-4\sin^2(\pi\tilde{k}_x/2)$
2-D Laplace $\mathcal{L}$	$\begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix}$	$-4\sin^2(\pi\tilde{k}_x/2) - 4\sin^2(\pi\tilde{k}_y/2)$
2-D Laplace $\mathcal{L}'$	$\dfrac{1}{4}\begin{bmatrix} 1 & 2 & 1 \\ 2 & -12 & 2 \\ 1 & 2 & 1 \end{bmatrix}$	$4\cos^2(\pi\tilde{k}_x/2)\cos^2(\pi\tilde{k}_y/2) - 4$

B Notation

Because of the multidisciplinary nature of digital image processing, a consistent and generally accepted terminology — as in other areas — does not exist. Two basic problems must be addressed.

- *Conflicting terminology.* Different communities use different symbols (and even names) for the same terms.
- *Ambiguous symbols.* Because of the many terms used in image processing and the areas it is related to, one and the same symbol is used for multiple terms.

There exists no trivial solution to this awkward situation. Otherwise it would be available. Thus conflicting arguments must be balanced. In this textbook, the following guidelines are used:

- *Stick to common standards.* As a first guide, the symbols recommended by international organizations (such as the International Organization for Standardization, ISO) were consulted and several major reference works were compared [46, 123, 128, 156]. Additionally cross checks were made with several standard textbooks from different areas [13, 61, 148, 158]. Only in a few conflicting situations deviations from commonly accepted symbols are used.
- *Use most compact notation.* When there was a choice of different notations, the most compact and comprehensive notation was used. In rare cases, it appeared useful to use more than one notation for the same term. It is, for example, sometimes more convenient to use indexed vector components ($\boldsymbol{x} = [x_1, x_2]^T$), and sometimes to use $\boldsymbol{x} = [x, y]^T$.
- *Allow ambiguous symbols.* One and the same symbol can have different meanings. This is not so bad as it appears at first glance because from the context the meaning of the symbol becomes unambiguous. Thus care was taken that ambiguous symbols were only used when they can clearly be distinguished by the context.

In order to familiarize readers coming from different backgrounds to the notation used in this textbook, we will give here some comments on deviating notations.

B. Jähne, Digital Image Processing
ISBN 3-540-24035-7

Wave number. Unfortunately, different definitions for the term *wave number* exist:

$$k' = \frac{2\pi}{\lambda} \quad \text{and} \quad k = \frac{1}{\lambda}. \tag{B.1}$$

Physicists usually include the factor 2π in the definition of the wave number: $k' = 2\pi/\lambda$, by analogy to the definition of the circular frequency $\omega = 2\pi/T = 2\pi\nu$. In optics and spectroscopy, however, it is defined as the inverse of the wavelength without the factor 2π (i. e., number of wavelengths per unit length) and denoted by $\tilde{\nu} = \lambda^{-1}$.

Imaginary unit. The imaginary unit is denoted here by i. In electrical engineering and related areas, the symbol j is commonly used.

Time series, image matrices. The standard notation for *time series* [148], $x[n]$, is too cumbersome to be used with multidimensional signals: $g[k][m][n]$. Therefore the more compact notation x_n and $g_{k,m,n}$ is chosen.

Partial derivatives. In cases were it does not lead to confusion, partial derivates are abbreviated by indexing: $\partial g/\partial x = \partial_x g = g_x$

Typeface	Description
e, i, d, w	Upright symbols have a special meaning; examples: e for the base of natural logarithm, $\mathrm{i} = \sqrt{-1}$, symbol for derivatives: $\mathrm{d}g$, $\mathrm{w} = \mathrm{e}^{2\pi\mathrm{i}}$
$a, b, \ldots$	Italic (not bold): *scalar*
$\boldsymbol{g}, \boldsymbol{k}, \boldsymbol{u}, \boldsymbol{x}, \ldots$	Lowercase italic bold: *vector*, i. e., a coordinate vector, a time series, row of an image, …
$\boldsymbol{G}, \boldsymbol{H}, \boldsymbol{J}, \ldots$	Uppercase italic bold: *matrix*, *tensor*, i. e., a discrete image, a 2-D convolution mask, a structure tensor; also used for signals with more than two dimensions
$\mathcal{B}, \mathcal{R}, \mathcal{F}, \ldots$	Caligraphic letters indicate a representation-independent *operator*
$\mathbb{N}, \mathbb{Z}, \mathbb{R}, \mathbb{C}$	Blackboard bold letters denote sets of numbers or other quantities

Accents	Description
$\bar{\boldsymbol{k}}, \bar{\boldsymbol{n}}, \ldots$	A bar indicates a *unit vector*
$\tilde{k}, \tilde{\boldsymbol{k}}, \tilde{\boldsymbol{x}}, \ldots$	A tilde indicates a *dimensionless normalized* quantity (of a quantity with a dimension)
$\hat{\boldsymbol{G}}, \hat{g}(k), \ldots$	A hat indicates a quantity in the *Fourier domain*

Subscript	Description
g_n	Element n of the vector $\boldsymbol{g}$
g_{mn}	Element m, n of the matrix $\boldsymbol{G}$
g_p	Compact notation for first-order partial derivative of the continuous function g into the direction p: $\partial g(\boldsymbol{x})/\partial x_p$
g_{pq}	Compact notation for second-order partial derivative of the continuous function $g(\boldsymbol{x})$ into the directions p and q: $\partial^2 g(\boldsymbol{x})/(\partial x_p \partial x_q)$

Superscript	Description
$\boldsymbol{A}^{-1}$, $\boldsymbol{A}^{-g}$	Inverse of a square matrix $\boldsymbol{A}$; generalized inverse of a (nonsquare) matrix $\boldsymbol{A}$
$\boldsymbol{A}^T$, $\boldsymbol{a}^T$	Transpose of a matrix or vector; (includes conjugation for complex numbers)
$\boldsymbol{a}^T\boldsymbol{b}$, $\langle \boldsymbol{a} \vert \boldsymbol{b} \rangle$	Scalar product of two vectors
$a^\star$	Conjugate complex
$\boldsymbol{A}^\star$	Conjugate complex and transpose of a matrix

Indexing	Description
K, L, M, N	Extension of discrete images in t, z, y, and x directions
k, l, m, n	Indices of discrete images in t, z, y, and x directions
r, s, u, v	Indices of discrete images in Fourier domain in t, z, y, and x directions
P	Number of components in a multichannel image; dimension of a feature space, number of components, pyramid planes or data points
Q	Number of quantization levels, number of object classes, or number of regression parameters
R	Size of masks for neighborhood operators
W	Dimension of an image or feature space
p, q, w	Indices of a component in a multichannel image, dimension in an image, quantization level or feature

Function	Description
$\cos(x)$	Cosine function
$\exp(x)$	Exponential function
$\mathrm{ld}(x)$	Logarithmic function to base 2
$\ln(x)$	Logarithmic function to base **e**
$\log(x)$	Logarithmic function to base 10
$\sin(x)$	Sine function
$\mathrm{sinc}(x)$	Sinc function: $\mathrm{sinc}(x) = \sin(\pi x)/(\pi x)$
$\det(\boldsymbol{G})$	Determinant of a square matrix
$\mathrm{diag}(\boldsymbol{G})$	Vector with diagonal elements of a square matrix
$\mathrm{trace}(\boldsymbol{G})$	Trace of a square matrix
$\mathrm{cov}(\boldsymbol{g})$	Covariance matrix of a random vector
$E(g)$, $\mathrm{var}(\boldsymbol{G})$	Expectation (mean value) and variance

Image operators	Description
$\cdot$	Pointwise multiplication of two images
$*$	Convolution
$\star$	Correlation
$\ominus, \oplus$	Morphological erosion and dilation operators
$\circ, \bullet$	Morphological opening and closing operators
$\otimes$	Morphological hit-miss operator
$\vee, \wedge$	Boolean *or* and *and* operators
$\cup, \cap$	Union and intersection of sets
$\subset, \subseteq$	Set is subset, subset or equal
$\circlearrowleft$	Shift operator
$\downarrow_s$	Sample or reduction operator: take only every sth pixel, row, etc.
$\uparrow_s$	Expansion or interpolation operator: increase resolution in every coordinate direction by a factor of s, the new points are interpolated from the available points

Symbol	Definition, [Units]	Meaning
Greek symbols		
α	$[\mathrm{m}^{-1}]$	Absorption coefficient
β	$[\mathrm{m}^{-1}]$	Scattering coefficient
$\delta(x)$, δ_n		Continuous, discrete δ distribution
Δ	$\sum_{w=1}^{W} \frac{\partial^2}{\partial x_w^2}$	Laplacian operator
ϵ	[1]	Specific emissivity
ϵ	[m]	Radius of blur disk
κ	$[\mathrm{m}^{-1}]$	Extinction coefficient, sum of absorption and scattering coefficient
$\boldsymbol{\nabla}$	$\left[\frac{\partial}{\partial x_1}, \ldots, \frac{\partial}{\partial x_W}\right]^T$	Gradient operator
λ	[m]	Wavelength
ν	$[\mathrm{s}^{-1}]$, [Hz] (hertz)	Frequency
$\boldsymbol{\nabla}\times$		Rotation operator
η	$n + \mathrm{i}\xi$, [1]	Complex index of refraction
η	[1]	Quantum efficiency
ϕ	[rad], [°]	Phase shift, phase difference
ϕ_e	[rad], [°]	Azimuth angle
Φ	[J/s], [W], $[\mathrm{s}^{-1}]$, [lm]	Radiant or luminous flux
Φ_e, Φ_p	[W], $[\mathrm{s}^{-1}]$, [lm]	Energy-based radiant, photon-based radiant, and luminous flux
ρ, $\rho_\parallel$, $\rho_\perp$	[1]	Reflectivity for unpolarized, parallel polarized, and perpendicularly polarized light
ρ	$[\mathrm{kg/m^3}]$	Density
σ_x		Standard deviation of the random variable x
σ	$5.6696 \cdot 10^{-8}\mathrm{Wm^{-2}K^{-4}}$	Stefan-Boltzmann constant
σ_s	$[\mathrm{m}^2]$	Scattering cross-section
τ	[1]	Optical depth (thickness)
τ	[1]	Transmissivity
τ	[s]	Time constant
θ	[rad], [°]	Angle of incidence
θ_b	[rad], [°]	Brewster angle (polarizing angle)
θ_c	[rad], [°]	Critical angle (for total reflection)
θ_e	[rad], [°]	Polar angle
θ_i	[rad], [°]	Angle of incidence

continued on next page

Symbol	Definition, [Units]	Meaning
continued from previous page		
Ω	[sr] (steradian)	Solid angle
ω	$\omega = 2\pi\nu$, [s^{-1}], [Hz]	Circular frequency
Roman symbols		
A	[m^2]	Area
$a, \boldsymbol{a}$	$\boldsymbol{a} = \boldsymbol{x}_{tt} = \boldsymbol{u}_t$, [$m/s^2$]	Acceleration
$\hat{b}(\tilde{\boldsymbol{k}})$		Transfer function of binomial mask
$\boldsymbol{B}$	[Vs/m^2]	Magnetic field
$\boldsymbol{B}$		Binomial filter mask
$\mathcal{B}$		Binomial convolution operator
c	$2.9979 \cdot 10^8\,ms^{-1}$	speed of light
$\mathbb{C}$		set of complex numbers
d	[m]	Diameter (aperture) of optics, distance
d'	[m]	Distance in image space
$\hat{d}(\tilde{\boldsymbol{k}})$		Transfer function of $\boldsymbol{D}$
D	[m^2/s]	Diffusion coefficient
$\boldsymbol{D}$		First-order difference filter mask
$\mathcal{D}$		First-order difference operator
e	$1.6022 \cdot 10^{-19}$ As	Elementary electric charge
e	$2.718281\ldots$	Base for natural logarithm
E	[W/m^2], [lm/m^2], [lx]	Radiant (irradiance) or luminous (illuminance) incident energy flux density
$\boldsymbol{E}$	[V/m]	Electric field
$\bar{\boldsymbol{e}}$	[1]	Unit eigenvector of a matrix
f, f_e	[m]	(Effective) focal length of an optical system
f_b, f_f	[m]	Back and front focal length
$\boldsymbol{f}$		Optical flow
$\boldsymbol{f}$		Feature vector
F	[N] (newton)	Force
$\boldsymbol{G}$		Image matrix
$\boldsymbol{H}$		General filter mask
h	$6.6262 \cdot 10^{-34}$ Js	Planck's constant (action quantum)
$\hbar$	$h/(2\pi)$ [Js]	
i	$\sqrt{-1}$	Imaginary unit
I	[W/sr], [lm/sr]	Radiant or luminous intensity
I	[A]	Electric current
		continued on next page

Symbol	Definition, [Units]	Meaning
continued from previous page		
$\boldsymbol{I}$		Identity matrix
$\mathcal{I}$		Identity operator
$\boldsymbol{J}$		Structure tensor, inertia tensor
k_B	$1.3806 \cdot 10^{-23}$ J/K	Boltzmann constant
k	$1/\lambda$, [m^{-1}]	Magnitude of wave number
$\boldsymbol{k}$	[m^{-1}]	Wave number (number of wavelengths per unit length)
$\tilde{k}$	$k\Delta x/\pi$	Wave number normalized to the maximum wave number that can be sampled (Nyquist wave number)
K_q	[l/mol]	Quenching constant
K_r	Φ_v/Φ_e, [lm/W]	Radiation luminous efficiency
K_s	Φ_v/P [lm/W]	Lighting system luminous efficiency
K_I	[1]	Indicator equilibrium constant
L	[W/(m^{2}sr)], [1/(m^{2}sr)], [lm/(m^{2}sr)], [cd/m^2]	Radiant (radiance) or luminous (luminance) flux density per solid angle
$\boldsymbol{L}$		Laplacian filter mask
$\mathcal{L}$		Laplacian operator
m	[kg]	Mass
m	[1]	Magnification of an optical system
$\boldsymbol{m}$		Feature vector
M	[W/m^2], [1/(s m^2)]	Excitant radiant energy flux density (excitance, emittance)
M_e	[W/m^2]	Energy-based excitance
M_p	[1/(s m^2)]	Photon-based excitance
$\mathbb{M}$		Feature space
n	[1]	Index of refraction
n_a	[1]	Numerical aperture of an optical system
n_f	f/d, [1]	Aperture of an optical system
$\bar{\boldsymbol{n}}$	[1]	Unit vector normal to a surface
$\mathbb{N}$		Set of natural numbers: $\{0, 1, 2, \ldots\}$
p	[kg m/s], [W m]	Momentum
p	[N/m^2]	Pressure
pH	[1]	pH value, negative logarithm of proton concentration
Q	[Ws] (joule), [lm s] number of photons	Radiant or luminous energy
Q_s	[1]	Scattering efficiency factor

continued on next page

Symbol	Definition, [Units]	Meaning
continued from previous page		
r	[m]	Radius
$\boldsymbol{r}_{m,n}$	$\boldsymbol{r}_{m,n} = [m\Delta x, n\Delta y]^T$	Translation vector on grid
$\hat{\boldsymbol{r}}_{p,q}$	$\hat{\boldsymbol{r}}_{p,q} = [p/\Delta x, q/\Delta y]^T$	Translation vector on reciprocal grid
R	Φ/s, [A/W]	Responsivity of a radiation detector
$\boldsymbol{R}$		Box filter mask
$\mathbb{R}$		Set of real numbers
s	[A]	Sensor signal
T	[K]	Absolute temperature
t	[s]	Time
t	[1]	Transmittance
u	[m/s]	Velocity
$\boldsymbol{u}$	[m/s]	Velocity vector
U	[V]	Voltage, electric potential
V	[m^3]	Volume
$V(\lambda)$	[lm/W]	Spectral luminous efficacy for photopic human vision
$V'(\lambda)$	[lm/W]	Spectral luminous efficacy for scotopic human vision
w	$\mathrm{e}^{2\pi\mathrm{i}}$	
w_N	$\exp(2\pi\mathrm{i}/N)$	
$\boldsymbol{x}$	$[x, y]^T, [x_1, x_2]^T$	Image coordinates in the spatial domain
$\boldsymbol{X}$	$[X, Y, Z]^T, [X_1, X_2, X_3]^T$	World coordinates
$\mathbb{Z}$, $\mathbb{Z}^+$		Set of integers, positive integers

Bibliography

[1] E. H. Adelson and J. R. Bergen. Spatio-temporal energy models for the perception of motion. *J. Opt. Soc. Am. A*, 2:284–299, 1985.

[2] E. H. Adelson and J. R. Bergen. The extraction of spatio-temporal energy in human and machine vision. In *Proceedings Workshop on Motion: Representation and Analysis, May 1986, Charleston, South Carolina*, pp. 151–155. IEEE Computer Society, Washington, 1986.

[3] A. V. Aho, J. E. Hopcroft, and J. D. Ullman. *The Design and Analysis of Computer Algorithms.* Addison Wesley, Reading, MA, 1974.

[4] A. V. Aho, J. E. Hopcroft, and J. D. Ullman. *The Design and Analysis of Computer Algorithms.* Addison-Wesley, Reading, MA, 1974.

[5] J. Anton. *Elementary Linear Algebra.* John Wiley & Sons, New York, 2000.

[6] G. R. Arce, N. C. Gallagher, and T. A. Nodes. Median filters: theory for one and two dimensional filters. JAI Press, Greenwich, USA, 1986.

[7] S. Beauchemin and J. Barron. The computation of optical flow. *ACM Computing Surveys*, 27(3):433–467, 1996.

[8] L. M. Biberman, ed. *Electro Optical Imaging: System Performance and Modeling.* SPIE, Bellingham, WA, 2001.

[9] J. Bigün and G. H. Granlund. Optimal orientation detection of linear symmetry. In *Proceedings ICCV'87, London*, pp. 433–438. IEEE, Washington, DC, 1987.

[10] C. M. Bishop. *Neural Networks for Pattern Recognition.* Clarendon, Oxford, 1995.

[11] R. Blahut. *Fast Algorithms for Digital Signal Processing.* Addison-Wesley, Reading, MA, 1985.

[12] M. Born and E. Wolf. *Principles of Optics.* Cambridge University Press, Cambridge, UK, 7th edn., 1999.

[13] R. Bracewell. *The Fourier Transform and its Applications.* McGraw-Hill, New York, 2nd edn., 1986.

[14] C. Broit. *Optimal registrations of deformed images.* Diss., Univ. of Pennsylvania, USA, 1981.

[15] I. N. Bronshtein, K. A. Semendyayev, G. Musiol, and H. Muehlig. *Handbook of Mathematics.* Springer, Berlin, 4th edn., 2004.

[16] H. Burkhardt, ed. *Workshop on Texture Analysis*, 1998. Albert-Ludwigs-Universität, Freiburg, Institut für Informatik.

[17] H. Burkhardt and S. Siggelkow. Invariant features in pattern recognition - fundamentals and applications. In C. Kotropoulos and I. Pitas, eds., *Nonlinear Model-Based Image/Video Processing and Analysis*, pp. 269–307. John Wiley & Sons, 2001.

[18] P. J. Burt. The pyramid as a structure for efficient computation. In A. Rosenfeld, ed., *Multiresolution image processing and analysis*, vol. 12 of *Springer Series in Information Sciences*, pp. 6–35. Springer, New York, 1984.

[19] P. J. Burt and E. H. Adelson. The Laplacian pyramid as a compact image code. *IEEE Trans. COMM*, 31:532–540, 1983.

[20] P. J. Burt, T. H. Hong, and A. Rosenfeld. Segmentation and estimation of image region properties through cooperative hierarchical computation. *IEEE Trans. SMC*, 11:802–809, 1981.

[21] J. F. Canny. A computational approach to edge detection. *PAMI*, 8:679–698, 1986.

[22] R. Chelappa. *Digital Image Processing*. IEEE Computer Society Press, Los Alamitos, CA, 1992.

[23] N. Christianini and J. Shawe-Taylor. *An Introduction to Support Vector Machines*. Cambridge University Press, Cambridge, 2000.

[24] C. M. Close and D. K. Frederick. *Modelling and Analysis of Dynamic Systems*. Houghton Mifflin, Boston, 1978.

[25] J. W. Cooley and J. W. Tukey. An algorithm for the machine calculation of complex Fourier series. *Math. of Comput.*, 19:297–301, 1965.

[26] T. H. Cormen, C. E. Leiserson, R. L. Rivest, and C. Stein. *Introduction to Algorithms*. MIT Press, Cambridge, MA, 2nd edn., 2001.

[27] J. Crank. *The Mathematics of Diffusion*. Oxford University Press, New York, 2nd edn., 1975.

[28] P.-E. Danielsson, Q. Lin, and Q.-Z. Ye. Efficient detection of second degree variations in 2D and 3D images. Technical Report LiTH-ISY-R-2155, Department of Electrical Engineering, Linköping University, S-58183 Linköping, Sweden, 1999.

[29] P. J. Davis. *Interpolation and Approximation*. Dover, New York, 1975.

[30] C. DeCusaris, ed. *Handbook of Applied Photometry*. Springer, New York, 1998.

[31] C. Demant, B. Streicher-Abel, and P. Waszkewitz. *Industrial Image Processing. Viusal Quality Control in Manufacturing*. Springer, Berlin, 1999. Includes CD-ROM.

[32] P. DeMarco, J. Pokorny, and V. C. Smith. Full-spectrum cone sensitivity functions for X-chromosome-linked anomalous trichromats. *J. of the Optical Society*, A9:1465–1476, 1992.

[33] J. Dengler. *Methoden und Algorithmen zur Analyse bewegter Realweltszenen im Hinblick auf ein Blindenhilfesystem*. Diss., Univ. Heidelberg, 1985.

[34] R. Deriche. Fast algorithms for low-level vision. *IEEE Trans. PAMI*, 12(1): 78–87, 1990.

[35] N. Diehl and H. Burkhardt. Planar motion estimation with a fast converging algorithm. In *Proc. 8th Int. Conf. Pattern Recognition, ICPR'86, October 27–31, 1986, Paris*, pp. 1099–1102. IEEE Computer Society, Los Alamitos, 1986.

[36] R. C. Dorf and R. H. Bishop. *Modern Control Systems*. Addison-Wesley, Menlo Park, CA, 8th edn., 1998.

[37] S. A. Drury. *Image Interpretation in Geology*. Chapman & Hall, London, 2nd edn., 1993.

[38] R. O. Duda, P. E. Hart, and D. G. Stork. *Pattern Classification*. Wlley, New York, 2nd edn., 2001.

[39] M. A. H. Elmore, W. C. *Physics of Waves*. Dover Publications, New York, 1985.

[40] A. Erhardt, G. Zinser, D. Komitowski, and J. Bille. Reconstructing 3D light microscopic images by digital image processing. *Applied Optics*, 24:194–200, 1985.

[41] J. F. S. Crawford. *Waves*, vol. 3 of *Berkely Physics Course*. McGraw-Hill, New York, 1965.

[42] O. Faugeras. *Three-dimensional Computer Vision. A Geometric Vewpoint*. MIT Press, Cambridge, MA, 1993.

[43] O. Faugeras and Q.-T. Luong. *The Geometry of Multiple Images*. MIT Press, Cambdridge, MA, 2001.

[44] M. Felsberg and G. Sommer. A new extension of linear signal processing for estimating local properties and detecting features. In G. Sommer, N. Krüger, and C. Perwass, eds., *Mustererkennung 2000, 22. DAGM Symposium, Kiel*, Informatik aktuell, pp. 195–202. Springer, Berlin, 2000.

[45] R. Feynman. *Lectures on Physics*, vol. 2. Addison-Wesley, Reading, Mass., 1964.

[46] D. G. Fink and D. Christiansen, eds. *Electronics Engineers' Handbook*. McGraw-Hill, New York, 3rd edn., 1989.

[47] M. A. Fischler and O. Firschein, eds. *Readings in Computer Vision: Issues, Problems, Principles, and Paradigms*. Morgan Kaufmann, Los Altos, CA, 1987.

[48] D. J. Fleet. *Measurement of Image Velocity*. Diss., University of Toronto, Canada, 1990.

[49] D. J. Fleet. *Measurement of Image Velocity*. Kluwer Academic Publisher, Dordrecht, 1992.

[50] D. J. Fleet and A. D. Jepson. Hierarchical construction of orientation and velocity selective filters. *IEEE Trans. PAMI*, 11(3):315–324, 1989.

[51] D. J. Fleet and A. D. Jepson. Computation of component image velocity from local phase information. *Int. J. Comp. Vision*, 5:77–104, 1990.

[52] J. D. Foley, A. van Dam, S. K. Feiner, and J. F. Hughes. *Computer Graphics, Principles and Practice*. Addison Wesley, Reading, MA, 2nd edn., 1995.

[53] W. Förstner. Image preprocessing for feature extraction in digital intensity, color and range images. In A. Dermanis, A. Grün, and F. Sanso, eds., *Geomatic Methods for the Analysis of Data in the Earth Sciences*, vol. 95 of *Lecture Notes in Earth Sciences*. Springer, Berlin, 2000.

[54] D. A. Forsyth and J. Ponce. *Computer Vision, a Modern Approach*. Prentice Hall, Upper Saddle River, NJ, 2003.

[55] W. T. Freeman and E. H. Adelson. The design and use of steerable filters. *IEEE Trans. PAMI*, 13:891–906, 1991.

[56] G. Gaussorgues. *Infrared Thermography*. Chapman & Hall, London, 1994.

[57] P. Geißler and B. Jähne. One-image depth-from-focus for concentration measurements. In E. P. Baltsavias, ed., *Proc. ISPRS Intercommission workshop from pixels to sequences, Zürich, March 22-24*, pp. 122–127. RISC Books, Coventry UK, 1995.

[58] J. Gelles, B. J. Schnapp, and M. P. Sheetz. Tracking kinesin driven movements with nanometre-scale precision. *Nature*, 331:450–453, 1988.

[59] F. Girosi, A. Verri, and V. Torre. Constraints for the computation of optical flow. In *Proceedings Workshop on Visual Motion, March 1989, Irvine, CA*, pp. 116–124. IEEE, Washington, 1989.

[60] H. Goldstein. *Classical Mechanics*. Addison-Wesley, Reading, MA, 1980.

[61] G. H. Golub and C. F. van Loan. *Matrix Computations*. The John Hopkins University Press, Baltimore, 1989.

[62] R. C. Gonzalez and R. E. Woods. *Digital image processing*. Prentice Hall, Upper Saddle River, NJ, 2nd edn., 2002.

[63] G. H. Granlund. In search of a general picture processing operator. *Comp. Graph. Imag. Process.*, 8:155–173, 1978.

[64] G. H. Granlund and H. Knutsson. *Signal Processing for Computer Vision*. Kluwer, 1995.

[65] L. D. Griffin and M. Lillhom, eds. *Scale Space Methods in Computer Vision*, vol. 2695 of *Lecture Notes in Computer Science*, 2003. 4th Int. Conf. Scale-Space'03, Springer, Berlin.

[66] M. Groß. *Visual Computing*. Springer, Berlin, 1994.

[67] E. M. Haacke, R. W. Brown, M. R. Thompson, and R. Venkatesan. *Magnetic Resonance Imaging: Physical Principles and Sequence Design*. John Wiley & Sons, New York, 1999.

[68] M. Halloran. 700×9000 imaging on an integrated CCD wafer - affordably. *Advanced Imaging*, Jan.:46–48, 1996.

[69] J. G. Harris. *The coupled depth/slope approach to surface reconstruction*. Master thesis, Dept. Elec. Eng. Comput. Sci., Cambridge, Mass., 1986.

[70] J. G. Harris. A new approach to surface reconstruction: the coupled depth/slope model. In *1st Int. Conf. Comp. Vis. (ICCV), London*, pp. 277–283. IEEE Computer Society, Washington, 1987.

[71] H. Haußecker. *Messung und Simulation kleinskaliger Austauschvorgänge an der Ozeanoberfläche mittels Thermographie*. Diss., University of Heidelberg, Germany, 1995.

[72] H. Haußecker. Simultaneous estimation of optical flow and heat transport in infrared imaghe sequences. In *Proc. IEEE Workshop on Computer Vision beyond the Visible Spectrum*, pp. 85–93. IEEE Computer Society, Washington, DC, 2000.

[73] H. Haußecker and D. J. Fleet. Computing optical flow with physical models of brightness variation. *IEEE Trans. PAMI*, 23:661–673, 2001.

[74] E. Hecht. *Optics*. Addison-Wesley, Reading, MA, 1987.

[75] D. J. Heeger. Optical flow from spatiotemporal filters. *Int. J. Comp. Vis.*, 1:279–302, 1988.

[76] E. C. Hildreth. Computations underlying the measurement of visual motion. *Artificial Intelligence*, 23:309–354, 1984.

[77] G. C. Holst. *CCD Arrays, Cameras, and Displays*. SPIE, Bellingham, WA, 2nd edn., 1998.

[78] G. C. Holst. *Testing and Evaluation of Infrared Imaging Systems*. SPIE, Bellingham, WA, 2nd edn., 1998.

[79] G. C. Holst. *Common Sense Approach to Thermal Imaging*. SPIE, Bellingham, WA, 2000.

[80] G. C. Holst. *Electro-optical Imaging System Performance*. SPIE, Bellingham, WA, 2nd edn., 2000.

[81] B. K. Horn. *Robot Vision*. MIT Press, Cambridge, MA, 1986.

[82] S. Howell. *Handbook of CCD Astronomy*. Cambridge University Press, Cambridge, 2000.

[83] T. S. Huang, ed. *Two-dimensional Digital Signal Processing II: Transforms and Median Filters*, vol. 43 of *Topics in Applied Physics*. Springer, New York, 1981.

[84] S. V. Huffel and J. Vandewalle. *The Total Least Squares Problem - Computational Aspects and Analysis*. SIAM, Philadelphia, 1991.

[85] K. Iizuka. *Engineering Optics*, vol. 35 of *Springer Series in Optical Sciences*. Springer, Berlin, 2nd edn., 1987.

[86] B. Jähne. Image sequence analysis of complex physical objects: nonlinear small scale water surface waves. In *Proceedings ICCV'87, London*, pp. 191–200. IEEE Computer Society, Washington, DC, 1987.

[87] B. Jähne. Motion determination in space-time images. In *Image Processing III, SPIE Proceeding 1135, international congress on optical science and engineering, Paris, 24-28 April 1989*, pp. 147–152, 1989.

[88] B. Jähne. *Spatio-temporal Image Processing*. Lecture Notes in Computer Science. Springer, Berlin, 1993.

[89] B. Jähne. *Handbook of Digital Image Processing for Scientific Applications*. CRC Press, Boca Raton, FL, 1997.

[90] B. Jähne. Vergleichende Analyse moderner Bildsensoren für die optische Messtechnik. In *Sensoren und Messsysteme 2004*, vol. 1829 of *VDI-Berichte*, pp. 317–324. VDI Verlag, Düsseldorf, 2004.

[91] B. Jähne, ed. *Image Sequence Analysis to Investigate Dynamic Processes*, Lecture Notes in Computer Science, 2005. Springer, Berlin.

[92] B. Jähne, E. Barth, R. Mester, and H. Scharr, eds. *Complex Motion, Proc. 1th Int. Workshop, Günzburg, Oct. 2004*, vol. 3417 of *Lecture Notes in Computer Science*, 2005. Springer, Berlin.

[93] B. Jähne and H. Haußecker, eds. *Computer Vision and Applications. A Guide for Students and Practitioners*. Academic Press, San Diego, 2000.

[94] B. Jähne, H. Haußecker, and P. Geißler, eds. *Handbook of Computer Vision and Applications. Volume I: Sensors and Imaging. Volume II: Signal Processing and Pattern Recognition. Volume III: Systems and Applications*.

Academic Press, San Diego, 1999. Includes three CD-ROMs.

[95] B. Jähne, J. Klinke, and S. Waas. Imaging of short ocean wind waves: a critical theoretical review. *J. Optical Soc. Amer. A*, 11:2197–2209, 1994.

[96] B. Jähne, H. Scharr, and S. Körgel. Principles of filter design. In B. Jähne, H. Haußecker, and P. Geißler, eds., *Computer Vision and Applications, volume 2, Signal Processing and Pattern Recognition*, chapter 6, pp. 125–151. Academic Press, San Diego, 1999.

[97] A. K. Jain. *Fundamentals of Digital Image Processing*. Prentice-Hall, Englewood Cliffs, NJ, 1989.

[98] R. Jain, R. Kasturi, and B. G. Schunck. *Machine Vision*. McGraw-Hill, New York, 1995.

[99] J. R. Janesick. *Scientific Charge-Coupled Devices*. SPIE, Bellingham, WA, 2001.

[100] J. T. Kajiya. The rendering equation. *Computer Graphics*, 20:143–150, 1986.

[101] M. Kass and A. Witkin. Analysing oriented patterns. *Comp. Vis. Graph. Im. Process.*, 37:362–385, 1987.

[102] M. Kass, A. Witkin, and D. Terzopoulos. Snakes: active contour models. In *Proc. 1st Int. Conf. Comp. Vis. (ICCV), London*, pp. 259–268. IEEE Computer Society, Washington, 1987.

[103] B. Y. Kasturi and R. C. Jain. *Computer Vision: Advances and Applications*. IEEE Computer Society, Los Alamitos, 1991.

[104] B. Y. Kasturi and R. C. Jain, eds. *Computer Vision: Principles*. IEEE Computer Society, Los Alamitos, 1991.

[105] J. K. Kearney, W. B. Thompson, and D. L. Boley. Optical flow estimation: an error analysis of gradient-based methods with local optimization. *IEEE Trans. PAMI*, 9 (2):229–244, 1987.

[106] M. Kerckhove, ed. *Scale-Space and Morphology in Computer Vision*, vol. 2106 of *Lecture Notes in Computer Science*, 2001. 3rd Int. Conf. Scale-Space'01, Vancouver, Canada, Springer, Berlin.

[107] R. Kimmel, N. Sochen, and J. Weickert, eds. *Scale-Space and PDE Methods in Computer Vision*, Lecture Notes in Computer Science, 2005. 5th Int. Conf. Scale-Space'05, Springer, Berlin.

[108] C. Kittel. *Introduction to Solid State Physics*. Wiley, New York, 1971.

[109] R. Klette, A. Koschan, and K. Schlüns. *Computer Vision. Three-Dimensional Data from Images*. Springer, New York, 1998.

[110] H. Knutsson. *Filtering and Reconstruction in Image Processing*. Diss., Linköping Univ., Sweden, 1982.

[111] H. Knutsson. Representing local structure using tensors. In *The 6th Scandinavian Conference on Image Analysis, Oulu, Finland, June 19-22, 1989*, 1989.

[112] H. E. Knutsson, R. Wilson, and G. H. Granlund. Anisotropic nonstationary image estimation and its applications: part I – restoration of noisy images. *IEEE Trans. COMM*, 31(3):388–397, 1983.

[113] J. J. Koenderink and A. J. van Doorn. Generic neighborhood operators. *IEEE Trans. PAMI*, 14(6):597–605, 1992.

[114] C. Koschnitzke, R. Mehnert, and P. Quick. *Das KMQ-Verfahren: Medienkompatible Übertragung echter Stereofarbabbildungen.* Forschungsbericht Nr. 201, Universität Hohenheim, 1983.

[115] P. Lancaster and K. Salkauskas. *Curve and Surface Fitting. An Introduction.* Academic Press, London, 1986.

[116] S. Lanser and W. Eckstein. Eine Modifikation des Deriche-Verfahrens zur Kantendetektion. In B. Radig, ed., *Mustererkennung 1991*, vol. 290 of *Informatik Fachberichte*, pp. 151–158. 13. DAGM Symposium, München, Springer, Berlin, 1991.

[117] Laurin. *The Photonics Design and Applications Handbook.* Laurin Publishing CO, Pittsfield, MA, 40th edn., 1994.

[118] D. C. Lay. *Linear Algebra and Its Applications.* Addison-Wesley, Reading, MA, 1999.

[119] R. Lenz. Linsenfehlerkorrigierte Eichung von Halbleiterkameras mit Standardobjektiven für hochgenaue 3D-Messungen in Echtzeit. In E. Paulus, ed., *Proc. 9. DAGM-Symp. Mustererkennung 1987, Informatik Fachberichte 149*, pp. 212–216. DAGM, Springer, Berlin, 1987.

[120] R. Lenz. Zur Genauigkeit der Videometrie mit CCD-Sensoren. In H. Bunke, O. Kübler, and P. Stucki, eds., *Proc. 10. DAGM-Symp. Mustererkennung 1988, Informatik Fachberichte 180*, pp. 179–189. DAGM, Springer, Berlin, 1988.

[121] M. Levine. *Vision in Man and Machine.* McGraw-Hill, New York, 1985.

[122] Z.-P. Liang and P. C. Lauterbur. *Principles of Magnetic Resonance Imaging: A Signal Processing Perspective.* SPIE, Bellingham, WA, 1999.

[123] D. R. Lide, ed. *CRC Handbook of Chemistry and Physics.* CRC, Boca Raton, FL, 76th edn., 1995.

[124] J. S. Lim. *Two-dimensional Signal and Image Processing.* Prentice-Hall, Englewood Cliffs, NJ, 1990.

[125] T. Lindeberg. *Scale-space Theory in Computer Vision.* Kluwer Academic Publishers, Boston, 1994.

[126] M. Loose, K. Meier, and J. Schemmel. A self-calibrating single-chip CMOS camera with logarithmic response. *IEEE J. Solid-State Circuits*, 36(4), 2001.

[127] D. Lorenz. *Das Stereobild in Wissenschaft und Technik.* Deutsche Forschungs- und Versuchsanstalt für Luft- und Raumfahrt, Köln, Oberpfaffenhofen, 1985.

[128] V. K. Madisetti and D. B. Williams, eds. *The Digital Signal Processing Handbook.* CRC, Boca Raton, FL, 1998.

[129] H. A. Mallot. *Computational Vision: Information Processing in Perception and Visual Behavior.* The MIT Press, Cambridge, MA, 2000.

[130] V. Markandey and B. E. Flinchbaugh. Multispectral constraints for optical flow computation. In *Proc. 3rd Int. Conf. on Computer Vision 1990 (ICCV'90), Osaka*, pp. 38–41. IEEE Computer Society, Los Alamitos, 1990.

[131] S. L. Marple Jr. *Digital Spectral Analysis with Applications.* Prentice-Hall, Englewood Cliffs, NJ, 1987.

[132] D. Marr. *Vision.* W. H. Freeman and Company, New York, 1982.

[133] D. Marr and E. Hildreth. Theory of edge detection. *Proc. Royal Society, London, Ser. B*, 270:187–217, 1980.

[134] E. A. Maxwell. *General Homogeneous Coordinates in Space of Three Dimensions*. University Press, Cambridge, 1951.

[135] C. Mead. *Analog VLSI and Neural Systems*. Addison-Wesley, Reading, MA, 1989.

[136] W. Menke. *Geophysical Data Analysis: Discrete Inverse Theory*, vol. 45 of *International Geophysics Series*. Academic Press, San Diego, 1989.

[137] C. D. Meyer. *Matrix Analysis and Applied Linear Algebra*. SIAM, Philadelphia, 2001.

[138] D. G. Mitchell and M. S. Cohen. *MRI Principles*. Saunders, Philadelphia, 2nd edn., 2004.

[139] A. Z. J. Mou, D. S. Rice, and W. Ding. VIS-based native video processing on UltraSPARC. In *Proc. IEEE Int. Conf. on Image Proc., ICIP'96*, pp. 153–156. IEEE, Lausanne, 1996.

[140] T. Münsterer. *Messung von Konzentrationsprofilen gelöster Gase in der wasserseitigen Grenzschicht*. Diploma thesis, University of Heidelberg, Germany, 1993.

[141] T. Münsterer, H. J. Mayer, and B. Jähne. Dual-tracer measurements of concentration profiles in the aqueous mass boundary layer. In B. Jähne and E. Monahan, eds., *Air-Water Gas Transfer, Selected Papers, 3rd Intern. Symp. on Air-Water Gas Transfer*, pp. 637–648. AEON, Hanau, 1995.

[142] H. Nagel. Displacement vectors derived from second-order intensity variations in image sequences. *Computer Vision, Graphics, and Image Processing (GVGIP)*, 21:85–117, 1983.

[143] Y. Nakayama and Y. Tanida, eds. *Atlas of Visualization III*. CRC, Boca Raton, FL, 1997.

[144] V. S. Nalwa. *A Guided Tour of Computer Vision*. Addison-Wesley, Reading, MA, 1993.

[145] M. Nielsen, P. Johansen, O. Olsen, and J. Weickert, eds. *Scale-Space Theories in Computer Vision*, vol. 1682 of *Lecture Notes in Computer Science*, 1999. 2nd Int. Conf. Scale-Space'99, Corfu, Greece, Springer, Berlin.

[146] H. K. Nishihara. Practical real-time stereo matcher. *Optical Eng.*, 23:536–545, 1984.

[147] J. Ohser and F. Mücklich. *Statistical Analysis of Microstructures in Material Science*. Wiley, Chicester, England, 2000.

[148] A. V. Oppenheim and R. W. Schafer. *Discrete-time Signal Processing*. Prentice-Hall, Englewood Cliffs, NJ, 1989.

[149] A. Papoulis. *Probability, Random Variables, and Stochastic Processes*. McGraw-Hill, New York, 3rd edn., 1991.

[150] J. R. Parker. *Algorithms for Image Processing and Computer Vision*. John Wiley & Sons, New York, 1997. Includes CD-ROM.

[151] P. Perona and J. Malik. Scale space and edge detection using anisotropic diffusion. In *Proc. IEEE comp. soc. workshop on computer vision (Miami Beach, Nov. 30-Dec. 2, 1987)*, pp. 16–20. IEEE Computer Society, Washington, 1987.

[152] Photobit. PB-MV13 20 mm CMOS Active Pixel Digital Image Sensor. Photobit, Pasadena, CA, August 2000. www.photobit.com.

[153] M. Pietikäinen and A. Rosenfeld. Image segmentation by texture using pyramid node linking. *SMC*, 11:822–825, 1981.

[154] I. Pitas. *Digital Image Processing Algorithms.* Prentice Hall, New York, 1993.

[155] I. Pitas and A. N. Venetsanopoulos. *Nonlinear Digital Filters. Principles and Applications.* Kluwer Academic Publishers, Norwell, MA, 1990.

[156] A. D. Poularikas, ed. *The Transforms and Applications Handbook.* CRC, Boca Raton, 1996.

[157] W. K. Pratt. *Digital image processing, PIKS Inside.* Wiley, New York, 3rd edn., 2001.

[158] W. H. Press, B. P. Flannery, S. A. Teukolsky, and W. T. Vetterling. *Numerical Recipes in C: The Art of Scientific Computing.* Cambridge University Press, New York, 1992.

[159] J. G. Proakis and D. G. Manolakis. *Digital Signal Processing. Principles, Algorithms, and Applications.* McMillan, New York, 1992.

[160] L. H. Quam. Hierarchical warp stereo. In *Proc. DARPA Image Understanding Workshop, October 1984, New Orleans, LA*, pp. 149–155, 1984.

[161] A. R. Rao. *A Taxonomy for Texture Description and Identification.* Springer, New York, 1990.

[162] A. R. Rao and B. G. Schunck. Computing oriented texture fields. In *Proceedings CVPR'89, San Diego, CA*, pp. 61–68. IEEE Computer Society, Washington, DC, 1989.

[163] T. H. Reiss. *Recognizing Planar Objects Using Invariant Image Features*, vol. 676 of *Lecture notes in computer science.* Springer, Berlin, 1993.

[164] J. A. Rice. *Mathematical Statistics and Data Analysis.* Duxbury Press, Belmont, CA, 1995.

[165] A. Richards. *Alien Vision: Exploring the Electromagnetic Spectrum with Imaging Technology.* SPIE, Bellingham, WA, 2001.

[166] J. A. Richards. *Remote Sensing Digital Image Analysis.* Springer, Berlin, 1986.

[167] J. A. Richards and X. Jia. *Remote Sensing Digital Image Analysis.* Springer, Berlin, 1999.

[168] M. J. Riedl. *Optical Design Fundamentals for Infrared Systems.* SPIE, Bellingham, 2nd edn., 2001.

[169] K. Riemer. *Analyse von Wasseroberflächenwellen im Orts-Wellenzahl-Raum.* Diss., Univ. Heidelberg, 1991.

[170] K. Riemer, T. Scholz, and B. Jähne. Bildfolgenanalyse im Orts-Wellenzahlraum. In B. Radig, ed., *Mustererkennung 1991, Proc. 13. DAGM-Symposium München, 9.-11. October 1991*, pp. 223–230. Springer, Berlin, 1991.

[171] A. Rosenfeld, ed. *Multiresolution Image Processing and Analysis*, vol. 12 of *Springer Series in Information Sciences.* Springer, New York, 1984.

[172] A. Rosenfeld and A. C. Kak. *Digital Picture Processing*, vol. I and II. Academic Press, San Diego, 2nd edn., 1982.

[173] J. C. Russ. *The Image Processing Handbook*. CRC, Boca Raton, FL, 4th edn., 2002.

[174] H. Samet. *Applications of Spatial Data Structures: Computer Graphics, Image processing, and GIS*. Addison-Wesley, Reading, MA, 1990.

[175] H. Samet. *The Design and Analysis of Spatial Data Structures*. Addison-Wesley, Reading, MA, 1990.

[176] H. Scharr and D. Uttenweiler. 3D anisotropic diffusion filtering for enhancing noisy actin filaments. In B. Radig and S. Florczyk, eds., *Pattern Recognition, 23rd DAGM Stmposium, Munich*, vol. 2191 of *Lecture Notes in Computer Science*, pp. 69–75. Springer, Berlin, 2001.

[177] H. Scharr and J. Weickert. An anisotropic diffusion algorithm with optimized rotation invariance. In G. Sommer, N. Krüger, and C. Perwass, eds., *Mustererkennung 2000*, Informatik Aktuell, pp. 460–467. 22. DAGM Symposium, Kiel, Springer, Berlin, 2000.

[178] T. Scheuermann, G. Pfundt, P. Eyerer, and B. Jähne. Oberflächenkonturvermessung mikroskopischer Objekte durch Projektion statistischer Rauschmuster. In G. Sagerer, S. Posch, and F. Kummert, eds., *Mustererkennung 1995, Proc. 17. DAGM-Symposium, Bielefeld, 13.-15. September 1995*, pp. 319–326. DAGM, Springer, Berlin, 1995.

[179] C. Schnörr and J. Weickert. Variational image motion computations: theoretical framework, problems and perspective. In G. Sommer, N. Krüger, and C. Perwass, eds., *Mustererkennung 2000*, Informatik Aktuell, pp. 476–487. 22. DAGM Symposium, Kiel, Springer, Berlin, 2000.

[180] B. Schöllkopf and A. J. Smola. *Learning with Kernels, Support Vector Machines, Regularization, Optimization, and Beyond*. MIT Press, Cambridge, MA, 2002.

[181] J. R. Schott. *Remote Sensing. The Image Chain Approach*. Oxford University Press, New York, 1997.

[182] J. Schürmann. *Pattern Classification*. John Wiley & Sons, New York, 1996.

[183] R. Sedgewick. *Algorithms in C, Part 1–4*. Addison-Wesley, Reading, MA, 3rd edn., 1997.

[184] J. Serra. *Image analysis and mathematical morphology*. Academic Press, London, 1982.

[185] J. Serra and P. Soille, eds. *Mathematical Morphology and its Applications to Image Processing*, vol. 2 of *Computational Imaging and Vision*. Kluwer, Dordrecht, 1994.

[186] L. G. Shapiro and G. C. Stockman. *Computer Vision*. Prentice Hall, Upper Saddle River, NJ, 2001.

[187] E. P. Simoncelli, W. T. Freeman, E. H. Adelson, and D. J. Heeger. Shiftable multiscale transforms. *IEEE Trans. IT*, 38(2):587–607, 1992.

[188] R. M. Simonds. Reduction of large convolutional kernels into multipass applications of small generating kernels. *J. Opt. Soc. Am. A*, 5:1023–1029, 1988.

[189] A. Singh. *Optic Flow Computation: a Unified Perspective*. IEEE Computer Society Press, Los Alamitos, CA, 1991.

[190] A. T. Smith and R. J. Snowden, eds. *Visual Detection of Motion*. Academic Press, London, 1994.

[191] W. J. Smith. *Modern Optical Design*. McGraw-Hill, New York, 3rd edn., 2000.

[192] P. Soille. *Morphological Image Analysis. Principles and Applications*. Springer, Berlin, 2nd edn., 2002.

[193] G. Sommer, ed. *Geometric Computing with Clifford Algebras*. Springer, Berlin, 2001.

[194] J. Steurer, H. Giebel, and W. Altner. Ein lichtmikroskopisches Verfahren zur zweieinhalbdimensionalen Auswertung von Oberflächen. In G. Hartmann, ed., *Proc. 8. DAGM-Symp. Mustererkennung 1986, Informatik-Fachberichte 125*, pp. 66–70. DAGM, Springer, Berlin, 1986.

[195] R. H. Stewart. *Methods of Satellite Oceanography*. University of California Press, Berkeley, 1985.

[196] T. M. Strat. Recovering the camera parameters from a transformation matrix. In *Proc. DARPA Image Understanding Workshop*, pp. 264–271, 1984.

[197] B. ter Haar Romeny, L. Florack, J. Koenderink, and M. Viergever, eds. *Scale-Space Theory in Computer Vision*, vol. 1252 of *Lecture Notes in Computer Science*, 1997. 1st Int. Conf., Scale-Space'97, Utrecht, The Netherlands, Springer, Berlin.

[198] D. Terzopoulos. Regularization of inverse visual problems involving discontinuities. *IEEE Trans. PAMI*, 8:413–424, 1986.

[199] D. Terzopoulos. The computation of visible-surface representations. *IEEE Trans. PAMI*, 10 (4):417–438, 1988.

[200] D. Terzopoulos, A. Witkin, and M. Kass. Symmetry-seeking models for 3D object reconstruction. In *Proc. 1st Int. Conf. Comp. Vis. (ICCV), London*, pp. 269–276. IEEE, IEEE Computer Society Press, Washington, 1987.

[201] D. H. Towne. *Wave Phenomena*. Dover, New York, 1988.

[202] S. Ullman. *High-level Vision. Object Recognition and Visual Cognition*. The MIT Press, Cambridge, MA, 1996.

[203] S. E. Umbaugh. *Computer Vision and Image Processing: A Practical Approach Using CVIPTools*. Prentice Hall PTR, Upper Saddle River, NJ, 1998.

[204] M. Unser, A. Aldroubi, and M. Eden. Fast B-spline transforms for continuous image representation and interpolation. *IEEE Trans. PAMI*, 13: 277–285, 1991.

[205] F. van der Heijden. *Image Based Measurement Systems. Object Recognition and Parameter Estimation*. Wiley, Chichester, England, 1994.

[206] W. M. Vaughan and G. Weber. Oxygen quenching of pyrenebutyric acid fluorescence in water. *Biochemistry*, 9:464, 1970.

[207] A. Verri and T. Poggio. Against quantitative optical flow. In *Proceedings ICCV'87, London*, pp. 171–180. IEEE, IEEE Computer Society Press, Washington, DC, 1987.

[208] A. Verri and T. Poggio. Motion field and optical flow: qualitative properties. *IEEE Trans. PAMI*, 11 (5):490–498, 1989.

[209] K. Voss and H. Süße. *Praktische Bildverarbeitung*. Hanser, München, 1991.

[210] B. A. Wandell. *Foundations of Vision*. Sinauer Ass., Sunderland, MA, 1995.

[211] A. Watt. *3D Computer Graphics*. Addison-Wesley, Workingham, England, 3rd edn., 1999.

[212] A. Webb. *Statistical Pattern Recognition*. Wiley, Chichester, UK, 2002.

[213] J. Weickert. *Anisotropic Diffusion in Image Processing*. Dissertation, Faculty of Mathematics, University of Kaiserslautern, 1996.

[214] J. Weickert. *Anisotropic Diffusion in Image Processing*. Teubner, Stuttgart, 1998.

[215] E. W. Weisstein. *CRC Concise Encyclopedia of Mathematics*. CRC, Boca Raton, FL, 2nd edn., 2002.

[216] I. Wells, W. M. Efficient synthesis of Gaussian filters by cascaded uniform filters. *IEEE Trans. PAMI*, 8(2):234–239, 1989.

[217] J. N. Wilson and G. X. Ritter. *Handbook of Computer Vision Algorithms in Image Algebra*. CRC, Boca Raton, FL, 2nd edn., 2000.

[218] G. Wiora. *Optische 3D-Messtechnik: Präzise Gestaltvermessung mit einem erweiterten Streifenprojektionsverfahren*. Dissertation, Fakultät für Physik und Astronomie, Universität Heidelberg, 2001. http://www.ub.uni-heidelberg.de/archiv/1808.

[219] G. Wolberg. *Digital Image Warping*. IEEE Computer Society, Los Alamitos, CA, 1990.

[220] R. J. Woodham. Multiple light source optical flow. In *Proc. 3rd Int. Conf. on Computer Vision 1990 (ICCV'90), Osaka*, pp. 42–46. IEEE Computer Society, Los Alamitos, 1990.

[221] P. Zamperoni. *Methoden der digitalen Bildsignalverarbeitung*. Vieweg, Braunschweig, 1989.

Index