

# A Reference Material

## Selection of CMOS imaging sensors (Section 1.7.1)

R1

C: charge saturation capacity in electrons, FR: frame rate in  $\text{s}^{-1}$ , PC: pixel clock in MHz, QE: peak quantum efficiency

Chip	Format $H \times V$	FR	PC	Pixel size $H \times V, \mu\text{m}$	Comments
Linear response					
Micron <sup>3</sup> MT9V403	$656 \times 491$	200	66	$9.9 \times 9.9$	QE 0.32 @ 520 nm
Fillfactory <sup>2</sup> IBIS54-1300	$1280 \times 1024$	30	40	$6.7 \times 6.7$	QE 0.30–0.35 @ 600 nm, C 60k
Fillfactory <sup>2</sup> IBIS4-4000	$2496 \times 1692$	4.5		$11.4 \times 11.4$	C 150k
Fast frame rate linear response					
Fillfactory <sup>2</sup> LUPA1300	$1280 \times 1024$	450	40	$12.0 \times 12.0$	16 parallel ports
Micron <sup>3</sup> MV40	$2352 \times 1728$	240	80	$7.0 \times 7.0$	16 parallel 10-bit ports
Micron <sup>3,5</sup> MT9M413	$1280 \times 1024$	600	80	$12.0 \times 12.0$	QE 0.27 @ 520 nm, C 63k, 10 parallel 10-bit ports
Micron <sup>4</sup> MV02	$512 \times 512$	4000	80	$16.0 \times 16.0$	16 parallel 10-bit ports
Logarithmic response					
IMS HDRC VGA <sup>4</sup>	$640 \times 480$	25	8	$12 \times 12$	
PhotonFocus <sup>1</sup>	$1024 \times 1024$	150	80	$10.6 \times 10.6$	QE 0.29 @ 600 nm, C 200k, linear response at low light levels with adjustable transition to logarithmic response

Sources:

<sup>1</sup> <http://www.photonfocus.com>

<sup>2</sup> <http://www.fillfactory.com>

<sup>3</sup> <http://www.photobit.com>

<sup>4</sup> <http://www.ims-chips.de>

<sup>5</sup> <http://www.pco.de>

**R2 Selection of CCD imaging sensors (Section 1.7.1)**

C: charge saturation capacity in electrons, eNIR: enhanced NIR sensitivity, FR: frame rate in  $\text{s}^{-1}$ , ID: image diagonal in mm, QE: peak quantum efficiency, Sony (ICX...) and Kodak (KAI...) sensors

Chip	Format H $\times$ V	FR	ID	Pixel size H $\times$ V, $\mu\text{m}$	Comments
Interlaced EIA video					
ICX278AL 1/4"	768 $\times$ 494	30	4.56	4.75 $\times$ 5.55	eNIR
ICX258AL 1/3"	768 $\times$ 494	30	6.09	6.35 $\times$ 7.4	eNIR
ICX248AL 1/2"	768 $\times$ 494	30	8.07	8.4 $\times$ 9.8	eNIR
ICX422AL 2/3"	768 $\times$ 494	30	11.1	11.6 $\times$ 13.5	
Interlaced CCIR video					
ICX279AL 1/4"	752 $\times$ 582	25	4.54	4.85 $\times$ 4.65	eNIR
ICX259AL 1/3"	752 $\times$ 582	25	6.09	6.5 $\times$ 6.25	eNIR
ICX249AL 1/2"	752 $\times$ 582	25	8.07	8.6 $\times$ 8.3	eNIR
ICX423AL 2/3"	752 $\times$ 582	25	10.9	11.6 $\times$ 11.2	
Progressive scanning interline					
ICX098AL 1/4"	659 $\times$ 494	30	4.61	5.6 $\times$ 5.6	
ICX424AL 1/3"	659 $\times$ 494	30	6.09	7.4 $\times$ 7.4	
ICX074AL 1/2"	659 $\times$ 494	40	8.15	9.9 $\times$ 9.9	C 32k, QE 0.43 @ 340 nm
ICX414AL 1/2"	659 $\times$ 494	50	8.15	9.9 $\times$ 9.9	C 30k, QE 0.40 @ 500 nm
ICX075AL 1/2"	782 $\times$ 582	30	8.09	8.3 $\times$ 8.3	
ICX204AL 1/3"	1024 $\times$ 768	15	5.95	4.65 $\times$ 4.65	
ICX205AL 1/2"	1360 $\times$ 1024	9.5	7.72	4.65 $\times$ 4.65	C 13 ke
ICX285AL 2/3"	1360 $\times$ 1024	10	11.0	6.45 $\times$ 6.45	C 18k, QE 0.65 @ 500 nm
ICX085AL 2/3"	1300 $\times$ 1030	12.5	11.1	6.7 $\times$ 6.7	C 20k, QE 0.54 @ 380 nm
ICX274AL 1/1.8"	1628 $\times$ 1236	12	8.99	4.4 $\times$ 4.4	
KAI-0340DM 1/3"	640 $\times$ 480	200	5.92	7.4 $\times$ 7.4	C 20k, QE 0.55 @ 500 nm
KAI-1010M	1008 $\times$ 1018	30	12.9	9.0 $\times$ 9.0	QE 0.37 @ 500 nm
KAI-1020M	1000 $\times$ 1000	49	10.5	7.4 $\times$ 7.4	C 42k, QE 0.45 @ 490 nm
KAI-2001M	1600 $\times$ 1200	30	14.8	7.4 $\times$ 7.4	C 40k, QE 0.55 @ 480 nm
KAI-4020M	2048 $\times$ 2048	15	21.4	7.4 $\times$ 7.4	C 40k, QE 0.55 @ 480 nm
KAI-10000M	4008 $\times$ 2672	3	43.3	9.0 $\times$ 9.0	C 60k, QE 0.50 @ 500 nm

Sources:

<http://www.framos.de>

<http://www.kodak.com/global/en/digital/ccd/>

<http://www.pco.de>

### Imaging sensors for the infrared (IR, Section 1.7.1)

R3

C: full well capacity in millions of electrons [Me], IT: integration time, NETD: noise equivalent temperature difference, QE: peak quantum efficiency

Chip	Format H × V	FR	PC	Pixel size H × V, $\mu\text{m}$	Comments
Near infrared (NIR)					
Indigo <sup>1</sup> InGaAs	320 × 256	345		30 × 30	0.9–1.68 $\mu\text{m}$ , C 3.5 Me
Mid wave infrared (MWIR)					
AIM <sup>2</sup> PtSi	640 × 486	50	12	24 × 24	3.0–5.0 $\mu\text{m}$ , NETD < 75 mK @ 33 ms IT
Indigo <sup>1</sup> InSb	320 × 256	345		30 × 30	2.0–5.0 $\mu\text{m}$ , C 18 Me
Indigo <sup>1</sup> InSb	640 × 512	100		25 × 25	2.0–5.0 $\mu\text{m}$ , C 11 Me
AIM <sup>2</sup> HgCdTe	384 × 288	120	20	24 × 24	3.0–5.0 $\mu\text{m}$ , NETD < 20 mK @ 2 ms IT
AIM <sup>2</sup> /IaF FhG <sup>3</sup> QWIP	640 × 512	30	18	24 × 24	3.0–5.0 $\mu\text{m}$ , NETD < 15 mK @ 20 ms IT
Long wave infrared (LWIR)					
AIM <sup>2</sup> HgCdTe	256 × 256	200	16	40 × 40	8–10 $\mu\text{m}$ , NETD < 20 mK @ 0.35 ms IT
Indigo <sup>1</sup> QWIP	320 × 256	345		30 × 30	8.0–9.2 $\mu\text{m}$ , C 18 Me, NETD < 30 mK
AIM <sup>2</sup> /IaF FhG <sup>3</sup> QWIP	256 × 256	200	16	40 × 40	8.0–9.2 $\mu\text{m}$ , NETD < 8 mK @ 20 ms IT
AIM <sup>2</sup> /IaF FhG <sup>3</sup> QWIP	640 × 512	30	18	24 × 24	8.0–9.2 $\mu\text{m}$ , NETD < 10 mK @ 30 ms IT
Uncooled sensors					
Indigo <sup>1</sup> Microbolometer	320 × 240	60		30 × 30	7.0–14.0 $\mu\text{m}$ , NETD < 120 mK

Sources:

<sup>1</sup> <http://www.indigosystems.com>

<sup>2</sup> <http://www.aim-ir.de>

<sup>3</sup> [http://www.iaf.fhg.de/tpqw/frames\\_d.htm](http://www.iaf.fhg.de/tpqw/frames_d.htm)

**R4 Properties of the  $W$ -dimensional Fourier transform (Section 2.3.4)**

$g(\mathbf{x}) \longleftrightarrow \hat{g}(\mathbf{k})$  and  $h(\mathbf{x}) \longleftrightarrow \hat{h}(\mathbf{k})$  are Fourier transform pairs:  $\mathbb{R}^W \mapsto \mathbb{C}$ :

$$\hat{g}(\mathbf{k}) = \int_{-\infty}^{\infty} g(\mathbf{x}) \exp(-2\pi i \mathbf{k}^T \mathbf{x}) d^W x = \langle \exp(2\pi i \mathbf{k}^T \mathbf{x}) | g(\mathbf{x}) \rangle;$$

$s$  is a real, nonzero number,  $a$  and  $b$  are complex constants;  $\mathbf{A}$  is a  $W \times W$  matrix,  $\mathbf{R}$  is an orthogonal rotation matrix ( $\mathbf{R}^{-1} = \mathbf{R}^T$ ,  $\det \mathbf{R} = 1$ )

Property	Spatial domain	Fourier domain
Linearity	$ag(\mathbf{x}) + bh(\mathbf{x})$	$a\hat{g}(\mathbf{k}) + b\hat{h}(\mathbf{k})$
Similarity	$g(s\mathbf{x})$	$\hat{g}(\mathbf{k}/s)/ s ^W$
Generalized similarity	$g(\mathbf{A}\mathbf{x})$	$\hat{g}((\mathbf{A}^{-1})^T \mathbf{k}) / \det \mathbf{A}$
Rotation	$g(\mathbf{R}\mathbf{x})$	$\hat{g}(\mathbf{R}\mathbf{k})$
Separability	$\prod_{w=1}^W g_w(x_w)$	$\prod_{w=1}^W \hat{g}_w(k_w)$
Shift in $x$ space	$g(\mathbf{x} - \mathbf{x}_0)$	$\exp(-2\pi i \mathbf{k}^T \mathbf{x}_0) \hat{g}(\mathbf{k})$
Finite difference	$g(\mathbf{x} + \mathbf{x}_0/2) - g(\mathbf{x} - \mathbf{x}_0/2)$	$2i \sin(\pi \mathbf{x}_0^T \mathbf{k}) \hat{g}(\mathbf{k})$
Shift in $k$ space	$\exp(2\pi i \mathbf{k}_0^T \mathbf{x}) g(\mathbf{x})$	$\hat{g}(\mathbf{k} - \mathbf{k}_0)$
Modulation	$\cos(2\pi \mathbf{k}_0^T \mathbf{x}) g(\mathbf{x})$	$(\hat{g}(\mathbf{k} - \mathbf{k}_0) + \hat{g}(\mathbf{k} + \mathbf{k}_0))/2$
Differentiation in $x$ space	$\frac{\partial g(\mathbf{x})}{\partial x_p}$	$2\pi i k_p \hat{g}(\mathbf{k})$
Differentiation in $k$ space	$-2\pi i x_p g(\mathbf{x})$	$\frac{\partial \hat{g}(\mathbf{k})}{\partial k_p}$
Definite integral, mean	$\int_{-\infty}^{\infty} g(\mathbf{x}') d^W x'$	$\hat{g}(\mathbf{0})$
Moments	$\int_{-\infty}^{\infty} x_p^m x_q^n g(\mathbf{x}) d^W x$	$\left(\frac{i}{2\pi}\right)^{m+n} \left(\frac{\partial^{m+n} \hat{g}(\mathbf{k})}{\partial k_p^m \partial k_q^n}\right) \Big _0$
Convolution	$\int_{-\infty}^{\infty} h(\mathbf{x}') g(\mathbf{x} - \mathbf{x}') d^W x'$	$\hat{h}(\mathbf{k}) \hat{g}(\mathbf{k})$
Spatial correlation	$\int_{-\infty}^{\infty} h(\mathbf{x}') g(\mathbf{x}' + \mathbf{x}) d^W x'$	$\hat{g}^*(\mathbf{k}) \hat{h}(\mathbf{k})$
Multiplication	$h(\mathbf{x}) g(\mathbf{x})$	$\int_{-\infty}^{\infty} \hat{h}(\mathbf{k}') \hat{g}(\mathbf{k} - \mathbf{k}') d^W k'$
Inner product	$\int_{-\infty}^{\infty} g^*(\mathbf{x}) h(\mathbf{x}) d^W x$	$\int_{-\infty}^{\infty} \hat{g}^*(\mathbf{k}) \hat{h}(\mathbf{k}) d^W k$

## Elementary transform pairs for the continuous Fourier transform

R5

2-D and 3-D functions are marked by † and ‡, respectively.

Space domain	Fourier domain
Delta, $\delta(x)$	const., 1
const., 1	Delta, $\delta(k)$
$\cos(k_0 x)$	$\frac{1}{2} (\delta(k - k_0) + \delta(k + k_0))$
$\sin(k_0 x)$	$\frac{i}{2} (\delta(k - k_0) - \delta(k + k_0))$
$\text{sgn}(x) = \begin{cases} 1 & x \geq 0 \\ -1 & x < 0 \end{cases}$	$\frac{-i}{\pi k}$
Box, $\Pi(x) = \begin{cases} 1 &  x  < 1/2 \\ 0 &  x  \geq 1/2 \end{cases}$	$\text{sinc}(k) = \frac{\sin(\pi k)}{\pi k}$
Disk, † $\frac{1}{\pi r^2} \Pi\left(\frac{ \mathbf{x} }{2r}\right)$	Bessel, $\frac{J_1(2\pi r \mathbf{k} )}{\pi r \mathbf{k} }$
Ball, ‡ $\Pi\left(\frac{ \mathbf{x} }{2}\right)$	$\frac{\sin( \mathbf{k} ) -  \mathbf{k}  \cos( \mathbf{k} )}{ \mathbf{k} ^3/(4\pi)}$
Bessel, $\frac{J_1(2\pi x)}{x}$	$2(1 - k)^{1/2} \Pi\left(\frac{k}{2}\right)$
$\exp(- x ), \exp(- \mathbf{x} )^\dagger$	$\frac{2}{1 + (2\pi k)^2}, \frac{2\pi}{(1 + (2\pi \mathbf{k} )^2)^{3/2}}^\dagger$

## Functions invariant under the Fourier transform

R6

Space domain	Fourier domain
Gaussian, $\exp(-\pi \mathbf{x}^T \mathbf{x})$	Gaussian, $\exp(-\pi \mathbf{k}^T \mathbf{k})$
$x_p \exp(-\pi \mathbf{x}^T \mathbf{x})$	$-ik_p \exp(-\pi \mathbf{k}^T \mathbf{k})$
$\text{sech}(\pi x) = \frac{1}{\exp(\pi x) + \exp(-\pi x)}$	$\text{sech}(\pi k) = \frac{1}{\exp(\pi k) + \exp(-\pi k)}$
Hyperbola, $ \mathbf{x} ^{-W/2}$	$ \mathbf{k} ^{-W/2}$
1-D $\delta$ comb, $\text{III}(x) = \sum_{n=-\infty}^{\infty} \delta(x - n)$	$\text{III}(k) = \sum_{v=-\infty}^{\infty} \delta(k - v)$

**R7 Properties of the 2-D DFT (Section 2.3.4)**

$\mathbf{G}$  and  $\mathbf{H}$  are complex-valued  $M \times N$  matrices,  $\hat{\mathbf{G}}$  and  $\hat{\mathbf{H}}$  their Fourier transforms,

$$\hat{g}_{u,v} = \frac{1}{MN} \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} g_{m,n} w_M^{-mu} w_N^{-nv}, \quad w_N = \exp(2\pi i/N)$$

$$g_{m,n} = \sum_{u=0}^{M-1} \sum_{v=0}^{N-1} \hat{g}_{u,v} w_M^{mu} w_N^{nv},$$

and  $a$  and  $b$  complex-valued constants. Stretching and replication by factors  $K, L \in \mathbb{N}$  yields  $KM \times LN$  matrices. For proofs see Cooley and Tukey [25], Poularikas [156].

Property	Space domain	Wave-number domain
Mean	$\frac{1}{MN} \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} G_{mn}$	$\hat{g}_{0,0}$
Linearity	$a\mathbf{G} + b\mathbf{H}$	$a\hat{\mathbf{G}} + b\hat{\mathbf{H}}$
Spatial stretching (up-sampling)	$g_{Km,Ln}$	$\hat{g}_{uv} / (KL)$ ( $\hat{g}_{kM+u,lN+v} = \hat{g}_{u,v}$ )
Replication (frequency stretching)	$g_{m,n} \quad (g_{kM+m,lN+n} = g_{m,n})$	$\hat{g}_{Ku,Lv}$
Shifting	$g_{m-m',n-n'}$	$w_M^{-m'u} w_N^{-n'v} \hat{g}_{uv}$
Modulation	$w_M^{u'm} w_N^{v'n} g_{m,n}$	$\hat{g}_{u-u',v-v'}$
Finite differences	$(g_{m+1,n} - g_{m-1,n})/2$ $(g_{m,n+1} - g_{m,n-1})/2$	$i \sin(2\pi u/M) \hat{g}_{uv}$ $i \sin(2\pi v/N) \hat{g}_{uv}$
Convolution	$\sum_{m'=0}^{M-1} \sum_{n'=0}^{N-1} h_{m'n'} g_{m-m',n-n'}$	$MN \hat{h}_{uv} \hat{g}_{uv}$
Spatial correlation	$\sum_{m'=0}^{M-1} \sum_{n'=0}^{N-1} h_{m'n'} g_{m+m',n+n'}$	$MN \hat{h}_{uv} \hat{g}_{uv}^*$
Multiplication	$g_{mn} h_{mn}$	$\sum_{u'=0}^{M-1} \sum_{v'=0}^{N-1} h_{u'v'} g_{u-u',v-v'}$
Inner product	$\sum_{m=0}^{M-1} \sum_{n=0}^{N-1} g_{mn}^* h_{mn}$	$\sum_{u=0}^{M-1} \sum_{v=0}^{N-1} \hat{g}_{uv}^* \hat{h}_{uv}$
Norm	$\sum_{m=0}^{M-1} \sum_{n=0}^{N-1}  g_{mn} ^2$	$\sum_{u=0}^{M-1} \sum_{v=0}^{N-1}  \hat{g}_{uv} ^2$

**Properties of the continuous 1-D Hartley transform (Section 2.4.2)****R8**

$g(x) \longleftrightarrow \hat{g}(k)$  and  $h(x) \longleftrightarrow \hat{h}(k)$  are Hartley transform pairs:  $\mathbb{R} \mapsto \mathbb{R}$ ,

$${}^h\hat{g}(k) = \int_{-\infty}^{\infty} g(x) \operatorname{cas}(2\pi kx) dx \longleftrightarrow g(x) = \int_{-\infty}^{\infty} {}^h\hat{g}(k) \operatorname{cas}(2\pi kx) dk$$

with

$$\operatorname{cas} 2\pi kx = \cos(2\pi kx) + \sin(2\pi kx).$$

$s$  is a real, nonzero number,  $a$  and  $b$  are real constants.

Property	Spatial domain	Fourier domain
Linearity	$ag(x) + bh(x)$	$a\hat{g}(k) + b\hat{h}(k)$
Similarity	$g(sx)$	$\hat{g}(k/s)/ s $
Shift in $x$ space	$g(x - x_0)$	$\cos(2\pi kx_0)\hat{g}(k) - \sin(2\pi kx_0)\hat{g}(-k)$
Modulation	$\cos(2\pi k_0x)g(x)$	$(\hat{g}(k - k_0) + \hat{g}(k + k_0))/2$
Differentiation in $x$ space	$\frac{\partial g(\mathbf{x})}{\partial x_p}$	$-2\pi k_p \hat{g}(-k)$
Definite integral, mean	$\int_{-\infty}^{\infty} g(\mathbf{x}') d\mathbf{x}'$	$\hat{g}(0)$
Convolution	$\int_{-\infty}^{\infty} h(x')g(x - x') dx'$	$[\hat{g}(k)\hat{h}(k) + \hat{g}(k)\hat{h}(-k) + \hat{g}(-k)\hat{h}(k) - \hat{g}(-k)\hat{h}(-k)]/2$
Multiplication	$h(x)g(x)$	$[\hat{g}(k) * \hat{h}(k) + \hat{g}(k) * \hat{h}(-k) + \hat{g}(-k) * \hat{h}(k) - \hat{g}(-k) * \hat{h}(-k)]/2$
Autocorrelation	$\int_{-\infty}^{\infty} g(x')g(x' + x) dx'$	$[\hat{g}^2(k) + \hat{g}^2(-k)]/2$

1. Fourier transform expressed in terms of the Hartley transform

$$\hat{g}(k) = \frac{1}{2} ({}^h\hat{g}(k) + {}^h\hat{g}(-k)) - \frac{i}{2} ({}^h\hat{g}(k) - {}^h\hat{g}(-k))$$

2. Hartley transform expressed in terms of the Fourier transform

$${}^h\hat{g}(k) = \Re[\hat{g}(k)] - \Im[\hat{g}(k)] = \frac{1}{2} (\hat{g}(k) + \hat{g}^*(k)) + \frac{i}{2} (\hat{g}(k) - \hat{g}^*(k))$$

R9

Probability density functions (PDFs, Section 3.4).

Definition, mean, and variance of some PDFs

Name	Definition	Mean	Variance
Discrete PDFs $f_n$			
Poisson $P(\mu)$	$\exp(-\mu)\frac{\mu^n}{n!}, n \geq 0$	$\mu$	$\mu$
Binomial $B(Q, p)$	$\frac{Q!}{n!(Q-n)!}p^n(1-p)^{Q-n}, 0 \leq n < Q$	$Qp$	$Qp(1-p)$
Continuous PDFs $f(x)$			
Uniform $U(a, b)$	$\frac{1}{b-a}$	$\frac{a+b}{2}$	$\frac{(b-a)^2}{12}$
Normal $N(\mu, \sigma)$	$\frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$	$\mu$	$\sigma^2$
Rayleigh $R(\sigma)$	$\frac{x}{\sigma^2} \exp\left(-\frac{x^2}{2\sigma^2}\right), x > 0$	$\sigma\sqrt{\pi/2}$	$\sigma^2(4-\pi)/2$
Chi-square $\chi^2(Q, \sigma)$	$\frac{x^{Q/2-1}}{2^{Q/2}\Gamma(Q/2)\sigma^Q} \exp\left(-\frac{x}{2\sigma^2}\right), x > 0$	$Q\sigma^2$	$2Q\sigma^4$

Addition theorems for independent random variables  $g_1$  and  $g_2$

PDF	$g_1$	$g_2$	$g_1 + g_2$
Binomial	$B(Q_1, p)$	$B(Q_2, p)$	$B(Q_1 + Q_2, p)$
Poisson	$P(\mu_1)$	$P(\mu_2)$	$P(\mu_1 + \mu_2)$
Normal	$N(\mu_1, \sigma_1)$	$N(\mu_2, \sigma_2)$	$N(\mu_1 + \mu_2, (\sigma_1^2 + \sigma_2^2)^{1/2})$
Chi-square	$\chi^2(Q_1, \sigma)$	$\chi^2(Q_2, \sigma)$	$\chi^2(Q_1 + Q_2, \sigma)$

PDFs of functions of independent random variables  $g_n$

PDF of variable	Function	PDF of function
$g_n: N(0, \sigma)$	$(g_1^2 + g_2^2)^{1/2}$	$R(\sigma)$
$g_n: N(0, \sigma)$	$\arctan(g_2^2/g_1^2)$	$U(0, 2\pi)$
$g_n: N(0, \sigma)$	$\sum_{n=1}^Q g_n^2$	$\chi^2(Q, \sigma)$



**Error propagation (Sections 3.2.3, 3.3.3, and 4.2.8)****R10**

$f_g$  is the PDF of the random variable (RV)  $g$ ,  $a$ , and  $b$  are constants,  $g' = p(g)$  a differentiable monotonic function with the derivative  $dp/dg$  and the inverse function  $g = p^{-1}(g')$ .

Let  $\mathbf{g}$  be a vector with  $P$  RVs with the covariance matrix  $\text{cov}(\mathbf{g})$ ,  $\mathbf{g}'$  a vector with  $Q$  RVs and with the covariance matrix  $\text{cov}(\mathbf{g}')$ ,  $\mathbf{M}$  a  $Q \times P$  matrix, and  $\mathbf{a}$  a column vector with  $Q$  elements.

1. PDF, mean, and variance of a linear function  $g' = ag + b$

$$f_{g'}(g') = \frac{f_g((g' - a)/b)}{|a|}, \quad \mu_{g'} = a\mu_g + b, \quad \sigma_{g'}^2 = a^2\sigma_g^2$$

2. PDF of monotonous differentiable nonlinear function  $g' = p(g)$

$$f_{g'}(g') = \frac{f_g(p^{-1}(g'))}{|dp(p^{-1}(g'))/dg|},$$

3. Mean and variance of differentiable nonlinear function  $g' = p(g)$

$$\mu_{g'} \approx p(\mu_g) + \frac{\sigma_g^2}{2} \frac{d^2 p(\mu_g)}{dg^2}, \quad \sigma_{g'}^2 \approx \left| \frac{dp(\mu_g)}{dg} \right|^2 \sigma_g^2$$

4. Covariance matrix of a linear combination of RVs,  $\mathbf{g}' = \mathbf{M}\mathbf{g} + \mathbf{a}$

$$\text{cov}(\mathbf{g}') = \mathbf{M} \text{cov}(\mathbf{g}) \mathbf{M}^T$$

5. Covariance matrix of a nonlinear combination of RVs,  $\mathbf{g}' = \mathbf{p}(\mathbf{g})$

$$\text{cov}(\mathbf{g}') \approx \mathbf{J} \text{cov}(\mathbf{g}) \mathbf{J}^T \quad \text{with the Jacobian matrix } \mathbf{J}, \quad j_{q,p} = \frac{\partial p_q}{\partial g_p}.$$

6. Homogeneous stochastic field: convolution of a random vector by the filter  $\mathbf{h}$   $\mathbf{g}' = \mathbf{h} * \mathbf{g}$  (Section 4.2.8)

- (a) With the autocovariance vector  $\mathbf{c}$

$$\mathbf{c}' = \mathbf{c} \star (\mathbf{h} \star \mathbf{h}) \quad \longrightarrow \quad \hat{\mathbf{c}}'(k) = \hat{\mathbf{c}}(k) \left| \hat{\mathbf{h}}(k) \right|^2.$$

- (b) With the autocovariance vector  $\mathbf{c} = \sigma^2 \delta_n$  (uncorrelated elements)

$$\mathbf{c}' = \sigma^2 (\mathbf{h} \star \mathbf{h}) \quad \longrightarrow \quad \hat{\mathbf{c}}'(k) = \sigma^2 \left| \hat{\mathbf{h}}(k) \right|^2.$$

**R11 1-D LSI filters (Sections 4.2.6, 11.2, and 12.3)**

1. Transfer function of a 1-D filter with an odd number of coefficients ( $2R + 1$ ,  $[h_{-R}, \dots, h_{-1}, h_0, h_1, \dots, h_R]$ )

(a) General

$$\hat{h}(\tilde{k}) = \sum_{v'=-R}^R h_{v'} \exp(-\pi i v' \tilde{k})$$

(b) Even symmetry ( $h_{-v} = h_v$ )

$$\hat{h}_v = h_0 + 2 \sum_{v'=1}^R h_{v'} \cos(\pi v' \tilde{k})$$

(c) Odd symmetry ( $h_{-v} = -h_v$ )

$$\hat{h}_v = -2i \sum_{v'=1}^R h_{v'} \sin(\pi v' \tilde{k})$$

2. Transfer function of a 1-D filter with an even number of coefficients ( $2R$ ,  $[h_{-R}, \dots, h_{-1}, h_1, \dots, h_R]$ , convolution results put on intermediate grid)

(a) Even symmetry ( $h_{-v} = h_v$ )

$$\hat{h}_v = 2 \sum_{v'=1}^R h_{v'} \cos(\pi(v' - 1/2)\tilde{k})$$

(b) Odd symmetry ( $h_{-v} = -h_v$ )

$$\hat{h}_v = -2i \sum_{v'=1}^R h_{v'} \sin(\pi(v' - 1/2)\tilde{k})$$

3. Transfer function of the two elementary filters

(a) Averaging of two neighboring points

$$\mathbf{B} = \begin{bmatrix} 1 & 1 \end{bmatrix} / 2 \quad \circ \longrightarrow \bullet \quad \hat{b}(\tilde{k}) = \cos(\pi \tilde{k} / 2)$$

(b) Difference of two neighboring points

$$\mathbf{D}_1 = \begin{bmatrix} 1 & -1 \end{bmatrix} \quad \circ \longrightarrow \bullet \quad \hat{d}_1(\tilde{k}) = 2i \sin(\pi \tilde{k} / 2)$$

**1-D recursive filters (Section 4.5).****R12**

## 1. General filter equation

$$g'_n = - \sum_{n''=1}^S a_{n''} g'_{n-n''} + \sum_{n'=-R}^R h_{n'} g_{n-n'}$$

## 2. General transfer function

$$\hat{h}(\tilde{k}) = \frac{\sum_{n'=-R}^R h_{n'} \exp(-\pi i n' \tilde{k})}{\sum_{n''=0}^S a_{n''} \exp(-\pi i n'' \tilde{k})}$$

3. Factorization of the transfer function using the  $z$  transform and the fundamental law of algebra

$$\hat{h}(z) = h_{-R} z^R \frac{\prod_{n'=1}^{2R} (1 - c_{n'} z^{-1})}{\prod_{n''=1}^S (1 - d_{n''} z^{-1})}$$

## 4. Relaxation filter

(a) Filter equation ( $|\alpha| < 1$ )

$$g'_n = \alpha g'_{n+1} + (1 - \alpha) g_n$$

## (b) Point spread function

$${}^{\pm}r_{\pm n} = \begin{cases} (1 - \alpha) \alpha^n & n \geq 0 \\ 0 & \text{else} \end{cases}$$

## (c) Transfer function of the symmetric filter (running filter successively in positive and negative direction)

$$\hat{r}(\tilde{k}) = \frac{1}{1 + \beta - \beta \cos \pi \tilde{k}}, \quad \left( \hat{r}(0) = 1, \hat{r}(1) = \frac{1}{1 + 2\beta} \right)$$

with

$$\beta = \frac{2\alpha}{(1 - \alpha)^2}, \quad \alpha = \frac{1 + \beta - \sqrt{1 + 2\beta}}{\beta}, \quad \beta \in ] -1/2, \infty[$$

5. Resonance filter with unit response at resonance wave number  $\tilde{k}_0$  in the limit of low damping  $1 - r \ll 1$

(a) Filter equation (damping coefficient  $r \in [0, 1[$ , resonance wave number  $\tilde{k}_0 \in [0, 1]$ )

$$g'_n = (1 - r^2) \sin(\pi \tilde{k}_0) g_n + 2r \cos(\pi \tilde{k}_0) g'_{n \mp 1} - r^2 g'_{n \mp 2}$$

(b) Point spread function

$$h_{\pm n} = \begin{cases} (1 - r^2) r^n \sin[(n + 1)\pi \tilde{k}_0] & n \geq 0 \\ 0 & n < 0 \end{cases}$$

(c) Transfer function of the symmetric filter (running filter successively in positive and negative direction)

$$\hat{s}(\tilde{k}) = \frac{\sin^2(\pi \tilde{k}_0) (1 - r^2)^2}{(1 - 2r \cos[\pi(\tilde{k} - \tilde{k}_0)] + r^2) (1 - 2r \cos[\pi(\tilde{k} + \tilde{k}_0)] + r^2)}$$

(d) For low damping, the transfer function can be approximated by

$$\hat{s}(\tilde{k}) \approx \frac{1}{1 + (\tilde{k} - \tilde{k}_0)^2 / \frac{(1 - r^2)^2}{4r^2\pi^2}} \quad \text{for } 1 - r \ll 1$$

(e) Halfwidth  $\Delta k$ , defined by  $\hat{s}(\tilde{k}_0 + \Delta k) = 1/2$

$$\Delta k \approx (1 - r)/\pi$$

### **R13 Gaussian and Laplacian pyramids (Section 5.2)**

1. Construction of the *Gaussian pyramid*  $\mathbf{G}^{(0)}, \mathbf{G}^{(1)}, \dots, \mathbf{G}^{(P)}$  with  $P + 1$  planes by iterative smoothing and subsampling by a factor of two in all directions

$$\mathbf{G}^{(0)} = \mathbf{G}, \quad \mathbf{G}^{(p+1)} = \mathcal{B}_{\downarrow 2} \mathbf{G}^{(p)}$$

2. Condition for smoothing filter to avoid aliasing

$$\hat{B}(\tilde{\mathbf{k}}) = 0 \quad \forall \tilde{k}_p \geq \frac{1}{2}$$

3. Construction of the *Laplacian pyramid*  $\mathbf{L}^{(0)}, \mathbf{L}^{(1)}, \dots, \mathbf{L}^{(P)}$  with  $P + 1$  planes from the Gaussian pyramid

$$\mathbf{L}^{(p)} = \mathbf{G}^{(p)} - \uparrow_2 \mathbf{G}^{(p+1)}, \quad \mathbf{L}^{(P)} = \mathbf{G}^{(P)}$$

The last plane of the Laplacian pyramid is the last plane of the Gaussian pyramid.

4. Interpolation filters for upsampling operation  $\uparrow_2$  (> R22)
5. Iterative reconstruction of the original image from the Laplacian pyramid. Compute

$$\mathbf{G}^{(p-1)} = \mathbf{L}^{(p-1)} + \uparrow_2 \mathbf{G}^{(p)}$$

starting with the highest plane ( $p = P$ ). When the same upsampling operator is used as for the construction of the Laplacian pyramid, the reconstruction is perfect except for rounding errors.

6. Directional decomposition in two directional components

$$\begin{aligned} \mathbf{G}^{(p+1)} &= \downarrow_2 \mathcal{B}_x \mathcal{B}_y \mathbf{G}^{(p)} \\ \mathbf{L}^{(p)} &= \mathbf{G}^{(p)} - \uparrow_2 \mathbf{G}^{(p+1)} \\ \mathbf{L}_x^{(p)} &= 1/2(\mathbf{L}^{(p)} - (\mathcal{B}_x - \mathcal{B}_y)\mathbf{G}^{(p)}) \\ \mathbf{L}_y^{(p)} &= 1/2(\mathbf{L}^{(p)} + (\mathcal{B}_x - \mathcal{B}_y)\mathbf{G}^{(p)}) \end{aligned}$$

### Basic properties of electromagnetic waves (Section 6.3)

**R14**

1. The *frequency*  $\nu$  (cycles per unit time) and *wavelength*  $\lambda$  (length of a period) are related by the *phase speed*  $c$  (in vacuum *speed of light*  $c = 2.9979 \times 10^8 \text{ m s}^{-1}$ ):

$$\lambda \nu = c$$

2. Classification of the ultraviolet, visible and infrared part of the electromagnetic spectrum (see also Fig. 6.6)

Name	Wavelength range	Comment
VUV (vacuum UV)	30–180 nm	Strongly absorbed by air; requires evacuated equipment
UV-C	100–280 nm	CIE standard definition
UV-B	280–315 nm	CIE standard definition
UV-A	315–400 nm	CIE standard definition
Visible (light)	400–700 nm	Visible by the human eye
VNIR (very near IR)	0.7–1.0 $\mu\text{m}$	IR wavelength range to which standard silicon image sensors respond
NIR (near IR)	0.7–3.0 $\mu\text{m}$	
TIR (thermal IR)	3.0–14.0 $\mu\text{m}$	Range of largest emission at environmental temperatures
MIR (middle IR)	3–100 $\mu\text{m}$	
FIR (far IR)	100–1000 $\mu\text{m}$	

3. Energy and momentum of particulate radiation such as  $\beta$  radiation (electrons),  $\alpha$  radiation (helium nuclei), neutrons, and photons (electromagnetic radiation):

$$\begin{aligned} \nu &= E/h && \text{Bohr frequency condition,} \\ \lambda &= h/p && \text{de Broglie wavelength relation.} \end{aligned}$$

**R15 Radiometric and photometric terms (Section 6.2)**

$dA_0$  is an element of area in the surface,  $\theta$  the angle of incidence,  $\Omega$  the solid angle. For energy-, photon-, and photometry-related terms, often the indices  $e$ ,  $p$ , and  $v$ , respectively, are used.

Term	Energy-related	Photon-related	Photometric quantity
Energy	Radiant energy $Q$ [Ws]	Photon number [1]	Luminous energy [lm s]
Energy flux (power)	Radiant flux $\Phi = \frac{dQ}{dt}$ [W]	Photon flux [s <sup>-1</sup> ]	Luminous flux [lumen (lm)]
Incident energy flux density	Irradiance $E = \frac{d\Phi}{dA_0}$ [W m <sup>-2</sup> ]	Photon irradiance [m <sup>-2</sup> s <sup>-1</sup> ]	Illuminance [lm/m <sup>2</sup> = lux [(lx)]
Excitant energy flux density	Radiant excitance (emittance) $M = \frac{d\Phi}{dA_0}$ [W m <sup>-2</sup> ]	Photon flux density [m <sup>-2</sup> s <sup>-1</sup> ]	Luminous excitance [lm/m <sup>2</sup> ]
Energy flux per solid angle	Radiant intensity $I = \frac{d\Phi}{d\Omega}$ [Wsr <sup>-1</sup> ]	Photon intensity [s <sup>-1</sup> sr <sup>-1</sup> ]	Luminous intensity [lm/sr = candela (cd)]
Energy flux density per solid angle	Radiance $L = \frac{d^2\Phi}{d\Omega dA_0 \cos \theta}$ [W m <sup>-2</sup> sr <sup>-1</sup> ]	Photon radiance [m <sup>-2</sup> s <sup>-1</sup> sr <sup>-1</sup> ]	Luminance [cd m <sup>-2</sup> ]
Energy/area	Energy density [W s m <sup>2</sup> ]	Photon density [m <sup>-2</sup> ]	Exposure [lm s m <sup>-2</sup> = lx s]

Computation of luminous quantities from the corresponding radiometric quantity by the *spectral luminous efficacy*  $V(\lambda)$  for daylight (photopic) vision:

$$Q_v = 683 \frac{\text{lm}}{\text{W}} \int_{380 \text{ nm}}^{780 \text{ nm}} Q(\lambda) V(\lambda) d\lambda$$

Table with the 1980 CIE values of the spectral luminous efficacy  $V(\lambda)$  for photopic vision

$\lambda$ [nm]	$V(\lambda)$	$\lambda$ [nm]	$V(\lambda)$	$\lambda$ [nm]	$V(\lambda)$
380	0.00004	520	0.710	660	0.061
390	0.00012	530	0.862	670	0.032
400	0.0004	540	0.954	680	0.017
410	0.0012	550	0.995	690	0.0082
420	0.0040	560	0.995	700	0.0041
430	0.0116	570	0.952	710	0.0021
440	0.023	580	0.870	720	0.00105
450	0.038	590	0.757	730	0.00052
460	0.060	600	0.631	740	0.00025
470	0.091	610	0.503	750	0.00012
480	0.139	620	0.381	760	0.00006
490	0.208	630	0.265	770	0.00003
500	0.323	640	0.175	780	0.000015
510	0.503	650	0.107		

### Color systems (Section 6.2.4)

R16

1. Human color vision based on three types of cones with maximal sensitivities at 445 nm, 535 nm, and 575 nm (Fig. 6.4b).
2. *RGB* color system; additive color system with the three primary colors red, green, and blue. This could either be monochromatic colors with wavelengths 700 nm, 646.1 nm, and 435.8 nm or red, green, and blue phosphor as used in *RGB* monitors (e. g., according to the European EBU norm). Not all colors can be represented by the *RGB* color system (see Fig. 6.5a).
3. Chromaticity diagram: reduction of the 3-D color space to a 2-D color plane normalized by the intensity:

$$r = \frac{R}{R + G + B}, \quad g = \frac{G}{R + G + B}, \quad b = \frac{B}{R + G + B}.$$

It is sufficient to use the two components  $r$  and  $g$ :  $b = 1 - r - g$ .

4. *XYZ* color system (Fig. 6.5c): additive color system with three virtual primaries  $X$ ,  $Y$ , and  $Z$  that can represent all possible colors and is given by the following linear transform from the EBU *RGB* color

system:

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} 0.490 & 0.310 & 0.200 \\ 0.177 & 0.812 & 0.011 \\ 0.000 & 0.010 & 0.990 \end{bmatrix} \begin{bmatrix} R \\ G \\ B \end{bmatrix}.$$

5. Color difference or *YUV* system: color system with an origin at the white point (Fig. 6.5b).
6. Hue-saturation (HSI) color system: color system using polar coordinates in a color difference system. The saturation is given by the radius and the hue by the angle.

### **R17 Thermal emission (Section 6.4.1)**

1. Spectral emittance (law of Planck)

$$M_e(\lambda, T) = \frac{2\pi hc^2}{\lambda^5} \frac{1}{\exp\left(\frac{hc}{k_B T \lambda}\right) - 1}$$

with

$$\begin{aligned} h &= 6.6262 \times 10^{-34} \text{ J s} && \text{Planck constant,} \\ k_B &= 1.3806 \times 10^{-23} \text{ J K}^{-1} && \text{Boltzmann constant, and} \\ c &= 2.9979 \times 10^8 \text{ m s}^{-1} && \text{speed of light in vacuum.} \end{aligned}$$

2. Total emittance (law of Stefan and Boltzmann)

$$M_e = \frac{2}{15} \frac{k_B^4 \pi^5}{c^2 h^3} T^4 = \sigma T^4 \quad \text{with} \quad \sigma \approx 5.67 \cdot 10^{-8} \text{ W m}^{-2} \text{ K}^{-4}$$

3. Wavelength of maximum emittance (Wien's law)

$$\lambda_m \approx \frac{2898 \text{ K } \mu\text{m}}{T}$$

### **R18 Interaction of radiation with matter (Section 6.4)**

1. *Snell's law* of *refraction* at the boundary of two optical media with the indices of refraction  $n_1$  and  $n_2$

$$\frac{\sin \theta_1}{\sin \theta_2} = \frac{n_2}{n_1}$$

$\theta_1$  and  $\theta_2$  are the angles of incidence and refraction, respectively.

2. *Reflectivity*  $\rho$ : ratio of the reflected radiant flux to the incident flux at the surface. *Fresnel's equations* give the reflectivity for parallel polarized light

$$\rho_{\parallel} = \frac{\tan^2(\theta_1 - \theta_2)}{\tan^2(\theta_1 + \theta_2)},$$



for perpendicular polarized light

$$\rho_{\perp} = \frac{\sin^2(\theta_1 - \theta_2)}{\sin^2(\theta_1 + \theta_2)},$$

and for unpolarized light

$$\rho = \frac{\rho_{\parallel} + \rho_{\perp}}{2}.$$

3. Reflectivity at normal incidence ( $\theta_1 = 0$ ) for all polarization states

$$\rho = \frac{(n_1 - n_2)^2}{(n_1 + n_2)^2} = \frac{(n - 1)^2}{(n + 1)^2} \quad \text{with} \quad n = n_1/n_2$$

4. *Total reflection.* When a ray enters into a medium with lower refractive index, beyond the critical angle  $\theta_c$  all light is reflected and none enters the optically thinner medium:

$$\theta_c = \arcsin \frac{n_1}{n_2} \quad \text{with} \quad n_1 < n_2$$

## Optical imaging

R19

1. Perspective projection with *pinhole camera* model

$$x_1 = -\frac{d' X_1}{X_3}, \quad x_2 = -\frac{d' X_2}{X_3}$$

Pinhole located at origin of world coordinate system  $[X_1, X_2, X_3]^T$ ,  $d'$  is distance of image plane to projection center,  $X_3$  axis aligned perpendicular to image plane.

2. Image equation (Newtonian and Gaussian form)

$$dd' = f^2 \quad \text{or} \quad \frac{1}{d' + f} + \frac{1}{d + f} = \frac{1}{f}$$

$d$  and  $d'$  are the distances of the object and image to the front and back focal points of the optical system, respectively (see Fig. 7.7).

3. Lateral magnification

$$m_l = \frac{x_1}{X_1} = \frac{f}{d} = \frac{d'}{f}$$

4. Axial magnification

$$m_a \approx \frac{d'}{d} = \frac{f^2}{d^2} = \frac{d'^2}{f^2} = m_l^2$$

5. The  $f$ -number  $n_f$  of an optical system is the ratio of the focal length and diameter of lens aperture

$$n_f = \frac{f}{2r}$$

6. Depth of focus (image space)

$$\Delta x_3 = 2n_f \left( 1 + \frac{d'}{f} \right) \epsilon = 2n_f (1 + m_l) \epsilon$$

7. Depth of field (object space)

$$\text{Distant objects } (\Delta X_3 \ll d) \quad \Delta X_3 \approx 2n_f \cdot \frac{1 + m_l}{m_l^2} \epsilon$$

$$d_{\min} \text{ for range including infinity} \quad d_{\min} \approx \frac{f^2}{4n_f \epsilon}$$

$$\text{Microscopy } (m_l \gg 1) \quad \Delta X_3 \approx \frac{2n_f \epsilon}{m_l}$$

8. Resolution of a diffraction-limited optical system: angular resolution

$$\text{Angular resolution} \quad \Delta \theta_0 = 0.61 \frac{\lambda}{r}$$

$$\text{Lateral resolution at image plane} \quad \Delta x = 0.61 \frac{\lambda}{n'_a}$$

$$\text{Lateral resolution at object plane} \quad \Delta X = 0.61 \frac{\lambda}{n_a}$$

The resolution is given by the Rayleigh criterion (see Fig. 7.15b);  $n_a$  and  $n_{a'}$  are the object-sided and image sided numerical aperture of the light cone entering the optical system:

$$n_a = n \sin \theta_0 = \frac{2n}{n_f} = \frac{nr}{f};$$

$n$  is the index of refraction.

9. Relation of the irradiance at image plane  $E'$  to the object radiance  $L$  (see Fig. 7.10)

$$E' = t\pi \left( \frac{r}{f + d'} \right)^2 \cos^4 \theta L \approx t\pi \frac{\cos^4 \theta}{n_f^2} L \quad \text{for } d \gg f$$

**Homogeneous point operation (Section 10.2)****R20**

Point operation that is independent of the position of the pixel

$$G'_{mn} = P(G_{mn})$$

1. Negative

$$P_N(q) = Q - 1 - q$$

2. Detection of underflow and overflow by a pseudocolor  $[r, g, b]$  mapping

$$P_{uo}(q) = \begin{cases} [0, 0, Q - 1] & \text{(blue)} & q = 0 \\ [q, q, q] & \text{(gray)} & q \in [1, Q - 2] \\ [Q - 1, 0, 0] & \text{(red)} & q = Q - 1 \end{cases}$$

3. Contrast stretching of range  $[q_1, q_2]$

$$P_{cs}(q) = \begin{cases} 0 & q < q_1 \\ \frac{(q - q_1)(Q - 1)}{q_2 - q_1} & q \in [q_1, q_2] \\ Q - 1 & q > q_2 \end{cases}$$

**Calibration procedures****R21**

1. Noise equalization (Section 10.2.3)

If the variance of the noise depends on the image intensity, it can be equalized by a nonlinear grayscale transformation

$$h(g) = \sigma_h \int_0^g \frac{dg'}{\sqrt{\sigma^2(g')}} + C$$

with the two free parameters  $\sigma_h$  and  $C$ . With a linear variance function (Section 3.4.5)

$$\sigma_g^2(g) = \sigma_0^2 + \alpha g$$

the transformation becomes for  $g \in [0, g_{\max}] \mapsto h \in [0, \gamma g_{\max}]$

$$h(g) = \gamma g_{\max} \frac{\sqrt{\sigma_0^2 + K g} - \sigma_0}{\sqrt{\sigma_0^2 + K g_{\max}} - \sigma_0}, \quad \sigma_h = \frac{\gamma K g_{\max} / 2}{\sqrt{\sigma_0^2 + K g_{\max}} - \sigma_0}.$$

2. Linear photometric two-point calibration (Section 10.3.3)

Two calibration images are taken, a dark image  $\mathbf{B}$  without any illumination and a reference image  $\mathbf{R}$  with an object of constant radiance. A normalized image corrected for both the fixed pattern noise and inhomogeneous sensitivity is given by

$$\mathbf{G}' = c \frac{\mathbf{G} - \mathbf{B}}{\mathbf{R} - \mathbf{B}}.$$

**R22 Interpolation (Section 10.5)**

1. Interpolation of continuous function from sampled points at distances  $\Delta x_w$  is an convolution operation:

$$g_r(\mathbf{x}) = \sum_{\mathbf{n}} g(\mathbf{x}_n) h(\mathbf{x} - \mathbf{x}_n).$$

Reproduction of the grid points results in the *interpolation condition*

$$h(\mathbf{x}_n) = \begin{cases} 1 & \mathbf{n} = \mathbf{0} \\ 0 & \text{otherwise.} \end{cases}$$

2. Ideal interpolation function

$$h(\mathbf{x}) = \prod_{w=1}^W \text{sinc}(x_w / \Delta x_w) \quad \longleftrightarrow \quad \hat{h}(\mathbf{k}) = \prod_{w=1}^W \Pi(\tilde{k}_w / 2)$$

3. Discrete 1-D interpolation filters for interpolation of intermediate grid points halfway between the existing points

Type	Mask	Transfer function
Linear	$\begin{bmatrix} 1 & 1 \end{bmatrix} / 2$	$\cos(\pi \tilde{k} / 2)$
Cubic	$\begin{bmatrix} -1 & 9 & 9 & -1 \end{bmatrix} / 16$	$\frac{9 \cos(\pi \tilde{k} / 2) - \cos(3\pi \tilde{k} / 2)}{8}$
Cubic B-spline	$\begin{bmatrix} 1 & 23 & 23 & 1 \end{bmatrix} / 48$ $\begin{bmatrix} 3 - \sqrt{3} & \sqrt{3} - 2 \end{bmatrix}^\dagger$	$\frac{23 \cos(\pi \tilde{k} / 2) + \cos(3\pi \tilde{k} / 2)}{16 + 8 \cos(\pi \tilde{k})}$

<sup>†</sup>Recursive filter applied in forward and backward direction, see Section 10.6.1

**Averaging convolution filters (Chapter 11)**

**R23**

1. Summary of general constraints for averaging convolution filters

Property	Space domain	Wave-number domain
Preservation of mean	$\sum_n h_n = 1$	$\hat{h}(\mathbf{0}) = 1$
Zero shift, even symmetry	$h_{-n} = h_n$	$\Im(\hat{h}(\mathbf{k})) = 0$
Monotonic decrease from one to zero	—	$\hat{h}(\tilde{k}_2) \leq \hat{h}(\tilde{k}_1)$ if $\tilde{k}_2 > \tilde{k}_1$ , $\hat{h}(\mathbf{k}) \in [0, 1]$
Isotropy	$h(\mathbf{x}) = h( \mathbf{x} )$	$\hat{h}(\mathbf{k}) = \hat{h}( \mathbf{k} )$

2. 1-D smoothing box filters

Mask	Transfer function	Noise suppression <sup>†</sup>
${}^3R = [1 \ 1 \ 1]/3$	$\frac{1}{3} + \frac{2}{3} \cos(\pi \tilde{k})$	$\frac{1}{\sqrt{3}} \approx 0.577$
${}^4R = [1 \ 1 \ 1 \ 1]/4$	$\cos(\pi \tilde{k}) \cos(\pi \tilde{k}/2)$	$1/2 = 0.5$
${}^R R = \underbrace{[1 \ \dots \ 1]}_{R \text{ times}}/R$	$\frac{\sin(\pi R \tilde{k}/2)}{R \sin(\pi \tilde{k}/2)}$	$\frac{1}{\sqrt{R}}$

<sup>†</sup>For white noise

3. 1-D smoothing binomial filters

Mask	TF	Noise suppression <sup>†</sup>
$B^2 = [1 \ 2 \ 1]/4$	$\cos^2(\pi \tilde{k}/2)$	$\sqrt{\frac{3}{8}} \approx 0.612$
$B^4 = [1 \ 4 \ 6 \ 4 \ 1]/16$	$\cos^4(\pi \tilde{k}/2)$	$\sqrt{\frac{35}{128}} \approx 0.523$
$B^{2R}$	$\cos^{2R}(\pi \tilde{k}/2)$	$\left(\frac{\Gamma(R+1/2)}{\sqrt{\pi} \Gamma(R+1)}\right)^{1/2} \approx \left(\frac{1}{R\pi}\right)^{1/4} \left(1 - \frac{1}{16R}\right)$

<sup>†</sup>For white noise

**R24 First-order derivative convolution filters (Chapter 12)**

- Summary of general constraints for a first-order derivative filter into the direction  $x_w$  for  $W$ -dimensional signals;  $w'$  denotes any of the possible directions and  $\mathbf{n}$  vector indexing (Section 4.2.1)

Property	Space domain	Wave-number domain
Zero mean	$\sum_{\mathbf{n}} h_{\mathbf{n}} = 0$	$\hat{h}(\tilde{\mathbf{k}}) \Big _{\tilde{\mathbf{k}}=0} = 0$
Zero shift, odd symmetry	$h_{n_1, \dots, -n_w, \dots, n_W} = -h_{n_1, \dots, n_w, \dots, n_W}$	$\Re(\hat{H}(\mathbf{k})) = 0$
First-order derivative	$\sum_{\mathbf{n}} n_{w'} h_{\mathbf{n}} = \delta_{w' - w}$	$\frac{\partial \hat{h}(\tilde{\mathbf{k}})}{\partial \tilde{k}_w} \Big _{\tilde{\mathbf{k}}=0} = \pi i \delta_{w' - w}$
Isotropy		$\hat{h}(\tilde{\mathbf{k}}) = \pi i \tilde{k}_w \hat{b}(\tilde{\mathbf{k}})$ with $\hat{b}(0) = 1, \nabla_{\tilde{\mathbf{k}}} \hat{b}(\tilde{\mathbf{k}}) = \mathbf{0}$

- First-order discrete difference filters

Name	Mask	Transfer function
$\mathcal{D}_x$	$\begin{bmatrix} 1 & -1 \end{bmatrix}$	$2i \sin(\pi \tilde{k}_x / 2)$
Symmetric difference, $\mathcal{D}_{2x}$	$\begin{bmatrix} 1 & 0 & -1 \end{bmatrix} / 2$	$i \sin(\pi \tilde{k}_x)$
Cubic B-spline $\mathcal{D}_{2x} \pm \mathcal{R}$	$\begin{bmatrix} 1 & 0 & -1 \end{bmatrix} / 2,$ $\begin{bmatrix} 3 - \sqrt{3}, & \sqrt{3} - 2 \end{bmatrix}^\dagger$	$i \frac{\sin(\pi \tilde{k}_x)}{2/3 + 1/3 \cos(\pi \tilde{k}_x)}$

<sup>†</sup>Recursive filter applied in forward and backward direction, see Section 10.6.1

3. Regularized first-order discrete difference filters

Name	Mask	Transfer function
$2 \times 2, \mathcal{D}_x \mathcal{B}_y$	$\frac{1}{2} \begin{bmatrix} 1 & -1 \\ 1 & -1 \end{bmatrix}$	$2i \sin(\pi \tilde{k}_x/2) \cos(\pi \tilde{k}_y/2)$
Sobel, $\mathcal{D}_{2x} \mathcal{B}_y^2$	$\frac{1}{8} \begin{bmatrix} 1 & 0 & -1 \\ 2 & 0 & -2 \\ 1 & 0 & -1 \end{bmatrix}$	$i \sin(\pi \tilde{k}_x) \cos^2(\pi \tilde{k}_y/2)$
Optimized Sobel $\mathcal{D}_{2x}(3\mathcal{B}_y^2 + 1)/4$	$\frac{1}{32} \begin{bmatrix} 3 & 0 & -3 \\ 10 & 0 & -10 \\ 3 & 0 & -3 \end{bmatrix}$	$i \sin(\pi \tilde{k}_x) (3 \cos^2(\pi \tilde{k}_y/2) + 1)/4$

4. Performance characteristics of edge detectors: angle error, magnitude error, and noise suppression for white noise. The three values in the two error columns give the errors for a wave number range of 0-0.25, 0.25-0.5, and 0.5-0.75, respectively.

Name	Angle error [°]	Magnitude error	Noise factor
$\mathcal{D}_x$			$\sqrt{2} \approx 1.414$
$\mathcal{D}_{2x}$	1.36 4.90 12.66	0.026 0.151 0.398	$1/\sqrt{2} \approx 0.707$
$\mathcal{D}_{2x} \pm \mathcal{R}$	0.02 0.33 2.26	0.001 0.023 0.220	$\sqrt{3 \ln 3 / \pi} \approx 1.024$
$\mathcal{D}_x \mathcal{B}_y$	0.67 2.27 5.10	0.013 0.079 0.221	1
$\mathcal{D}_{2x} \mathcal{B}_y^2$	0.67 2.27 5.10	0.012 0.053 0.070	$\sqrt{3}/4 \approx 0.433$
$\mathcal{D}_{2x}(3\mathcal{B}_y^2 + 1)/4$	0.15 0.32 0.72	0.003 0.005 0.047	$\sqrt{59}/16 \approx 0.480$

**R25 Second-order derivative convolution filters (Chapter 12)**

- Summary of general constraints for a second-order derivative filter into the direction  $x_w$  for  $W$ -dimensional signals;  $w'$  denotes any of the possible directions and  $\mathbf{n}$  vector indexing (Section 4.2.1)

Property	Space domain	Wave-number domain
Zero mean	$\sum_{\mathbf{n}} h_{\mathbf{n}} = 0$	$\hat{h}(\tilde{\mathbf{k}}) \big _{\tilde{\mathbf{k}}=\mathbf{0}} = 0$
Zero slope	$\sum_{\mathbf{n}} n_{w'} h_{\mathbf{n}} = 0$	$\frac{\partial \hat{h}(\tilde{\mathbf{k}})}{\partial \tilde{k}_{w'}} \bigg _{\tilde{\mathbf{k}}=\mathbf{0}} = 0$
Zero shift, even symmetry	$h_{-\mathbf{n}} = h_{\mathbf{n}}$	$\Im(\hat{H}(\mathbf{k})) = 0$
2nd-order derivative	$\sum_{\mathbf{n}} n_{w'}^2 h_{\mathbf{n}} = 2\delta_{w'-w}$	$\frac{\partial^2 \hat{h}(\tilde{\mathbf{k}})}{\partial \tilde{k}_{w'}^2} \bigg _{\tilde{\mathbf{k}}=\mathbf{0}} = -2\pi^2 \delta_{w'-w}$
Isotropy		$\hat{h}(\tilde{\mathbf{k}}) = -(\pi \tilde{k}_w)^2 \hat{b}( \tilde{\mathbf{k}} )$ with $\hat{b}(0) = 1, \nabla_{\mathbf{k}} \hat{b}( \tilde{\mathbf{k}} ) = \mathbf{0}$

- Second-order discrete difference filters

Name	Mask	Transfer function
1-D Laplace $\mathcal{D}_x^2$	$\begin{bmatrix} 1 & -2 & 1 \end{bmatrix}$	$-4 \sin^2(\pi \tilde{k}_x/2)$
2-D Laplace $\mathcal{L}$	$\begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix}$	$-4 \sin^2(\pi \tilde{k}_x/2) - 4 \sin^2(\pi \tilde{k}_y/2)$
2-D Laplace $\mathcal{L}'$	$\frac{1}{4} \begin{bmatrix} 1 & 2 & 1 \\ 2 & -12 & 2 \\ 1 & 2 & 1 \end{bmatrix}$	$4 \cos^2(\pi \tilde{k}_x/2) \cos^2(\pi \tilde{k}_y/2) - 4$



## B Notation

Because of the multidisciplinary nature of digital image processing, a consistent and generally accepted terminology — as in other areas — does not exist. Two basic problems must be addressed.

- *Conflicting terminology.* Different communities use different symbols (and even names) for the same terms.
- *Ambiguous symbols.* Because of the many terms used in image processing and the areas it is related to, one and the same symbol is used for multiple terms.

There exists no trivial solution to this awkward situation. Otherwise it would be available. Thus conflicting arguments must be balanced. In this textbook, the following guidelines are used:

- *Stick to common standards.* As a first guide, the symbols recommended by international organizations (such as the International Organization for Standardization, ISO) were consulted and several major reference works were compared [46, 123, 128, 156]. Additionally cross checks were made with several standard textbooks from different areas [13, 61, 148, 158]. Only in a few conflicting situations deviations from commonly accepted symbols are used.
- *Use most compact notation.* When there was a choice of different notations, the most compact and comprehensive notation was used. In rare cases, it appeared useful to use more than one notation for the same term. It is, for example, sometimes more convenient to use indexed vector components ( $\mathbf{x} = [x_1, x_2]^T$ ), and sometimes to use  $\mathbf{x} = [x, y]^T$ .
- *Allow ambiguous symbols.* One and the same symbol can have different meanings. This is not so bad as it appears at first glance because from the context the meaning of the symbol becomes unambiguous. Thus care was taken that ambiguous symbols were only used when they can clearly be distinguished by the context.

In order to familiarize readers coming from different backgrounds to the notation used in this textbook, we will give here some comments on deviating notations.

**Wave number.** Unfortunately, different definitions for the term *wave number* exist:

$$k' = \frac{2\pi}{\lambda} \quad \text{and} \quad k = \frac{1}{\lambda}. \quad (\text{B.1})$$

Physicists usually include the factor  $2\pi$  in the definition of the wave number:  $k' = 2\pi/\lambda$ , by analogy to the definition of the circular frequency  $\omega = 2\pi/T = 2\pi\nu$ . In optics and spectroscopy, however, it is defined as the inverse of the wavelength without the factor  $2\pi$  (i.e., number of wavelengths per unit length) and denoted by  $\tilde{\nu} = \lambda^{-1}$ .

**Imaginary unit.** The imaginary unit is denoted here by  $i$ . In electrical engineering and related areas, the symbol  $j$  is commonly used.

**Time series, image matrices.** The standard notation for *time series* [148],  $x[n]$ , is too cumbersome to be used with multidimensional signals:  $g[k][m][n]$ . Therefore the more compact notation  $x_n$  and  $g_{k,m,n}$  is chosen.

**Partial derivatives.** In cases where it does not lead to confusion, partial derivatives are abbreviated by indexing:  $\partial g / \partial x = \partial_x g = g_x$

Typeface	Description
e, i, d, w	Upright symbols have a special meaning; examples: e for the base of natural logarithm, $i = \sqrt{-1}$ , symbol for derivatives: $dg$ , $w = e^{2\pi i}$
<i>a, b, ...</i>	Italic (not bold): <i>scalar</i>
<b><i>g, k, u, x, ...</i></b>	Lowercase italic bold: <i>vector</i> , i.e., a coordinate vector, a time series, row of an image, ...
<b><i>G, H, J, ...</i></b>	Uppercase italic bold: <i>matrix</i> , <i>tensor</i> , i.e., a discrete image, a 2-D convolution mask, a structure tensor; also used for signals with more than two dimensions
$\mathcal{B}, \mathcal{R}, \mathcal{F}, \dots$	Caligraphic letters indicate a representation-independent <i>operator</i>
$\mathbb{N}, \mathbb{Z}, \mathbb{R}, \mathbb{C}$	Blackboard bold letters denote sets of numbers or other quantities

Accents	Description
$\bar{\mathbf{k}}, \bar{\mathbf{n}}, \dots$	A bar indicates a <i>unit vector</i>
$\tilde{k}, \tilde{\mathbf{k}}, \tilde{\mathbf{x}}, \dots$	A tilde indicates a <i>dimensionless normalized</i> quantity (of a quantity with a dimension)
$\hat{G}, \hat{g}(k), \dots$	A hat indicates a quantity in the <i>Fourier domain</i>

Subscript	Description
$g_n$	Element $n$ of the vector $\mathbf{g}$
$g_{mn}$	Element $m, n$ of the matrix $\mathbf{G}$
$g_p$	Compact notation for first-order partial derivative of the continuous function $g$ into the direction $p$ : $\partial g(\mathbf{x})/\partial x_p$
$g_{pq}$	Compact notation for second-order partial derivative of the continuous function $g(\mathbf{x})$ into the directions $p$ and $q$ : $\partial^2 g(\mathbf{x})/(\partial x_p \partial x_q)$
Superscript	Description
$\mathbf{A}^{-1}, \mathbf{A}^{-g}$	Inverse of a square matrix $\mathbf{A}$ ; generalized inverse of a (non-square) matrix $\mathbf{A}$
$\mathbf{A}^T, \mathbf{a}^T$	Transpose of a matrix or vector; (includes conjugation for complex numbers)
$\mathbf{a}^T \mathbf{b}, \langle \mathbf{a}   \mathbf{b} \rangle$	Scalar product of two vectors
$a^*$	Conjugate complex
$\mathbf{A}^*$	Conjugate complex and transpose of a matrix
Indexing	Description
$K, L, M, N$	Extension of discrete images in $t, z, y$ , and $x$ directions
$k, l, m, n$	Indices of discrete images in $t, z, y$ , and $x$ directions
$r, s, u, v$	Indices of discrete images in Fourier domain in $t, z, y$ , and $x$ directions
$P$	Number of components in a multichannel image; dimension of a feature space, number of components, pyramid planes or data points
$Q$	Number of quantization levels, number of object classes, or number of regression parameters
$R$	Size of masks for neighborhood operators
$W$	Dimension of an image or feature space
$p, q, w$	Indices of a component in a multichannel image, dimension in an image, quantization level or feature

Function	Description
$\cos(x)$	Cosine function
$\exp(x)$	Exponential function
$\text{ld}(x)$	Logarithmic function to base 2
$\ln(x)$	Logarithmic function to base $e$
$\log(x)$	Logarithmic function to base 10
$\sin(x)$	Sine function
$\text{sinc}(x)$	Sinc function: $\text{sinc}(x) = \sin(\pi x) / (\pi x)$
$\det(\mathbf{G})$	Determinant of a square matrix
$\text{diag}(\mathbf{G})$	Vector with diagonal elements of a square matrix
$\text{trace}(\mathbf{G})$	Trace of a square matrix
$\text{cov}(\mathbf{g})$	Covariance matrix of a random vector
$E(\mathbf{g}), \text{var}(\mathbf{G})$	Expectation (mean value) and variance
Image operators	Description
$\cdot$	Pointwise multiplication of two images
$*$	Convolution
$\star$	Correlation
$\ominus, \oplus$	Morphological erosion and dilation operators
$\circ, \bullet$	Morphological opening and closing operators
$\otimes$	Morphological hit-miss operator
$\vee, \wedge$	Boolean <i>or</i> and <i>and</i> operators
$\cup, \cap$	Union and intersection of sets
$\subset, \subseteq$	Set is subset, subset or equal
$\cup$	Shift operator
$\downarrow_s$	Sample or reduction operator: take only every $s$ th pixel, row, etc.
$\uparrow_s$	Expansion or interpolation operator: increase resolution in every coordinate direction by a factor of $s$ , the new points are interpolated from the available points

Symbol	Definition, [Units]	Meaning
<b>Greek symbols</b>		
$\alpha$	$[\text{m}^{-1}]$	Absorption coefficient
$\beta$	$[\text{m}^{-1}]$	Scattering coefficient
$\delta(x), \delta_n$		Continuous, discrete $\delta$ distribution
$\Delta$	$\sum_{w=1}^W \frac{\partial^2}{\partial x_w^2}$	Laplacian operator
$\epsilon$	[1]	Specific emissivity
$\epsilon$	[m]	Radius of blur disk
$\kappa$	$[\text{m}^{-1}]$	Extinction coefficient, sum of absorption and scattering coefficient
$\nabla$	$\left[ \frac{\partial}{\partial x_1}, \dots, \frac{\partial}{\partial x_W} \right]^T$	Gradient operator
$\lambda$	[m]	Wavelength
$\nu$	$[\text{s}^{-1}]$ , [Hz] (hertz)	Frequency
$\nabla \times$		Rotation operator
$\eta$	$n + i\xi$ , [1]	Complex index of refraction
$\eta$	[1]	Quantum efficiency
$\phi$	[rad], $[\circ]$	Phase shift, phase difference
$\phi_e$	[rad], $[\circ]$	Azimuth angle
$\Phi$	[J/s], [W], $[\text{s}^{-1}]$ , [lm]	Radiant or luminous flux
$\Phi_e, \Phi_p$	[W], $[\text{s}^{-1}]$ , [lm]	Energy-based radiant, photon-based radiant, and luminous flux
$\rho, \rho_{\parallel}, \rho_{\perp}$	[1]	Reflectivity for unpolarized, parallel polarized, and perpendicularly polarized light
$\rho$	$[\text{kg}/\text{m}^3]$	Density
$\sigma_x$		Standard deviation of the random variable $x$
$\sigma$	$5.6696 \cdot 10^{-8} \text{Wm}^{-2}\text{K}^{-4}$	Stefan-Boltzmann constant
$\sigma_s$	$[\text{m}^2]$	Scattering cross-section
$\tau$	[1]	Optical depth (thickness)
$\tau$	[1]	Transmissivity
$\tau$	[s]	Time constant
$\theta$	[rad], $[\circ]$	Angle of incidence
$\theta_b$	[rad], $[\circ]$	Brewster angle (polarizing angle)
$\theta_c$	[rad], $[\circ]$	Critical angle (for total reflection)
$\theta_e$	[rad], $[\circ]$	Polar angle
$\theta_i$	[rad], $[\circ]$	Angle of incidence

*continued on next page*

Symbol	Definition, [Units]	Meaning
<i>continued from previous page</i>		
$\Omega$	[sr] (steradian)	Solid angle
$\omega$	$\omega = 2\pi\nu$ , [s <sup>-1</sup> ], [Hz]	Circular frequency
<b>Roman symbols</b>		
$A$	[m <sup>2</sup> ]	Area
$\mathbf{a}, \mathbf{a}$	$\mathbf{a} = \mathbf{x}_{tt} = \mathbf{u}_t$ , [m/s <sup>2</sup> ]	Acceleration
$\hat{b}(\tilde{\mathbf{k}})$		Transfer function of binomial mask
$\mathbf{B}$	[Vs/m <sup>2</sup> ]	Magnetic field
$\mathcal{B}$		Binomial filter mask
$\mathcal{B}$		Binomial convolution operator
$c$	$2.9979 \cdot 10^8$ ms <sup>-1</sup>	speed of light
$\mathbb{C}$		set of complex numbers
$d$	[m]	Diameter (aperture) of optics, distance
$d'$	[m]	Distance in image space
$\hat{d}(\tilde{\mathbf{k}})$		Transfer function of $\mathbf{D}$
$D$	[m <sup>2</sup> /s]	Diffusion coefficient
$\mathbf{D}$		First-order difference filter mask
$\mathcal{D}$		First-order difference operator
$e$	$1.6022 \cdot 10^{-19}$ As	Elementary electric charge
$e$	$2.718281 \dots$	Base for natural logarithm
$E$	[W/m <sup>2</sup> ], [lm/m <sup>2</sup> ], [lx]	Radiant (irradiance) or luminous (illuminance) incident energy flux density
$\mathbf{E}$	[V/m]	Electric field
$\bar{\mathbf{e}}$	[1]	Unit eigenvector of a matrix
$f, f_e$	[m]	(Effective) focal length of an optical system
$f_b, f_f$	[m]	Back and front focal length
$\mathbf{f}$		Optical flow
$\mathbf{f}$		Feature vector
$F$	[N] (newton)	Force
$\mathbf{G}$		Image matrix
$\mathbf{H}$		General filter mask
$h$	$6.6262 \cdot 10^{-34}$ Js	Planck's constant (action quantum)
$\hbar$	$h/(2\pi)$ [Js]	
$i$	$\sqrt{-1}$	Imaginary unit
$I$	[W/sr], [lm/sr]	Radiant or luminous intensity
$I$	[A]	Electric current
<i>continued on next page</i>		

Symbol	Definition, [Units]	Meaning
<i>continued from previous page</i>		
$I$		Identity matrix
$\mathcal{I}$		Identity operator
$J$		Structure tensor, inertia tensor
$k_B$	$1.3806 \cdot 10^{-23} \text{ J/K}$	Boltzmann constant
$k$	$1/\lambda, [\text{m}^{-1}]$	Magnitude of wave number
$\mathbf{k}$	$[\text{m}^{-1}]$	Wave number (number of wave-lengths per unit length)
$\tilde{k}$	$k\Delta x/\pi$	Wave number normalized to the maximum wave number that can be sampled (Nyquist wave number)
$K_q$	$[\text{l/mol}]$	Quenching constant
$K_r$	$\Phi_v/\Phi_e, [\text{lm/W}]$	Radiation luminous efficiency
$K_s$	$\Phi_v/P [\text{lm/W}]$	Lighting system luminous efficiency
$K_I$	$[1]$	Indicator equilibrium constant
$L$	$[\text{W}/(\text{m}^2\text{sr})], [1/(\text{m}^2\text{sr})], [\text{lm}/(\text{m}^2\text{sr})], [\text{cd}/\text{m}^2]$	Radiant (radiance) or luminous (luminance) flux density per solid angle
$L$		Laplacian filter mask
$\mathcal{L}$		Laplacian operator
$m$	$[\text{kg}]$	Mass
$m$	$[1]$	Magnification of an optical system
$\mathbf{m}$		Feature vector
$M$	$[\text{W}/\text{m}^2], [1/(\text{s m}^2)]$	Excitant radiant energy flux density (excitance, emittance)
$M_e$	$[\text{W}/\text{m}^2]$	Energy-based excitance
$M_p$	$[1/(\text{s m}^2)]$	Photon-based excitance
$\mathbb{M}$		Feature space
$n$	$[1]$	Index of refraction
$n_a$	$[1]$	Numerical aperture of an optical system
$n_f$	$f/d, [1]$	Aperture of an optical system
$\hat{\mathbf{n}}$	$[1]$	Unit vector normal to a surface
$\mathbb{N}$		Set of natural numbers: $\{0, 1, 2, \dots\}$
$p$	$[\text{kg m/s}], [\text{W m}]$	Momentum
$p$	$[\text{N}/\text{m}^2]$	Pressure
pH	$[1]$	pH value, negative logarithm of proton concentration
$Q$	$[\text{Ws}] (\text{joule}), [\text{lm s}]$ number of photons	Radiant or luminous energy
$Q_s$	$[1]$	Scattering efficiency factor
<i>continued on next page</i>		

Symbol	Definition, [Units]	Meaning
<i>continued from previous page</i>		
$r$	[m]	Radius
$\mathbf{r}_{m,n}$	$\mathbf{r}_{m,n} = [m\Delta x, n\Delta y]^T$	Translation vector on grid
$\hat{\mathbf{r}}_{p,q}$	$\hat{\mathbf{r}}_{p,q} = [p/\Delta x, q/\Delta y]^T$	Translation vector on reciprocal grid
$R$	$\Phi/s$ , [A/W]	Responsivity of a radiation detector
$\mathbf{R}$		Box filter mask
$\mathbb{R}$		Set of real numbers
$s$	[A]	Sensor signal
$T$	[K]	Absolute temperature
$t$	[s]	Time
$t$	[1]	Transmittance
$u$	[m/s]	Velocity
$\mathbf{u}$	[m/s]	Velocity vector
$U$	[V]	Voltage, electric potential
$V$	[m <sup>3</sup> ]	Volume
$V(\lambda)$	[lm/W]	Spectral luminous efficacy for photopic human vision
$V'(\lambda)$	[lm/W]	Spectral luminous efficacy for scotopic human vision
$w$	$e^{2\pi i}$	
$w_N$	$\exp(2\pi i/N)$	
$\mathbf{x}$	$[x, y]^T, [x_1, x_2]^T$	Image coordinates in the spatial domain
$\mathbf{X}$	$[X, Y, Z]^T, [X_1, X_2, X_3]^T$	World coordinates
$\mathbb{Z}, \mathbb{Z}^+$		Set of integers, positive integers



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