Chapter 11

SUSTAINABLE CYBERNETICS SYSTEMS

Backbones of Ambient Intelligent Environments

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Introduction

In artificial intelligence, cognitive science and cognitive engineering a main challenge for the next decades will be building sustainable cybernetic systems that can individually and/or collectively anticipate and attend to their own or environmental dynamics. The anticipation and attention measures foreseen and taken by cybernetic systems will determine decisively whether they can achieve their goals and accommodate their own and environmental changes. The current abundance and omnipresence of ICT architectures and infrastructures enable ubiquitous, pervasive, sentient, and ambient intelligent computing, communication, cooperation and competition of both artificial and societal organizations and structures. However, they appear to us as merely nice to haves in a rather unstructured and unorganized ICT infrastructure. In general they still lack cognitive engineering capabilities, namely those for anticipation and selection of attention. Instead of perpetually handcrafting standalone ICT infrastructures and integrating them, smart human-system network interaction paradigms are needed such that cybernetic systems can continuously select and embody (after reinforcement learning) suitable anticipatory and selection of attention schemes to bring those novel integrated ICT architectures and infrastructures to life. In short new paradigms for co-existence and co-evolution of humans, machines and their extensions are needed in order to simultaneously sustain both types of schemes. In this article we present a mathematical-physical framework that allows to model and deploy sustainable cybernetic systems.

A (sustainable) cybernetic system should realize its current states in terms of physical structures and organizations by taking into account, besides its past and present states, also its foreseen potential future states that can lead to the highest chance of fulfilling its current and future goals. Such states are embedded and predicted by the system itself and/or enforced and communicated by its environment. Thus a cybernetic system should instruct itself to restructure and to reorganize itself in order to maximally achieve its own goals whatever that maybe. The goals in turn may be in line with constraints and opportunities put forward by such a system itself or by its environment. In this way a self-organization of the cybernetic system comes about that may guarantee the system's sustainability despite its possibly predestined own and environmental evolutionary dynamics [4]. The explorative and goal-directed behavior of a cybernetic system then displays itself not only as (reinforcement) learning, understanding and assessment of the system itself and its environment, but also as a functional re-organization and physical restructuring of a large number of its (imaginary) current and future states and/or organizations. Summarising continuous self-constrained functional reorganization and physical restructuring is a necessity - given its objectives/goals/tasks, internal and external states, and constrained and/or engendered by its co-evolving environment. Thus the question rises what causes self-structuring/assembly and self-organization of a cybernetic system and how can such a system embody that itself.

As stated, cybernetic systems should be endowed with anticipatory and attentive capabilities and capacities in order to tackle and cope with their own internal and environmental evolutionary pressures. The viability and sustainability of a cybernetic system in those aspects can be assessed on the basis of so-called fitness or intelligence measures for the anticipatory and selection of attention schemes, respectively, in dealing with dynamical changes of the system itself and its environment [4]. Up to today determining and making explicit proper fitness and intelligence measures are outstanding cybernetics problems with respect to self-organization and natural selection that are hardly ever satisfactory addressed. Fitness measures here we define as measures for the intertwining, entanglement and entrainment of (non-) local structures and/or organizations of the cybernetic system and those of its environment. Intelligence measures refer to the anticipatory and attentive (non-) local potentials (in solving existing, hidden and rising problems), embodied in the structure and the organization of the cybernetic system, relative to those potentials in its environment. Thus all this asks for smart anticipatory and attentive cybernetic systems that can simultaneously counterbalance or fuse both the phenotypic (evolutionary) and genotypic (stationary) dynamics of both the system itself and its environment. Therewith our initial problem of self-structuring and self-organization by the cybernetic system itself can be rephrased in terms of what are and how to embody/embed proper fitness and intelligence measures for anticipatory and attentive purposes.

Many scientists have proposed to define the above fitness and intelligence measures in terms of indicators or utility measures for self-organized critical states of social, biological, physical and ICT networks. Such indicators or utility measures relate to scaling laws (self-similarity) and symmetry breaking mechanisms of punctuated and far-off equilibrium network dynamics, respectively. These physical laws and mechanisms spell out which strategies are the most valuable ones that co-evolving systems could follow and apply during phases of self-organization and natural selection of anticipation and selection of attention schemes.

Having mastered such physical laws and mechanisms a cybernetic system should, besides anticipate, also know why, when and how to capture, to direct and to change attention towards relevant dynamical phenotypic and genotypic issues while enacting on itself, its collective and/or its environment. In this respect a cybernetic system should allow for the emergence of smart anticipatory and selection of attention schemes at appropriate spatio-temporal and dynamic scales. Of course, the latter schemes should couple to inference and association schemes that in turn are indispensable during anticipatory cycles, which certainly also include the common (explorative and intentional) perception-decision-action cycles occurring at lower dynamic evolutionary scales. Summarizing anticipatory or selection of attention (pre-) schemes of cybernetic systems together with their fitness and intelligence measures should upon enaction allow for the emergence of hierarchies of relevant niches for their own and environmental dynamics. Nevertheless, the question remains how to effectively embed and embody those schemes in networks of artificial systems, humans or their extensions.

How to realize (pre-) schemes for natural anticipation and selection of attention (NASA) in cybernetic systems is a problem that is hardly ever satisfactory tackled in computer and cognitive science. However, Ilya Prigogine in his Nobel Lecture [2] touches upon the fundamental perception-decision-action problem in science. Furthermore,Roger Penrose and Stuart Hameroff in their seminal work [8] dare to explain consciousness as a nonlocal quantum phenomenon in the brain. They emphasize the importance of first unraveling the relevant physical laws and mechanisms involved in cybernetic systems before one even can think of reaching any sensible levels of consciousness (awareness), understanding or intelligence.

Obviously, the fact that the evolution of a cybernetic system and its environment are difficult to access or to predict, makes a faithful structural embedding and functional embodiment of this natural system into a cybernetic system a real tour de force [3, 10]. Actually the perception-decision-action problem for cybernetic systems cannot be disentangled from the following more fundamental physical problems:

- Problem of measurement, calibration and gauging of an evolving cybernetic system: it relates to the problems of natural anticipation and selection of attention, causality, learning.

- Problem of complexity versus resolution of an evolving cybernetic systems: it relates to the problems of disorder versus order,predictability, renormalisation and sustainability (including scalability).

The above perception-decision-action problem has been adressed separately for one or a pair of modalities from a mono- or multi-disciplinary perspective. In [5, 10, 12] we proposed a mathematical physics framework that supports development and deployment of sustainable cybernetic systems. Our approach distinguishes itself from the mono- or multi-disciplinary approaches in the sense that the statistical physics geometry of the interacting environment, user and system are conceptually as well as data-driven physics-based. The other approaches advocate representing e.g. spatio- temporal ordering relations and derived geometric properties in terms of heuristic Euclidean invariant measures. Such measures are generally totally inadequate to capture in a robust and reliable way the statistical physics and geometry underlying interacting sustainable intelligent multimodal systems (SIMS) [13]. Furthermore, such approaches do not address possible coupling and associative (pre-) schemes between multimodalities. Our framework does not only allow robust and reliable modeling of complex systems, but also sustains acquisition of natural anticipation and selection of attention (pre-) schemes needed during co-evolution of humans and systems.

Intelligent agent systems are indispensable to create and sustain NASA (pre-) schemes. Specialization and leverage of these (pre-) schemes can be realized by collective intelligent human and software agent systems (CIA) [9]. At higher levels of network complexity similar systems can be posited for such purposes. Organizations, groups, individuals, ICT and knowledge systems all have limited capacities and capabilities. They need to free time and resources for differentiating, diversifying and integrating information and knowledge.

This chapter is organized as follows. In section 1 we present our mathematical-physical framework for encoding interplay and evolution of natural and cybernetic systems. In section 2 we show how to sustain natural and cybernetic systems by means of collective intelligent agent system architectures. We define measures for sustainability - the true measures for ambient intelligence - as topological currents supporting co-evolving (symbiotic) natural and cybernetic systems.

1. Encoding Interplay and Co-Evolution

In the sequel we present our mathematical physical framework to encode natural and cybernetic systems. In Section 1.1 we consider the machines of a natural system relevant for encoding a cybernetic system, whereas in Section 1.2 we consider those for co-evolving natural and cybernetic systems.

1.1 Encoding Interplay between Natural and Cybernetic Systems

Let us model an encoding I of an observable as a transduction (with losses) of a vector-valued current m representing a natural system M onto a vector-valued current n representing an (induced) cybernetic system N, and vice versa:

DEFINITION 11.1 *An encoding I of natural system M onto cybernetic system N is defined by a transduction I:*

$$I : M \rightarrow N; \ I(m) = n.$$

Each current is a density field for a physical system either being a natural or cybernetic system. The cybernetic system current is a superposition of several currents representing or consistent with a natural system and a dissipation current. Note that, the number of components of a cybernetic system will normally be less than that of a natural system. Under encoding there always occurs a high level of structural and functional abstraction that is controlled by natural and cybernetic system requirements. Furthermore, the problem of optimal transduction coincides with the perception-decision-action problem.

Now an encoding of other observables derived from observable m follows upon finding a set of so-called equivalences of encoding (Definition 11.1) that is invariant under a gauge group:

DEFINITION 11.2 *A gauge group G on encoding is a set of transformations leaving invariant the encoding (Definition 11.1) of the natural system M, the transduction I and the cybernetic system N.*

The gauge group related to the encoding is normally given by a system of partial integro-differential equations with initial-boundary value conditions on *I*, *M* and *N*. In Section 1.2 we make the gauge group explicit as the evolution of the natural system. We observe that this group and the evolution of a natural system cannot be disentangled. Thus observables do not have any meaning before having that evolution empirically derived and verified. Furthermore, that a gauge group does not have to satisfy at all the standard group properties as the existence of an inverse. For example, if the gauge group is covered by a renormalisation group that in turn is defined by a particular diffusion equation, then the flow is uniquely directed with no possibility of return or defining an inverse. Therefore, it might be more suitable to rename in the future a gauge group into a gauge renormalisation functor category. Again in Section 1.2 and in Section 2 we elaborate on the implications of these issues; in the sequel we just assume that a gauge group possesses all the group properties.

Now the question arises how to derive a set of equivalences of encoding (Definition 11.1) invariant under a gauge group. There exist in the mathematical literature several rich methods to systematically retrieve such a set of equivalences based on (for references see [10]:

- Differential and integral geometry,
- Lie group theory applied to system of partial integro-differential equations with initial-boundary value conditions,
- Algebraic and differential topology.

In the following we briefly summarise the differential and integral geometric method for obtaining such a set of equivalences. For applications of this method to computer vision problems the reader is referred to [5, 6, 12]: the mathematical-physical objects referred to in the sequel are therein made explicit Analogously the same methods can be applied to multimodal interaction and other problems [13]. However, in Section 1.2 we will see that only a set of equivalences consistent with an encoding of the evolution of a natural system can come about upon application of the other methods as well.

A set of equivalences of the encoding (Definition 11.1) invariant under a gauge group (Definition 11.2) come about after setting up a (co)frame field, metric and/or connection invariant under the gauge group.

Let us make explicit the structure and function of these physical objects in a cybernetic system.

DEFINITION 11.3 *A frame field* (ϕ_{p}) *is a section of the tangent bundle T of encoding (Definition 11.1).*

By exponentiating the frame vector field ϕ_p one obtains a parametrisation or labelling of the natural system *M*, the transduction *I* and the cybernetic system *N*.

DEFINITION 11.4 *A coframe field* (dv^{p}) *is a section of the cotangent bundle T* of encoding (definition 11.1).*

The coframe field in combination with the frame field allow the measurement of actions of the gauge group (Definition 11.3) on the encoding (Definition 11.1).
Frame field (Definition 11.3) and coframe field (Definition 11.4) then satisfy not necessarily a duality constraint:

DEFINITION 11.5 *A frame field (Definition 11.3) and a coframe field (Definition 11.4) of the encoding (Definition 11.1) are their duals, if and only if:*

$$d\phi^p(\phi_q) = \delta^p_q,$$

where δ *is the Kronecker delta-function.*

DEFINITION 11.6 *A metric tensor* γ *is a (non-degenerate) bilinear form on* $T \times T$ *such that*

$$\gamma : T \times T \to \mathrm{K};\ \ \gamma(\phi_{\mathrm{p}}, \phi_{\mathrm{q}}) = \gamma_{\mathrm{pq}},$$

where γ_{pq} *are the components of the metric tensor, and K is the field of numbers measuring distances and angles of actions of the gauge group (Definition 11.2).*

A metric not only allows to measure classical notions of distance and angle such as Euclidean distance between points. Differences in other more complex gauge group actions related to e.g. energy or vector potentials can also be read out.

DEFINITION 11.7 *A connection* Γ *on encoding (Definition 11.1) is defined by one-forms* o^q_p *on its tangent bundle T:*

$$\nabla^\Gamma : T \to T \times T^*; \nabla^\Gamma \phi_p = \omega^q_p \otimes \phi_q,\ \ \omega^q_p(v_r) \in \mathrm{K},$$

where $\otimes$ *denotes the tensor product, and* ∇^Γ *is the covariant derivative on encoding (Definition 11.1), and* K *a field of scalar numbers representing physical observations related to gauge group actions in (Definition 11.2).*

The metric (Definition 11.6) and the connection (Definition 11.7) are in general assumed to be compatible with each other.

DEFINITION 11.8 *The metric (Definition 11.6) and the connection (Definition 11.5) are compatible if and only if:*

$$\nabla^\Gamma g = 0.$$

This means that, e.g., the angles between and lengths of vectors measured by the metric tensor under the parallel transport associated with the connection are being preserved. Note that, as will become clear shortly, it is not necessary to require the existence of a so-called metric connection (connection is fully determined by a metric; connection coefficients expressible in terms of derivatives of the metric components with respect to the frame fields) to derive a suitable set of equivalences of the encoding (Definition 11.1). The stipulation of a connection or a Lie derivative is in general necessary and sufficient.

In order to categorise encoding (Definition 11.1) so-called curvatures of frame fields in (Definition 11.3) are read out.

DEFINITION 11.9 *The curvature Φ_i of a frame vector field ϕ_i in (Definition 11.3) at point p on a two-dimensional surface S parametrised by frame field (Definition 11.3) is defined by:*

$$\Phi_i(p, S) = \oint_C \nabla^\Gamma \phi_i,$$

where the sense of traversing circuit $p \in C$ is chosen such that the interior of the circuit C on surface S is to its left.

Using Stokes' theorem curvature (Definition 11.9) can be expressed as:

$$\Phi_i(p, S) = \int_{C^\circ \subset S} \nabla^\Gamma \wedge \nabla^\Gamma \phi_i = \int_{C^\circ \subset S} \Omega_i^j \phi_j,$$

where C° is the interior of the circuit C in S, $\nabla^\Gamma \wedge$ the covariant exterior derivative in which $\wedge$ is the wedge product consistent with metric (Definition 11.6) and/or connection (Definition 11.7), and Ω_i^j represent the so-called curvature two-forms. These curvature two-forms measure the inhomogeneity of the gauge group actions (Definition 11.2). They are quite common in differential geometry, defect theory, gauge field theory and general relativity. Only lately theur were introduced in computer vision and cybernetics in general [5, 13]. Note that they should not be confused with the curvatures that appear as invariant functions (zero-forms) under a gauge group in equivalence problems concerning, e.g., planar curves under the group of Euclidean movements. Actually they form the desired topological charges, potentials and curvatures for pinpointing down the structure and function of natural system, transduction and cybernetic system.

From these curvatures we can in turn derive higher order curvatures $\Phi_{i;j_1 \ldots j_k}$ by taking successively covariant derivatives $\nabla_{v_{j_l}}$ with respect to frame vector fields ϕ_{j_l}. Together they form the first set of locally and directionally equivalences that quantify encoding (Definition 11.1).

EQUIVALENCE 1 *The set of directional equivalences of encoding (Definition 11.1) is given by:*

$$\Phi_{i;j_1 \ldots j_k} = \nabla^\Gamma_{\phi_{j_k}} \cdot \ldots \cdot \nabla^\Gamma_{\phi_{j_1}} \Phi_i,$$

where ; indicates taking a covariant derivative.

If there are symmetries underlying the encoding, then it is worthwhile to try to find the irreducible equivalences [5]. (Equivalence 1) allows the quantification of the inhomogeneity or coherence of the encoding (Definition 11.1): it allows even to bridge the heterogeneity in encodings.

If we consider a set of circuits, *{C}*, on a set of related surfaces, *{S}*, through point p, then (Equivalence 1) at p satisfies obviously a local conservation law (superposition principle) such that the directional information will become obsolete.

EQUIVALENCE 2 *A set of local equivalences of encoding (Definition 11.1) is given by:*

$$\bar{\Phi}_{i;j_1\dots j_k}(p,\{S\}) = \sum_{\{S\}} \Phi_{i;j_1\dots j_k}(p,S),$$

being total curvatures of the vector fields ϕ_i in frame field (Definition 11.3) over the set of all surfaces, {S}, each of which contains one corresponding circuit C, through point p.

These local equivalences explain the problem of describing, for example, encodings solely on the basis of a local analysis. A local analysis, namely, maps a sequence of directionally equivalences onto one number. Recall, that point p is an initial point on C and that this point and the diametrically located end-point of C in curvature (Definition 11.10) determine actually a direction.

(Equivalence 2) can be complemented by a set of global equivalences for a region U of the encoding (Definition 11.1).

EQUIVALENCE 3 *A set of global equivalences of encoding (Definition 11.1) is given by:*

$$\tilde{\Phi}_{i;j_1\dots j_k}(\{S\},U) = \int_U \bar{\Phi}_{i;j_1\dots j_k}(p,\{S\})dU,$$

in which U is a region on N not necessarily of constant dimension nor simply connected to point p.

Up to now we unravelled only sets of local and directionally equivalences over either simply or multiply connected regions. The question arises what are possible interactions between such currents?

First of all there would be a gauge group transformation needed to bring the (co)frame fields and connections at two (more than two) positions in line. The latter gauging will be noticeable for an external observer. An internal observer, the point of view chosen in our framework, will not be aware of this action as he or she is falling freely along some kind of geodesic. Now we are in the position to carry out the comparison by establishing a set of joint equivalences, comparable to semi-differential, multi-local, simultaneous or joint (differential) invariants [5, 12].

Assuming the considered gauge group to be living on the whole encoding (if different gauge groups would be applicable over subencodings the analysis does not deviate) the most immediate construction follows upon computing the structure functions for a multiple point set of morphisms. The latter set should not be confused with the gauge group. The elements of this set of transformations are generated by a set of gauge invariant propagators:

$$\Psi_{pj_1\ldots j_k} = \Phi^{p}_{i;j_1\ldots j_k} \nabla^{\Gamma(p)}_{\phi_i(p)},$$

where p labels a selected point. The Lie algebra of this set of propagators determines the sought set of joint equivalences.

EQUIVALENCE 4 *A set of joint equivalences of encoding (Definition 11.1) is given by the structure functions* Φ^r_{pq}*:*

$$[\Psi_p, \Psi_q] = \Phi^r_{pq}\Psi_r,$$

where Ψ*'s are gauge invariant propagators.*

Note that ratios or differences of the components of the aforementioned equivalences are also joint equivalences but can be retained by simple algebra on the set of local and directional equivalences.

Up to now we have only considered circuits *C*. Applying homotopy and homology theory it is clear that among the set of local and directional equivalences there are also topological invariants such as winding numbers for higher order groups of homotopy, and Betti-numbers for higher order groups of cohomology. These invariants require the analysis of a normalised hypervolume-form on a hypersphere rather than a circle *C* and higher order differential forms both invariant under the gauge group (Definition 11.2) and consistent with encoding (Definition 11.1). For recent applications of homotopy and homology theory to computer vision and cybernetics the reader is referred to [5, 7, 12, 13].

More importantly, in establishing sets of local, global and joint equivalences we restricted ourselves to a mere integration of comparable structural equivalences over an object or to some multi-local interaction between them, respectively. Being aware of (self)-interactions in a natural system one would like in addition to retrieve a set of functional equivalences of encoding (Definition 11.1). Among these equivalences are (generalised) Vassiliev invariants, linking number and Möbius energies for knots, links, braids and more general CW-complexes but as will see also co-evolving systems.

EQUIVALENCE 5 *A set of functional equivalences* $\tilde{V}$ *of encoding (Definition 11.1) are (generalised) Vassiliev invariants, linking numbers and Möbius energies of CW-complexes.*

In Section 1.2 these and the other type of equivalences appear to be just (topological) invariants of specific homotopy and cohomology groups.

An important property of a physical object or process *F* contained in encoding (Definition 11.1) is its invariance under the gauge group (Definition 11.2).

DEFINITION 11.10 *A physical object or process F, consistent with encoding (Definition 11.1), is invariant under gauge group (Definition 11.2) if and only if:*

$$GF = F.$$

All above mentioned physical objects and (either reversibel or irreversible) processes are by definition unaffected by the corresponding gauge group.

THEOREM 11.11 *The frame field (Definition 11.3), coframe field (Definition 11.4), metric (Definition 11.5), connection (Definition 11.6), and equivalences (Equivalence 1), (Equivalence 2), (Equivalence 3), (Equivalence 4) and (Equivalence 5) are invariant under gauge group (Definition 11.2).*

PROOF 1 *Proofs follow simply by applying gauge group to the induced encoding (Definition 11.1), computing the geometric objects and comparing them with the initial ones.*

Note that in gauge field theories one normally defines a gauge transformation as an inhomogeneous invertible linear transformation of the frame field. This transformations leads subsequently to a transformation of the gauge fields, i.e. the connection one-forms. The field strengths of the gauge fields, i.e. the curvature two-form valued vector fields, behave then covariantly. The only equivalences of the encoding numerically invariant under this gauge group are topological invariants that correspond to the Jordan normal forms of the matrix representations of the curvature two-forms. These topological invariants concisely represent groups. The operationalisation, i.e. quantification and qualification, of the functor category of such groups plays an important role in the actual encoding of the evolution of a natural system (see Section 1.2). Also we come back therein why we don't make a distinction between reversible and irreversible process.

Summarising, we have presented a method for encoding the interplay among cybernetic and natural systems in terms of equivalences corresponding to a frame field, coframe field, metric and connection invariant under a gauge group. This method allows us to retrieve local, multi-local, nonlocal, global and topological information about the encoded observable and evolution of the encoding. The latter information can be of help in order to steer policies and ensure the sustainability of the natural system as well as the cybernetic system. The observables and their equivalences invariant under the gauge group is fully determined by the evolution of the natural system. This implies also that the gauge group to be considered is intimately related to that evolution. Moreover, that the analysis of the evolution of the natural system in terms of measures not invariant under a gauge group consistent with this evolution cannot be of any use: they can never form the proper reference variables or indicators for sustainable cybernetic systems. This principle of gauge invariance or equivalence is by the way at the very heart of any relativity or renormalisation theory and forms an integral part in the encoding of the co-evolution of a natural system and a cybernetic system in the next section.

1.2 Encoding Co-Evolution of Natural and Cybernetic Systems

In cybernetic system theory it is common use during modeling phase of the natural or artificial system to postulate a posteori some hypotheses concerning its dimensionality and the laws underlying its evolution. One fixes the number of independent and dependent variables or observables, and the evolutionary laws expressed in these observables. We like to coin latter systems as closed. Next to these closed systems are open systems that are characterised over time by varying dimensionality and changing evolution laws. Closed systems are like their simulations reactive and very rigid in their formulation, whereas open systems are characterised by fuzzy anticipatory and predictive controls of their states and laws. Nevertheless, an open system can be covered by a closed system during a certain period.

Let us give some examples to illustrate the above classification of natural and cybernetic systems.

EXAMPLE 11.12 *Meteorological and climatological systems are in general modelled as closed. The systems are represented by a fixed set ofobservables governed by a fixed set ofconservation laws normally represented by a deterministic system of PDES with initial-boundary conditions.*

EXAMPLE 11.13 *Anthropological systems such as those for politics, economy and technology are very open. The economical system over time shows emergence and annihilation ofobservables such as currencies, and ofevolutionary lawsfor them. E.g.,one has left the gold standardfor the dollar as reference currency. Furthermore, the trading rules with respect to derivates have become ever more intricate due to changes in market rules and in legislation.*

EXAMPLE 11.14 *Biological systems are from the very onset designed to be open and in particular to be adaptive to changing environmental conditions and to become anticipatory and thus predictive both by learning to ensure persistence of the systems themselves.*

In closed systems the notion of the observables is clear, whereas for open ones they seem to become more and more obscure due to fact that the evolution laws also change over time. However, over time also open systems display changes in the encoding. These changes are caused by a disaggregation of the observables and the deterministic system as a whole, and by a true evolution of the deterministic system from one to another.

In Section 1.2.0 we explain how to arrive at a functor category of evolutions that can serve as framework for encoding the evolution of natural and cybernetic systems. In Section 1.2.0 we propose to use this framework to quantify a path of natural transformations between different evolutionary phases of the system, analogously a renormalisation theory in theoretical physics does with the difference that we are not forced to choose a group-like relation between the evolutionary phases. Structural and functional transitions in the dynamics are very well possible and are in sustainable systems certainly to be welcomed.

For the proofs of the theorems and alike in the remainder of this section the reader is kindly referred to the listed references in [10].

Preliminaries. In the sequel we merely briefly point out how to arrive at a functor categorification of co-evolving natural and cybernetic systems. In this context the decision problem in topology plays a central role. This problem concerns whether or not two topological spaces are homeomorphic, i.e. whether there exists an invertible one-to-one and onto mapping between them that is continuous. This problem can be solved by associating topological invariants (groups) to such spaces and comparing them. This solution to the decision problem is based on a so-called decategorification, i.e. forgetting about morphisms or in this case homeomorphisms ofthe topological spaces. We will observe that there are different categorifications possible, namely as homotopy groups and cohomology groups. But we will also realise that these categorifications, functors, can constitute objects in a functor category with natural transformations between them as morphisms. Finally, we carry this process of categorification over onto that of co-evolving natural and cybernetic systems.

Let us first start dwelling on the notion of a category.

DEFINITION 11.15 *A category consists of objects and morphisms between the objects.*

A category of all topological spaces consists of objects, i.e. all topological spaces M belonging to a particular type of mathematical universe U, and of morphisms, i.e. all continuous mappings f, of pairs $M, N \in U$ from M to N. Besides this category in the sequel also a category of all groups G in U is essential in the process of encoding evolution. It consists of individual groups as objects and group homomorphisms f^{***} between the groups as morphisms.

Categories can be studied by means of structure preserving mappings between them called functors.

DEFINITION 11.16 *Let $\mathcal{A}$ and $\mathcal{B}$ be categories. $F : \mathcal{A} \to \mathcal{B}$ is called a covariant functor, if $F(a, f^a) = (b, f^b)$ with $f^b = F(f^a) : F(A) \to F(B)$ and F respects the rule of composition of mappings: $f^{cb} f^{ba} : A \to C$ implies $F(f^{cb} f^{ba}) = F(f^{cb}) F(f^{ba}) : F(A) \to F(C)$ and identity mapping $F : A \to A$ implies identity mapping $F(f) : F(A) \to F(A)$.*

Analogously one defines a contravariant functor $\tilde{F} : B \to A$.

In order to relate and to compare functors F_1 and F_2 one uses the notion of natural equivalence and natural transformation.

DEFINITION 11.17 *Let $\mathcal{A}$ and $\mathcal{B}$ be categories. Functors $F_1, F_2 : \mathcal{A} \to \mathcal{B}$ are naturally equivalent if for all pair of objects (A, B) in category pair $(\mathcal{A}, \mathcal{B})$ there exists isomorphisms $\psi(A, B)$ and $\phi(A, B)$ in category $\mathcal{B}$, such that $\phi(A, B)(F_1(f)) = F_2(f)(\psi(A, B))$.*

DEFINITION 11.18 *Let $\mathcal{A}$ and $\mathcal{B}$ be categories. Functors $F_1, F_2 : \mathcal{A} \to \mathcal{B}$ are naturally transformations of one another if for all pair of objects (A, B) in category pair $(\mathcal{A}, \mathcal{B})$ there exists transformations, i.e. no isomorphisms, $\psi(A, B)$ and $\phi(A, B)$ in category $\mathcal{B}$, such that $\phi(A, B)(F_1(f)) = F_2(f)(\psi(A, B))$.*

Thus by means of the natural equivalence or transformation defined by the set $\{(\psi, \phi)\}$ it is possible to map the encoding of properties of the functor F_1 into those of functor F_2.

Now we are ready to define the functor category on the basis of the set of functors F_1 and F_2 from category $\mathcal{A}$ to category $\mathcal{B}$ and the set of all natural transformations from functor F_1 to functor F_2.

DEFINITION 11.19 *The functor category consists of functors as objects and natural transformations as morphisms.*

Categorification follows upon adding to the topological spaces or groups the morphisms between them. It is based on the analogy between sets and categories. There is the analogy of equations between elements and isomorphisms between objects, sets and categories, functions and functors, and equations between functions and natural transformations between functors.

In the next paragraphs we illustrate the process of functor categorification for higher order homotopy groups and higher order cohomology groups of topological spaces. We will see that these groups can be decategorified by a system of winding numbers and a set of Betti numbers, respectively.

Higher order homotopy groups. In order to define equivalences of a topological space Y the homotopy of mappings between a topological space X and Y is essential.

DEFINITION 11.20 *Two continuous mappings*

$$f, g : X \to Y$$

from topological space X onto Y are homotopic,

$$f \sim g,$$

if there exists a mapping

$$F : X \times [0, 1] \to Y,$$

such that

$$F(x, 0) = f(x), \;\; F(x, 1) = g(x).$$

The space of the equivalence classes of mappings is denoted by $[X, Y]$. Obviously, here, X serves as some reference space in order to assess structures of Y.

The most important homotopy groups are the fundamental group and higher order homotopy groups of a n-dimensional manifold $Y = M$. In order to define these groups we need first a notion of loops X and next that of a product of loops.

DEFINITION 11.21 *A k-loop is a mapping l of cube I^k in M that maps the boundary ∂I^k on a fixed point $x_0 \in M$:*

$$\begin{aligned} l(t) &\in M, \;\; t_i \in [0,1] \\ l(t) &= x_0, \;\; t_i = 0, 1. \end{aligned}$$

Note that, homotopy of k-loops is a conjugacy of k-loops, i,e. there exists a continuous mapping ψ of the k-cube I^k on the space of k-loops l such that two k-loops l and l^* are joined by ψ with $\psi(t_i = 0) = l$ and $\psi(t_i = 1) = l^*$.

DEFINITION 11.22 *The product of two k-loops l and m is defined by:*

$$(l \cdot m)(t) = \begin{cases} l(2t_1, \ldots, t_k) & t_1 \in [0, \frac{1}{2}] \\ m(2t_1 - 1, \ldots, t_k) & t_1 \in [\frac{1}{2}, 1] \end{cases}$$

A unit element, associativity and an inverse under the above multiplication rule for k-loops are now readily defined. Furthermore, the set of homotopy classes of k-loops with this multiplication $[l] \cdot [m] = [l \cdot m]$ can easily be proven to form a group. Thus we are in the position to define the higher order homotopy groups.

DEFINITION 11.23 *The k-th order homotopy group $\pi_k(l(x_0), M)$ is the set of homotopy classes of k-loops with multiplication $[l] \cdot [m] = [l \cdot m]$.*

If topological space M is simply connected, then it can be shown that $\pi_k(l(x_0), M) = \pi_k(l(x_1), M), \forall x_0, x_1 \in M$. Furthermore, that $\pi_k(l(x_0), M_1 \times M_2) = \pi_k(l_1(x_0), M_1) \times \pi_k(l_2(x_0), M_2)$.

It's straightforward to define whenever two spaces are homotopic and which conditions have to be satisfied.

DEFINITION 11.24 *Two spaces M and N are of the same homotopy type, $M \sim N$, if there exist mappings $f : M \rightarrow N$ and $g : N \rightarrow M$ such that the compositions $f \odot g \sim id_N$ and $g \odot f \sim id_M$.*

If this homotopy holds it's readily proven that $\pi_k(l_m, M) = \pi_k(l_n, N)$.

The higher order homotopy groups $\pi_k(l(x_0), M)$ can be decategorified in terms of winding numbers.

DEFINITION 11.25 *With the k-th order homotopy groups $\pi_k(l(x_0), M)$ are associated winding numbers ν_k of mappings $f : S^k \rightarrow M$ defined by:*

$$\nu(f) = \int_{S^k} f^* \omega,$$

where S^k a k-sphere, ω is a normalised volume-form on S^k and f^* is the pull-back of the mapping f.

Note that these winding numbers for the higher order homotopy groups are just a small subset of the class of functional equivalences (Equivalence 5) defined in Section 1.1. For applications of such a decategorification within the realm of computer vision the interested reader should turn to [5, 7, 12].

Higher order cohomology groups. In order to retrieve topological invariants of a manifold M one may study function spaces on M and in particular the (de Rham) cohomology groups on M. In order to make these cohomology groups explicit we need first to define the space of closed and exact k-forms:

DEFINITION 11.26 *The space* $Z^k(M)$ *of closed k-forms* $\alpha \in \Omega^k(M)$ *on manifold* M *is defined by:*

$$Z^k(M) = \left\{\alpha \in \Omega^k \| d\alpha = 0\right\},,$$

where d *denotes the ordinary exterior derivative operator.*

In applications we cannot restrict ourselves to these ordinary derivatives; adaptation of the derivatives and the function spaces considered on the manifold M to the considered gauge group (Definition 11.2) becomes really indispensable.

DEFINITION 11.27 *The space* $B^k(M)$ *of exact k-forms* $\alpha \in \Omega^k(M)$ *on manifold M is defined by:*

$$B^k(M) = \left\{\alpha \in \Omega^k(M) \middle\| \exists\beta\Omega^{k-1}(M)\alpha = d\beta\right\}.$$

Obviously, here, exactness implies closedness.

Although every closed form can be locally written as an exact form there exist global topological obstructions determined by the cohomology groups.

DEFINITION 11.28 *The cohomology groups* $H^k(M); k = 0, \dots, n$ *on manifold M is defined by:*

$$H^k(M) = Z^k(M)/B^k(M).$$

Thus with each $\alpha \in Z^k(M)$ is associated an equivalence class $[\alpha] \in H^k(M)$ with $[\alpha] = [\alpha + d\beta], \beta\Omega^{k-1}(M)$.

Now the cohomology groups $H^k(M)$ are topological invariants of manifold M:

THEOREM 11.29 *If* $\phi : M \to N$ *a diffeomorphism, then* $H^k(M) \sim H^k(N)$.

Note that every closed form can locally be written as an exact form on the basis of the cohomology ring $H^*(M) = \bigoplus_{k=0}^n H^k(M)$.

The topological invariants of a compact manifold M can subsequently be represented by Betti-numbers.

DEFINITION 11.30 *The Betti-numbers* b_k *of a manifold M are defined by:*

$$b_k = \dim H^k(M).$$

For example, b_0 measures the number of path-connected components of M. If $b_0 = b_n = 1$, then the manifold M is simply connected as well as orientable. In that case the other Betti-numbers are related to each other through the Poincaé duality: $b_k = b_{n-k}$. Moreover, we can define the so called Euler characteristic.

DEFINITION 11.31 *The Euler characteristic* χ *of manifold M is defined by:*

$$\chi(M) = \sum_{k=0}^{n}(-1)^k b_k.$$

The latter topological information can be cast into the Poincaré polynomial $P_t(M) = \sum_{k=0}^{n} b_k t^k$,i.e. $\chi(M) = P_{-1}(M)$. For this polynomial the so-called Künneth formula holds: $P_t(M \times N) = P_t(M)P_t(N)$ for a product-space of manifolds.

For applications of this kind of decategorification in computer vision or cybernetics in general the interested reader may turn to [5].

Functor Category. In the sequel we show that the homotopy and cohomology groups are generated by a pair of corresponding functors on the category of topological spaces. Moreover, we will observe that this pair of functors are related through a natural transformation.

Homeomorphic topological spaces can be associated isomorphic higher order homotopy groups.

THEOREM 11.32 *If the k-th order homotopy group $\pi_k(M)$ on topological space M is isomorphic with the k-th order homotopy group $\pi_k(N)$ on topological space N, then M is homeomorphic with N (Necessary condition).*

If f a homeomorphims between M and N, then $f^* = \pi_k(f)$ is a group homomorphism between $\pi_k(M)$ and $\pi_k(N)$. Here π_k are functors associating with any topological space its corresponding higher order homotopy group $\pi_k(M)$. Summarising, one has a conjugacy relation f^* between higher order homotopy groups that are associated to topological spaces that are related to each other through a homeomorphism f. Note that, this conjugacy differs from the conjugacy of k-loops mentioned above.

Similarly, homeomorphic topological spaces can be associated isomorphic cohomology groups.

THEOREM 11.33 *If the k-th order cohomology group $H^k(M)$ on topological space M is isomorphic with the k-th order cohomology group $H^k(N)$ on topological space N, then M is homeomorphic with N (Necessary condition).*

If f a homeomorphims between M and N, then $f^{**} = H^k(f)$ is a group homomorphism between $H^k(M)$ and $H^k(N)$. Here H^k are functors associating with any topological space their corresponding higher order cohomology groups. Summarising, one has a conjugacy relation f^{**} between higher order cohomology groups that are associated to topological spaces that are related to each other through a homeomorphism f.

Thus rules π_k and H^k are both covariant functors of the category of topological spaces into the category of groups. More precisely, topological spaces are associated with groups and homeomorphisms with group homomorphisms by both functors. The above pair of functors and the natural transformation f^{***} between them constitutes an object and a morphism in a functor category. This categorification permits us in turn to quantify the extent in which the modeling of a topological space M by π_k deviates from that by H^k.

Categorification of Encoding Interplaying Systems. Up to now we have applied category theory over topological spaces without further specifying these spaces. The question arises how such a category looks like over natural and cybernetic systems. For a fixed space-time region the natural and cybernetic system can be considered closed and its categorification can be based on several classification methods.

The developmental dynamics of the natural and cybernetic systems can then be represented by a system of partial integro-differential equations with initial and boundary value conditions.

LAW 1 *The developmental dynamics of the closed cybernetic system is governed by the following law:*

$$\frac{d\psi}{dt} = e(\psi; \alpha),$$

with suitable initial and boundary value conditions and where t *is time,* ψ *is a set of encodings e denotes the driving force on the encodings and a specifying a classification of the developmental dynamics.*

The classification a of the closed natural and cybernetic systems can be brought about by considering (see [10] and references therein):

- Symmetries,
- Curvatures,
- Conservation laws.

Consequently the classification α can be stated as follows:

CLASSIFICATION 1 *The classification* α *of a closed cybernetic system is given in terms of symmetries, curvatures and conservation laws underlying the developmental dynamics in (Law 1).*

THEOREM 11.34 *(Classification 1) is gauge invariant under the symmetries of the developmental dynamics in (Law 1).*

One retains the symmetries, curvatures and conservation laws for those systems, which are not necessarily in divergence form, through the use of symbolic packages. These classifications yield a set of equivalences as in Section 1.1 but now for each closed system another set. Obviously the system category consists of natural and cybernetic systems and of each their symmetries. Actually the functors used in the previous paragraphs can be shown to generate the topological invariants of (Classification 1) of the encoding of the natural and cybernetic systems. The classification comprises besides aspects of the developmental dynamics of the natural and cybernetic systems also initial and boundary value conditions that can form constraints within the system.

For one equivalence given (Classification 1), for example, a conservation law, it is possible to derive a hierarchy of infinitely many equivalences. The methods for finding related conserved densities and fluxes should then not be confused with those for finding the equivalences in a standard setting as in

Section 1.1. The gauge group and thus the related equivalences are namely bound to be consistent with the symmetries of the developmental dynamics of the cybernetic system. The use of recursion operators that can generate a hierarchy of infinitely many symmetries then come into play to derive novel conservation laws [12].

Now the natural and cybernetic systems categorification consists of defining those systems as systems of partial integro-differential equations with initial-boundary value conditions, isomorphisms between them, functors over them and natural transformations between those functors. The decategorification occurs upon forgetting about the isomorphisms and retaining the equivalences of the classification. In reality our encoded natural and cybernetic systems trace out some region of the system categorification: there is an evolution of the natural and cybernetic systems.

The cybernetic system, however, can only partly follow over time a natural system, or vice versa, due to limited resources. This optimal abstraction, as reckoned in the introduction, implies a certain level of resolution of the encoded natural and cybernetic system dynamics, i.e. the resolution with which equivalences supported by those systema are observable or foreseeable. Thus in order to stay as close as possible to the natural system it's essential to couple the cybernetic system in an optimal way to the natural one, an vice versa. In doing so a higher resolution cybernetic system should be projectable onto a lower resolution one while satisfying superposition principles for the equivalences. The reader rightfully can object that it's sheer impossible to predict such a path or sweeping out system cateorification space given the counterintuitive behaviour of subsystems such as that of the anthroposphere. However, as we will demonstrate in the next section behaviour of a cybernetic system can and should be modelled simultaneously in order to be consistent with the natural system itself. From the standpoint of scenario analysis the latter behaviour manifesting itself in terms of bifurcations in the evolution of the cybernetic system can be of great importance in their own right too. But a suitable average of these bifurcating evolutions may be equally well valuable (see also Section 2). Note that emergence and annihilation of cybernetic system dynamics is no real problem as they can be readily be covered by natural transformations as will be demonstrated in the next section.

Categorification of Encoding Co-Evolving Systems. The decategorification of the natural and cybernetic systems, (Classification 1), obviously, is a function of time-parameter t. The evolution of this decategorification depends on the developments perceivable on a larger time-span. Thus different scenarios, for instance, will cause in general different evolutions of (Classification 1). In the context of sustainable development of above systems this evolutionary dynamics asks for a suitable selection mechanism. This mechanism in turn requires measures of sustainability β weighting the different evolutionary dynamics. As the natural and cybernetic systems can be viewed as essentially open because of the variability of (Classification 1), these systems can again be represented by systems of partial integro-differential equations with initial and boundary value conditions.

LAW 2 *The developmental and evolutionary dynamics of the open cybernetic system is governed by the following law:*

$$\begin{aligned} \frac{d\psi}{dt} &= e(\psi;\alpha), \\ \frac{d\alpha}{dt} &= E(e,\alpha;\beta), \end{aligned}$$

with suitable initial and boundary value conditions and where t *is time,* ψ *is a set of encodings,* e *denotes the driving force on* ψ*,* α *specifying (Classification 1), E the driving force on (Classification 1) and* β *denotes the sustainability measures.*

A major challenge for future research is to make such a developmental and evolutionary dynamics explicit through an effective scheme to obtain objective classifications, sustainability measures β and driving forces E on these classifications given e and β.

Analogous to the developmental dynamics of a closed cybernetic system the developmental and evolutionary dynamics of open natural and cybernetic systems can be classified. From analogy with the developmental dynamics it's clear that in this case the measures of sustainability β in this case characterise both the open natural and cybernetic systems.

CLASSIFICATION 2 *The classification* β *of open natural and cybernetic system is given in terms of symmetries, curvatures and conservation laws underlying the developmental and evolutionary dynamics in (Law 2).*

THEOREM 11.35 *(Classification 2) is gauge invariant under the symmetries of the developmental and evolutionary dynamics in (Law 2).*

The above description of a cybernetic system parallels that for biological systems [3], in which (Classification 1) represents the genomic and environmental characteristics and (Classification 2) represents measures of adaptation of the biological systems as a whole to a particular class of phenotypical and genomical dynamics. Moreover, many other natural systems, e.g. learning systems generating predictive models, are subjected to similar modeling relations. Above it's silently assumed that all these systems form also an integral part of sustainable systems.

Contrary to Rosen's exposition [3] in our framework the natural and cybernetic systems as a whole cannot be disentangled from their subsystems. They form in general each an integral part of one another. For example, an organism and a cell nor an individual and a society can be dissolved from one another. This inseparability of a system and its subsystems is manifest in the coupling between developmental and evolutionary dynamics in (Law 2). This inseparability also manifests itself as the fact that natural and cybernetic systems in essence are nonintegrable and are part of and subjected to irreversible processes.

A cybernetic or natural system with a parasitic developmental and evolutionary dynamics is bound to become extinct because of e.g. environmental deprivation of resources. In order to prevent such catastrophes from happening it is necessary and sufficient to incorporate constraints or control mechanisms in the cybernetic system. The latter build in restrictions on the developmental and evolutionary dynamics permit then the survival of the subsystems and the systems as a whole. But the latter controlled natural or cybernetic system obviously asks for a anticipatory, learning and attentive system, such that an evaluation and validation of (Classification 2) is feasible, and such that in the end their outcomes can be used to direct their actions and evolutions. The essence of a predictive system, however, is that it's also capable to foresee the nonintegrable changes in the developmental and evolutionary dynamics in (Law 2) of natural systems. Deploying cybernetic system related to a natural system asks thus for the application of renormalisation theory [12].

From our above exposition it's more than reasonable to link the various measures of sustainability, (Classification 2), to measures of structural and functional stability of natural and cybernetic systems under genomic or evolutionary changes, and to link (Classification 1) to measures of Lyapunov stability under perturbations of the observables. In Section 2 we introduce a method to filter natural and cybernetic systems in developmental as well as evolutionary sense, such that a stable classification of the cybernetic system can be retained despite possible perturbations within the developmental and evolutionary dynamics.

2. Sustaining Ambient Intelligence

In the analysis of co-evolving natural and cybernetic systems it makes no sense to stick to the highest resolution information about their behaviours, because their developmental and evolutionary dynamics are depending on the specific intricacies of their encoding procedures. However, larger scale aspects of these dynamics will be partially equivalent or even the same despite the fact that cybernetic and natural systems will display at highest resolution levels very different characteristics. A cybernetic system consistent filtration of the developmental as well as the evolutionary dynamics, according to a dynamic scale-space paradigm [5, 10, 12] just ensures the (partial) equivalence of both the cybernetic and the natural system dynamics. In Section 2.1 we introduce an exchange principle that allows us to diffuse and stabilise the encoded evolution of the natural and cybernetic systems in order to achieve simultaneously Lyapunov and structural stability for the developmental and evolutionary dynamics, respectively. In Section 2.2 we recapitulate the methods of the previous section for retrieving reliable indicators for sustainability of natural and cybernetic systems.

2.1 Propagating Structure and Function

Considering an ensemble of natural and cybernetic systems one notices that they are in a modern geometric, topological and dynamical sense perturbed versions ofeach other. It's clear that we can consider the following perturbations of natural and cybernetic systems:

- Perturbations of their encoded observables,
- Perturbations of their encoded developmental and evolutionary dynamics.

The first type of perturbation consists of non-integrable and integrable deformations of the frame fields, coframe fields, metrics and/or connections resulting in a change of the curvatures and even of the induced physical field equations. The non-integrable deformations due to noise (irreversible processes) and relative resolution differences in the encodings cause changes in (Equivalence 1), (Equivalence 2), (Equivalence 3), (Equivalence 4) and (Equivalence 5). The integrable deformations consistent with the postulated gauge group on the contrary do not cause curvature changes. However, in practice the encoding of natural and cybernetic systems are the result of a density field requiring the integrable deformations to belong to a certain class in order not after measurement to give rise to non-integrable deformations.

The second type of perturbations directly affects the natural and cybernetic systems as characterised by (Classification 1) and (Classification 2). Here also the perturbation can be such that the natural and cybernetic systems stay within the realm of a particular class of dynamics and that only a deformation of the constitutive parameters, α and β, is necessary to ensure that the systems are conjugate. However, in general a change in the cybernetic system dynamics cannot be captured in terms of an integrable deformation. In this case nonintegrable deformations or natural transformations (see Section 1.2.0) are necessary to relate the cybernetic systems.

In order to extract from natural and cybernetic systems a stable and reproducible set of equivalences and classifications despite these perturbations we proposed in the realm of computer vision [5, 12] a dynamic scale-space paradigm controlled by the equivalences and classifications themselves. Essential in the context of this paradigm is the derivation of a proper induced exchange principle for equivalences and classifications between among natural and cybernetic systems.

From an encoding of a natural system, (Definition 11.1), governed by developmental and evolutionary dynamics, (Law 1) and (Law 2), respectively, a stable and reproducible set of equivalences and classifications despite above types of perturbations can be retained by coupling the exchange principle intrinsically to the encoded natural system dynamics. Essential in this context is the assessment of the topological interactions activated among natural and cybernetic systems. These interactions can be quantified by these co-evolving systems system as topological currents.

For a complete and irreducible set of equivalences and classifications a committed ordering of the activated cybernetic system can be succinctly formulated through the use of a statistical partition function Z related to a free energy F for (Equivalence 1), (Equivalence 2), (Equivalence 3), (Equivalence 4) and (Equivalence 5), and (Classification 1) and (Classification 2).

EQUIVALENCE 6 *The statistical partition function Z related to free energy F for irreducible (Equivalence 1), (Equivalence 2), (Equivalence 3), (Equivalence 4) and (Equivalence 5), and (Classification 1) and (Classification 2) of the cybernetic system is defined by:*

$$Z = \sum_{V} \prod_{x,i} \exp\left[-F\left[V_i(x)\right]\right],$$

with

$$\begin{aligned} F\left[V_i(x)\right] &= -\log Z \\ &= \sum_{i,k,p} dv^p \left(\tilde{V}_{i;\pi_k(g_1 \dots g_k)}(x, \tau_{i;\pi_k(g_1 \dots g_k)})\right), \end{aligned}$$

where x labels any state, i.e. a space-time region with some dynamics, giving rise to equivalence or classification V, π_k a permutation of a sequence of $k \geq 0$ integers $(g_1 \dots g_k)$ with $k = 0$ for labeling frame vector fields v_{g_k} and $\tau_{\alpha;\pi_k(g_1 \dots g_k)}$ (inner) dynamic scales consistent with gauge group (Definition 11.2) and the equivalences and classification $\tilde{V}_{i;\pi_k(g_1 \dots g_k)}$.

Again gauge invariance of the statistical partition function for the developmental and evolutionary dynamics holds.

THEOREM 11.36 *(Equivalence 6) is invariant under gauge group (Definition 11.2).*

PROOF 2 *Follows immediately from the gauge invariance of (Equivalence 1), (Equivalence 2), (Equivalence 3), (Equivalence 4) and (Equivalence 5), and (Classification 1 and (Classification 2).*

The partition function can be conceived as a measure of the topological, geometric and dynamical complexity of the developmental and evolutionary dynamics. The advantage of our measure is that it readily substantiates and extends information theoretical measures as proposed in computer vision [1]. Furthermore, the new measures of complexity are to be preferred for their conciseness. Moreover, the dynamic scale-space paradigm, therewith, falls nicely in the realm of modern theory of dynamical systems.

Besides the induced gauge invariant canonical parametrisation of space-time and dynamics also a topological interaction is needed to ensure a filtration yielding a hierarchy of partially equivalent ensembles of cybernetic systems that are slight perturbations of one another. This topological current is in our dynamic scale-space paradigm brought about by the statistical partition function (Equivalence 6). Studying two 'local' factors $Z(p_1)$ and $Z(p_2)$ in the statistical partition function going from state p_i to state p_j involves a factor $k(i,j)$ to generate $Z(p_j)$ from $Z(p_i)$, whereas going from p_j to p_i requires a factor $K(i,j)$ (assume $k \leq K$) to generate $Z(p_i)$ from $Z(p_j)$, such that $kK = 1$, i.e., the notable Artin-Whaples formula in disguise. Realising that the interaction can only be defined through the interactions between two adjacent states, it is more than reasonable to let a topological current between states p_i and p_j to be controlled by the partition function Z^r for two-state interactions capturing all the possible couplings between the states of all pairs of states:

$$\begin{aligned} Z^r &= \prod_{i \neq j} Z^r_{ij} = \prod_{i \neq j} \left(\frac{K(i,j) + K^{-1}(i,j)}{2} \right) \\ &= \prod_{i \neq j} \cosh\left(F(p_i) - F(p_j)\right). \end{aligned}$$

Note that these interactions between two states do not exclude long range forcings: $Z(p_i)$ can instantaneously incorporate natural and cybernetic systems properties such as (Equivalence 5) and (Classification 1) and (Classification 2). The finiteness of the transport velocity need not to be in contradiction with the fact that the action is instantaneous and nonlocal!

With this two-state coupling partition function, Z^r, there's associated also a two-state coupling free energy F^r:

$$F^r = -\log Z^r.$$

Assuming the co-evolving natural and cybernetic systems to be a closed system for a particular region of space-time, then the change in the state of the co-evolving system can be realised by a change in the two-state coupling free energy F^r. Keeping in mind that the free energy, see (Equivalence 6), should be preserved, i.e., $dF(p_i) = -dF(p_j)$, this change in the two-state coupling free energy, dF^r, is given by:

$$dF^r = -\sum_{i \neq j} \tanh\left(F(p_i) - F(p_j)\right) dF(p_i).$$

Thus the geometric or topological charges of the developmental and evolutionary dynamics have become the generators of an induced filtration of the dynamics of the co-evolving system. Now let us consider again the interaction mechanisms between pairs of states p_i and p_j, and define the topological current to be the curl of the induced connection on the two-state coupling free energy:

DEFINITION 11.37 *The topological current for the free energy on the activated co-evolving system is defined by:*

$$\begin{aligned} j^F &= \nabla^\Gamma \wedge dF^r \\ &= -\frac{\nabla^\Gamma_{v_s} F}{\cosh^2\left(\sqrt{g(\nabla^\Gamma_{v_s} F, \nabla^\Gamma_{v_s} F)}\right)} dv^s \wedge dF, \end{aligned}$$

where v_s is connecting free energy states $F(p_i)$ and $F(p_j)$ of the co-evolving system.

Note that the topological current is steered by (Equivalence 1), (Equivalence 2), (Equivalence 3), (Equivalence 4) and (Equivalence 5), (Classification 1) and (Classification 2).

As the free energy (Equivalence 6) should be preserved the dynamic exchange principle for free energy is made manifest through a physical law involving the topological current (Definition 11.37):

LAW 3 *The dynamic exchange principle for free energy says that the change per unit scale τ in the free energy (Equivalence 6) in a region Ω of the cybernetic system is equal to the exchange of free energy F between this region and its surrounding across their (common) boundary $S = \partial\Omega$ quantised by topological current (Definition 11.37):*

$$\delta_\tau F = -j^F,$$

with suitable initial and boundary conditions.

THEOREM 11.38 *The dynamic exchange principle residing in (Law 3) is gauge invariant.*

PROOF 3 *Proof follows those of (Theorem 11.11) and (Theorem 11.36).*

This law is in perfect harmony with the second law of thermodynamics that states that the entropy of a closed co-evolving system with time is only increasing if elementary subsystems not all in their ground states are permitted to interact. Here the equivalences and classifications living on the cellular regions are recombined causing a substantial simplification of the natural and cybernetic systems.

2.2 Indicators of Sustainability

In the previous section we have presented the exchange principle co-evolving natural and cybernetic systems. The question arises how do we quantify and qualify the multi-scale characteristics of such co-evolving systems.

It is clear that for the multi-scale co-evolving natural and cybernetic systems again similar analysis methods are available as for a closed and open systems (see Section 1.2).

CLASSIFICATION 3 *The classification γ of a filtered co-evolving natural and cybernetic systems is given in terms of symmetries, curvatures and conservation laws underlying the filtered developmental and evolutionary dynamics in (Law 3).*

THEOREM 11.39 *(Classification 3) is gauge invariant under the symmetries of the filtered developmental and evolutionary dynamics in (Law 3).*

This classification may form the basis for a set ofindicators for spatio-temporally as well dynamically Lyapunov and structural stability. A subset may serve as sustainability measures for co-evolving natural and cybernetic systems.

2.3 Collective Intelligent Agents

Architectures for collective intelligent agents (CIA) can automate and sustain NASA (pre-) schemes within SIMS [13]. The agents in a CIA environment are best breed agents selected from possibly distributed CIA development and deployment platforms: they essentially behave like co-evolving natural and cybernetic systems. Both platforms and environments may communicate with physical actors being either individual humans or groups with their own NASA-SIMS capacities and capabilities. The (off-spring) actor software agents support the actors to interact, communicate and collaborate with each other in an ever-complex multimodal way. The development and deployment environment looks after embedding and embodiment of the diversification of the intelligence of NASA (pre-) schemes in SIMS and environment following the laws for propagating structure and function of section 2.1. Thereto, the actor agents use (not necessarily language determined) agent communication languages (ACLs) and negotiation strategies to set up interactions among humans and systems: they could talk physics. Furthermore, the CIA development and deployment platforms generate genetic algorithms (GAs) with sustainability measures for NASA and other self-organization (pre-) schemes as proposed in section 2.3. By registering and assessing fitness and utility of those (pre-) schemes during operations within the CIA environment reinforcement learning within SIMS can be effectuated. Here again reinforcemnt learning may follow similar dynamic exchange principles as in section 2.1. Note that NASA-ing in SIMS by means of ACLs, negotiation strategies, GAs with sustainability measures are most efficiently developed and deployed by means of computer algebra systems.

3. Conclusion

We have laid down a framework for modelling and analysing the sustainable development of co-evolving natural and cybernetic systems. The modelling consists of the encoding of observables and their evolutions. The observables are gauge invariant equivalences that are consistent with the developmental and evolutionary dynamics with respect to these observables. The classification of latter dynamics is of eminent importance in the derivation of those equivalences as well as their classification. In order ot retain stable measures of sustainability for the dynamics as a whole an adapted dynamic scale-space paradigm has been proposed.

It is obvious that our presented framework asks for a very sophisticated problem solving environment. First of all the developmental and evolutionary dynamics of the co-evolving natural and cybernetic systems together with a predictive and (reinforcement) learning model have to be put in place. In this context it is essential to let the problem solving environment to be too integral parts of the co-evolving and natural cybernetic systems.

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