Chapter

5 Navigation

the process of directing a vehicle so as to reach the intended destination
IEEE Standard 172-1983

Robot navigation is the problem of guiding a robot towards a goal. The human approach to navigation is to make maps and erect signposts, and at first glance it seems obvious that robots should operate the same way. However many robotic tasks can be achieved without any map at all, using an approach referred to as *reactive navigation*. For example heading towards a light, following a white line on the ground, moving through a maze by following a wall, or vacuuming a room by following a random path. The robot is reacting directly to its environment: the intensity of the light, the relative position of the white line or contact with a wall. Grey Walter's tortoise Elsie from page 61 demonstrated "life-like" behaviours – she *reacted* to her environment and could seek out a light source. Today more than 5 million Roomba vacuum cleaners are cleaning floors without using any map of the rooms they work in. The robots work by making random moves and sensing only that they have made contact with an obstacle.

The more familiar human-style *map-based navigation* is used by more sophisticated robots. This approach supports more complex tasks but is itself more complex. It imposes a number of requirements, not the least of which is a map of the environment. It also requires that the robot's position is always known. In the next chapter we will discuss how robots can determine their position and create maps. The remainder of this chapter discusses the reactive and map-based approaches to robot navigation with a focus on wheeled robots operating in a planar environment.

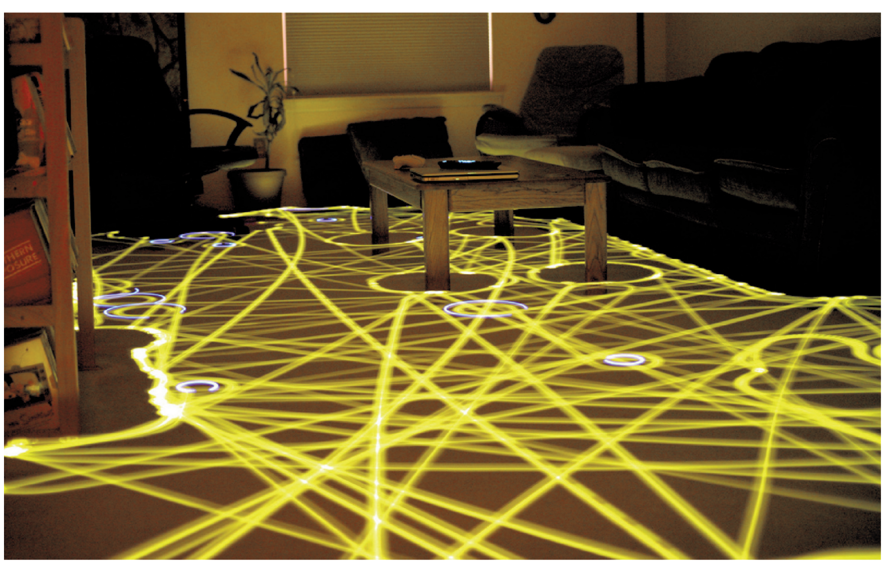
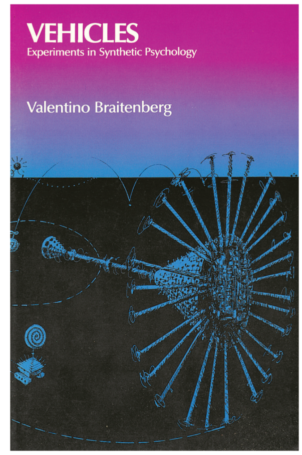


Fig. 5.1.
Time lapse photograph of a Roomba robot cleaning a room (photo by Chris Bartlett)

Valentino Braitenberg (1926–) is an Italian-Austrian neuro-scientist and cyberneticist, and former director at the Max Planck Institute for Biological Cybernetics in Tübingen, Germany. His 1986 book "*Vehicles: Experiments in Synthetic Psychology*" (image on right is of the cover this book, published by The MIT Press, ©MIT 1984) describes reactive goal-achieving vehicles, and such systems are now commonly known as Braitenberg Vehicles.

A Braitenberg vehicle is an automaton or robot which combines sensors, actuators and their direct interconnection to produce goal-oriented behaviors. Grey Walter's tortoise predates the use of this term but is nevertheless an example of such a vehicle.

These vehicles are described as conceptually as analog circuits, but more recently small robots based on a digital realization of the same principles have been developed.



5.1 Reactive Navigation

Surprisingly complex tasks can be performed by a robot even if it has no map and no real *idea* about where it is. As already mentioned robotic vacuum cleaners use only random motion and information from contact sensors to perform a complex task as shown in Fig. 5.1. Insects such as ants and bees gather food and return it to the nest based on input from their senses, they have far too few neurons to create any kind of mental map of the world and plan paths through it. Even single-celled organisms such as flagellate protozoa exhibited goal seeking behaviours. In this case we need to revise our earlier definition of a robot to

a goal oriented machine that can sense, ~~plan~~ and act.

The manifestation of complex behaviours by simple organisms was of interest to early researchers in cybernetics. Grey Walter's robotic tortoise demonstrated that it could moves toward a light source, a behaviour known as phototaxis. This was an important result in the then emerging scientific field of cybernetics.

More generally a *taxis* is the response of an organism to a stimulus gradient.

5.1.1 Braitenberg Vehicles

A very simple class of goal achieving robots are known as Braitenberg vehicles and are characterised by direct connection between sensors and motors. They have no explicit internal representation of the environment in which they operates and nor do they make explicit plans.

This is a fine philosophical point, the plan could be considered to be implicit in the details of the connections between the motors and sensors.

Consider the problem of a robot moving in two dimensions that is seeking the maxima of a scalar field – the field could be light intensity or the concentration of some chemical. The Simulink® model

This is similar to the problem of moving to a point discussed in Sect. 4.2.1.

```
>> sl_braitenberg
```

shown in Fig. 5.2 achieves this using a steering signal derived directly from the sensors.

This is similar to Braitenberg's Vehicle 4a.

William Grey Walter (1910–1977) was a neurophysiologist and pioneering cyberneticist born in Kansas City, Missouri and studied at King's College, Cambridge. Unable to obtain a research fellowship at Cambridge he worked on neurophysiological research in hospitals in London and from 1939 at the Burden Neurological Institute in Bristol. He developed electroencephalographic brain topography which used multiple electrodes on the scalp and a triangulation algorithm to determine the amplitude and location of brain activity. (Image: courtesy of the Reuben Hoggett Collection)

Walter was influential in the then new field of cybernetics. He built robots to study how complex reflex behavior could arise from neural interconnections. His tortoise Elsie (of the species *Machina Speculatrix*) is shown, without its shell, on page 61. Built in 1948 Elsie was a three-wheeled robot capable of phototaxis that could also find its way to a recharging station. A second generation tortoise (from 1951) is in the collection of the Smithsonian Institution. He published popular articles in "Scientific American" (1950 and 1951) and a book "The Living Brain" (1953). He was badly injured in a car accident in 1970 from which he never fully recovered. (Image courtesy Reuben Hoggett collection)

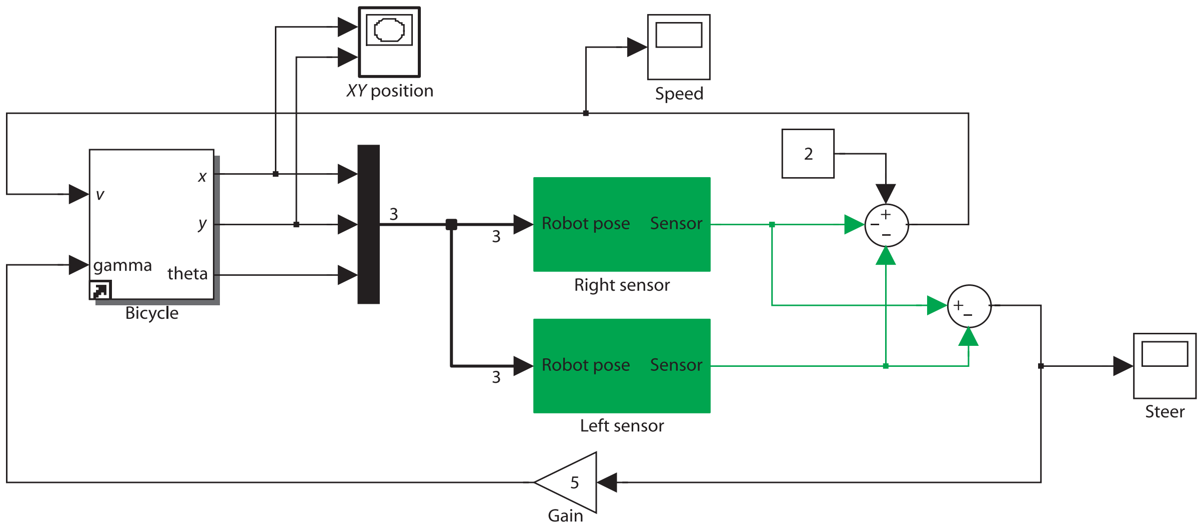


Fig. 5.2. The Simulink® model `sl_braitenberg` drives the vehicle toward the maxima of a provided scalar function. The vehicle plus controller is an example of a Braitenberg vehicle

To ascend the gradient we need to estimate the gradient direction at the current location and this requires at least two measurements of the field. In this example we use two sensors, bilateral sensing, with one on each side of the robot's body. The sensors are modelled by the green sensor blocks shown in Fig. 5.2 and are parameterized by the position of the sensor with respect to the robot's body, and the sensing function. In this example the sensors are at ± 2 units in the vehicle's lateral or y-direction.

We can make the measurements simultaneously using two spatially separated sensors or from one sensor over time as the robot moves.

The field to be sensed is a simple inverse square field defined by

```
function sensor = sensorfield(x, y)
    xc = 60; yc = 90;
    sensor = 200./((x-xc).^2 + (y-yc).^2 + 200);
```

which returns the sensor value $s(x, y) \in [0, 1]$ which is a function of the sensor's position in the plane. This particular function has a peak value at the point (60, 90).

The vehicle speed is

$$v = 2 - s_R - s_L$$

where s_R and s_L are the right and left sensor readings respectively. At the goal, where $s_R = s_L = 1$ the velocity becomes zero.

Steering angle is based on the difference between the sensor readings

$$\gamma = k(s_R - s_L)$$

so when the field is equal in the left- and right-hand sensors the robot moves straight ahead.

Similar strategies are used by moths whose two antennae are exquisitely sensitive odor detectors that are used to steer a male moth toward a pheromone emitting female.

We start the simulation from the Simulink® menu or the command line

```
>> sim('sl_braitenberg');
```

and the path of the robot is shown in Fig. 5.3. The starting pose can be changed through the parameters of the `Bicycle` block. We see that the robot turns toward the goal and slows down as it approaches, asymptotically achieving the goal position.

This particular sensor-action control law results in a specific robotic behaviour. We could add additional logic to the robot to detect that it had arrived near the goal and then switch to a stopping behaviour. An obstacle would block this robot since its only behaviour is to steer toward the goal, but an additional behaviour could be added to handle this case and drive around an obstacle. We could add another behaviour to search randomly for the source if none was visible. Grey Walter's tortoise had four behaviours and switching was based on light level and a touch sensor.

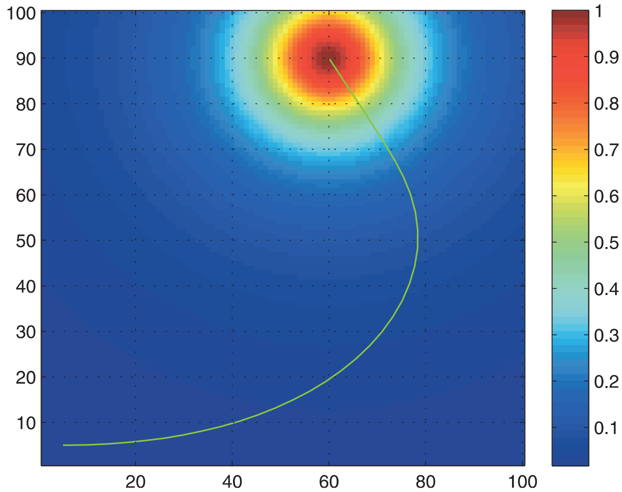


Fig. 5.3.
Path of the Braitenberg vehicle moving toward (and past) the maximum of a 2D scalar field whose magnitude is shown color coded

Multiple behaviours and the ability to switch between them leads to an approach known as behaviour-based robotics. The subsumption architecture was proposed as a means to formalize the interaction between different behaviours. Complex, some might say *intelligent looking*, behaviours can be manifested by such systems. However as more behaviours are added the complexity of the system grows rapidly and interactions between behaviours become more complex to express and debug. Ultimately the penalty of not using a map becomes too great.

5.1.2 Simple Automata

Another class of reactive robots are known as *bugs* – simple automata that perform goal seeking in the presence of non-driveable areas or obstacles. There are a large number of *bug* algorithms and they share the ability to sense when they are in proximity to an obstacle. In this respect they are similar to the Braitenberg class vehicle, but the *bug* includes a state machine and other logic in between the sensor and the motors. The automata have memory which our earlier Braitenberg vehicle lacked.▸ In this section we will investigate a specific *bug* algorithm known as *bug2*.

Braitenberg's book describes a series of increasingly complex vehicles, some of which incorporate memory. However the term *Braitenberg vehicle* has become associated with the simplest vehicles he described.

We start by loading an obstacle field to challenge the robot

```
>> load map1
```

which defines a 100 × 100 matrix variable `map` in the workspace. The elements are zero or one representing free space or obstacle respectively and this is shown in Fig. 5.4. Tools to generate such maps are discussed on page 92. This matrix is an example of an occupancy grid which will be discussed further in the next section.

At this point we state some assumptions. Firstly, the robot operates in a grid world and occupies one grid cell. Secondly, the robot does not have any non-holonomic constraints and can move to any neighbouring grid cell. Thirdly, it is able to determine its position on the plane which is a non-trivial problem that will be discussed in detail in Chap. 6. Finally, the robot can only sense its immediate locale and the goal. The robot does not use the map – the map is used by the simulator to provide sensory inputs to the robot.

We create an instance of the `bug2` class

```
>> bug = Bug2(map);
```

and the goal is

```
>> bug.goal = [50; 35];
```

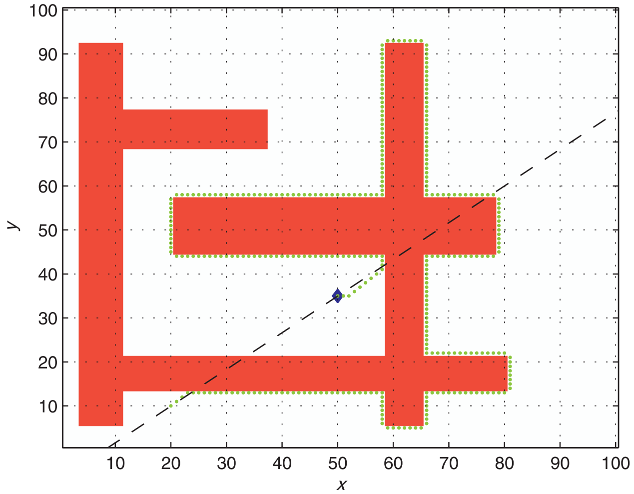


Fig. 5.4. The path taken by the *bug2* algorithm is marked by green dots. The goal is a blue diamond, the black dashed line is the M-line, the direct path from the start to the goal. Obstacles are indicated by red pixels

The simulation is run using the `path` method

```
>> bug.path([20; 10]);
```

where the argument is the initial position of the robot. The method displays an animation of the robot moving toward the goal and the path is shown as a series of green dots in Fig. 5.4.

The strategy of the *bug2* algorithm is quite simple. It moves along a straight line towards its goal. If it encounters an obstacle it moves around the obstacle (always counter-clockwise) until it encounters a point that lies along its original line that is closer to the goal than where it first encountered the obstacle.

It could be argued that the M-line represents an explicit plan. Thus *bug* algorithms occupy a position somewhere between Braitenberg vehicles and map-based planning systems in the spectrum of approaches to navigation.

If an output argument is specified

```
>> p = bug.path([20; 10]);
```

it returns the path as a matrix `p`

```
>> about(p)
p [double] : 332x2 (5312 bytes)
```

which has one row per point, and comprises 332 points for this example. Invoking the function without the starting position

```
>> p = bug.path();
```

will prompt for a starting point to be selected by clicking on the plot.

The *bug2* algorithms has taken a path that is clearly not optimal. It has wasted time by continuing to follow the perimeter of the obstacle until it rejoined its original line. It would also have been quicker in this case to go clockwise around the obstacle. Many variants of the *bug* algorithm have been developed, but while they improve the performance for one type of environment they can degrade performance in others. Fundamentally the robot is limited by not using a map. It cannot see the big picture and therefore takes paths that are locally, rather than globally, optimal.

5.2 Map-Based Planning

Beyond the trivial straight line case.

The key to achieving the *best* path between points A and B, as we know from everyday life, is to use a map. Typically best means the shortest distance but it may also include some penalty term or cost related to traversability which is how easy the terrain is to drive over – it might be quicker to travel further but over better roads. A more sophisticated planner might also consider the kinematics and dynamics of the

Making a map. An occupancy grid is a matrix that corresponds to a region of 2-dimensional space. Elements containing zeros are free space where the robot can move, and those with ones are obstacles where the robot cannot move. We can use many approaches to create a map. For example we could create a matrix filled with zeros (representing all free space)

```
>> map = zeros(100, 100);
```

and use MATLAB® operations such as

```
>> map(40:50,20:80) = 1;
```

to create an obstacle but this is quite cumbersome. Instead we can use the Toolbox map editor `makemap` to create more complex maps using a simple interactive editor

```
>> map = makemap(100)
makeworld:
  left button, click and drag to create a rectangle
  p - draw polygon
  c - draw circle
  e - erase map
  u - undo last action
  q - leave editing mode
```

which allows you to add rectangles, circles and polygons to an occupancy grid, in this example the grid is 100×100.

Note that the occupancy grid is a matrix whose coordinates are conventionally expressed as (row, column) and the row is the vertical dimension of a matrix. We use the Cartesian convention of a horizontal x-coordinate first, followed by the y-coordinate therefore the matrix is always indexed as `y,x` in the code.

vehicle and avoid paths that involve turns that are tighter than the vehicle can execute. Recalling our earlier definition of a robot as a

goal oriented machine that can sense, plan and act,

this section concentrates on planning.

There are many ways to represent a map and the position of the vehicle within the map. One approach is to represent the vehicle position as $(x, y) \in \mathbb{R}^2$ and the the driveable regions or obstacles as polygons, each comprising lists of vertices or edges. This is potentially a very compact format but determining potential collisions between the robot and obstacles may involve testing against long lists of edges.

A simpler and very computer-friendly representation is the occupancy grid. As its name implies the world is treated as a grid of cells and each cell is marked as occupied or unoccupied. We use zero to indicate an unoccupied cell or free space where the robot can drive. A value of one indicates an occupied or non-driveable cell. The size of the cell depends on the application. The memory required to hold the occupancy grid increases with the spatial area represented and inversely with the cell size. However for modern computers this representation is very feasible. For example a cell size 1×1 m requires► just 125 kbyte km^{-2}.

Considering a single bit to represent each cell. The occupancy grid could be compressed or could be kept on a disk with only the local region in memory.

In the remainder of this section we use code examples to illustrate several different planners and all are based on the occupancy grid representation. To create uniformity the planners are all implemented as classes derived from the `Navigation` superclass which is briefly described on page 93. The *bug2* class we used previously was also an instance of this class so the remaining examples follow a familiar pattern.

Once again we state some assumptions. Firstly, the robot operates in a grid world and occupies one grid cell. Secondly, the robot does not have any non-holonomic constraints and can move to any neighbouring grid cell. Thirdly, it is able to determine its position on the plane. Fourthly, the robot is able to use the map to compute the path it will take.

Navigation superclass. The examples in this chapter are all based on classes derived from the `Navigation` class which is designed for 2D grid-based navigation. Each example consists of essentially the following pattern. Firstly we create an instance of an object derived from the `Navigation` class by calling the class constructor.

```
>> nav = MyNavClass(world)
```

which is passed the occupancy grid. Then a plan to reach the goal is computed

```
>> nav.plan(goal)
```

The plan can be visualized by

```
>> nav.visualize()
```

and a path from an initial position to the goal is computed by

```
>> p = nav.path(start)
>> p = nav.path()
```

where `p` is the path, a sequence of points from `start` to `goal`, one row per point, and each row comprises the *x*- and *y*-coordinate. If `start` is not specified, as in the second example, the user is prompted to interactively click the start point. If no output argument is provided an animation of the robot's motion is displayed.

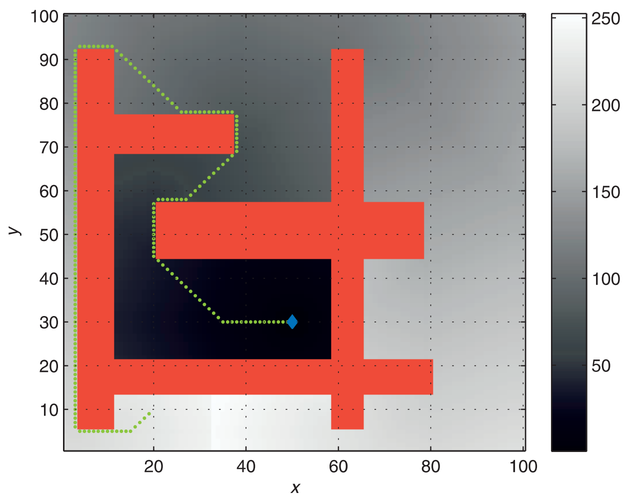


Fig. 5.5. The distance transform path. Obstacles are indicated by red cells. The background grey intensity represents the cell's distance from the goal in units of cell size as indicated by the scale on the right-hand side

In all examples we use the following parameters

```
>> goal = [50; 30];
>> start = [20; 10];
>> load map1
```

for the goal position, start position and world map respectively. The world map is loaded into the workspace variable `map`. These parameters can be varied, and the occupancy grid changed using the tools described on page 92.

5.2.1 Distance Transform

Consider a matrix of zeros with just a single non-zero element representing the goal. The distance transform of this matrix is another matrix, of the same size, but the value of each element is its distance from the original non-zero pixel. For robot path planning we use the default Euclidean distance. The distance transform is actually an image processing technique and will be discussed further in Chap. 12.

The distance between two points (x_1, y_1) and (x_2, y_2) where $\Delta_x = x_2 - x_1$ and $\Delta_y = y_2 - y_1$ can be Euclidean $\sqrt{\Delta_x^2 + \Delta_y^2}$ or CityBlock (also known as Manhattan) distance $|\Delta_x| + |\Delta_y|$.

To use the distance transform for robot navigation we create a `DXform` object, which is derived from the `Navigation` class

```
>> dx = DXform(map);
```

and create a plan to reach the specified goal

```
>> dx.plan(goal)
```

which can be visualized

```
>> dx.visualize()
```

as shown in Fig. 5.5. We see the obstacle regions in red and the background grey level at any point indicates the distance from that point to the goal, taking into account travel *around* the obstacle.

The hard work has been done and finding a path from *any* point to the goal is now very simple. Wherever the robot starts, it moves to the neighbouring cell that has the smallest distance to the goal. The process is repeated until the robot reaches a cell with a distance value of zero which is the goal. For example to find a path from to the goal from position `start` is

```
>> dx.path(start);
```

which displays an animation of the robot moving toward the goal. The path is indicated by a series of green dots as shown in Fig. 5.5.

If the `path` method is called with an output argument the animation is skipped and the path

```
>> p = dx.path(start);
```

is returned as a matrix, one row per point, which we can visualize

```
>> dx.visualize(p)
```

The path comprises

```
>> numrows(p)
ans =
   205
```

points which is shorter than the path found by *bug2*. Unlike *bug2* this planner has found the shorter clockwise path around the obstacle.

This navigation algorithm has exploited its global view of the world and has, through exhaustive computation, found the shortest possible path. In contrast, *bug2* without the global view has just bumped its way through the world. The penalty for achieving the optimal path is computational cost. The distance transform is iterative. Each iteration has a cost of $O(N^2)$ and the number of iterations is at least $O(N)$, where N is the dimension of the map.

We can visualize the iterations of the distance transform by

```
>> dx.plan(goal, show 0.1);
```

which shows the distance values propagating as a wavefront outward from the goal. The wavefront moves upward, splits to the left and right, moves downward and the two fronts collide at the bottom of the map along the line $x = 32$. The last argument specifies a pause of 0.1 s between frames. Although the plan is expensive to create, once it has been created it can be used to plan a path from *any* initial point to the goal.

We have converted a fairly complex planning problem into one that can now be handled by a Braitenberg-class robot that makes local decisions based on the distance to the goal. Effectively the robot is rolling *downhill* on the distance function which we can plot as a 3D surface

```
>> dx.visualize3d(p)
```

shown in Fig. 5.6 with the robot's path overlaid.

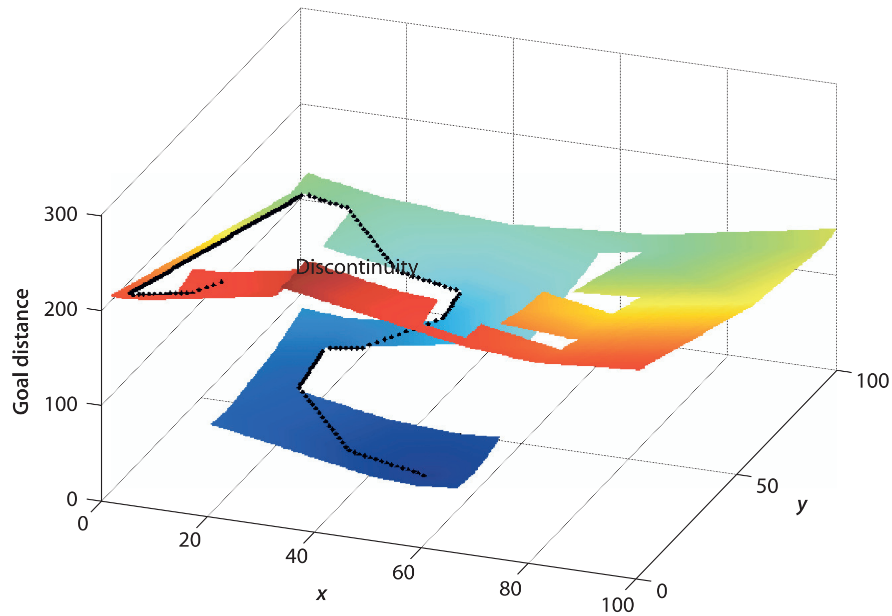


Fig. 5.6. The distance transform as a 3D function, where height is distance from the goal. Navigation is simply a downhill run. Note the discontinuity in the distance transform where the split wavefronts met

For large occupancy grids this approach to planning will become impractical. The roadmap methods that we discuss later in this chapter provide an effective means to find paths in large maps at greatly reduced computational cost.

5.2.2 D*

D* is an extension of the A* algorithm for finding minimum cost paths through a graph, see Appendix J.

A popular algorithm for robot path planning is called D*, and it has a number of features that are useful for real-world applications. D* generalizes the occupancy grid to a cost map which represents the cost $c \in \mathbb{R}$, $c > 0$ of traversing each cell in the horizontal or vertical direction. The cost of traversing the cell diagonally is $c\sqrt{2}$. For cells corresponding to obstacles $c = \infty$ (`Inf` in MATLAB®).

D* finds the path which minimizes the total cost of travel. If we are interested in the shortest time to reach the goal then cost is the time to drive across the cell and is inversely related to traversability. If we are interested in minimizing damage to the vehicle or maximizing passenger comfort then cost might be related to the roughness of the terrain within the cell. The costs assigned to cells will depend on the characteristics of the vehicle: a large 4-wheel drive vehicle may have a finite cost to cross a rough area whereas for a small car that cost might be infinite.

The key feature of D* is that it supports incremental replanning. This is important if, while we are moving, we discover that the world is different to our map. If we discover that a route has a higher than expected cost or is completely blocked we can incrementally replan to find a better path. The incremental replanning has a lower computational cost than completely replanning as would be required using the distance transform method just discussed.

To implement the D* planner using the Toolbox we use a similar pattern and first create a D* navigation object

```
>> ds = Dstar(map);
```

The D* planner converts the passed occupancy grid `map` into a cost map which we can retrieve

```
>> c = ds.costmap();
```

where the elements of `c` will be 1 or ∞ representing free and occupied cells respectively.

A plan for moving to the goal is generated by

```
>> ds.plan(goal);
```

which creates a very dense directed graph (see Appendix J). Every cell is a graph vertex and has a cost, a distance to the goal, and a link to the neighbouring cell that is closest to the goal. Each cell also has a state $t \in$ {NEW, OPEN, CLOSED}. Initially every cell is in the NEW state, the cost of the goal cell is zero and its state is OPEN. We can consider the set of all cells in the OPEN state as a wavefront propagating outward from the goal. The cost of reaching cells that are neighbours of an OPEN cell is computed and these cells in turn are set to OPEN and the original cell is removed from the open list and becomes CLOSED. In MATLAB® this initial planning phase is quite slow and takes tens of seconds and

```
>> ds.niter
ans =
       10558
```

iterations of the planning loop.

The path from an arbitrary starting point to the goal

```
>> ds.path(start);
```

is shown in Fig. 5.7. The robot has again taken the short path to the left of the obstacles and is almost the same as that generated by the distance transform.

The real power of D* comes from being able to efficiently change the cost map during the mission. This is actually quite a common requirement in robotics since real sensors have a finite range and a robot discovers more of world as it proceeds. We inform D* about changes using the `modify_cost` method, for example

```
>> for y=78:85
>>     for x=12:45
>>         ds.modify_cost([x,y], 2);
>>     end
>> end
```

where we have raised the cost to 2 for a small rectangular region to simulate a patch of terrain with lower traversability. This region is indicated by the white dashed rectangle in Fig. 5.8. The other driveable cells have a cost of 1. The plan is updated by invoking the planning algorithm again

```
>> ds.plan();
```

but this time the number of iterations is only

```
>> ds.niter
ans =
        3178
```

The distance transform also evolves as a wavefront outward from the goal. However D* represents the frontier efficiently as a list of cells whereas the distance transform computes the frontier on a per-pixel basis at every iteration – the frontier is implicitly where a cell with infinite cost (the initial value of all cells) is adjacent to a cell with finite cost.

D* is more efficient than the distance transform but it executes more slowly because it is implemented entirely in MATLAB® code whereas the distance transform is a MEX-file written in C.

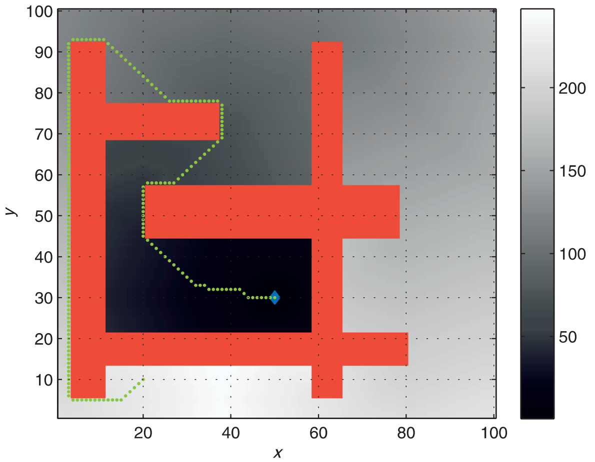


Fig. 5.7.
The D* planner path. Obstacles are indicated by red cells and all driveable cells have a cost of 1. The background grey intensity represents the cell's distance from the goal in units of cell size as indicated by the scale on the right-hand side

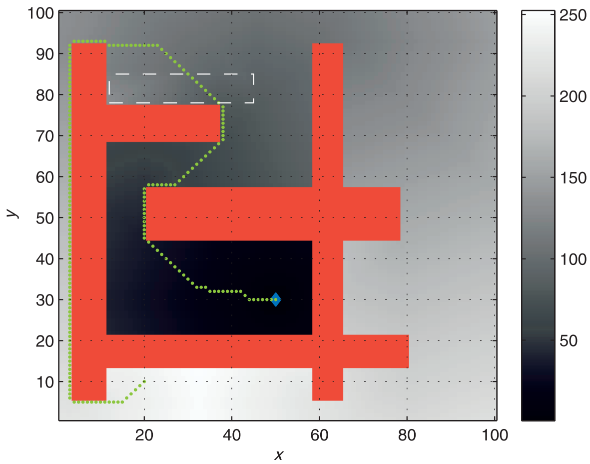
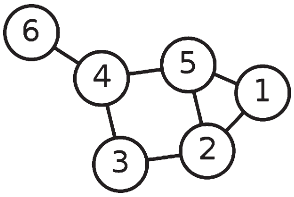


Fig. 5.8.
Path from D* planner with modified map. The higher-cost region is indicated by the white dashed rectangle and has changed the path compared to Fig. 5.7



A graph is an abstract representation of a set of objects connected by links typically denoted $G(V, E)$ and depicted diagrammatically as shown to the left. The objects, V, are called vertices or nodes, and the links, E, that connect some pairs of vertices are called edges or arcs. Edges can be directed (arrows) or undirected as in this case. Edges can have an associated weight or cost associated with moving from one of its vertices to the other. A sequence of edges from one vertex to another is a path. Graphs can be used to represent transport or communications networks and even social relationships, and the branch of mathematics is graph theory. Minimum cost path between two nodes in the graph can be computed using well known algorithms such as Dijstrka's method or A*.

The navigation classes use a simple MATLAB® graph class called `PGraph`, see Appendix J.

which is 30% of that required to create the original plan. The new path for the robot

```
>> ds.path(start);
```

is shown in Fig. 5.8. The cost change is relatively small but we notice that the increased cost of driving within this region is indicated by a subtle brightening of those cells – in a cost sense these cells are now further from the goal. Compared to Fig. 5.7 the robot's path has moved to the right in order to minimize the distance it travels through the high-cost region. D* allows updates to the map to be made at any time while the robot is moving. After replanning the robot simply moves to the adjacent cell with the lowest cost which ensures continuity of motion even if the plan has changed.

5.2.3 Voronoi Roadmap Method

In planning terminology the creation of a plan is referred to as the *planning phase*. The *query phase* uses the result of the planning phase to find a path from A to B. The two previous planning algorithms, distance transform and D*, require a significant amount of computation for the planning phase, but the query phase is very cheap. However the plan depends on the goal. If the goal changes the expensive planning phase must be re-executed. Even though D* allows the path to be recomputed as the costmap changes it does not support a changing goal.

The disparity in planning and query costs has led to the development of roadmap methods where the query can include both the start and goal positions. The planning phase provides analysis that supports changing starting points and changing goals. A good analogy is making a journey by train. We first find a local path to the nearest train station, travel through the train network, get off at the station closest to our goal, and then take a local path to the goal. The train network is invariant and planning a path through the train network is straightforward. Planning paths to and from the entry and exit stations respectively is also straightforward since they are, ideally, short

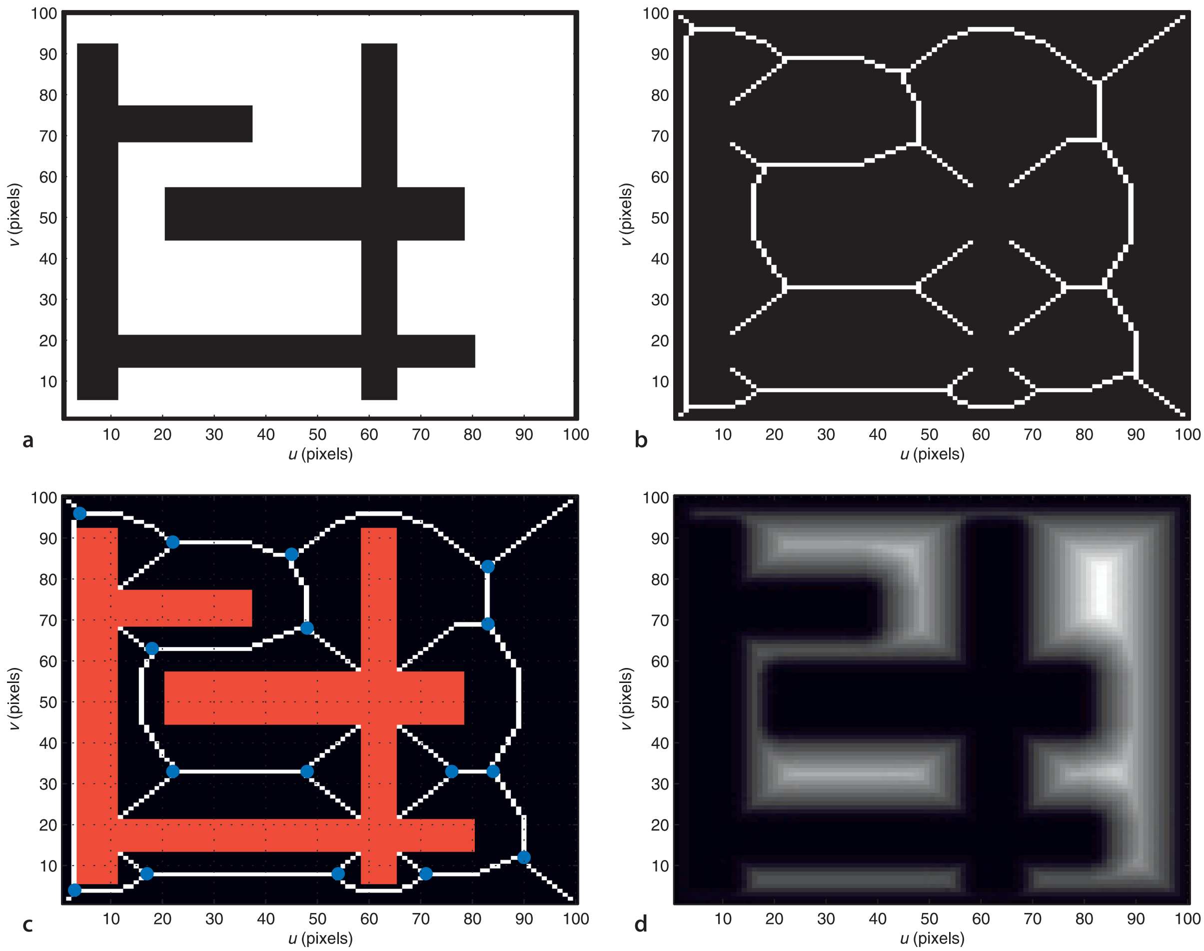


Fig. 5.9. Steps in the creation of a Voronoi roadmap. **a** Free space is indicated by white cells, **b** the skeleton of the free space is a network of adjacent cells no more than one cell thick, **c** the skeleton with the obstacles overlaid in red and roadmap junction points indicated in blue. **d** the distance transform of the obstacles, pixel values correspond to distance to the nearest obstacle

paths. The robot navigation problem then becomes one of building a network of obstacle free paths through the environment which serve the function of the train network. In the literature such a network is referred to as a roadmap. The roadmap need only be computed once and can then be used like the train network to get us from any start location to any goal location.

We will illustrate the principles by creating a roadmap from the occupancy grid's free space using some image processing techniques. The essential steps in creating the roadmap are shown in Fig. 5.9. The first step is to find the free space in the map which is simply the complement of the occupied space

```
>> free = 1 - map;
```

and is a matrix with non-zero elements where the robot is free to move. The boundary is also an obstacle so we mark the outermost cells as being not free

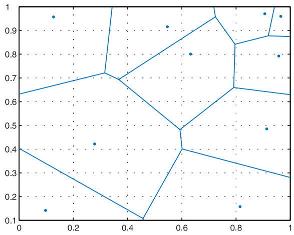
```
>> free(1,:) = 0; free(100,:) = 0;
>> free(:,1) = 0; free(:,100) = 0;
```

and this map is shown in Fig. 5.9a where free space is depicted as white.

The topological skeleton of the free space is computed by a morphological image processing algorithm known as thinning applied to the free space of Fig. 5.9a

```
>> skeleton = ithin(free);
```

and the result is shown in Fig. 5.9b. We see that the obstacles have grown and the free space, the white cells, have become a thin network of connected white cells which are



The Voronoi tessellation of a set of planar points, known as sites, is a set of Voronoi cells as shown to the left. Each cell corresponds to a site and consists of all points that are closer to its site than to any other site. The edges of the cells are the points that are equidistant to the two nearest sites. A generalized Voronoi diagram comprises cells defined by measuring distances to objects rather than points. In MATLAB® we can generate a Voronoi diagram by

```
>> sites = rand(10,2)
>> voronoi(sites(:,1), sites(:,2))
```

Georgy Voronoi (1868–1908) was a Russian mathematician, born in what is now Ukraine. He studied at Saint Petersburg University and was a student of Andrey Markov. One of his students Boris Delaunay defined the eponymous triangulation which has dual properties with the Voronoi diagram.

equidistant from the boundaries of the original obstacles. Image processing functions, and morphological operations in particular, will be explained more fully in Chap. 12.

Figure 5.9c shows the free space network overlaid on the original map. We have created a network of paths that span the space and which can be used for obstacle-free travel around the map. These paths are the edges of a generalized Voronoi diagram. We could obtain a similar result by computing the distance transform of the obstacles, Fig. 5.9a, and this is shown in Fig. 5.9d. The value of each pixel is the distance to the nearest obstacle and the ridge lines correspond to the skeleton of Fig. 5.9b. Thinning or skeletonization, like the distance transform, is a computationally expensive iterative algorithm but it illustrates well the principles of finding paths through free space. In the next section we will examine a cheaper alternative.

The junctions in the roadmap are indicated by blue circles. The junctions, or triple points, are identified using the morphological image processing function `triplepoint`.

5.2.4 Probabilistic Roadmap Method

The high computational cost of the distance transform and skeletonization methods makes them infeasible for large maps and has led to the development of probabilistic methods. These methods sparsely sample the world map and the most well known of these methods is the probabilistic roadmap or PRM method.

To use the Toolbox PRM planner for our problem we first create a `PRM` object

```
>> prm = PRM(map)
```

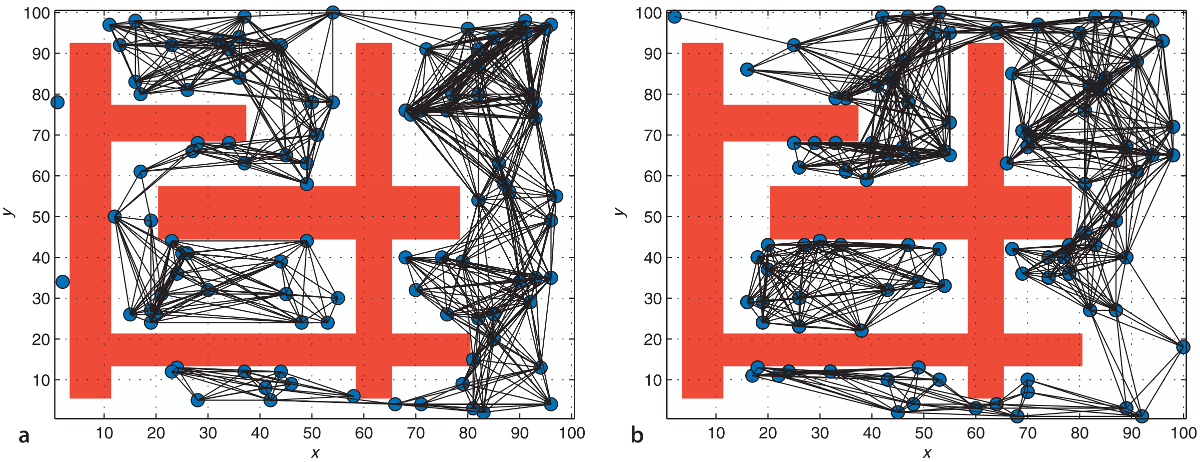
and then create the plan

```
>> prm.plan()
```

To replicate the following result be sure to initialize the random number generator first using `randinit`. See p. 101.

Note that in this case we do not pass `goal` as an argument to the planner since the plan is independent of the goal. Creating the path is a two phase process: planning, and query. The planning phase finds N random points, 100 by default, that lie in free space. Each point is connected to its nearest neighbours by a straight line path that does not cross any obstacles, so as to create a network, or graph, with a minimal number of disjoint components and no cycles. The advantage of PRM is that relatively few points need to be tested to ascertain that the points and the paths between them are obstacle free. The resulting network is stored within the `PRM` object and a summary can be displayed

```
>> prm
prm =
PRM: 100x100
  graph size: 100
  dist thresh: 30.000000
  100 vertices
  712 edges
  3 components
```



which indicates the number of edges and connected components in the graph. The graph can be visualized

```
>> prm.visualize()
```

as shown in Fig. 5.10a. The dots represent the randomly selected points and the lines are obstacle-free paths between the points. Only paths less than 30 cells long are selected which is the distance threshold parameter of the `PRM` class. Each edge of the graph has an associated cost which is the distance between its two nodes. The color of the node indicates which component it belongs to and each component is assigned a unique color. We see two nodes on the left-hand side that are disconnected from the bulk of the roadmap.

Fig. 5.10. Probablistic roadmap (PRM) planner and the random graphs produced in the planning phase. **a** Almost fully connected graph, apart from two nodes on the left-hand edge, **b** graph with a large disconnected component

The query phase is to find a path from the start point to the goal. This is simply a matter of moving to the closest node in the roadmap, following the roadmap, and getting off at the node closest to the goal and

```
>> prm.path(start, goal)
```

shows an animation of the robot moving through the graph and the path followed is shown in Fig. 5.11. Note that this time we provide the start and the goal position to the query phase. The next node on the roadmap is indicated in yellow and a line of green dots shows the robot's path. Travel along the roadmap involves moving toward the neighbouring node which has the lowest cost, that is, closest to the goal. We repeat the process until we arrive at the node in the graph closest to the goal, and from there we move directly to the goal.

An advantage of this planner is that once the roadmap is created by the planning phase we can change the goal and starting points very cheaply, only the query phase needs to be repeated.

However the path is not optimal and the distance travelled

```
>> p = prm.path(start, goal);
>> numcols(p)
ans =
   299
```

is greater than the optimal value found by the distance transform but less than that found by *bug2*.

There are some important tradeoffs in achieving this computational efficiency. Firstly, the underlying random sampling of the free space means that a different graph is created every time the planner is run, resulting in different paths and path lengths. For example rerunning the planner

```
>> prm.plan();
```

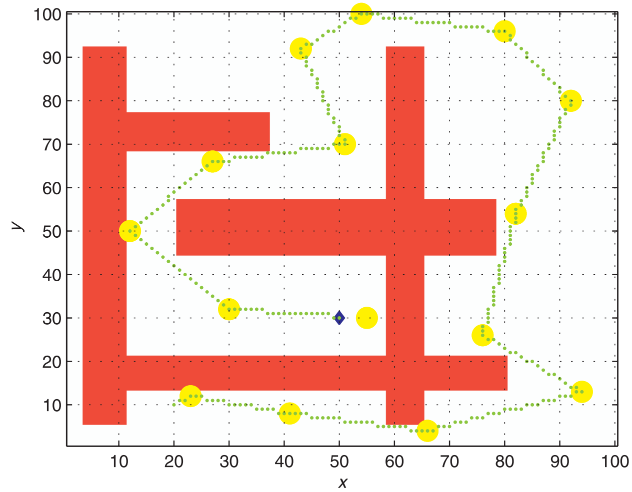


Fig. 5.11. Probablistic roadmap (PRM) planner showing the path taken by the robot, shown as green dots. The nodes of the roadmap that are visited are highlighted in yellow

Random numbers. The MATLAB® random number generator (used for `rand` and `randn`) generates a very long sequence of numbers that are an excellent approximation to a random sequence. The generator maintains an internal state which is effectively the position within the sequence. After startup MATLAB® always generates the following random number sequence

```
>> rand
ans =
     0.8147
>> rand
ans =
     0.9058
>> rand
ans =
     0.1270
```

Many algorithms discussed in this book make use of random numbers and this means that the results can never be repeated. Before all such examples in this book is an invisible call to `randinit` which resets the random number generator to a known state

```
>> randinit
>> rand
ans =
     0.8147
     >> rand
ans =
     0.9058
```

and we see that the random sequence has been restarted.

A real robot is not a point. We have assumed that the robot is a point, occupying a single cell in the occupancy grid. Some of the resulting paths which hug the sides of obstacles are impractical for a robot larger than a single cell. Rather than change the planning algorithms which are powerful and work very well we transform the obstacles. The Minkowski sum is used to *inflate* the obstacles to accomodate the worst-case pose of the robot in close proximity. The obstacles are replaced by virtual obstacles which are union of the obstacle and the robot in all its possible poses and just touching the boundary. The robot's various poses can be conservatively modelled as a circle that contains the robot.

If we consider the occupancy grid as an image then obstacle inflation can be achieved using the image processing operation known as dilation (discussed further in Chap. 12.). To inflate the obstacles with a circle of radius 3 cells the Toolbox command would be

```
>> map_new = imorph(map, kcircle(3), 'max');
```

produces the graph shown in Fig. 5.10b which has nodes at different locations and has a different number of edges.

Secondly, the planner can fail by creating a network consisting of disjoint components. The graph in Fig. 5.10a shows some disjoint components on the left-hand side, while the graph in Fig. 5.10b has a large disconnected component. If the start and goal positions are not connected by the roadmap, that is, they are close to different components the `path` method will report an error. The only solution is to rerun the planner.

Thirdly, long narrow gaps between obstacles are unlikely to be exploited since the probability of randomly choosing points that lie along such gaps is very low. In this example the planner has taken the longer counter-clockwise path around the obstacle unlike the optimal distance transform planner which was able to exploit the narrow vertical path on the left-hand side.

5.2.5 RRT

The next, and final, planner that we introduce is able to take into account the motion model of the vehicle, relaxing the assumption that the robot is capable of omni-directional motion.

Figure 5.12 shows a family of paths that the bicycle model of Eq. 4.2 would follow for discrete values of velocity, forward and backward, and steering wheel angle over a fixed time interval. This demonstrates clearly the subset of all possible configurations that a non-holonomic vehicle can reach from a given initial configuration. In this discrete example, from the initial pose we have computed 22 poses that the vehicle could achieve. From each of these we could compute another 22 poses that the vehicle could reach after two periods, and so on. After just a few periods we would have a very large number of possible poses.

For any desired goal pose we could find the closest precomputed pose, and working backward toward the starting pose we could determine the sequence of steering angles and velocities needed to move from initial to the goal pose. This has some similarities to the roadmap methods discussed previously, but the limiting factor is the combinatoric explosion in the number of possible poses.

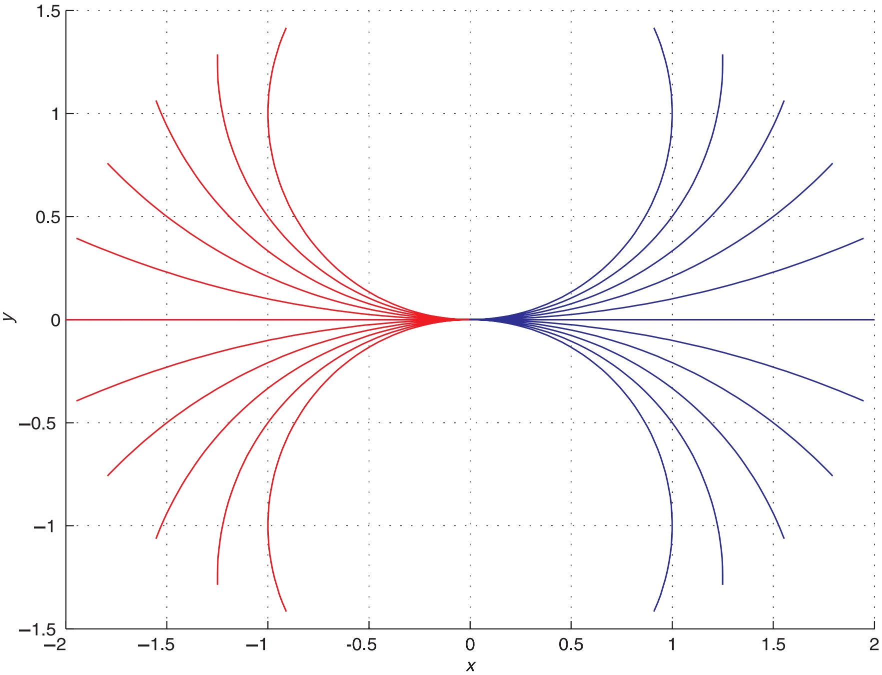


Fig. 5.12. A set of possible paths that the bicycle model robot could follow from an initial configuration of $(0, 0, 0)$. For $v = \pm 1$, $\alpha \in [-1, 1]$ over a 2 s period. Red lines correspond to $v < 0$

The particular planner that we discuss is the Rapidly-exploring Random Tree or RRT. Like PRM it is a probabilistic algorithm and the main steps are as follows. A graph of robot configurations is maintained and each node is a configuration $\xi \in SE(2)$ which is represented by a 3-vector $\xi \sim (x, y, \theta)$. The first node in the graph is some initial configuration of the robot. A random configuration ξ_{rand} is chosen, and the node with the closest configuration ξ_{near} is found – this point is near in terms of a cost function that includes distance and orientation. A control is computed that moves the robot from ξ_{near} toward ξ_{rand} over a fixed period of time. The point that it reaches is ξ_{new} and this is added to the graph.

The distance measure must account for a difference in position and orientation and requires appropriate weighting of these quantities. From a consideration of units this is not quite proper since we are adding metres and radians.

We create an RRT roadmap for an obstacle free environment by following our familiar programming pattern. We create an RRT object

```
>> rrt = RRT()
```

which includes a default bicycle kinematic model, velocity and steering angle limits. We create a plan and visualize the results

```
>> rrt.plan();
>> rrt.visualize();
```

which are shown in Fig. 5.13. We see how the paths have a good coverage of the configuration space, not just in the x- and y-directions but also in orientation, which is why the algorithm is known as *rapidly exploring*.

An important part of the RRT algorithm is computing the control input that moves the robot from an existing point in the graph to ξ_{rand}. From Sect. 4.2 we understand the difficulty of driving a non-holonomic vehicle to a specified pose. Rather than the complex non-linear controller of Sect. 4.2.4 we will use something simpler that fits with the randomized sampling strategy used in this class of planner. The controller randomly chooses whether to drive forwards or backwards and the steering angle,

Uniformly randomly distributed between the steering angle limits.

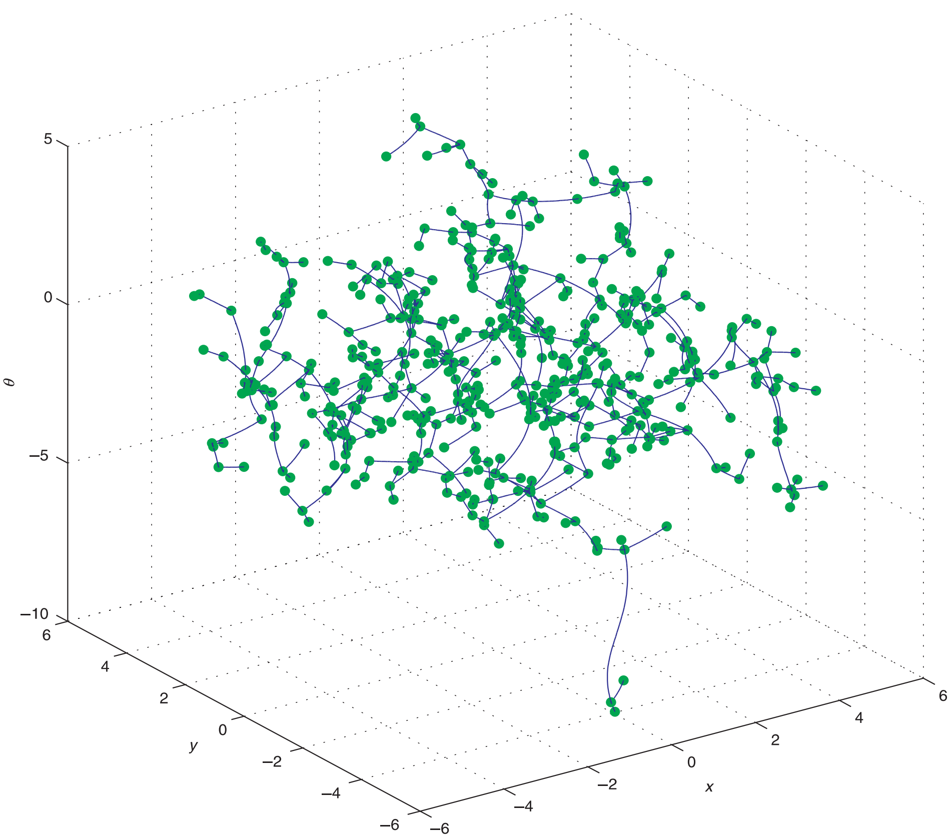


Fig. 5.13. An RRT computed for the bicycle model with a velocity of $\pm 1\ \text{m s}^{-1}$, steering angle limits of ± 1.2 rad, integration period of 1 s, and initial configuration of $(0, 0, 0)$. Each node is indicated by a green circle in the 3-dimensional space of vehicle poses (x, y, θ)

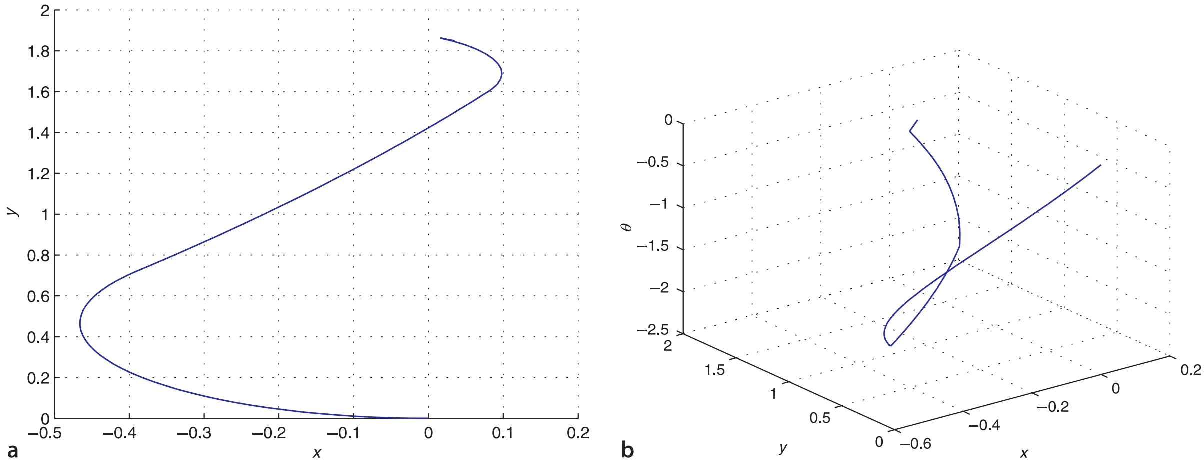


Fig. 5.14. The path computed by RRT that translates the non-holonomic vehicle sideways. **a** Path in the xy-plane; **b** path in (x, y, θ) space

simulates motion of the bicycle model for a fixed period of time, and computes the closest distance to ξ_{rand}. This is repeated multiple times and the control input with the best performance is chosen. The point on its path that was closest to ξ_{rand} is chosen as ξ_{near} and becomes a new node in the graph.

Handling obstacles with the RRT is quite straightforward. The point ξ_{rand} is discarded if it lies within an obstacle, and the point ξ_{near} will not be added to the graph if the path from ξ_{near} toward ξ_{rand} intersects an obstacle. The result is a set of paths, a roadmap, that is collision free and driveable with this non-holonomic vehicle.

We will illustrate this with the challenging problem of moving a non-holonomic vehicle sideways. Specifically we want to find a path to move the robot 2 m in the lateral direction with its final heading angle the same as its initial heading angle

```
>> p = rrt.path([0 0 0], [0 2 0]);
```

The result is a continuous path

```
>> about(p)
p [double] : 3x126 (3024 bytes)
```

which we can plot

```
>> plot2(p')
>> iprint('rrt_path2')
```

and the result is shown in Fig. 5.14. This is a smooth path that is feasible for the non-holonomic vehicle. The robot has an initial heading angle of 0 which means it is facing in the positive x-direction, so here it has driven backwards to the desired pose. Note also that the motion does not quite finish at the desired pose but at the node in the tree closest to the desired pose. This could be remedied by computing a denser tree, this one had 500 nodes, some adjustment of the steering command on the last segment of the motion, or using a local motion planner to move from the end of this path to the goal pose.

Although the steering angle is not continuous.

5.3 Wrapping Up

Robot navigation is the problem of guiding a robot towards a goal and we have covered a spectrum of approaches. The simplest was the purely reactive Braitenberg-type vehicle. Then we added limited memory to create state machine based automata such as *bug2* which can deal with obstacles, however the paths that it find are far from optimal.

A number of different map-based planning algorithms were then introduced. The distance transform is a computationally intense approach that finds an optimal path to the goal. D* also finds a optimal path, but accounts for traversibility of individual cells rather than considering them as either free space or obstacle. D^* also supports computationally cheap incremental replanning for small changes in the map. PRM reduces the computational burden by probabilistic sampling but at the expense of less optimal paths. In particular it may not discover narrow routes between areas of free space. Another sampling method is RRT which uses a kinematic model of the vehicle to create paths which are feasible to drive, and can readily account for the orientation of the vehicle as well as its position. All the map-based approaches require a map and knowledge of the robot's location, and these are both topics that we will cover in the next chapter.

Further Reading

The defining book in cybernetics was written by Wiener in 1948 and updated in 1965 (Wiener 1965). Grey Walter published a number of popular articles (1950, 1951) and a book (1953) based on his theories and experiments with robotic tortoises.

The definitive reference for Braitenberg vehicles is Braitenberg's own book (1986) which is a whimsical, almost poetic, set of thought experiments. Vehicles of increasing complexity (fourteen vehicle families in all) are developed, some including non-linearities, memory and logic to which he attributes anthropomorphic characteristics such as love, fear, agression and egotism. The second part of the book outlines the factual basis of these machines in the neural structure of animals. The *bug1* and *bug2* algorithms were described by Lumelsky and Stepanov (1986). More recently eleven variations of Bug algorithm were implemented and compared for a number of different environments (Ng and Bräunl 2007).

Early behaviour-based robots included the Johns Hopkins Beast, built in the 1960s, and Genghis (Brooks 1989) built in 1989. Behaviour-based robotics are covered in the book by Arkin (1999) and the Robotics Handbook (Siciliano and Khatib 2008, § 38). Matarić's Robotics Primer (Matarić 2007) and associated comprehensive web-based resources is also an excellent introduction to reactive control, behaviour based control and robot navigation. A rich collection of archival material about early cybernetic machines, including Gray-Walter's tortoise and the Johns Hopkins Beast can be found at the Cybernetic Zoo http://cyberneticzoo.com.

The distance transform is well described by Borgefors (1986) and its early application to robotic navigation was explored by Jarvis and Byrne (1988). The D^* algorithm is an extension of the classic A^* algorithm for graphs (Nilsson 1971). It was proposed by Stentz (1994) and later extensions include Field D^* (Ferguson and Stentz 2006) and D^* lite (Koenig and Likhachev 2002). D^* was used by many vehicles in the DARPA challenges (Buehler et al. 2007, 2010).

The ideas behind PRM started to emerge in the mid 1990s and it was first described by Kavraki et al. (1996). Geraerts and Overmars (2004) compare the efficacy of a number of subsequent variations that have been proposed to the basic PRM algorithm. Approaches to planning that incorporate the vehicles dynamics include state-space sampling (Howard et al. 2008), and the RRT which is described in LaValle (1998, 2006) as well as http://msl.cs.uiuc.edu/rrt.

Two recent books provide almost encyclopedic coverage of planning for robots. The book on robot motion by Choset et al. (2005) covers geometric and probabilistic approaches to planning as well as the application to robots with dynamics and non-holonomic constraints. The book on robot planning by LaValle (2006) covers motion planning, planning under uncertainty, sensor-based planning, reinforcement learning, nonlinear systems, trajectory planning and nonholonomic planning. The powerful planning techniques discussed in these books can be applied beyond robotics to very high order systems such as complex mechanisms or even the shape of molecules. More succinct coverage is provided by Siegwart et al. (2011), the Robotics Handbook (Siciliano and Khatib 2008, § 35), and also in Spong et al. (2006) and Siciliano et al. (2008).

Exercises

1. Braitenberg vehicles (page 88)
   a) Experiment with different starting configurations and control gains.
   b) Modify the signs on the steering signal to make the vehicle light-phobic.
   c) Modify the `sensorfield` function so that the peak moves with time.
   d) The vehicle approaches the maxima asymptotically. Add a stopping rule so that the vehicle stops when the when either sensor detects a value greater than 0.95.
   e) Create a scalar field with two peaks. Can you create a starting pose where the robot gets confused?
2. Bug algorithms (page 90)
   a) Using the function `makemap` create a new map to challenge *bug2*. Try different starting points. Is it possible to trap *bug2*?
   b) Create an obstacle map that contains a maze. Can *bug2* solve the maze?
   c) Implement other bug algorithms such as *bug1* and *tangent bug*. Do they perform better or worse?
3. At 1 m cell size how much memory is required to represent the surface of the Earth? How much memory is required to represent just the land area of Earth? What cell size is needed in order for a map of your country to fit in 1 Gbyte of memory?
4. Distance transform (page 93). A real robot has finite dimensions and a common technique for use with the point-robot planning methods is to grow the obstacles by half the radius of the robot. Use the Toolbox function `imorph` (see page 317) to dilate the obstacles by 4 grid cells.
5. For the D^* planner (page 95) increase the cost of the rough terrain and observe what happens. Add a region of very low-cost terrain (less than one) near the robot's path and observe what happens.
6. PRM planner (page 99)
   a) Run the PRM planner 100 times and gather statistics on the resulting path length.
   b) Vary the value of the distance threshold parameter and observe the effect.
   c) Implement a non-grid based version of PRM. The robot is represented by an arbitrary polygon as are the obstacles. You will need functions to determine if a polygon intersects or is contained by another polygon (see the Toolbox `Polygon` class). Test the algorithm on the piano movers problem.
7. RRT planner (page 102)
   a) Find a path to implement a 3-point turn.
   b) Define an obstacle field and repeat the planning.
   c) Experiment with RRT parameters such as the number of points, the vehicle steering angle limits, and the path integration time.
   d) The current RRT chooses the steering angle as a uniform distribution between the steering angle limits. People tend to drive more gently, what happens if you choose a Gaussian distribution for the steering angle?
   e) Additional information in the node of each graph holds the control input that was computed to reach the node. Plot the steering angle and velocity sequence required to move from start to goal pose.
   f) Add a local planner to move from initial pose to the closest vertex, and from the final vertex to the goal pose.
   g) Determine a path through the graph that minimizes the number of reversals of direction.
   h) Add a more sophisticated collision detector where the vehicle is a finite sized rectangle and the world has polygonal obstacles. You will need functions to determine if a polygon intersects or is contained by another polygon (see the Toolbox `Polygon` class).