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# Logistic Regression

## Muti-class in text classification

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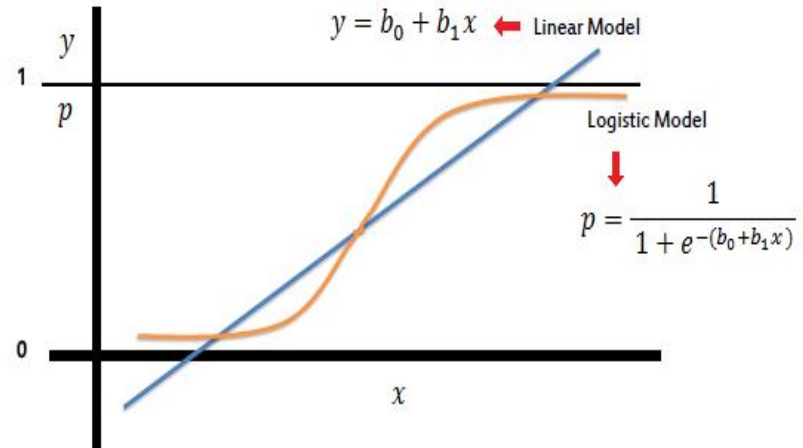
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# 1. logistic regression?

Logistic regression is the appropriate regression analysis to conduct when the dependent variable is dichotomous (binary

Like all regression analyses, the logistic regression is a predictive analysis.

Logistic regression is used to describe data and to explain the relationship between one dependent binary variable and one or more nominal, ordinal, interval or ratio-level independent variables.

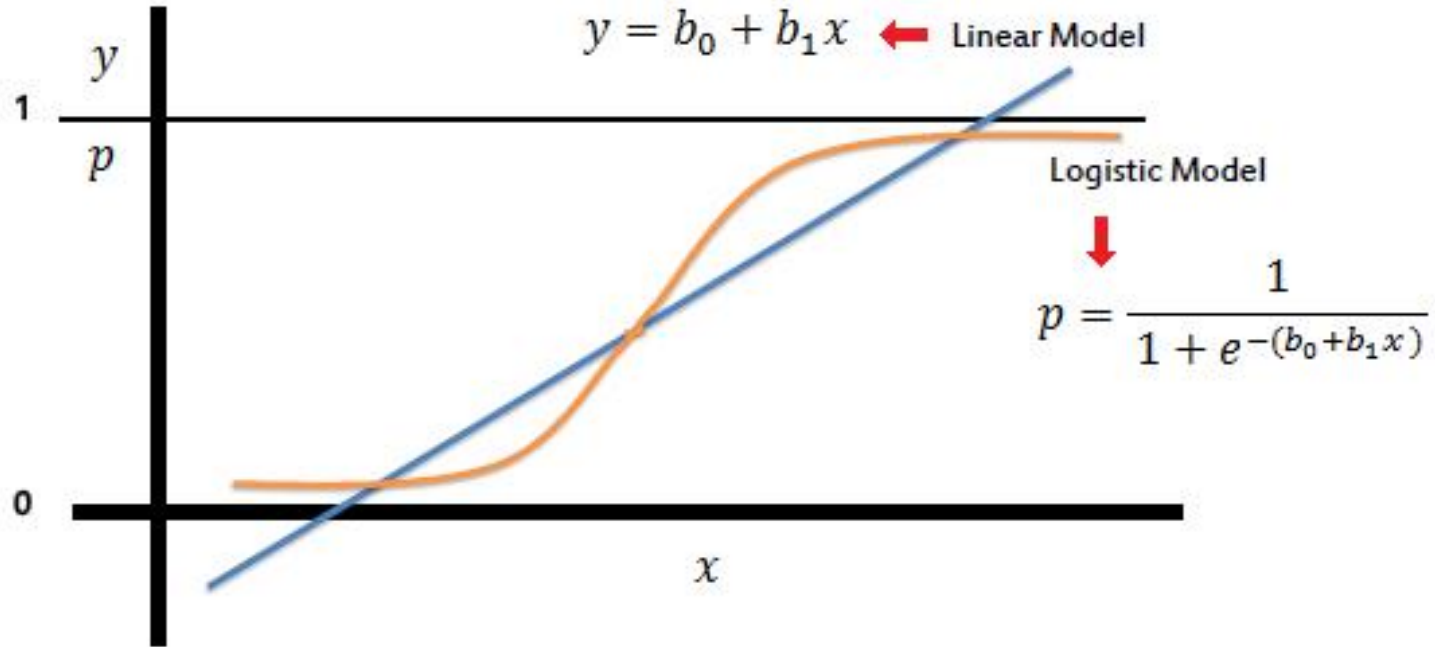




# Intro

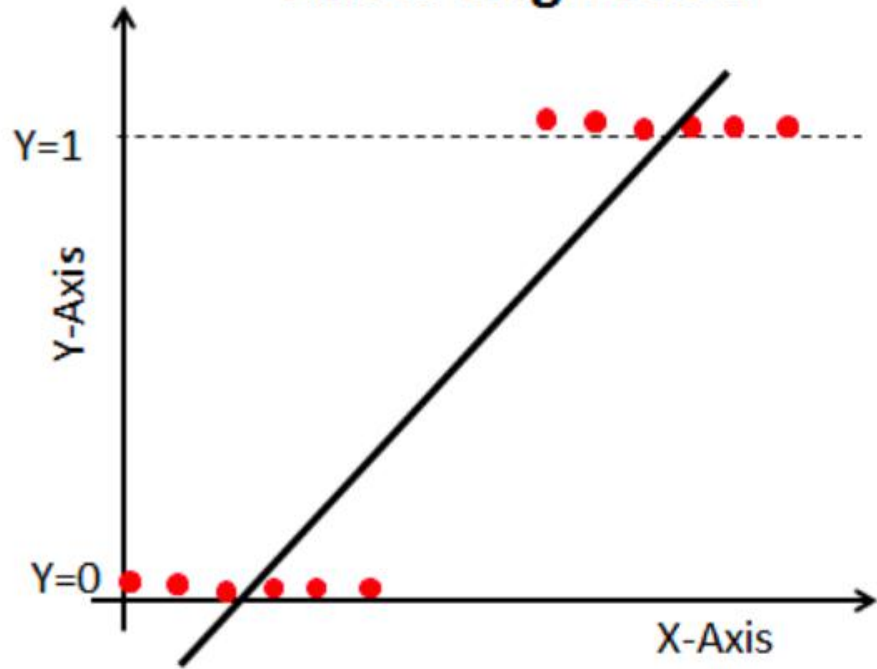
- Logistic Regression Model
- Sigmoid function
- Loss function
- Stochastic Gradient Descent
- Logistic Sigmoid Regression
- Demo

# – Logistic Regression Model

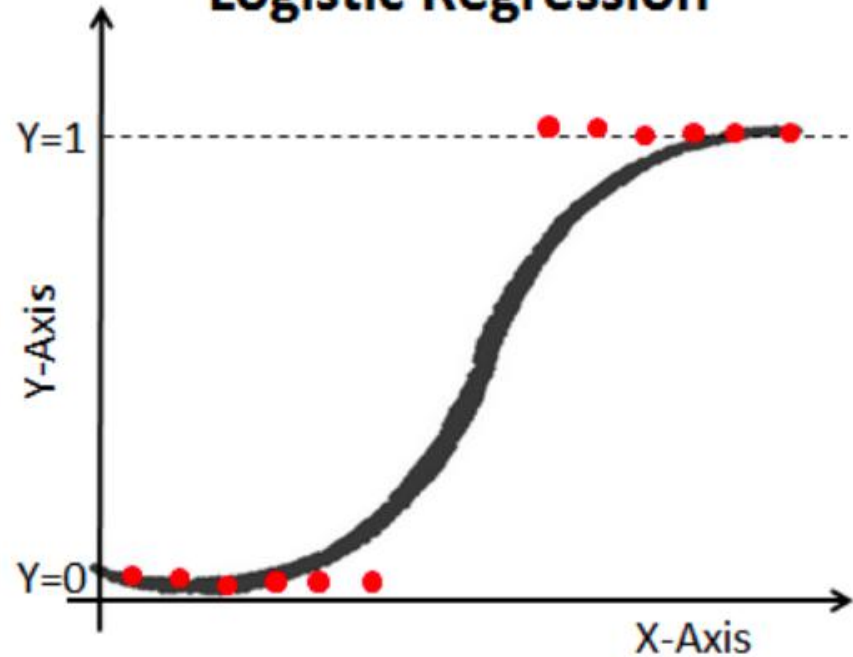


# – Logistic Regression Model

**Linear Regression**

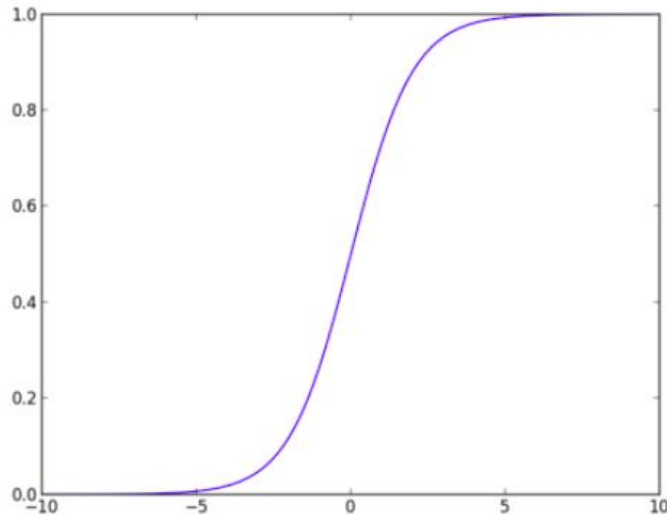


**Logistic Regression**



# – Sigmoid function

$$f(x) = \frac{1}{1 + e^{-(x)}}$$



*Sigmoid function*  $f(s) = \frac{1}{1+e^{-s}}$

$$\begin{aligned} f'(s) &= \frac{e^{-s}}{(1+e^{-s})^2} = \frac{1}{1+e^{-s}} \cdot \frac{e^{-s}}{1+e^{-s}} = \frac{1}{1+e^{-s}} \cdot \frac{e^{-s}+1-1}{1+e^{-s}} \\ &= \frac{1}{1+e^{-s}} \cdot \left( \frac{e^{-s}+1}{1+e^{-s}} - \frac{1}{1+e^{-s}} \right) \\ &= f(s) \cdot (1 - f(s)) \end{aligned}$$

$$\lim_{s \rightarrow -\infty} f(s) = 0; \quad \lim_{s \rightarrow +\infty} f(s) = 1$$

## Loss function

$$P(y_i = 1 | \mathbf{x}_i; \mathbf{w}) = f(\mathbf{w}^T \mathbf{x}_i) \quad (1)$$

$$P(y_i = 0 | \mathbf{x}_i; \mathbf{w}) = 1 - f(\mathbf{w}^T \mathbf{x}_i) \quad (2)$$

$$z_i = f(\mathbf{w}^T \mathbf{x}_i)$$

$$P(y_i | \mathbf{x}_i; \mathbf{w}) = z_i^{y_i} (1 - z_i)^{1-y_i}$$



# Loss function

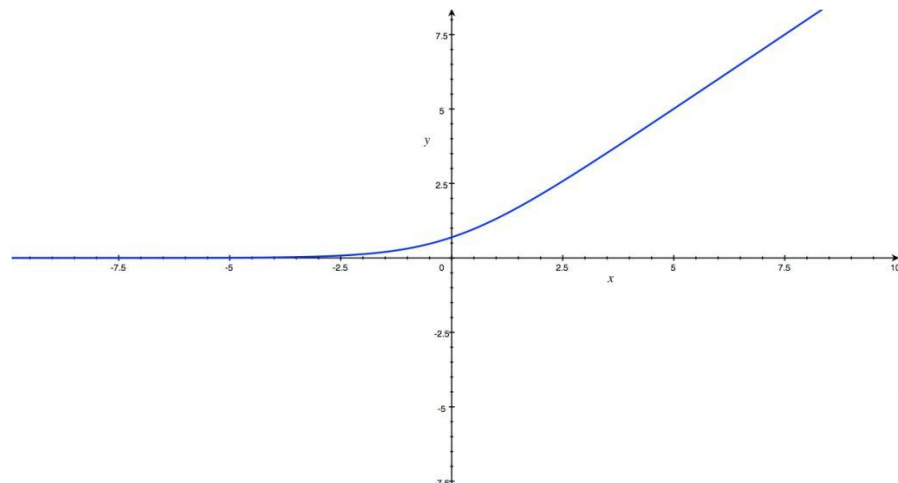
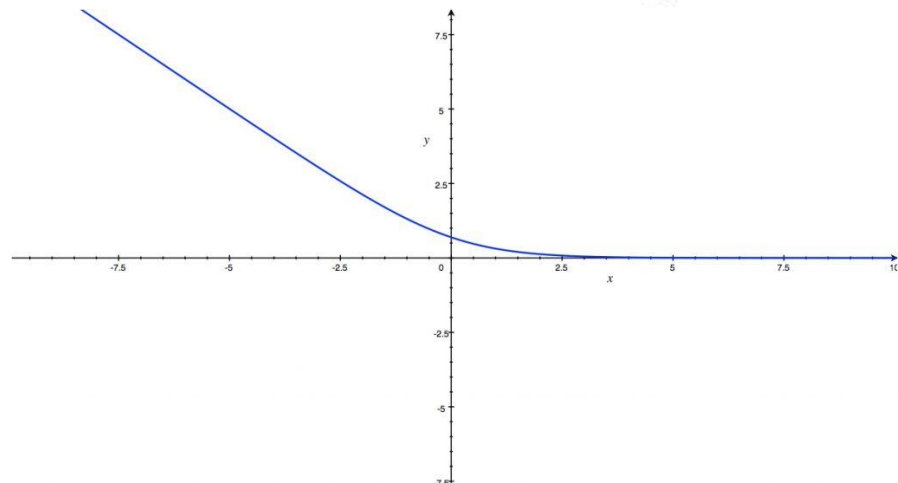
$$\begin{aligned} P(\mathbf{y}|\mathbf{X}; \mathbf{w}) &= \prod_{i=1}^N P(y_i | \mathbf{x}_i; \mathbf{w}) \\ &= \prod_{i=1}^N z_i^{y_i} (1 - z_i)^{1-y_i} \end{aligned}$$

## *Loss function*

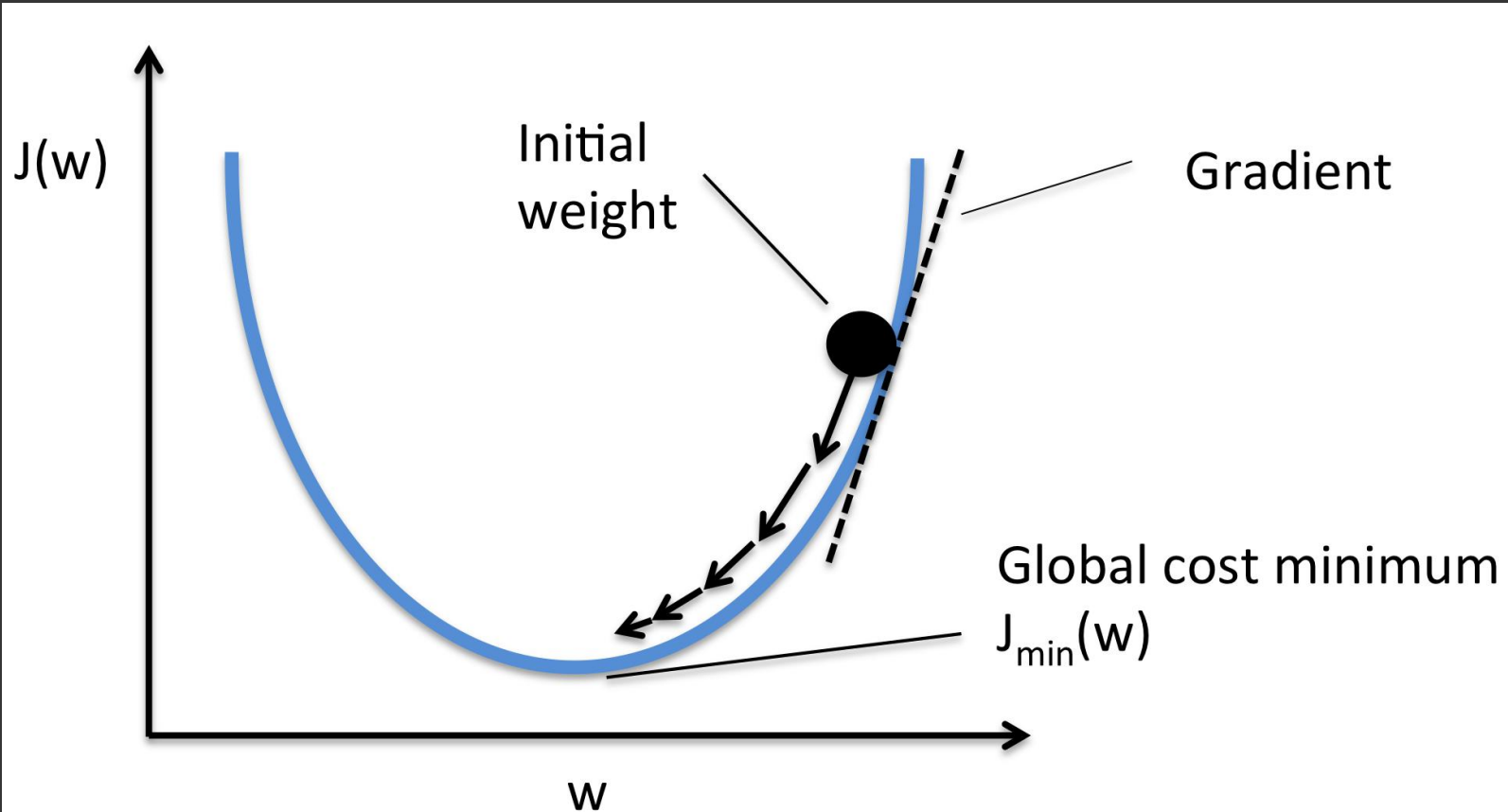
$$\begin{aligned} J(\mathbf{w}) &= -\log P(\mathbf{y}|\mathbf{X}; \mathbf{w}) \\ &= -\sum_{i=1}^N (y_i \log z_i + (1 - y_i) \log(1 - z_i)) \end{aligned}$$

# Loss function

$$J(\theta) = \begin{cases} -\log(h(x)) & (\text{if } y = 1) \\ -\log(1 - (h(x))) & (\text{if } y = 0) \end{cases}$$



# Stochastic Gradient Descent



# *Stochastic Gradient Descent*

$$J(\mathbf{w}; \mathbf{x}_i, y_i) = -(y_i \log z_i + (1 - y_i) \log(1 - z_i))$$

$$\begin{aligned} \frac{\partial J(\mathbf{w}; \mathbf{x}_i, y_i)}{\partial \mathbf{w}} &= -\left(\frac{y_i}{z_i} - \frac{1 - y_i}{1 - z_i}\right) \frac{\partial z_i}{\partial \mathbf{w}} \\ &= \frac{z_i - y_i}{z_i(1 - z_i)} \frac{\partial z_i}{\partial \mathbf{w}} \end{aligned}$$

# Stochastic Gradient Descent

Chain rule:  $\frac{\partial z_i}{\partial \mathbf{w}} = \frac{\partial z_i}{\partial s} \frac{\partial s}{\partial \mathbf{w}} = \frac{\partial z_i}{\partial s} \mathbf{x}$

$$\frac{\partial J(\mathbf{w}; \mathbf{x}_i, y_i)}{\partial \mathbf{w}} = (z_i - y_i) \mathbf{x}_i$$

$$\mathbf{w} = \mathbf{w} + \eta(y_i - z_i) \mathbf{x}_i$$



## 2. Demo

# Maximum Entropy Model

feature

$$\frac{1}{|\mathcal{D}|} \sum_{d \in \mathcal{D}} f_i(d, c(d)) = \sum_d P(d) \sum_c P(c|d) f_i(d, c). \quad (1)$$

$$\frac{1}{|\mathcal{D}|} \sum_{d \in \mathcal{D}} f_i(d, c(d)) = \frac{1}{|\mathcal{D}|} \sum_{d \in \mathcal{D}} \sum_c P(c|d) f_i(d, c). \quad (2)$$

$$f_{w,c'}(d, c) = \begin{cases} 0 & \text{if } c \neq c' \\ \frac{N(d,w)}{N(d)} & \text{Otherwise,} \end{cases}$$