TRƯỜNG ĐẠI HỌC BÁCH KHOA HÀ NỘI
HANOI UNIVERSITY OF SCIENCE AND TECHNOLOGY

Database
Lesson 3. Relational algebra

Ba-Lam Do

Learning Map

Sequence	Title
1	Introduction to Databases
2	Relational Databases
3	Relational Algebra
4	Structured Query Language – Part 1
5	Structured Query Language – Part 2
6	Constraints and Triggers
7	Entity Relationship Model
8	Functional Dependency
9	Normalization
10	Storage - Indexing
11	Query Processing
12	Transaction Management – Part 1
13	Transaction Management – Part 2

Outline

1. Introduction to relational algebra
2. Set operators
3. Relational operators: Projection, Selection, Rename, Join
4. Common extensions

Learning objectives

- Upon completion of this lesson, students will be able to:
  - Understand relational algebra operators
  - Write relational algebraic expressions

Keywords

Relation	Is thought of as a table of values, each row in the table represents a collection of related data values.
Set	A collection of objects
Operator	Is a special token that represent computations such as union, minus, selection, join, etc.
Expression	A mathematical phrase that is built up from operators and operands.

Database Schema

student(**student_id**, first_name,last_name, dob, gender,address,note,*clazz_id*)

clazz(**clazz_id**, name, *lecturer_id*, *monitor_id*)

subject(**subject_id**, name, credit, percentage_final_exam)

enrollment(***student_id*, *subject_id*, *semester***, midterm_score, final_score)

lecturer(**lecturer_id**, first_name, last_name, dob, gender, address, email)

teaching(***subject_id*, *lecturer_id***)

grade(**code**, from_score, to_score)

1.1. Different data models

- Hierarchical database model
- Network model
- Object-oriented database model
- Relational model
- Entity-relationship model
- Document model
- …

Database

student

student_id	first_name	last_name	dob	…	clazz_id
20160001	Ngọc An	Bùi	3/18/1987	…	
20160002	Anh	Hoàng	5/20/1987	…	20162101
20160003	Thu Hồng	Trần	6/6/1987	…	20162101
20160004	Minh Anh	Nguyễn	5/20/1987	…	20162101
20170001	Nhật Ánh	Nguyễn	5/15/1988	…	20172201

subject

subject_id	name	credit	percentage_final_exam
IT1110	Tin học đại cương	4	60
IT3080	Mạng máy tính	3	70
IT3090	Cơ sở dữ liệu	3	70
IT4857	Thị giác máy tính	3	60
IT4866	Học máy	2	70

clazz

clazz_id	name	lecturer_id	monitor_id
20162101	CNTT1.01-K61	02001	20160003
20162102	CNTT1.02-K61		
20172201	CNTT2.01-K62	02002	20170001
20172202	CNTT2.02-K62		

enrollment

student_id	subject_id	semester	midterm_score	final_score
20160001	IT1110	20171	9	8.5
20160001	IT3080	20172	8	
20160001	IT3090	20172	6	9
20160001	IT4857	20172	7.5	9
20160001	IT4866	20172	7	9
20160002	IT3080	20172	9	
20160003	IT1110	20171	7	6
20160004	IT1110	20171	6	5

1. Introduction to relational algebra

- Relational algebra providing a theoretical foundation for relational databases, particularly query languages for relational databases
- Relational algebra expression is composed of one or several relational algebraic operators
  - Operator: represent computations
    - Input: one or two relation
    - Output: a relation
  - - Unary operator (one input) vs. binary operator (two inputs)

2. Set operators

- Union
- Intersection
- Difference
- Cartesian product

2.1. Set operators: Union

- Union: combining the tuples from two input union-compatible relations (having the same set of attributes)

clazz

clazz_id	name	lecturer_id	monitor_id
20162101	CNTT1.01-K61	02001	20160003

∪

clazz 2

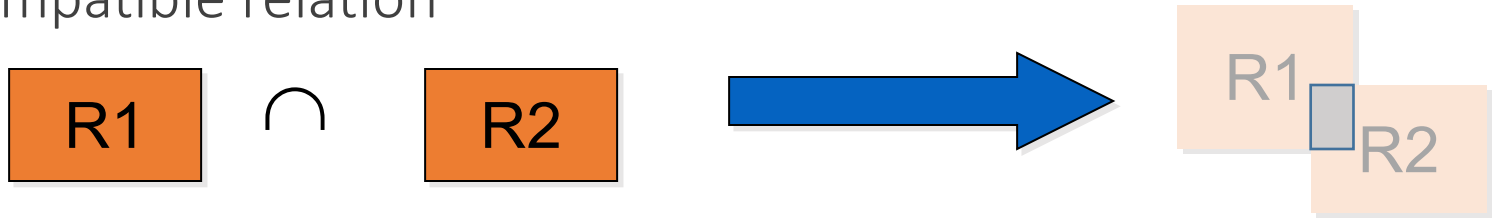
clazz_id	name	lecturer_id	monitor_id
20172201	CNTT2.01-K62	02002	20170001

result

clazz_id	name	lecturer_id	monitor_id
20162101	CNTT1.01-K61	02001	20160003
20172201	CNTT2.01-K62	02002	20170001

2.2. Set operators: Intersection

- Intersection: Keeping only common tuples from 2 input union-compatible relation



clazz

clazz_id	name	lecturer_id	monitor_id
20162102	CNTT1.02-K61		
20172201	CNTT2.01-K62	02002	20170001
20172202	CNTT2.02-K62		

∩

clazz 2

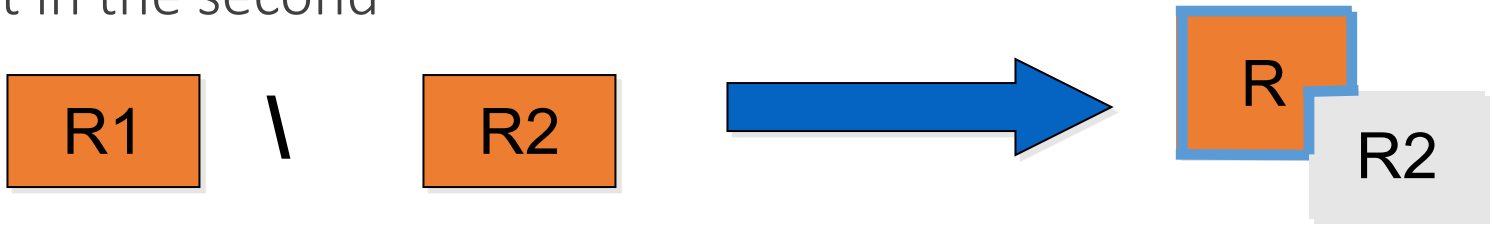
clazz_id	name	lecturer_id	monitor_id
20162101	CNTT1.01-K61	02001	20160003
20162102	CNTT1.02-K61		
20172201	CNTT2.01-K62	02002	20170001

result

clazz_id	name	lecturer_id	monitor_id
20162102	CNTT1.02-K61		
20172201	CNTT2.01-K62	02002	20170001

2.3. Set operators: Difference

- Difference: containing tuples occurred in the first relation but not in the second



clazz

clazz_id	name	lecturer_id	monitor_id
20162101	CNTT1.01-K61	02001	20160003
20162102	CNTT1.02-K61		
20172201	CNTT2.01-K62	02002	20170001
20172202	CNTT2.02-K62		

\

clazz 2

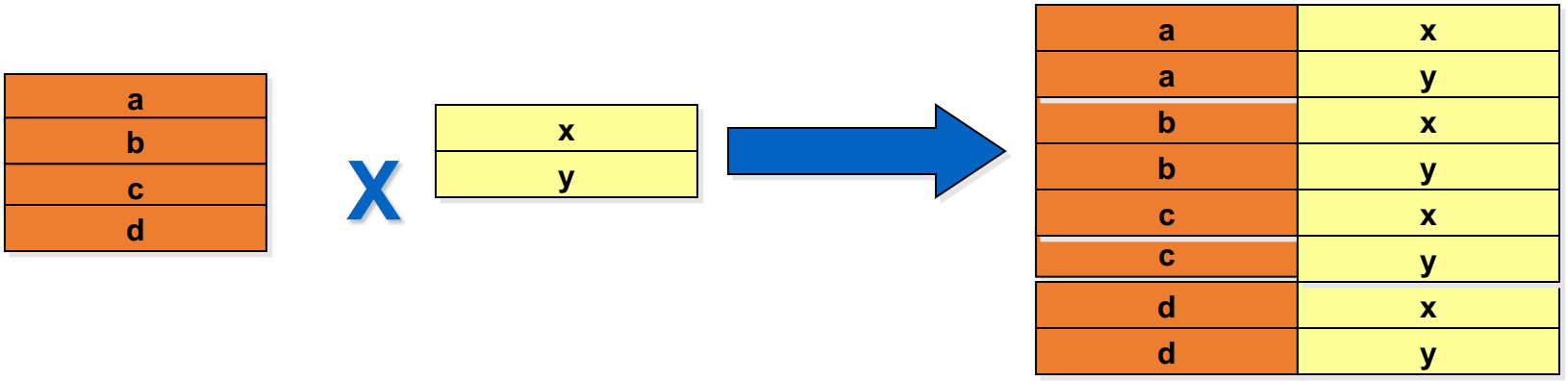
clazz_id	name	lecturer_id	monitor_id
20172202	CNTT2.02-K62		
20162102	CNTT1.02-K61		

result

clazz_id	name	lecturer_id	monitor_id
20162101	CNTT1.01-K61	02001	20160003
20172201	CNTT2.01-K62	02002	20170001

2.4. Set operators: Cartesian product

- Cartesian Product: the concatenation of every tuple of one relation with every tuple of the other relation.



3. Relational algebraic operators

- Selection
- Projection
- Rename
- Join
- Division

3.1. Relational algebraic operators: Selection

- Selection: choose from R each tuple where the condition holds.

$$\sigma_{<condition>}(R)$$

clazz

clazz_id	name	lecturer_id	monitor_id
20162101	CNTT1.01-K61	02001	20160003
20162102	CNTT1.02-K61		
20172201	CNTT2.01-K62	02002	20170001
20172202	CNTT2.02-K62		

$$\sigma_{\text{lecturer\_id != null}}(\text{clazz})$$

result

clazz_id	name	lecturer_id	monitor_id
20162101	CNTT1.01-K61	02001	20160003
20172201	CNTT2.01-K62	02002	20170001

3.2. Relational algebraic operators: Projection

- Projection: Choose some attributes

$$\prod_{A1,A2,\ldots}(R)$$

clazz

clazz_id	name	lecturer_id	monitor_id
20162101	CNTT1.01-K61	02001	20160003
20162102	CNTT1.02-K61		
20172201	CNTT2.01-K62	02002	20170001
20172202	CNTT2.02-K62		

$$\prod_{\text{clazz\_id, name}}(\text{clazz})$$

result

clazz_id	name
20162101	CNTT1.01-K61
20172201	CNTT2.01-K62

3.3. Relational algebraic operators: Rename

- Rename: result is identical to R except that the b attribute in all tuples is renamed to an a attribute

$$\rho_{a|b}(R)$$

clazz

clazz_id	name	lecturer_id	monitor_id
20162101	CNTT1.01-K61	02001	20160003
20162102	CNTT1.02-K61		
20172201	CNTT2.01-K62	02002	20170001
20172202	CNTT2.02-K62		

$$\rho_{\text{name of class} \mid \text{name}}(\text{clazz})$$

clazz

clazz_id	Name of class	lecturer_id	monitor_id
20162101	CNTT1.01-K61	02001	20160003
20162102	CNTT1.02-K61		
20172201	CNTT2.01-K62	02002	20170001
20172202	CNTT2.02-K62		

3.4. Relational algebraic operators: Join

- Join: Combine attributes from 2 tables

$$R_1 \triangleright\!\triangleleft_{<join\_condition>} R_2$$

3.4. Relational algebraic operators: Join example

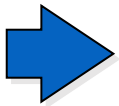
Student ⋈ clazz

student

student_id	first_name	last_name	…	clazz_id
20160001	Ngọc An	Bùi		
20160002	Anh	Hoàng		20162101
20160003	Thu Hồng	Trần		20162101
20160004	Minh Anh	Nguyễn		20162101
20170001	Nhật Ánh	Nguyễn		20172201

clazz

clazz_id	name	lecturer_id	monitor_id
20162101	CNTT1.01-K61	02001	20160003
20162102	CNTT1.02-K61		
20172201	CNTT2.01-K62	02002	20170001
20172202	CNTT2.02-K62		

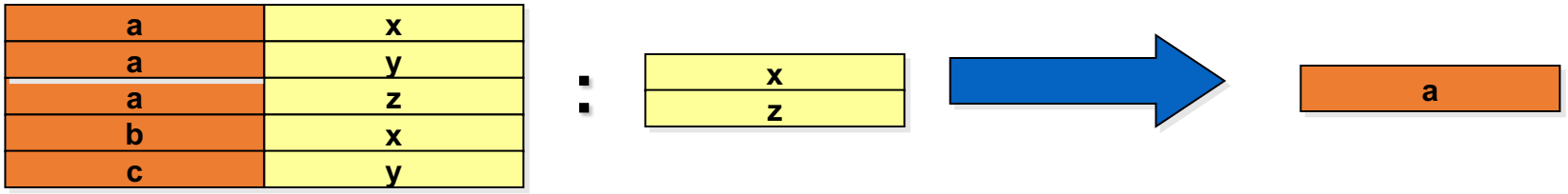


result

student_id	first_name	last_name	name
20160002	Anh	Hoàng	CNTT1.01-K61
20160003	Thu Hồng	Trần	CNTT1.01-K61
20160004	Minh Anh	Nguyễn	CNTT1.01-K61
20170001	Nhật Ánh	Nguyễn	CNTT2.01-K62

3.5. Relational algebraic operators: Division

- Division: divides a dividend relation R1 or degree m+n by a divisor relation R2 of degree n, and produces a quotient relation of degree m.



3.5. Relational algebraic operators: Division example

- List student_id who enroll in all subjects

enrollment

student_id	subject_id	semester	midterm_ score	final_ score
20160001	IT1110	20171	9	8.5
20160001	IT3080	20172	8	
20160001	IT3090	20172	6	9
20160001	IT4857	20172	7.5	9
20160001	IT4866	20172	7	9
20160002	IT3080	20172	9	
20160003	IT1110	20171	7	6
20160004	IT1110	20171	6	5

subject

subject_id	name	credit	percentage_ final_exam
IT1110	Tin học đại cương	4	60
IT3080	Mạng máy tính	3	70
IT3090	Cơ sở dữ liệu	3	70
IT4857	Thị giác máy tính	3	60
IT4866	Học máy	2	70

3.6. Relational algebraic operators: Division example [2]

- List student_id who enroll in all subjects

enrollment

student_id	subject_id	semester	midterm_ score	final_ score
20160001	IT1110	20171	9	8.5
20160001	IT3080	20172	8	
20160001	IT3090	20172	6	9
20160001	IT4857	20172	7.5	9
20160001	IT4866	20172	7	9
20160002	IT3080	20172	9	
20160003	IT1110	20171	7	6
20160004	IT1110	20171	6	5

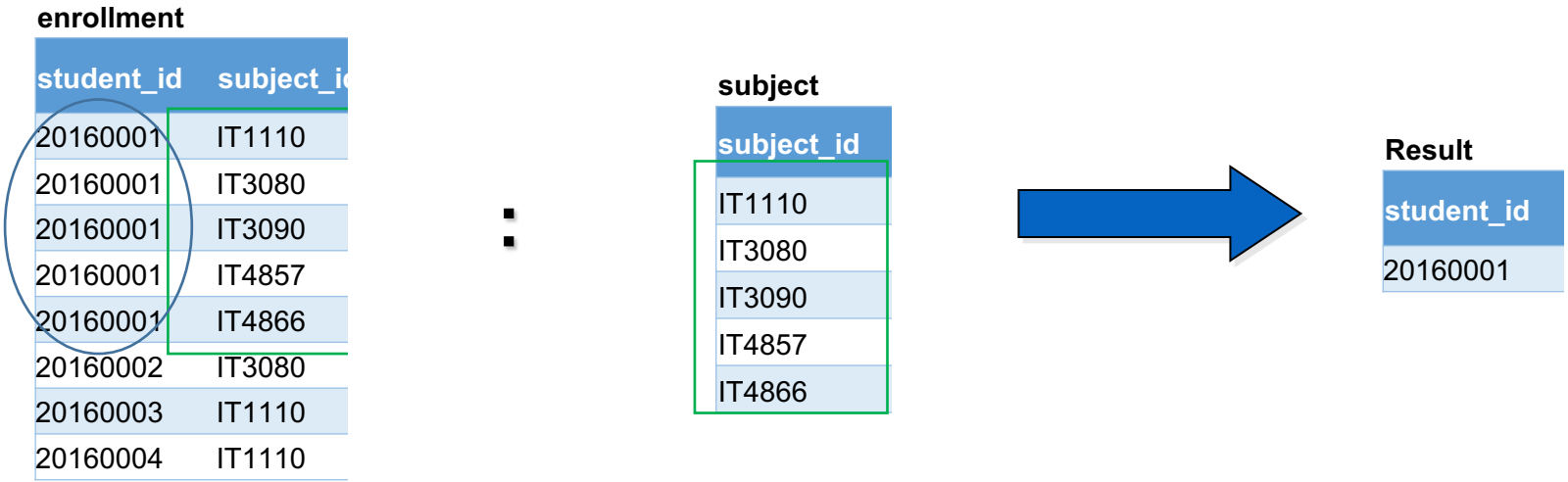
subject

subject_id	name	credit	percentage_ final_exam
IT1110	Tin học đại cương	4	60
IT3080	Mạng máy tính	3	70
IT3090	Cơ sở dữ liệu	3	70
IT4857	Thị giác máy tính	3	60
IT4866	Học máy	2	70

$\Pi_{\text{student\_id, subject\_id}}$ (enrollement)

$\Pi_{\text{subject\_id}}$ (subject)

3.6. Relational algebraic operators: Division example [3]



$(\Pi_{\text{student\_id, subject\_id}}(\text{enrollement})) \div (\Pi_{\text{subject\_id}}(\text{subject}))$

4. Common extension

- Natural join
- Outer join
- Aggregation

4.1. Common extension: Natural join

- Natural join: Special join operation with equal join condition on their common attributes, noted *

student

student_id	first_name	last_name	dob	…	clazz_id
20160001	Ngọc An	Bùi	3/18/1987	…	
20160002	Anh	Hoàng	5/20/1987	…	20162101
20160003	Thu Hồng	Trần	6/6/1987	…	20162101
20160004	Minh Anh	Nguyễn	5/20/1987	…	20162101
20170001	Nhật Ánh	Nguyễn	5/15/1988	…	20172201

clazz

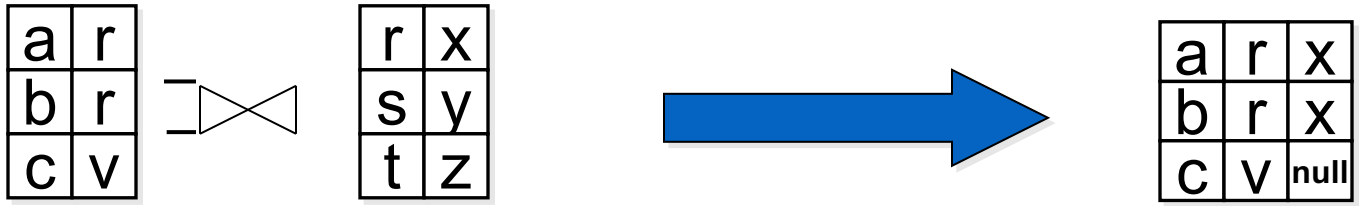
clazz_id	name	lecturer_id	monitor_id
20162101	CNTT1.01-K61	02001	20160003
20162102	CNTT1.02-K61		
20172201	CNTT2.01-K62	02002	20170001
20172202	CNTT2.02-K62		

results

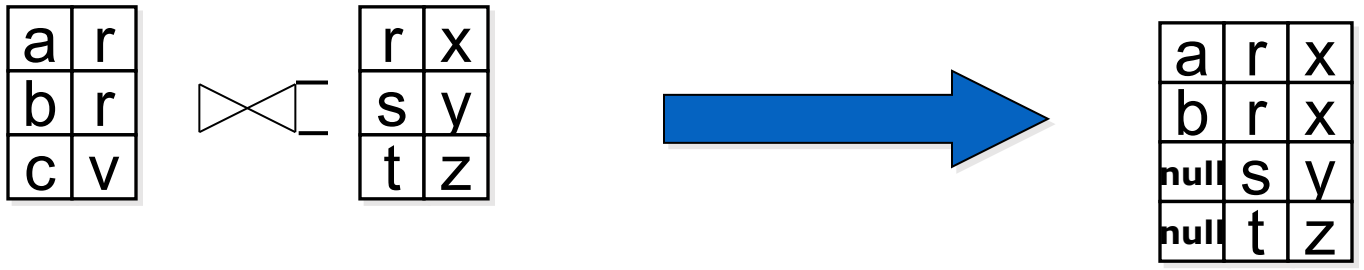
student_id	first_name	last_name	dob	…	clazz_id	Name	lecturer_id	Monitor_id
20160002	Anh	Hoàng	5/20/1987	…	20162101	CNTT1.01-K61	02001	20160003
20160003	Thu Hồng	Trần	6/6/1987	…	20162101	CNTT1.01-K61	02001	20160003
20160004	Minh Anh	Nguyễn	5/20/1987	…	20162101	CNTT1.01-K61	02001	20160003
20170001	Nhật Ánh	Nguyễn	5/15/1988	…	20172201	CNTT2.01-K62	02002	20170001

4.2. Common extension: Outer join

- Outer join
  - Outer join left



  - Outer join right



4.2. Common extension: Outer join example

- Example of left-outer join: List all students and class information if any

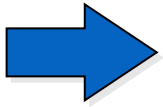
student

student_id	first_name	last_name	dob	…	clazz_id
20160001	Ngọc An	Bùi	3/18/1987	…	
20160002	Anh	Hoàng	5/20/1987	…	20162101
20160003	Thu Hồng	Trần	6/6/1987	…	20162101
20160004	Minh Anh	Nguyễn	5/20/1987	…	20162101
20170001	Nhật Ánh	Nguyễn	5/15/1988	…	20172201

clazz

clazz_id	name	lecturer_id	monitor_id
20162101	CNTT1.01-K61	02001	20160003
20162102	CNTT1.02-K61		
20172201	CNTT2.01-K62	02002	20170001
20172202	CNTT2.02-K62		

results



student_id	first_name	last_name	dob	…	clazz_id	Name	lecturer_id	Monitor_id
20160001	Ngọc An	Bùi	3/18/1987					
20160002	Anh	Hoàng	5/20/1987	…	20162101	CNTT1.01-K61	02001	20160003
20160003	Thu Hồng	Trần	6/6/1987	…	20162101	CNTT1.01-K61	02001	20160003
20160004	Minh Anh	Nguyễn	5/20/1987	…	20162101	CNTT1.01-K61	02001	20160003
20170001	Nhật Ánh	Nguyễn	5/15/1988	…	20172201	CNTT2.01-K62	02002	20170001

4.3. Common extension: Aggregation

- Aggregation

$$G_1, G_2, \ldots G_n \; \mathcal{G} \; F_1(A_1), F_2(A_2), \ldots F_n(A_n) \; (\mathcal{R})$$

- G1, G2, … Gn is a list of attributes on which to group
- F1(A1), F2(A2),….Fn(An) is a list of aggregation function on attribute A1 ,A2 , .. An

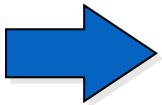
4.3. Common extension: Aggregation example

- Example of Aggregation

$$G_{\text{student\_id}}\ \mathcal{G}\ \text{count(subject\_id)}\ (\text{enrollment})$$

enrollment

student_id	subject_id	semester	midterm_ score	final_ score
20160001	IT1110	20171	9	8.5
20160001	IT3080	20172	8	
20160001	IT3090	20172	6	9
20160001	IT4857	20172	7.5	9
20160001	IT4866	20172	7	9
20160002	IT3080	20172	9	
20160003	IT1110	20171	7	6
20160004	IT1110	20171	6	5



results

student_id	count(subject_id)
20160001	5
20160002	1
20160003	1
20160004	1

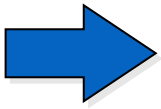
4.3. Common extension: Aggregation example [2]

- Example of Aggregation

$$\mathcal{G}_{\text{count(student\_id)}}(\text{student})$$

student

student_id	first_name	last_name	dob	…	clazz_id
20160001	Ngọc An	Bùi	3/18/1987	…	
20160002	Anh	Hoàng	5/20/1987	…	20162101
20160003	Thu Hồng	Trần	6/6/1987	…	20162101
20160004	Minh Anh	Nguyễn	5/20/1987	…	20162101
20170001	Nhật Ánh	Nguyễn	5/15/1988	…	20172201



value not a relation

5

Remarks

- Relational algebra
  - operators
  - expressions
- Set operators
- Relational algebraic operators
- Common extension : not standard

Next lesson: Structured query language

- Raghu Ramakrishnan and Johannes Gehrke, Database Management S ystems, 3rd edition, Mc Graw Hill, 2003.
- Elmasri and Navathe, Fundamentals of Database Systems, 6th edition , Addison-Wesley, 2011.