War games

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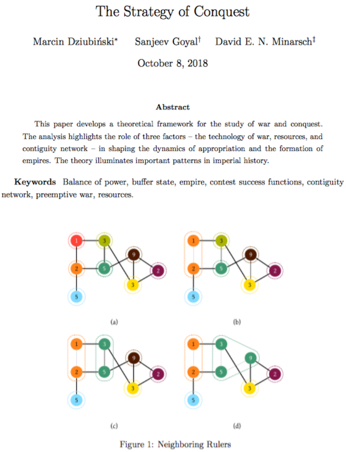
Origins


The Strategy of Conquest

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Abstract

This paper develops a theoretical framework for the study of war and conquest. The analysis highlights the role of three factors – the technology of war, resources, and contiguity network – in shaping the dynamics of appropriation and the formation of empires. The theory illuminates important patterns in imperial history.

Keywords Balance of power, buffer state, empire, contest success functions, contiguity network, preemptive war, resources.

(a) (b)

(c) (d)

Figure 1: Neighboring Rulers

- Inspiration comes from Sanjeev Goyal's work on conflict and networks.
- We extended his model to a more complex setting.
- We (Mario, Carlos, Yamir, and me) are currently focused on studying war game dynamics with $N = 2$

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Combatants

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THE EMPIRE

The challenger empress: endowed with R_1 resources, rushing to attack.

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The crown prince: with R_2 resources awaiting for the offensive.

THE KINGDOM

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THE KINGDOM

We set: $R_1 > R_2$ and define $r \equiv R_2/R_1$

Battle on two fronts

The Rich-Rewarding Front:

- RR-technology $\gamma_R > 1$.
- Empire's deployment: $\alpha_{R,1} R_1$
- Kingdom's deployment: $\alpha_{R,2} R_2$
- Empire's victory probability: $p_{\gamma_R}(R_1, R_2) = \frac{1}{1+\left(\frac{\alpha_{R,2} R_2}{\alpha_{R,1} R_1}\right)^{\gamma_R}}$
- It holds: $u_1 > R_1$ at equilibrium.

The Poor-Rewarding Front:

- PR-technology $1 > \gamma_P > 0$.
- Empire's deployment: $\alpha_{P,1} R_1 = (1 - \alpha_{R,1}) R_1$
- Kingdom's deployment: $\alpha_{P,2} R_2 = (1 - \alpha_{R,2}) R_2$
- Empire's victory probability: $p_{\gamma_R}(R_1, R_2) = \frac{1}{1+\left(\frac{\alpha_{P,2} R_2}{\alpha_{P,1} R_1}\right)^{\gamma_P}}$

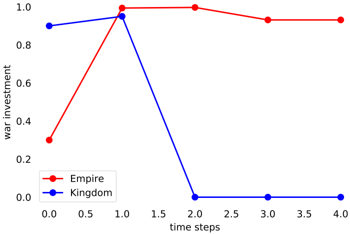
A crucial decission

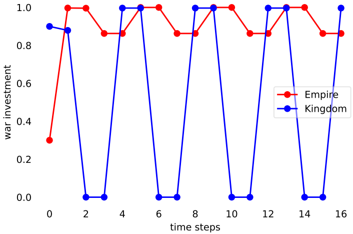
How should factions deploy resources at every front?

- Rulers are cold and competent: seek to maximize their benefit.
- Very good spies, tricksters and rogues: Perfect information.
- Best-response dynamics determines $(\alpha^*_{R,1}, \alpha^*_{R,2})$: Nash equilibrium existence.

Sleepwalking into war...

- Empire & Kingdom maximize their respective expected gains regarding previous information from their adversary.
- They check their bet and change it if necessary until...





- Left: Nash equilibrium successfully attained: $(\alpha^*_{R,1}, \alpha^*_{R,2})$.
- Right: Failure to reach equilibrium. Peace prevails.

The fate of war & aftermaths

A: Empire dominates, takes all resources with $P_A = p_{\gamma_R} p_{\gamma_P}$

C: Empire loses RRF, wins PRF, with $P_C = (1 - p_{\gamma_R}) p_{\gamma_P}$

- Opposite of scenario B.

B: Empire wins RRF, loses PRF, with $P_B = p_{\gamma_R}(1 - p_{\gamma_P})$

- Both factions stand but...
- Empire net gains: $\alpha_{R,2} R_2 - \alpha_{P1} R_1$

D: Kingdom dominates, takes all resources with $P_D = (1 - p_{\gamma_R})(1 - p_{\gamma_P})$

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Invoking the oracle

What can we expect of the outcome of a war?

Given the pair (γ_R, γ_P), some questions arise

- For which values of r, factions are going to wage war?
- Given an initial r, what will be the fate of war?

Giving up a front

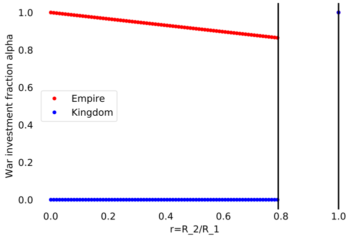


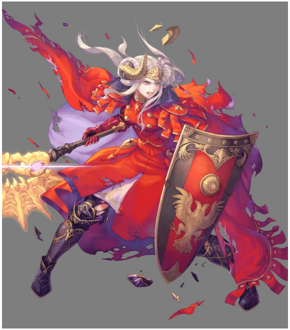
Figure: Equilibrium values for $\alpha_{R,1} = \alpha_{R,1}(r)$ (red) and $\alpha_{R,2} = \alpha_{R,2}(r)$ (blue). Vertical lines: r_b and $r = 1$ thresholds. For $(\gamma_R = 5, \gamma_P = 0.5)$.

- War and Peace, and a threshold r_b.
- The kingdom retires from the RRF: $\alpha_{R,2} = 0$.
- $P_C = P_D = 0$ and
- $r_{\text{eff}} = \frac{\alpha_{P,2} R_2}{\alpha_{P,1} R_1} \equiv$ constant.
- In war regime:

$$\boxed{\alpha_{R,1} = 1 - \frac{r}{r_{\text{eff}}}.} \qquad (1)$$

- Redefine $P_A = P_{\text{ext}}(r_{\text{eff}})$, $P_B = P_{\text{sur}}(r_{\text{eff}})$.

Total triumph: Empire's hegemony. Kingdom annihilated. All resources go to the Empire ($r_t \to r_{t+1} = 0$).



Pyhrric victory: Empire undermined, Kingdom resists. $R_1^{t+1} = \alpha_{R,1} R_1^t + \alpha_{R,2} R_2$ ($r_t \to \Delta r_t = r_{t+1} - r_t > 0$)

We look for r_b in the technology parameter space:

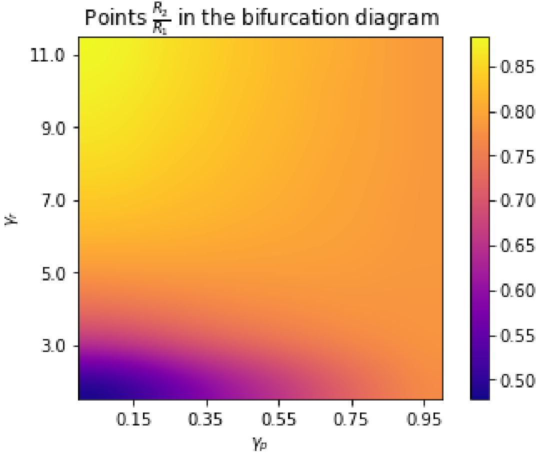


Figure: Color code represents threshold r-values above which Nash equilibrium disappears and thus the peaceful regime sets in.

A thousand wars

- We define a coexistence metric ρ:
  - $\rho = 0$ if hegemony,
  - $\rho = 1$ if peace.
- Scanning for r we simulated thousands of wars.
- For every initial r, we have a measure $\langle \rho \rangle$.

The coexistence ladder

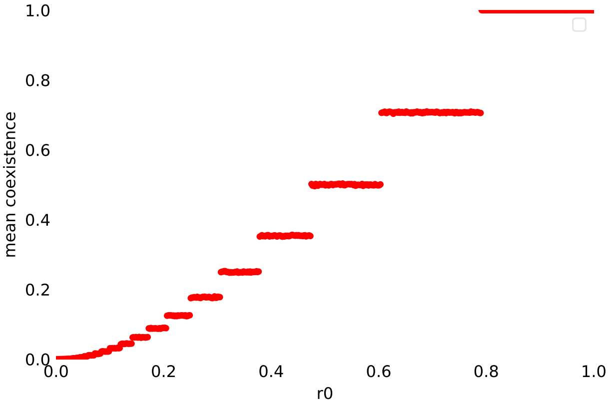
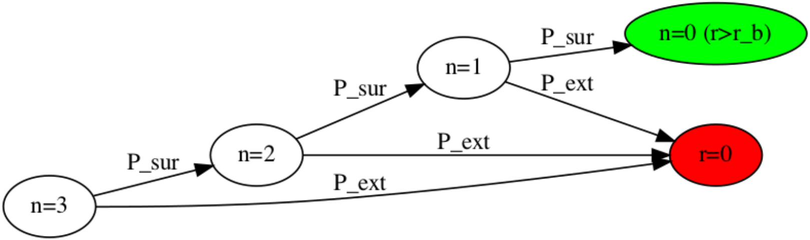


Figure: Mean coexistence $\langle \rho \rangle$ diagram. With $\gamma_R = 5.0, \gamma_P = 0.5$. Every point represents an average over 10^5 simulations.

The chain of war

- Consider every combat as a Markov chain.
- Two absorbing states: $r = 0$ and $r > r_b$.
- Pyhrric transition: $\Delta r_t = r_{t+1} - r_t > 0$ with P_{sur}.



Enhancing our understanding

- $P_{\rm ext}$ (or $P_{\rm sur}$) $\approx$ constant lead to

$$\boxed{P_{\mathrm{heg},n(r)} = \sum_{i=0}^{n(r)} (1 - P_{\mathrm{sur}}) P_{\mathrm{sur}}^{i}.} \tag{2}$$

(where $n \equiv$ steps from ceil) And thus the peaceful solution probability is

$$\boxed{P_{\mathrm{peace},n(r)} = 1 - P_{\mathrm{heg},n(r)}.} \tag{3}$$

- Backwards map for r assuming only Pyhrric-transitions:

$$\boxed{r_{t-1} = b_p(r_b) = \frac{r_{\mathrm{eff}}\, r_b}{r_b + r_{\mathrm{eff}} + 1}.} \tag{4}$$

It turned out to be true

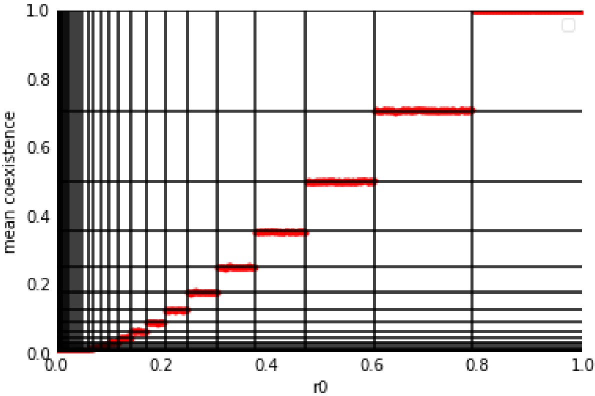


Figure: Mean coexistence together with survival probabilities for 15 levels and step borders. Duel played under Tullock Type B mixed war with combined technologies: $\gamma_R = 5.0, \gamma_P = 0.5$. Every point represents an average over 10^5 simulations.

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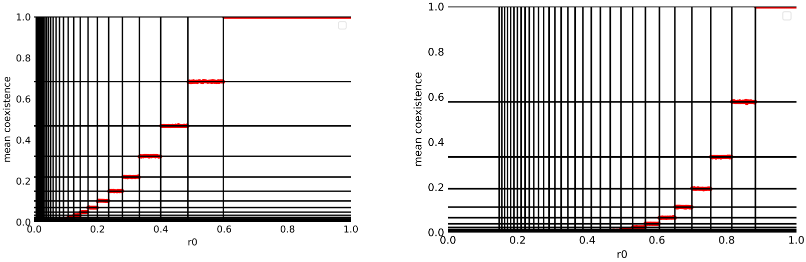

Figure: Mean coexistence diagram together with survival probabilities and step borders for 15 levels. Duel player under Tullock Type B mixed war. Left: $(\gamma_R, \gamma_P) = (2.0, 0.4)$. Right: $(\gamma_R, \gamma_P) = (11.0, 0.1)$. Every point represents an average over 10^4 simulations.

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Some lessons from (this) war (game)

- Two regimes separated by threshold r_b.
- Two equilibria: hegemony and peace.
- Pyhrric transitions tend to equalize factions.

- The Empire cannot be defeated... but might be undermined.
- Analytical work may brings us greater insights when moving to networks.