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Association Rules Mining

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Items & Users

- Market basket/shopping cart (physical or online)
  - Items: Products
  - Users: Customers
- Library
  - Items: Books
  - Users: Library visitors
- News site
  - Items: News articles
  - Users: Readers

Recommender Systems

Suggest personalised and relevant items

- Cover the entire spectrum of the user's interests
- Take the user's context into account
- Avoid suggestions from what is already known
- Expand the user's range of interests

Why Recommender Systems?

Aproaches for Recommender Systems

- Popularity-based recommendation
- Frequent itemsets & association rules mining
- Content-based recommendation
- Collaborative filtering

Popularity-based recommendations

Example: The New York Times Online

Pros:

- Small complexity
- Explainable

Cons:

- No personalisation
- Item properties not taken into account

Association Rules Mining

Synonym: Market basket analysis

Source: www.quickmeme.com

Itemsets

- Items I: The set of available items $I = \{i_1, \ldots, i_M\}$
- Itemset X: A set of items $X \subseteq I$
- Itemset size K: The number of items in the itemset
- K-itemset: An itemset of size K
- Items are ordered:

$$X_n = \{x_1, x_2, \ldots, x_K\}, \text{ such that } x_1 \leq x_2 \leq \ldots \leq x_K$$

Transactions

- Transaction: $\mathrm{T_n} = (\mathrm{tid}, \mathrm{X_{tid}})$
- Transactions in database: $\{\mathrm{T_1}, \mathrm{T_2}, \ldots, \mathrm{X_N}\}$

tid	itemset
1	{apple, bread, honey, milk, peanuts}
2	{bread, chips, coke, honey, milk}
3	{bread, chips, coke, honey, steak}
…	
8	{apple, bread, cheese, milk, peanuts}

Support

- $s(X)$: fraction of transactions that contain X

$$s(\{bread, milk\}) = \ ?$$

$$s(\{chips, coke\}) = \ ?$$

Example

tid	itemset
1	{apple, bread, honey, milk, peanuts}
2	{bread, chips, coke, honey, milk}
3	{bread, chips, coke, honey, steak}
4	{apple, coke, honey, milk, peanuts}
5	{bread, chips, coke, honey, milk}
6	{apple, chips, coke, milk}
7	{apple, bread, coke, milk, peanuts}
8	{apple, bread, cheese, milk, peanuts}

Frequent Itemsets Mining (FIM)

- An itemset X is frequent if its support in the DB is greater or equal than a minimum support threshold t_s:

$$s(X) \geq t_s$$

- Given:
  - Set of items I
  - Transaction database over I
  - Minimum support threshold t_s
- Goal: Find all frequent itemsets $\{X \subseteq I | s(X) \geq t_s\}$

FIM - example

	items
2000	{A,B,C}
1000	{A,C}
4000	{A,D}
5000	{B,E,F}

- Support of 1-itemsets:

  (A): 75%, (B), (C): 50%, (D), (E), (F): 25%

- Support of 2-itemsets:

  (A, C): 50%,
  (A,B), (A,D), (B,C), (B,E), (B,F), (E,F): 25%

Association rules

Let X, Y be two itemsets: $X, Y \subseteq I$ and $X \cap Y = \emptyset$.

- Association rules represent implications of the form $X \rightarrow Y$
- Support of a rule: The fraction of transactions containing $X \cup Y$

$$s(X \rightarrow Y) = s(X \cup Y)$$

- Confidence c of a rule: the fraction of transactions containing $X \cup Y$ in the set of transactions containing X.

$$c(X \rightarrow Y) = \frac{s(X \cup Y)}{s(X)}$$

Association Rule Mining (ARM)

- Given:
  - Set of items I
  - Transaction database over I
  - Minimum support threshold t_s and a minimum confidence threshold t_c
- Goal: Find all association rules $X \rightarrow Y$ in DB with minimum support threshold and a minimum confidence i.e.:

$$\{X \rightarrow Y | s(X \cup Y) \geq t_s, c(X \rightarrow Y) \geq t_c\}$$

These rules are called strong.

ARM - example

	items
2000	{A,B,C}
1000	{A,C}
4000	{A,D}
5000	{B,E,F}

Association rules:

- $A \rightarrow C$:

  $$s(A \rightarrow C) = 0.50, c(A \rightarrow C) = 0.67$$

- $C \rightarrow A$:

  $$s(C \rightarrow A) = 0.50, c(C \rightarrow A) = 1.0$$

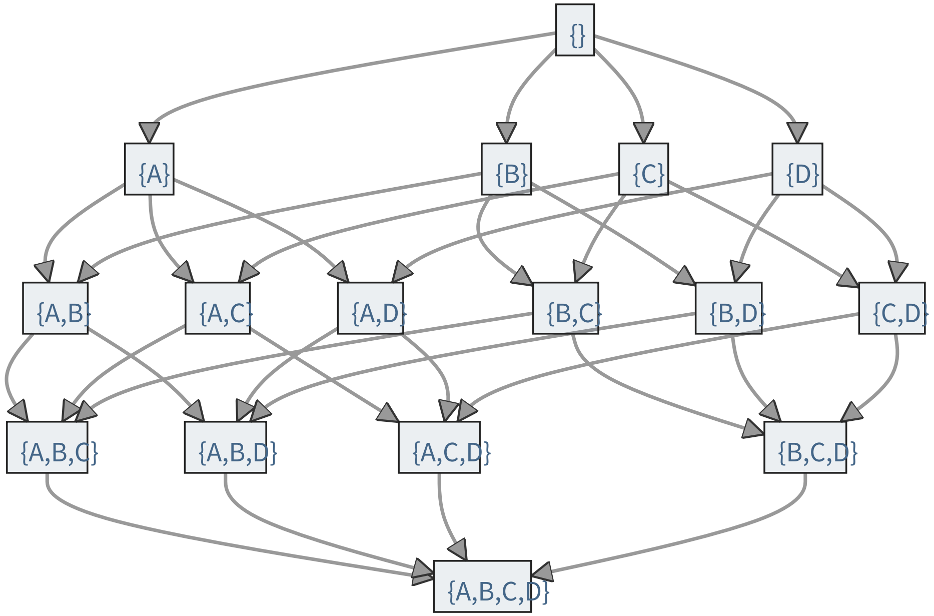
Finding Strong Association Rules

1. FIM: Find the frequent itemsets w.r.t. minimum support threshold.
2. ARM: Find strong association rules among the frequent itemsets.

FRM by brute-force is inefficient

Example with small set of items: $I = \{A, B, C, D\}$

- # 1-itemsets: $\binom{4}{1} = \frac{4!}{1!(4-1)!} = \frac{4!}{3!} = 4$
- # 2-itemsets: $\binom{4}{2} = \frac{4!}{2!(4-2)!} = \frac{4!}{2!2!} = 6$
- # 3-itemsets: $\binom{4}{3} = \frac{4!}{3!(4-3)!} = \frac{4!}{3!} = 4$
- # 4-itemsets: $\binom{4}{4} = \frac{4!}{4!(4-4)!} = 1$



In general for $|I|$ items: $\binom{|I|}{1} + \binom{|I|}{2} + \ldots + \binom{|I|}{K} = 2^{|I|} - 1$ itemsets

Apriori Algorithm

Reduces the candidate itemsets to be tested: If an itemset is frequent, then all of its subsets must also be frequent. And if an itemset is infrequent, its supersets must not be tested.

Initialise: $k = 1$. Scan DB to get frequent 1-itemsets

Repeat:

1. Set $k = k + 1$
2. generate length k candidate itemsets from length $k - 1$ frequent itemsets
3. test the candidates against DB to get frequent k-itemset
4. Stop when no frequent or candidate set was generated in 3.

Example

- Database with 9 transactions
- Minimum support $t_s = 22\%$
- Minimum confidence $t_c = 70\%$
- 1. Identify frequent itemsets using Apriori
- 2. Identify association rules

Database	
tid	**items**
1	{chips, coke, whiskey}
2	{beer, chips}
3	{chips, ice}
4	{beer, chips, coke}
5	{coke, ice}
6	{chips, ice}
7	{coke, ice}
8	{chips, ice, coke, whiskey}
9	{chips, coke, ice}

Example - $k = 1$

Database	
tid	**items**
1	{chips, coke, whiskey}
2	{beer, chips}
3	{chips, ice}
4	{beer, chips, coke}
5	{coke, ice}
6	{chips, ice}
7	{coke, ice}
8	{chips, ice, coke, whiskey}
9	{chips, coke, ice}

Candidates C_1

itemset	s
{coke}	67%
{chips}	78%
{ice}	67%
{beer}	22%
{whiskey}	22%

Frequent itemsets L_1

itemset	s
{coke}	67%
{chips}	78%
{ice}	67%
{beer}	22%
{whiskey}	22%

Generate candidates

C_k is generated by

1. Self-joining $L_{k-1} : L_{k-1} \cdot L_{k-1}$. Two $(k-1)$-itemsets are joined, if they agree in the first $(k-2)$ items
2. Pruning all k-itemsets with a $(k-1)$-subset that is not frequent, i.e. not in L_{k-1}

Example: $L_3 = \{abc, abd, acd, ace, bcd\}$

1. $C_4 = L_3 \cdot L_3 = \{abc \cdot abd = abcd, acd \cdot ace = acde\}$
2. $acde$ is pruned since cde is not in L_3

Example - $k = 2$

Generate C_2 by self-joining L_1, determine s and prune by support threshold $\rightarrow L_2$

Database

tid	items
1	{chips, coke, whiskey}
2	{chips, beer}
3	{chips, ice}
4	{coke, chips, beer}
5	{coke, ice}
6	{chips, ice}
7	{coke, ice}
8	{coke, chips, ice, whiskey}
9	{coke, chips, ice}

Candidates C_2

itemset	s
{beer, chips}	22%
{beer, coke}	11%
{beer, ice}	0%
{beer, whiskey}	0%
{chips, coke}	44%
{chips, ice}	44%
{chips, whiskey}	22%
{coke, ice}	44%
{coke, whiskey}	22%
{ice, whiskey}	11%

Frequent itemsets L_2

itemset	s
{beer, chips}	22%
{chips, coke}	44%
{chips, ice}	44%
{chips, whiskey}	22%
{coke, ice}	44%
{coke, whiskey}	22%

Example - $k = 3$

Generate C_3 by self-joining L_2, determine s and prune by support threshold $\rightarrow L_3$

Candidates C_3

itemset	s
{chips, coke, ice}	22%
{chips, coke, whiskey}	22%
{coke, ice, whiskey}	11%

Frequent itemsets L_3

itemset	s
{chips, coke, ice}	22%
{chips, coke, whiskey}	22%

Example - $\mathrm{k} = 4$

Self-joining $\rightarrow \mathrm{C}_3 = \emptyset \rightarrow$ stop

Identify Association Rules

- For every frequent itemset X
  - For every subset $Y : Y \neq \emptyset, Y \neq X$, form the rule $Y \to (X - Y)$
  - Remove rules with $c(Y \to (X - Y)) = \frac{s(X)}{s(Y)} < t_c$

Example - Association Rules

From L_2:

- $\{beer\} \to \{chips\} : c = \frac{s(\{beer, chips\})}{s(\{beer\})} = \frac{22\%}{22\%} = 100\%$
- $\{beer\} \to \{chips\} : c = \frac{s(\{beer, chips\})}{s(\{chips\})} = \frac{22\%}{78\%} = 28\%$
- $\{chips\} \to \{coke\} : c = \frac{s(\{chips, coke\})}{s(\{chips\})} = \frac{44\%}{78\%} = 56\%$
- …

From L_3:

- $\{chips, coke\} \to \{ice\} : c = \frac{s(\{chips, coke, ice\})}{s(\{chips, coke\})} = \frac{22\%}{44\%} = 50\%$
- $\{chips, ice\} \to \{ccoke\} : c = \frac{s(\{chips, coke, ice\})}{s(\{chips, ice\})} = \frac{22\%}{44\%} = 50\%$

  …
- $chips\} \to \{coke, ice\} : c = \frac{s(\{chips, coke, ice\})}{s(\{chips\})} = \frac{22\%}{78\%} = 28\%$

  …
- $whiskey \to \{chips, coke\} : c = s(\{chips, coke, whiskey\}) = 22\% = 100\%$

The Efficient Apriori Package

```python
from efficient_apriori import apriori
transactions = [('eggs', 'bacon', 'soup'),
                ('eggs', 'bacon', 'apple'),
                ('soup', 'bacon', 'banana')]
itemsets, rules = apriori(transactions, min_support=0.5,  min_confidence=1)
print(rules)  # [{eggs} -> {bacon}, {soup} -> {bacon}]
```

Beyond the Apriori Algorithm

Computational challenge: Multiple scans of transaction database required. Problematic with growing number of transactions and candidate itemsets.

Possible improvements:

- Reduce database scans
- shrink number of candidate itemsets

Frequent Pattern Tree: Allows frequent itemsets discovery without candidates generation.

Summary Association Rules Mining

- identifes item combinations frequently appearing together across all transactions, not grouped by users
- does not take into account ratings
- does not allow to build a latent representation of users and/or items
- → does not offer personalisation