Q 1.1)

One of the minimal sequences is

- Use a small explosive on building 3, resulting in [2,3,4,1].
- Spend $1000 to use the wrecking ball, resulting in heights [1,2,3,0]. Building 4 was demolished, so the company receives $1000.
- Use the wrecking ball again, resulting in heights [0,1,2,0]. Building 1 was demolished, so the company receives $1000.
- Use the wrecking ball again, resulting in heights [0,0,1,0]. Building 2 was demolished, so the company receives $1000.
- Use the wrecking ball again, resulting in heights [0,0,0,0]. Building 3 was demolished, so the company receives $1000.

Since using the wrecking ball requires less actions to reduce the height of many buildings comparing to small explosives because a wrecking ball could reduce the height of all buildings at a time but an explosive couldn't. Which means for any array H, a sequence of actions using only or as many wrecking balls possible would be one of the minimal sequences.

In order to use most wrecking balls, we need to earn highest money to spend on them. The highest money earnt is equal to the number of buildings (*1000) from finishing demolition of them with the wrecking ball (we couldn't spend last $1000 from demolishing last building but we have $1000 initially). That means for each of the buildings, we need to use the wrecking ball on any building with 1 height. However, the highest number of using the wrecking ball doesn't necessary to be the number of buildings, once the highest building is less than the number of buildings, we would be able to use only that many wrecking balls because all buildings would be demolished before we could use any more than that amount.

The sequence above is a minimal sequence because it earns and uses highest money ($4000) and wrecking balls (4) possible from finishing demolition each of the buildings with the wrecking ball.

Q 1.2)

One of the ways to get a minimal sequence of actions is using the wrecking balls as many as possible (as mentioned above) and the only impact explosives could make in order to have that happens is to be used to prepare each building to be able to be demolished by the wrecking ball.

All uses of the explosive could occur only at first or in the middle of sequence since the last action would always be using the wrecking ball to demolish last building(s) (according to my logic in Q1.1).

Using small explosive require nothing so it could be used at any time without any condition making its order of action being able to be moved from at the middle to at first affecting nothing as long as we try to get a minimal sequence using Q1.1 logic (only using the wrecking ball to demolish a building). (Q1.4 algorithm could be used to prove this as well).

That is, there is always a minimal sequence of actions where all uses of the explosives (sometimes 0) occur before any uses of the wrecking ball.

Q 1.3)

Any array H would fit the criterion when after it is sorted in an ascending order, for all elements in the array, $H[i] \leq i \mid i \in [1, n]$. Return yes if an array fits the criterion, no otherwise.

A sequence of using only the wrecking ball is obviously a minimal sequence as mentioned in Q1.1.

We would be able to use only the wrecking ball if it fits the criterion because when we use i number of the wrecking ball, the height of i^{th} building would become $H[i] - i$ which would be demolished if $H[i] \leq i$. Which means at i^{th} times of using the wrecking ball, up to i^{th} building would be demolished if $H[i] \leq i$ and earn money back to spend on the next wrecking ball. It is possible to gain more than $1000 when we have many buildings of the same height but since the array is sorted, we would still earn (cumulatively) up to $(i) * 1000$ at any $H[i]$. So as long as $H[i] \leq i$, at any i^{th} building, we would always have enough money to spend on the i^{th} wrecking ball to finish the demolition of that building.

Q 1.4)

Begin by sorting an array $H[1 \dots n]$ in an ascending order in $O(n \log n)$ time and having a queue $sequence$ for collecting actions. (Storing original index positions during sorting and mapping it back at the end).

For each building in a sorted array H, checking if $H[i] \leq i$. If not then pushing $H[i] - i$ times of the use of small explosive on that building (using its original index that is mapped) into $sequence$ in $O(1)$ time (similar to using explosives to prepare each building to be able to be demolished by the wrecking ball to fit Q1.3 criterion).

After iterate through all elements in the array (fit Q1.3 criterion at this point), push $H[n]$ times of the use of wrecking ball into $sequence$ if $H[n] \leq n$ (we could use only that many wrecking balls because all buildings would be demolished before we could use any more than that amount). Otherwise, push n times of the use of wrecking ball into $sequence$ instead (use as many wrecking balls possible n times from money earnt $n * 1000$ when the highest building is higher than the number of buildings). $sequence$ is the minimal sequence of actions that all uses of explosive occur before any uses of the wrecking ball (pop to see first action and so on).

Whenever $H[i] > i$ means that the building couldn't be demolished within the i^{th} wrecking ball, so the uses of explosive are required to reduce the height to be equal to i that is using $H[i] - i$ small explosives. And the wrecking balls would be used to demolish every single building.

$sequence$ is a minimal sequence for all reasons mentioned in Q1.1, Q1.2 and Q1.3.

Overall time complexity is $O(n \log n)$. Sorting n length array takes $O(n \log n)$, iterating through n length array, checking conditions and inserting elements into a queue takes $O(n * 1)$, so $O(n \log n + n) \cong O(n \log n)$.