Q 1.1)

* Use a small explosive on building 3, resulting in [2,3,4,1].
* Spend $1000 to use the wrecking ball, resulting in heights [1,2,3,0]. Building 4 was demolished, so the company receives $1000.
* Use the wrecking ball again, resulting in heights [0,1,2,0]. Building 1 was demolished, so the company receives $1000.
* Use the wrecking ball again, resulting in heights [0,0,1,0]. Building 2 was demolished, so the company receives $1000.
* Use the wrecking ball again, resulting in heights [0,0,0,0]. Building 3 was demolished, so the company receives $1000.

Using the wrecking ball requires less actions to reduce the height of many buildings comparing to small explosives because a wrecking ball could reduce the height of all buildings at a time but an explosive couldn’t. Which means for any array , using as many wrecking balls possible would be one of the minimal sequence of actions.

The most wrecking balls that could be used for these heights are 4 because the most budget earn possible is $4000 from 4 building. And we could only spend $3000 of it plus initial $1000 on wrecking balls because we have 1 highest building which would be demolished last that we couldn’t spend that $1000 earn from it since there is no buildings left (we would be able to spend less if there are 2 or more highest buildings for the same reason).

Thus, this is one of the minimal sequence of actions because of using the most wrecking balls possible.

Q 1.2)

One of the ways to get a minimal sequence of actions is using the wrecking balls as many as possible (as mentioned above).

So, the order of using small explosives wouldn’t matter as long as we could use as many wrecking balls possible.

Since small explosives could be used at any time without any condition. It wouldn’t make differences of how much money we would earn by moving those actions to occur first before anything because (as mentioned in Q1.1) small explosives are only used to prepare buildings to be ready to finish demolishing within another wrecking ball to earn money from it. Which means if there exists a way of using explosives after any uses of wrecking ball, those actions could also be done before any uses of the wrecking ball resulting in the same number of buildings to be done demolishing with wrecking balls with the same number of actions.

That is, there is always a minimal sequence of actions where all uses of the explosives (sometimes 0) occur before any uses of the wrecking ball.

Q 1.3)

By having a counter starts from 1 (indicating the budget of the company which equal to $1000) and a counter starts from 0 (indicating the number of the wrecking balls used and it is the current height that should be demolished).

When iterating through the sorted array, would be decreased when current element is not equal to previous element (or when there is no previous elements) by the amount of for each element we iterate through (because if 2 or more buildings are at the same height, 1 wrecking ball that was used for the first amongst the buildings would affect this building as well). would be increased by 1 after every time is decreased. Whenever and , increase by and set back to 1 (similar to spending budget to use the wrecking balls to reduce the height of all buildings). Lastly, before moving to next element, would be increased by 1 if (similar to gaining money whenever a building is demolished). We then iterate to the next element and so on until we reach the last element.

Any array would fit the criterion when after it is sorted in an ascending order, it’s never fall down less than 0.

Because whenever , that means we are not gaining money back from using last wrecking ball affecting having no enough budget to spend on more wrecking balls and need to use small explosives instead.

If it fits the criterion then yes, there would be a minimal sequence using only the wrecking ball because after spending $1000 for the first one, you would gain that amount or higher (when there are elements of the same height) back and that amount of money would always be enough to use that many wrecking balls until the next building is demolished. And do it again until the last building is demolished. No, otherwise.

Q 1.4)

Begin by sorting array in an ascending order. We then need a counter starts from 1 and a counter starts from 0. But this time we would need a counter as well, in order to track the number of small explosives used. And we need a queue for collecting actions.

Then we iterate through the sorted array, decreasing when current element is not equal to previous element (or when there is no previous elements) by the amount of for each element we iterate through. Increasing by 1 after every time is decreased. Whenever and , increase by and set back to 1. Lastly, before moving to next element, increasing by 1 if . We then iterate to the next element and so on until we reach the last element. (Similar to question 1.3).

Whenever , we increase by the amount of (indicating when you spend that number of actions using explosives because we wouldn’t have enough budget to continue spending on wrecking balls if we couldn’t spend the last $1000 to finish demolish any building with the height of 1) and set back to 1 (because we would gain at least $1000 from finish demolishing at least 1 building). When this happens, adding a log into that we use explosive on that (index) building by times.

The number of minimal sequence of actions of the given array is . Because is here to make sure that once we use the wrecking balls, we would have enough budget to spend on the next balls until the last building is demolished. The amount of wrecking ball used is as mentioned in question 1.3. We finally add a log into that we use wrecking ball by times. And itself is the minimum sequence of actions.

This guarantees that we would earn and spend most budget possible to use as many wrecking balls possible making the sequence one of the minimal ones. (Since we would always have $1000 to use a wrecking ball to finish demolishing of any possible buildings to gain some back and would spend all excess budget (>$1000) on wrecking balls if there are some buildings left). The uses of explosive are for preparing buildings to be finished demolish with the uses of wrecking ball to earn highest money possible.

Overall time complexity is . Sorting length array takes , iterating through length array and inserting elements into a queue takes , so .