Phat Tang

June 06, 2018

Physics 116W

# Final Report

## Abstract

The goal of this experiment was to build a nonlinear oscillator with a spring and magnet. To get nonlinear oscillations in a spring, we attached a magnet to a spring and placed the magnet inside two inductive coils which were driven with currents coming from two power supplies. One of the coils had a constant current while the other coil had an alternating current to initiate the oscillations of the spring. The constant current in the first coil was varied from 0.9 amps to 1.2 amps and then to 1.5 amps to measure the frequency dependence of the field strength. To control the power supplies, we used a function generator, Labview, and a DAQ card. To measure the position of the magnet, we constructed a system that has an LED pointing at a linear filter that is attached above the magnet. The linear filter is to vary the light intensity of the light delivered by the LED, which is a white yellow-emitting diode. The light that is passed through the filter excites the phototransistor and the position of the magnet can be measured by the voltage drop across a 470 ohm resistor that is in series with the photodetector. The set-up can be seen in Figure 1. The voltage was recorded using the DAQ card and Labview. The measurements were written onto a spreadsheet and then analyzed using Matlab. The Fast-Fourier transform of the data was done to get the frequency dependence of the field strength and graphs were constructed to see that the system was behaving like an an-harmonic oscillator inside a double-well potential.

## Introduction

To understand a nonlinear oscillator, it is first important to understand how a linear oscillator behaves. A simple example of a harmonic linear oscillator is a system that has a mass on a spring. When the mass is pulled down from its equilibrium position, it will start oscillating up and down until it returns to its rest position. An ideal spring should exhibit a linear restoring force which means the force of the spring will be proportional to how far away it is from the equilibrium point. The position of the mass can be described using the equation

$$y = Acos(2\pi * f * t) \tag{1}$$

where the frequency is given by $f = \frac{1}{2}\sqrt{\frac{K}{m}}$. The amplitude which is A, is the maximum vertical distance the mass can travel, m is the mass of the object, and K is a coefficient which shows the stiffness of the spring (Boas). For this simple oscillator, only one frequency is present and the period of the sinusoidal is the inverse of it. We can now take a look at nonlinear oscillators which has more than one frequency. Harmonics are sine waves that has frequencies of integer multiples of the lowest frequency, which is known as the fundamental of the harmonic series. That is the main difference between a linear oscillator and a nonlinear oscillator. The motion of the nonlinear oscillator is a superposition of complex oscillation which are made up of the harmonics of the lowest frequency. The two main properties that were discovered in this experiment is that the nonlinear oscillating system we built had two resting positions after oscillating and that the strength of the magnetic field will vary the resting position of the magnet.

## Theoretical

The magnetic force a coil can be calculated using

$$\bar{F} = \int io\, \bar{dl} \times \bar{B} = io2\pi aB(x,z) \quad (2)$$

where F is the magnetic force, io is current applied to the coil, and B is the radial component of a magnet field at vertical coordinate z and radial distance a. After some simplifying, equation (2) becomes equation (3) which is the approximation for a single dipole of a magnetic field (Griffiths).

$$F(z) = \frac{\mu N a^2 m}{2L} * io\{\frac{1}{[a^2+\left(\frac{L}{2}-z\right)^2]^{\frac{3}{2}}} - \frac{1}{[a^2+(\frac{L}{2}+z)^2]^{3/2}}\} \quad (3)$$

N is the number of turns in the coil and m is the magnet magnetic dipole, L is the length of the coil. More calculations can be seen on the journal *Anharmonic oscillations of a spring-magnet system inside a magnetic coil* (Donoso).

## Experimental Method/Procedure

The system that was constructed is shown in figure 1, the yellow LED was powered by a breadboard using 5.0 volts, the phototransistor (model BPW77N) is connected to a 470-ohm resistor and is powered also by the breadboard. The voltage across the 470-ohm resistor is connected to the DAQ card to record the data in Labview.

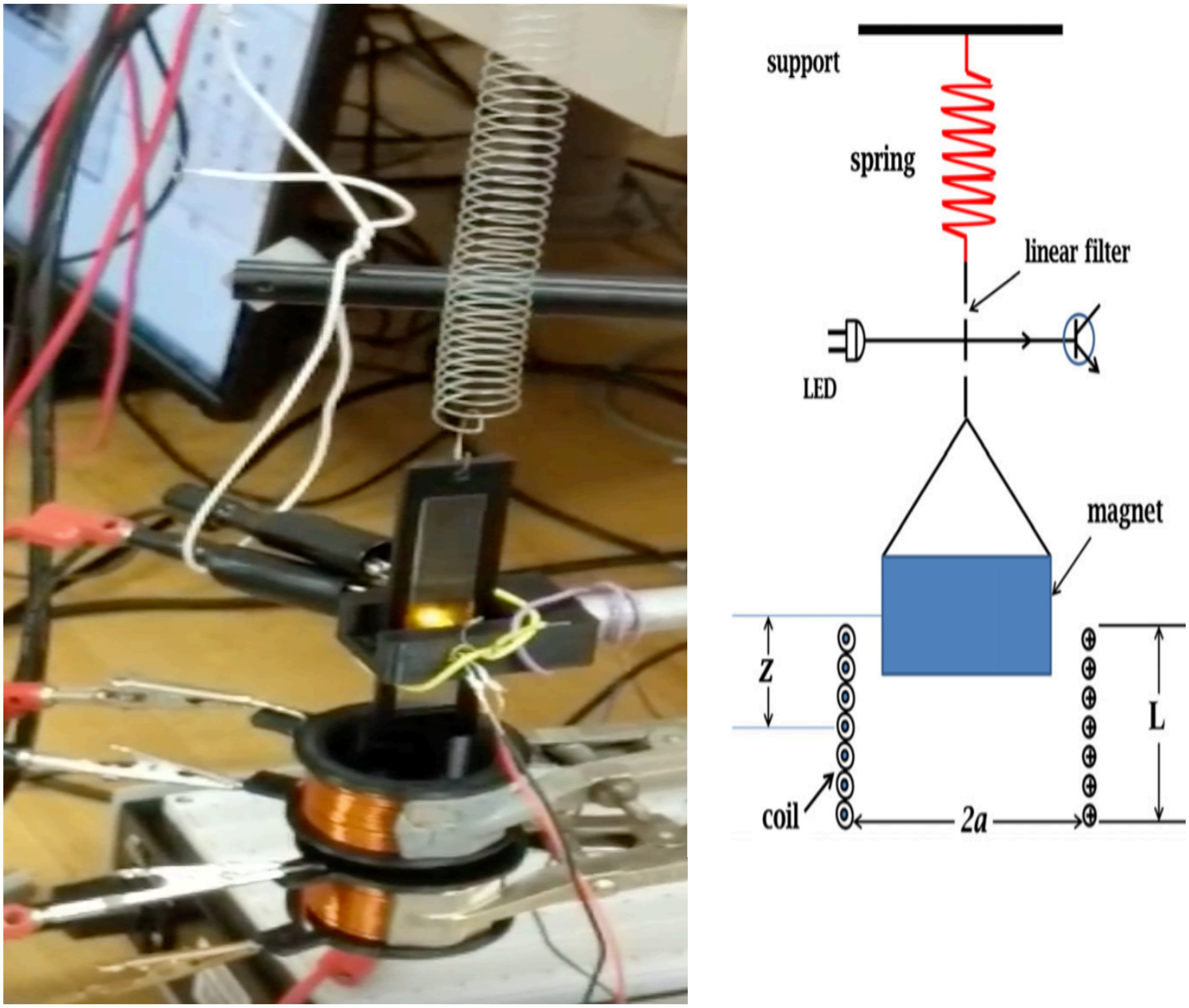



**Figure 1:** Setup of experiment for nonlinear oscillator.

The VI in Labview also controlled a function generator, and the output of the function generator controlled the constant current of the power supply. The power supply was connected to the top coil to start the kick of the oscillation. When the VI started, it set the function generator to apply a sin waveform with an amplitude of 5 Volts peak-to-peak and a frequency of 1.5 Hertz to the top coil for a random duration between 5 to 10 seconds. (Shown in Figure 2). This allows the spring to start oscillating using the top coil.

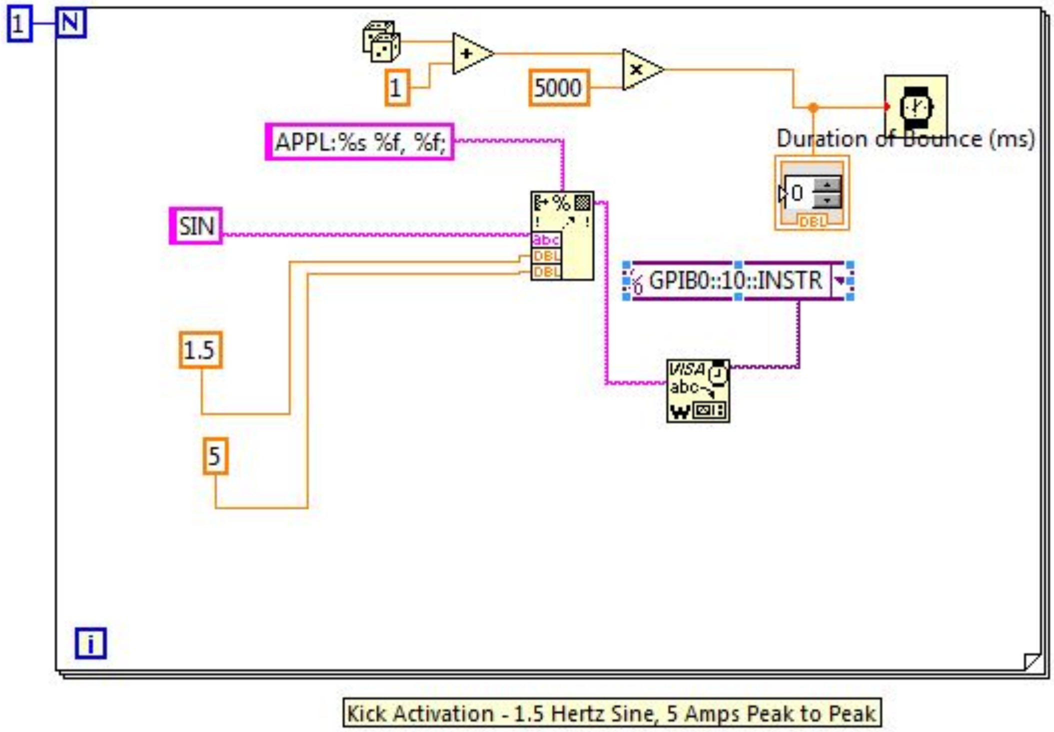
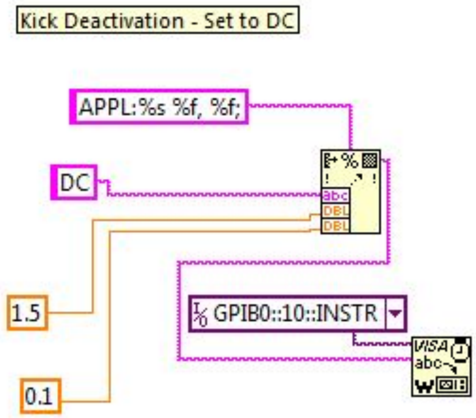



**Figure 2:** First sequence to start the oscillation

After the random time duration, the VI ran the second sequence which told the function generator to apply a DC of 0.1 Volts to the coil, which is a negligible amount. It turned off the kick and the magnet oscillated freely (Shown in Figure 3). Then the VI controlled the DAQ card to apply the constant field strength to the bottom coil to create the double-well potential for the magnet. The constant current we applied to the bottom coil was varied from 0.9 amps, 1.2 amps, and 1.5 amps (Shown in Figure 4). The fourth sequence of the program took the data using the DAQ card, and the data that was taken was the voltage across the 470-ohm resistor which was connected to the phototransistor.



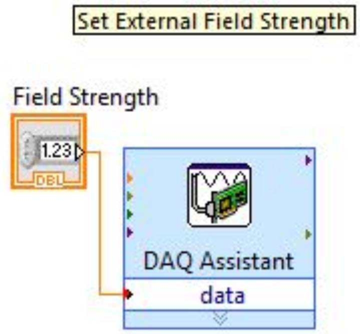




**Figure 3:** Second Sequence to turn off kick

**Figure 4:** Third Sequence to turn on bottom coil

After the program took all the data, we saved the spreadsheet into a text file and the data was then analyzed using Matlab. Using Matlab helped showing the properties of a double-well potential and a frequency dependence on this system. I used Matlab analyze the data we took for 0.9 amps, 1.2 amps, and 1.5 amps for the bottom coil. We ran the program 5 times for 0.9 amps because we saw that the equilibrium position of the magnet was in the same spot we assumed the current was too weak to create a double-well. We then ran the program 10 times for 1.2 amps and noticed the equilibrium position was still unchanged. At 1.5 amps we saw that there were two equilibrium positions. For 1.5 amps we ran the program 20 times. When we were taking data, we ran the program a total of 35 times and noted that a double-well was only created when the constant current was 1.5 amps or higher. The maximum current that we were able to apply in the laboratory was 1.5 amps.

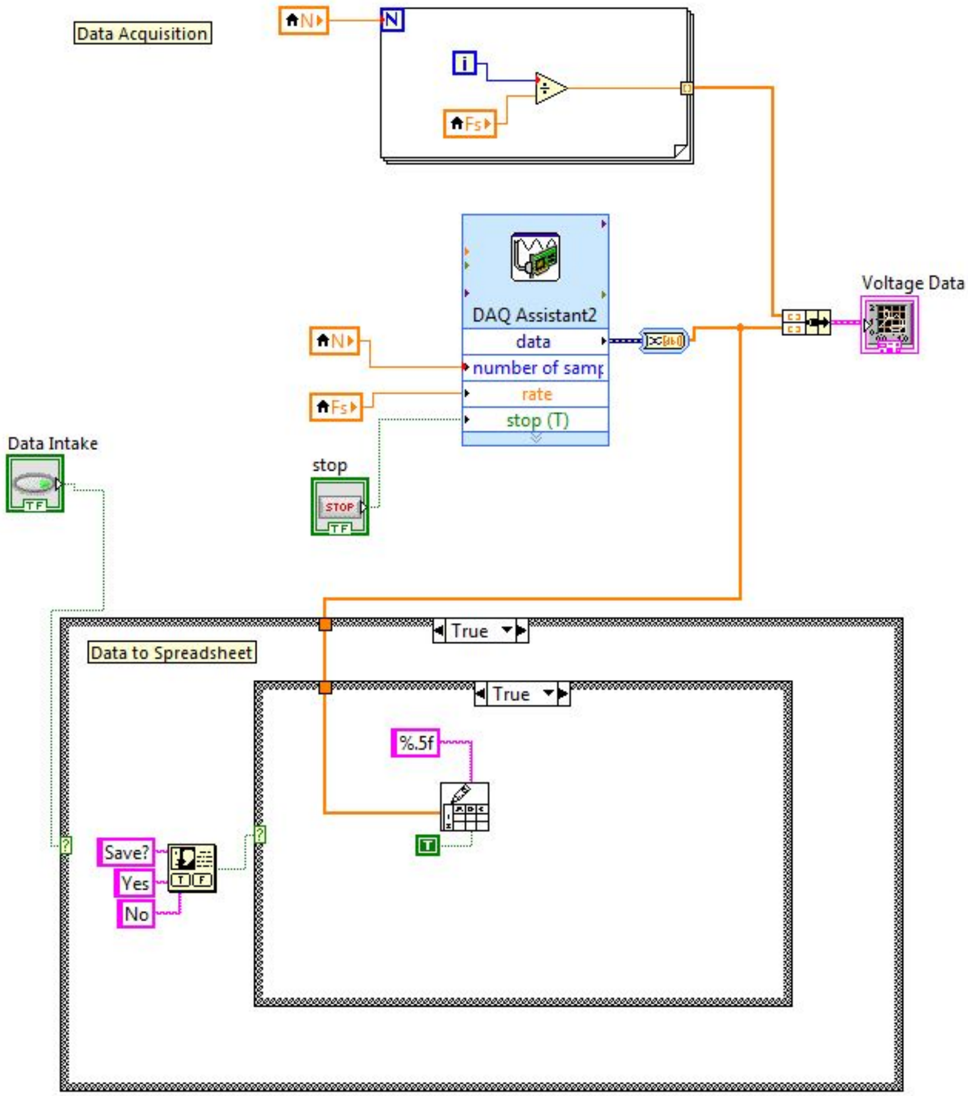



**Figure 5:** Fourth sequence, took data from DAQ card. Graphed the data and wrote to spreadsheet

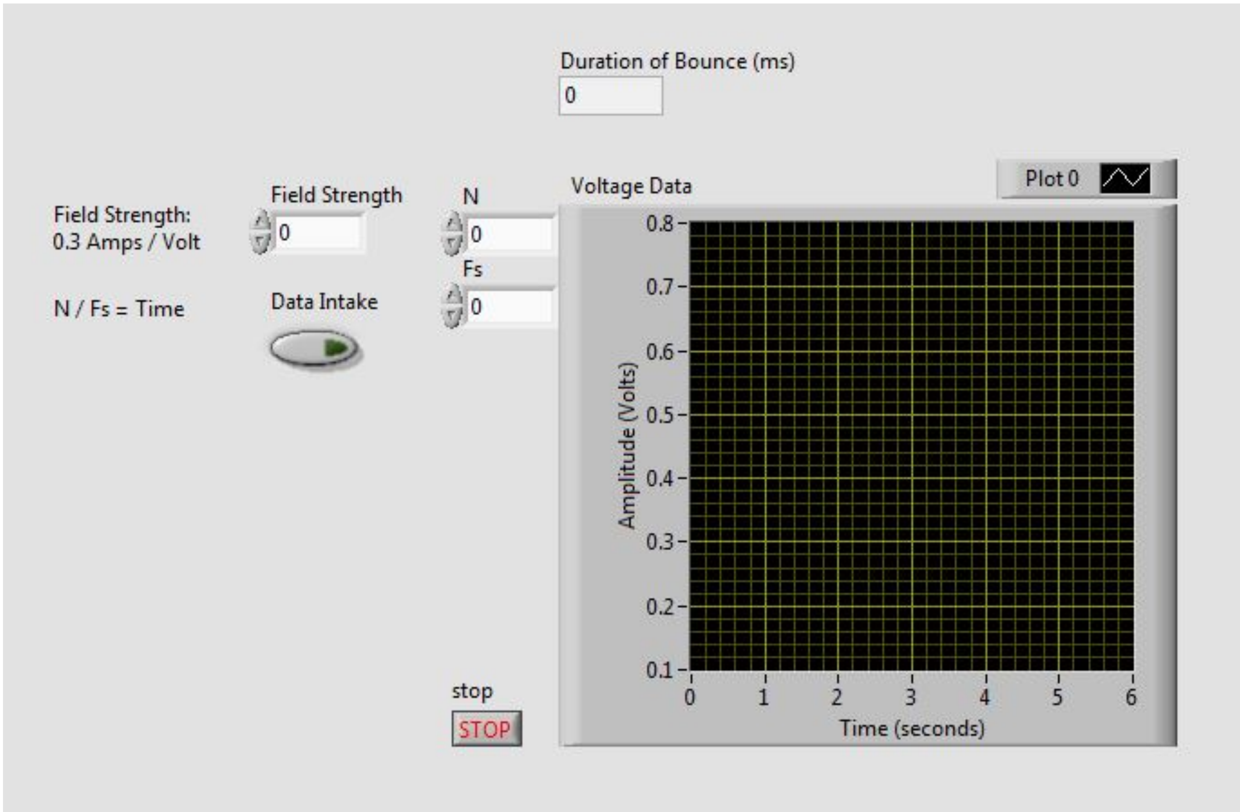



**Figure 6:** Front Panel of VI program using Labview

## Results

Using the data when the bottom coil had a constant current of 1.5 amps, I calculated the minimum and maximum voltages that were recorded to find the minimum and maximum position of the magnet. Figure 7 shows the first 8 trials and shows where the voltage where the two well potentials can be seen. I was able to calculate the voltage range, the equilibrium position, and position of the potential wells.

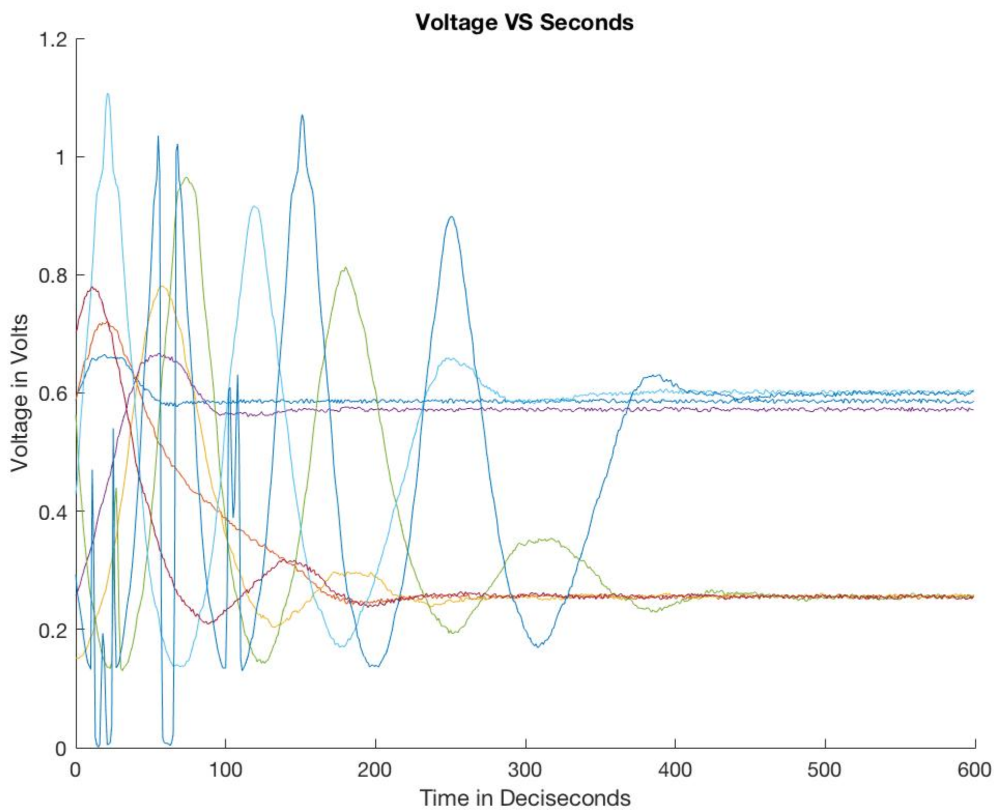



**Figure 7:** These oscillations were from data sets 1 through 8.

The code used to analyze the 20 data trials is shown below and it corresponds to Zminimum = 0.186 V, Zmaximum = 0.864 V, LowerWellPotential = 0.3152 V, and UpperWellPotential = 0.5667 V. This means that when the magnet is at its lowest position, the linear encoder will filter most of the light from the LED and the phototransistor will read the lowest voltage. Vice versa for the highest position. I used the built-in mean, min, and max functions inside Matlab to find these points. For the minimum and maximum positions, I used all the data points and for the two well potentials I only used the data points from 400 deci seconds to 600 deci seconds because that's when the magnet was inside of one of the potential wells.

```
% Finding the highest and lowest voltage for 4 seconds when the magnet was
% in one of the wells

HighVoltage = [MeanAveragePositionData(1),MeanAveragePositionData(4),MeanAveragePositionData(6),MeanAvera(
LowVoltage = [MeanAveragePositionData(2),MeanAveragePositionData(3),MeanAveragePositionData(5),MeanAverage

% Finding where there top and bottom well potential is

UpperWellPotential = mean(HighVoltage);
LowerWellPotential = mean(LowVoltage);

%Finding the Maximum and Minimum values in the data to find the lowest and
%highest point of the magnet
MaxValue = max(ArrayData);
MinValue = min(ArrayData);

Zminimum = mean(MaxValue);
ZMaximum = mean(MinValue);
```

**Figure 8:** Finding the voltage for different positions of the magnet.

After finding these values, I created a vector, named ‘Arbitrarylength’, that had 600 elements where the voltage range was from Zminimum to Zmaximum with equally spaced increments. This was done to match the position of the magnet to a corresponding voltage. The position vector is called, ‘Arbitrarylengthpt2’, starts from -1 to +1 with also 600 elements. The -1 shows that the magnet is at its lowest position and the +1 shows that the magnet is at its highest position. Each voltage in the first vector matches the position of the second vector. This code can be seen below.

```
% Movig Data from Voltage to position (VTP)

% VTP data(1-4)
for i=1:600
    ArbitraryLength(i) = 0.186 + (VoltageRange/6)*i;
end

 ArbitraryLengthpt2 = linspace(-1,1,600)';
VoltagePositionMatrix = [ArbitraryLength ,ArbitraryLengthpt2]

for s1=1:600
    [~,idx] = min(abs(VarName1(s1)-ArbitraryLength));
    Data1(s1) = ArbitraryLengthpt2(idx);
end

for s2=1:600
    [~,idx] = min(abs(VarName2(s2)-ArbitraryLength));
    Data2(s2) = ArbitraryLengthpt2(idx);
end
```

**Figure 9:** Vectors created to make a voltage range with a corresponding position range.

When the vectors were created, I went through all the 20 data trials and converted the voltages into positions between -1 and 1, which are arbitrary lengths and is length of the linear filter with the center being 0. Data sets 1 through 8 are shown in Figure 11 to show the position of the magnet with time. We can see that the position exceeds the minimum and maximum, those values are considered saturated and are outliers in the data sets. I also plotted the two arbitrary vectors to find the equilibrium position of the magnet when there is external magnetic field. The equilibrium position happens at 0.525 V.

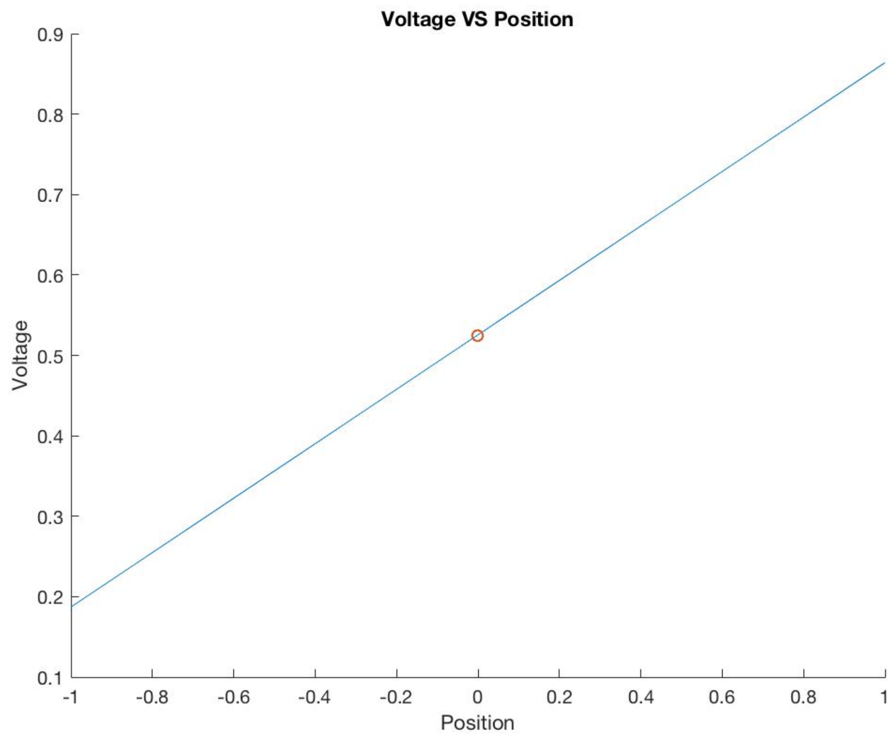



**Figure 10:** This plot is used to find the relative position of the magnet according to the voltage seen by the load resistor. The equilibrium position is at the red circle and it shows at 0.525 Volts.

The data was converted to its position and the graphs were analyzed to find where the double-well potential lie with respect to equilibrium position. The upper well potential for these trials was at 18% above the equilibrium position and the lower well potential was at -80% below the equilibrium position. This shows that there was some error because the two wells should be equally spaced out from one another and from the graph, it looks like the equilibrium position should have been around -0.3. The error could have been due to the position of the coils, a nonlinear filter, or an intensity-varying LED. A double-well potential was still observed with 1.5 amps to the bottom coil and it can be seen that the oscillations were damped and ended in either one of the sub-wells.

The data with 0.9 and 1.2 amps to the constant coil was also analyzed and the equilibrium positions were also found using Matlab. For 0.9 amps we ran 5 trials and found that there was a single equilibrium position at 0.3705 V and for 1.2 amps we ran 10 trials and also found a single equilibrium position which was at 0.334 V. From these observations, I assumed the coil was too weak to create two sub-wells and it was only able to create a single well potential. This also indicates that the actual equilibrium position lies around 0.3 V compared to 0.52 V which was found earlier. When looking at Figure 7, it shows that the center of the two wells lies around 0.3V.

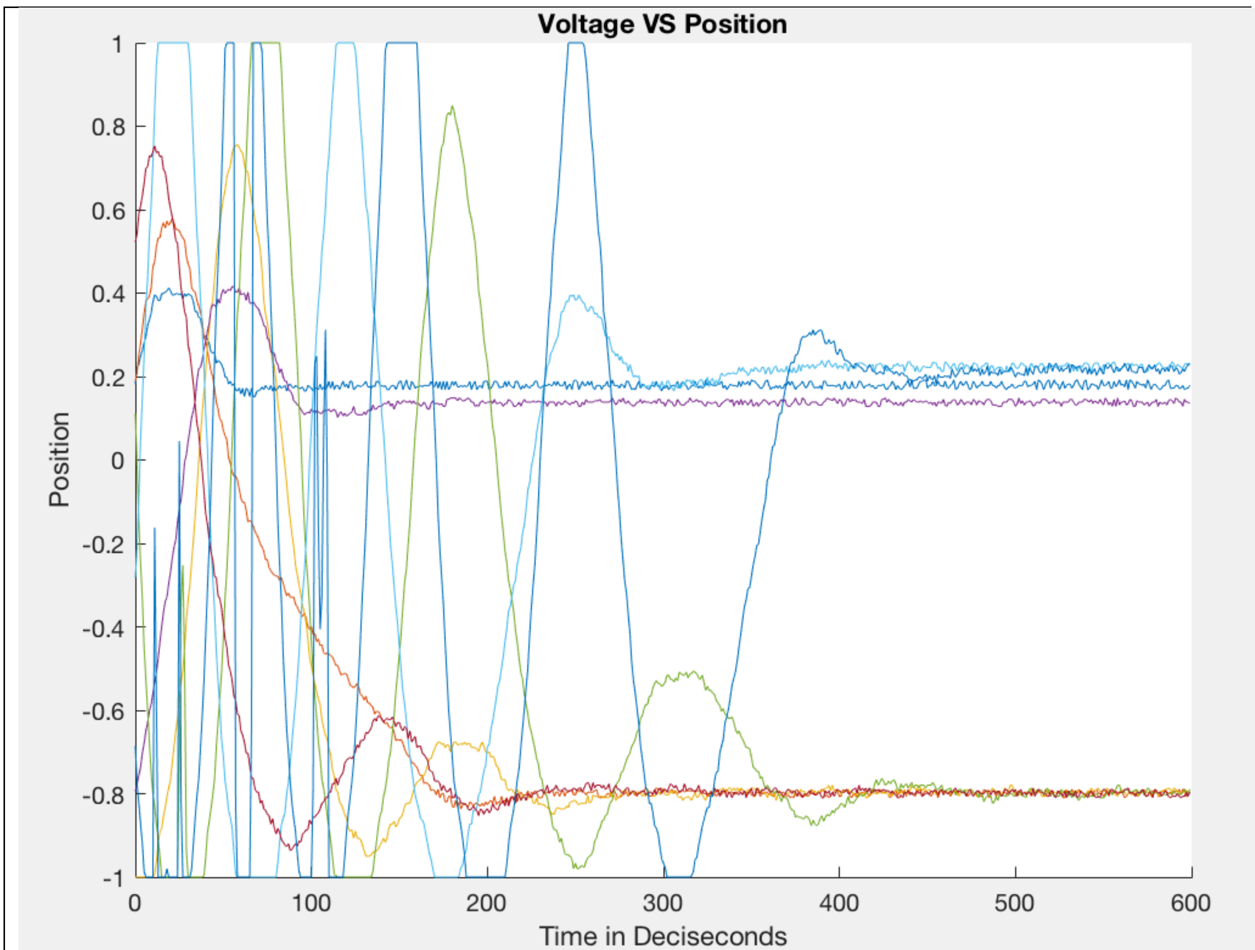


**Figure 11:** Position of the magnet in time for trials 1 through 8.

The double-well potential was analyzed and observed, then the next property was analyzed which is the frequency dependence on the field strength. The field strength that we used were 0.9, 1.2, and 1.5 amps. For these different currents applied to the bottom coil, a corresponding angular frequency of the oscillator can be seen. Using the Fast-Fourier Transform on the data points can show the frequencies that this system oscillates in. Using the fft function inside Matlab I plotted the Fast-Fourier Transform of the data points, which is shown below.

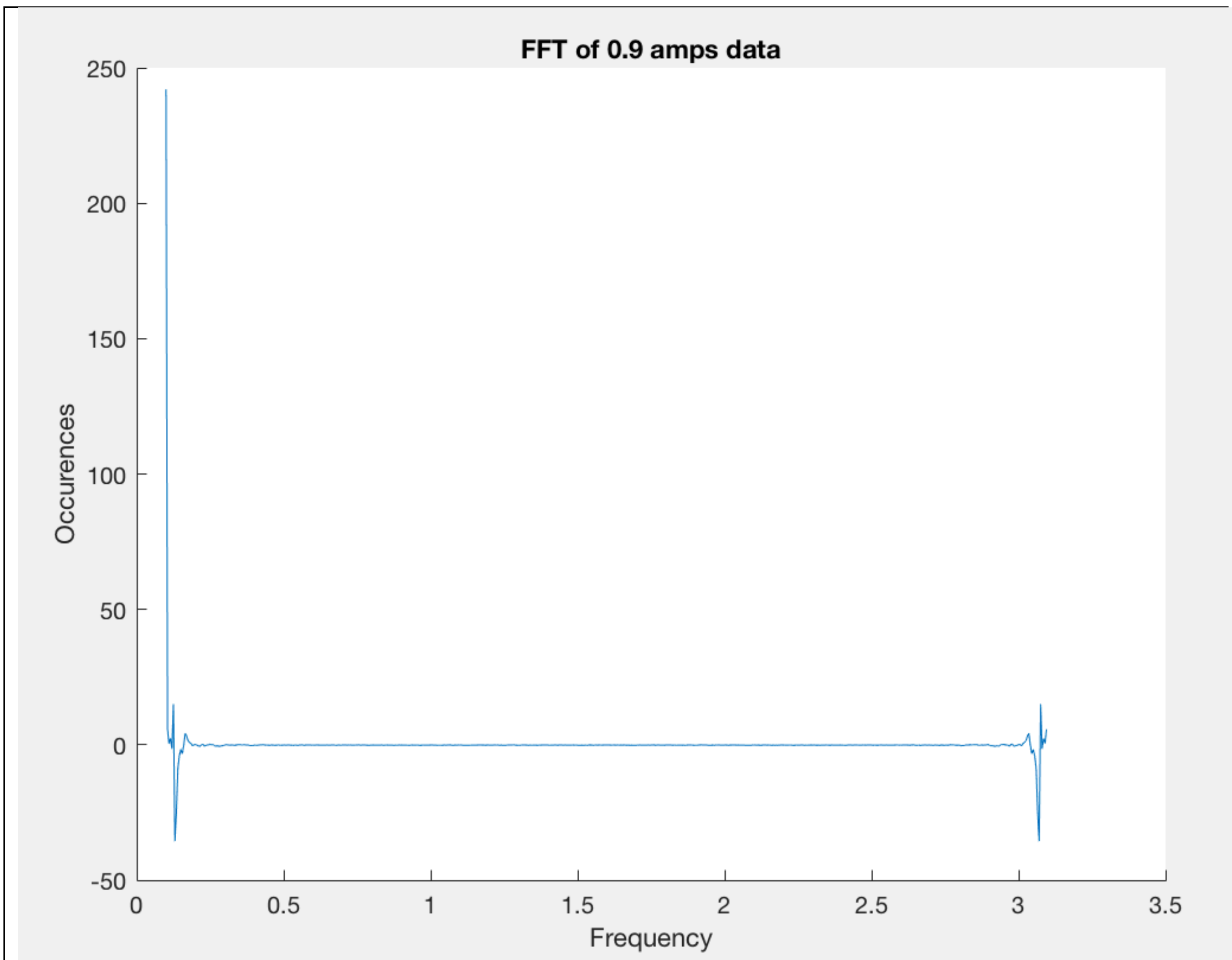


**Figure 12:** Using the Fast-Fourier Transform function in Matlab, the angular frequency was found for 0.9 amps.

Figure 12 shows that the angular frequency of this oscillator is at 1.5 Hz, when a current of 0.9 amps is applied to the bottom coil. This was done for 1.2 amps and 1.5 amps as well and the frequencies that were found were also 1.5 Hz. Varying the excitation current going into the coil should have shown a different normalized frequency but in this experiment the frequencies were the same. This shows that either the frequencies were too close together that they were indistinguishable or the current going into the coil was not varied enough to see changes in the frequencies. To understand the frequency dependence more data points should have been taken for different amounts of current and it would have been nice to be able to vary from 0 to 3 amps but our maximum current output was 1.5 amps for the power supply. Using Labview and Matlab, I was able to show that a double-well potential is created for this nonlinear oscillator and that there was no frequency dependence on the magnetic field even though there should have been one.

**Conclusion**

This experiment was done using the main components such as two coils, a magnet, a linear filter, an LED, and a phototransistor. This setup was connected to a DAQ card and is then processed in Labview and Matlab to show that this setup has the two properties of a double-well potential and a frequency dependence when an external magnetic field is present. The double-well is shown in the results section and is only present when a constant current of 1.5 amps is delivered to the bottom coil. We observed that when the current is lower than that, it created only a single-well potential. Using the 20 sets of data trials, I calculated the position of the two wells with respect to it resting position. If the assumption that the linear filter is a length of 2L, the upper-well potential is 0.18L above the the midpoint of the filter and the lower-well potential is 0.80L below the midpoint. The midpoint of linear filter is also the midpoint of the magnet and it is called the equilibrium position. The theoretical equilibrium position corresponded to a voltage of 0.525 V but the experimental value was 0.3 volts, which shows a percent error of 42.86%. I also found out that the minimum position of the magnet corresponds to a voltage of 0.186V and the maximum position corresponds to a voltage of 0.864 V. The frequency of the oscillation was found to be 1.5 Hz and the resonant frequency of the spring was said to be between 1 to 2 Hz. This experiment was done to show the two properties of this nonlinear oscillator. After completing this experiment, I learned the behavior of a nonlinear oscillator using a magnetic field as my dissipative force. I also learned how to use the tools necessary to trigger the system, acquire the data, analyze the data and then describe the data that was observed. The most important lessons I learned was how to use Labview in order to start the oscillation inside the system and using the DAQ card to record the data onto a text file. I also learned how to use Matlab to calculate the needed parameters and graphing the data to interpret the plots. Other systems can be constructed using different dissipative forces such as a buoyance force. I imagine that the results would be the same but creating a two well potential would be harder because the forces are not concentric like magnetic forces.

**References**

[1] Boas L. Mary 2006 *Mathematical Methods in the Physical Sciences* (Depaul University, Wiley)

[2] Donoso Guillermo, Ladera L. Celso 13 July 2012 *Anharmonic oscillations of a spring-magnet system inside a magnetic coil* (European Journal of Physics)

[3] Griffiths D 1981 *Introduction to Electrodynamics* (Englewood Cliff, NJ: Prentice-Hall)