Calculation of circle parameters in 2D given 3 points

majstr

Aug 2021

1 Derivation

Assuming we have 3 points $\mathbf{A}$,$\mathbf{B}$ and $\mathbf{C}$, we are looking for x_0, y_0 and r, i.e.the parameters of our circle, on which the points lie.

$$\underset{2\times1}{\mathbf{x}} = \begin{bmatrix} x_0 \\ y_0 \end{bmatrix} \underset{2\times1}{\mathbf{A}} = \begin{bmatrix} a_0 \\ a_1 \end{bmatrix} \underset{2\times1}{\mathbf{B}} = \begin{bmatrix} b_0 \\ b_1 \end{bmatrix} \underset{2\times1}{\mathbf{C}} = \begin{bmatrix} c_0 \\ c_1 \end{bmatrix}$$

$$r^2 = (\mathbf{A} - \mathbf{x})^T(\mathbf{A} - \mathbf{x}) = (\mathbf{B} - \mathbf{x})^T(\mathbf{B} - \mathbf{x}) = (\mathbf{C} - \mathbf{x})^T(\mathbf{C} - \mathbf{x})$$

$$\begin{aligned} (\mathbf{A^T} - \mathbf{B^T})\mathbf{x} &= \frac{1}{2}(\mathbf{A^T}\mathbf{A} - \mathbf{B^T}\mathbf{B^T}) \\ (\mathbf{A^T} - \mathbf{C^T})\mathbf{x} &= \frac{1}{2}(\mathbf{A^T}\mathbf{A} - \mathbf{C^T}\mathbf{C^T}) \end{aligned} \tag{1}$$

Or in matrix from

$$\mathbf{W}\mathbf{x} = \mathbf{b}$$

$$\begin{bmatrix} \mathbf{A^T} - \mathbf{B^T} \\ \mathbf{A^T} - \mathbf{C^T} \end{bmatrix} \begin{bmatrix} x_0 \\ y_0 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} \mathbf{A^T}\mathbf{A} - \mathbf{B^T}\mathbf{B} \\ \mathbf{A^T}\mathbf{A} - \mathbf{C^T}\mathbf{C} \end{bmatrix} \tag{2}$$

If $\mathbf{W}$ is not of full rank the point configuration is degenerate, i.e. the points are colinear/overlapping. Linear dependency of rows of $\mathbf{W}$ can be interpreted as point colinearity. If any row of $\mathbf{W}$ is the null vector the corresponding points overlap.