

Calculation of circle parameters in 2D given 3 points

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1 Derivation

Assuming we have 3 points \mathbf{A}, \mathbf{B} and \mathbf{C} , we are looking for x_0, y_0 and r , i.e. the parameters of our circle, on which the points lie.

$$\begin{aligned} \mathbf{x}_{2 \times 1} &= \begin{bmatrix} x_0 \\ y_0 \end{bmatrix} \mathbf{A}_{2 \times 1} = \begin{bmatrix} a_0 \\ a_1 \end{bmatrix} \mathbf{B}_{2 \times 1} = \begin{bmatrix} b_0 \\ b_1 \end{bmatrix} \mathbf{C}_{2 \times 1} = \begin{bmatrix} c_0 \\ c_1 \end{bmatrix} \\ r^2 &= (\mathbf{A} - \mathbf{x})^T (\mathbf{A} - \mathbf{x}) = (\mathbf{B} - \mathbf{x})^T (\mathbf{B} - \mathbf{x}) = (\mathbf{C} - \mathbf{x})^T (\mathbf{C} - \mathbf{x}) \\ (\mathbf{A}^T - \mathbf{B}^T) \mathbf{x} &= \frac{1}{2} (\mathbf{A}^T \mathbf{A} - \mathbf{B}^T \mathbf{B}) \\ (\mathbf{A}^T - \mathbf{C}^T) \mathbf{x} &= \frac{1}{2} (\mathbf{A}^T \mathbf{A} - \mathbf{C}^T \mathbf{C}) \end{aligned} \tag{1}$$

Or in matrix from

$$\mathbf{W} \mathbf{x} = \mathbf{b}$$

$$\begin{bmatrix} \mathbf{A}^T - \mathbf{B}^T \\ \mathbf{A}^T - \mathbf{C}^T \end{bmatrix} \begin{bmatrix} x_0 \\ y_0 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} \mathbf{A}^T \mathbf{A} - \mathbf{B}^T \mathbf{B} \\ \mathbf{A}^T \mathbf{A} - \mathbf{C}^T \mathbf{C} \end{bmatrix} \tag{2}$$

If \mathbf{W} is not of full rank the point configuration is degenerate, i.e. the points are colinear/overlapping. Linear dependency of rows of \mathbf{W} can be interpreted as point colinearity. If any row of the augmented matrix $[\mathbf{W}|\mathbf{b}]$ is the null vector the corresponding points overlap.