Statistics for MFEs – Problem Set 1 (NOT graded!)

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1. For any three events A, B, and D, such that $Pr(D) > 0$, prove that $Pr(A \cup B|D) = Pr(A|D) + Pr(B|D) - Pr(A \cap B|D)$.

2. Consider a machine that produces items in sequence. Under normal operating conditions, the items are independent with probability 0.01 of being defective. However, it is possible for the machine to develop a "memory" in the following sense: After each defective item, and independent of anything that happened earlier, the probability that the next item is defective is 2/5. After each non-defective item, and independent of anything that happened earlier, the probability that the next item is defective is 1/165. Assume that the machine is either operating normally for the whole time we observe or has a memory for the whole time that we observe. Let B be the event that the machine is operating normally, and assume that $Pr(B) = 2/3$. Let D_i be the event that the i^{th} item inspected is defective. Assume that D_1 is independent of B.

   (a) Prove that $Pr(D_i) = 0.01$ for all i. Hint: Use induction.

   (b) Assume that we observe the first six items and the event that occurs is $E = D_1^c \cap D_2^c \cap D_3 \cap D_4 \cap D_5^c \cap D_6^c$. That is, the third and fourth items are defective, but the other four are not. Compute $Pr(B|E)$.

3. Suppose that the PDF of a random variable X is as follows:

$$f(x) = \begin{cases} ce^{-2x} & for\ x > 0, \\ 0 & otherwise. \end{cases}$$

   (a) Find the value of the constant c and sketch the PDF.

   (b) Find the value of $Pr(1 < X < 2)$.

4. Suppose that $X \sim \text{Lognormal}(\mu, \sigma^2)$. Find the distribution of $1/X$.