

# Statistics for MFEs – Problem Set 8

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**Due February 7, 2024, 2:00pm PST, to be submitted via bCourses**

Note: Use basic Python commands (e.g. matrix multiplication) for all questions in this problem set. Do NOT use the built-in package (e.g. statsmodels or pandas regression commands)!

1. Replicate the Monte Carlo simulation on page 77 of the week 8 lecture notes. Add the random walk case of  $\phi = 1$  to the list of  $\phi$ 's.
  - (a) Compute the theoretical distribution of the OLS  $\hat{\phi}$  under the assumptions of the classic OLS model.
  - (b) In addition to the table of means as in the lecture notes, construct the same table for the standard deviation of the distribution of  $\hat{\phi}$  for each combination of  $T, \phi$ .
  - (c) Plot the distribution of the OLS  $\hat{\phi}$ 's for each combination of  $T, \phi$ .
  - (d) Compare the results of the simulations to the theoretical properties of the OLS estimation you derived in part (a).
2. Download data of the S&P 500 index from Yahoo Finance. The Python package `yfinance` can access the Yahoo Finance API and download data without the need to save the data to your computer. Use the ticker `^GSPC` to access S&P 500 data. Use adjusted closing prices to compute a monthly time series of the S&P index,  $P_t$ .
  - (a) Plot  $P_t$  and  $p_t = \log P_t$ . What do you learn from the plots?
  - (b) Construct and plot returns  $R_{t+1} = P_{t+1}/P_t$  and log returns  $r_{t+1} = p_{t+1} - p_t$ . What do you learn from the plots?
  - (c) Compute and plot the autocorrelation functions of  $P_t, p_t, R_t, r_t$ . What do you learn from the autocorrelation functions?
  - (d) Estimate AR(0) (i.e. the only regressor is a constant), AR(1) and AR(2) models for  $p_t$  and  $r_t$  by OLS. Compare the specifications and describe the results. Which AR model is best for prices? Which AR model is best for returns?
  - (e) Perform the OLS diagnostic checks that were included in problem set 7. For each diagnostic, describe the results and explain your conclusion.