Statistics for MFEs – Problem Set 4

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Due January 10, 2023, 2:00pm PST, to be submitted via bCourses

1. DGS ch. 9.2 Problem 8
2. DGS ch. 9.5 Problem 10
3. DGS ch. 9.6 Problem 4
4. DGS ch. 9.6 Problem 6. What is the distribution of the test statistic if $m, n \to \infty$?
5. DGS ch. 7.2 problem 10
6. DGS ch. 7.3 problem 18
7. DGS ch. 7.4 problem 6
8. DGS ch. 7.4 problem 12
9. Consider the Bayesian estimation of the parameter θ of a Poisson sample with a Gamma prior (Example 5 in Lecture Notes 4). Assume that the true $\theta = 2$ and that the prior is $\Gamma(3, 1)$. Write a Jupyter notebook with the following Monte Carlo simulation. Try different draws and check how your results change but you only have to report the results of one simulation.
   (a) Plot the PDF of the prior $\Gamma(3, 1)$ distribution.
   (b) Plot the PDF of Poisson distributions with θ set to the prior mean, the prior mean plus 1 standard deviation of the prior, and the prior mean minus 1 standard deviation.
   (c) Generate a sample of the data of size 500, i.e. draw 500 data points from the Poisson distribution with $\theta = 2$. Assume that you observe the 500 observations sequentially. For each $i = 1, ..., 500$, compute the posterior distribution. Plot the parameters of the i-th posterior as well as the posterior mean and standard deviation.
   (d) For each $i = 1, ..., 500$, compute and plot the mean and standard deviation of posterior distribution.
   (e) Plot the posteriors for $i = 5, 10, 20, 50, 100, 500$.
   (f) For each $i = 1, ..., 500$, compute and plot the Bayes estimator for the quadratic loss function. Also plot the 90%, 95%, and 99% confidence intervals around the Bayes estimator.

(g) Derive the maximum likelihood estimator, its standard error, and limiting distribution for the Poisson sample.

(h) For each $i = 1, ..., 500$, compute and plot the MLE, its standard deviation, and the 90%, 95%, and 99% confidence intervals.

(i) Compare the Bayes and ML estimators and their distributions. Consider the effect of the sample size.