Task 3: Covariance Propagation

$$\begin{bmatrix} x \\ y \end{bmatrix}_t = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}_{t-1} + \begin{bmatrix} \Delta t & 0 \\ 0 & \Delta t \end{bmatrix} \begin{bmatrix} v_x \\ v_y \end{bmatrix}_t + \begin{bmatrix} \eta_x \\ \eta_y \end{bmatrix}_t ;$$

$$\begin{bmatrix} \eta_x \\ \eta_y \end{bmatrix}_t \sim \mathcal{N}\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0{,}1 & 0 \\ 0 & 0{,}1 \end{bmatrix} \right).$$

$\Delta t = 0{,}5.$

---

a) Write the equations corresponding to the mean and covariance after a single propagation:

1) In this equation we have some function around a gaussian for noise. From the L02 we had the following result:

$$y \sim \mathcal{N}(f(\mu), J \Sigma J^T)$$

1) Mean: $f\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} x + 0{,}5 v_x \\ y + 0{,}5 v_y \end{bmatrix}$

2) Covariance:

$$J = \begin{bmatrix} \frac{\partial f_1}{\partial x} & \frac{\partial f_1}{\partial y} \\ \frac{\partial f_2}{\partial x} & \frac{\partial f_2}{\partial y} \end{bmatrix} =$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$\Rightarrow$ covariance does not change!

$$\boxed{y \sim N(f(\mu), \Sigma)}$$

- We can use this result so that we only follow the mean of our position, while cov is the same.