Faster Approximate Pattern Matching: A Unified Approach

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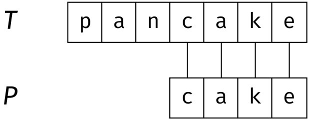
FOCS 2020

Approximate Pattern Matching

Pattern Matching

Given a text T and a pattern P, identify the occurrences of P as a substring of T.



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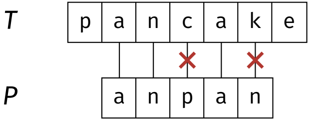

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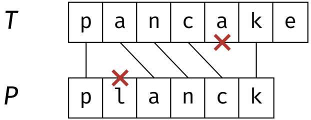

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Grammar Compression

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For a string T, a grammar compression of T is a context-free grammar G_T that generates $\{T\}$. The grammar G_T is wlog. a straight-line program or SLP.

Known Results

Problem	uncompressed text and pattern	SLP text and pattern $n = \Omega(\log N)$, $m = \Omega(\log M)$
Pattern Matching	$O(N+M)$ [KMP'77]	$\tilde{O}(n+m)$ [Jeż'15]
PM with k Mismatches	$\tilde{O}(N + k^2 \cdot N/M)$, $\tilde{O}(N + kN/\sqrt{M})$ [CFPSS'16] [GU'18]	$\tilde{O}(nk^4 + Mk)$ [BKW'19]
PM with k Errors	$O(N + k^4 \cdot N/M)$ [CH'02]	$O(nm \operatorname{poly}(k))$ [BLRSSW'15]

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Improvements obtained via improved/new structural insight in solution structure.

Known Structural Results for PM with Mismatches

Theorem [BKW'19]

Given a pattern P of length M, a text T of length $N \leq {}^3/_2\, M$, and a threshold $k \leq M$, at least one of the following holds:

- The number of k-mismatch occurrences of P in T is at most $O(k^2)$.
- The pattern P is almost periodic (at HD $\leq 6k$ to a string Q with period $O(M/k)$).

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- Additional insights into the almost periodic case

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Main Structural Theorem (HD)

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Are these bounds optimal?—Yes, see long version.

What about PM with errors?

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Main Structural Theorem (ED)

Given a pattern P of length M, a text T of length $N \le {}^3\!/_2\, M$, and a threshold $k \le M$, at least one of the following holds:

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In this talk, Hamming distance only.

Proofs for edit distance work similarly.

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Key Lemma (Analyze)

For each string P of length M, at least one of the following holds:

- P contains $2k$ disjoint breaks; each break has length $M/8k$ and period $> M/128k$.
- P contains disjoint repetitive regions R_i with total length $\ge 3/8 \cdot M$;
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Long version:

- Key Lemma $\Longrightarrow$ Main Structural Theorem
- Proof idea for Key Lemma

How do we turn the structural insights into algorithms?

Obtaining Faster Algorithms

- All of our algorithms rely on a small set of essential operations:
  - `LCP`(S, T): Compute the length of the longest common prefix of S and T.
  - `LCP`^{R}(S, T): Compute the length of the longest common suffix of S and T.
  - `IPM`(P, T): Compute all exact matches of P in T.
  - `Length`(S): Compute the length $|S|$ of S.
  - `Access`(S, i): Retrieve the character $S[i]$.
  - `Extract`(S, ℓ, r): Extract the fragment (or substring) $S[\ell\,..\,r)$ from S.

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Theorem

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+

Theorem

Fast PILLAR Implementation

=

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This work		This work: Combination of known results for 3 settings *(see long version)*		

Obtaining Fast Algorithms: The Fully-Compressed Setting

The `PILLAR` operations: `LCP`, $\texttt{LCP}^R$, `IPM`, `Length`, `Access`, `Extract`

SLPs: G_P of size m generating pattern P, G_T of size n generating text T.

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For any positive threshold $k \leq |P|$, we can compute the number of all k-mismatch occ's of P in T in time $O(m \log(|P| + |T|) + nk^2 \log^3(|P| + |T|))$.

(Reporting of all occ's takes time linear in the number of occ's.)

Theorem (Algorithm for PM w/ Errors)

For any positive threshold $k \leq |P|$, we can compute the number of all k-error occ's of P in T in time $O(m \log(|P| + |T|) + nk^4 \log^3(|P| + |T|))$.

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Thank You!

Full paper: `arxiv.org/abs/2004.08350`

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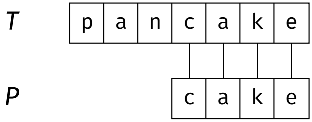

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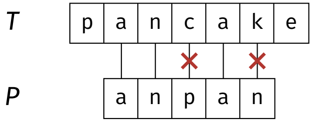

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Matching (conditional) lower bound (for combinatorial algorithms) [GU'18]

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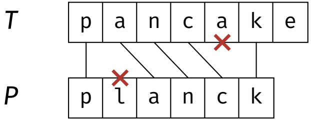

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Not in this talk: How to shrink the gap.

What if the text and the pattern are *huge*?

o	f	s	w	e	e	t	b	e	e	n	c	e	n	t	e	r	e	d	i	n	t	h	e	a	n	p	a	n	f	o	r	e	x	a	m

n	p	a	s	t	e	a	n	p	a	n	c	a	n	a	l	s	o	b	e	p	r	e	p	a	r	e	d	w	i	t	h	o	t	h	e

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Straight-Line Program (SLP)

An SLP G_T is a set of non-terminals $\{T_1, \dots, T_n\}$ and productions of the form $T_i \to a, a \in \Sigma$ or $T_i \to T_\ell T_r$, where $\ell, r < i$. The starting symbol is T_n.

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$T_1 \to \text{a}; \quad T_2 \to \text{n}; \quad T_3 \to \text{p}$

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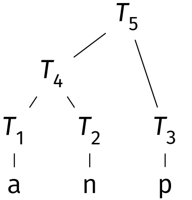

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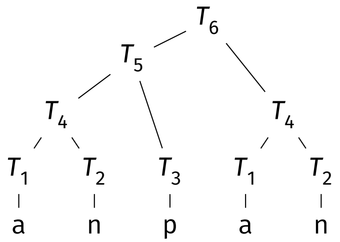
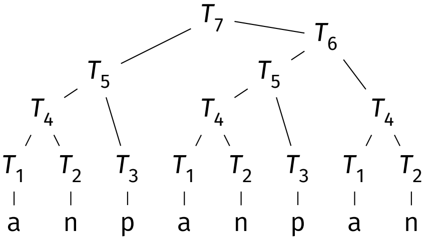


Grammar Compression

Straight-Line Program (SLP)

An SLP G_T is a set of non-terminals $\{T_1, \ldots, T_n\}$ and productions of the form $T_i \to a, a \in \Sigma$ or $T_i \to T_\ell T_r$, where $\ell, r < i$. The starting symbol is T_n.

$$T_1 \to \mathrm{a}; \quad T_2 \to \mathrm{n}; \quad T_3 \to \mathrm{p}$$
$$T_4 \to T_1 T_2; \qquad T_5 \to T_4 T_3$$
$$T_6 \to T_5 T_4; \qquad T_7 \to T_5 T_6$$



Structure of Exact Pattern Matching

Fact (Folklore)

Let text T and pattern P, $|T| \leq {}^3/_2\,|P|$, be given such that there are ≥ 2 matches of P in T that together match T completely. Then, both P and T are periodic with some period x and every match of P in T starts at a position $1 + i \cdot |x|$.

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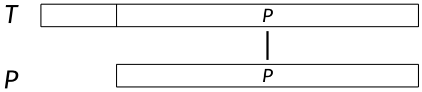

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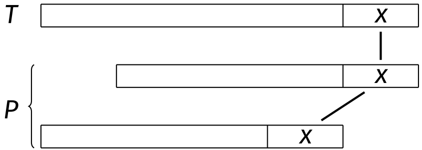

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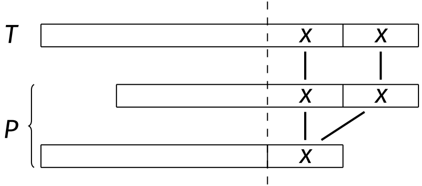

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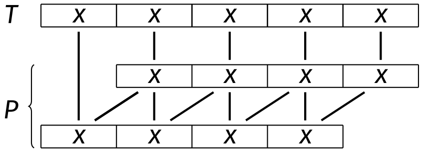

Structural Results for PM with Mismatches

Main Structural Theorem (HD)

Given a pattern P of length M, a text T of length $N \le {}^3\!/_2\, M$, and a threshold $k \le M$, at least one of the following holds:

- The number of k-mismatch occurrences of P in T is at most $\cancel{O(k^2)}$ $O(k)$.
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Are these bounds optimal?

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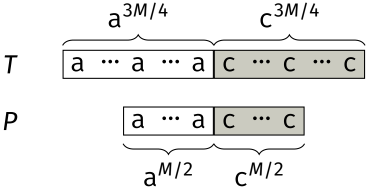

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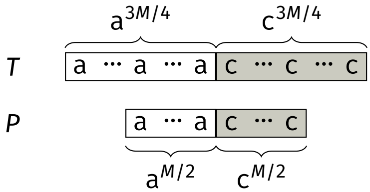

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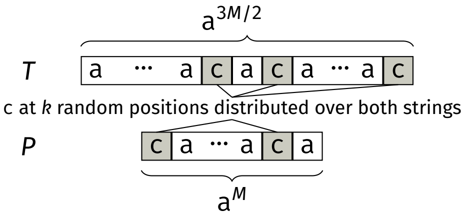

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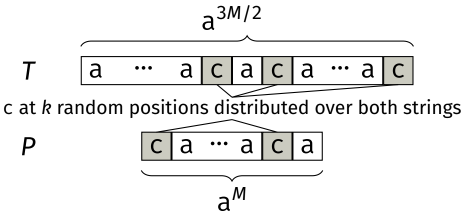

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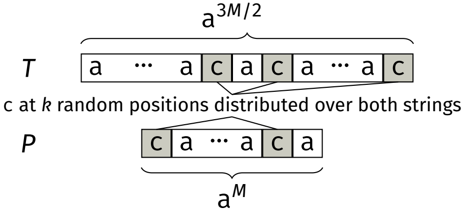

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Observation [BKW'19, refined]

If P has HD $\ge 2k$ and < $8k$ to a string w/ period $O(M/k)$, there are $O(k)$ k-mism. occ's of P in T.

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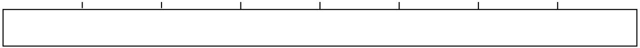

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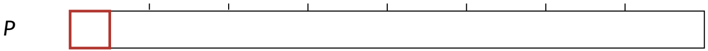

- Process P from left to right, $M/8k$ new characters at a time.

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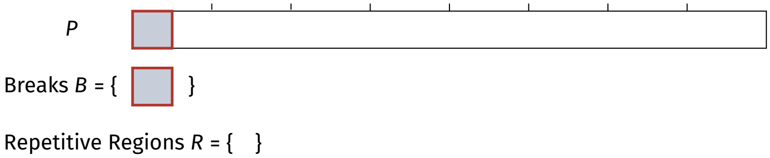

- If a fragment has a period $> M/128k$, add it to the found breaks.

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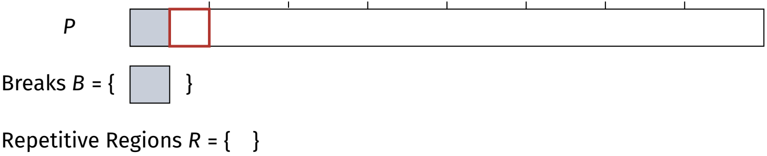

- Otherwise, find the shortest prefix (longer than $M/8k$) that is a repetitive region.

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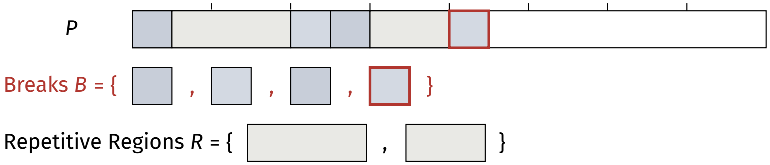

- If we found $2k$ breaks, return the breaks.

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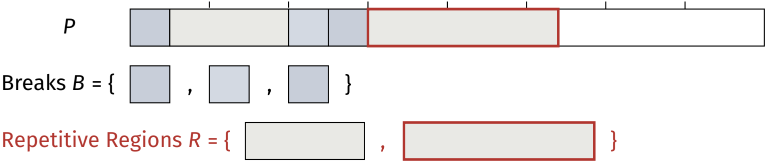

- If the total length of the repetitive regions is $> 3/8 \cdot M$, return the repetitive regions.

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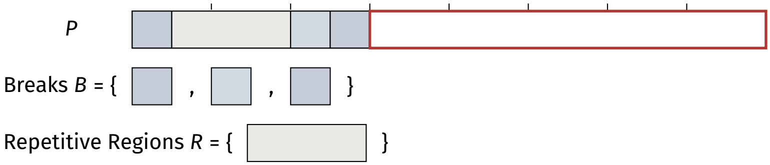

- If we reach the end of P, try to find a single repetitive region starting from the end.

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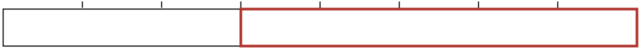

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- If we found a repetitive region, return it.

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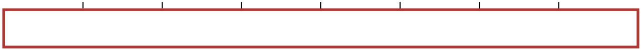

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Repetitive Regions $R = \{\quad\}$

- If we again don't obtain a repetitive region, P is almost periodic.

Analyzing the Pattern, Proof Idea

Key Lemma (Analyze) ✓

For each string P of length M, at least one of the following holds:

- P contains $2k$ disjoint breaks; each break has length $M/8k$ and period $> M/128k$.
- P contains disjoint repetitive regions R_i with total length $\geq 3/8 \cdot M$;
  each region has length $\geq M/8k$ and is almost periodic with HD exactly $8k/M \cdot |R_i|$.
- P is almost periodic (at HD $< 8k$ to a string with period $M/128k$).

Obtaining Fast Algorithms: Fast PILLAR Algorithms

The PILLAR operations: LCP, LCP^R, IPM, Length, Access, Extract

Theorem (PILLAR Alg. for PM w/ Mism.)

Given a pattern P of length m, a text T of length n, and a positive threshold $k \le m$, we can compute (a representation of) all k-mismatch occurrences of P in T using $O(n/m \cdot k^2 \log\log k)$ time plus $O(n/m \cdot k^2)$ PILLAR operations.

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Theorem (PILLAR Alg. for PM w/ Errors)

Given a pattern P of length m, a text T of length n, and a positive threshold $k \le m$, we can compute (a representation of) all k-error occurrences of P in T using $O(n/m \cdot k^4)$ PILLAR operations.

Obtaining Fast Algorithms: The Standard Setting

The PILLAR operations: LCP, LCP^R, IPM, Length, Access, Extract

Uncompressed strings: pattern P of length M, text T of length N.

Obtaining Fast Algorithms: The Standard Setting

The `PILLAR` operations: `LCP`, $\texttt{LCP}^R$, `IPM`, `Length`, `Access`, `Extract`

Uncompressed strings: pattern P of length M, text T of length N.

We can implement each operation in $O(1)$ time. (After $O(N + M)$ preprocessing.)

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Theorem (Algorithm for PM w/ Mism.)

For any positive threshold $k \le M$,
we can compute all k-mismatch occ's of P in T in time $O(N + N/M \cdot k^2 \log\log k)$.

Theorem (Algorithm for PM w/ Errors)

For any positive threshold $k \le M$,
we can compute (starting positions of) all k-error occ's of P in T in time $O(N + N/M \cdot k^4)$.

Obtaining Fast Algorithms: The Fully-Compressed Setting

The PILLAR operations: LCP, LCP^R, IPM, Length, Access, Extract

SLPs: G_P of size m generating pattern P, G_T of size n generating text T.

Obtaining Fast Algorithms: The Fully-Compressed Setting

The PILLAR operations: LCP, LCP^R, IPM, Length, Access, Extract

SLPs: G_P of size m generating pattern P, G_T of size n generating text T.

Using Recompression [Jeż'15], we can implement each operation in $O(\log^3(|P| + |T|))$ time.

(After $O((n + m) \log(|P| + |T|))$ preprocessing.)

Obtaining Fast Algorithms: The Fully-Compressed Setting

The `PILLAR` operations: `LCP`, $\texttt{LCP}^R$, `IPM`, `Length`, `Access`, `Extract`

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Theorem (Algorithm for PM w/ Mism.)

For any positive threshold $k \leq |P|$, we can compute the number of all k-mismatch occ's of P in T in time $O(m\log(|P| + |T|) + nk^2\log^3(|P| + |T|))$.

(Reporting of all occ's takes time linear in the number of occ's.)

Theorem (Algorithm for PM w/ Errors)

For any positive threshold $k \leq |P|$, we can compute the number of all k-error occ's of P in T in time $O(m\log(|P| + |T|) + nk^4\log^3(|P| + |T|))$.

(Reporting of all occ's takes time linear in the number of occ's.)

Obtaining Fast Algorithms: The Dynamic Setting

The `PILLAR` operations: `LCP`, $\mathtt{LCP}^R$, `IPM`, `Length`, `Access`, `Extract`

Dynamic maintenance of a collection of (non-empty persistent) strings $\mathcal{X}$ of total length N; supporting `makestring`, `concat`, `split`.

Obtaining Fast Algorithms: The Dynamic Setting

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Dynamic maintenance of a collection of (non-empty persistent) strings X of total length N; supporting makestring, concat, split.

Using Optimal Dynamic Strings [Gawrychowski et al.'18], we can implement each PILLAR operation in (w.h.p) $O(\log^2 N)$ time.

Obtaining Fast Algorithms: The Dynamic Setting

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Theorem (Algorithm for PM w/ Mism.)

For any two strings $P, T \in X$ and any threshold k, we support the additional operation "Find all k-mismatch occ's of P in T" in (w.h.p) $O(|T|/|P| \cdot k^2 \log^2 N)$ time.

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For any two strings $P, T \in X$ and any threshold k, we support the additional operation "Find all k-error occ's of P in T" in (w.h.p) $O(|T|/|P| \cdot k^4 \log^2 N)$ time.

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