Inter-University Research Institute Corporation
Research Organization of Information and Systems
National Institute of Informatics

Revitalizing Research on Approximate String Matching Algorithms

Philip Wellnitz

National Institute of Informatics

Based on joint works with Karl Bringmann, Alejandro Cassis, Panagiotis Charalampopoulos, Tomasz Kociumaka, Marvin Künnemann, and Jakob Nogler.

Research Focus: Theoretical Guarantees for Fundamental Problems

Approximate String Matching

[BK**W**,SODA’19]
[CK**W**,FOCS’20,’22,’25]
[CK**W**,FOCS’23]
[NK**W**,STOC’24,
SODA’25]

Counting Structures in Networks

[DR**W**,ICALP’19][R**W**,SODA’20]
[RS**W**,FOCS’20,ICALP’21]
[DRS**W**,MFCS’19]
[DM**W**,STOC’24,
SODA’25]

Dense Subset Sum

[B**W**,SODA’20]

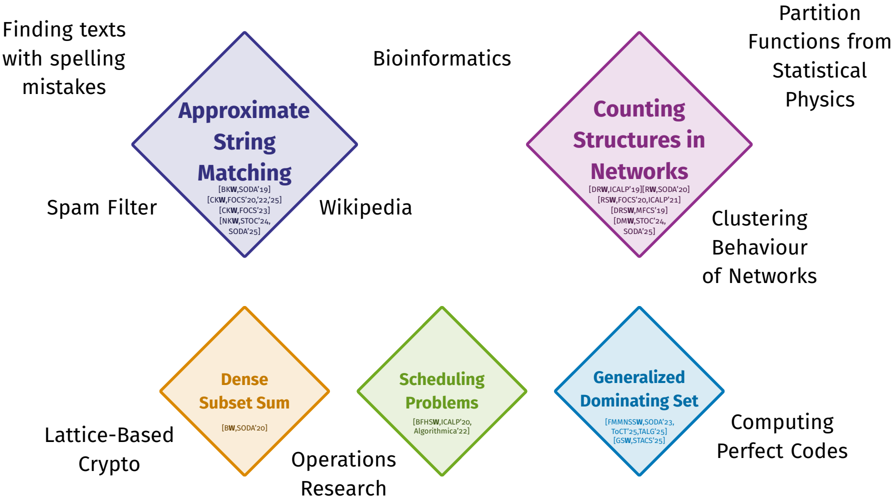
Scheduling Problems

[BFHS**W**,ICALP’20,
Algorithmica’22]

Generalized Dominating Set

[FMMNSS**W**,SODA’23,
ToCT’25,TALG’25]
[GS**W**,STACS’25]

Research Focus: Theoretical Guarantees for Fundamental Problems of Practical Relevance



Approximate String Matching
[BK**W**,SODA'19]
[CK**W**,FOCS'20,'22,'25]
[CK**W**,FOCS'23]
[NK**W**,STOC'24,
SODA'25]

Finding texts with spelling mistakes

Bioinformatics

Spam Filter

Wikipedia

Counting Structures in Networks
[DR**W**,ICALP'19][R**W**,SODA'20]
[RS**W**,FOCS'20,ICALP'21]
[DRS**W**,MFCS'19]
[DM**W**,STOC'24,
SODA'25]

Partition Functions from Statistical Physics

Clustering Behaviour of Networks

Dense Subset Sum
[B**W**,SODA'20]

Lattice-Based Crypto

Scheduling Problems
[BFHS**W**,ICALP'20,
Algorithmica'22]

Operations Research

Generalized Dominating Set
[FMMNSS**W**,SODA'23,
ToCT'25,TALG'25]
[GS**W**,STACS'25]

Computing Perfect Codes

Inter-University Research Institute Corporation
Research Organization of Information and Systems
National Institute of Informatics

Approximate String Matching

An Example

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An Example

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✿✿✿ ✿ ✿✿ (TU Kaiserslautern).

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✿✿ ✿✿ ✿ ✿ ✿ (Simons Institute and UC Berkeley), ✿ ✿✿✿ ✿✿✿ (University of Salzburg), ✿✿✿✿✿✿ ✿ (University of Salzburg).

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(University of Pennsylvania), ✿✿ ✿ ✿ ✿ ✿ (University of Pennsylvania), ✿✿ ✿ (University of Pennsylvania).

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✿✿ ✿✿✿✿✿ ✿✿✿✿✿✿✿✿✿✿✿✿✿✿ (Reichman University, Herzliya, Israel and Birkbeck, University of London), ✿✿✿✿ ✿✿✿✿✿✿ (UC Berkeley and Max Planck Institute for Informatics, SIC, **Saarbrücken**, Germany), ✿ ✿✿ ✿✿ (Max Planck Institute for Informatics, SIC, **Saarbrücken**, Germany).

✿✿ ✿✿ ✿✿✿✿✿ ✿ ✿✿✿✿✿✿✿✿ ✿✿✿✿✿✿✿✿✿✿

✿ ✿✿ ✿ ✿✿ (National Institute of Informatics).

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✿✿✿✿✿✿✿ ✿ ✿✿✿✿ (UniversitÃ© Paris CitÃ©, CNRS, IRIF, F-75013, Paris, France); ✿✿✿✿ ✿ ✿ ✿✿✿ (Universitat PolitÃ¨cnica de Catalunya)

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✿✿✿✿ ✿ ✿ ✿ (ETH Zurich); ✿✿ ✿✿ ✿✿✿✿✿✿ ✿ ✿✿ (Toyota Technological Institute at Chicago)

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✿✿✿✿ (Merton College, University of Oxford, United Kingdom); ✿✿ ✿ ✿✿✿ ✿ (Mathematical Institute, University of Bonn, Germany); ✿ ✿✿ ✿✿ (Max Planck Institute for Informatics, Saarland Informatics Campus (SIC), Saarbr|cken, Germany)

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✿✿ ✿✿✿✿✿ ✿ (The Academic College of Tel Aviv-Yafo), ✿✿✿✿ ✿✿✿✿✿✿✿ (UC Berkeley), ✿✿✿ ✿✿✿ (Weizmann Institute of Science), ✿✿✿✿ ✿ (University of California San Diego).

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✿✿ ✿ ✿ (Alibaba Group); ✿✿ ✿✿ ✿ (Microsoft Research); ✿ ✿✿✿ ✿ (Washington University in St. Louis); ✿ ✿ (Columbia University); ✿✿ ✿✿✿ (University of Chicago)

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✿✿✿ ✿✿✿ (Stony Brook University); ✿✿✿✿ ✿✿✿✿✿✿ (Max Planck Institute for Informatics, Saarbruecken)

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✿✿ ✿✿✿✿✿✿ (Univ. Bordeaux, CNRS, Bordeaux INP, LaBRI, France); ✿✿ ✿✿✿✿ ✿ ✿✿ (University of Warsaw, Poland); ✿✿✿ ✿ ✿✿ (Saarland University and Max Planck Institute for Informatics, SaarbrÃ¼cken, Germany); ✿ ✿ ✿✿✿✿✿ ✿ ✿ (University of Warsaw, Poland)

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✿✿✿✿✿ ✿ ✿ (ETH ZÃ¼rich); ✿✿ ✿✿ ✿✿✿✿ ✿ ✿✿✿ ✿ (TU Berlin)

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An Example

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An Example

Task: Find **Saarbrücken** in a text.

An Example

Task: Find **Saarbrücken** in a text.

Or Saarbruecken.

An Example

Task: Find **Saarbrücken** in a text.

Or Saarbruecken. Or Sarrebruck.

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Task: Find **Saarbrücken** in a text.

Or Saarbruecken. Or Sarrebruck. Or Saarbrucken, Saarbr|cken, SaarbrÃ¼cken, ...

The Approximate String Matching Problem

early 1980’s

Approximate String Matching

Given a text T, a pattern P, and an integer k, identify the (starting positions of) substrings of T that are at **edit distance** of at most k to P.

The Approximate String Matching Problem

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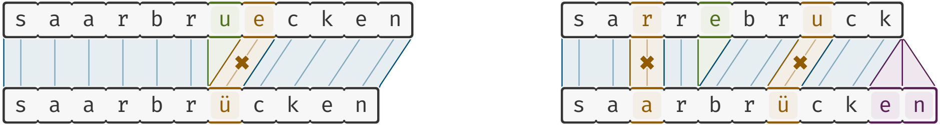
Given a text T, a pattern P, and an integer k, identify the (starting positions of) substrings of T that are at **edit distance** of at most k to P.

Edit distance: minimum number of insertions, deletions, or substitutions of single characters to transform one string into another string



Outline

Basic Tricks and Tools, Previous Work

New Algorithms via Structural Insights

Spotlight Extension: Quantum Algorithms for ASM

Spotlight Extension: String Matching with Weighted Edits

Open Problems and Future Directions

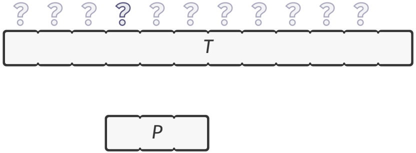
Basic Tricks and Tools: The Standard Trick

early 1980's

Approximate String Matching

Given: text T, pattern P, threshold k; Find: (starting pos. of) substrings of T at edit distance ≤ k to P.

Focus: Obtain starting positions of occurrences



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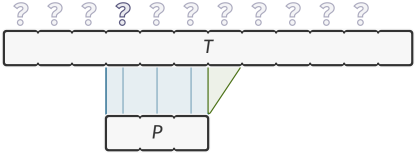

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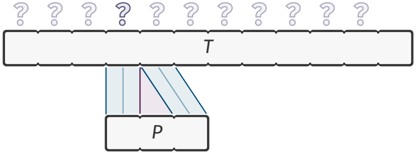

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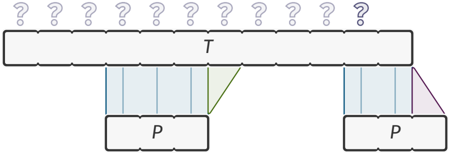

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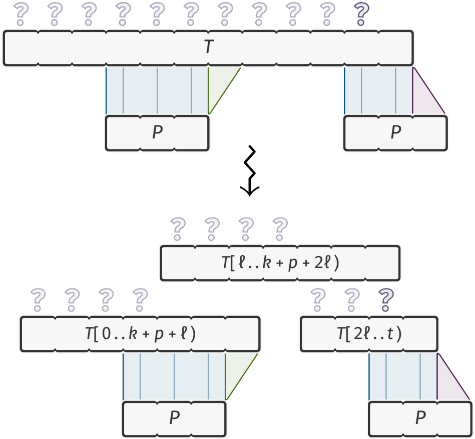
"Standard Trick": write $t := |T|$, $p := |P|$

Split T into overlapping fragments of len $\ell + p + k$

$\rightsquigarrow$ $O(t/\ell)$ instances,
each "responsible" for its first ℓ positions



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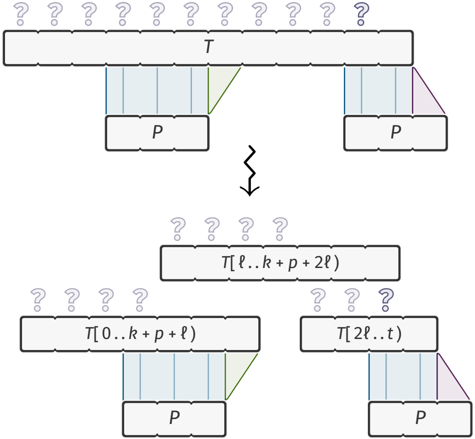

Basic Tricks and Tools: (Filter and) Verify

early 1980's

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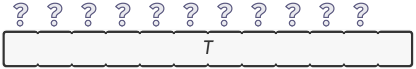

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"Filter and Verify Paradigm"

Basic Tricks and Tools: (Filter and) Verify

early 1980's

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Given: text T, pattern P, integer k; Find: (starting pos. of) substrings of T at edit distance ≤ k to P.

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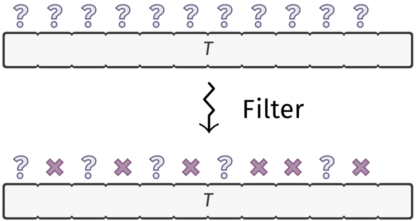
“Filter and Verify Paradigm”

Step 1, Filter: (typically fast)
Compute (small) superset of starting positions



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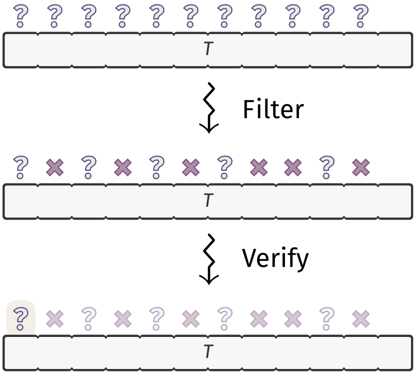
Check for occ at each remaining position



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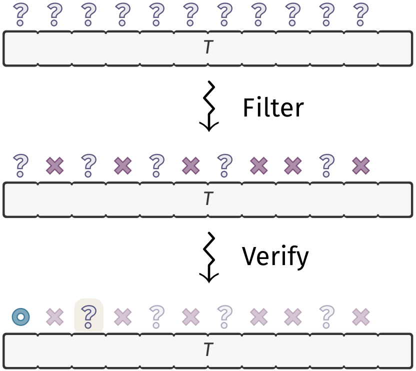

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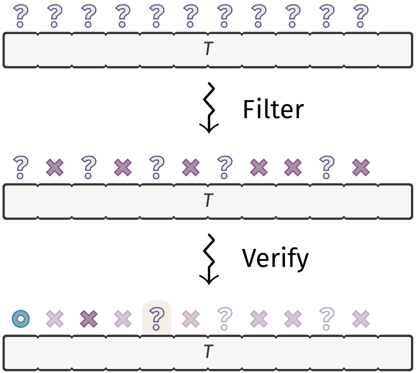

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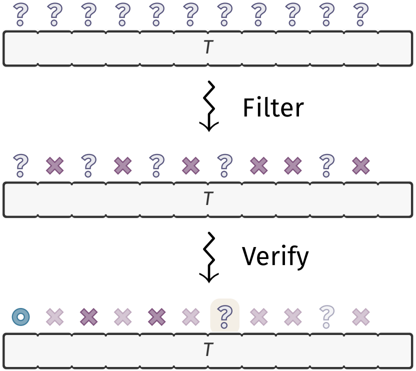

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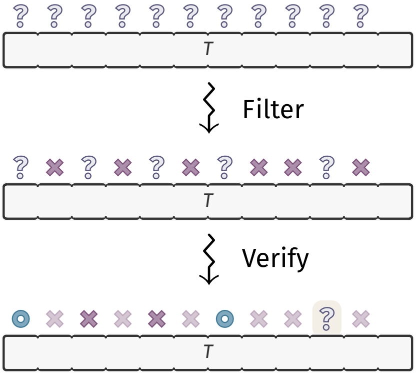

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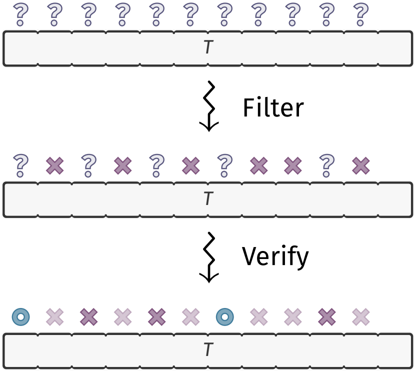

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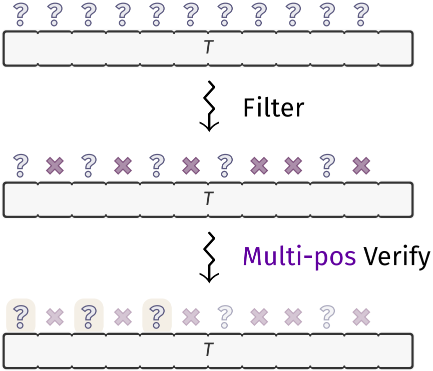
Step 2', Multi-pos Verify: (typically slow)
Check for occ at each remaining position,
multiple positions at once



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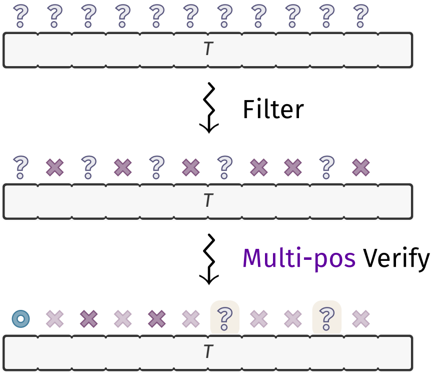

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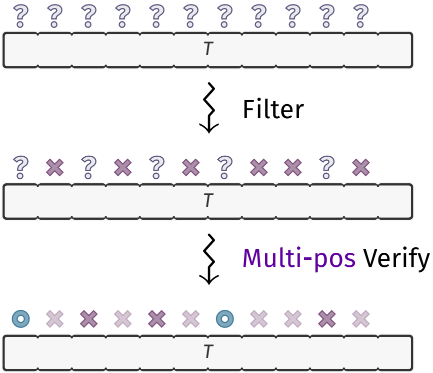
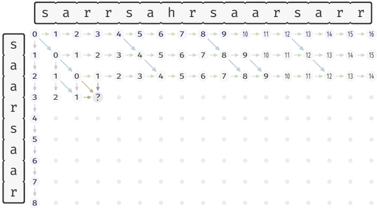


early 1980's

Approximate String Matching

Given: text T, pattern P, threshold k; Find: (starting pos. of) substrings of T at edit distance $\leq k$ to P.

Edit dist/Verify Algorithm	ASM Algorithm	($t := \lvert T \rvert, p := \lvert P \rvert$)
Textbook DP ⇝ $O(tp)$ [Sellers, 1980] and others	Verify pos 1 by 1 ⇝ $O(t^2p)$	substr



$e_{i,0} = i$ (delete $T[0..i)$)

$e_{0,j} = j$ (insert $P[0..j)$)

$$e_{i,j} = \min \begin{cases} e_{i-1,j} + 1 & \text{(del from } T\text{)} \\ e_{i,j-1} + 1 & \text{(ins in } T\text{)} \\ e_{i-1,j-1} + [T[i] \neq P[j]] & \text{(match/subst)} \end{cases}$$

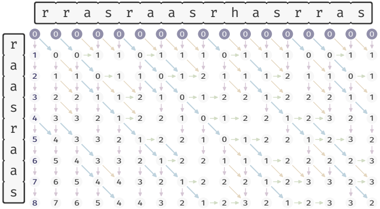
Classical Algorithms

Approximate String Matching

early 1980's

Given: text T, pattern P, threshold k; Find: (starting pos. of) substrings of T at edit distance $\leq k$ to P.

Edit dist/Verify Algorithm	ASM Algorithm	$(t := \|T\|, p := \|P\|)$
Textbook DP $\rightsquigarrow$ $O(tp)$ [Sellers, 1980] and others	Verify pos 1 by 1 $\rightsquigarrow$ $O(t^2p)$ Verify all pos at once $\rightsquigarrow$ $O(tp)$	substr start pos



		r	r	a	s	r	a	a	s	r	h	a	s	r	r	a	s
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
r	1	0	0	1	1	0	1	1	1	0	1	1	1	0	0	1	1
a	2	1	1	0	1	1	0	1	2	1	1	1	2	1	1	0	1
a	3	2	2	1	1	2	1	0	1	2	2	1	2	2	2	1	1
s	4	3	3	2	1	2	2	1	0	1	2	2	1	2	3	2	1
r	5	4	3	3	2	1	2	2	1	0	1	2	2	1	2	3	2
a	6	5	4	3	3	2	1	2	2	1	1	1	2	2	2	2	3
a	7	6	5	4	4	3	2	1	2	2	2	1	2	3	3	2	3
s	8	7	6	5	4	4	3	2	1	2	3	2	1	2	3	3	2

$e_{i,0} = 0$ (delete $T[\,0\,..\,i\,)$)

$e_{0,j} = j$ (insert $P[\,0\,..\,j\,)$)

$$e_{i,j} = \min \begin{cases} e_{i-1,j} + 1 & \text{(del from } T\text{)} \\ e_{i,j-1} + 1 & \text{(ins in } T\text{)} \\ e_{i-1,j-1} + [T[\,i\,] \neq P[\,j\,]] & \text{(match/subst)} \end{cases}$$

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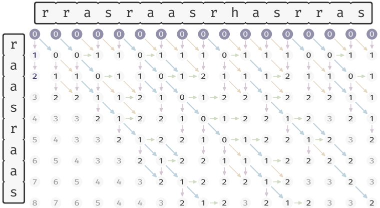
DP, diagonally compute only entries $\leq k$ $\rightsquigarrow O(t + kp)$	Verify pos 1 by 1 $\rightsquigarrow O(tkp)$ Verify $\ell \leq t$ pos at once $\rightsquigarrow O(t/\ell \cdot (\ell + k)p) = O(tp)$	substr start pos



		r	r	a	s	r	a	a	s	r	h	a	s	r	r	a	s
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
r	1	0	0	1	1	0	1	1	1	0	1	1	1	0	0	1	1
a	2	1	1	0	1	1	0	1	2	1	1	1	2	1	1	0	1
a	3	2	2	1	1	2	1	0	1	2	2	1	2	2	2	1	1
s	4	3	3	2	1	2	2	1	0	1	2	2	1	2	3	2	1
r	5	4	3	3	2	1	2	2	1	0	1	2	2	1	2	3	2
a	6	5	4	3	3	2	1	2	2	1	1	1	2	2	2	2	3
a	7	6	5	4	4	3	2	1	2	2	2	1	2	3	3	2	3
s	8	7	6	5	4	4	3	2	1	2	3	2	1	2	3	3	2

Prune whenever $e_{i,j} > k$.

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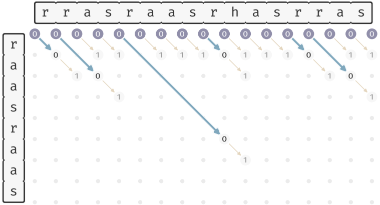

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Approximate String Matching

early 1980's

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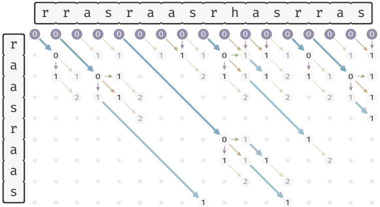

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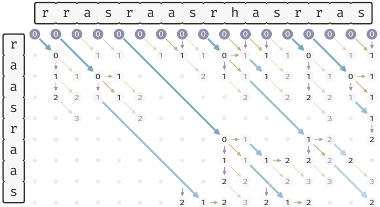

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⇝ Didn't use filter yet...

Classical Algorithms

early 1980's

Approximate String Matching

Given: text T, pattern P, integer k; Find: (starting pos. of) substrings of T at edit distance $\leq k$ to P.

Idea: Can we filter out diagonals that will never lead to an occurrence?

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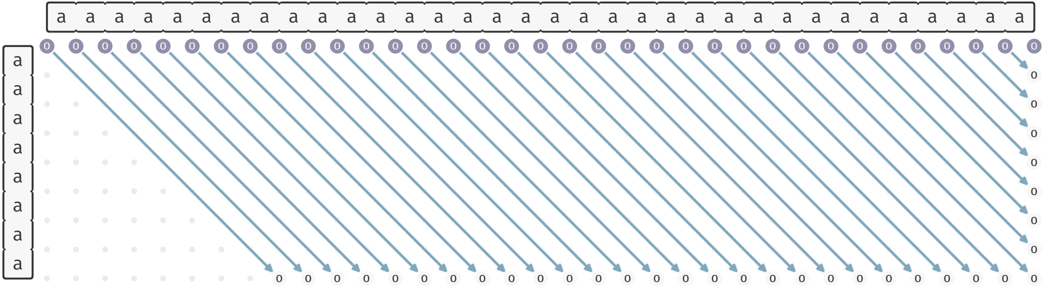

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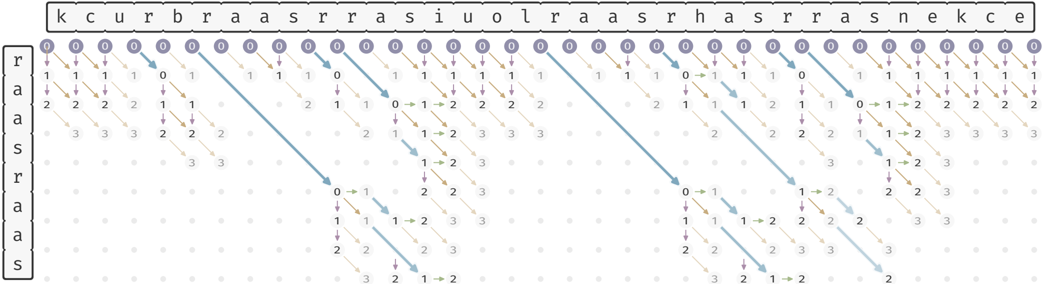
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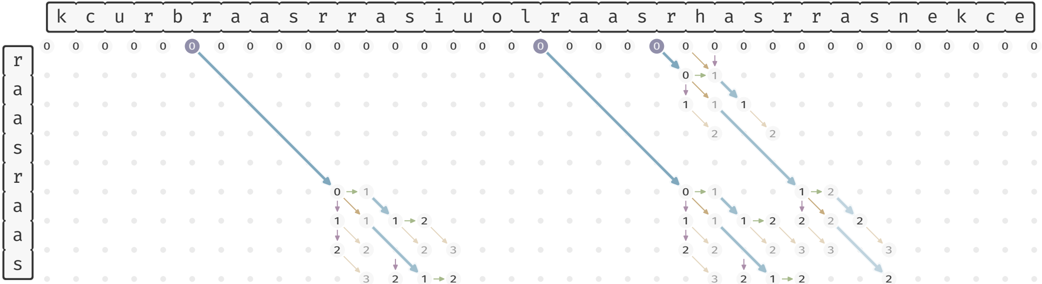
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Classical Algorithms

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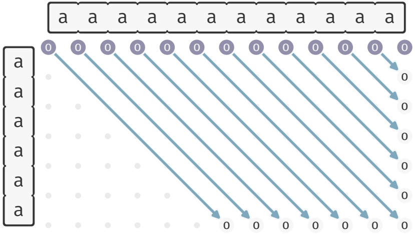

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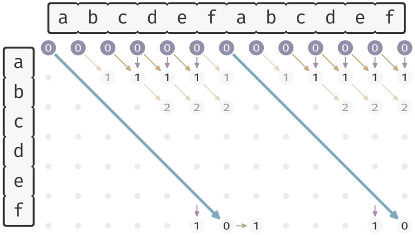

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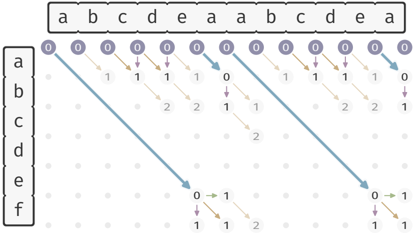

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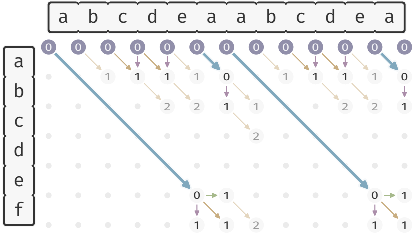
  - ⇝ Use > k disjoint breaks



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Suppose we have: $2k$ disjoint breaks $B_1, \dots, B_{2k}$ in P such that

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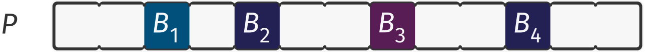

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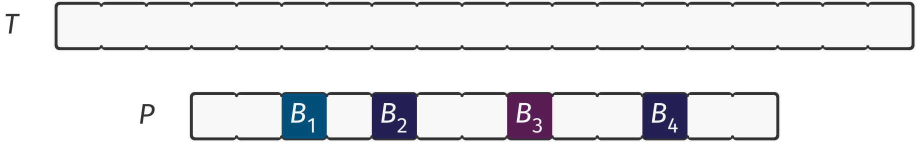

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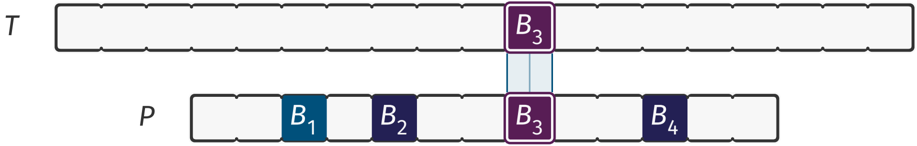

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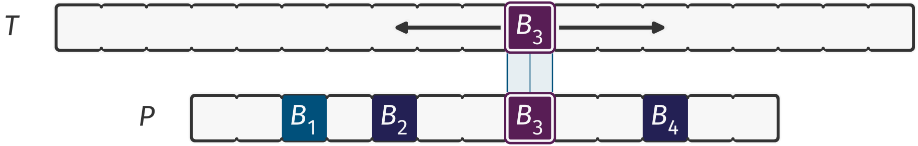

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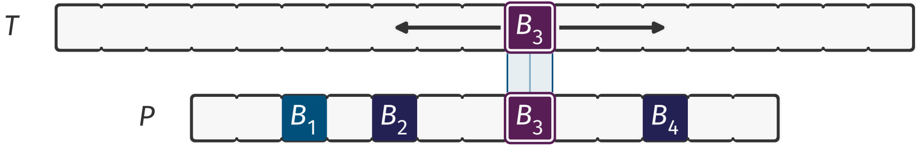

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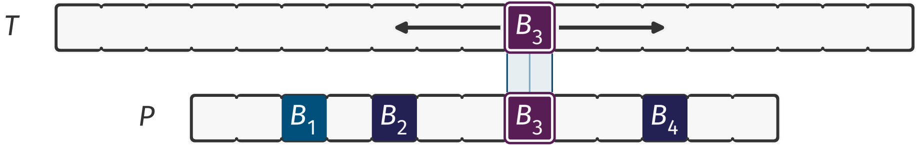

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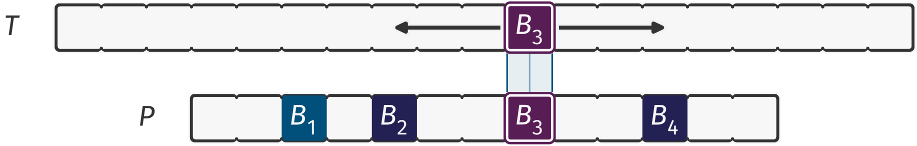
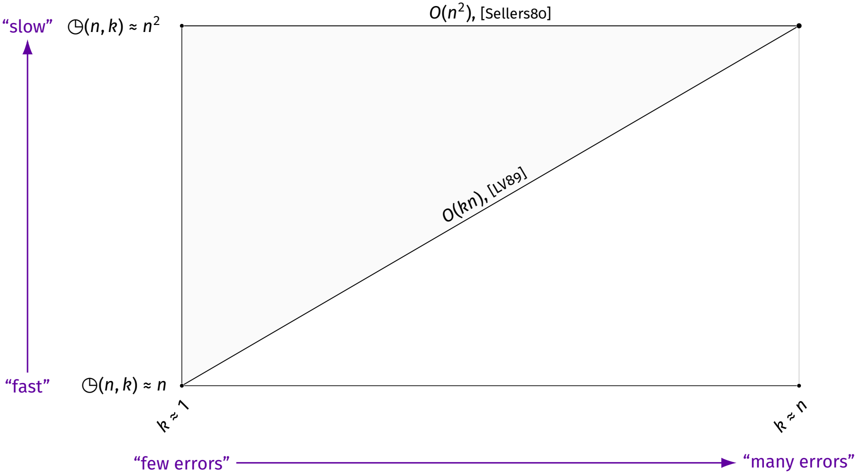
- Can even be improved to $O(k^3 t/p)$; but not-enough breaks case is also $O(k^4 t/p)$.



Recap: State-of-the-Art Algorithms



Existing algorithms for the case $n \approx t \approx 2p$

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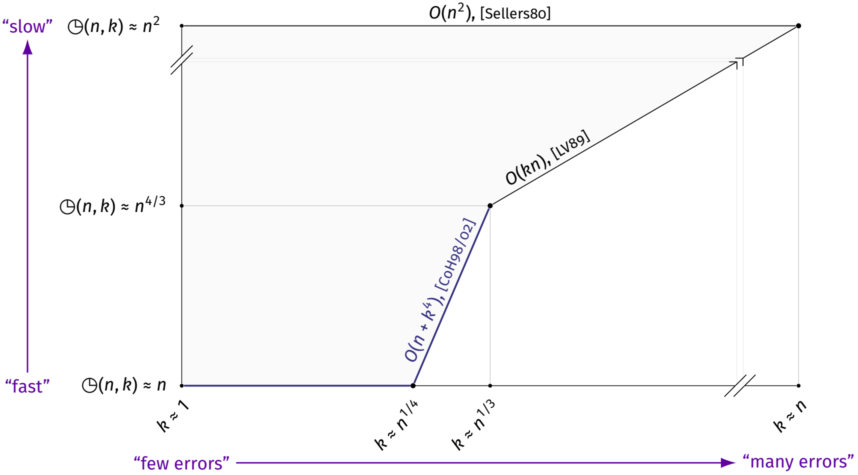

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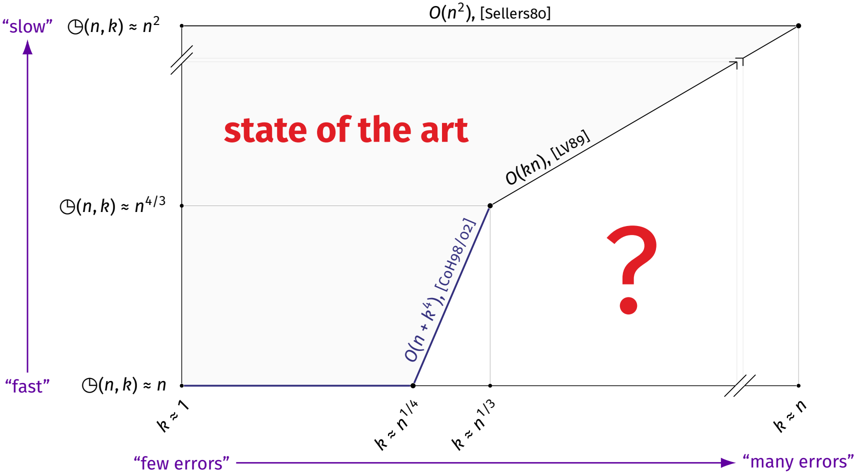

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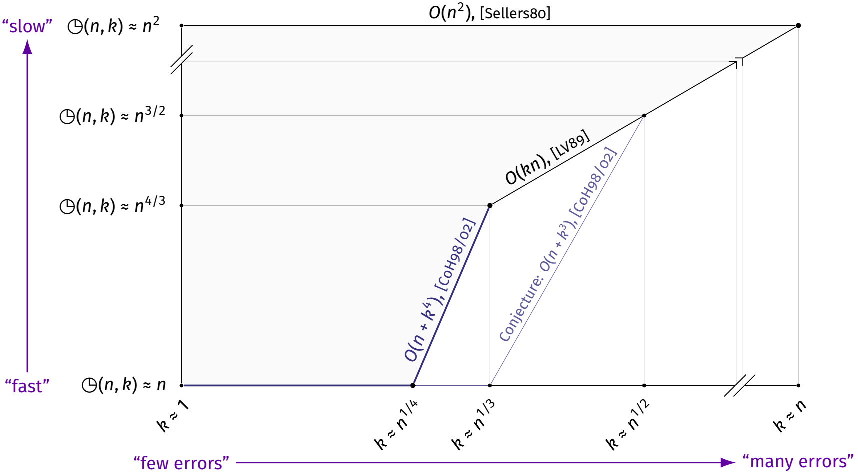

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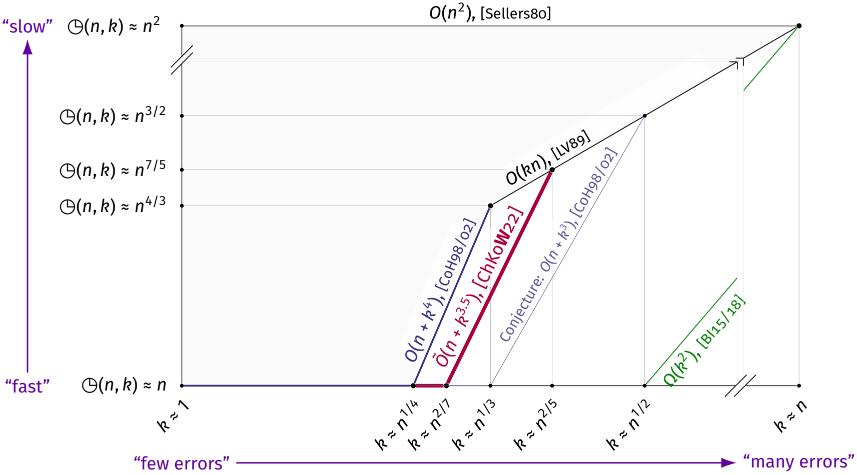

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Recap: State-of-the-Art Algorithms

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Detour: Why did a new improvement for Approximate String Matching take 24 years?

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Computing the Edit Distance of to strings of length n is not possible in $O(n^{2-\varepsilon})$ time.

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⇝ Yields lower bound of $O(t + k^2 \cdot t/p)$ for Approximate String Matching.

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$\rightsquigarrow$ String Matching with Mismatches was “fully” understood only in 2017.

Beating Cole and Hariharan’s Algorithm

How do we obtain faster algorithms?

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How do we obtain faster algorithms?

New insights into the solution structure of
Approximate String Matching

Think: Better filter + more structure when filter fails

Step 0:

What is the solution structure of Exact String Matching?

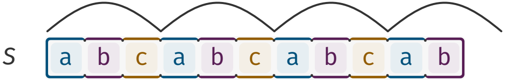
The Solution Structure of Exact String Matching

(The) Period of a String

$p > 0$ is a period of S $:\iff$ $S[i] = S[i+p]$ for all $i = 1, \dots, |S| - p$

The period of S $:\iff$ smallest period of S, write $\mathrm{per}(S)$

S is periodic $:\iff$ $\mathrm{per}(S) \le |S|/2$



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The Solution Structure of Exact String Matching

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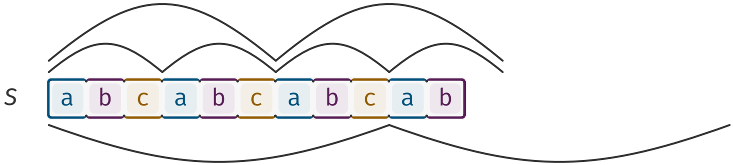

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6 and 9 also periods of S

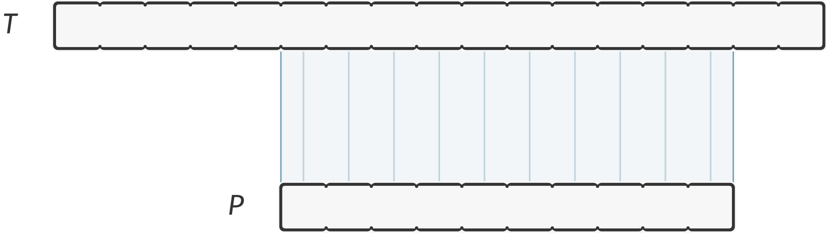
The Solution Structure of Exact String Matching

(Folklore)

"Periodicity Lemma"

Text T, pattern P with $t \leq {}^{3}\!/_{2}\, p$;   one of the following holds:

- P appears ≤ 1 times in T.



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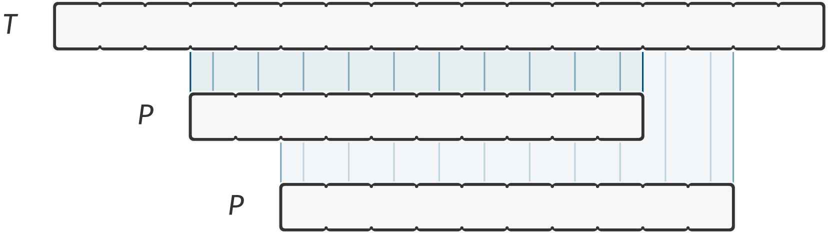

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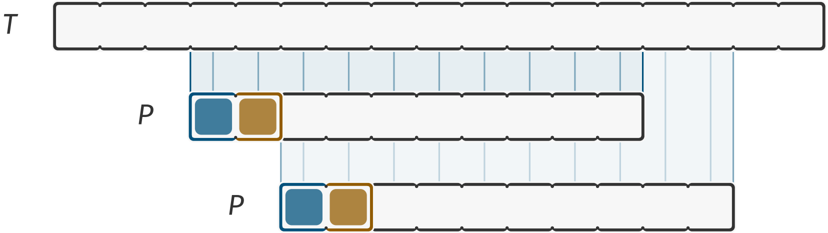

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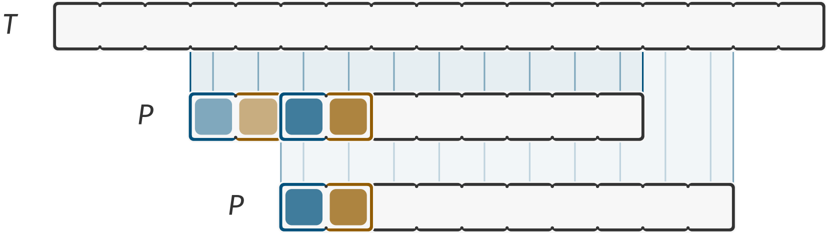
- P appears ≤ 1 times in T.
- P (and the relevant part of T) are periodic with some period Q.



The Solution Structure of Exact String Matching

(Folklore)

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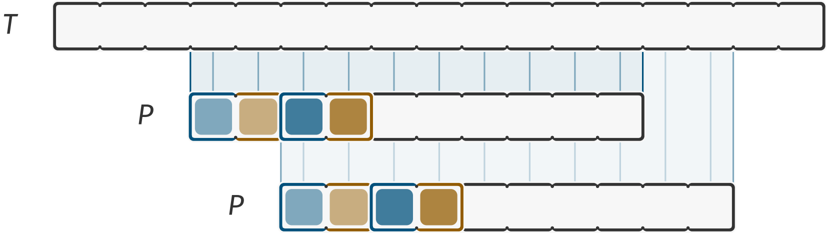

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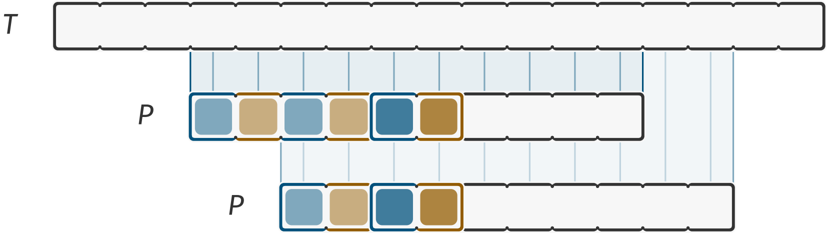

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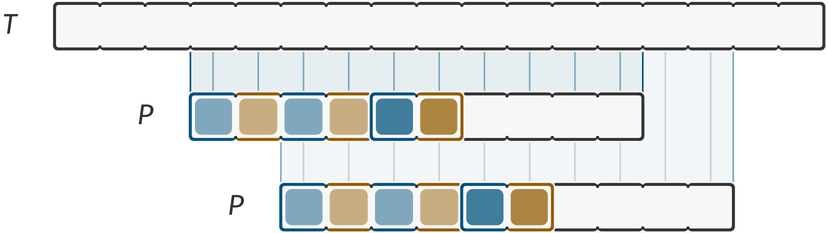

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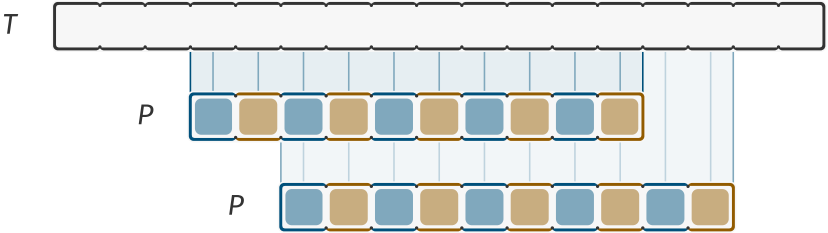

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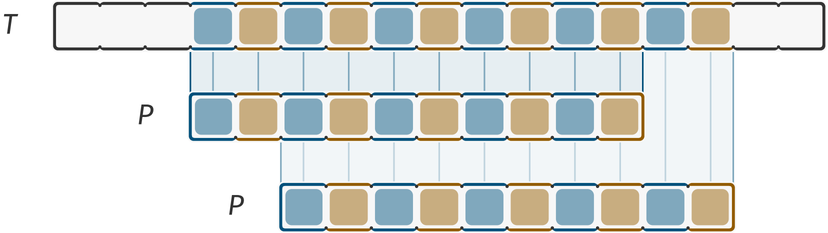

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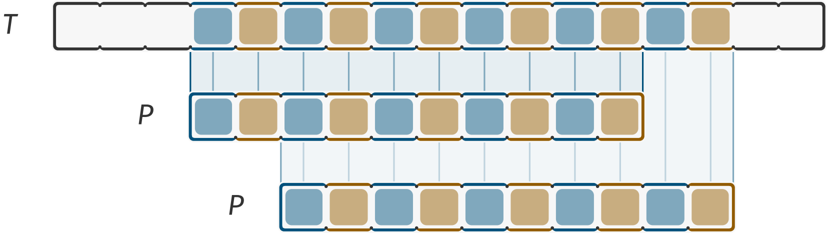


Standard Trick is useful here:

For $t \gg p$ consider separately $O(t/p)$ fragments of T; then Periodicity Lemma applies

Step 1:

What is the solution structure of String Matching with Mismatches?

Structural Results for Approximate String Matching

(Folklore)

Periodicity Lemma

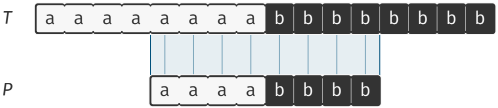
Text T, pattern P with $t \le {}^{3}/_{2}\, p$; P appears ≤ 1 times in T or P is periodic with some period Q.

[BKüW'19; ChKoW'20]

Main Result (Mismatches)

Text T, pattern P with $t \le {}^{3}/_{2}\, p$; threshold k, one of the following holds

P appears ≤ $O(k)$ times in T



P appears $2k$ times w/ ≤ k mism.

but P far from periodic

Structural Results for Approximate String Matching

Periodicity Lemma

(Folklore)

Text T, pattern P with $t \le {}^{3}/_{2}\, p$; P appears ≤ 1 times in T or P is periodic with some period Q.

Main Result (Mismatches)

[BKüW'19; ChKoW'20]

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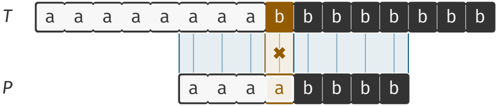

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Structural Results for Approximate String Matching

(Folklore)

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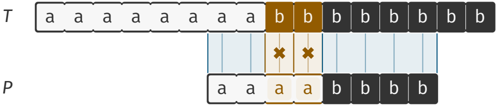

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Structural Results for Approximate String Matching

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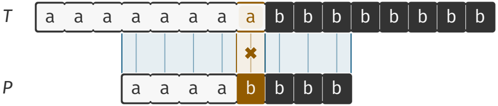

T a a a a a a a a b b b b b b b b

P a a a a b b b b

P appears $2k$ times w/ $\le k$ mism.
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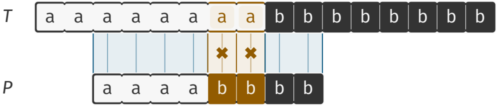

T	a	a	a	a	a	a	a	a	b	b	b	b	b	b	b	b
							✖	✖								
P			a	a	a	a	b	b	b	b						

P appears $2k$ times w/ $\le k$ mism.

but P far from periodic

Structural Results for Approximate String Matching

Periodicity Lemma

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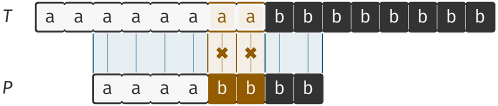
Text T, pattern P with $t \leq {}^{3}\!/_{2}\, p$; P appears ≤ 1 times in T or P is periodic with some period Q.

Main Result (Mismatches)

[BKüW’19; ChKoW’20]

Text T, pattern P with $t \leq {}^{3}\!/_{2}\, p$; threshold k, one of the following holds

P appears ≤ $O(k)$ times in T or P is almost periodic with some period Q



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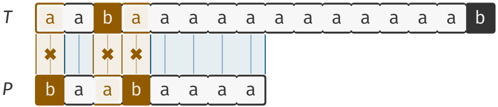

P and T have approximate period a
(are at HD ≤ $2k$ from aa…aa)

Structural Results for Approximate String Matching

Periodicity Lemma

(Folklore)

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Main Result (Mismatches)

[BKüW’19; ChKoW’20]

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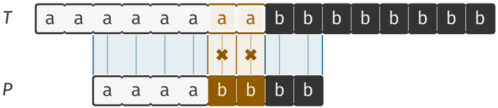

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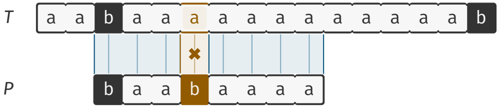

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Structural Results for Approximate String Matching

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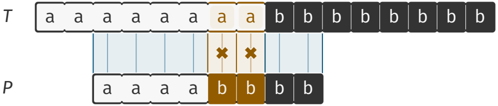

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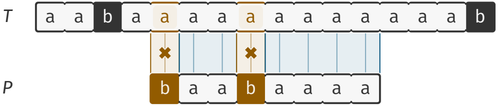

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Structural Results for Approximate String Matching

Periodicity Lemma

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Main Result (Mismatches)

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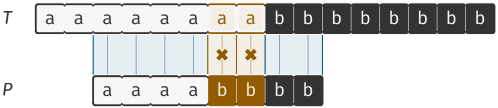

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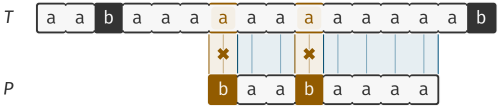

P and T have approximate period a
(are at HD ≤ $2k$ from aa…aa)

Structural Results for Approximate String Matching

Periodicity Lemma

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Main Result (Mismatches)

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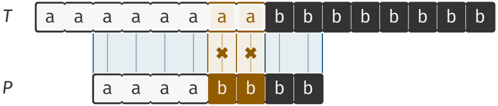

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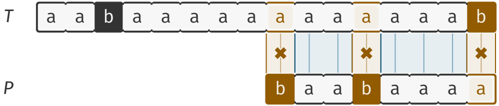


P and T have approximate period a
(are at HD ≤ $2k$ from aa…aa)

Step 1.5:

The solution structure of
Approximate String Matching.

Structural Results for Approximate String Matching

[BKüW'19; ChKoW'20]

Main Result (Mismatches)

T, P with $t \le {}^3/_2\, p$; threshold k, P appears $O(k)$ times in T or P is almost periodic with some period Q

[ChKoW'20]

Main Result (Edits)

Text T, pattern P with $t \le {}^3/_2\, p$; threshold k, one of the following holds

or P is almost periodic with some period Q

Structural Results for Approximate String Matching

Main Result (Mismatches)

[BKüW'19; ChKoW'20]

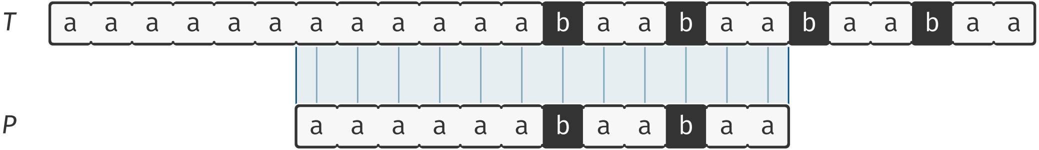
T, P with $t \le {}^3\!/_2\, p$; threshold k, P appears $O(k)$ times in T or P is almost periodic with some period Q

Main Result (Edits)

[ChKoW'20]

Text T, pattern P with $t \le {}^3\!/_2\, p$; threshold k, one of the following holds

P appears $\le O(k^2)$ times in T or P is almost periodic with some period Q



T: a a a a a a a a a a a a b a a b a a b a a b a a

P: a a a a a a b a a b a a

P appears $\approx k^2$ times with $\le k$ edits but P and T far from (approx.) periodic

Structural Results for Approximate String Matching

Main Result (Mismatches)

[BKüW'19; ChKoW'20]

T, P with $t \le 3/2\, p$; threshold k, P appears $O(k)$ times in T or P is almost periodic with some period Q

Main Result (Edits)

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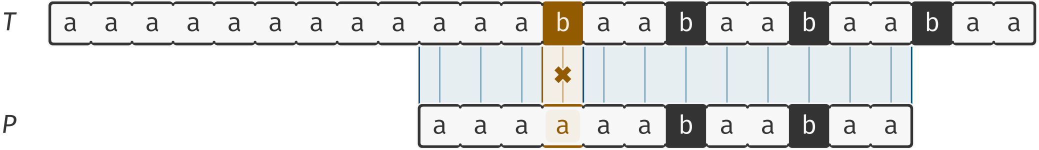

P appears $\approx k^2$ times with $\le k$ edits but P and T far from (approx.) periodic

Structural Results for Approximate String Matching

Main Result (Mismatches)

[BKüW'19; ChKoW'20]

T, P with $t \le {}^3/_2\, p$; threshold k, P appears $O(k)$ times in T or P is almost periodic with some period Q

Main Result (Edits)

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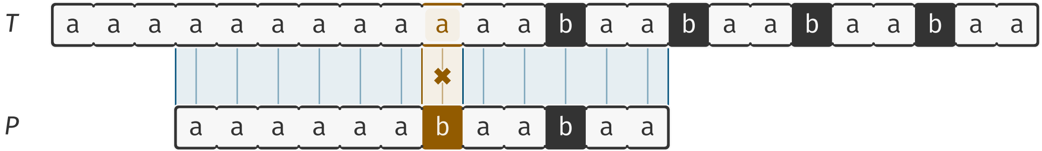

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Structural Results for Approximate String Matching

Main Result (Mismatches)

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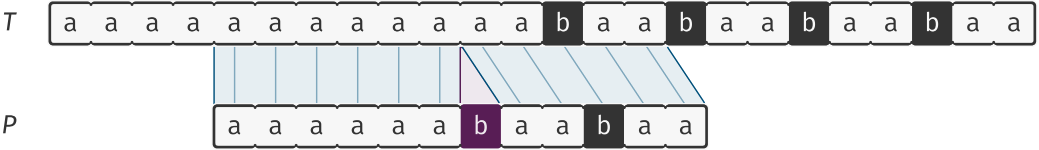

T a a a a a a a a a a a a b a a b a a b a a b a a

P a a a a a a b a a b a a

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Structural Results for Approximate String Matching

Main Result (Mismatches)

[BKüW'19; ChKoW'20]

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Main Result (Edits)

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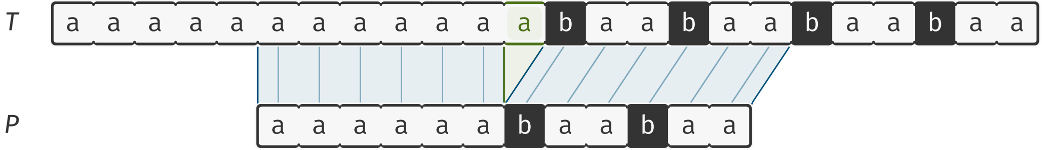

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Structural Results for Approximate String Matching

[ChKoW'20]

Main Result (Edits)

T, P with $t \leq {}^{3}\!/_{2}\, p$; threshold k, P appears $O(k^2)$ times in T or P is almost periodic with some period Q

Structural Results for Approximate String Matching

Main Result (Edits)

[ChKoW'20]

T, P with $t \leq {}^3/_2\, p$; threshold k, P appears $O(k^2)$ times in T or P is almost periodic with some period Q

Key Intermediate Result (Analyze)

[ChKoW'20]

Every string P satisfies at least one of

- P is almost periodic. aaacaaaaaaaaacaaaaaaa

Structural Results for Approximate String Matching

[ChKoW’20]

Main Result (Edits)

T, P with $t \leq {}^3\!/_2\, p$; threshold k, P appears $O(k^2)$ times in T or P is almost periodic with some period Q

[ChKoW’20]

Key Intermediate Result (Analyze)

Every string P satisfies at least one of

- P has $2k$ disjoint, long breaks c❃✿❁❁❃❋a❃✿❁❁❃❋a
- P is almost periodic. aaacaaaaaaaaacaaaaaaa

Structural Results for Approximate String Matching

[ChKoW'20]

Main Result (Edits)

T, P with $t \le {}^{3}/_{2}\, p$; threshold k, P appears $O(k^2)$ times in T or P is almost periodic with some period Q

[ChKoW'20]

Key Intermediate Result (Analyze)

Every string P satisfies at least one of

- P has $2k$ disjoint, long breaks c❃✿❁❁❃❋a❃✿❁❁❃❋a
- P has disjoint repetitive regions that cover ${}^{3}/_{8}\, P$ ❃❋aaaaaa❃❋ccaccc❃❋
- P is almost periodic. aaacaaaaaaaaacaaaaaaa

Structural Results for Approximate String Matching

Main Result (Edits)

[ChKoW'20]

T, P with $t \le {}^{3}\!/_{2}\, p$; threshold k, P appears $O(k^2)$ times in T or P is almost periodic with some period Q

Key Intermediate Result (Analyze)

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Every string P satisfies at least one of

- P has $2k$ disjoint, long breaks — c❉✿❁❁❉❋a❉✿❁❁❉❋a
- P has disjoint repetitive regions that cover ${}^{3}\!/_{8}\, P$ — ❉❋aaaaaa❉❋ccaccc❉❋
- P is almost periodic. — aaacaaaaaaaaacaaaaaaa

Analyze implies Main Result:

Almost periodicity ⇝ as in Main Result

Breaks, repetitive regions ⇝ good filter (as in [Cole, Hariharan'98])

Key Intermediate Result (Analyze)

[ChKoW'20]

Every string P satisfies at least one of

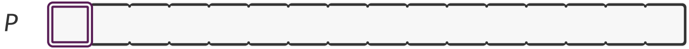
- P has $2k$ disjoint, long breaks c✻✿❁❁✻✻a✻✿❁❁✻✻a
- P has disjoint repetitive regions that cover $^3/_8$ P ✻✻aaaaaa✻✻ccaccc✻✻
- P is almost periodic. aaacaaaaaaaaacaaaaaaa

Key Intermediate Result (Analyze)

[ChKoW'20]

Every string P satisfies at least one of

- P has $2k$ disjoint, long breaks — c❃✿❁❁❃❃a❃✿❁❁❃❃a
- P has disjoint repetitive regions that cover $^3/_8\ P$ — ❃❃aaaaaa❃❃ccaccc❃❃
- P is almost periodic. — aaacaaaaaaaaacaaaaaaa



- Process P from left to right, $p/8k$ new characters at a time.

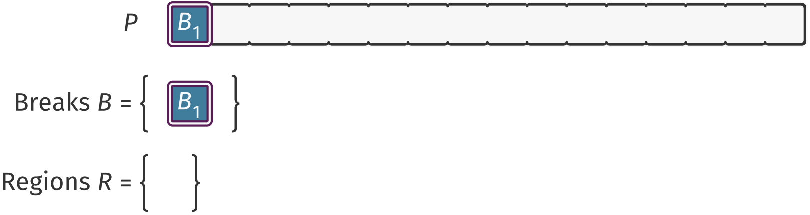
Structural Results for Approximate String Matching

Key Intermediate Result (Analyze)

[ChKoW'20]

Every string P satisfies at least one of

- P has $2k$ disjoint, long breaks c✾✿❁❁✾❋a✾✿❁❁✾❋a
- P has disjoint repetitive regions that cover ${}^{3}/_{8}$ P ✾❋aaaaaa✾❋ccaccc✾❋
- P is almost periodic. aaacaaaaaaaaacaaaaaaa



- If a fragment is a break, add it to the found breaks.

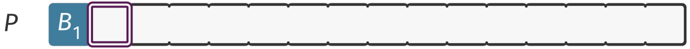
Structural Results for Approximate String Matching

[ChKoW'20]

Key Intermediate Result (Analyze)

Every string P satisfies at least one of

- P has $2k$ disjoint, long breaks   c✽✿❁❁✽✽a✽✿❁❁✽✽a
- P has disjoint repetitive regions that cover $^3/_8$ P ❃❃aaaaaa❃❃ccaccc❃❃
- P is almost periodic.   aaacaaaaaaaaacaaaaaaa



Breaks B = { B_1 }

Repetitive Regions R = { }

- Otherwise, find the shortest prefix (longer than $p/8k$) that is a repetitive region.

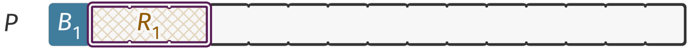
Structural Results for Approximate String Matching

[ChKoW'20]

Key Intermediate Result (Analyze)

Every string P satisfies at least one of

- P has $2k$ disjoint, long breaks c✾✿❁❁✾✾a✾✿❁❁✾✾a
- P has disjoint repetitive regions that cover ${}^{3}/_{8}$ P ✾✾aaaaaa✾✾ccaccc✾✾
- P is almost periodic. aaacaaaaaaaaacaaaaaaa



Breaks $B = \{ B_1 \}$

Repetitive Regions $R = \{ R_1 \}$

- Otherwise, find the shortest prefix (longer than $p/8k$) that is a repetitive region.

Structural Results for Approximate String Matching

Key Intermediate Result (Analyze)

[ChKoW'20]

Every string P satisfies at least one of

- P has $2k$ disjoint, long breaks — c❃✿❁❁❃❃a❃✿❁❁❃❃a
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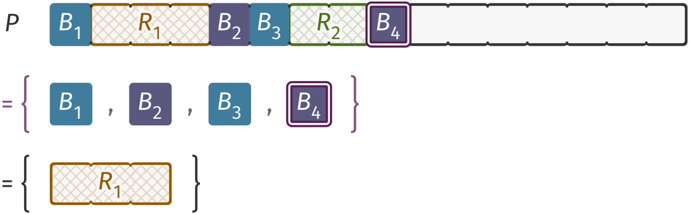

Breaks $B = \{B_1, B_2, B_3, B_4\}$

Repetitive Regions $R = \{R_1\}$

- If we found $2k$ breaks, return the breaks.

Structural Results for Approximate String Matching

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- P has $2k$ disjoint, long breaks — c❃✿❁❁❃❋a❃✿❁❁❃❋a
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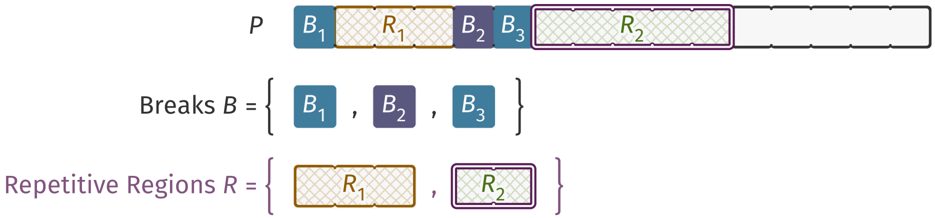

P
B_1
R_1
B_2
B_3
R_2
Breaks B = { B_1 , B_2 , B_3 }
Repetitive Regions R = { R_1 , R_2 }

- If the total length of the repetitive regions is > $3/8 \cdot p$, return the repetitive regions.

Structural Results for Approximate String Matching

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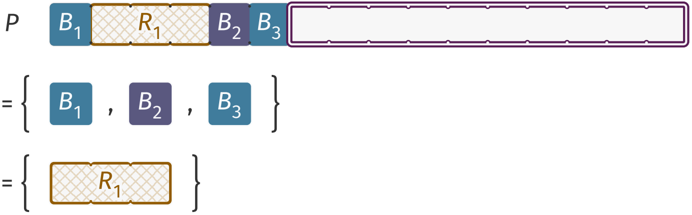

P B_1 R_1 B_2 B_3

Breaks $B = \{ B_1 , B_2 , B_3 \}$

Repetitive Regions $R = \{ R_1 \}$

- If we reach the end of P, try to find a single repetitive region starting from the end.

Structural Results for Approximate String Matching

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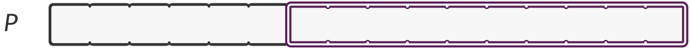

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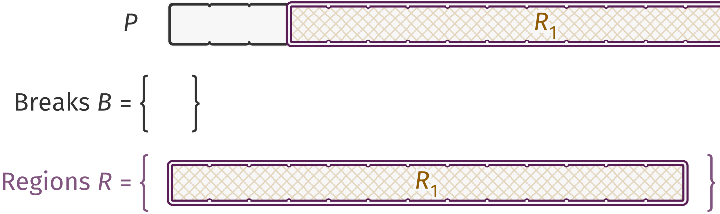

- If we found a repetitive region, return it.

Structural Results for Approximate String Matching

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P

Breaks $B = \{ \quad \}$

Repetitive Regions $R = \{ \quad \}$

- If we again don’t obtain a repetitive region, P is almost periodic.

Structural Results for Approximate String Matching

[ChKoW'20]

Key Intermediate Result (Analyze) ✓

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New Insights, Same Old Problems

How do we turn our insights
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New Insights, Same Old Problems

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Need to tackle three cases.

- P contains $2k$ disjoint breaks;
- P contains disjoint repetitive regions R_i;
- P is almost periodic

New Insights, Same Old Problems

How do we turn our insights into faster algorithms?

Need to tackle ~~three~~ two cases.

- P contains $2k$ disjoint breaks;
- ~~P contains disjoint repetitive regions R_i;~~ ⇝ Follows from the other two cases.
- P is almost periodic

New Insights, Same Old Problems

How do we turn our insights into faster algorithms?

Need to tackle ~~three~~ one case.

- ~~P contains $2k$ disjoint breaks;~~ ⇝ Adaption of [Cole,Hariharan'98] yields $O(|T| + |T|/|P| \cdot k^3)$.
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Research Organization of Information and Systems
National Institute of Informatics

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[ChKoW'22]

Reduction to Periodic Patterns

Algorithm for almost periodic case in time $\tilde{O}(|T| + k^a \cdot |T|/|P|)$, for $a \geq 3$

$\implies$ Algorithm for general case in time $\tilde{O}(|T| + k^a \cdot |T|/|P|)$

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- In [ChKo**W**'20], use elaborate marking scheme for $O(|T| + |T|/|P| \cdot k^4)$ algo

  $\leadsto$ Not faster than [Cole,Hariharan'98]

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  - New algorithm based on “Seaweed Technology” of [Tiskin'10,'15]

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    New in [ChKo**W**’25]: Simpler plug-in replacement for “Seaweed Technology” based on SMAWK (efficient (min, +)-multiplication of Monge matrices)

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Beyond Strings: The PILLAR Model

- All of our algorithms rely on a small set of essential operations:
  - LCP(S, T): Compute the length of the longest common prefix of S and T.
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For $t \le {}^{3}/_{2}\, p + k$; ASM can be solved using $O(k^{3.5})$ PILLAR operations

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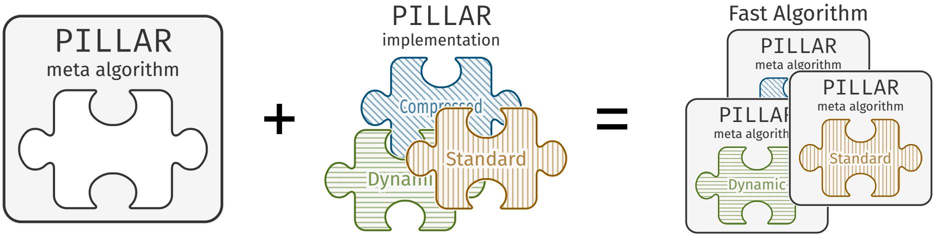

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- Since then: quantum setting (more details soon!), packed setting, read-only setting, ...
  PILLAR algorithms for other related problems, etc. [ChKoW'20] has > 60 citations by now
- Bottom-line, useful design principle: meta algorithm (PILLAR) + simple-to-solve subproblems

Spotlight Extension: Quantum Algorithms for ASM

early 1980's

Approximate String Matching

Given: text T, pattern P, integer k; Find: (starting pos. of) substrings of T at edit distance ≤ k to P.

- P/T given as oracle ⇝ queries possible in superposition
- Query complexity $Q(n)$: number of oracle queries
- Time complexity $T(n)$: number of elementary gates

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early 1980’s

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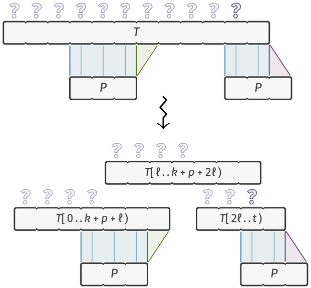
Toy example: “Standard Trick” for ASM decision version (∃? occ)
Split T into overlapping fragments of len $\ell + p + k$
⇝ $O(t/\ell)$ instances, each “responsible” for its first ℓ positions
⇝ Classically: $O(t/\ell)$ time overhead (evaluate each inst. 1 by 1)



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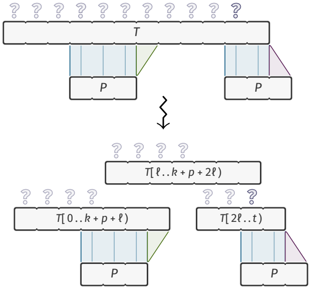
Quantum setting: Use amplitude amp. / Grover's algo.
$\tilde{O}(\sqrt{t/\ell})$ time and $O(\sqrt{t/\ell})$ evaluations of single inst.



Amplitude Amplification

[Gro96, BHMT02]

Given function $f : [0..n) \to \{0,1\}$
Can (wp $\geq 2/3$) obtain $x \in f^{-1}(1)$ in $\tilde{O}(\sqrt{n})$ time + $O(\sqrt{n})$ queries to f

Plugging Together a First Algorithm

early 1980's

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Idea: Use existing quantum algorithms to implement PILLAR operations.

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Quantum PILLAR **Impl**

implied by [HV'03]

For length-n strings,
can implement all PILLAR ops
in $\tilde{O}(\sqrt{n})$ time and queries

Yields $\tilde{O}(t/p \cdot k^{3.5} \cdot \sqrt{p})$ quantum time algo for ASM; ($\tilde{O}(\sqrt{t/p} \cdot k^{3.5} \cdot \sqrt{p})$ for $\exists$?) (algos are correct whp.)

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Are these algorithms optimal? ⇝ No.

If You Want to Be Fast, Take a Detour

Approximate String Matching

early 1980's

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If You Want to Be Fast, Take a Detour

early 1980's

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  ⇝ Can use PILLAR algorithm for fully-compressed setting on lossy compression!

Quantum ASM: Bottom-Lines

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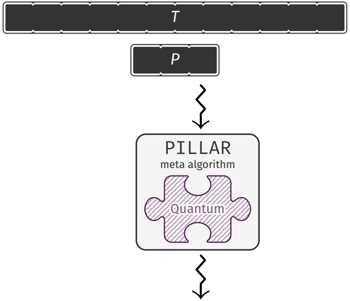

T

P

PILLAR

meta algorithm

Quantum

Easy Result

ASM on T, P w/ $t \le {}^{3}/_{2}\, p + k$ admits qu-algo using $\tilde{O}(k^{3.5}\sqrt{n})$ queries/time

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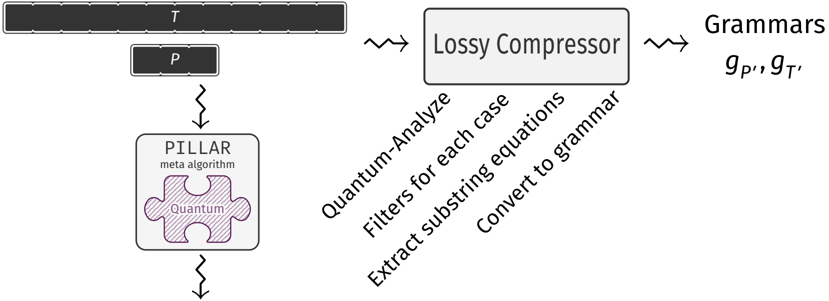

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Lossy Compressor

Grammars
$g_{P'}, g_{T'}$

Quantum-Analyze

Filters for each case

Extract substring equations

Convert to grammar

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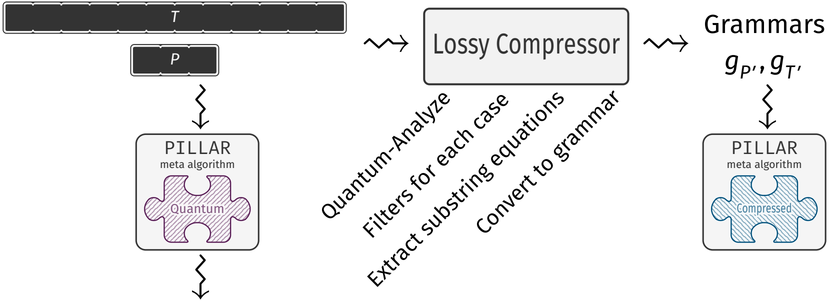

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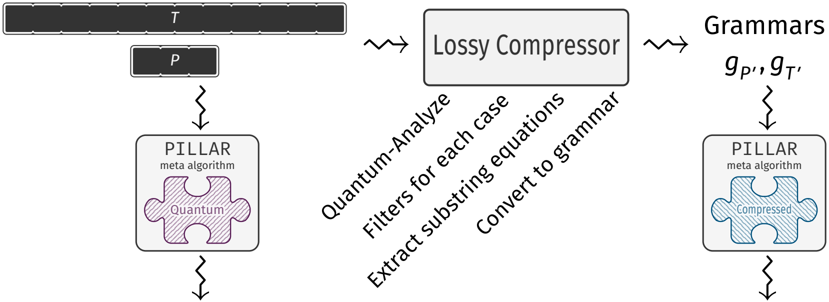

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Better Result

[KoNW'24,'25]

ASM on T, P w/ $t \leq {}^{3}\!/_{2}\, p + k$ admits qu-algo using $\hat{O}(\sqrt{kn})$ queries + $\hat{O}(\sqrt{kn} + k^{3.5})$ time

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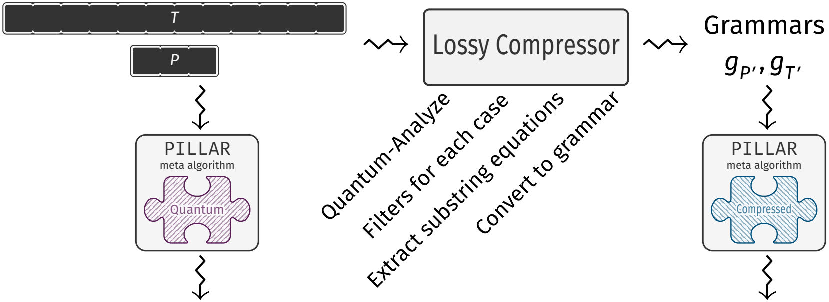

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⇝ Queries are (near) optimal [KoNW'24], time overhead from PILLAR (but future-proof)

Using Analyze + PILLAR for Obtaining New Algorithms 2

Spotlight Extension: String Matching with Weighted Edits

An Example

How similar are two strings X and Y?

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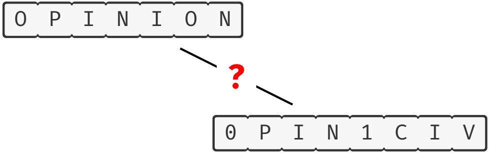
How similar are two strings *X* and *Y*?



O P I N I O N

?

0 P I N 1 C I V

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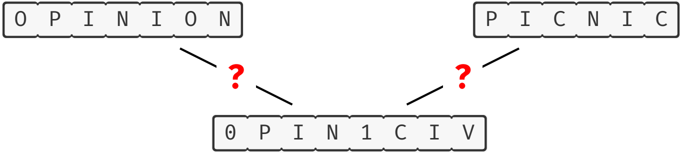

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Edit Distance, Once More

Edit Distance $\mathrm{ED}(X,Y)$

Min number of character insertions, deletions, and substitutions that transform X to Y

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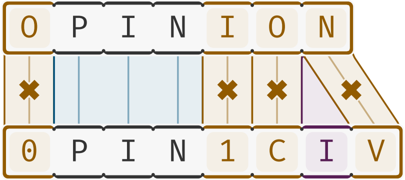

ED(0PIN1CIV, OPINION) = 5

Edit Distance, Once More

ED(X, Y)

Edit Distance

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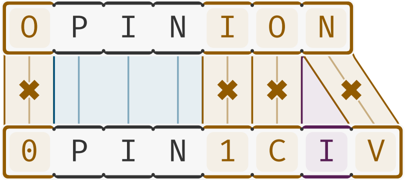

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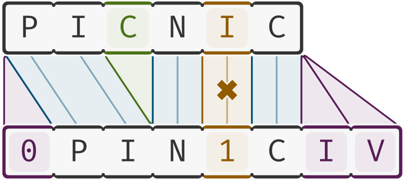

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Edit Distance, Once More

$\mathrm{ED}^w(X, Y)$

Weighted Edit Distance

Min cost of transforming X to Y using character edits, where:

- inserting y costs $w(\varepsilon, y)$;
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$w(\mathtt{0}, \mathtt{O}) := 1 \quad w(\mathtt{1}, \mathtt{I}) := 1 \quad w(\mathtt{C}, \mathtt{O}) := 1 \qquad w(*, *) := 2 \quad w(*, \varepsilon) := 1 \quad w(\varepsilon, *) := 10$

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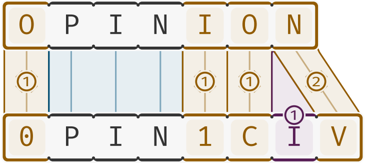

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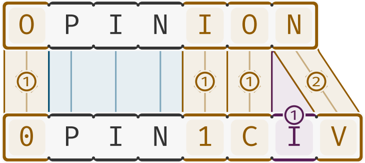

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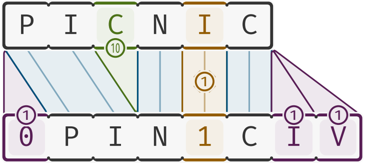

$\mathrm{ED}^w(\mathtt{0PIN1CIV}, \mathtt{PICNIC}) \le 14$

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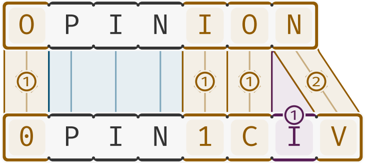

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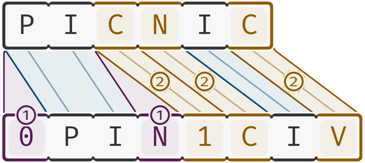

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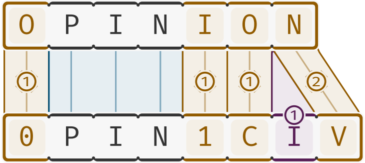

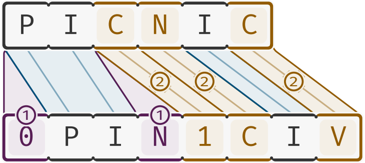

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Justified Assumption: w is normalized, $w(x, y) \geq 1$ for all $x \neq y$.

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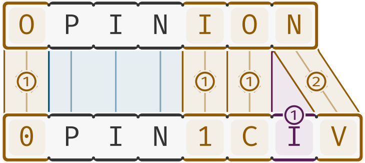

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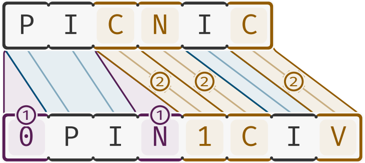

$\mathrm{ED}^{w}(\mathtt{0PIN1CIV}, \mathtt{PICNIC}) = 8$

Justified Assumption: w is normalized, $w(x, y) \geq 1$ for all $x \neq y$.

Otherwise: could scale weights and condition $\mathrm{ED}^{w}_{\leq k}(X, Y) \leq k$ (e.g. in ASM) becomes meaningless.

Computing String Matching with (Weighted) Edits

Recall crucial subroutine for ASM:

Verify that occurrence starts at given (interval of) position of T

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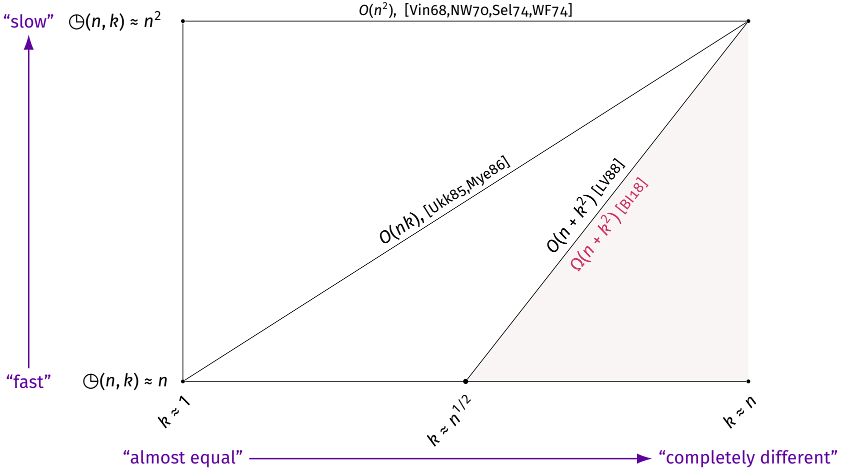
$\rightsquigarrow$ Need to compute Bounded Weighted Edit Distance

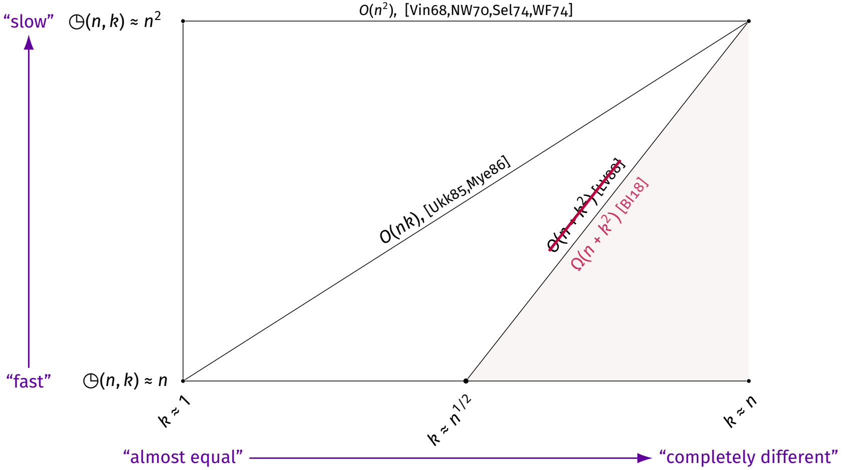
Recall: Algorithms for (Bounded) Edit Distance / Verify



Existing algorithms for Edit Distance $\mathrm{ED}(X, Y)$, where $|X|, |Y| \leq n$

What about Weighted Edit Distance?

Algorithms for (Bounded) Weighted Edit Distance / Weighted Verify



$O(n^2)$, [Vin68,NW70,Sel74,WF74]

"slow" ◷$(n, k) \approx n^2$

"fast" ◷$(n, k) \approx n$

$O(nk)$, [Ukk85,Mye86]

~~$O(n + k^2)$ [LV88]~~

$\Omega(n + k^2)$ [BI18]

$k \approx 1$

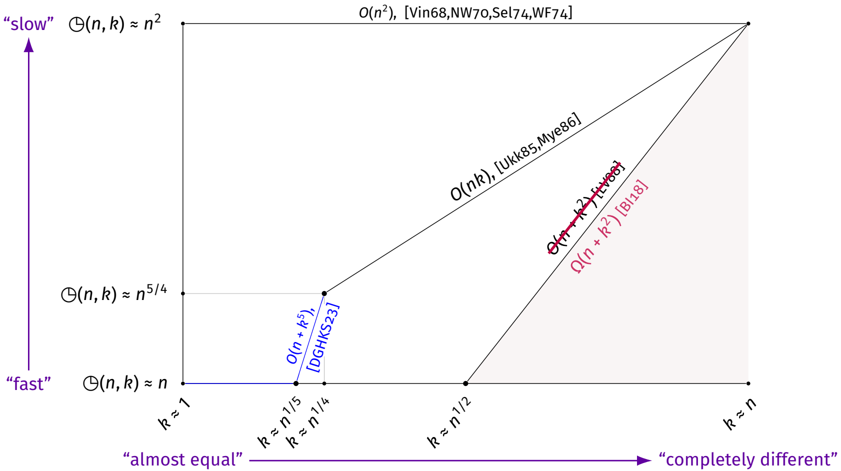
$k \approx n^{1/2}$

$k \approx n$

"almost equal" → "completely different"

Existing algorithms for Weighted Edit Distance $\mathrm{ED}^w(X, Y)$, where $|X|, |Y| \leq n$

Algorithms for (Bounded) Weighted Edit Distance / Weighted Verify



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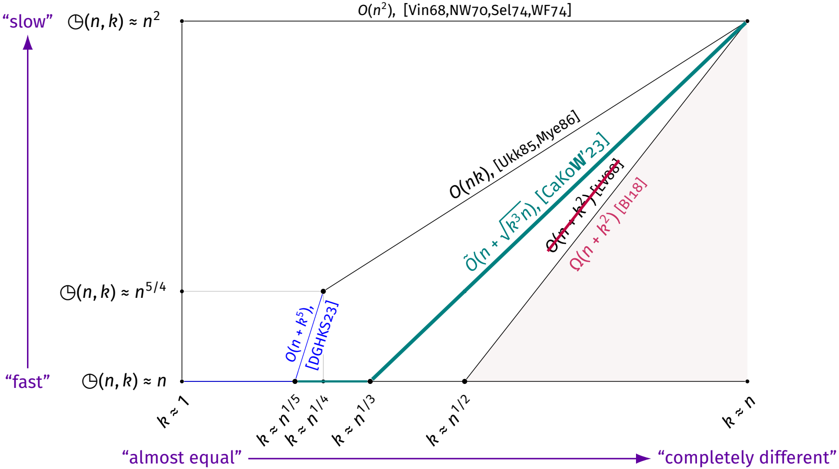

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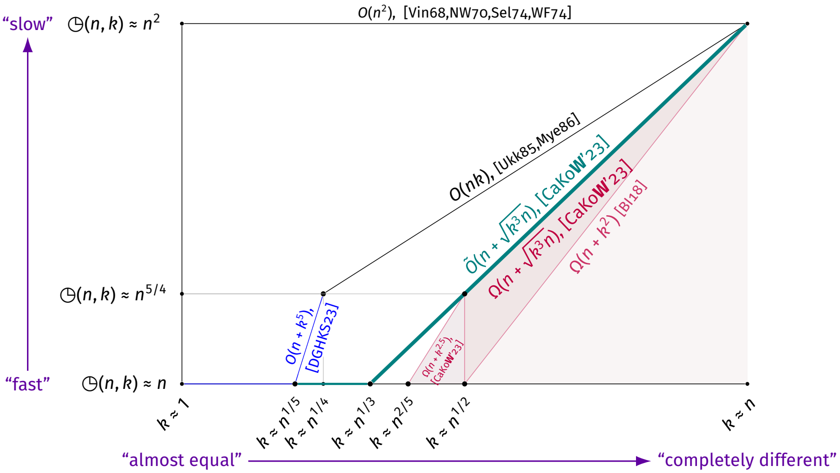

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Algorithms for (Bounded) Weighted Edit Distance / Weighted Verify

[CaKoW'23]

Main Theorem (Upper Bound)

Strings X, Y each of length at most n

Oracle access to (normalized) weight function w

Can compute $k = \mathrm{ED}^w(X, Y)$ in time $O(n + \sqrt{nk^3} \log^3 n)$

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$\rightsquigarrow$ What about $\tilde{O}(kt)$-time for weighted ASM?

With Standard Trick, UB yields $\tilde{O}(t/k \cdot \sqrt{pk^3}) = \tilde{O}(t\sqrt{pk})$

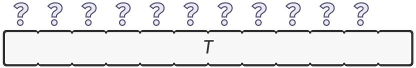
LB rules out $\tilde{O}(kt)$ via better WED algo $\rightsquigarrow$ need different approach

Searching for $\tilde{O}(kt)$-time Algorithms for Weighted ASM

Weighted Approximate String Matching

Given: T, P, k, oracle to w; Find: (starting pos. of) substr. of T at weighted ED ≤ k to P.

Focus: Obtain starting positions of occurrences



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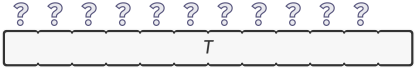

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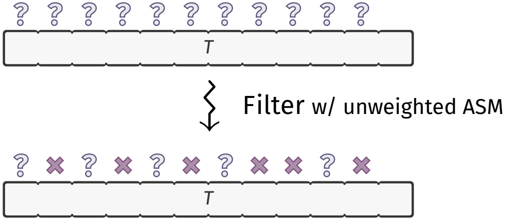

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If $\mathrm{ED}(P, T[\,a\,..\,b\,]) \leq k$, also get cheap alignment
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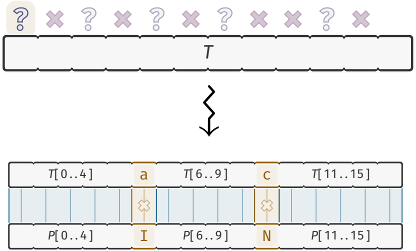

Searching for $\tilde{O}(kt)$-time Algorithms for Weighted ASM

Weighted Approximate String Matching

Given: T, P, k, oracle to w; Find: (starting pos. of) substr. of T at weighted ED ≤ k to P.

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All such alignments very similar to each other
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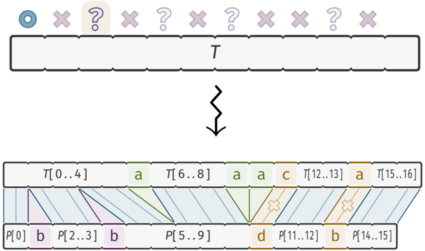

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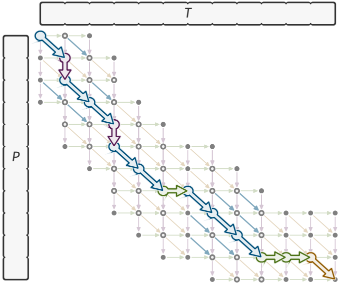

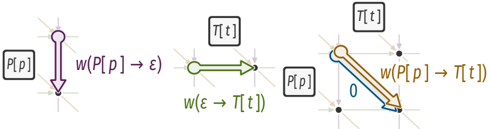

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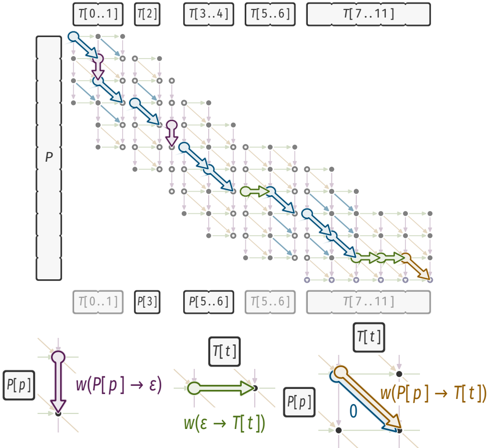

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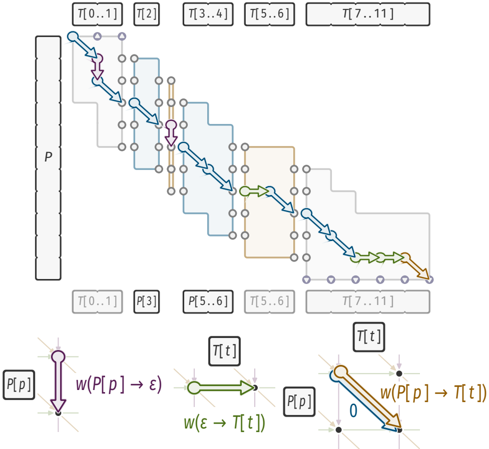

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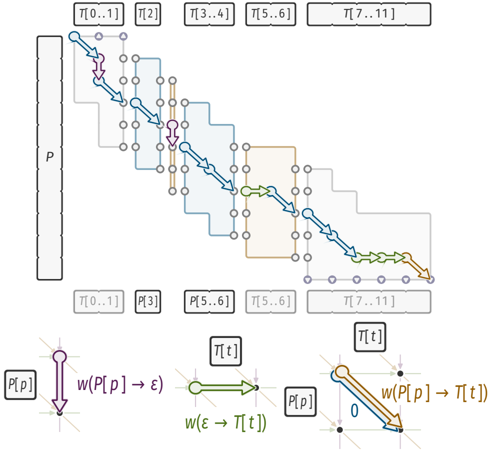
Precompute all P vs P blocks [Klein'05];
⇝ Verify $\le k$ pos in $\tilde{O}(k^2)$ after $\tilde{O}(kp)$ prep. ⇝ $\tilde{O}(kt)$ total



Weighted ASM: Bottom Lines

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Given: T, P, k, oracle to w; Find: (starting pos. of) substr. of T at weighted ED $\leq k$ to P.

Main Theorem

[ChKoW'25]

Weighted ASM is in time $\tilde{O}(t + t/p \cdot k^4)$ and in $O(k^4)$ $\texttt{PILLAR}$ time for $t < {}^3\!/_2\, p + k$.

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But, also interesting non-PILLAR algorithms out there.

Open Problems and Future Directions

- Big Question: Close gap between UB $O(n + k^{3.5})$ / LB $\Omega(n + k^2)$ for ASM.
  - An $\tilde{O}(k^2)$-time `PILLAR` algo would imply optimal algorithms in many other settings (e.g. quantum, compressed, etc.)
  - First step: $\tilde{O}(k^3)$-time `PILLAR` algo
  - Stronger (conditional) LB would require new techniques
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