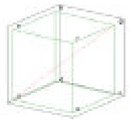


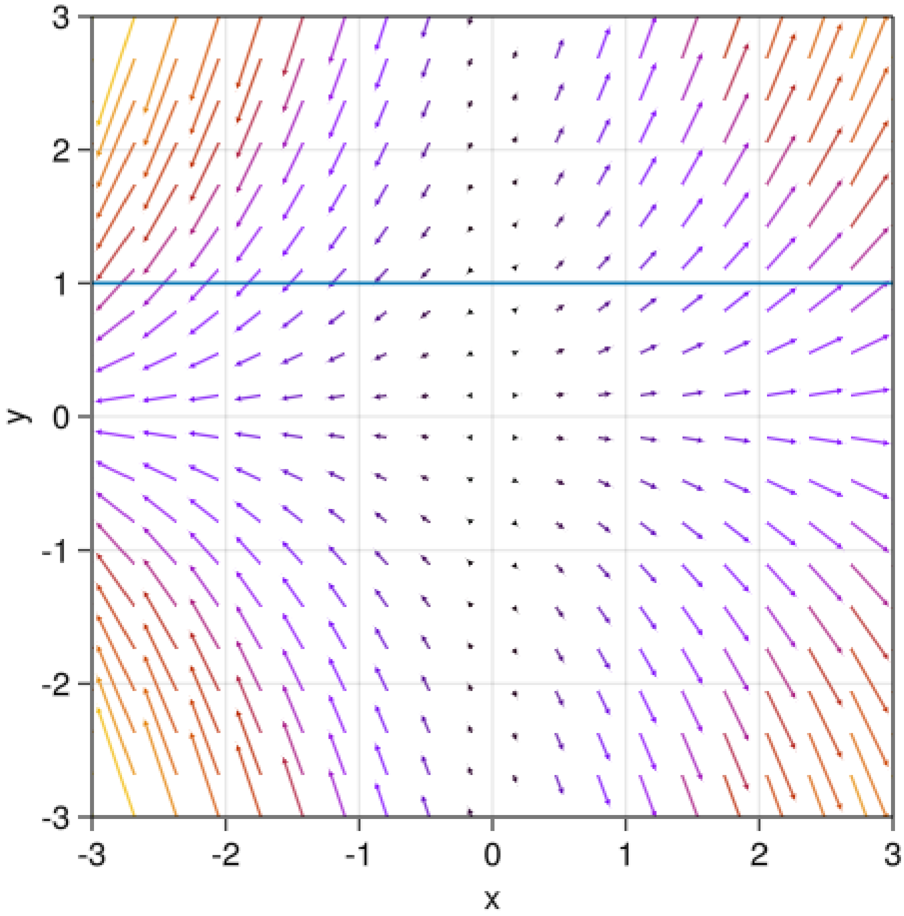
```
PlutoUI.Resource("https://tva1.sinaimg.cn/thumbnail/e6c9d24egy1h2alsw1tzxj20m80gomxn.jpg")
```

ch11 sec11.2 斜率场

📕 Table of Contents

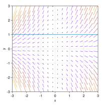
首先来看一下 $\frac{dy}{dx} = y$ 的斜率场

下面绘制了上述导数的斜率场



导数的意义是当x 变化dx个单位的时候, y 如何变化. 对于空间的一个坐标$\{x, y\}$, y分量就是随着x 变化时 y 的变化. 如果画一条水平直线($y = 1$), 那么这个直线上的所有点的斜率都是一样的,因为所有的y 值相同, 当y 越靠近0, 变化趋势越小,远离0 点, 变化速度越快

注意,这里实际画的是向量场, 斜率是一致的, 但是当x 取正值和负值的时候,y的方向是相反的

Dict("fields1" ⇒ )

```
let
    #   dy/dx=y 的向量场
    xs = LinRange(-3, 3, 20)
    ys = LinRange(-3, 3, 20)
    us = [x for x in xs, y in ys]
    vs = [y*x for x in xs, y in ys]
    strength = vec(sqrt.(us .^ 2 .+ vs .^ 2))
    cmap = :gnuplot
    fig = Figure(resolution = (640, 480))
    ax = Axis(fig[1,1], xlabel = "x", ylabel = "y", aspect = DataAspect())
    arrows!(ax, xs, ys, us, vs, arrowsize = 5, lengthscale = 0.1,
        arrowcolor = strength, linecolor = strength, colormap = cmap)
    hlines!(1,lw=0.5)

    limits!(ax, -3,3,-3,3)
    colsize!(fig.layout, 1, Aspect(1, 1.0))
    fig
    save("fields1",fig)
end
```

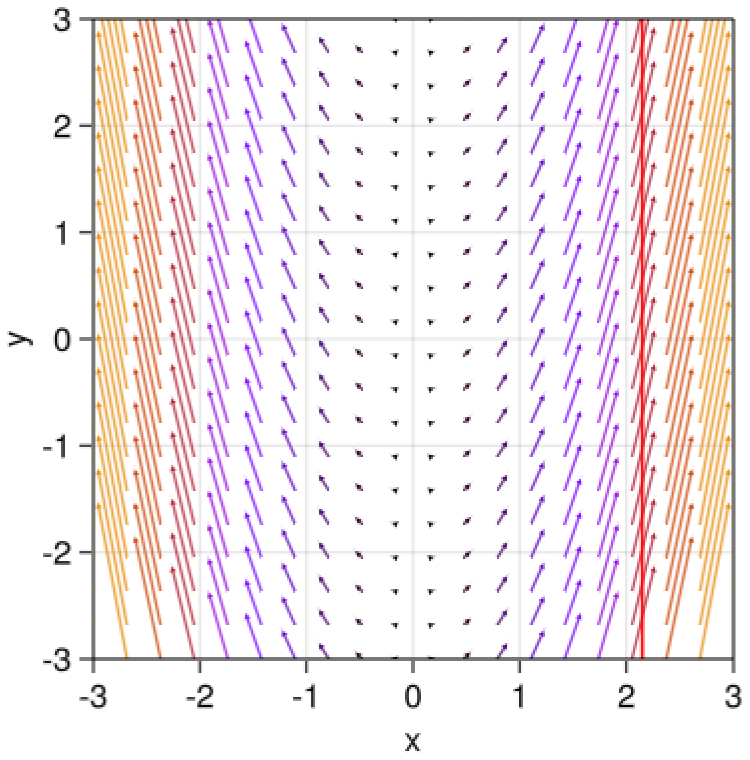
Example

example 1

根据微分方程 $dy/dx = 2x$ 的斜率场 描述

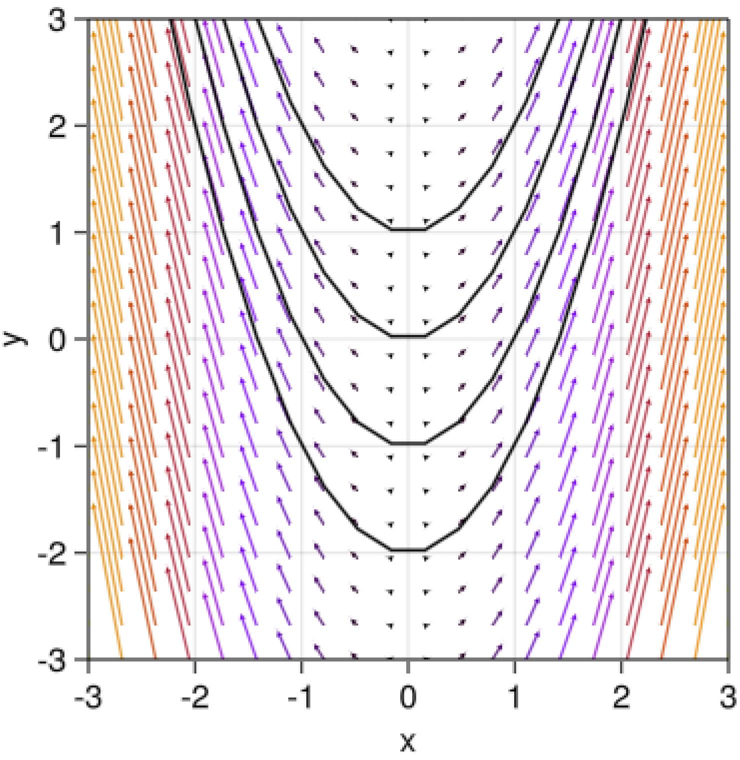
- a. 斜率的变化情况
- b. 在斜率场比较叠加解曲线的情况

a. 斜率场绘图如下



从微分方程可以看到,y 的变化是和x有关的, 变化率为$2x$,所以在x 越大的位置, 变化率越大. 但是在同一x 值的垂直线上, 所有点的斜率是一样的. 如上图里红色线直线所示

b. 叠加解曲线,如下图



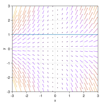
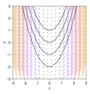
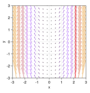
因为对导函数$dy/dx = 2x$ 求不定积分结果如下:

$$y = \int 2x dx = x^2 + C$$

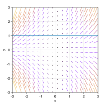
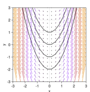
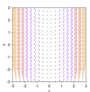
所以这个斜率场描述的就是函数 $f(x) = x^2 + C$ 曲线簇各个点的斜率变化

```
md"""
!!! example

    example 1

    根据微分方程 $dy/dx=2x$ 的斜率场 描述

    - a. 斜率的变化情况

    - b. 在斜率场比较叠加解曲线的情况

a.  斜率场绘图如下

  $(store["fields2"])


从微分方程可以看到,$y$ 的变化是和$x$有关的, 变化率为$2x$,所以在$x$ 越大的位置, 变化率越大.
但是在同一$x$ 值的垂直线上, 所有点的斜率是一样的. 如上图里红色线直线所示

b. 叠加解曲线,如下图

  $(store["fields3"])



因为对导函数$dy/dx=2x$ 求不定积分结果如下:

```

```
$y=\int 2xdx=x^2+C$

所以这个斜率场描述的就是函数 $f(x)=x^2+C$ 曲线簇各个点的斜率变化

"""
```

Dict("fields1" ⇒  , "fields3" ⇒  , "fields2" ⇒ )

```
let
    # f(x,y)=x2+2y2
    xs = LinRange(-3, 3, 20)
    ys = LinRange(-3, 3, 20)
    us = [x for x in xs, y in ys]
    vs = [(2x)*x for x in xs, y in ys]
    strength = vec(sqrt.(us .^ 2 .+ vs .^ 2))
    cmap = :gnuplot
    fig = Figure(resolution = (600, 400))
    ax = Axis(fig[1,1], xlabel = "x", ylabel = "y", aspect = DataAspect())
    arrows!(ax, xs, ys, us, vs, arrowsize = 5, lengthscale = 0.1,
        arrowcolor = strength, linecolor = strength, colormap = cmap)
    vlines!(2.15,lw=1,color=:red)
    limits!(ax, -3,3,-3,3)
    colsize!(fig.layout, 1, Aspect(1, 1.0))
    save("fields2",fig)
end
```

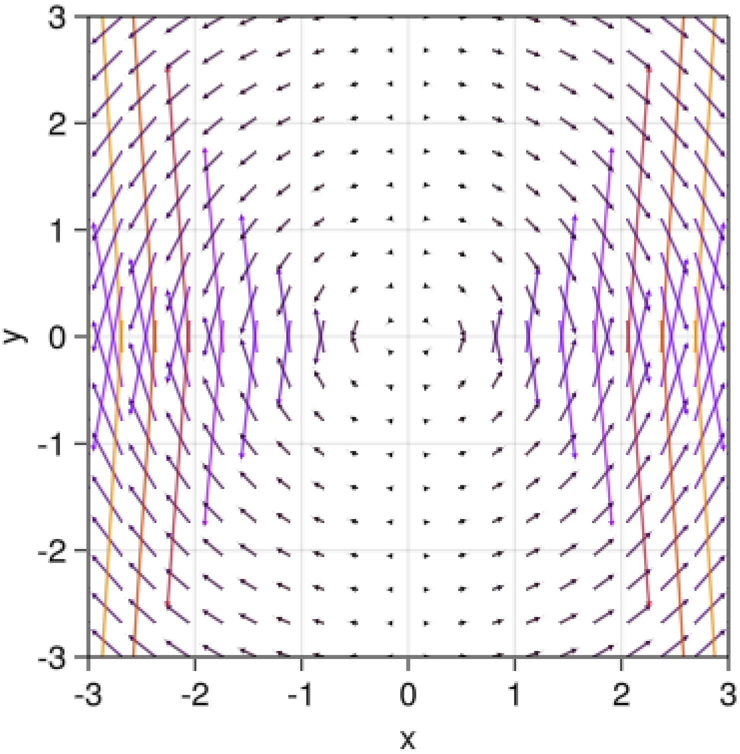
Dict("fields1" ⇒  , "fields3" ⇒  , "fields2" ⇒ )

```
let
    # f(x,y)=x2+2y2
    xs = LinRange(-3, 3, 20)
    ys = LinRange(-3, 3, 20)
    us = [x for x in xs, y in ys]
    vs = [(2x)*x for x in xs, y in ys]
    strength = vec(sqrt.(us .^ 2 .+ vs .^ 2))
    cmap = :gnuplot
    fig = Figure(resolution = (600, 400))
    ax = Axis(fig[1,1], xlabel = "x", ylabel = "y", aspect = DataAspect())
    arrows!(ax, xs, ys, us, vs, arrowsize = 5, lengthscale = 0.1,
        arrowcolor = strength, linecolor = strength, colormap = cmap)
    lines!(ax, xs, x -> x^2; color = :black,lw=1)
    lines!(ax, xs, x -> x^2-1; color = :black,lw=1)
    lines!(ax, xs, x -> x^2-2; color = :black,lw=1)
    lines!(ax, xs, x -> x^2+1; color = :black,lw=1)
    limits!(ax, -3,3,-3,3)
    colsize!(fig.layout, 1, Aspect(1, 1.0))
     save("fields3",fig)
end
```

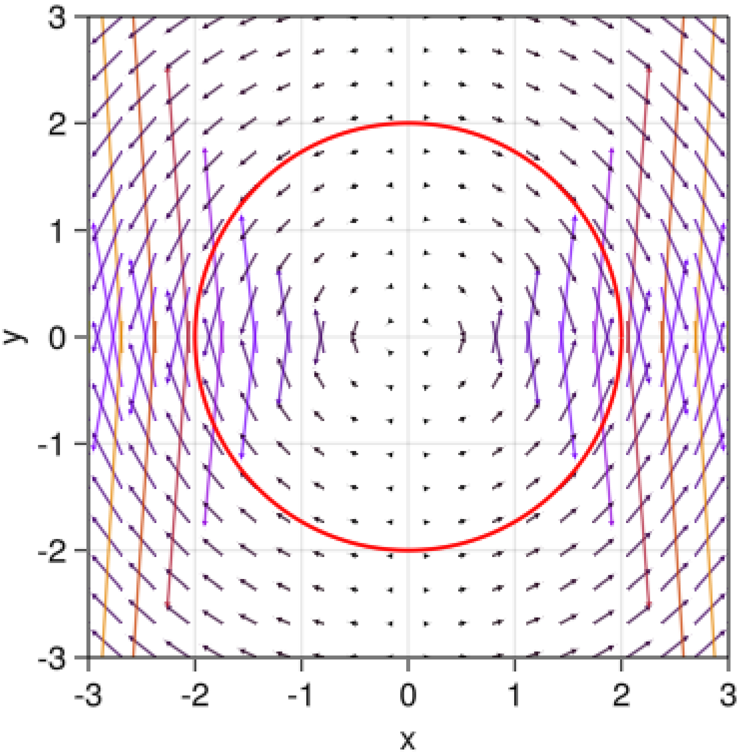
Example

example 2

绘制微分方程 $\frac{dy}{dx} = \frac{-x}{y}$



这个向量场描述的是同心圆的变化情况, 我们叠加一个半径为2 的圆, 看看



对圆的方程求导也可以得到 $\frac{dy}{dx} = \frac{-x}{y}$,具体推导参看 page 592

```
md"""
!!! example

    example 2

    绘制微分方程 $\frac{dy}{dx}=\frac{-x}{y}$


 $(store["fields4"])

 这个向量场描述的是同心圆的变化情况，我们叠加一个半径为$2$ 的圆，看看

 $(store["fields5"])

对圆的方程求导也可以得到 $\frac{dy}{dx}=\frac{-x}{y}$ ,具体推导参看 page 592

"""
```

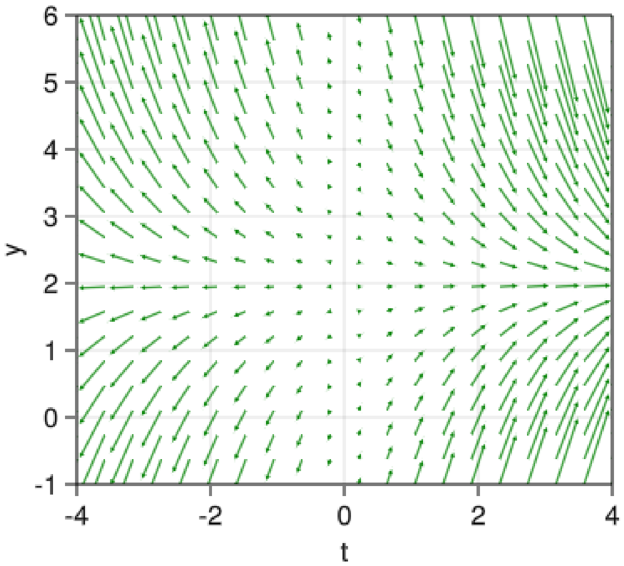
上面的斜率场图都是一组函数簇的斜率组成的, 怎么确定我们想要的某个特定方程? 这时我们需要更多的信息, 在微分方程中, 这些信息被称为初值. 当我们把一组初始的输入,输出值带入微分方程的通解, 就可以确定函数的最终形式.

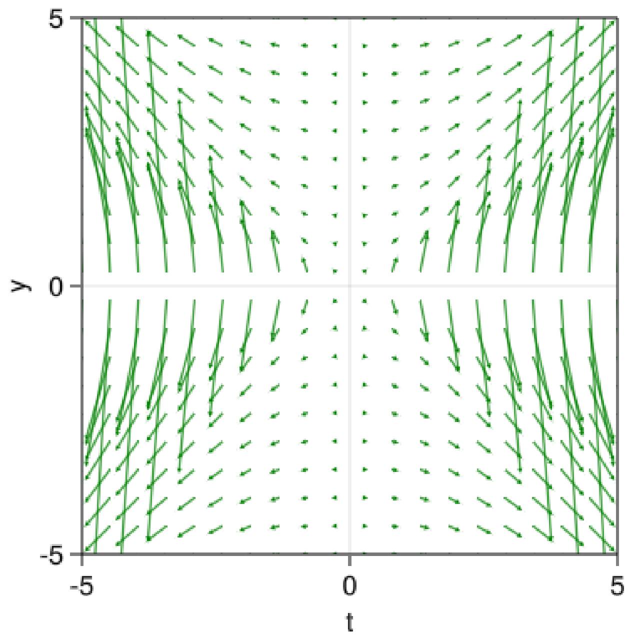
Example

example 3

导数为 $dy/dt = 2 - y$ 和$dy/dt = t/y$ 的斜率场如下图:

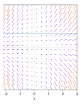
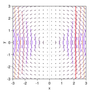
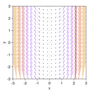
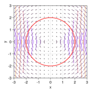
- a. 根据初始条件绘出特解曲线:
  - (i). 当$t = 0$时, $y = 1$
  - (ii). 当$t = 1$时, $y = 0$
  - (iii). 当$t = 0$时, $y = 3$
- b. 根据解曲线,描述函数的长期行为



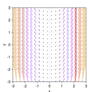
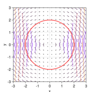
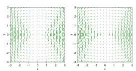
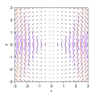



```
md"""
上面的斜率场图都是一组函数簇的斜率组成的, 怎么确定我们想要的某个特定方程?  这时我们需要更多的信息,
在微分方程中, 这些信息被称为初值. 当我们把一组初始的输入,输出值带入微分方程的通解, 就可以确定函数的
最终形式.

!!! example

    example 3

    导数为 $dy/dt=2-y$ 和$dy/dt=t/y$ 的斜率场如下图:
    - a. 根据初始条件绘出特解曲线:
        - (i). 当$t=0$时, $y=1$

        - (ii). 当$t=1$时, $y=0$

        - (iii). 当$t=0$时, $y=3$
    - b. 根据解曲线,描述函数的长期行为

$(store["fields6"])

```

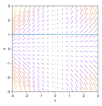
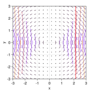
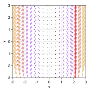
```
"""
```

 , "fields3" ⇒  , "fields2" ⇒  , "fields5" ⇒  , "fields6" =

```
let
    # f(x,y)=x2+y2
    xs = LinRange(-3, 3, 20)
    ys = LinRange(-3, 3, 20)
    us = [x for x in xs, y in ys]
    vs = [(-x/y)*x for x in xs, y in ys]
    strength = vec(sqrt.(us .^ 2 .+ vs .^ 2))
    cmap = :gnuplot
    fig = Figure(resolution = (600, 400))
    ax = Axis(fig[1,1], xlabel = "x", ylabel = "y", aspect = DataAspect())
    arrows!(ax, xs, ys, us, vs, arrowsize = 5, lengthscale = 0.1,
        arrowcolor = strength, linecolor = strength, colormap = cmap)

    limits!(ax, -3,3,-3,3)
    colsize!(fig.layout, 1, Aspect(1, 1.0))
    save("fields4",fig)
end
```

lds2" ⇒  , "fields5" ⇒  , "fields6" ⇒  , "fields4" ⇒ )

```
let
    # f(x,y)=x2+y2
    tspan=-2pi:0.02:2pi
    radius=2
    rx=[radius*cos(t) for t in tspan]
    ry=[radius*sin(t) for t in tspan]
    xs = LinRange(-3, 3, 20)
    ys = LinRange(-3, 3, 20)
    us = [x for x in xs, y in ys]
    vs = [(-x/y)*x for x in xs, y in ys]
    strength = vec(sqrt.(us .^ 2 .+ vs .^ 2))
    cmap = :gnuplot
    fig = Figure(resolution = (600, 400))
    ax = Axis(fig[1,1], xlabel = "x", ylabel = "y", aspect = DataAspect())
    arrows!(ax, xs, ys, us, vs, arrowsize = 5, lengthscale = 0.1,
        arrowcolor = strength, linecolor = strength, colormap = cmap)
    lines!(rx,ry, lw=1, color=:red)
    limits!(ax, -3,3,-3,3)
    colsize!(fig.layout, 1, Aspect(1, 1.0))
    save("fields5",fig)
end
```

Dict("fields1" ⇒  , "fields3" ⇒  , "fields2" ⇒  , "fields5" =

```
let
    xs1,ys1 = LinRange(-4, 4, 20),LinRange(-1, 6, 20)
    xs2,ys2 = LinRange(-5, 5, 20),LinRange(-5, 5, 20)
    us1 = [x for x in xs1, y in ys1]
    vs1 = [(2-y)*x for x in xs1, y in ys1]
    us2 = [x for x in xs2, y in ys2]
    vs2 = [(x/y)*x for x in xs2, y in ys2]

    cmap = :gnuplot
    fig = Figure(resolution = (800, 400))
    ax1 = Axis(fig[1,1], xlabel = "t", ylabel = "y", aspect = DataAspect())
    ax2 = Axis(fig[1,2], xlabel = "t", ylabel = "y", aspect = DataAspect())
    arrows!(ax1, xs1, ys1, us1, vs1, arrowsize = 5, lengthscale = 0.1,
        arrowcolor =:green, linecolor = :green, colormap = cmap)
    arrows!(ax2, xs2, ys2, us2, vs2, arrowsize = 5, lengthscale = 0.1,
        arrowcolor =:green, linecolor = :green, colormap = cmap)

    limits!(ax1, -4,4,-1,6)
    limits!(ax2, -5,5,-5,5)
    colsize!(fig.layout, 1, Aspect(1, 1.0))
    save("fields6",fig)
end
```

read (generic function with 1 method)

```
begin
    store=Dict()

    function save(key::String, dict)
        return merge!(store,Dict(key=>dict))
    end

    function read(key::String)
        return  store[key]
    end
end

```

```

        @htl("""<script
src="https://cdn.bootcdn.net/ajax/libs/mathjax/3.2.0/es5/tex-svg-full.min.js">
</script>
        """)
```