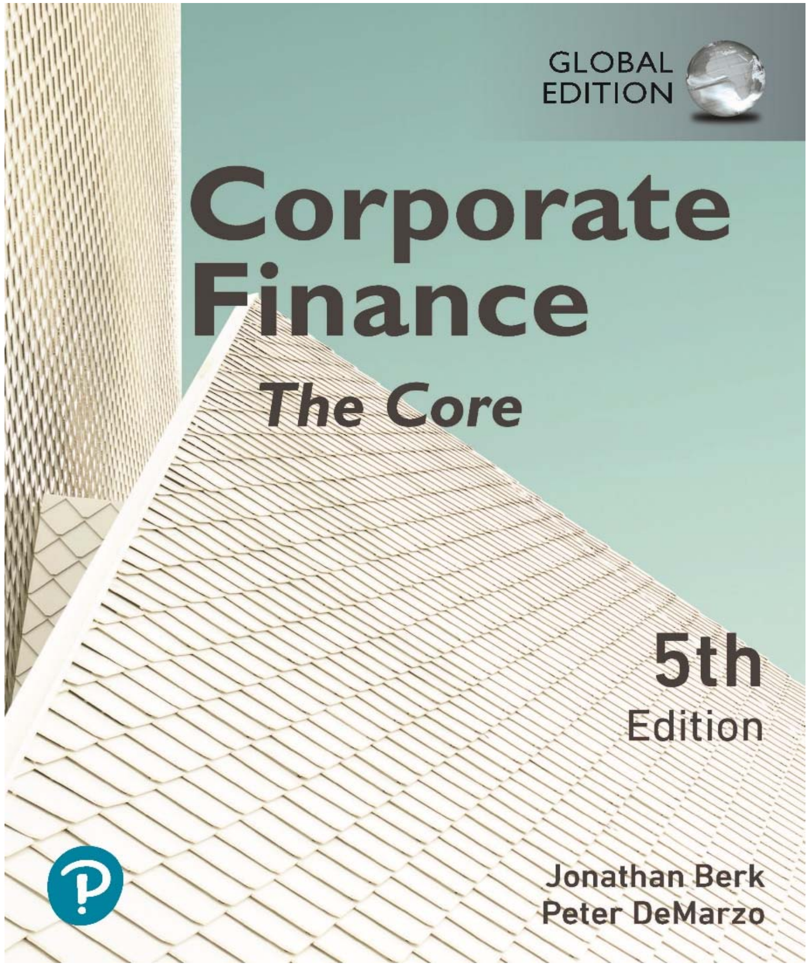
Corporate Finance: The Core

Fifth Edition, Global Edition



Chapter 11

Optimal Portfolio Choice
and the Capital Asset
Pricing Model

Pearson

Chapter Outline (1 of 2)

11.1 The Expected Return of a Portfolio

11.2 The Volatility of a Two-Stock Portfolio

11.3 The Volatility of a Large Portfolio

11.4 Risk Versus Return: Choosing an Efficient Portfolio

11.5 Risk-Free Saving and Borrowing

11.6 The Efficient Portfolio and Required Returns

Chapter Outline (2 of 2)

11.7 The Capital Asset Pricing Model

11.8 Determining the Risk Premium

Appendix

Learning Objectives (1 of 7)

- Given a portfolio of stocks, including the holdings in each stock and the expected return in each stock, compute the following:
  - Portfolio weight of each stock (Eq. 11.1)
  - Expected return on the portfolio (Eq. 11.3)
  - Covariance of each pair of stocks in the portfolio (Eq. 11.5)
  - Correlation coefficient of each pair of stocks in the portfolio (Eq. 11.6)
  - Variance of the portfolio (Eq. 11.8)
  - Standard deviation of the portfolio

Learning Objectives (2 of 7)

- Compute the variance of an equally weighted portfolio, using Eq. 11.12.
- Describe the contribution of each security to the portfolio.
- Use the definition of an efficient portfolio from Chapter 10 to describe the efficient frontier.
- Explain how an individual investor will choose from the set of efficient portfolios.

Learning Objectives (3 of 7)

- Describe what is meant by a short sale, and illustrate how short selling extends the set of possible portfolios.
- Explain the effect of combining a risk-free asset with a portfolio of risky assets, and compute the expected return and volatility for that combination.

Learning Objectives (4 of 7)

- Illustrate why the risk-return combinations of the risk-free investment and a risky portfolio lie on a straight line.
- Define the Sharpe ratio, and explain how it helps identify the portfolio with the highest possible expected return for any level of volatility, and how this information can be used to identify the tangency (efficient) portfolio.

Learning Objectives (5 of 7)

- Calculate the beta of investment with a portfolio.
- Use the beta of a security, expected return on a portfolio, and the risk-free rate to decide whether buying shares of that security will improve the performance of the portfolio.
- Explain why the expected return must equal the required return.

Learning Objectives (6 of 7)

- Use the risk-free rate, the expected return on the efficient (tangency) portfolio, and the beta of a security with the efficient portfolio to calculate the risk premium for an investment.
- List the three main assumptions underlying the Capital Asset Pricing Model.

Learning Objectives (7 of 7)

- Explain why the CAPM implies that the market portfolio of all risky securities is the efficient portfolio.
- Compare and contrast the capital market line with the security market line.
- Define beta for an individual stock and for a portfolio.

11.1 The Expected Return of a Portfolio (1 of 3)

- Portfolio Weights
  - The fraction of the total investment in the portfolio held in each individual investment in the portfolio
    - The portfolio weights must add up to 1.00 or 100%.

$$x_i = \frac{\text{Value of investment } i}{\text{Total value of portfolio}}$$

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11.1 The Expected Return of a Portfolio (2 of 3)

- Then the return on the portfolio, R_p is the weighted average of the returns on the investments in the portfolio, where the weights correspond to portfolio weights.

$$R_P = x_1 R_1 + x_2 R_2 + \cdots + x_n R_n = \sum_i x_i R_i$$

Textbook Example 11.1 (1 of 2)

Calculating Portfolio Returns

Problem

Suppose you buy 200 shares of Dolby Laboratories at $30 per share of Coca-Cola stock at $40 per share. If Dolby's share price goes up to $36 and Coca-Cola' s falls to $38, what is the new value of the portfolio, and
what return did it earn? Show that Eq. 11.2 holds.
After the price change, what are the portfolio weights?

Pearson

Textbook Example 11.1 (2 of 2)

Solution

The new value of the portfolio is $200 \times \$36 + 100 \times \$38 = \$11{,}000$, for a gain of \$1000 or a 10% return on your \$10,000 investment. Disney's return was $36/30 - 1 = 20\%$, and Coca-Cola's was $38/40 - 1 = -5\%$. Given the initial portfolio weights of 60% Disney and 40% Coca-Cola, we can also compute the portfolio's return from Eq. 11.2:

$$R_P = x_D R_D + x_C R_C = 0.6 \times (20\%) + 0.4 \times (-5\%) = 10\%$$

After the price change, the new portfolio weights are

$$x_D = \frac{200 \times \$36}{\$11{,}000} = 65.45\%, \qquad x_C = \frac{100 \times \$38}{\$11{,}000} = 34.55\%$$

Without trading, the weights increase for those stocks whose returns exceed the portfolio's return.

11.1 The Expected Return of a Portfolio (3 of 3)

- The expected return of a portfolio is the weighted average of the expected returns of the investments within it.

$$E[R_P] = E\left[\sum_i x_i R_i\right] = \sum_i E[x_i R_i] = \sum_i x_i E[R_i]$$

11.2 The Volatility of a Two-Stock Portfolio (1 of 4)

- Combining Risks

Table 11.1 Returns for Three Stocks, and Portfolios of Pairs of Stocks

	Stock Returns			Portfolio Returns	
Year	North Air	West Air	Tex Oil	$1/2R_N + 1/2R_W$	$1/2R_W + 1/2R_T$
2013	21%	9%	−2%	15.0%	3.5%
2014	30%	21%	−5%	25.5%	8.0%
2015	7%	7%	9%	7.0%	8.0%
2016	−5%	−2%	21%	−3.5%	9.5%
2017	−2%	−5%	30%	−3.5%	12.5%
2018	9%	30%	7%	19.5%	18.5%
Average Return	10.0%	10.0%	10.0%	10.0%	10.0%
Volatility	13.4%	13.4%	13.4%	12.1%	5.1%

11.2 The Volatility of a Two-Stock Portfolio (2 of 4)

- Combining Risks
  - While the three stocks in the previous table have the same volatility and average return, the pattern of their returns differs.
    - For example, when the airline stocks performed well, the oil stock tended to do poorly, and when the airlines did poorly, the oil stock tended to do well.

11.2 The Volatility of a Two-Stock Portfolio (3 of 4)

- Combining Risks
  - Consider the portfolio which consists of equal investments in West Air and Tex Oil.
    - The average return of the portfolio is equal to the average return of the two stocks.
    - However, the volatility of 5.1% is much less than the volatility of the two individual stocks.

11.2 The Volatility of a Two-Stock Portfolio (4 of 4)

- Combining Risks
  - By combining stocks into a portfolio, we reduce risk through diversification.
  - The amount of risk that is eliminated in a portfolio depends on the degree to which the stocks face common risks and their prices move together.

Determining Covariance and Correlation (1 of 3)

- To find the risk of a portfolio, one must know the degree to which the stocks' returns move together.

Determining Covariance and Correlation (2 of 3)

- Covariance
  - The expected product of the deviations of two returns from their means.
  - Covariance between Returns R_i and R_j

$$Cov(R_i, R_j) = E[(R_i - E[R_i])\,(R_j - E[R_j])]$$

  - Estimate of the Covariance from Historical Data

$$Cov(R_i, R_j) = \frac{1}{T-1}\sum_t (R_{i,t} - \bar{R}_i)\,(R_{j,t} - \bar{R}_j)$$

    - If the covariance is positive, the two returns tend to move together.
    - If the covariance is negative, the two returns tend to move in opposite directions.

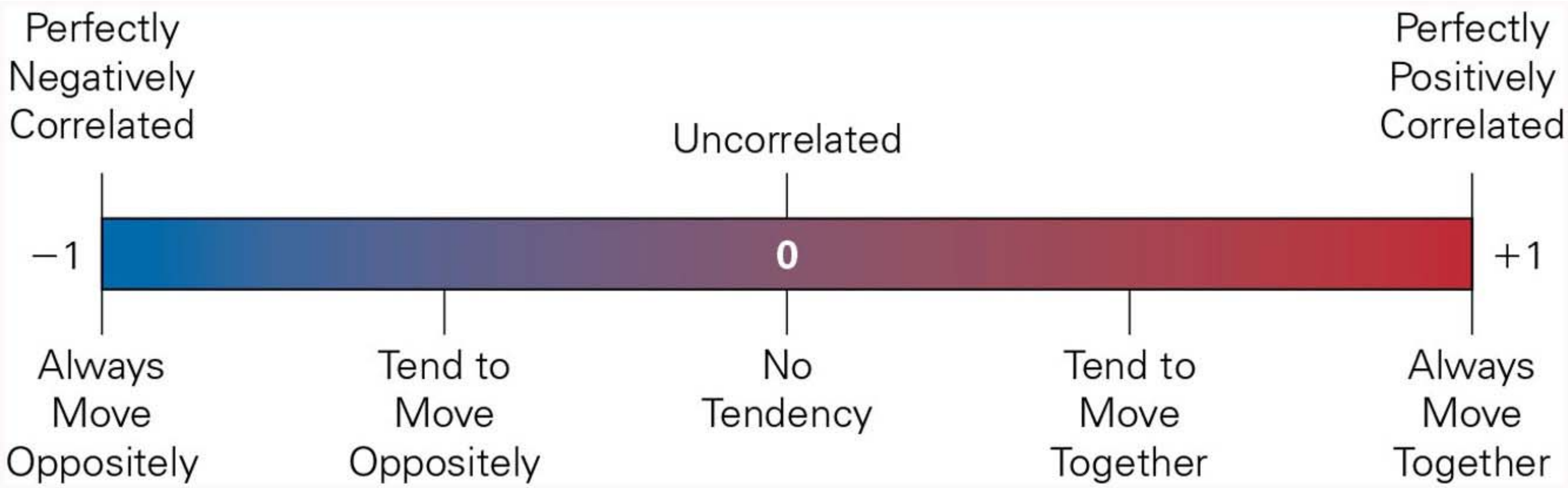
Determining Covariance and Correlation (3 of 3)

- Correlation
  - A measure of the common risk shared by stocks that does not depend on their volatility

$$\text{Corr}(R_i, R_j) = \frac{\text{Cov}(R_i, R_j)}{SD(R_i)\, SD(R_j)}$$

    - The correlation between two stocks will always be between -1 and +1.

Figure 11.1 Correlation



Textbook Example 11.3 (1 of 2)

The Covariance and Correlation of a Stock with Itself

Problem

What are the covariance and the correlation of a stock's return with itself?

Textbook Example 11.3 (2 of 2)

Solution

Let R be the stock's return. Form the definition of the covariance,

$$\mathrm{Cov}(R_{S,}R_S)=E[(R_S-E[R_S])(R_S-E[(R_S-E[R_S])]=E[R_S-E[R_S])^2]$$
$$=\mathrm{Var}(R_S)$$

where the last Eq. follows from the definition of the variance. That is, the covariance of a stock with itself is simply its variance. Then,

$$\mathrm{Corr}(R_{S,}R_S)=\frac{\mathrm{Cov}(R_{S,}R_S)}{SD(R_S)SD(R_S)}=\frac{\mathrm{Var}(R_S)}{SD(R_S)^2}=1$$

Where the last Eq. follows from the definition of the standard deviation. That is, a stock's return is perfectly correlated with itself, as it always moves together with itself in perfect synchrony.

Table 11.2 Computing the Covariance and Correlation Between Pairs of Stocks

Year	Deviation from Mean $(R_N - \bar{R}_N)$	$(R_W - \bar{R}_W)$	$(R_T - \bar{R}_T)$	North Air and West Air $(R_N - \bar{R}_N)(R_W - \bar{R}_W)$	West Air and Tex Oil $(R_W - \bar{R}_W)(R_T - \bar{R}_T)$
2013	11%	−1%	−12%	−0.0011	0.0012
2014	20%	11%	−15%	0.0220	−0.0165
2015	−3%	−3%	−1%	0.0009	0.0003
2016	−15%	−12%	11%	0.0180	−0.0132
2017	−12%	−15%	20%	0.0180	−0.0300
2018	−1%	20%	−3%	−0.0020	−0.0060
			$\text{Sum} = \sum_t (R_{i,t} - \bar{R}_i)(R_{j,t} - \bar{R}_j) =$	0.0558	−0.0642
Covariance:			$Cov(R_i, R_j) = \frac{1}{T-1}\text{Sum} =$	0.0112	−0.0128
Correlation:			$Corr(R_i, R_j) = \frac{Cov(R_i, R_j)}{SD(R_i)SD(R_j)} =$	0.624	−0.713

Table 11.3 Historical Annual Volatilities and Correlations for Selected Stocks

	Microsoft	HP	Alaska Air	Southwest Airlines	Ford Motor	Kellogg	General Mills
Volatility (SD) **Correlation with**	32%	36%	36%	31%	47%	19%	17%
Microsoft	1.00	0.40	0.18	0.22	0.27	0.04	0.10
HP	0.40	1.00	0.27	0.34	0.27	0.10	0.06
Alaska Air	0.18	0.27	1.00	0.40	0.15	0.15	0.20
Southwest Airlines	0.22	0.34	0.40	1.00	0.30	0.15	0.21
Ford Motor	0.27	0.27	0.15	0.30	1.00	0.17	0.08
Kellogg	0.04	0.10	0.15	0.15	0.17	1.00	0.55
General Mills	0.10	0.06	0.20	0.21	0.08	0.55	1.00

Textbook Example 11.4 (1 of 2)

Computing the Covariance and Correlation

Problem

Using the data in Table 11.1, What are the correlation between North Air and West Air? Between West Air and Tex Oil?

Textbook Example 11.4 (2 of 2)

Solution

Given the returns in Table 11.1, we deduct the mean return (10%) from each and compute the product of these deviations between the pairs of stocks. We then sum them and divide by $T - 1 = 5$ to compute the covariance, as in Table 11.2.

From the table, we see that North Air and West Air have a positive covariance, indicating a tendency to move together, whereas West Air and Tex Oil have a negative covariance, indicating a tendency to move oppositely. We can assess the strength of these tendencies from correlation, obtained by dividing the covariance by the standard deviation of each stock (13.4%). The correlation for North Air and West Air is 62.4%; the correlation for West Air and Tex Oil is -71.3%.

Textbook Example 11.5 (1 of 2)

Computing the Covariance from the Correlation

Problem

Using the data from Table 11.3, what is the covariance between Microsoft and HP?

Textbook Example 11.5 (2 of 2)

Solution

We can rewrite Eq. 11.6 to solve for the covariance

$$\begin{aligned}\text{Cov}(R_M, R_{HP}) &= \text{Corr}(R_M, R_{HP})\,SD(R_M)\,SD(R_{HP}) \\ &= (0.40)(0.32)(0.36) = 0.0461\end{aligned}$$

Alternative Example 11.5 (1 of 2)

Problem

- Using the data from Table 11.3, what is the covariance between General Mills and Ford?

Alternative Example 11.5 (2 of 2)

Solution

$$\begin{aligned}Cov(R_{\text{General Mills}}, R_{\text{Ford}}) &= Corr(R_{\text{General Mills}}, R_{\text{Ford}})SD(R_{\text{General Mills}})SD(R_{\text{Ford}}) \\ &= (0.08)(0.17)(0.47) = .00639\end{aligned}$$

X, Y: random variable

X~ (E[X], var[X]), Y~ (E[Y], var[Y])

Define Z such as Z= aX+bY where a, b: constant

E(Z)=?

E(Z)=E(aX+bY)=E(aX) + E(bY)= aE(X) +bE(Y)

Var(Z) =?

Var(Z)= Var(aX+bY)= Var(aX) + Var(bY)
= aVar (X) +bVar(Y)

Var(Z)= a^2Var (X) +b^2Var(Y) +2abCov(X, Y)

Computing a Portfolio's Variance and Volatility

- For a two security portfolio,

$$\begin{aligned} Var(R_P) &= Cov(R_P, R_P) \\ &= Cov(x_1R_1 + x_2R_2, x_1R_1 + x_2R_2) \\ &= x_1x_1Cov(R_1,R_1) + x_1x_2Cov(R_1,R_2) + x_2x_1Cov(R_2,R_1) + x_2x_2Cov(R_2,R_2) \end{aligned}$$

  – The Variance of a Two-Stock Portfolio

$$Var(R_P) = x_1^2 Var(R_1) + x_2^2 Var(R_2) + 2x_1x_2Cov(R_1,R_2)$$

Textbook Example 11.6 (1 of 2)

Computing the Volatility of a Two-Stock Portfolio

Problem

Using the data from Table 11.3, what is the volatility of a portfolio with equal amounts invested in Microsoft and Hewett-Packard stock? What is the volatility of a portfolio with equal amounts invested in Microsoft and Alaska Air stock?

Textbook Example 11.6 (2 of 2)

Solution

- With portfolio weights of 50% each in Microsoft and Hewlett-Packard stock, from Eq. 11.9, the portfolio's variance is

$$\begin{aligned} Var(R_P) &= x_M^2 SD(R_M)^2 + x_{HP}^2 SD(R_{HP})^2 + 2x_M x_{HP} Corr(R_M, R_{HP}) SD(R_M) SD(R_{HP}) \\ &= (0.50)^2(0.32)^2 + (0.50)^2(0.36)^2 + 2(0.50)(0.50)(0.40)(0.32)(0.36) \\ &= 0.0810 \end{aligned}$$

The volatility is therefore $SD(R_P) = \sqrt{Var(R)} = \sqrt{0.0810} = 28.5\%$

11.3 The Volatility of a Large Portfolio

- The variance of a portfolio is equal to the weighted average covariance of each stock with the portfolio:

$$Var(R_P) = Cov(R_P, R_P) = Cov\left(\sum_i x_i R_i, R_P\right) = \sum_i x_i Cov(R_i, R_P)$$

  - which reduces to

$$Var(R_P) = \sum_i x_i Cov(R_i, R_P) = \sum_i x_i Cov(R_i, \Sigma_j x_j R_j)$$
$$= \sum_i \sum_j x_i x_j Cov(R_i, R_j)$$

  - The Variance of a Two-Stock Portfolio

$$Var(R_P) = x_1^2 Var(R_1) + x_2^2 Var(R_2) + 2x_1 x_2 Cov(R_1, R_2)$$

Table 7-2 The Variance-Covariance Matrix Involved in Calculating the Standard Deviation of a Portfolio

Two securities:

	$\sigma_{1,1}$	$\sigma_{1,2}$
	$\sigma_{2,1}$	$\sigma_{2,2}$

Four securities:

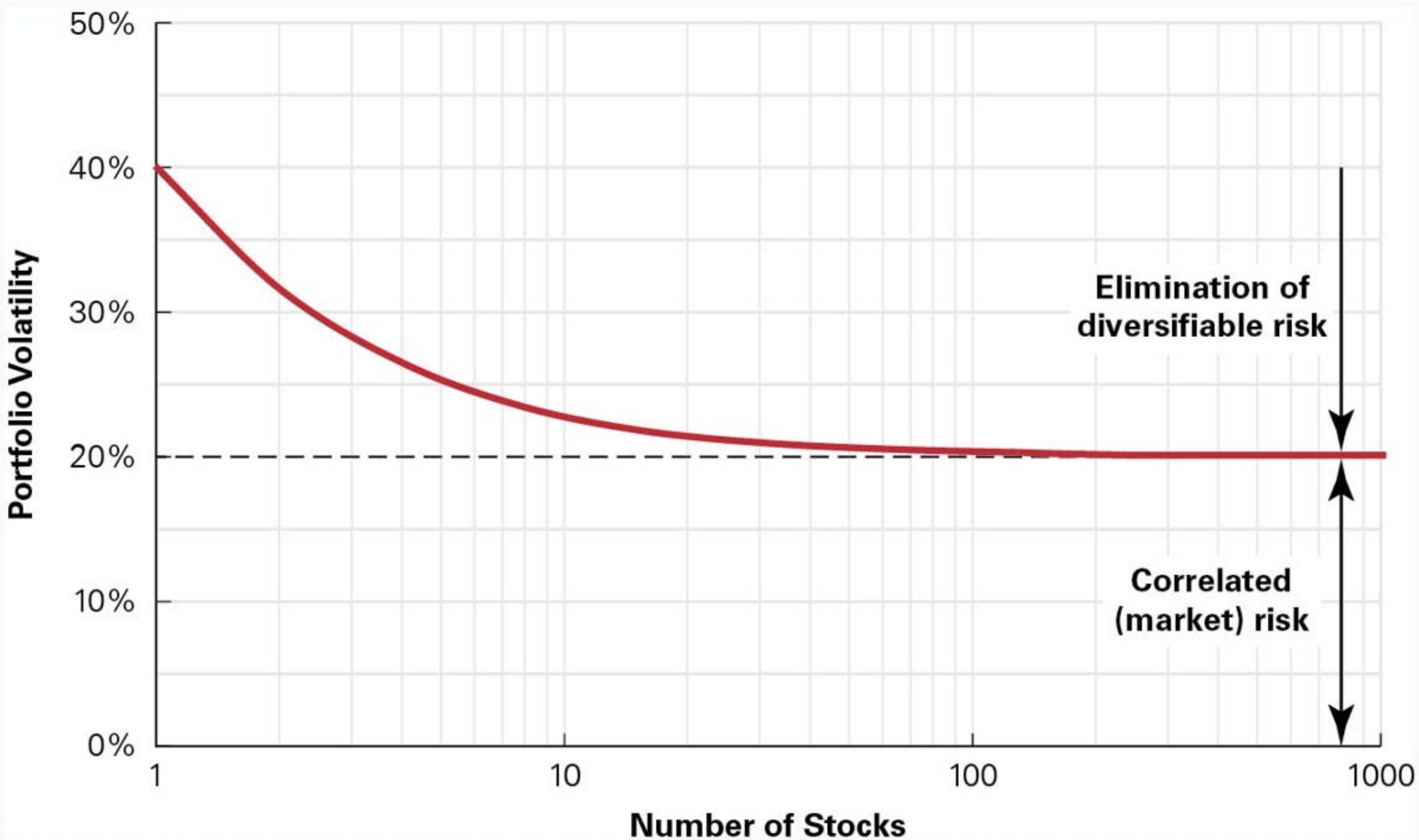
	$\sigma_{1,1}$	$\sigma_{1,2}$	$\sigma_{1,3}$	$\sigma_{1,4}$
	$\sigma_{2,1}$	$\sigma_{2,2}$	$\sigma_{2,3}$	$\sigma_{2,4}$
	$\sigma_{3,1}$	$\sigma_{3,2}$	$\sigma_{3,3}$	$\sigma_{3,4}$
	$\sigma_{4,1}$	$\sigma_{4,2}$	$\sigma_{4,3}$	$\sigma_{4,4}$

Diversification with an Equally Weighted Portfolio

- Equally Weighted Portfolio
  - A portfolio in which the same amount is invested in each stock
- Variance of an Equally Weighted Portfolio of *n* Stocks

$$\mathrm{Var}(R_P) = \frac{1}{n}(\text{Average Variance of the Individual Stocks}) + \left(1 - \frac{1}{n}\right)(\text{Average Covariance between the Stocks})$$

Figure 11.2 Volatility of an Equally Weighted Portfolio Versus the Number of Stocks



Textbook Example 11.7 (1 of 2)

Diversification using Different Types of Stocks

Problem

Stock within a single industry tend to have higher correlation than stocks in different industries. Likewise, stocks in different countries have lower correlation on average than stocks within the United States.

What is the volatility of a very large portfolio of stocks within an industry in which the stocks have a volatility of 40% and a correlation of 60%? What is the volatility of a very large portfolio of international stocks with a volatility of 40% and a correlation of 10%?

Textbook Example 11.7 (2 of 2)

Solution

From Eq. 11.12, the volatility of the industry portfolio as $n \to \infty$ is given by

$$\sqrt{\text{Average Covariance}} = \sqrt{0.60 \times 0.40 \times 0.40} = 31.0\%$$

This volatility is higher than when using stocks from different industries as in Figure 11.2. Combining stocks from the same industry that are more highly correlated therefore provides less diversification. We can achieve superior diversification using international stocks. In this case,

$$\sqrt{\text{Average Covariance}} = \sqrt{0.10 \times 0.40 \times 0.40} = 12.6\%$$

Textbook Example 11.8 (1 of 2)

Volatility when Risks Are Independent

Problem

What is the volatility of an equally weighted average of *n* independent risks?

Textbook Example 11.8 (2 of 2)

Solution

- If risks are independent, they are uncorrelated and their covariance is zero. Using Eq. 11.12, the volatility of an equally weighted portfolio of the risks is

$$SD(R_P) = \sqrt{Var(R_P)} = \sqrt{\frac{1}{n} Var(\text{individual Risk})} = \frac{SD(\text{individual Risk})}{\sqrt{n}}$$

- This result coincides with Eq. 10.8, which we used earlier to evaluate independent risks. Note that as $n \to \infty$, the volatility goes to 0—that is, a very large portfolio will have **no** risk. In this case, we can eliminate all risks because there is no common risk.

Diversification with General Portfolios

- For a portfolio with arbitrary weights, the standard deviation is calculated as follows:
  - Volatility of a Portfolio with Arbitrary Weights

$$SD(R_P) = \sum_i \overbrace{\underset{\substack{\uparrow \\ \text{Amount} \\ \text{of } i \text{ held}}}{x_i} \times \underset{\substack{\uparrow \\ \text{Total} \\ \text{risk of } i}}{SD(R_i)} \times \underset{\substack{\uparrow \\ \text{Fraction of } i\text{'s} \\ \text{risk that is} \\ \text{common to } P}}{Corr(R_i, R_p)}}^{\text{Security } i\text{'s contribution to the volatility of the portfolio}}$$

- Unless all of the stocks in a portfolio have a perfect positive correlation of +1 with one another, the risk of the portfolio will be lower than the weighted average volatility of the individual stocks:

$$SD(R_P) = \sum_i x_i SD(R_i)\ Corr(R_i, R_p) < \sum_i x_i SD(R_i)$$

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11.4 Risk Versus Return: Choosing an Efficient Portfolio (1 of 3)

- Efficient Portfolios with Two Stocks
  - Identifying Inefficient Portfolios
    - In an **inefficient portfolio**, it is possible to find another portfolio that is better in terms of both expected return and volatility.
  - Identifying Efficient Portfolios
    - Recall from Chapter 10, in an **efficient portfolio** there is no way to reduce the volatility of the portfolio without lowering its expected return.

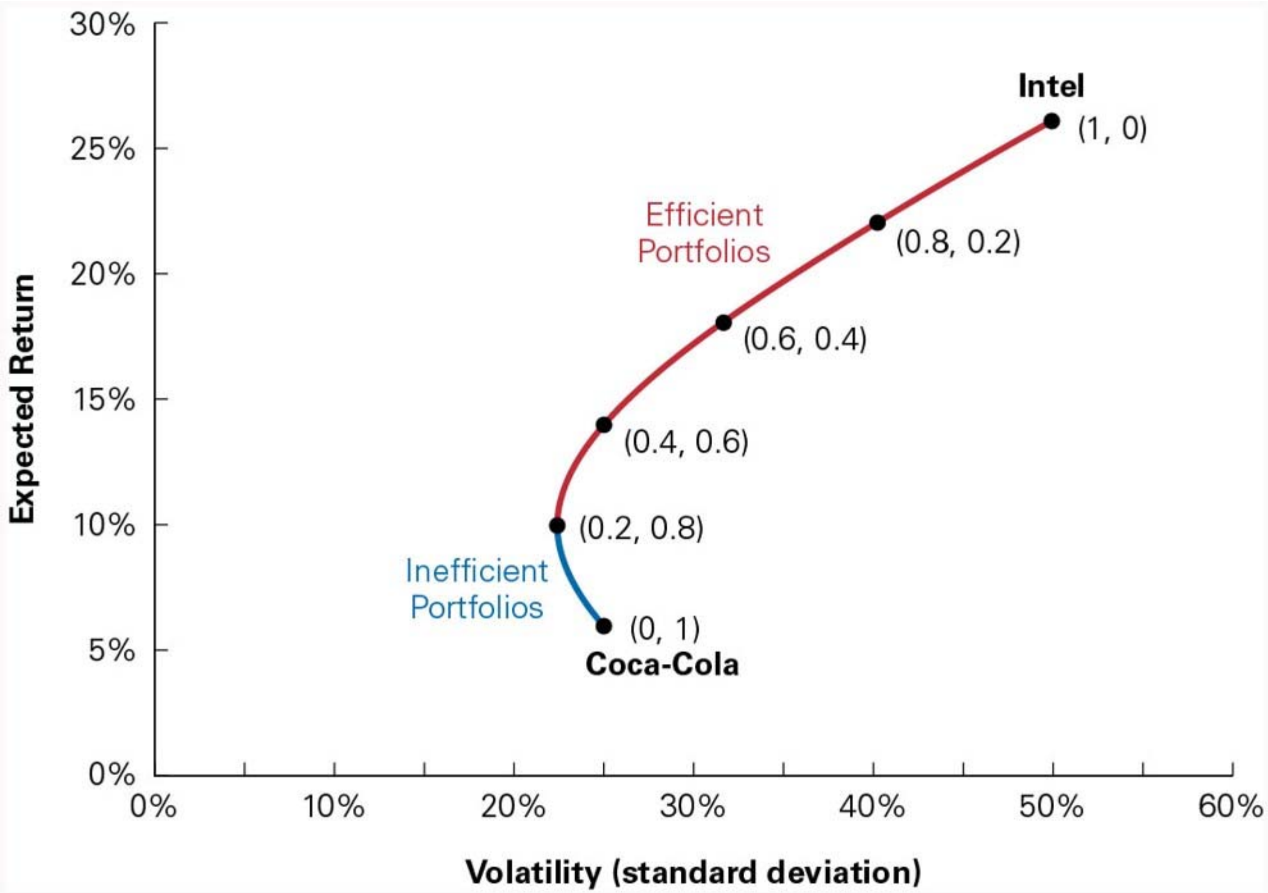
11.4 Risk Versus Return: Choosing an Efficient Portfolio (2 of 3)

- Efficient Portfolios with Two Stocks
  - Consider a portfolio of Intel and Coca-Cola.

Table 11.4 Expected Returns and Volatility for Different Portfolios of Two Stocks

Portfolio Weights		Expected Return (%)	Volatility (%)
x_I	x_C	$E[R_P]$	$SD[R_P]$
1.00	0.00	26.0	50.0
0.80	0.20	22.0	40.3
0.60	0.40	18.0	31.6
0.40	0.60	14.0	25.0
0.20	0.80	10.0	22.4
0.00	1.00	6.0	25.0

Figure 11.3 Volatility Versus Expected Return for Portfolios of Intel and Coca-Cola Stock



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11.4 Risk Versus Return: Choosing an Efficient Portfolio (3 of 3)

- Efficient Portfolios with Two Stocks
  - Consider investing 100% in Coca-Cola stock.
  - As shown in on the previous slide, other portfolios—such as the portfolio with 20% in Intel stock and 80% in Coca-Cola stock—make the investor better off in two ways:
    - It has a higher expected return, and it has lower volatility.
  - As a result, investing solely in Coca-Cola stock is inefficient.

Textbook Example 11.9 (1 of 2)

Improving Returns with an Efficient portfolio

Problem

Sally Ferson has invested 100% of her money in coca–cola stock and is seeking investment advice. She would like to earn the highest expected return possible without in increasing her volatility. Which portfolio would you recommend?

Textbook Example 11.9 (2 of 2)

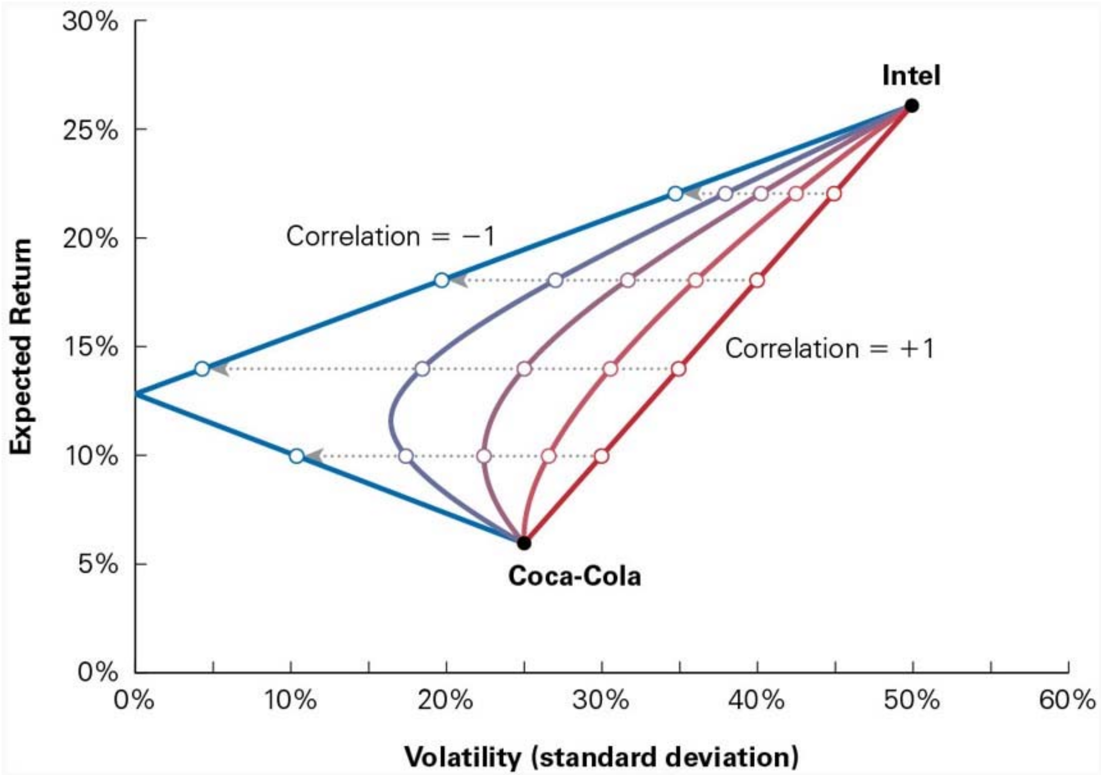
Solution

In Figure 11.3, we can see that Sally can invest up to 40% in Intel stock without increasing her volatility. Because stock has a higher expected return than coca-cola stock, she will earn higher expected returns by putting more money in Intel stock. Therefore, you should recommend that sally put 40% of her money in Intel stock, leaving 60% in coca-cola stock. This portfolio has the same volatility of 25%, but an expected return of 14% rather than the 6% she has now.

The Effect of Correlation

- Correlation has no effect on the expected return of a portfolio.
- However, the volatility of the portfolio will differ depending on the correlation.
- The lower the correlation, the lower the volatility we can obtain.
- As the correlation decreases, the volatility of the portfolio falls.
- The curve showing the portfolios will bend to the left to a greater degree as shown on the next slide.

Figure 11.4 Effect on Volatility and Expected Return of Changing the Correlation Between Intel and Coca-Cola Stock



Short Sales

- Long Position
  - A positive investment in a security
- Short Position
  - A negative investment in a security
  - In a short sale, you sell a stock that you do not own and then buy that stock back in the future.
  - Short selling is an advantageous strategy if you expect a stock price to decline in the future.

Textbook Example 11.10 (1 of 3)

Expected Return and Volatility with a Short Sale

Problem

Suppose you have $20,000 in cash to invest. You decide to short sell $10,000 worth of Coca-Cola stock and invest the proceeds from your short sale, plus your $20,000, in Intel. What is the expected return and volatility of your portfolio?

Textbook Example 11.10 (2 of 3)

Solution

- We can think of our short sale as a negative investment of \$10,000 in Coca-Cola stock. In addition, we invested + \$30,000 in Intel stock, for a total net investment of \$30,000 - \$10,000 = \$20,000 cash. The corresponding portfolio weights are

$$x_1 = \frac{\text{Value of investment in Intel}}{\text{Total value of portfolio}} = \frac{30{,}000}{20{,}000} = 150\%$$

$$x_C = \frac{\text{Value of investment in Coca-Cola}}{\text{Total valueof portfolio}} = \frac{-10{,}000}{20{,}000} = -50\%$$

- Note that the portfolio weights still add up to 100%. Using these portfolio weights, we can calculate the expected return and volatility of the portfolio using Eq. 11.3 and Eq. 11.8 as before:

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Textbook Example 11.10 (3 of 3)

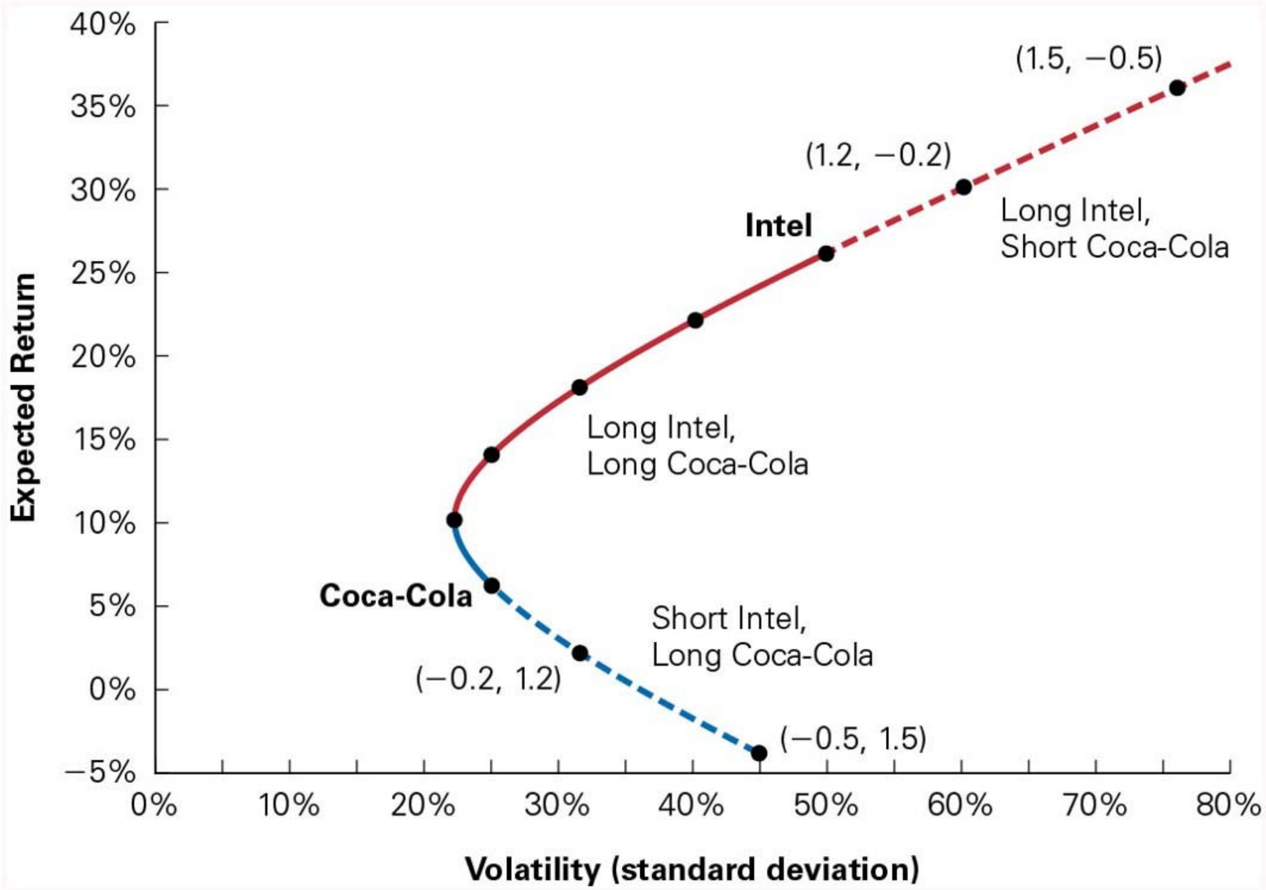
$$E[R_P] = x_I E[R_I] + x_C E[R_C] = 1.50 \times 26\% + (-0.50) \times 6\% = 36\%$$

$$SD(R_P) = \sqrt{Var(R_P)} = \sqrt{x_1^2 Var(R_I) + x_C^2 Var(R_C) + 2x_I x_C Cov(R_I, R_C)}$$

$$= \sqrt{1.5^2 \times 0.50^2 + (-0.5)^2 \times 0.25^2 + 2(1.5)(-0.5)(0)} = 76.0\%$$

- Note that in this case, short selling increases the expected return of your portfolio, but also its volatility, above those of the individual stocks.

Figure 11.5 Portfolios of Intel and Coca-Cola Allowing for Short Sales



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Efficient Portfolios with Many Stocks

- Consider adding Bore Industries to the two-stock portfolio:

				Correlation with	
Stock	**Expected Return**	**Volatility**	**Intel**	**Coca-Cola**	**Bore Ind.**
Intel	26%	50%	1.0	0.0	0.0
Coca-Cola	6%	25%	0.0	1.0	0.0
Bore Industries	2%	25%	0.0	0.0	1.0

  – Although Bore has a lower return and the same volatility as Coca-Cola, it still may be beneficial to add Bore to the portfolio for the diversification benefits.

Figure 11.6 Expected Return and Volatility for Selected Portfolios of Intel, Coca-Cola, and Bore Industries Stocks

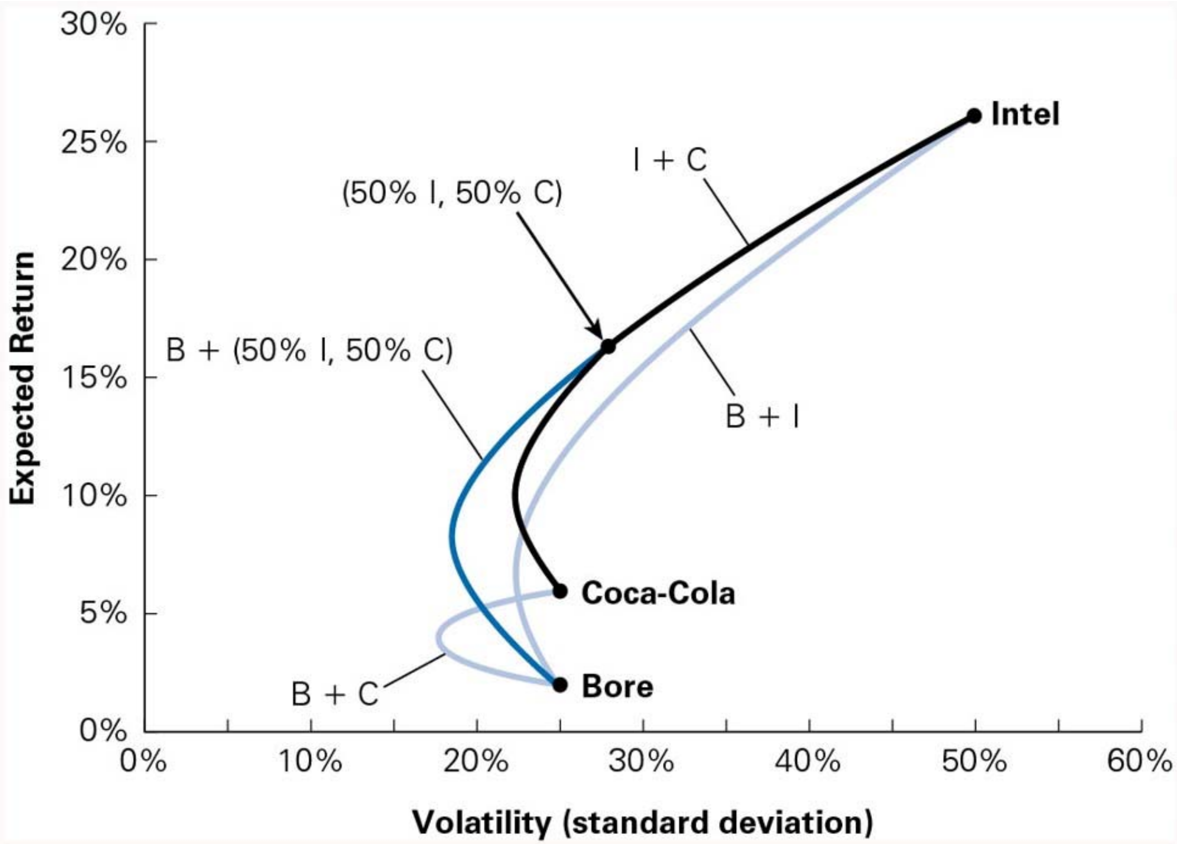
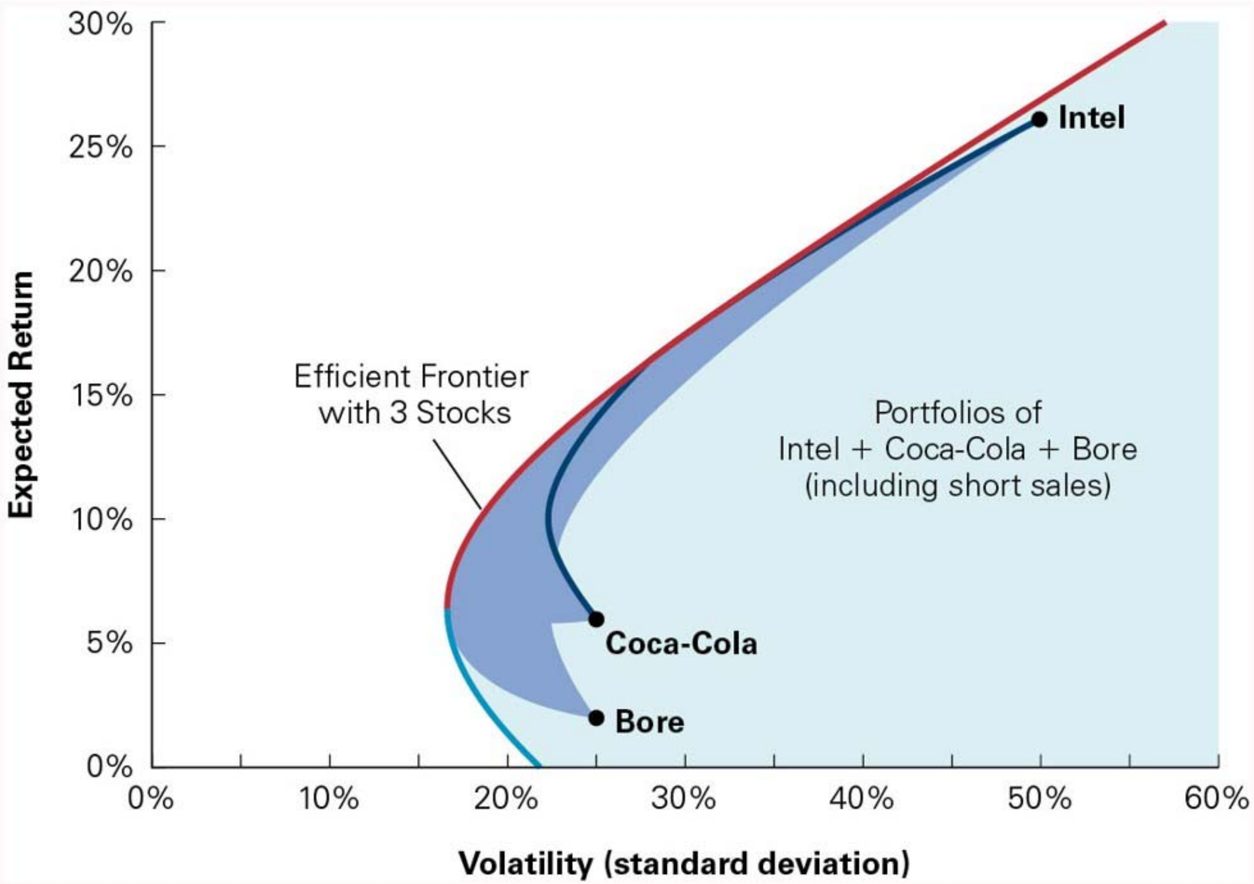


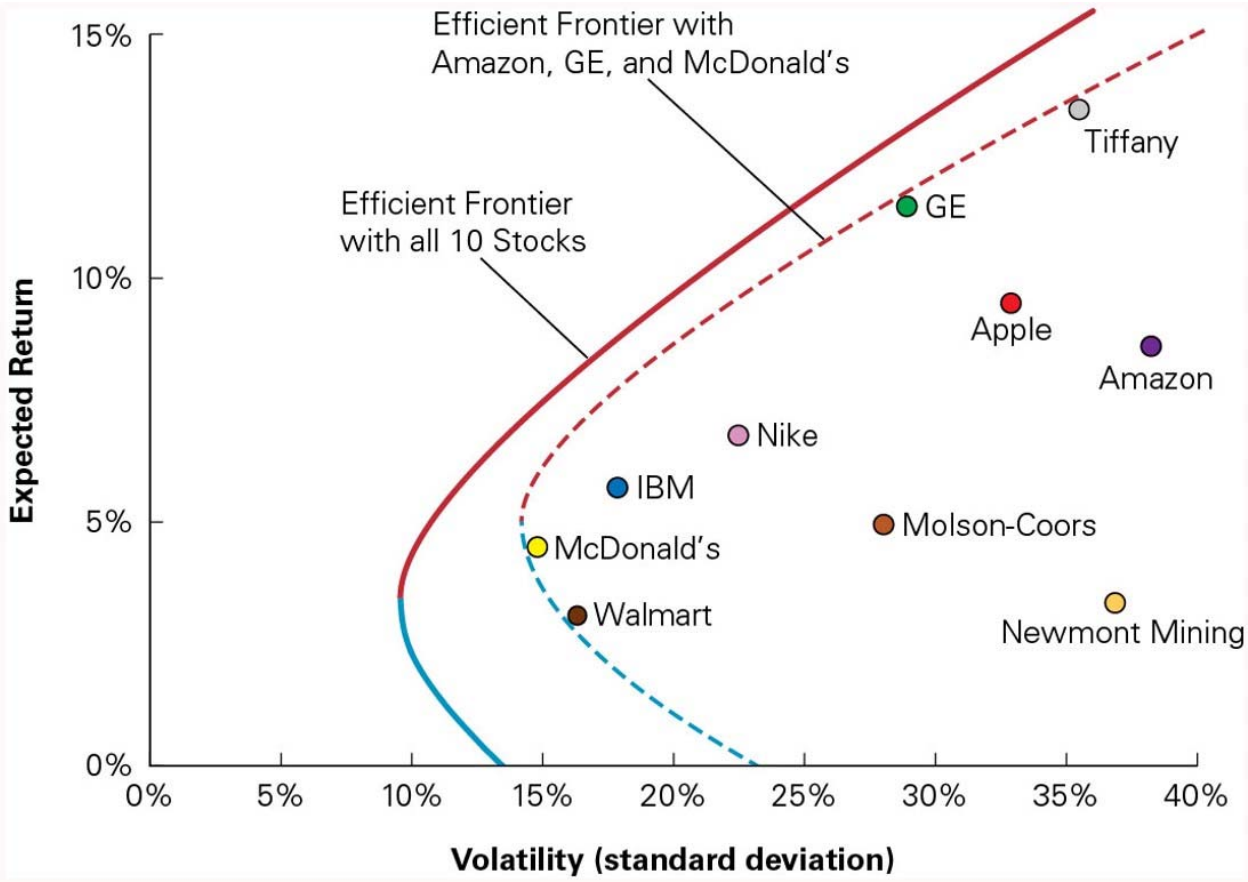
Figure 11.7 The Volatility and Expected Return for All Portfolios of Intel, Coca-Cola, and Bore Stock



Risk Versus Return: Many Stocks

- The efficient portfolios, those offering the highest possible expected return for a given level of volatility, are those on the northwest edge of the shaded region, which is called the **efficient frontier** for these three stocks.
  - In this case, none of the stocks, on its own, is on the efficient frontier, so it would not be efficient to put all our money in a single stock.

Figure 11.8 Efficient Frontier with Three Stocks Versus Ten Stocks



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11.5 Risk-Free Saving and Borrowing

- Risk can also be reduced by investing a portion of a portfolio in a risk-free investment, like T-Bills.
  - However, doing so will likely reduce the expected return.
- On the other hand, an aggressive investor who is seeking high expected returns might decide to borrow money to invest even more in the stock market.

Investing in Risk-Free Securities (1 of 2)

- Consider an arbitrary risky portfolio and the effect on risk and return of putting a fraction of the money *(x)* in the portfolio, while leaving the remaining fraction *(1-x)* in risk-free Treasury bills.
  - The expected return would be

$$E[R_{xP}] = (1 - x)r_f + xE[R_P]$$
$$= r_f + x(E[R_P] - r_f)$$

Investing in Risk-Free Securities (2 of 2)

- The standard deviation of the portfolio would be calculated as follows:

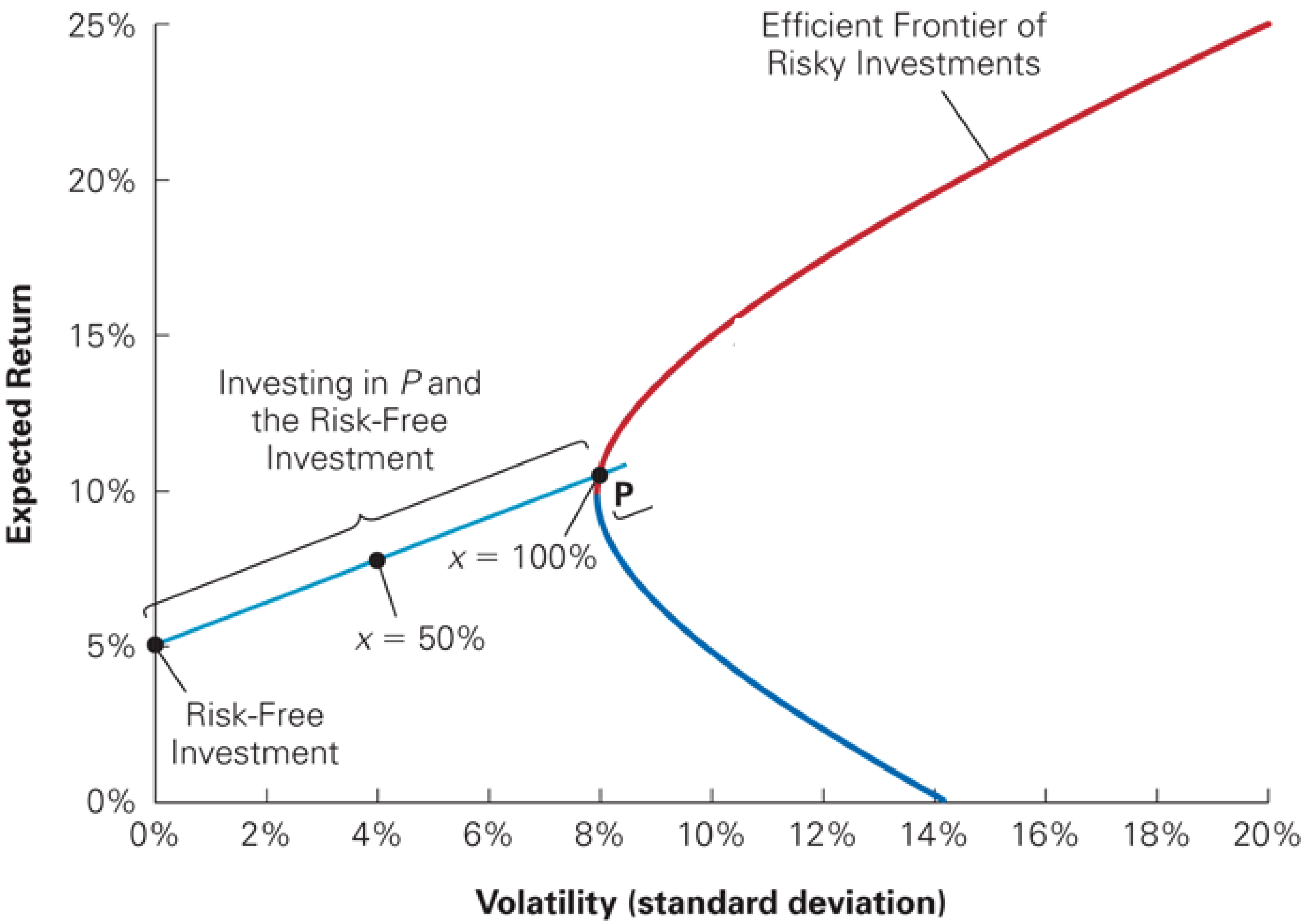
$$
\begin{aligned}
SD(R_{xP}) &= \sqrt{(1-x)^2 \underbrace{Var(r_f)}_{0} + x^2 Var(R_P) + 2(1-x)x \underbrace{Cov(r_f, R_P)}_{0}} \\
&= \sqrt{x^2 Var(R_P)} \\
&= x\,SD(R_P) \qquad (11.16)
\end{aligned}
$$

  – Note: The standard deviation is only a fraction of the volatility of the risky portfolio, based on the amount invested in the risky portfolio.

Borrowing and Buying Stocks on Margin

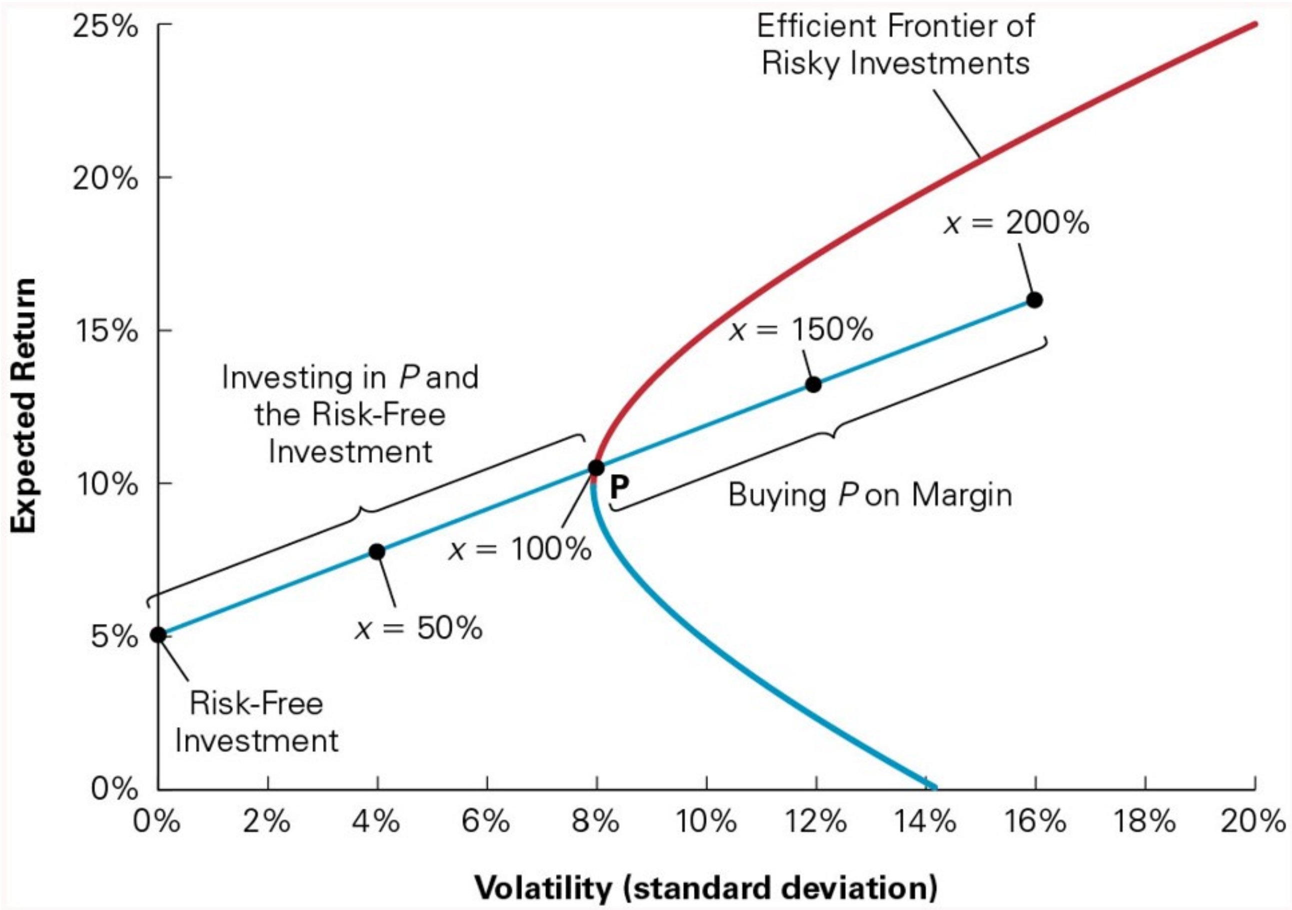
- Buying Stocks on Margin
  - Borrowing money to invest in a stock
  - A portfolio that consists of a short position in the risk-free investment is known as a levered portfolio.
    - Margin investing is a risky investment strategy.

Figure 11.9 The Risk–Return Combinations from Combining a Risk-Free Investment and a Risky Portfolio



Pearson

Figure 11.9 The Risk–Return Combinations from Combining a Risk-Free Investment and a Risky Portfolio



Textbook Example 11.11 (1 of 2)

Margin Investing

Problem

Suppose you have $10,000 in cash, and you decide to borrow another $10,000 at a 5% interest rate in order to invest $20,000 in portfolio *Q* which has a 10% expected return and volatility of your investment? What is your realized return if *Q* goes up 30% over the year ? What if *Q* falls by 10%?

Textbook Example 11.11 (2 of 2)

Solution

You have doubled your investment in Q using margin, so $x = 200\%$. From Eq. 11.15 and Eq. 11.16, we see that you have increased both your expected return and your risk relative to the portfolio Q:

$$E(R_{xQ}) = r_f + x(E[R_Q] - r_f) = 5\% + 2 \times (10\% - 5\%) = 15\%$$

$$SD(R_{xQ}) = xSD(R_Q) = 2 \times (20\%) = 40\%$$

If Q goes up 30%, your investment will be worth \$26,000, but you will owe $\$10,000 \times 1.05 = \$10,500$ on your loan, for a net payoff of \$15,500 or a 55% return on your \$10,000 initial investment. If Q drops by 10%, you are left with $\$18,000 - \$10,500 = \$7500$, and your return is -25%. Thus the use of margin doubled the range of your returns ($55\% - (-25\%) = 80\%$ versus $30\% - (-10\%) = 40\%$), corresponding to the doubling of the volatility of the portfolio.

Identifying the Tangent Portfolio (1 of 4)

- To earn the highest possible expected return for any level of volatility we must find the portfolio that generates the steepest possible line when combined with the risk-free investment.

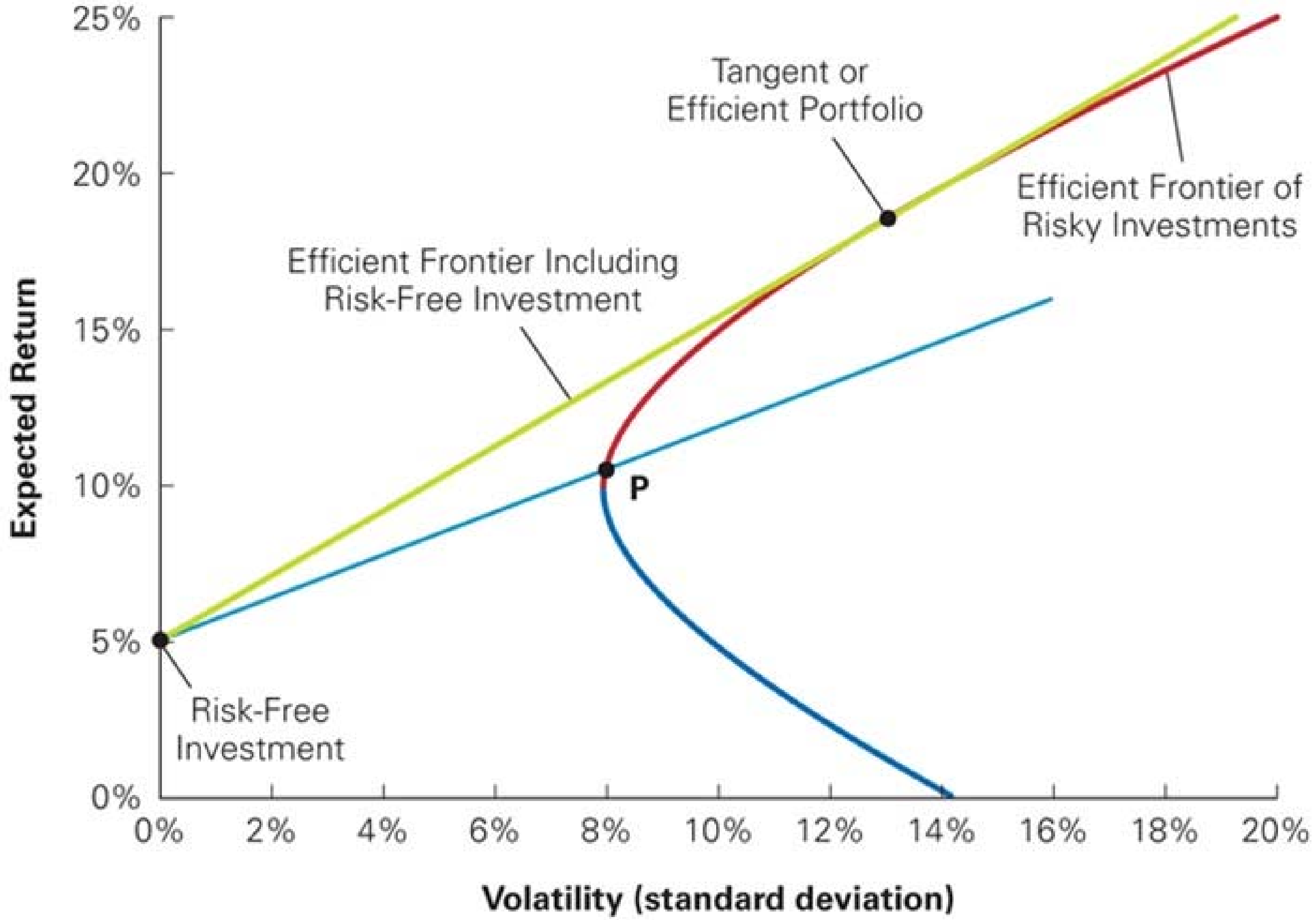
Identifying the Tangent Portfolio (2 of 4)

- Sharpe Ratio
  - Measures the ratio of reward-to-volatility provided by a portfolio

$$\text{Sharpe Ratio} = \frac{\text{Portfolio Excess Return}}{\text{Portfolio Volatility}} = \frac{E[R_P] - r_f}{SD(R_P)}$$

    - The portfolio with the highest Sharpe ratio is the portfolio where the line with the risk-free investment is tangent to the efficient frontier of risky investments.
    - The portfolio that generates this tangent line is known as the **tangent portfolio**.

Figure 11.10 The Tangent or Efficient Portfolio



Identifying the Tangent Portfolio (3 of 4)

- Combinations of the risk-free asset and the tangent portfolio provide the best risk and return trade-off available to an investor.
  - This means that the tangent portfolio is efficient and that all efficient portfolios are combinations of the risk-free investment and the tangent portfolio.
  - Every investor should invest in the tangent portfolio **independent of his or her taste for risk**.

Identifying the Tangent Portfolio (4 of 4)

- An investor's preferences will determine only **how much** to invest in the tangent portfolio versus the risk-free investment.
  - Conservative investors will invest a small amount in the tangent portfolio.
  - Aggressive investors will invest more in the tangent portfolio.
  - Both types of investors will choose to hold the same portfolio of risky assets, the tangent portfolio, which is the **efficient portfolio**.

Textbook Example 11.12 (1 of 2)

Optimal Portfolio Choice

Problem

Your uncle asks for investment advice. Currently, he has $100,000 invested in portfolio *p* in Figure 11.10, which has an expected return of 10.5% and a volatility of 8%. Suppose the risk-free rate is 5%, and the tangent portfolio has an expected return of 18.5% and a volatility of 13%.

1) To maximize his expected return without increasing his volatility, which portfolio would you recommend?
2) If your uncle prefers to keep his expected return the same but minimize his risk, which portfolio would you recommend?

Textbook Example 11.12 (2 of 2)

Solution

- In either case the best portfolios are combination of the risk-free investment and the tangent portfolio. *T*, using Eq. 11.16, the expected return and volatility are

$$E[R_{xT}]=r_f+x(E[R_T]-r_f)=5\%+x(18.5\%-5\%)$$

$$SD(R_{xT})=xSD(R_T)=x(13\%)$$

- So, to maintain the volatility at 8%, $x=\frac{8\%}{13\%}=61.5\%$. In this case, your uncle should invest $61,500 in the tangent portfolio, and the remaining $38,500 in the risk-free investment. His expected return will then be $5\% + (61.5\%)(13.5\%) = 13.3\%$, 13.3%, the highest possible given his level of risk.

Alternatively, to keep the expected return equal to the current value of 10.5%, *x* must satisfy $5\% + x(13.5\%) = 10.5\%$, so $x = 40.7\%$. Now your uncle should invest $40,700 in the tangent portfolio and $59,300 in the risk-free investment, lowering his volatility level to $(40.7\%)(13\%) = 5.29\%$, the lowest possible given his expected return.

11.6 The Efficient Portfolio and Required Returns (1 of 5)

- Portfolio Improvement: Beta and the Required Return
  - Assume there is a portfolio of risky securities, *P*
    - To determine whether *P* has the highest possible Sharpe ratio, consider whether its Sharpe ratio could be raised by adding more of some investment *i* to the portfolio.
    - The contribution of investment *i* to the volatility of the portfolio depends on the risk that *i* has in common with the portfolio, which is measured by i's volatility multiplied by its correlation with *P.*

11.6 The Efficient Portfolio and Required Returns (2 of 5)

- Portfolio Improvement: Beta and the Required Return
  - If you were to purchase more of investment *i* by borrowing, you would earn the expected return of *i* minus the risk-free return.
  - Thus adding *i* to the portfolio *P* will improve our Sharpe ratio if

$$\underbrace{E[R_i] - r_f}_{\substack{\text{Additional return} \\ \text{from investment } i}} > \underbrace{SD(R_i) \times Corr(R_i, R_P)}_{\substack{\text{Incremental volatility} \\ \text{from investment } i}} \times \underbrace{\frac{E[R_P] - r_f}{SD(R_P)}}_{\substack{\text{Return per unit of volatilty} \\ \text{available from portfolio } P}}$$

11.6 The Efficient Portfolio and Required Returns (3 of 5)

- Portfolio Improvement: Beta and the Required Return
  - Beta of Portfolio *i* with Portfolio *P*

$$\beta_i^P = \frac{SD(R_i) \times Corr(R_i, R_P)}{SD(R_P)} = \frac{Cov(R_i, R_P)}{Var(R_P)}$$

11.6 The Efficient Portfolio and Required Returns (4 of 5)

- Portfolio Improvement: Beta and the Required Return
  - Increasing the amount invested in *i* will increase the Sharpe ratio of portfolio *P* if its expected return $E[R_i]$ exceeds the required return r_i, which is given by

$$r_i = r_f + \beta_i^P \times (E[R_P] - r_f)$$

11.6 The Efficient Portfolio and Required Returns (5 of 5)

- Portfolio Improvement: Beta and the Required Return
  - Required Return of *i*
    - The expected return that is necessary to compensate for the risk investment *i* will contribute to the portfolio.

Textbook Example 11.13 (1 of 2)

The Required Return of a New Investment

Problem

You are currently invested in the omega Fund, a broad-based fund with an expected return of 15% and a volatility of 20%, as well as in risk-free Treasuries paying 3%. Your broker suggests that you add a real estate fund to your portfolio. The real estate fund has an expected return of 9%, a volatility of 35%, and a correlation of 0.10 with the omega fund. Will adding the real estate fund improve your portfolio?

Textbook Example 11.13 (2 of 2)

Solution

Let R_{re} be the return of the real estate fund and R_O be the return of the Omega Fund. From Eq. 11.19, the beta of the real estate fund with the Omega Fund is

$$\beta_{re}^{O} = \frac{SD(R_{re})\,Corr(R_{re}, R_O)}{SD(R_O)} = \frac{35\% \times 0.10}{20\%} = 0.175$$

We can then use Eq. 11.20 to determine the required return that makes the real estate fund an attractive addition to our portfolio:

$$r_{re} = r_f + \beta_{re}^{O}(E[R_O] - r_f) = 3\% + 0.175 \times (15\% - 3\%) = 5.1\%$$

Because its expected return of 9% exceeds the required return of 5.1%, investing some amount in the real estate fund will improve our portfolio's Sharpe ratio.

Expected Returns and the Efficient Portfolio

- Expected Return of a Security

$$\boxed{E[R_i] = r_i \equiv r_f + \beta_i^{eff} \times (E[R_{eff}] - r_f)}$$

  – A portfolio is efficient if and only if the expected return of every available security equals its required return

Textbook Example 11.14 (1 of 3)

Identifying the Efficient Portfolio

Problem

Consider the Omega Fund and real estate fund of Example 11.13. Suppose you have $100 million invested in the Omega Fund. In addition to this position, how much should you invest in the real estate fund to form an efficient portfolio of these two funds?

Textbook Example 11.14 (2 of 3)

Solution

- Suppose that for each $1 invested in the Omega Fund, we borrow x_{re} dollars (or sell x_{re} worth of treasury bills) to invest in the real estate fund. Then our portfolio has a return of $R_P = R_0 + x_{re}(R_{re} - r_f)$, where R_o is the return of the Omega Fund and R_{re} is the return of the real estate fund. Table 11.5 shows the change to the expected return and volatility of our portfolio as we increase the investment x_{re} in the real estate fund, using the formulas

$$Var(R_p) = Var[R_O + x_{re}(R_{re} - rf)] = Var(R_O) + x_{re}^2 Var(R_{re}) + 2x_{re}Cov(R_{re}, R_O)$$

$$E[R_p] = E[R_O] + x_{re}(E[R_{re}] - r_f)$$

Textbook Example 11.14 (3 of 3)

- Adding the real estate fund initially improves the Sharpe ratio of the portfolio, as defined by Eq. 11.17. As we add more of the real estate fund, however, its correlation with our portfolio rises, computed as

$$Corr(R_{re}, R_p) = \frac{Cov(R_{re}, R_p)}{SD(R_{re})SD(R_p)} = \frac{Cov(R_{re}, R_O + x_{re}(R_{re} - r_f))}{SD(R_{re})SD(R_p)}$$

$$= \frac{x_{re}Var(R_{re}) + Cov(R_{re}, R_O)}{SD(R_{re})SD(R_p)}$$

- The beta of the estate fund—computed from Eq. 11.19—also rise, increasing the required return. The required return equals the 9% expected return of the estate fund at about $x_{re} = 11\%$, which is the same level of investment that maximizes the Sharpe ratio. Thus, the efficient portfolio of these two funds includes \$0.11 in the real estate fund per \$1 invested in the Omega Fund.

Pearson

Table 11.5 Sharpe Ratio and Required Return for Different Investments in the Real Estate Fund

X_{re}	$E[R_p]$	$SD(R_p)$	Sharpe Ratio	Corr (R_{re}, R_p)	β_{re}^{P}	Required return r_{re}
0%	15.00%	20.00%	0.6000	10.0%	0.18	5.10%
4%	15.24%	20.19%	0.6063	16.8%	0.29	6.57%
8%	15.48%	20.47%	0.6097	23.4%	0.40	8.00%
10%	15.60%	20.65%	0.6103	26.6%	0.45	8.69%
11%	15.66%	20.74%	0.6104	28.2%	0.48	9.03%
12%	15.72%	20.84%	0.6103	29.7%	0.50	9.35%
16%	15.96%	21.30%	0.6084	35.7%	0.59	10.60%

Pearson

11.7 The Capital Asset Pricing Model

- The Capital Asset Pricing Model (CAPM) allows us to identify the efficient portfolio of risky assets without having any knowledge of the expected return of each security.
- Instead, the CAPM uses the optimal choices investors make to identify the efficient portfolio as the market portfolio, the portfolio of all stocks and securities in the market.

The CAPM Assumptions (1 of 3)

- Three Main Assumptions
  - Assumption 1
    - Investors can buy and sell all securities at competitive market prices (without incurring taxes or transaction costs) and can borrow and lend at the risk-free interest rate.

The CAPM Assumptions (2 of 3)

- Three Main Assumptions
  - Assumption 2
    - Investors hold only efficient portfolios of traded securities—portfolios that yield the maximum expected return for a given level of volatility.

The CAPM Assumptions (3 of 3)

- Three Main Assumptions
  - Assumption 3
    - Investors have **homogeneous expectations** regarding the volatilities, correlations, and expected returns of securities.
    - Homogeneous Expectations
      - All investors have the same estimates concerning future investments and returns.

Supply, Demand, and the Efficiency of the Market Portfolio

- Given homogeneous expectations, all investors will demand the same efficient portfolio of risky securities.
- The combined portfolio of risky securities of all investors must equal the efficient portfolio.
- Thus, if all investors demand the efficient portfolio, and the supply of securities is the market portfolio, the demand for market portfolio must equal the supply of the market portfolio.

Textbook Example 11.15 (1 of 2)

Portfolio Weights and the Market Portfolio

Problem

Suppose that after much research, you have identified the efficient portfolio. As part of your holdings, you have decided to invest $10,000 in Microsoft, and $5000 in Pfizer stock. Suppose your friend, who is a wealthier but more conservative investor, has $2000 invested in Pfizer. If your friend's portfolio is also efficient, how much has she invested in Microsoft? If all investors are holding efficient portfolios, what can you conclude about Microsoft's market capitalization, compared to Pfizer's?

Textbook Example 11.15 (2 of 2)

Solution

Because all efficient portfolios are combination of the risk-free investment and the tangent portfolio, they share the same proportions of risky stocks. Thus, since you have invested twice as much in Microsoft as in Pfizer, the same must be true for your friend; therefore, she has invested $4000 in Microsoft stock. If all investors hold efficient portfolios, the same must be true of each of their portfolios. Because, collectively, all investors own all shares of Microsoft and Pfizer, Microsoft's market capitalization must therefore be twice that of Pfizer's.

Optimal Investing: The Capital Market Line (1 of 2)

- When the CAPM assumptions hold, an optimal portfolio is a combination of the risk-free investment and the market portfolio.
  - When the tangent line goes through the market portfolio, it is called the **capital market line (CML)**.

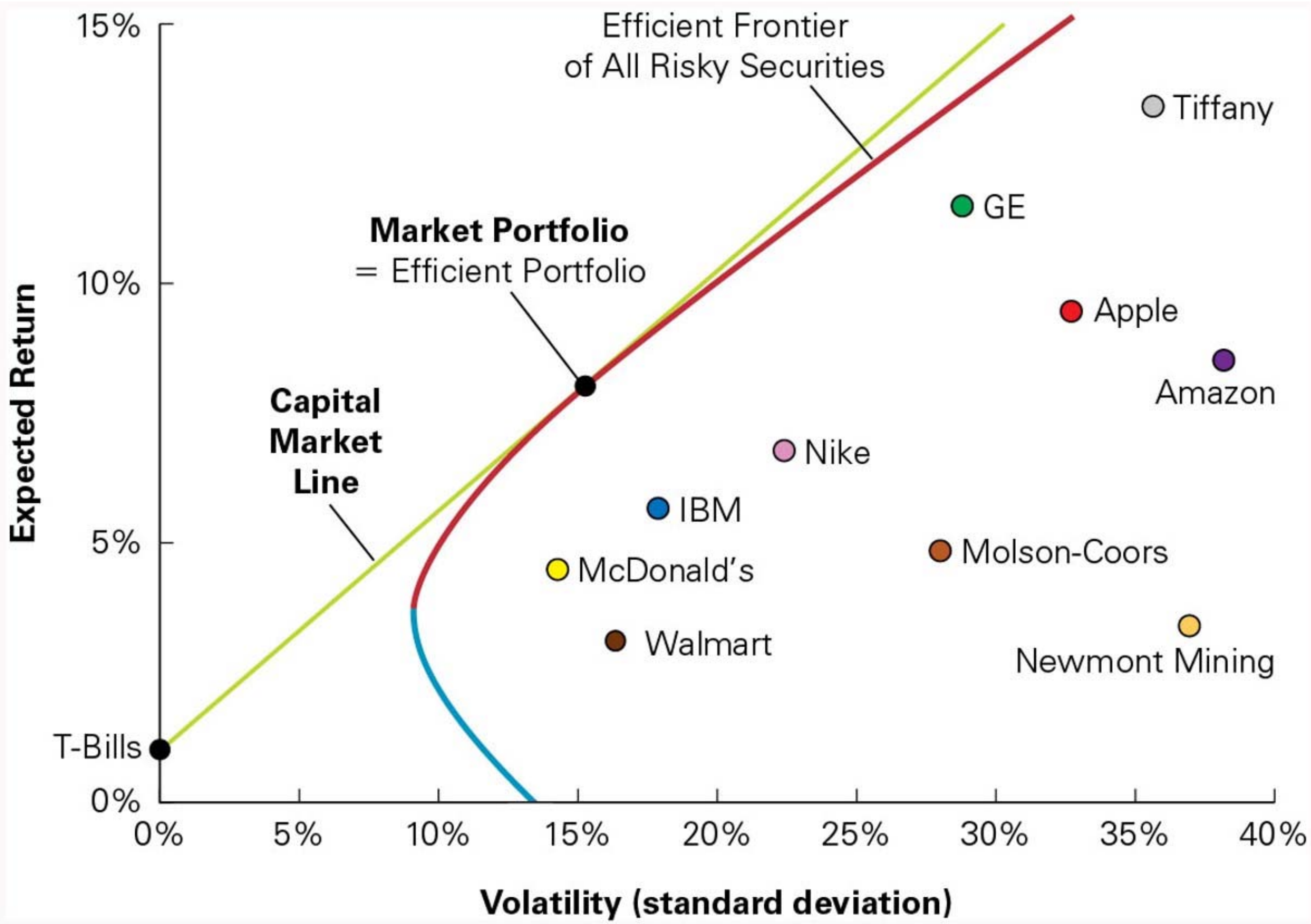
Optimal Investing: The Capital Market Line (2 of 2)

- The expected return and volatility of a capital market line portfolio are

$$E[R_{xCML}] = (1 - x)r_f + xE[R_{Mkt}] = r_f + x(E[R_{Mkt}] - r_f)$$

$$SD(R_{xCML}) = xSD(R_{Mkt})$$

Figure 11.11 The Capital Market Line



11.8 Determining the Risk Premium

- Market Risk and Beta
  – Given an efficient market portfolio, the expected return of an investment is

$$E[R_i] = r_i = r_f + \underbrace{\beta_i (E[R_{Mkt}] - r_f)}_{\text{Risk premium for security i}}$$

  – The beta is defined as follows:

$$\beta_i^{Mkt} \equiv \beta_i = \frac{\overbrace{SD(R_i) \times Corr(R_i, R_{Mkt})}^{\text{Volatility of } i \text{ that is common with the market}}}{SD(R_{Mkt})} = \frac{Cov(R_i, R_{Mkt})}{Var(R_{Mkt})}$$

Textbook Example 11.16 (1 of 3)

Computing the Expected Return for a stock

Problem

Suppose the risk-free return is 4% and the market portfolio has an expected return of 10% and a volatility of 16%. 3M stock has a 22% volatility and a correlation with the market of 0.50. What is 3M's beta with the market? What capital market line portfolio has equivalent market risk, and what is the expected return?

Textbook Example 11.16 (2 of 3)

Solution

- We can compute beta using Eq. 11.23:

$$\beta_{MMM} = \frac{SD(R_{MMM})Corr(R_{MMM}, R_{Mkt})}{SD(R_{Mkt})} = \frac{22\%?\ \ .50}{16\%} = 0.69$$

- That is, for each 1% move of the market portfolio, 3M stock tends to move 0.69%. We could obtain the same sensitivity to market risk by investing 69% in the market portfolio, and 31% in the risk-free security. Because it has the same market risk, 3M's stock should have the same expected return as this portfolio, which is (using Eq. 11.15 with $x = 0.69$),

Textbook Example 11.16 (3 of 3)

$$E[R_{MMM}] = r_f + x(E[R_{Mkt}] - r_f) = 4\% + 0.69(10\% - 4\%)$$
$$= 8.1\%$$

- Because $x = \beta_{MMM}$, this calculation is precisely the CAPM Eq. 11.22. Thus, investors will require an expected return of 8.1% to compensate for the risk associated with 3M stock.

Textbook Example 11.17 (1 of 2)

A Negative-Beta stock

Problem

Suppose the stock of Bankruptcy Auction Services, Inc.(BAS), has a negative beta of −0.30. How does its expected return compare to the risk-free rate, according to the CAMP? Does this result make sense?

Textbook Example 11.17 (2 of 2)

Solution

- Because the expected return of the market is higher than the risk-free rate. For example, if the risk-free rate is 4% and the expected return on the market is 10%

$$E[R_{BAS}]=4\%-0.30(10\%-4\%)=2.2\%$$

- This result seems odd: Why would investors be willing to accept a 2.2% expected return on this stock when they can invest in a safe investment and earn 4%? A savvy investor will not hold BAS alone; instead, she will hold it in combination with other securities fall, BAS portfolio. Because BAS will tend to rise when the market and most other securities as part of a well-diversified provides "recession insurance" for the portfolio, and investors pay for the insurance by accepting an expected return below the risk-free rate.

The Security Market Line (1 of 2)

- There is a linear relationship between a stock's beta and its expected return (See figure on next slide).
- The **security market line (SML)** is graphed as the line through the risk-free investment and the market.
  - According to the CAPM, if the expected return and beta for individual securities are plotted, they should all fall along the SML.

Figure 11.12 The Capital Market Line and the Security Market Line

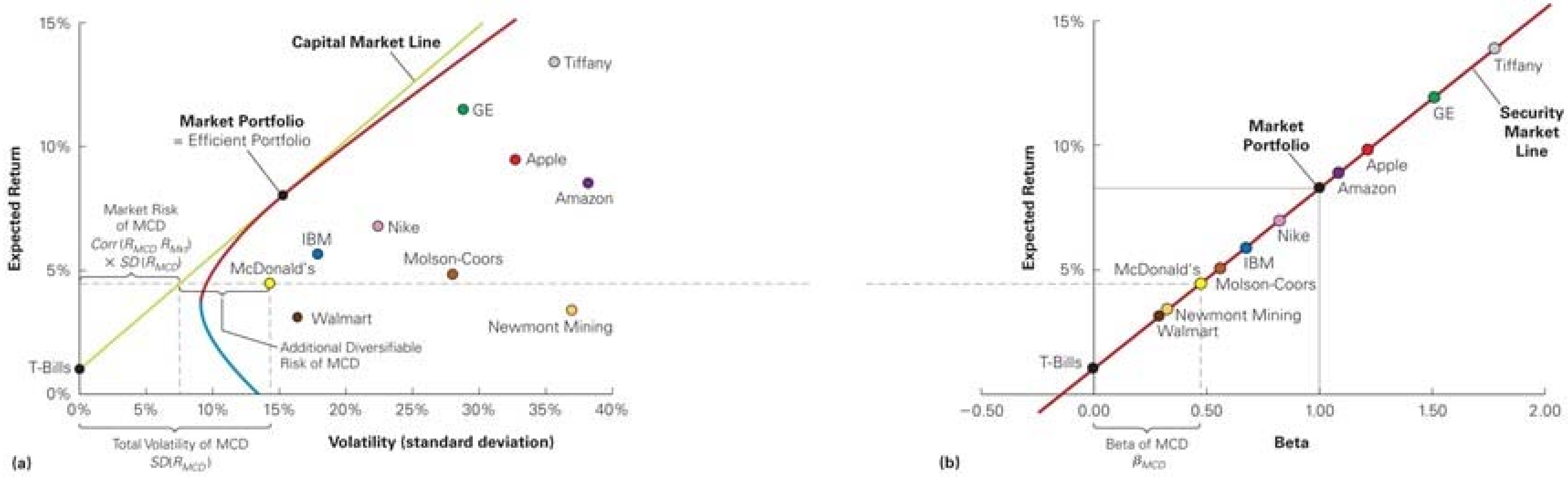


Figure 11.12 The Capital Market Line and the Security Market Line, Panel (a)

(a) The CML depicts portfolios combining the risk-free investment and the efficient portfolio, and shows the highest expected return that we can attain for each level of volatility. According to the CAPM, the market portfolio is on the CML and all other stocks and portfolios contain diversifiable risk and lie to the right of the CML, as illustrated for Exxon Mobil (XOM).

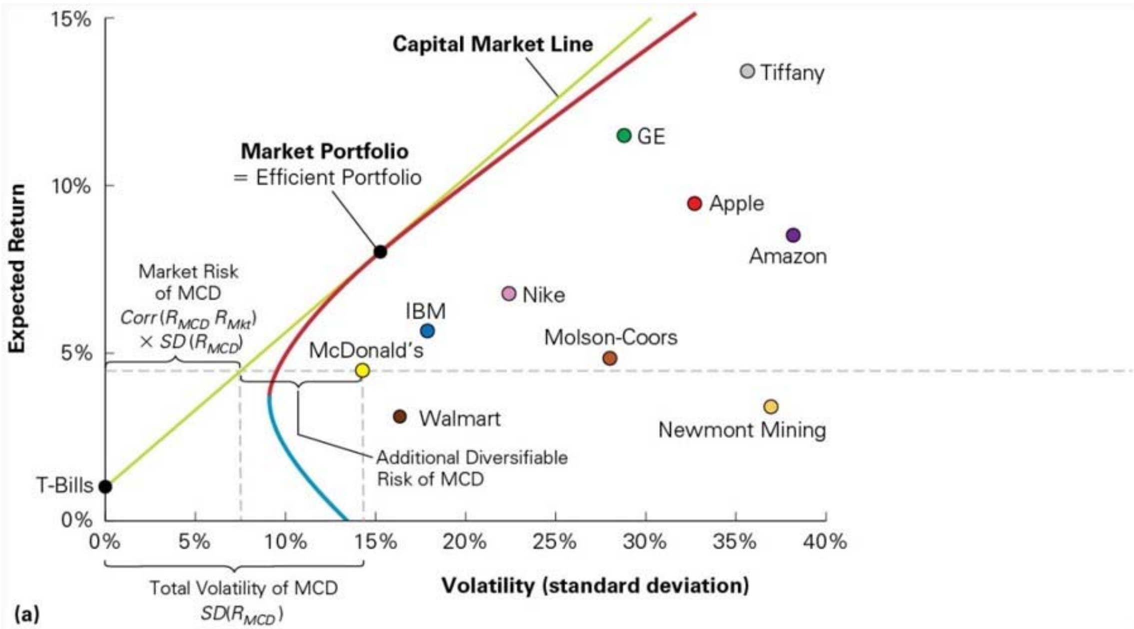
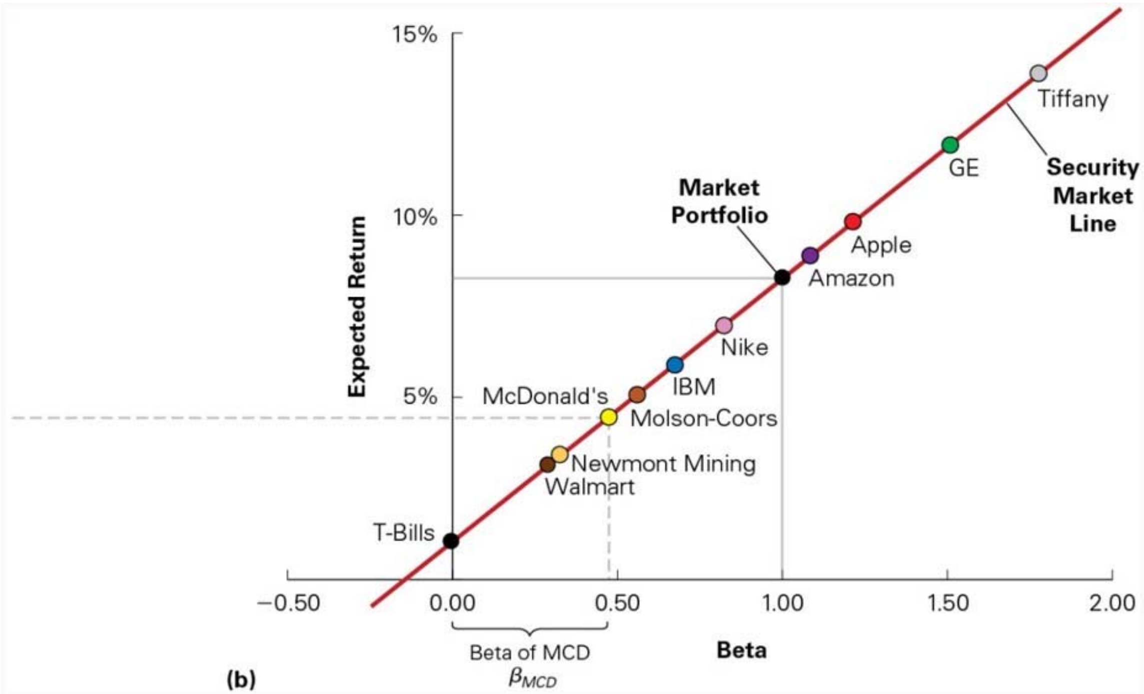


Figure 11.12 The Capital Market Line and the Security Market Line, Panel (b)

(b) The SML shows the expected return for each security as a function of its beta with the market. According to the CAPM, the market portfolio is efficient, so all stocks and portfolios should lie on the SML



The Security Market Line (2 of 2)

- Beta of a Portfolio
  - The beta of a portfolio is the weighted average beta of the securities in the portfolio.

$$\beta_P = \frac{Cov(R_P, R_{Mkt})}{Var(R_{Mkt})} = \frac{Cov\left(\sum_i x_i R_i, R_{Mkt}\right)}{Var(R_{Mkt})} = \sum_i x_i \frac{Cov(R_i, R_{Mkt})}{Var(R_{Mkt})} = \sum_i x_i \beta_i$$

Textbook Example 11.18 (1 of 3)

Textbook Example 11.18

Problem

Suppose Kraft Foods' stock has a beta of 0.50, whereas Boeing's beta is 1.25. If the risk-free rate is 4%, and the expected return of the market portfolio is 10%, what is the expected return of an equally weighted portfolio of Kraft Foods and Boeing stocks, according to the CAPM?

Textbook Example 11.18 (2 of 3)

Solution

- We can compute the expected return of the portfolio in two ways. First, we can use the SML to compute the expected return of Kraft Foods (KFT) and Boeing (BA) separately:

$$E[R_{KFT}] = r_f + \beta_{KFT}(E[r_{Mkt}] - r_f) = 4\% + 0.50(10\% - 4\%) = 7.0\%$$

$$E[R_{BA}] = r_f + \beta_{BA}(E[R_{Mkt}] - r_f) = 4\% + 1.25(10\% - 4\%) = 11.5\%$$

- Then, the expected return of the equally weighted portfolio *P is*

$$E[R_P] = \frac{1}{2}E[R_{KFT}] + \frac{1}{2}E[R_{BA}] = \frac{1}{2}(7.0\%) + \frac{1}{2}(11.5\%) = 9.25\%$$

Textbook Example 11.18 (3 of 3)

- Alternatively, we can compute the beta of the portfolio using Eq. 11.24:

$$\beta_P = \frac{1}{2}\beta_{KFT} + \frac{1}{2}\beta_{BA} = \frac{1}{2}(0.50) + \frac{1}{2}(1.25) = 0.875$$

- We can then find the portfolio's expected return from the SML:

$$E[R_P] = r_f + \beta_P(E[R_{Mkt}] - r_f) = 4\% + 0.875(10\% - 4\%) = 9.25\%$$

Summary of the Capital Asset Pricing Model

- The market portfolio is the efficient portfolio.
- The risk premium for any security is proportional to its beta with the market.