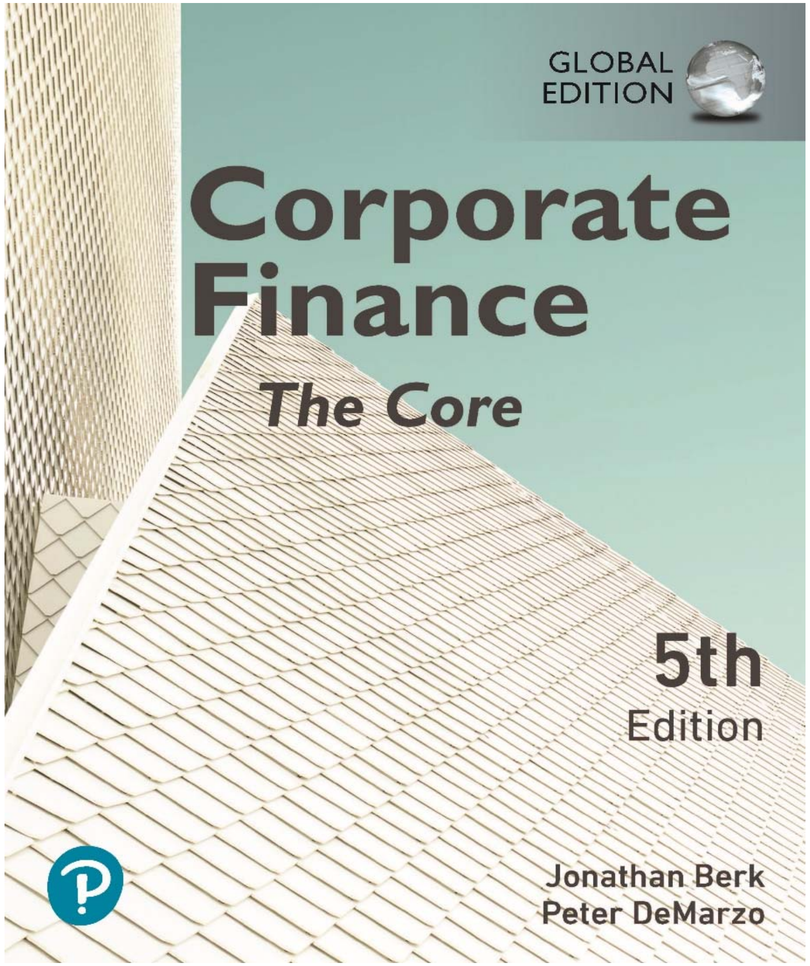
Corporate Finance: The Core

Fifth Edition, Global Edition



Chapter 5

Interest Rates

Chapter Outline

Learning Objectives (1 of 3)

- Define effective annual rate and annual percentage rate.
- Given an effective annual rate, compute the *n*-period effective annual rate.
- Convert an annual percentage rate into an effective annual rate, given the number of compounding periods.
- Describe the relation between nominal and real rates of interest.

Learning Objectives (2 of 3)

- Given two of the following, compute the third: nominal rate, real rate, and inflation rate.
- Describe the effect of higher interest rates on net present values in the economy.
- Explain how to choose the appropriate discount rate for a given stream of cash flows, according to the investment horizon.

Learning Objectives (3 of 3)

- Discuss the determinants of the shape of the yield curve.
- Explain why Treasury securities are considered risk-free, and describe the impact of default risk on interest rates.
- Given the other two, compute the third: after-tax interest rate, tax rate, and before-tax interest rate.

5.1 Interest Rate Quotes and Adjustments (1 of 2)

- The Effective Annual Rate
  - Indicates the total amount of interest that will be earned at the end of one year
  - Considers the effect of compounding
    - Also referred to as the effective annual yield (EAY) or annual percentage yield (APY)

5.1 Interest Rate Quotes and Adjustments (2 of 2)

- Adjusting the Discount Rate to Different Time Periods
  - Earning a 5% return annually is **not** the same as earning 2.5% every six months.
- General Equation for Discount Rate Period Conversion

$$\text{Equivalent } n\text{-Period Discount Rate} = (1+r)^n - 1$$

  - $(1.05)^{1/2} - 1 = 1.0247 - 1 = .0247 = 2.47\%$
    - Note: $n = 0.5$ since we are solving for the six-month (or half year) rate.

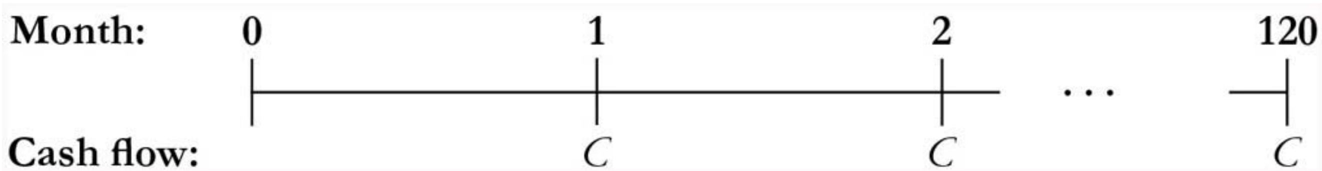
Textbook Example 5.1 (1 of 3)

Valuing Monthly Cash flows

- **Problem**
  - Suppose your bank account pays interest monthly with the interest rate quoted as an effective annual rate (EAR) of 6%. What amount of interest will you earn each month? If you have no money in the bank today, how much will you need to save at the end of each month to accumulate $100,000 in 10 years?

Textbook Example 5.1 (2 of 3)

- From Eq. 5.1, a 6% EAR is equivalent to earning $(1.06)^{\frac{1}{12}} - 1 = 0.4868\%$ per month. We can write the timeline for our savings plan using **monthly** periods as follows:



- That is, we can view the savings plan as a monthly annuity with $10 \times 12 = 120$ monthly payments. We can calculate the total amount saved as the future value of this annuity, using Eq. 4.10:

$$FV(\text{annuity}) = C \times \frac{1}{r}\left[(1+r)^n - 1\right]$$

- We can solve for the **monthly** payment C using the equivalent **monthly** interest rate r = 0.4868%, and n = 120 months:

Textbook Example 5.1 (3 of 3)

- We can also compute this result using the annuity spreadsheet:

	NPER	RATE	PV	PMT	FV	Excel Formula
Given	120	0.4868%	0	-	100,000	-
Solve for PMT	-	-	-	-615.47	-	= PMT(0.004868,120,0,100000)

- Thus, if we save $615.47 per month and we earn interest monthly at an effective annual rate of 6%, we will have $100,000 in 10 years.

Annual Percentage Rates (1 of 4)

- The **annual percentage rate (APR)**, indicates the amount of simple interest earned in one year.
  - **Simple interest** is the amount of interest earned **without** the effect of compounding.
  - The APR is typically less than the effective annual rate (EAR).

Pearson

Annual Percentage Rates (2 of 4)

- **The APR itself cannot be used as a discount rate.**
  - The APR with k compounding periods is a way of quoting the actual interest earned each compounding period:

$$\text{Interest Rate per Compounding Period} = \frac{\text{APR}}{\left(\dfrac{\text{k periods}}{\text{year}}\right)}$$

Annual Percentage Rates (3 of 4)

- Converting an APR to an EAR

$$\boxed{1 + EAR = \left(1 + \frac{APR}{k}\right)^k}$$

  – The EAR increases with the frequency of compounding.
    - **Continuous compounding** is compounding every instant.

Annual Percentage Rates (4 of 4)

Table 5.1 Effective Annual Rates for a 6% APR with Different Compounding Periods

Compounding Interval	Effective Annual Rate
Annual	$\left(\frac{1+0.06}{1}\right)^{1}-1=6\%$
Semiannual	$\left(\frac{1+0.06}{2}\right)^{2}-1=6.09\%$
Monthly	$\left(\frac{1+0.06}{12}\right)^{12}-1=6.1678\%$
Daily	$\left(\frac{1+0.06}{365}\right)^{365}-1=6.1831\%$

– A 6% APR with continuous compounding results in an EAR of approximately 6.1837%.

Textbook Example 5.2 (1 of 4)

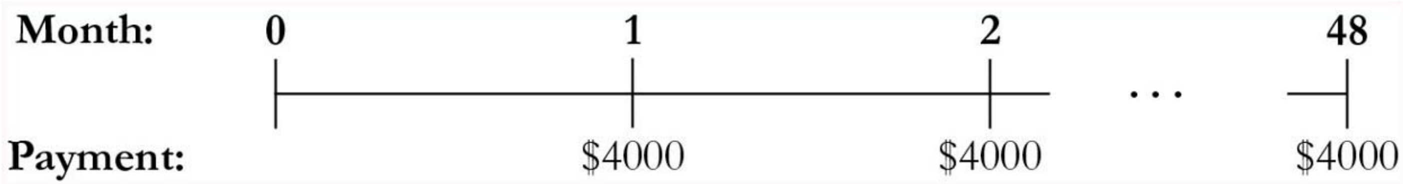
Converting the APR to a Discount Rate

- **Problem**
  - Your firm is purchasing a new telephone system, which will last for four years. You can purchase the system for an upfront cost of $150,000, or you can lease the system from the manufacturer for $4,000 paid at the end of each month.
  - Your firm can borrow at an interest rate of 5% APR with semiannual compounding. Should you purchase the system outright or pay $4,000 per month?

Textbook Example 5.2 (2 of 4)

Solution

- The cost of leasing the system is a 48-month annuity of $4,000 per month:



- We can compute the present value of the lease cash flows using the annuity formula, but first we need to compute the discount rate that corresponds to a period length of one month. To do so, we convert the borrowing cost of 5% APR with semiannual compounding to a monthly discount rate. Using Eq. 5.2,

the APR corresponds to a six-month discount rate of $\frac{5\%}{2} = 2.5\%$. To convert a six-month discount into a one-month discount rate, we compound the six-month rate by $\frac{1}{6}$ using Eq. 5.1:

$$(1.025)^{\frac{1}{6}} - 1 = 0.4124\% \text{ per month}$$

Textbook Example 5.2 (3 of 4)

$$1 + EAR = \left(1 + \frac{APR}{k}\right)^k$$

- (Alternatively, we could first use Eq. 5.3 to convert the APR to an *EAR*.

$1 + EAR = (1 + \frac{0.05}{2})^2 = 1.050625$. Then we can convert the *EAR* to a monthly rate using Eq. 5.1: $(1.050625)^{\frac{1}{12}}$ -1 = 0.4124% month.)

Given this discount rate, we can use the annuity formula (Eq. 4.9) to compute the present value of the 48 monthly payments:

$$PV = 4000 \times \frac{1}{0.004124^{48}}\left(1 - \frac{1}{1.004124^{48}}\right) = \$173,867$$

- We can also use the annuity spreadsheet:

	NPER	RATE	PV	PMT	FV	Excel Formula
Given	48	0.4124%	-	- 4,000	0	-
Solve for PV	-	-	**173,867**	-	-	= PV(0.004124,48,4000,0)

Textbook Example 5.2 (4 of 4)

- Thus, paying $4,000 per month for 48 months is equivalent to paying a present value of $173,867 today. This cost is $\$173{,}867 - \$150{,}000 = \$23{,}867$ higher than the cost of purchasing the system, so it is better to pay $150,000 for the system rather than lease it.

  We can interpret this result as meaning that at a 5% APR with semiannual compounding, by promising to repay $4,000 per month, your firm can borrow $173,867 today. With this loan it could purchase the phone system and have an additional $23,867 to use for other purposes.

5.2 Application: Discount Rates and Loans (1 of 2)

- Computing Loan Payments
  - Payments are made at a set interval, typically monthly.
  - All payments are equal and the loan is fully repaid with the final payment.
  - Each payment made includes the interest on the loan plus some part of the loan balance.

5.2 Application: Discount Rates and Loans (2 of 2)

- Computing Loan Payments
  - Consider a $30,000 car loan with 60 equal monthly payments, computed using a 6.75% APR with monthly compounding.
    - 6.75% APR with monthly compounding corresponds to a one-month discount rate of $\frac{6.75\%}{12} = 0.5625\%$.

$$C = \frac{P}{\frac{1}{r}\left(1 = \frac{1}{(1+r)^N}\right)} = \frac{30{,}000}{\frac{1}{0.005625}\left(1 - \frac{1}{(1+0.005625)^{60}}\right)} = \$590.50$$

Textbook Example 5.3 (1 of 3)

Computing the Outstanding Loan Balance

- **Problem**
  - Two years ago your firm took out a 30-year amortizing loan to purchase a small office building. The loan has a 4.80% APR with monthly payments of $2,623.33. How much do you owe on the loan today? How much interest did the firm pay on the loan in the past year?

Textbook Example 5.3 (2 of 3)

Solution

- After two years, the loan has 28 years, or 336 months, remaining:

- The remaining balance on the loan is the present value of these remaining payments, using the loan rate of $\frac{4.8\%}{12} = 0.4\%$ per month:

$$\text{Balance after 2 years} = \$2623.33 \times \frac{1}{0.004}\left(1 - \frac{1}{1.004^{336}}\right) = \$484{,}332$$

- During the past year, your firm made total payments of $\$2{,}623.33 \times 12 = \$31{,}480$ on the loan. To determine the amount that was interest, it is easiest to first determine the amount that was used to repay the principal. Your loan balance one year ago, with 29 years (348 months) remaining, was

Textbook Example 5.3 (3 of 3)

$$\text{Balance after one year} = \$2623.33 \times \left(1 - \frac{1}{1.004^{348}}\right) = \$492{,}354$$

Therefore, the balance declined by $\$492{,}354 - \$484{,}332 = \$8{,}022$ in the past year. Of the total payments made, $\$8{,}022$ was used to repay the principal and the remaining $\$31{,}480 - \$8{,}022 = \$23{,}458$ was used to pay interest.

5.3 The Determinants of Interest Rates (1 of 2)

- Inflation and Real Versus Nominal Rates
  - **Nominal Interest Rate:** The rates quoted by financial institutions and used for discounting or compounding cash flows
  - **Real Interest Rate:** The rate of growth of your purchasing power, after adjusting for inflation

5.3 The Determinants of Interest Rates (2 of 2)

$$\text{Growth in Purchasing Power} = 1 + r_r = \frac{1+r}{1+i} = \frac{\text{Growth of Money}}{\text{Growth of Prices}}$$

- The Real Interest Rate

$$r_r = \frac{r-i}{1+i} \approx r - i$$

Textbook Example 5.4 (1 of 3)

Calculating The Real Interest Rate

- **Problem**
  - In May of 2014, one-year U.S. government bond rates were about 0.1%, while the rate of inflation over the following year was around -0.05% (deflation). At the start of 2017, one-year interest rates were about 0.8%, and inflation over the following year was approximately 2.1%. What were the real interest rates in May 2014 and in 2017?

Textbook Example 5.4

- **Solution**
  - Using Eq. 5.5, the real interest rate in May 2014 was

$$\frac{(0.1\% + 0.05\%)}{(0.9995)} = 0.15\%.$$

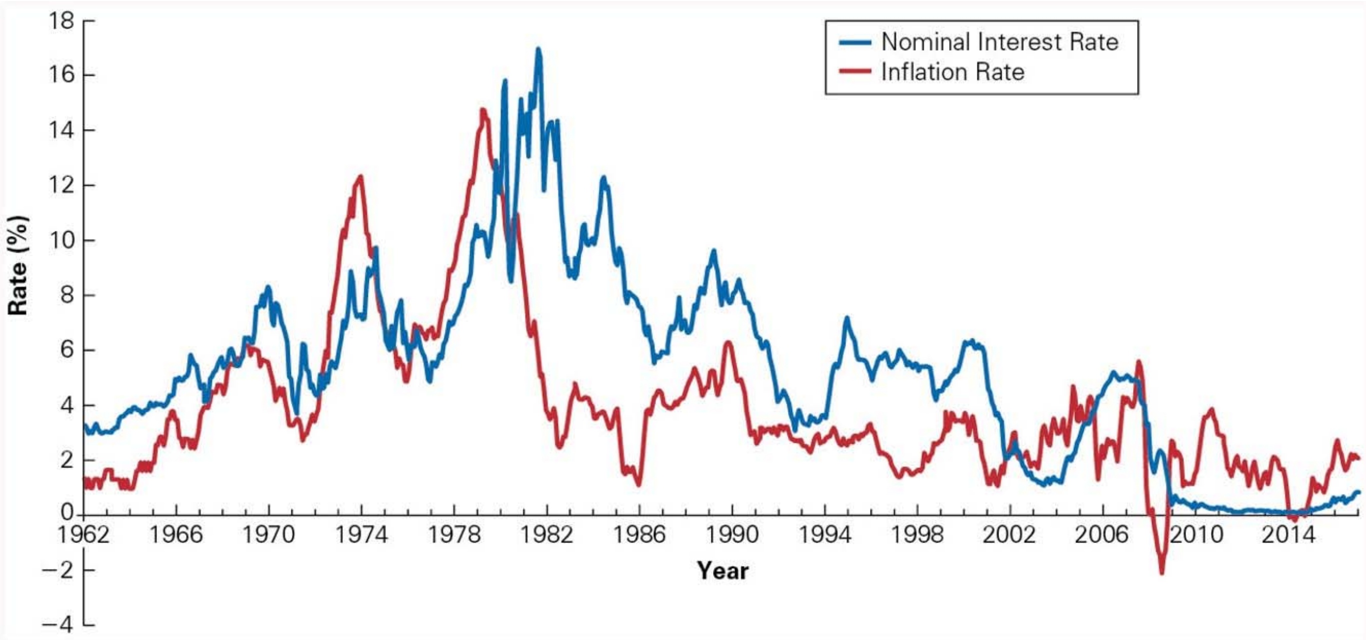
In 2017, the real interest rate was

$$\frac{(0.8\% - 2.1\%)}{(1.021)} = -1.27\%.$$

Textbook Example 5.4 (3 of 3)

- **Solution**
  - Note that the real interest rate was negative in 2017, indicating that interest rates were insufficient to keep up with inflation: Investors in U.S. government bonds were able to buy less at the end of the year than they could have purchased at the start of the year. On the other hand, because prices actually decreased (deflation) in the year following May 2014, the real interest rate briefly exceeded the nominal interest rate.

Figure 5.1 U.S. Interest Rates and Inflation Rates, 1962–2017



Interest rates are one-year Treasury rates, and inflation rates are the increase in the U.S. Bureau of Labor Statistics' consumer price index over the coming year, with both series computed on a monthly basis. The difference between them thus reflects the approximate real interest rate earned by holding Treasuries. Note that interest rates tend to be high when inflation is high.

Investment and Interest Rate Policy (1 of 2)

- An increase in interest rates will typically reduce the NPV of an investment.
  - Consider an investment that requires an initial investment of $10 million and generates a cash flow of $3 million per year for four years. If the interest rate is 5%, the investment has an NPV of

$$NPV = -10 + \frac{3}{1.05} + \frac{3}{1.05^2} + \frac{3}{1.05^3} + \frac{3}{1.05^4} = \$0.638 \text{ million}$$

Investment and Interest Rate Policy (2 of 2)

- If the interest rate rises to 9%, the NPV becomes negative and, the investment is no longer profitable:

$$NPV = -10 + \frac{3}{1.09} + \frac{3}{1.09^2} + \frac{3}{1.09^3} + \frac{3}{1.09^4} = \$0.281\ million$$

Monetary Policy, Deflation, and the 2008 Financial Crisis

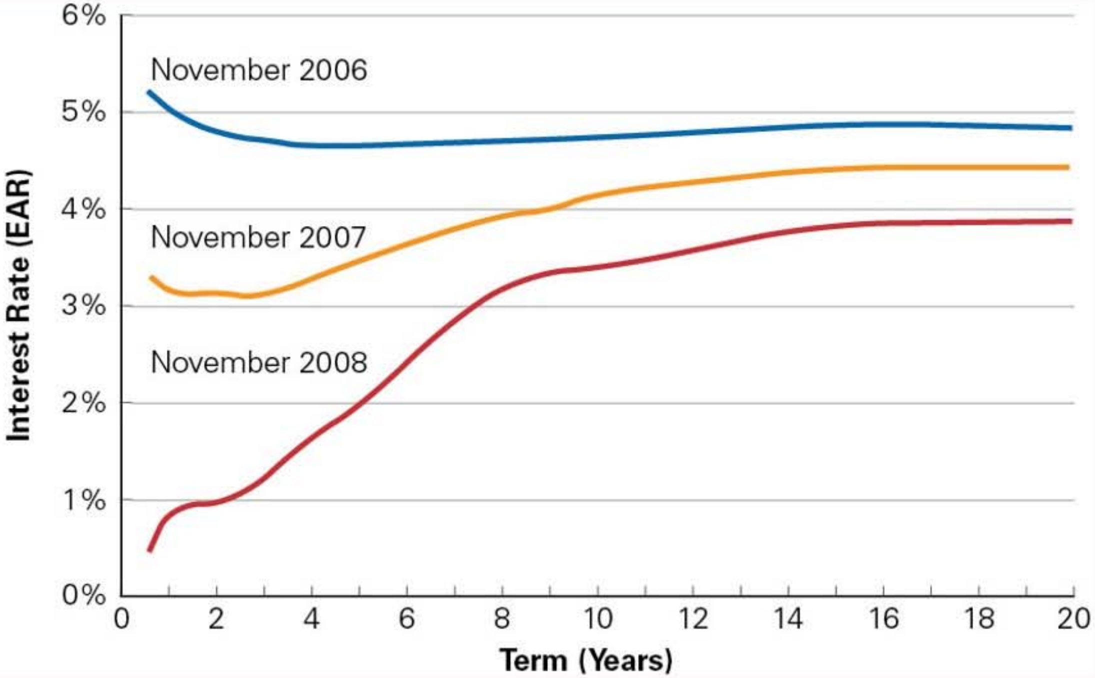
- When the 2008 financial crisis struck, the Federal Reserve responded by cutting its short-term interest rate target to 0%.
- While this use of monetary policy is generally quite effective, because consumer prices were falling in late 2008, the inflation rate was negative, and so even with a 0% nominal interest rate, the real interest rate remained positive.

The Yield Curve and Discount Rates (1 of 2)

- **Term Structure:** The relationship between the investment term and the interest rate
- **Yield Curve:** A graph of the term structure

Figure 5.2 Term Structure of Risk-Free U.S. Interest Rates, November 2006, 2007, and 2008

Term (years)	Date Nov-06	Nov-07	Nov-08
0.5	5.23%	3.32%	0.47%
1	4.99%	3.16%	0.91%
2	4.80%	3.16%	0.98%
3	4.72%	3.12%	1.26%
4	4.63%	3.34%	1.69%
5	4.64%	3.48%	2.01%
6	4.65%	3.63%	2.49%
7	4.66%	3.79%	2.90%
8	4.69%	3.96%	3.21%
9	4.70%	4.00%	3.38%
10	4.73%	4.18%	3.41%
15	4.89%	4.44%	3.86%
20	4.87%	4.45%	3.87%



The Yield Curve and Discount Rates (2 of 2)

- The term structure can be used to compute the present and future values of a risk-free cash flow over different investment horizons.

$$PV = \frac{C_n}{(1 + r_n)^n}$$

  – Present Value of a Cash Flow Stream Using a Term Structure of Discount Rates

$$PV = \frac{C_1}{1 + r_1} + \frac{C_2}{(1 + r_2)^2} + \cdots + \frac{C_N}{(1 + r_N)^N} = \sum_{n=1}^{N} \frac{C_N}{(1 + r_n)^n}$$

Textbook Example 5.5 (1 of 2)

Using the Term ‘Structure’ to Compute Present Values

- **Problem**
  - Compute the present value in November 2008 of a risk-free five-year annuity of $1,000 per year, given the yield curve for November 2008 in Figure 5.2

Textbook Example 5.5 (2 of 2)

Solution

To compute the present value, we discount each flow by the corresponding interest rate:

$$PV = \frac{1000}{1.0091} + \frac{1000}{1.0098^2} + \frac{1000}{1.0126^3} + \frac{1000}{1.0169^4} + \frac{1000}{1.0201^5} = \$4775.25$$

Note that we cannot use the annuity formula here because the discount rates differ for each cash flow.

The Yield Curve and the Economy (1 of 2)

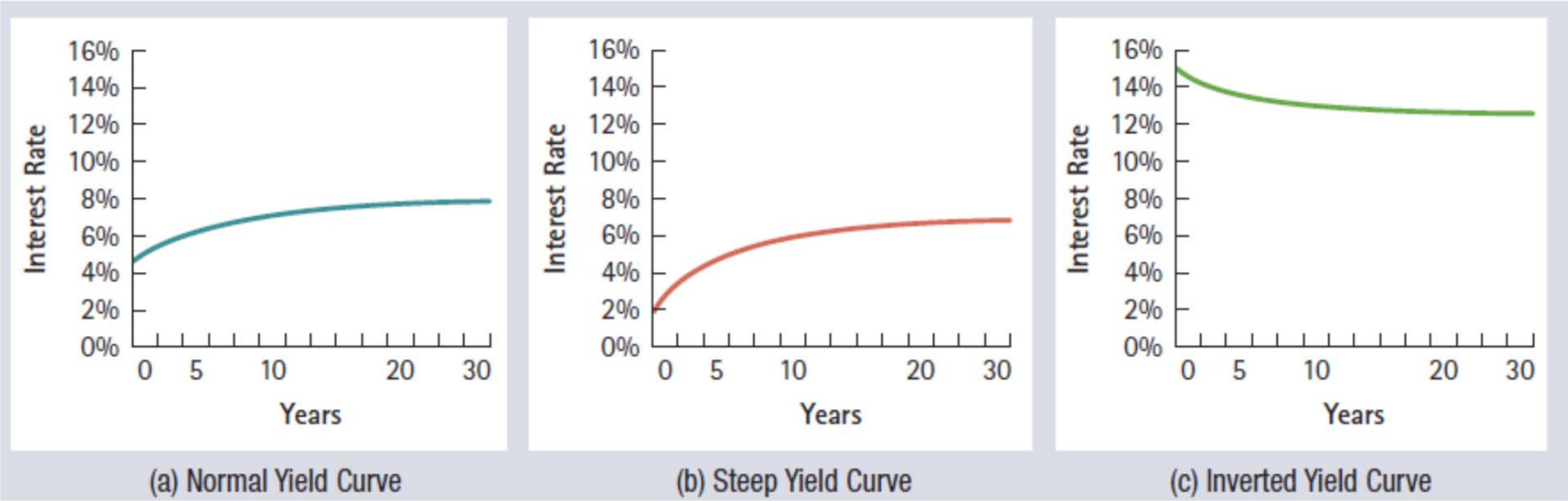
- Interest Determination
  - The Federal Reserve determines very short-term interest rates through its influence on the **federal funds rate**, which is the rate at which banks can borrow cash reserves on an overnight basis.
  - All other interest rates on the yield curve are set in the market and are adjusted until the supply of lending matches the demand for borrowing at each loan term.

Federal Reserve Chairman
Jerome Powel

Former Federal Reserve Chair
Janet Yellen

The Yield Curve and the Economy (2 of 2)

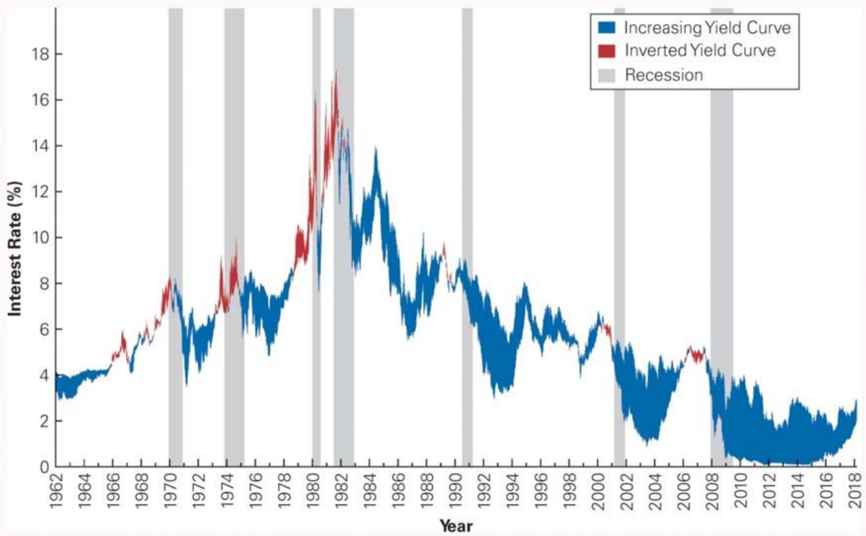
- Interest Rate Expectations
  - The shape of the yield curve is influenced by interest rate expectations.



The Yield Curve and the Economy (2 of 2)

- Interest Rate Expectations
  - The shape of the yield curve is influenced by interest rate expectations.
    - An inverted yield curve indicates that interest rates are expected to decline in the future.
      - Because interest rates tend to fall in response to an economic slowdown, an inverted yield curve is often interpreted as a negative forecast for economic growth.
        - Each of the last six recessions in the United States was preceded by a period in which the yield curve was inverted.
        - The yield curve tends to be sharply increasing as the economy comes out of a recession, and interest rates are expected to rise.

Figure 5.3 Short-Term Versus Long-Term U.S. Interest Rates and Recessions



One-year and ten-year U.S. Treasury rates are plotted, with the spread between them shaded in blue if the shape of the yield curve is increasing (the one-year rate is below the ten-year rate) and in red if the yield curve is inverted (the one year rate exceeds the ten-year rate). Gray bars show the dates of U.S. recessions as determined by the National Bureau of Economic Research. Note that inverted yield curves tend to precede recessions by 12–18 months. In recessions, interest rates tend to fall, with short-term rates dropping further. As a result, the yield curve tends to be steep coming out of a recession.

Textbook Example 5.6 (1 of 4)

Comparing Short- and Long-Term Interest Rates

- **Problem**
  - Suppose the current one-year interest rate is 1%. If it is known with certainty that the one-year interest rate will be 2% next year and 4% the following year, what will the interest rates $r_1, r_2,$ and r_3 of the yield curve be today? Is the yield curve flat, increasing, or inverted?

Textbook Example 5.6 (2 of 4)

Solution

We are told already that the one-year rate $r_1 = 1\%$.

To find the two-year rate, note that if we invest $1 for one-year at the current one-year rate and then reinvest next year at the new one-year rate, after two-years we will earn:

$$\$1 \times (1.01) \times (1.02) = \$1.0302$$

We should earn the same payoff if we invest for two-years at the current two-year rate r_2:

$$\$1 \times \left(1 + r_2\right)^2 = \$1.0302$$

Textbook Example 5.6 (3 of 4)

Otherwise, there would be an arbitrage opportunity: if investing at the two-year rate led to a higher payoff, investors could invest for two-years and borrow each year at the one-year rate. Investing at the two-year rate could led to a lower payoff. Investors could invest each year at the one-year rate and borrow at the two-year rate.

Solving for r_2, we find that

$$r_2 = (1.032)^{\frac{1}{2}} - 1 = 1.499\%$$

Textbook Example 5.6 (4 of 4)

Similarly, investing for three years at the one-year rates should have the same payoff as investing at the current three-year rate:

$$(1.01)\times(1.02)\times(1.04) = 1.0714 = \left(1 + r^{3}\right)^{3}$$

We can solve for $r_3 = (1.0714)^{\frac{1}{3}} - 1 = 2.326\%.$

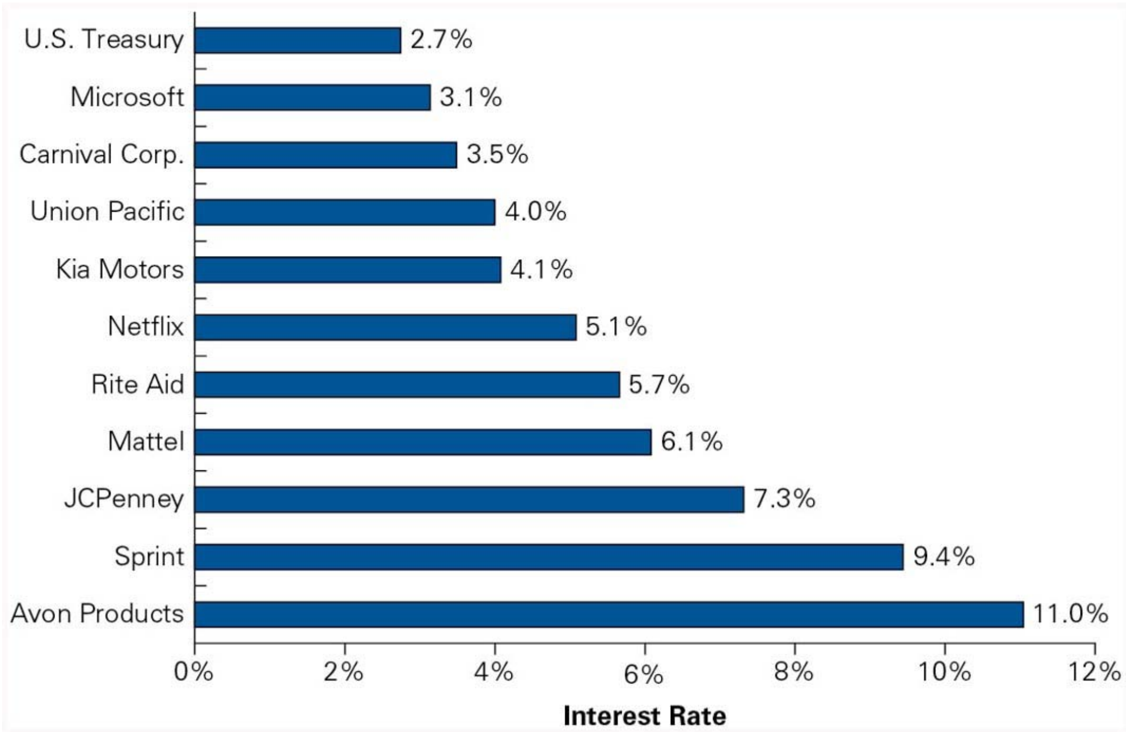
Therefore, the current yield curve has $r_1 = 1\%, r_2 = 1.499\%,$

and $r_3 = 2.326\%$ The yield curve is increasing as a result of the anticipated higher interest rates in the future.

5.4 Risk and Taxes

- Risk and Interest Rates
  - U.S. Treasury securities are considered “risk-free.” All other borrowers have some risk of default, so investors require a higher rate of return.

Figure 5.4 Interest Rates on Five-Year Loans for Various Borrowers, July 2018



Source: FINRA.org.

Pearson

Textbook Example 5.7 (1 of 2)

Discounting Risky Cash Flows

- **Problem**
  - Suppose the U.S. government owes your firm $1,000 to be paid in five years. Based on the interest rates in Figure 5.4, what is the present value of this cash flow? Suppose instead JC Penney owes your firm $1,000. Estimate the present value in this case.

Textbook Example 5.7 (2 of 2)

Solution

Assuming we can regard the government's obligation as risk free (there is no chance you won't be paid), then we discount the cash flow using the risk-free Treasury interest rate of 2.7%:

$$PV = \$1000 \div (1.027)^5 = \$875.28$$

The obligation from JC Penney is not risk-free. JCPenney may face financial difficulties and fail to pay the $1,000. Because the risk of this obligation is likely to be comparable to the five-year bond quoted in Figure 5.4, the 7.3% interest rate of the loan is a more appropriate discount rate to use to compute the present value in this case:

$$PV = \$1000 \div (1.073)^5 = \$703.07$$

Note the substantially lower present value of JC Penney's debt compared to the government debt due to JC Penney's higher risk of default.

After-Tax Interest Rates

Taxes reduce the amount of interest an investor can keep, and we refer to this reduced amount as the **after-tax interest rate**.

$$r - (\tau \times r) = r(1 - \tau)$$

Textbook Example 5.8 (1 of 3)

Comparing After-tax Interest Rates

- **Problem**
  - Suppose you have a credit card with a 14% APR with monthly compounding, a bank savings account paying 5% EAR, and a home equity loan with a 7% APR with monthly compounding. Your income tax rate is 40%. The interest on the savings account is taxable, and the interest on the home equity loan is tax deductible. What is the effective after-tax interest rate of each instrument, expressed as an EAR? Suppose you are purchasing a new car and are offered a car loan with a 4.8% APR and monthly compounding (which is not tax deductible). Should you take the car loan?

Textbook Example 5.8 (2 of 3)

Solution

Because taxes are typically paid annually, we first convert each interest rate to an EAR to determine the actual amount of interest earned or paid during the year. The savings account has a 5% EAR. Using Eq. 5.3, the EAR of the credit card is

$$\left(1+\frac{0.14}{12}\right)^{12}-1=14.93\%,$$

and the EAR of the home equity loan is $\left(1+\frac{0.07}{12}\right)^{12}-1=7.23\%.$

Next, we compute the after-tax rate for each. Because the credit card interest is not tax deductible, its after after-tax interest rate is the same as its pre-tax interest rate, 14.93%. The after-tax interest rate on the home equity loan ,which is tax deductible, is $7.23\% \times (1-0.40) = 4.34\%.$

The after-tax interest rate that we will earn on the savings account is $5\% \times (1-0.40) = 3\%.$

Textbook Example 5.8 (3 of 3)

Now consider the car loan. Its EAR is

$$\left(1+\frac{0.048}{12}\right)^{12}-1=4.91\%.$$

It is not tax deductible, so this rate is also its after-tax interest rate. Therefore, the car loan is not our cheapest source of funds. It would be best to use savings, which has an opportunity cost of foregone after-tax interest of 3%. If we don't have sufficient savings, we should use the home equity loan, which has an after-tax cost of 4.34%. And we should certainly not borrow using the credit card!

5.5 The Opportunity Cost of Capital

- **Investor's Opportunity Cost of Capital:** The best available expected return offered in the market on an investment of comparable risk and term to the cash flow being discounted
  - Also referred to as **Cost of Capital**