Performance Modeling and Optimization of the `bicg` SPAPT kernel

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1 Introduction

The objective of this report is to analyze data from our CCGRID paper, and perform new experiments as needed, in order to better understand the statiscal analysis and the performance achieved by our DoE-based approach. We would like to understand how a user could interfere and guide search space restriction and exploration, by intervening at each ANOVA step, for example.

We are also interested in exploring new design construction metrics, in order to understand which factors are best estimated at each step and possibly discard designs not capable of estimating parameters of interest.

We chose to start with the `bicg` SPAPT kernel. It was one of the kernels where our approach achieved similar-performing solutions using less experiments, in comparison with a uniform random sampling approach, across 10 repetitions of an experiment with a fixed budget of 400 experiments and a limit of 4 iterations, or steps, of our approach.

The following sections describe our ongoing exploration of the performance modeling and optimization of the `bicg` kernel.

2 Removing Cubic Terms

2.1 Iterations that found the Best Configuration

Figure 1 shows the iterations where the best configuration of each of the 10 runs was found, for uniform sampling and for our approach.

The results are the same as the ones presented in the paper, but the data are not the same. The data for the figures in this report were obtained with a new set of 10 repetitions the same experiment, but using a performance model with only linear and quadratic terms, removing the cubic terms used in the paper's experiments.

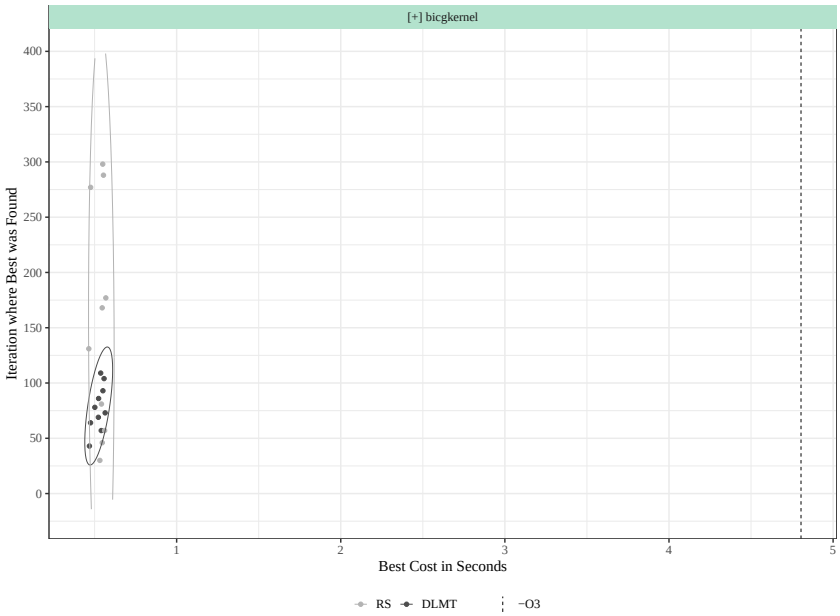

Figure 1: Iteration where best was found

The results are very similar from the ones in the paper, but our approach found slightly better configurations slightly faster. Random sampling also found a much better configuration much faster than before in one experiment.

Table 1:

experiment_id	technique	step	cost_mean	T1_I	T1_J	U_J	U_I	T2_I	T2_J	U1_I	RT_I	RT_J	OMP	SCR	VEC1	VEC2
parasilo-1	DLMT	4	0.5244322	2048	64	1	30	1	1	25	1	1	True	True	False	True
parasilo-10	DLMT	4	0.55769	8	2	1	9	2048	2048	2	4	32	True	True	False	True
parasilo-11	DLMT	3	0.4753558	2048	1024	30	1	1	1024	16	4	4	True	True	False	False
parasilo-13	DLMT	3	0.5016148	1	256	21	1	2048	256	1	8	4	True	False	False	False
parasilo-14	DLMT	3	0.5404449	2048	1	1	13	2048	2048	4	8	4	True	True	False	True
parasilo-17	DLMT	4	0.5371861	2048	1	29	1	2048	1024	7	4	8	True	True	True	False
parasilo-19	DLMT	4	0.5507874	2048	2	1	13	2048	128	15	1	1	True	True	True	True
parasilo-2	DLMT	3	0.5229024	1024	1	1	1	2048	1024	2	8	1	True	True	True	False
parasilo-20	DLMT	3	0.5643309	1	2048	1	12	2048	2048	4	1	8	True	True	False	True
parasilo-9	DLMT	2	0.4685077	2048	128	1	16	1	256	19	1	1	True	True	True	True
paravance-50	RS	NA	0.5685274	512	512	19	1	2048	1024	27	4	1	True	True	True	False
paravance-51	RS	NA	0.5467725	1024	32	10	1	2048	512	9	8	8	True	True	True	True
paravance-52	RS	NA	0.5417953	2048	8	9	1	2048	128	30	8	16	True	True	True	True
paravance-53	RS	NA	0.5612387	2048	64	16	1	2048	256	9	16	2	True	True	True	True
paravance-54	RS	NA	0.5322531	2048	8	1	9	1	1	19	1	1	True	True	True	False
paravance-55	RS	NA	0.5540684	32	4	1	5	2048	256	4	16	1	True	True	False	False
paravance-56	RS	NA	0.4648199	2048	1	1	23	1	512	25	8	1	True	True	False	True
paravance-57	RS	NA	0.5463944	256	16	1	15	2048	512	2	2	2	True	True	False	False
paravance-58	RS	NA	0.5496127	32	1	1	14	2048	2048	24	8	4	True	True	True	True
paravance-59	RS	NA	0.4758884	64	16	29	1	2048	1	27	16	4	True	True	True	True

2.2 Eliminated Factors and Fixed Values

Figure 2 shows the *count of model terms* that were eliminated at each color-coded DLMT step. As before, we see that `OMP` and `SCR` were the most eliminated factors, especially on the first step.

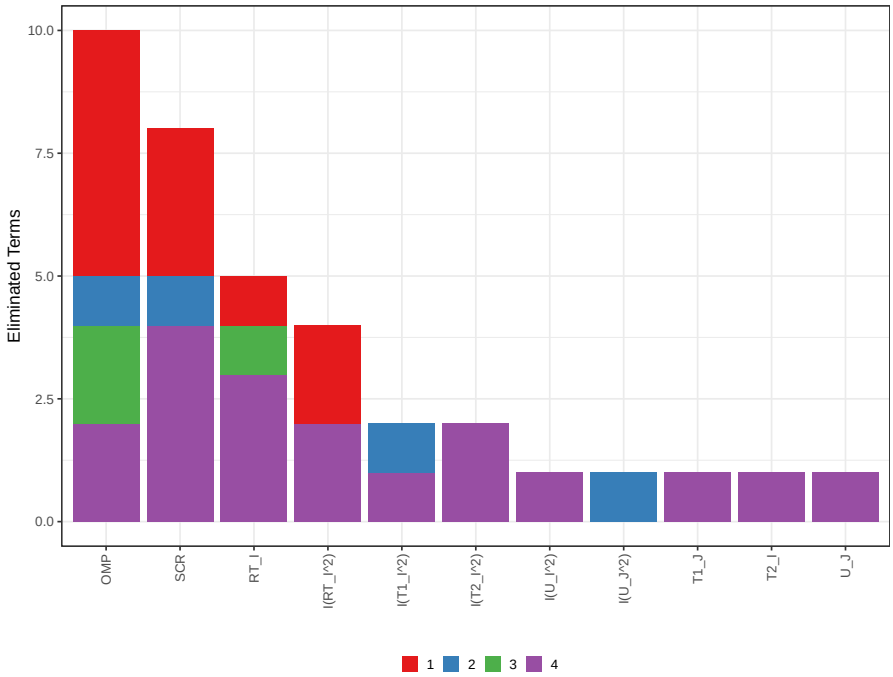

Figure 2: Eliminated terms

2.3 Fixed Factors & Values in the Explored Spaces

Figure 3 shows the factors that were identified as significant by ANOVA, with significance threshold of 0.01, at each DLMT step. Identifying a factor removes the corresponding parameter from the model, regardless of the model term that was identified. Figures are grouped by each of the 10 experiments. Figure headers identify each run, and correspond to the names of the Grid5000 machines where experiments were run.

It is interesting that only *parasilo-19*, *parasilo-20*, and *parasilo-9* eliminated any factor other than `OMP` and `SCR` on the first step where any factor was eliminated, and also that those runs fixed the factor `RT_I` to the same value.

We can also see that `OMP` seems to be the parameter behind the most extreme changes in the execution time of tested configurations. This is specially clear at *parasilo-13* and *parasilo-10*, where the explored configurations have relatively high execution time until after step 3, where `OMP` is fixed.

A very slight worsening of execution times, an increase, that is, can be seen at the fourth step at *parasilo-11*, after `RT_I` was fixed. The minimum execution time seems to be higher than in the third step.

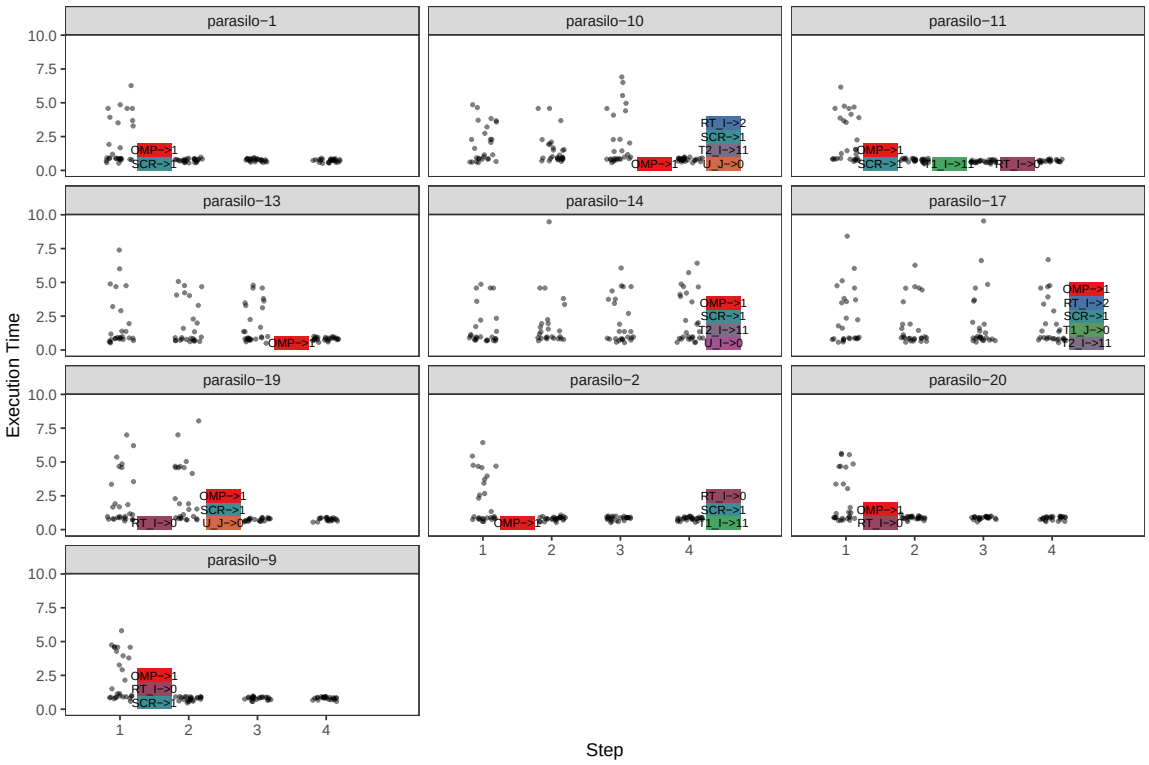

Figure 3: Execution time of Searched Space, step-by-step, by experiment

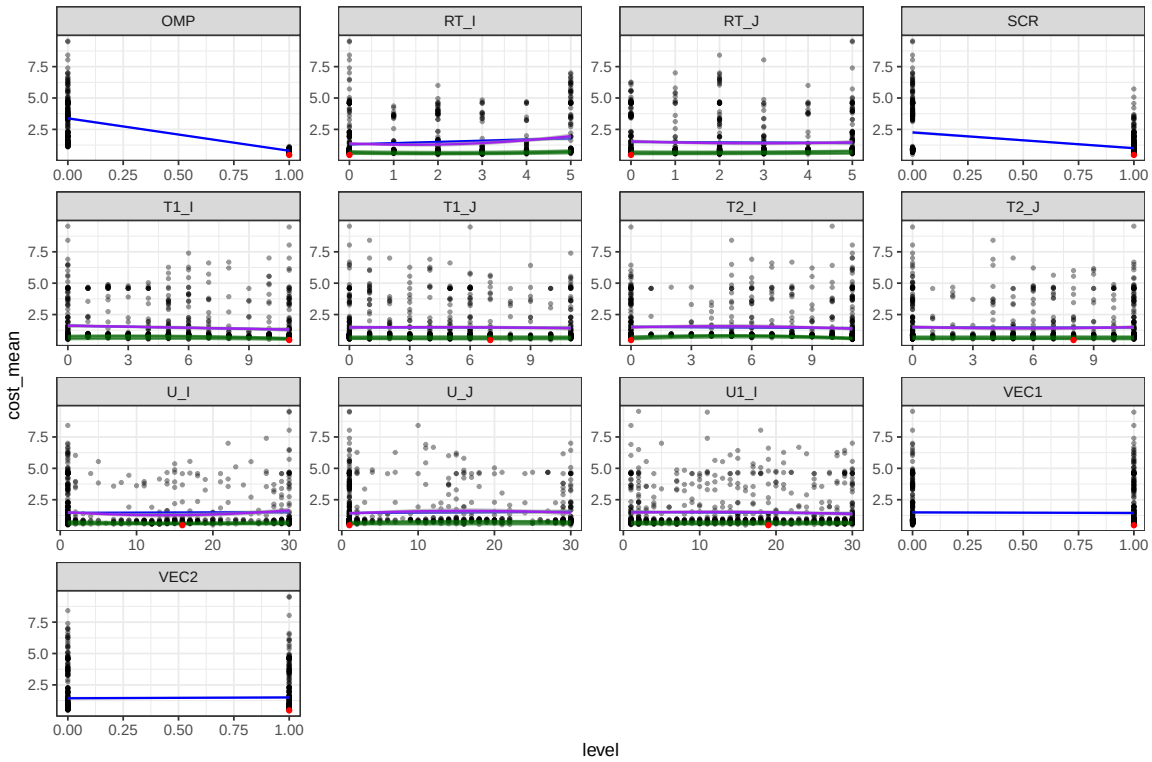

Figure 4: Fitting `lm` linear and cubic models to separate factors for the DLMT runs

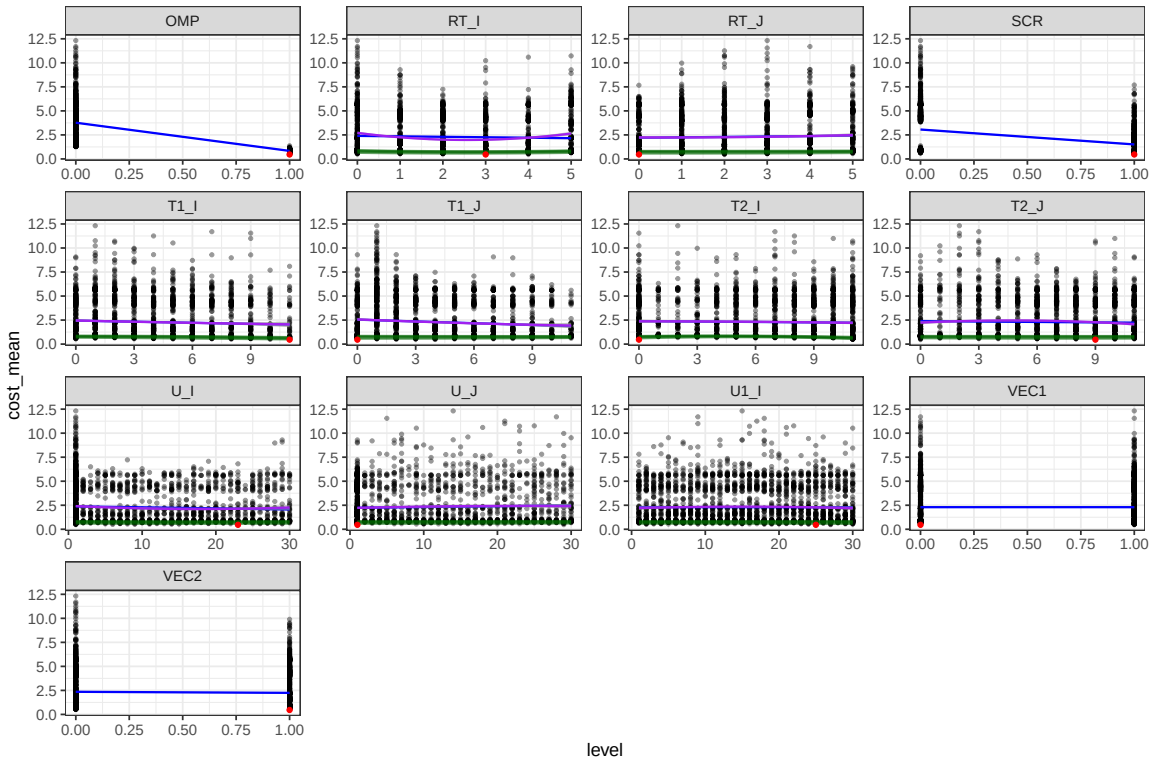

Figure 5: Fitting `lm` linear and cubic models to separate factors for the RS runs

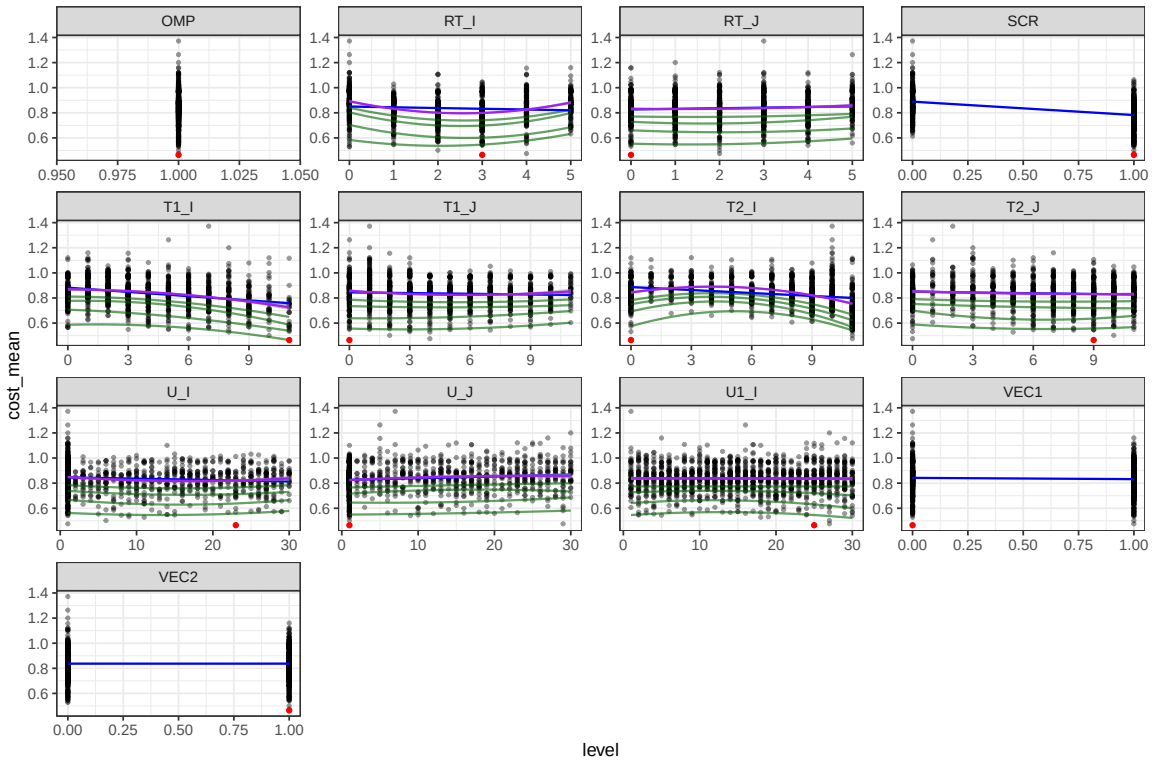

Figure 6: Fitting `lm` linear and cubic models to separate factors for the RS runs, with `OMP` fixed to `true`

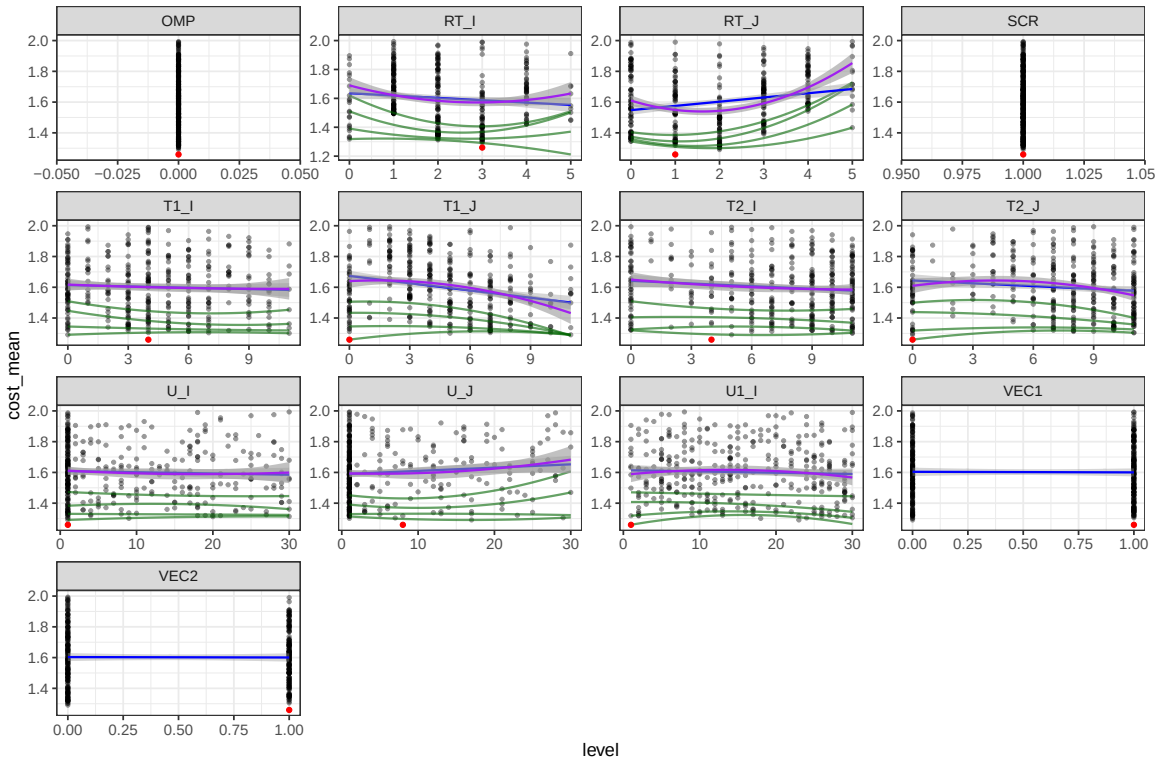

Figure 7: Fitting `lm` linear and cubic models to separate factors for the RS runs, with `OMP` fixed to `false`

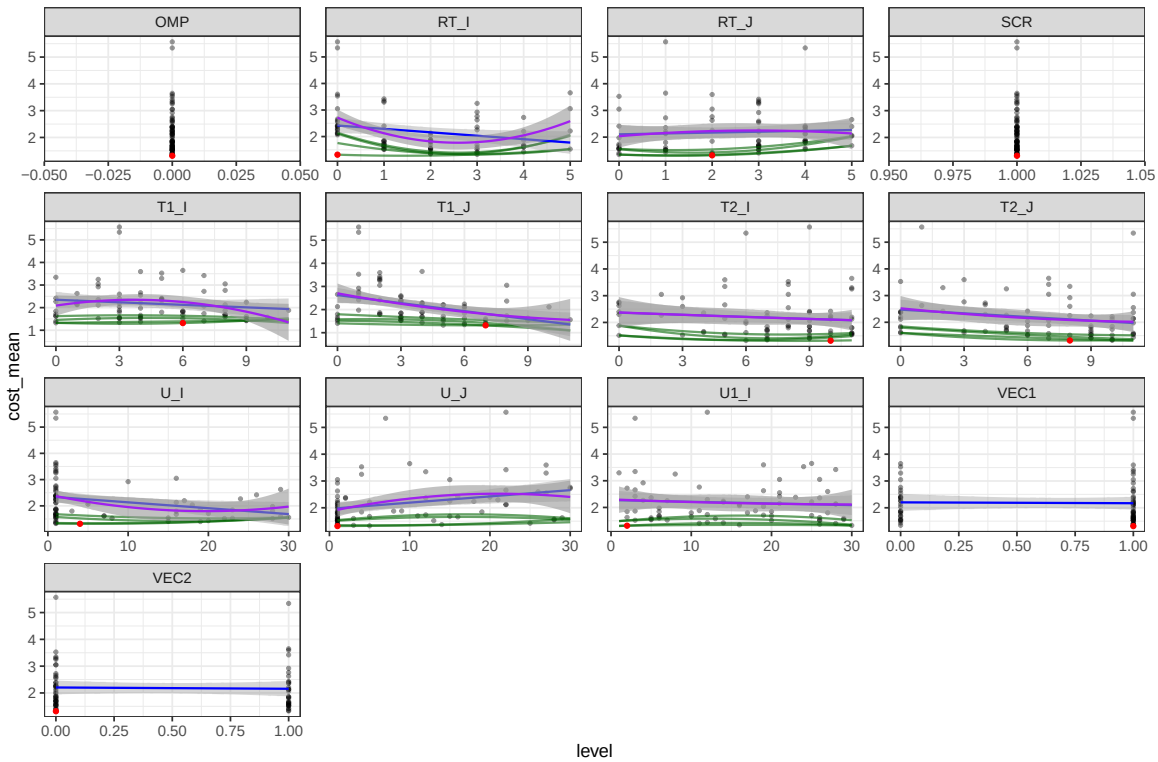

Figure 8: Fitting `lm` linear and cubic models to separate factors for the RS runs, with `OMP` fixed to `true`, for `paravance-58`

2.4 Predicted Performance at Each Step

Figure 9 compares the performance *predicted by the fitted model*, represented by the green line, with the performance of *the best point found at each design*, represented by the blue line. The red line marks the best point found so far.

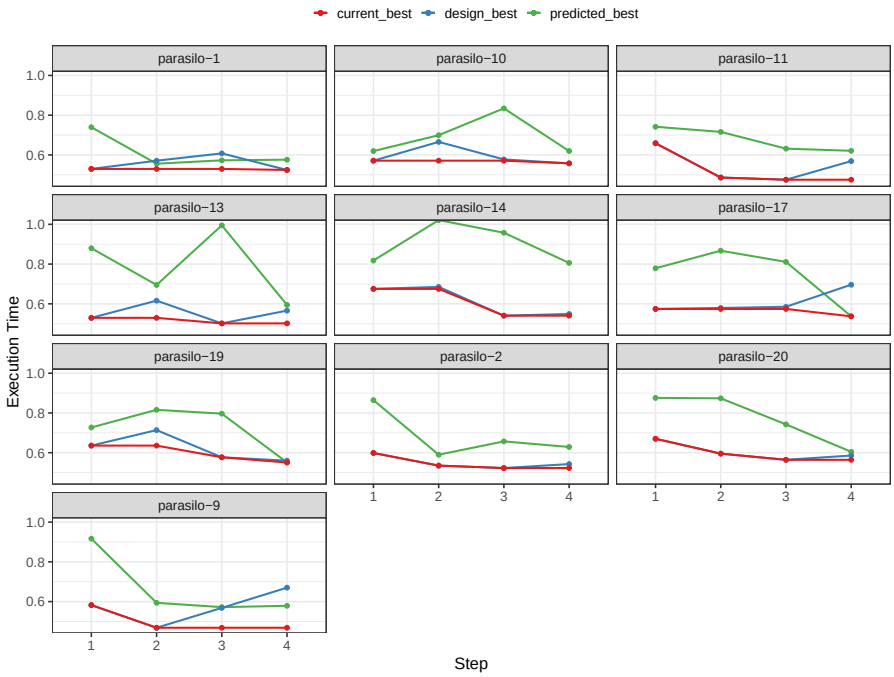

Figure 9: Performance of best points, step-by-step, by experiment

It is only at the fourth step of *parasilo-17* that the point with the best predicted performance was better than the best point on the design for that step, while also being better than the best point found so far. Although we seem to be effectively restricting the search space with our exploration, which is evidenced by the improvement that occurs as steps progress and by the best points being found inside designs, the models fit using experiment data are not able to improve the current best on the majority of cases.

2.5 D-Optimality

2.5.1 D-Criterion

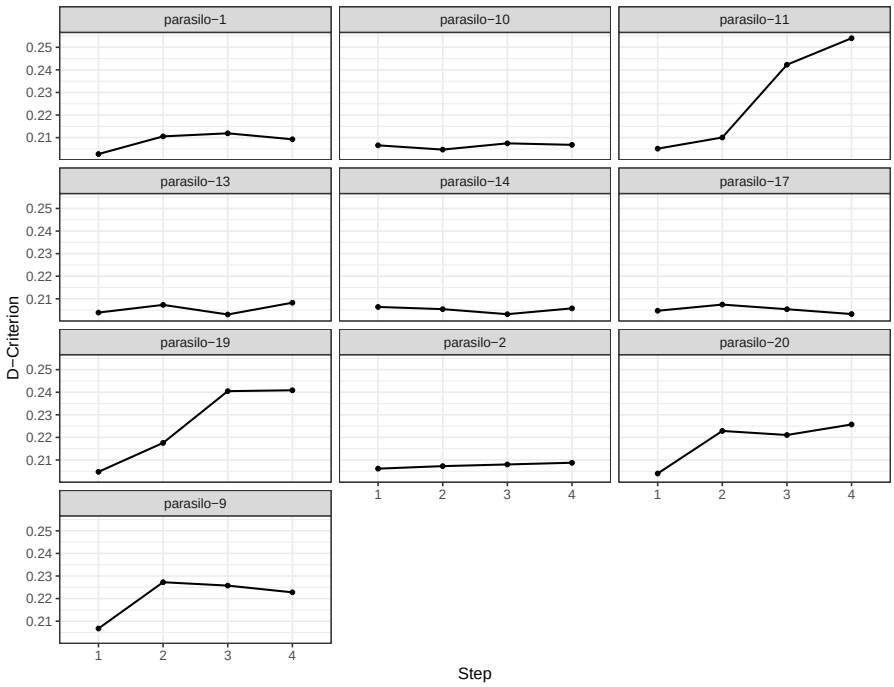

Figure 10: D-Criterion, step-by-step, by experiment

3 Reuse Data for `aov` and `lm`

Significant changes were performed on the initial DLMT implementation. We decided that there was no good reason to not reuse the data obtained from evaluating designs at each step, and the various samples of the search space taken at different points. Now, all evaluated experiments compose a single growing design used by `aov` to identify the best factors, and all samples from the search space compose a single data set used by `optFederov` to select experiments. The data set is pruned in both `aov` and `lm` analyses, to guarantee only experiments with the correct levels of fixed factors are used. This is crucial for both analyses, since having different fixed values of factors that are not in the model would imply that we would have inexplicable variance in the data set.

Using all experimental data on `aov` is interesting because it is always worth it to consider additional information on the factors that are currently being studied. On one hand, it might not allow for enough "flexibility" when we consider regression only on a small restricted subspace, because points outside the subspace would impact regression, and we would be interested in the significance of the factor inside the subspace at the moment. On the other hand, using all data available makes sense because we are also interested in exploring any global structures of the search space, and keeping points from other subspaces would increase the chance of "catching" the significance of a factor globally.

Using all sampled space, across all steps, as a candidate for `optFederov` has no downsides, provided we prune the search space to account for current constraints on factor levels fixed on previous steps. We increase the size of the available set of configurations that can compose a new design each time we sample a new subspace. This would hopefully improve the quality of designs produced as we progress.

3.1 Iterations that found the Best Configuration

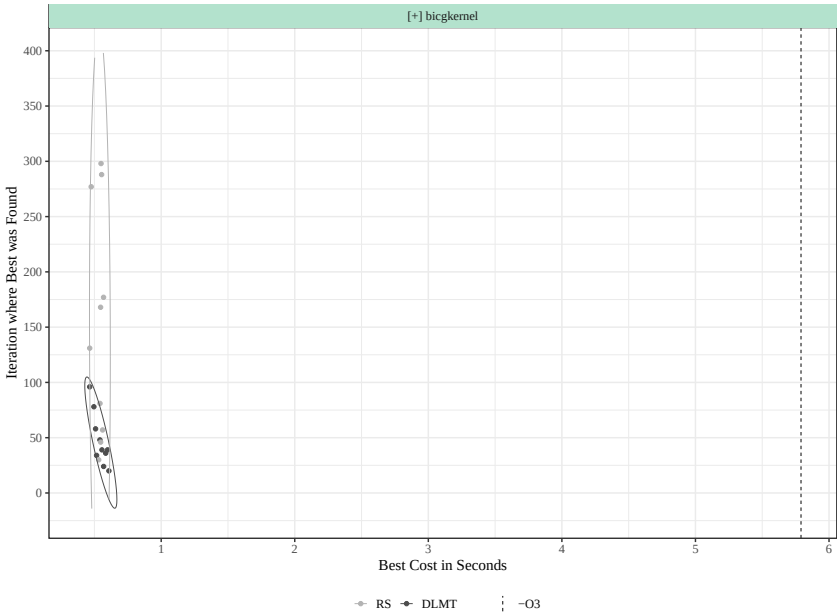

Figure 11: Iteration where best was found

Table 2:

experiment_id	technique	step	cost_mean	T1_I	T1_J	U_J	U_I	T2_I	T2_J	U1_I	RT_I	RT_J
graoully-13	DLMT	1	0.6084833	512	1	1	10	2048	2048	2	8	2
graoully-15	DLMT	2	0.5412621	512	16	1	21	2048	1	1	2	32
graoully-16	DLMT	1	0.5691611	2048	512	1	1	1	1024	5	8	8
graoully-3	DLMT	2	0.5163602	2048	2048	1	28	1	2048	30	4	4
graoully-4	DLMT	2	0.5845903	16	32	1	30	2048	1	8	2	4
graoully-5	DLMT	4	0.4961376	1024	2048	30	1	2048	2048	13	2	4
graoully-6	DLMT	2	0.5985619	2048	32	1	3	2048	256	16	8	1
graoully-7	DLMT	2	0.5565135	64	2048	1	18	2048	2048	1	4	8
graoully-8	DLMT	4	0.4646318	2048	2	1	2	1	1	30	4	4
graoully-9	DLMT	3	0.5087977	2048	2048	1	11	2048	2048	1	4	4
paravance-50	RS	NA	0.5685274	512	512	19	1	2048	1024	27	4	1
paravance-51	RS	NA	0.5467725	1024	32	10	1	2048	512	9	8	8
paravance-52	RS	NA	0.5417953	2048	8	9	1	2048	128	30	8	16
paravance-53	RS	NA	0.5612387	2048	64	16	1	2048	256	9	16	2
paravance-54	RS	NA	0.5322531	2048	8	1	9	1	1	19	1	1
paravance-55	RS	NA	0.5540684	32	4	1	5	2048	256	4	16	1
paravance-56	RS	NA	0.4648199	2048	1	1	23	1	512	25	8	1
paravance-57	RS	NA	0.5463944	256	16	1	15	2048	512	2	2	2
paravance-58	RS	NA	0.5496127	32	1	1	14	2048	2048	24	8	4
paravance-59	RS	NA	0.4758884	64	16	29	1	2048	1	27	16	4

3.2 Eliminated Factors and Fixed Values

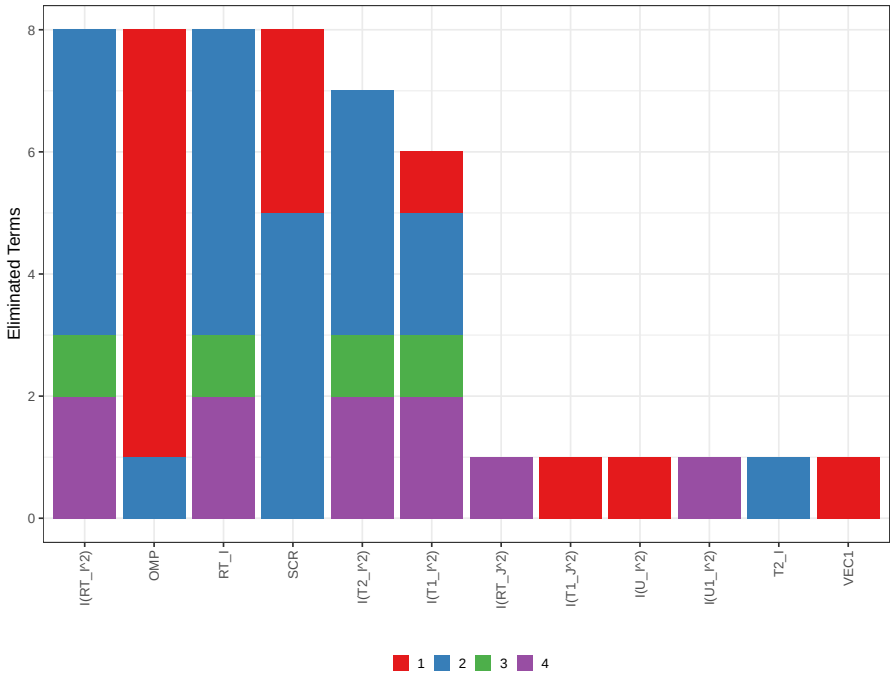

Figure 12: Eliminated terms

3.3 Fixed Factors & Values in the Explored Spaces

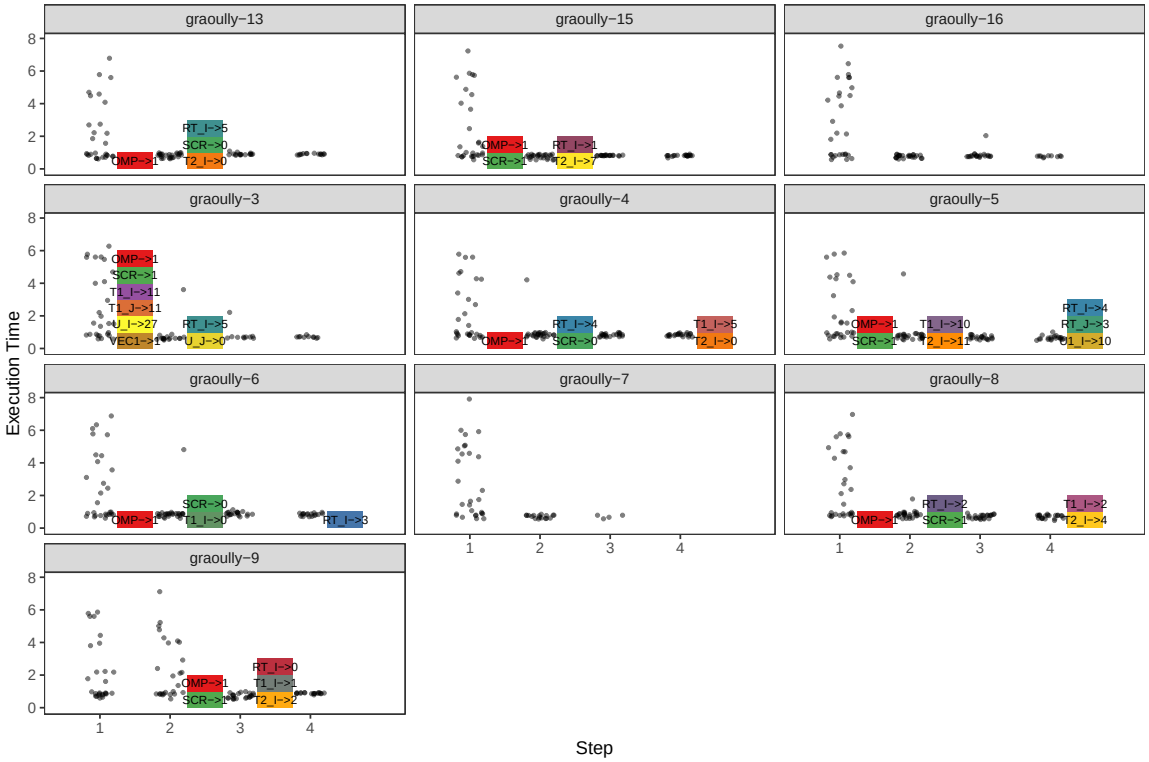

Figure 13: Execution time of Searched Space, step-by-step, by experiment

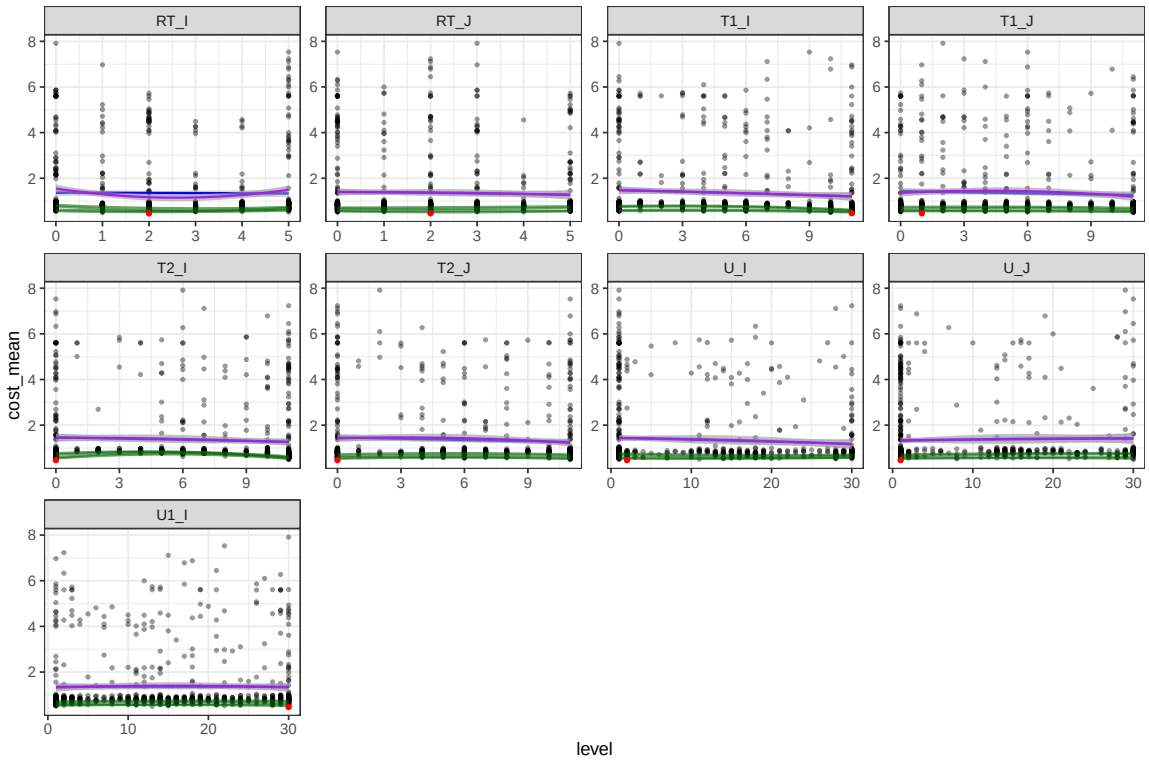

Figure 14: Fitting `lm` linear and cubic models to separate factors for the DLMT runs

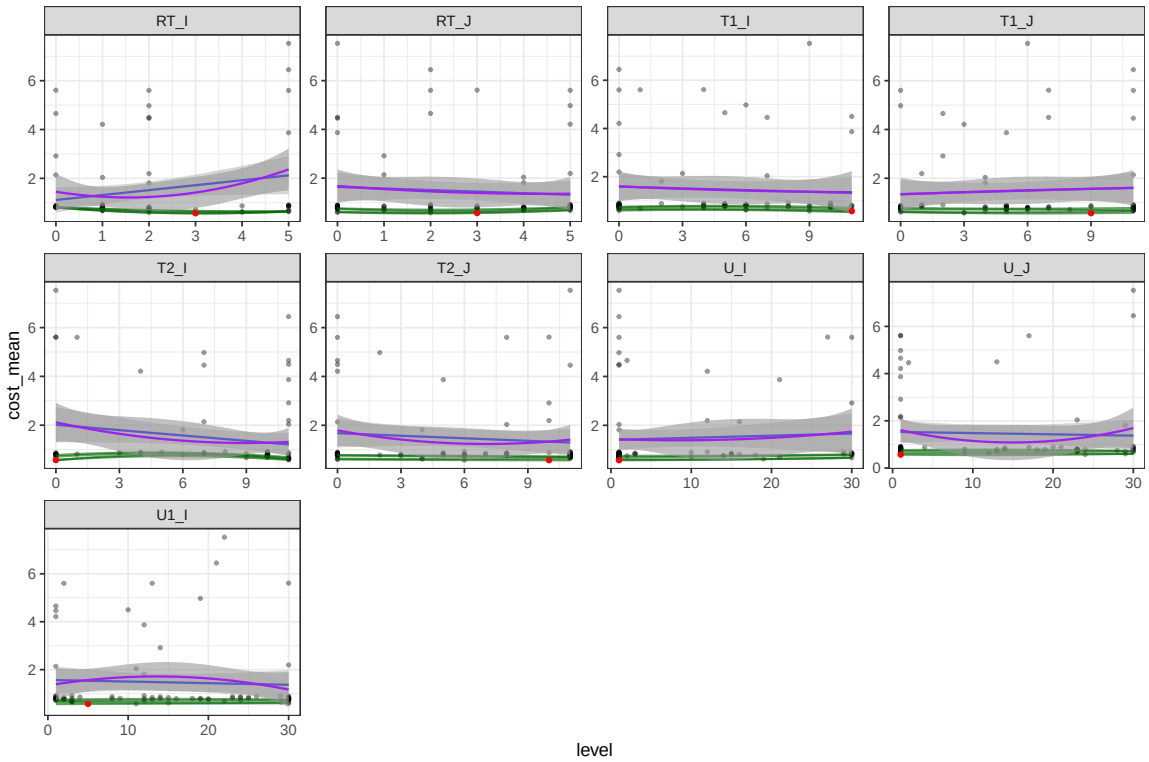

Figure 15: Fitting `lm` linear and cubic models to separate factors for the DLMT runs, for `graoully-16`

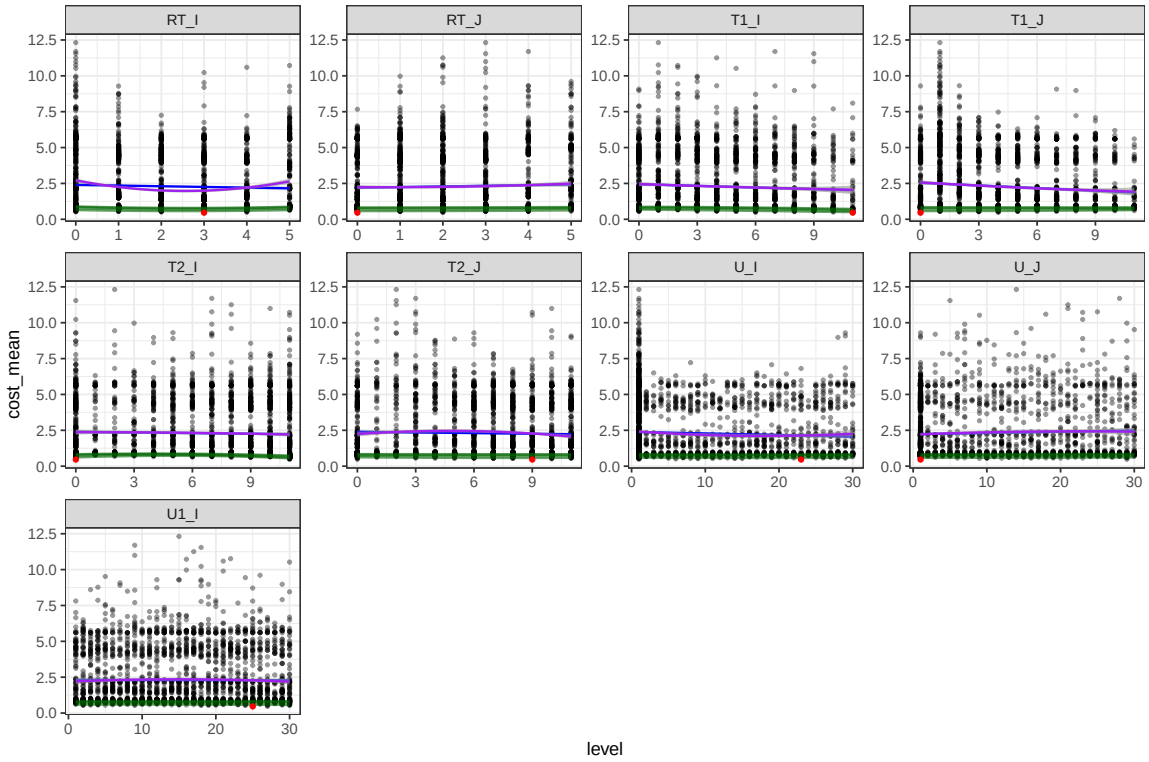

Figure 16: Fitting `lm` linear and cubic models to separate factors for the RS runs

3.4 Predicted Performance at Each Step

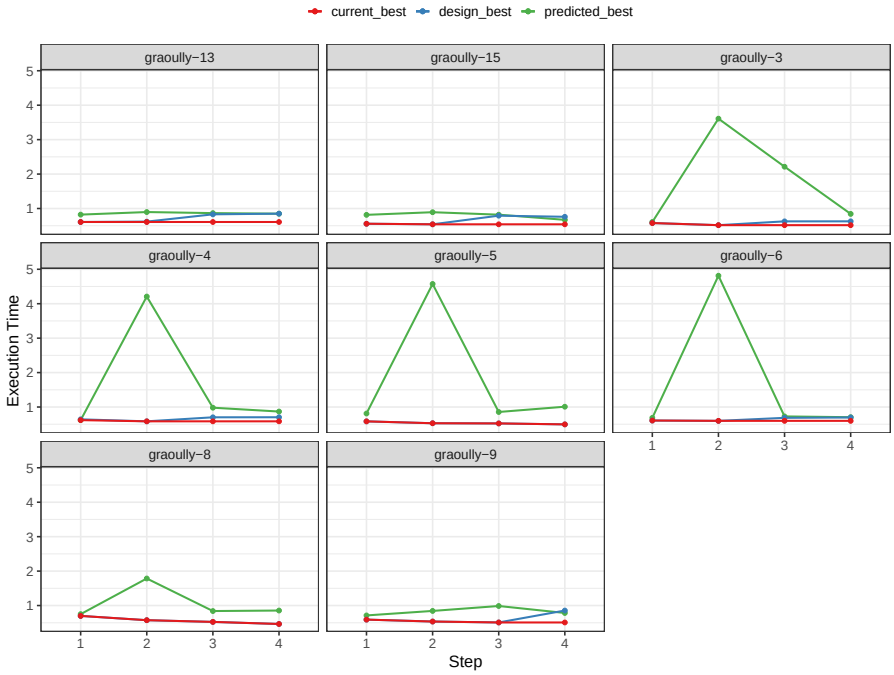

Figure 17: Performance of best points, step-by-step, by experiment

3.5 D-Optimality

3.5.1 D-Criterion

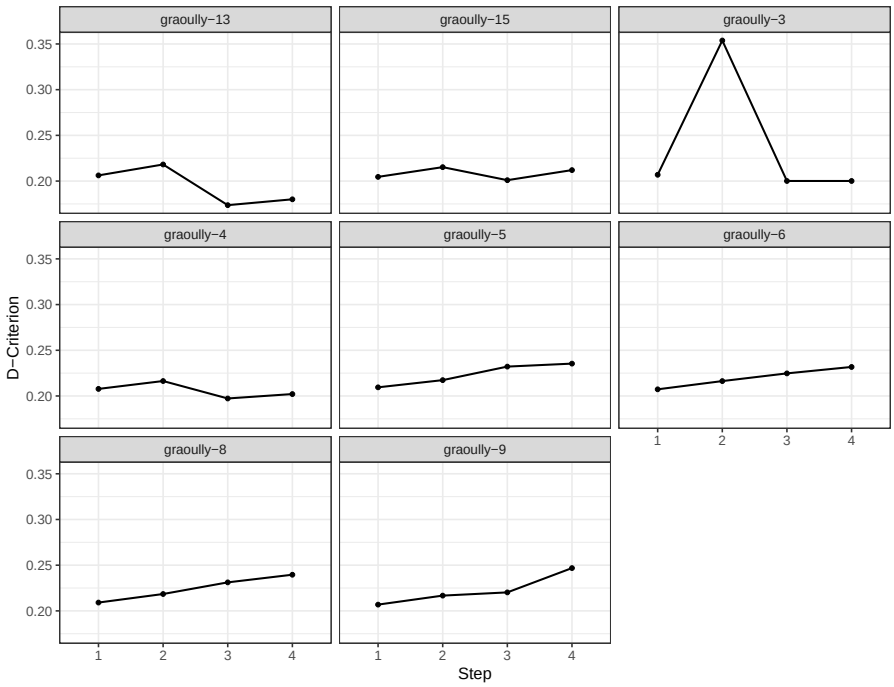

Figure 18: D-Criterion, step-by-step, by experiment

4 Reuse Data for `aov` and `lm`: Binary set to `False`

This experiment uses an initial model with linear and quadratic terms for all factors. All binary parameters were fixed to its `False` value. We hoped to determine if the effects of binary factors, which are proportionally

much larger than the effects of the other factors, were preventing the detection of the significance of the other factors.

4.1 Iterations that found the Best Configuration

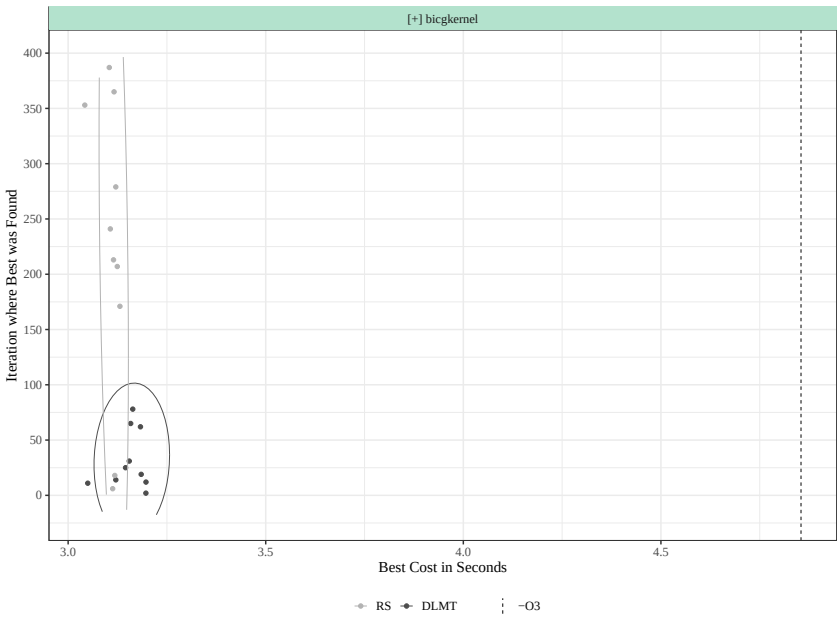

Figure 19: Iteration where best was found

Table 3:

experiment_id	technique	step	cost_mean	T1_I	T1_J	U_J	U_I	T2_I	T2_J	U1_I	RT_I	RT_J
paravance-11	DLMT	1	3.184848	512	512	1	15	512	1	30	1	4
paravance-14	DLMT	1	3.196773	256	1	1	18	2048	64	30	1	8
paravance-19	DLMT	1	3.04968	1	2	1	15	1	128	21	4	1
paravance-25	DLMT	3	3.183022	1	1	1	14	128	2048	30	4	2
paravance-26	DLMT	2	3.154706	32	2048	1	29	2048	2048	2	16	8
paravance-27	DLMT	2	3.145299	1	1	1	30	256	2048	1	16	2
paravance-28	DLMT	1	3.120673	1	2048	1	30	2048	2048	30	16	4
paravance-29	DLMT	4	3.16354	32	1	1	1	2048	1	14	4	2
paravance-3	DLMT	3	3.158316	128	1	1	1	128	1	28	4	2
paravance-32	DLMT	1	3.197007	32	64	1	10	32	2048	1	1	2
paravance-16	RS	1	3.118132	1	256	25	1	32	512	20	16	4
paravance-18	RS	1	3.042194	1	2	1	19	1	256	22	4	1
paravance-2	RS	1	3.131061	1	1024	1	6	256	2048	20	16	2
paravance-20	RS	1	3.120612	256	1024	13	1	1	1024	26	16	2
paravance-21	RS	1	3.112708	32	1	30	1	64	512	20	16	2
paravance-22	RS	1	3.124333	256	1	1	12	256	1024	28	8	2
paravance-23	RS	1	3.11485	512	512	25	1	2048	2048	3	16	4
paravance-24	RS	1	3.106786	128	1	12	1	512	1	23	8	2
paravance-30	RS	1	3.104083	32	2048	12	1	1	2048	8	16	2
paravance-31	RS	1	3.116196	2048	256	1	19	2048	1	24	8	2

4.2 Eliminated Factors and Fixed Values

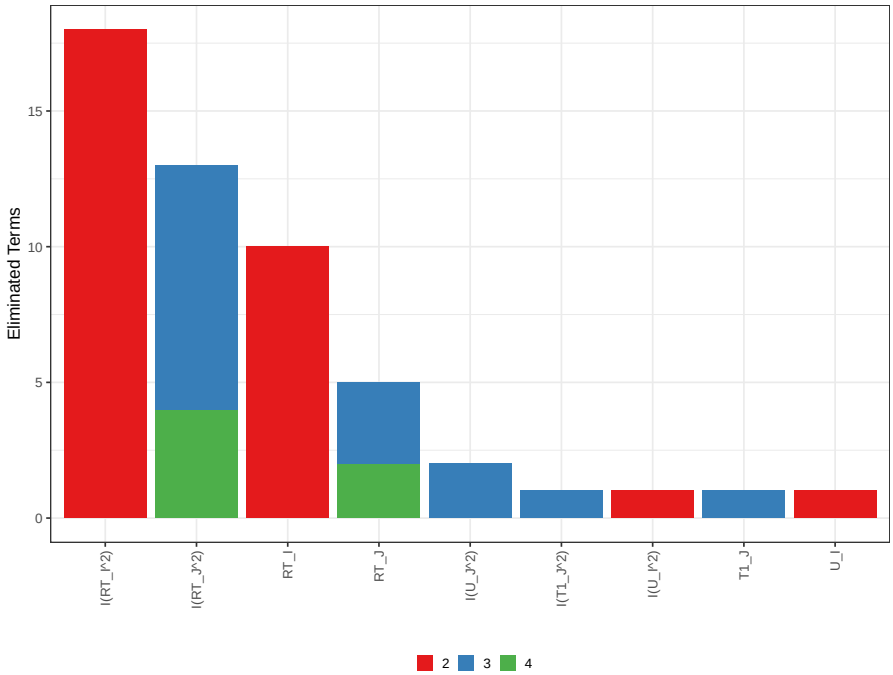

Figure 20: Eliminated terms

4.3 Fixed Factors & Values in the Explored Spaces

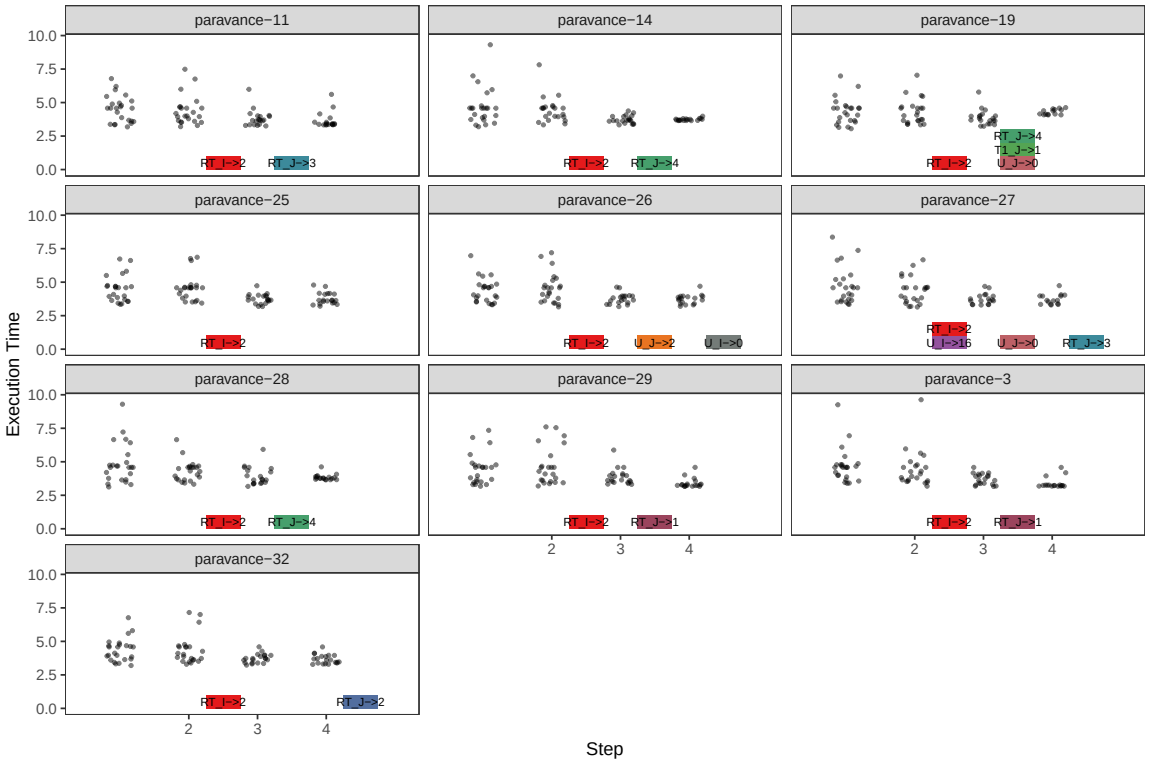

Figure 21: Execution time of Searched Space, step-by-step, by experiment

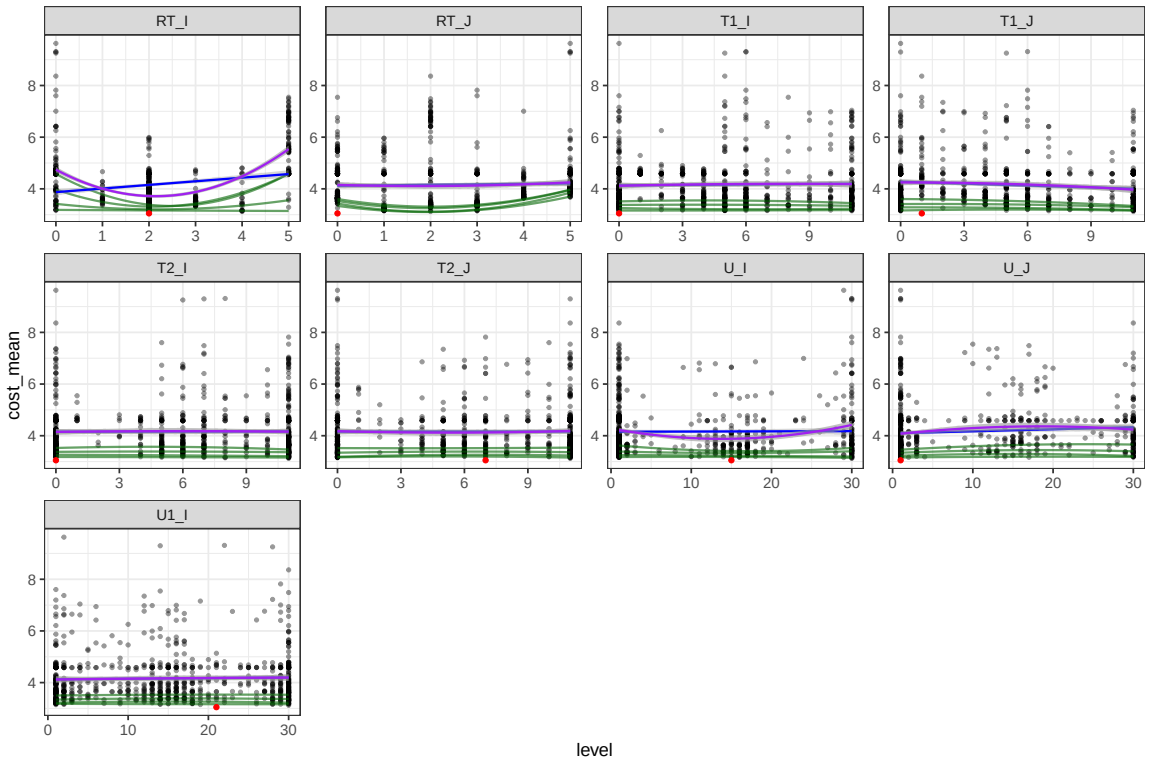

Figure 22: Fitting `lm` linear and cubic models to separate factors for the DLMT runs

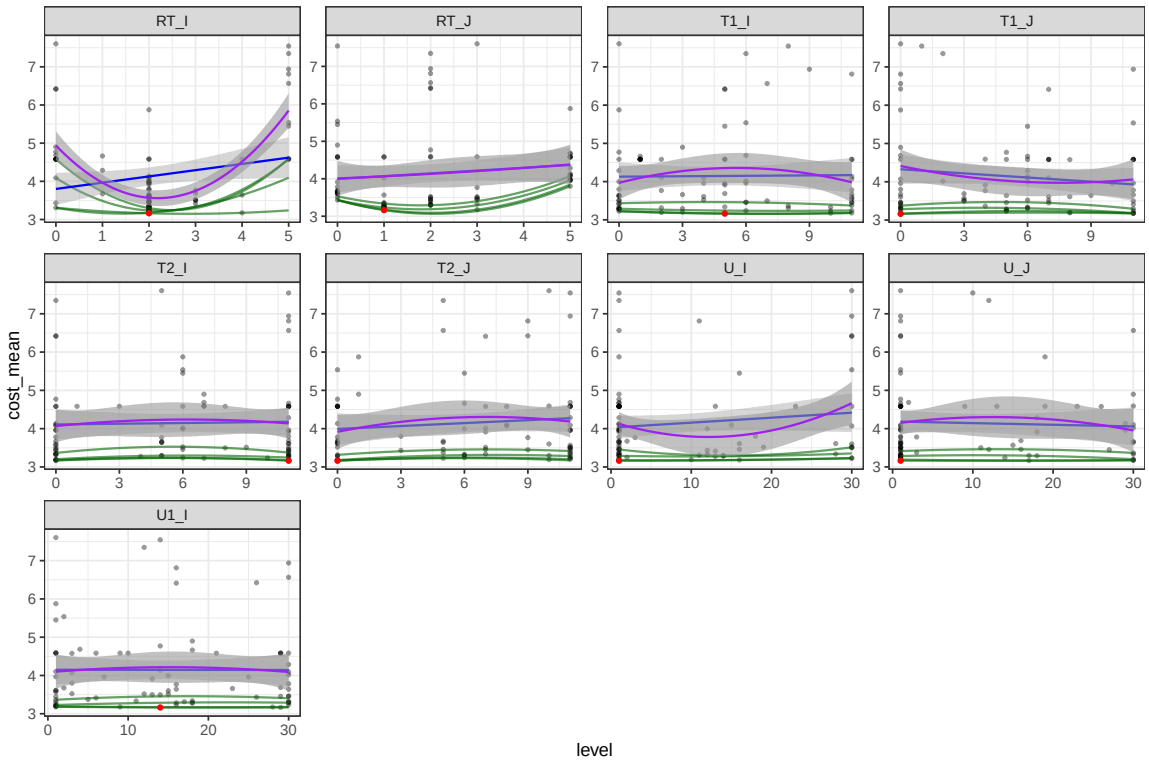

Figure 23: Fitting `lm` linear and cubic models to separate factors for the DLMT runs, for `paravance-29`

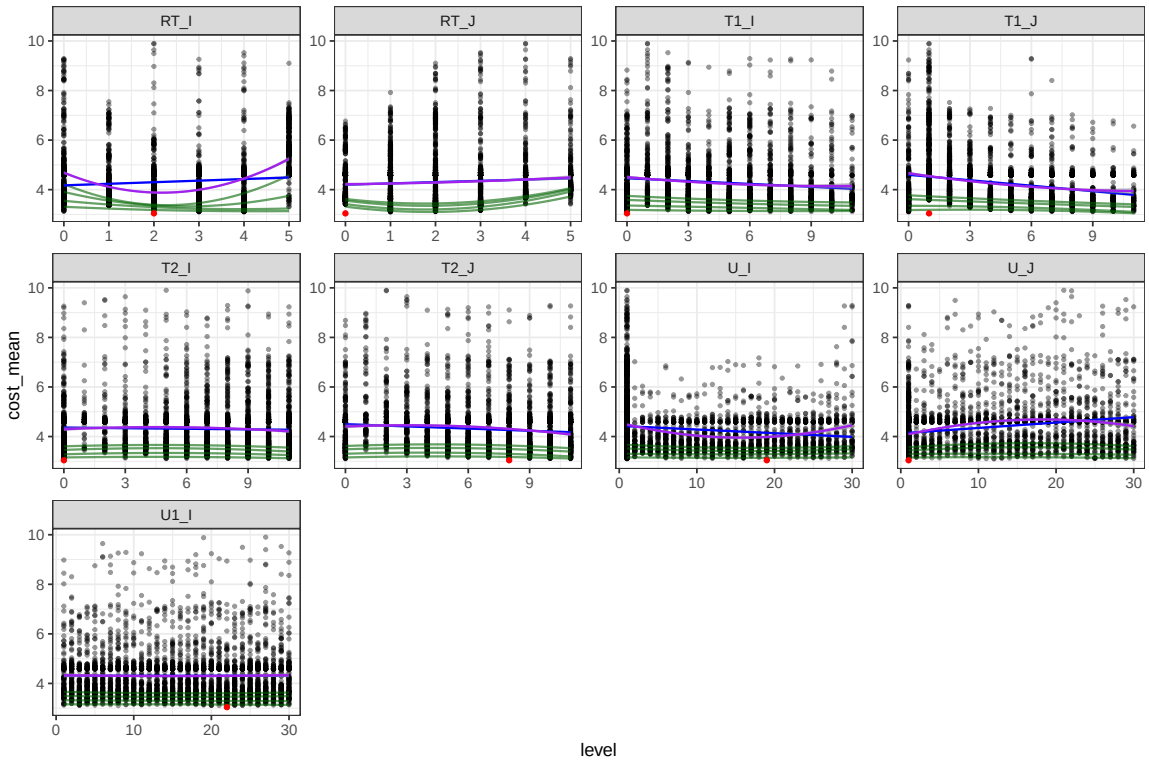

Figure 24: Fitting `lm` linear and cubic models to separate factors for the RS runs

4.4 Predicted Performance at Each Step

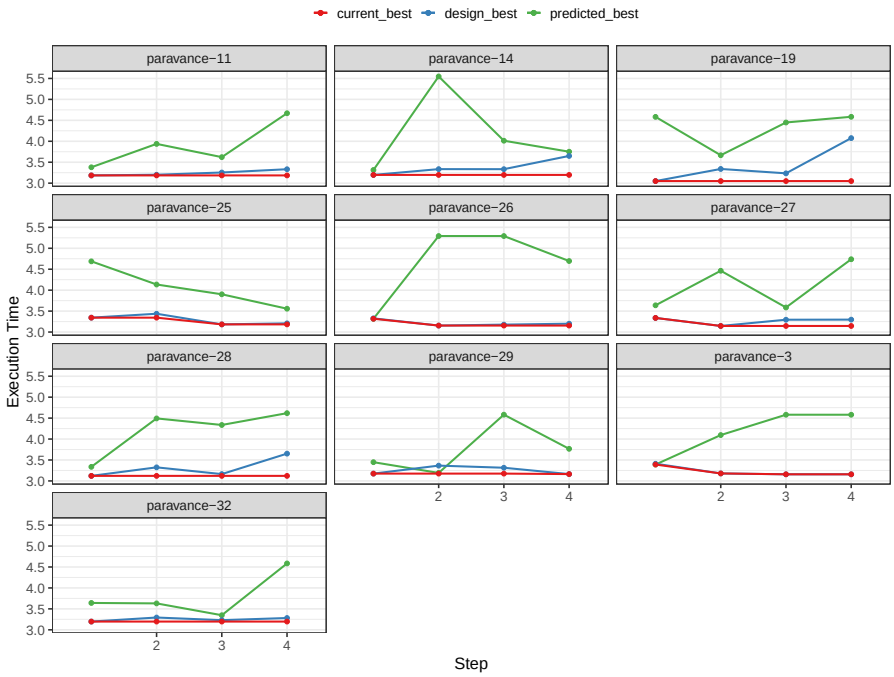

Figure 25: Performance of best points, step-by-step, by experiment

4.5 D-Optimality

4.5.1 D-Criterion

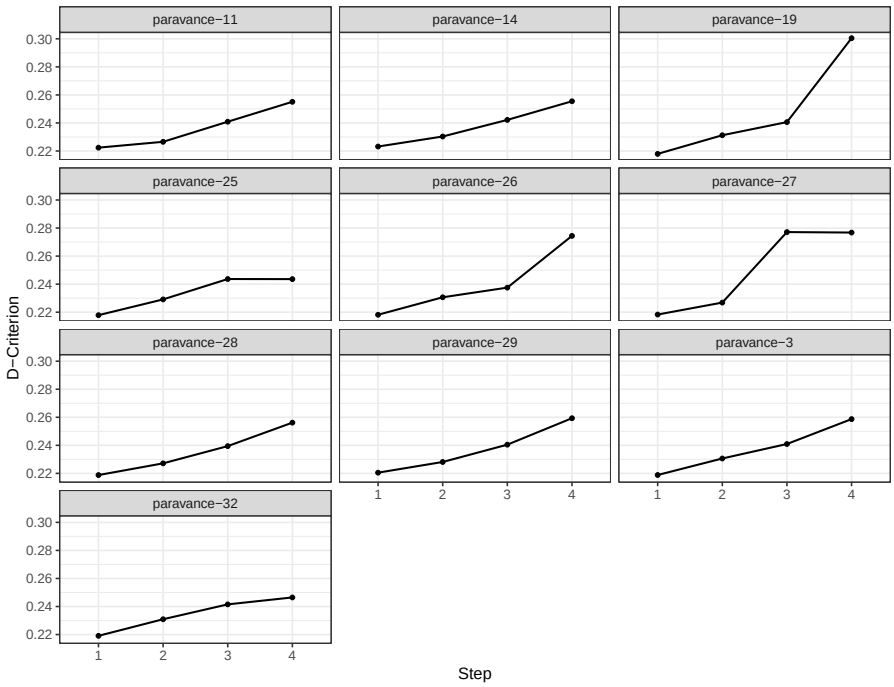

Figure 26: D-Criterion, step-by-step, by experiment

5 Updated DLMT with Quantile Regression ($\tau = 0.05$, Binary Parameters)

5.1 Iterations that found the Best Configuration

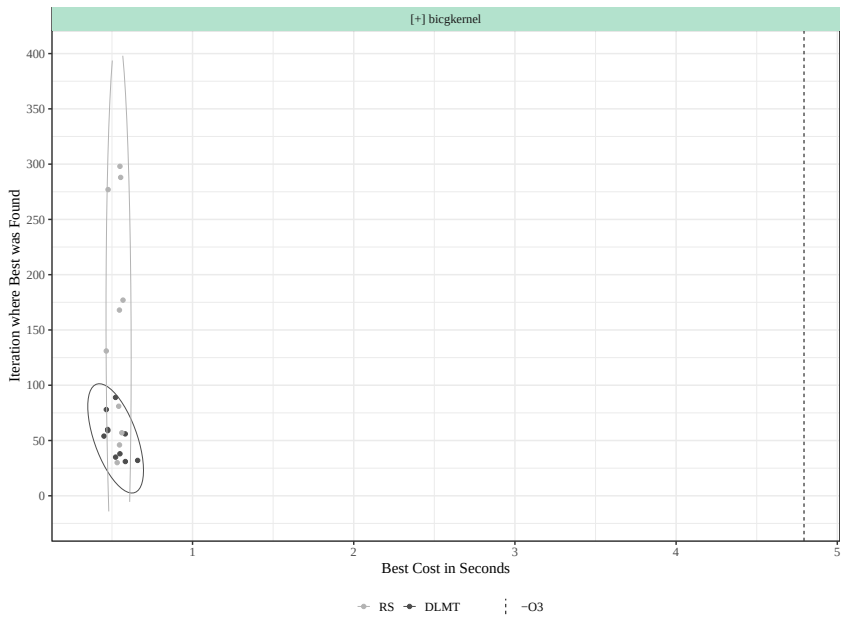

Figure 27: Iteration where best was found

Table 4:

experiment_id	technique	step	cost_mean	T1_I	T1_J	U_J	U_I	T2_I	T2_J	U1_I	RT_I	RT_J	OMP	SCR	VEC1	VEC2
paravance-44	DLMT	4	0.5222684	2048	2048	18	1	1	1	1	16	1	True	True	False	False
paravance-49	DLMT	2	0.5829423	2048	2048	3	1	2048	1	29	4	32	True	True	True	False
paravance-58	DLMT	4	0.4648992	2048	2048	1	30	1	2048	30	4	1	True	True	True	False
paravance-61	DLMT	3	0.4732099	1	2048	15	1	2048	2048	1	4	1	True	True	False	False
paravance-62	DLMT	3	0.582568	1	2048	17	1	2048	2048	20	4	4	True	True	False	True
paravance-63	DLMT	2	0.5494125	2048	16	1	30	2048	512	28	4	1	True	True	False	False
paravance-64	DLMT	3	0.4743967	1	2048	1	14	2048	1	30	16	4	True	True	True	False
paravance-65	DLMT	2	0.6593068	2048	1	1	7	2048	2048	8	16	4	True	True	False	False
paravance-66	DLMT	2	0.4504963	1	128	1	7	2048	256	9	1	4	True	True	True	True
paravance-71	DLMT	2	0.5226401	2048	2048	30	1	1	2048	12	8	8	True	True	False	False
paravance-50	RS	NA	0.5685274	512	512	19	1	2048	1024	27	4	1	True	True	True	False
paravance-51	RS	NA	0.5467725	1024	32	10	1	2048	512	9	8	8	True	True	True	True
paravance-52	RS	NA	0.5417953	2048	8	9	1	2048	128	30	8	16	True	True	True	True
paravance-53	RS	NA	0.5612387	2048	64	16	1	2048	256	9	16	2	True	True	True	True
paravance-54	RS	NA	0.5322531	2048	8	1	9	1	1	19	1	1	True	True	True	False
paravance-55	RS	NA	0.5540684	32	4	1	5	2048	256	4	16	1	True	True	False	False
paravance-56	RS	NA	0.4648199	2048	1	1	23	1	512	25	8	1	True	True	False	True
paravance-57	RS	NA	0.5463944	256	16	1	15	2048	512	2	2	2	True	True	False	False
paravance-58	RS	NA	0.5496127	32	1	1	14	2048	2048	24	8	4	True	True	True	True
paravance-59	RS	NA	0.4758884	64	16	29	1	2048	1	27	16	4	True	True	True	True

5.2 Eliminated Factors and Fixed Values

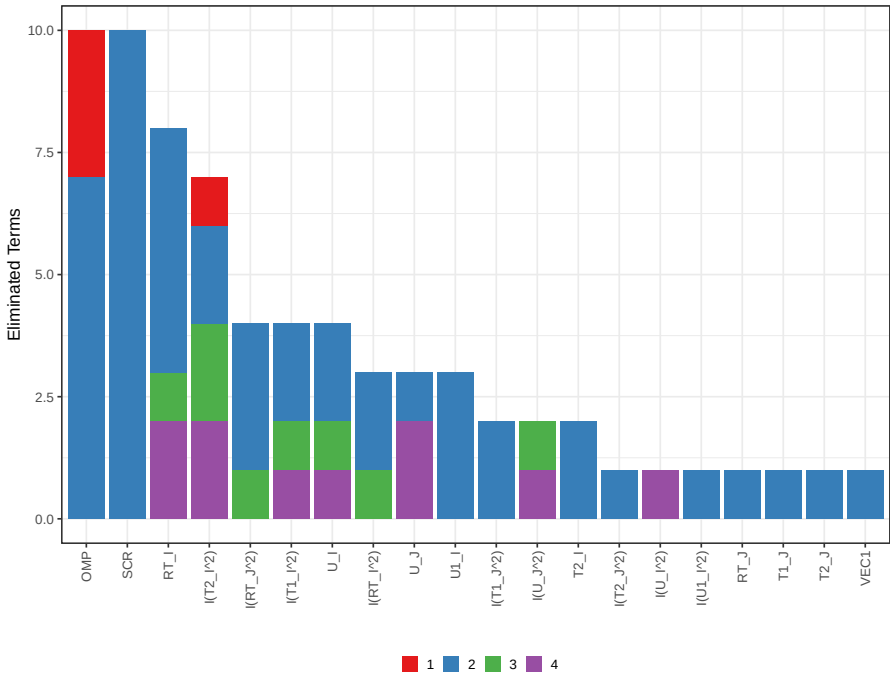

Figure 28: Eliminated terms

5.3 Fixed Factors & Values in the Explored Spaces

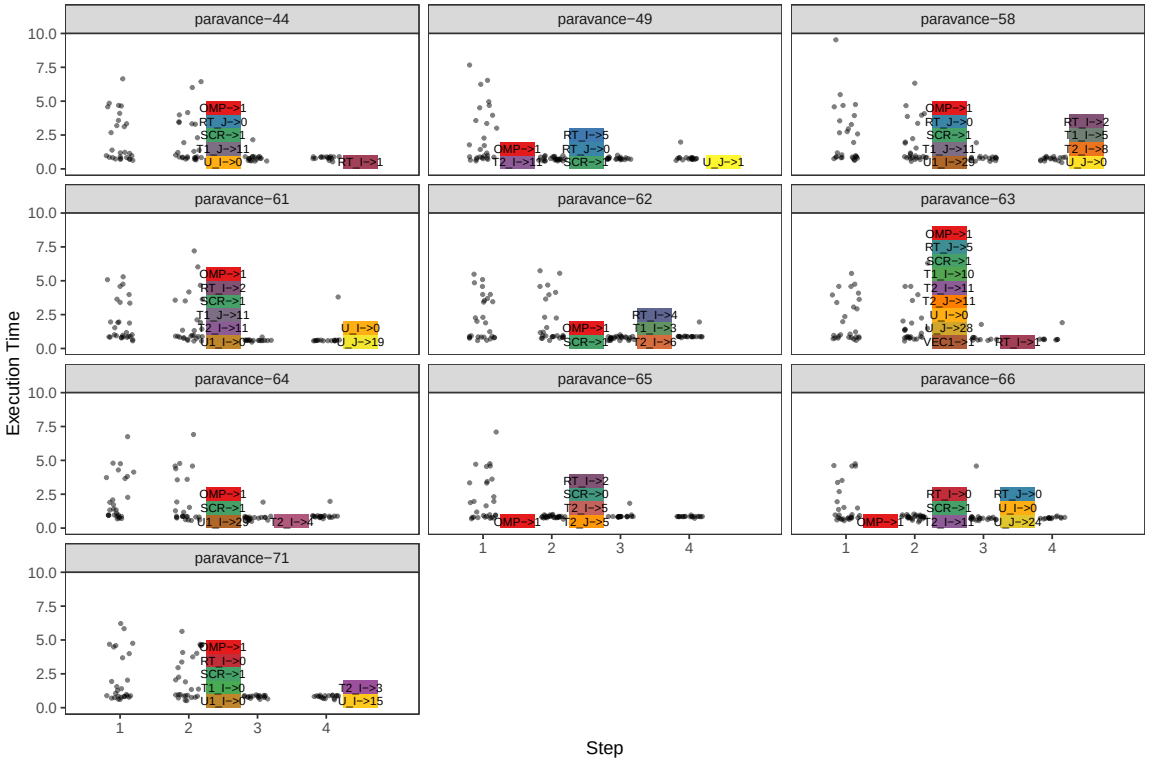

Figure 29: Execution time of Searched Space, step-by-step, by experiment

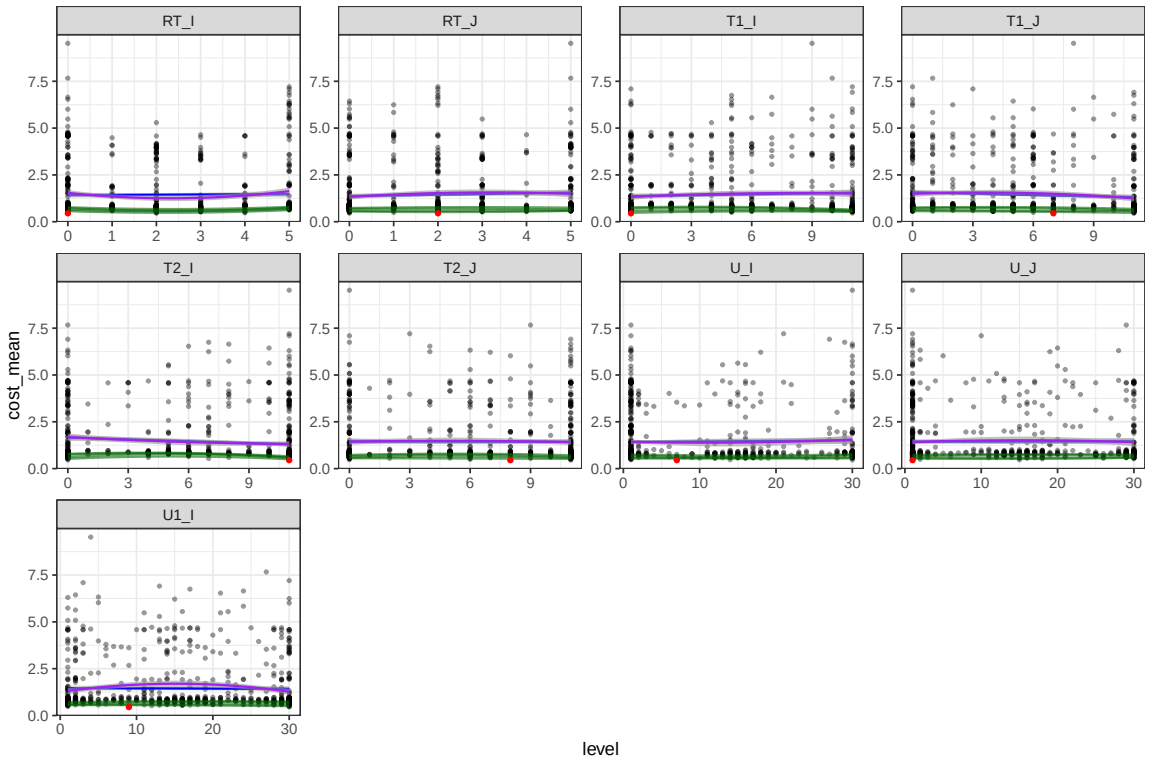

Figure 30: Fitting `lm` linear and cubic models to separate factors for the DLMT runs

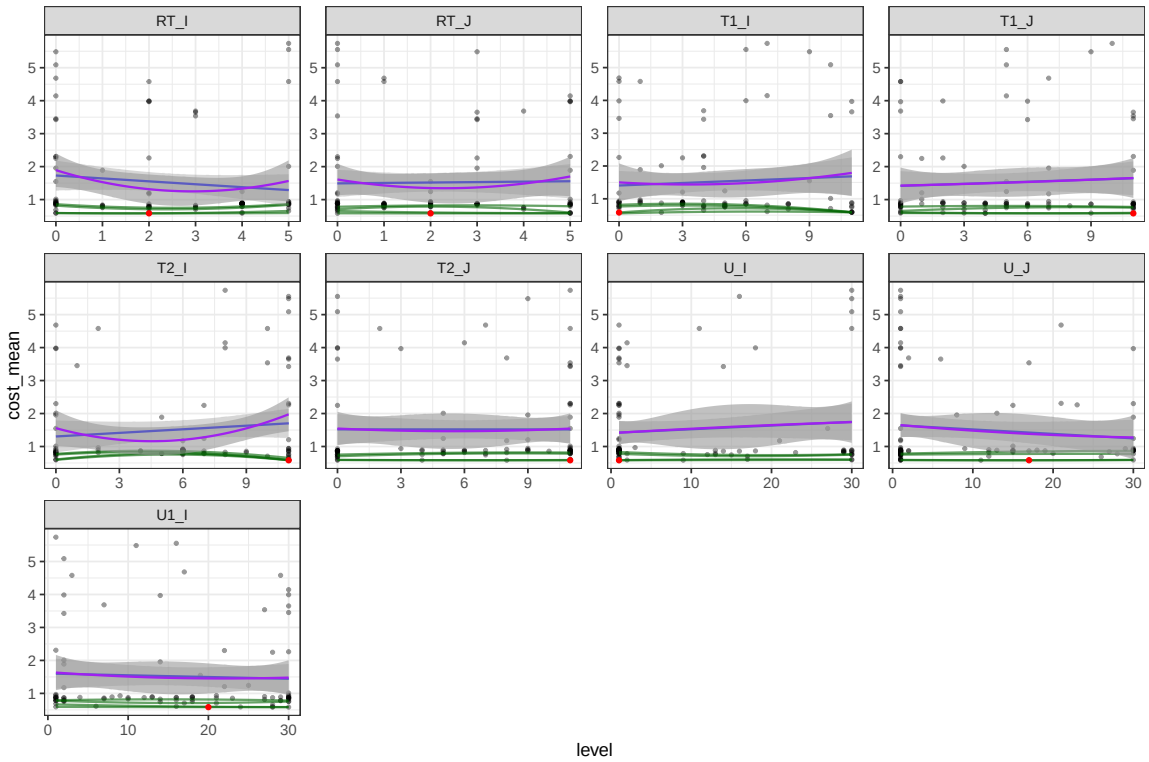

Figure 31: Fitting `lm` linear and cubic models to separate factors for the DLMT runs, for `paravance-62`

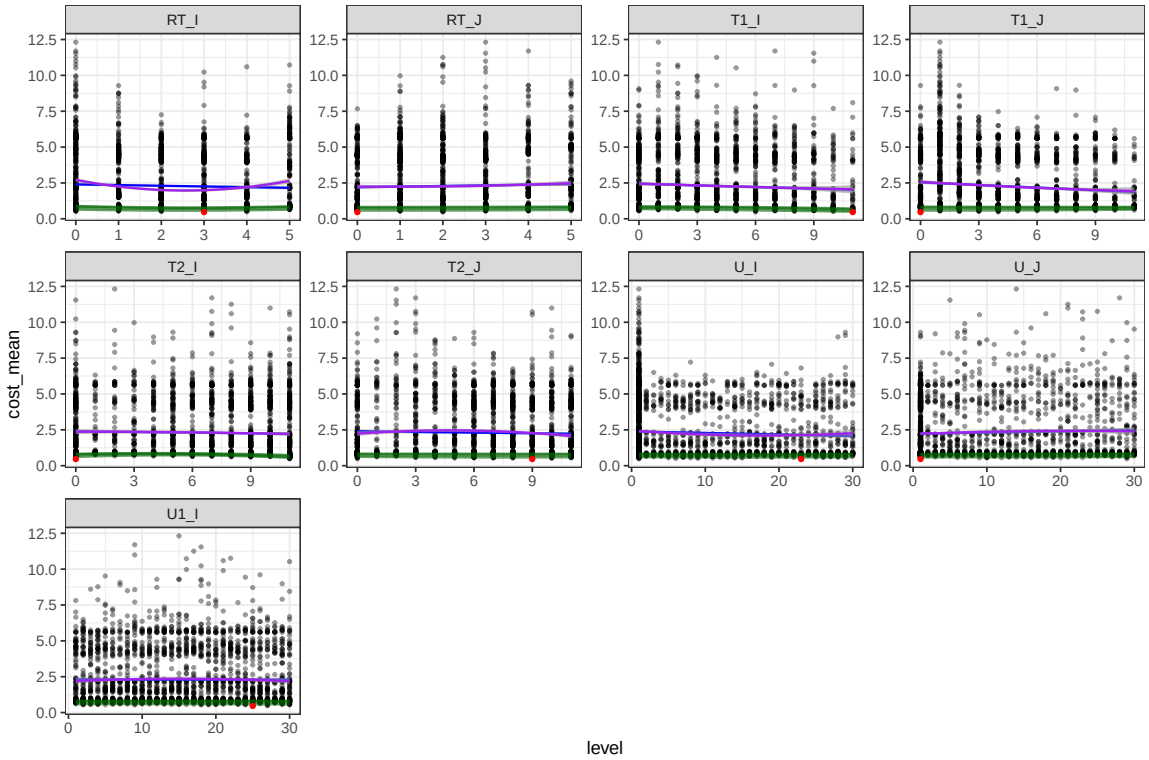

Figure 32: Fitting `lm` linear and cubic models to separate factors for the RS runs

5.4 Predicted Performance at Each Step

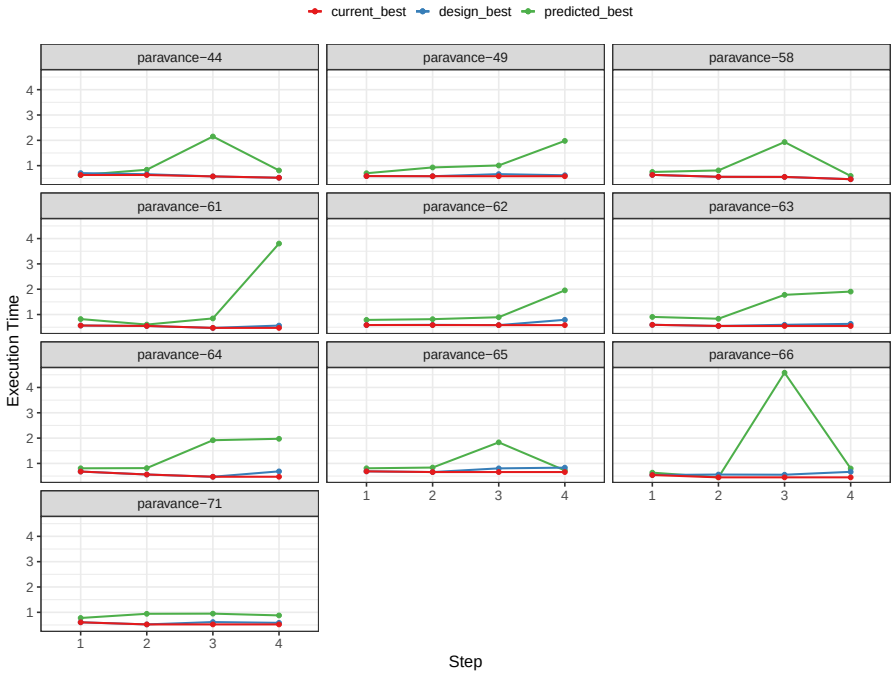

Figure 33: Performance of best points, step-by-step, by experiment

5.5 D-Optimality

5.5.1 D-Criterion

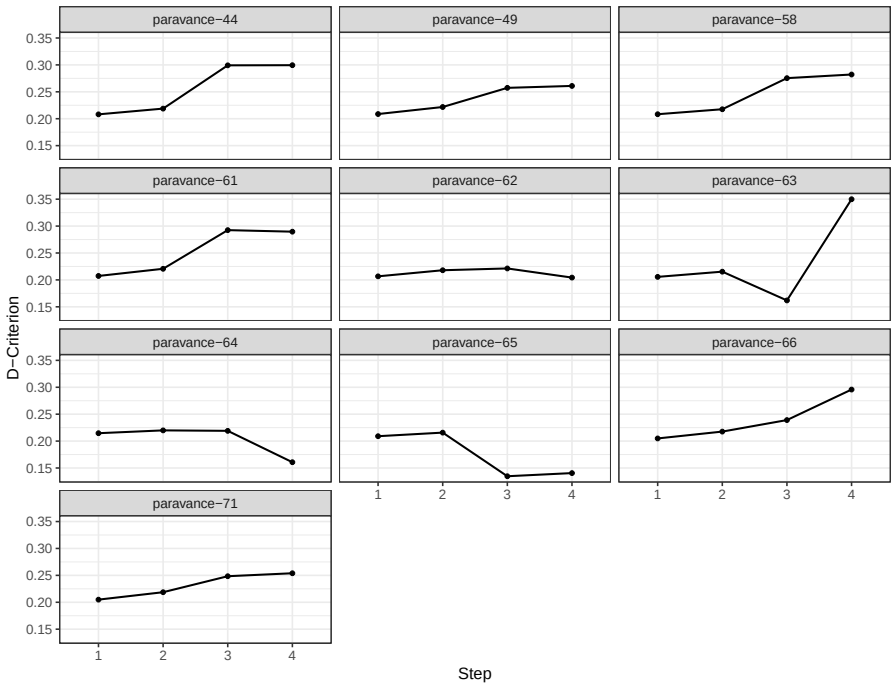

Figure 34: D-Criterion, step-by-step, by experiment

6 Updated DLMT with Quantile Regression ($\tau = 0.05$)

6.1 Iterations that found the Best Configuration

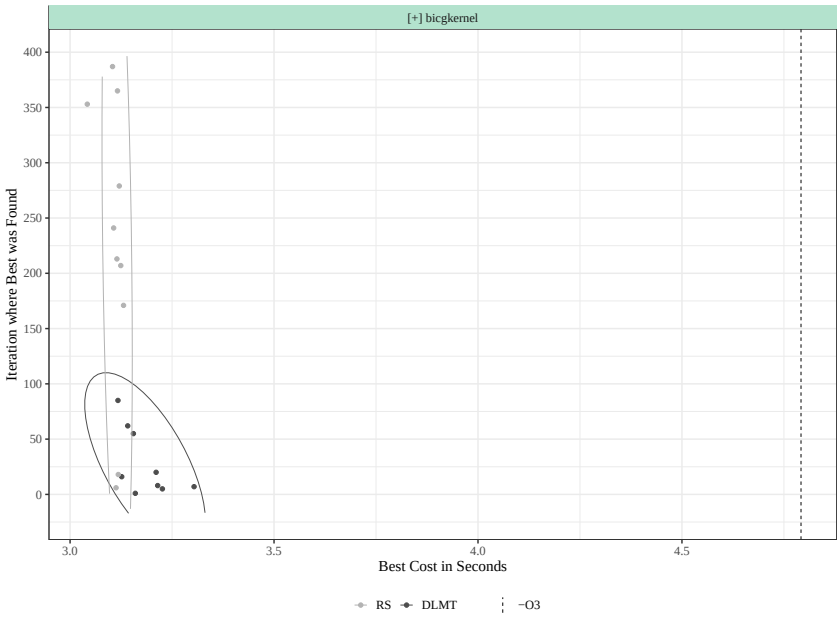

Figure 35: Iteration where best was found

Table 5:

experiment_id	technique	step	cost_mean	T1_I	T1_J	U_J	U_I	T2_I	T2_J	U1_I	RT_I	RT_J
paravance-40	DLMT	1	3.126611	1	128	1	30	128	1	2	8	2
paravance-41	DLMT	1	3.303955	1	16	1	21	2048	1	1	1	4
paravance-54	DLMT	1	3.22632	4	32	1	1	64	1	17	4	2
paravance-59	DLMT	1	3.214804	1	64	1	12	1	256	1	1	2
paravance-67	DLMT	1	3.21091	32	2048	11	1	2048	1	1	16	4
paravance-68	DLMT	3	3.141319	16	1	1	17	64	2048	1	8	2
paravance-69	DLMT	1	3.159918	4	1	8	1	1	1	20	4	2
paravance-70	DLMT	3	3.155196	1	1	1	27	2048	1	2	4	2
paravance-72	DLMT	4	3.117389	64	2048	1	2	128	2048	1	16	4
paravance-16	RS	1	3.118132	1	256	25	1	32	512	20	16	4
paravance-18	RS	1	3.042194	1	2	1	19	1	256	22	4	1
paravance-2	RS	1	3.131061	1	1024	1	6	256	2048	20	16	2
paravance-20	RS	1	3.120612	256	1024	13	1	1	1024	26	16	2
paravance-21	RS	1	3.112708	32	1	30	1	64	512	20	16	2
paravance-22	RS	1	3.124333	256	1	1	12	256	1024	28	8	2
paravance-23	RS	1	3.11485	512	512	25	1	2048	2048	3	16	4
paravance-24	RS	1	3.106786	128	1	12	1	512	1	23	8	2
paravance-30	RS	1	3.104083	32	2048	12	1	1	2048	8	16	2
paravance-31	RS	1	3.116196	2048	256	1	19	2048	1	24	8	2

6.2 Eliminated Factors and Fixed Values

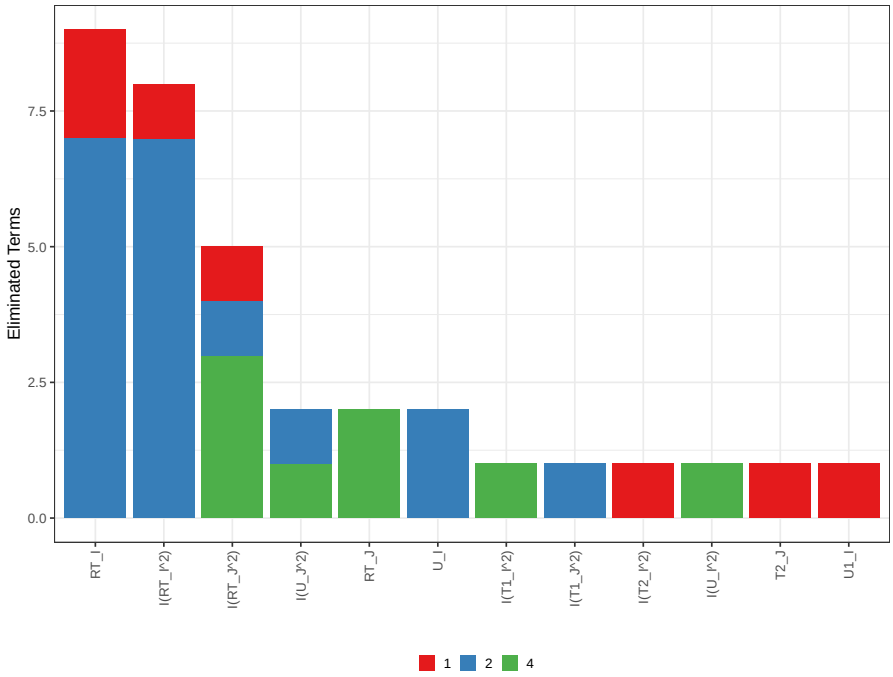

Figure 36: Eliminated terms

6.3 Fixed Factors & Values in the Explored Spaces

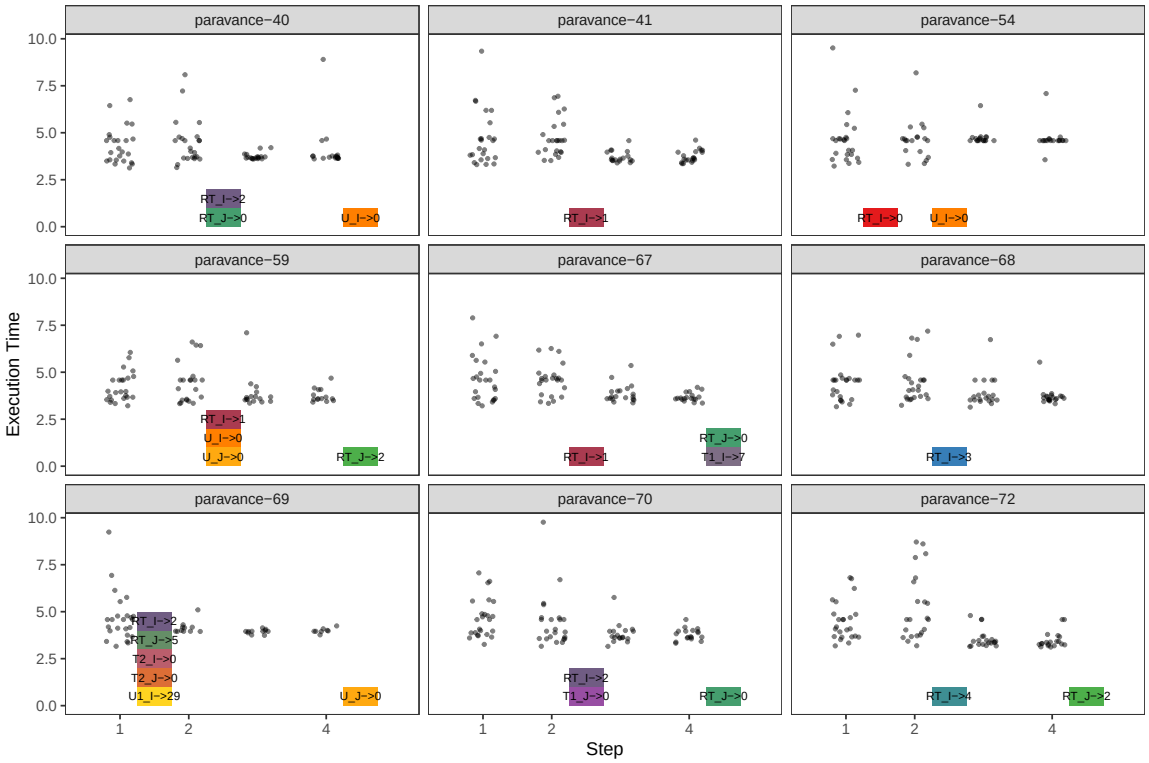

Figure 37: Execution time of Searched Space, step-by-step, by experiment

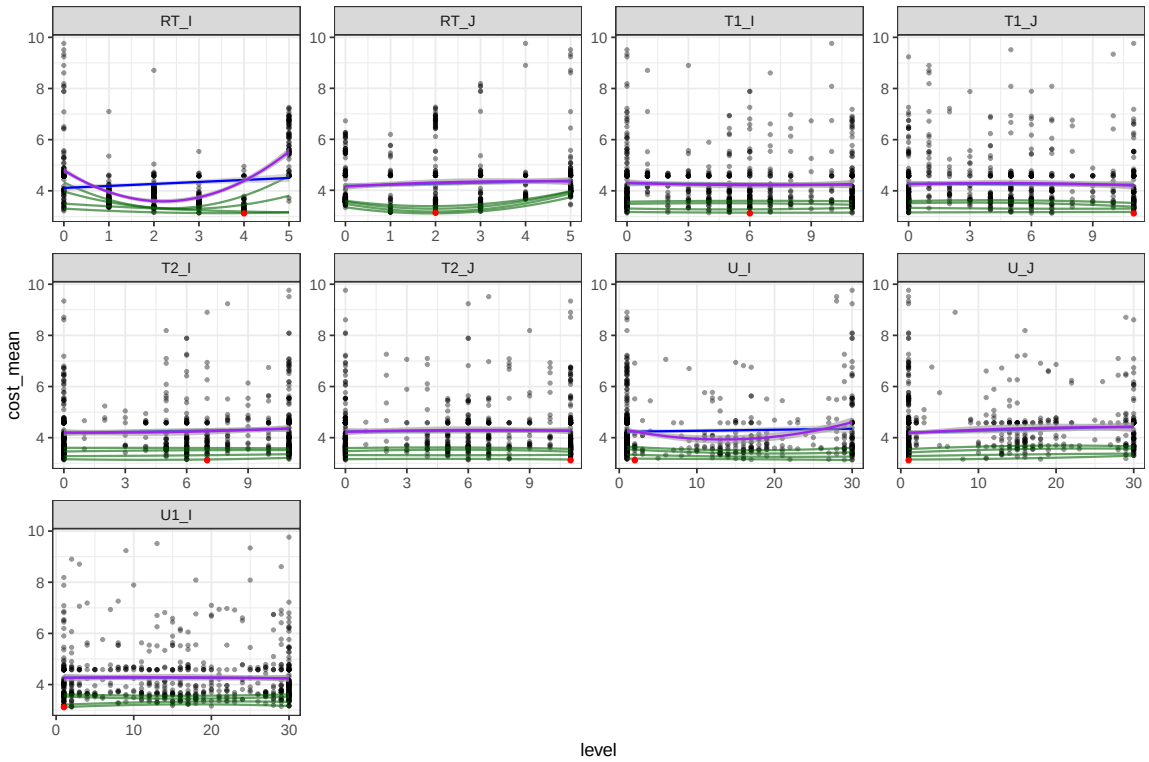

Figure 38: Fitting `lm` linear and cubic models to separate factors for the DLMT runs

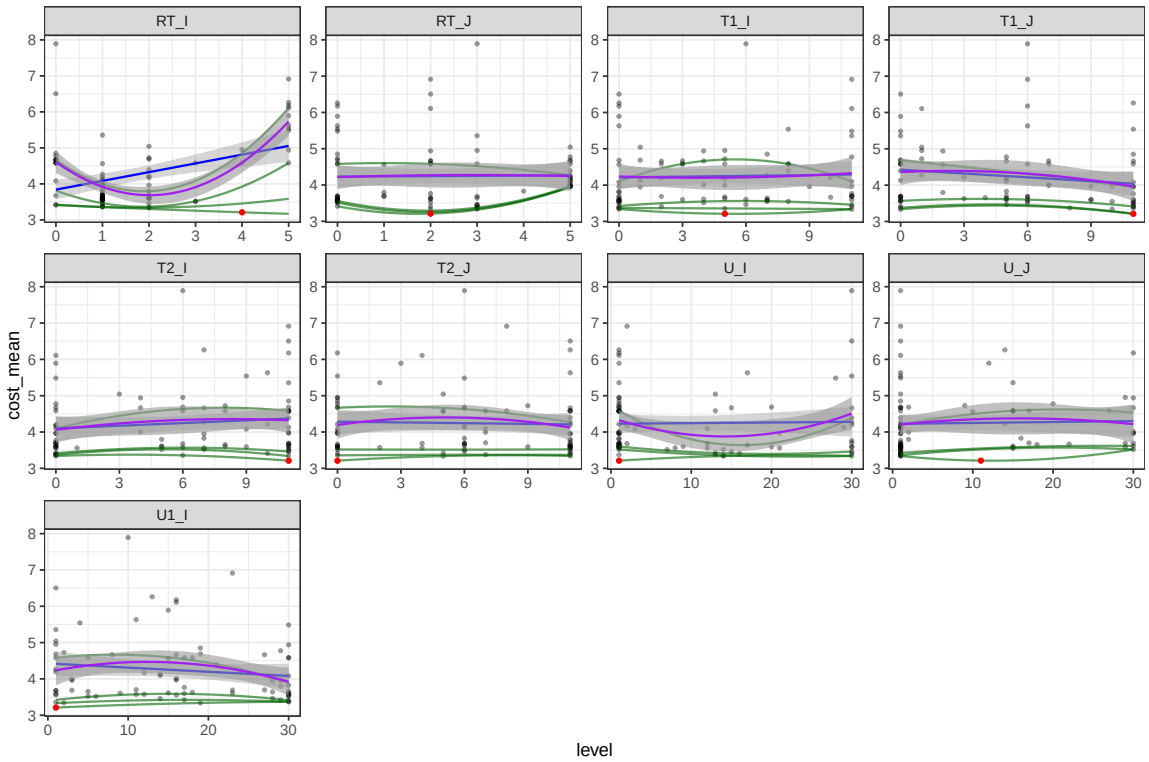

Figure 39: Fitting `lm` linear and cubic models to separate factors for the DLMT runs, for `paravance-67`

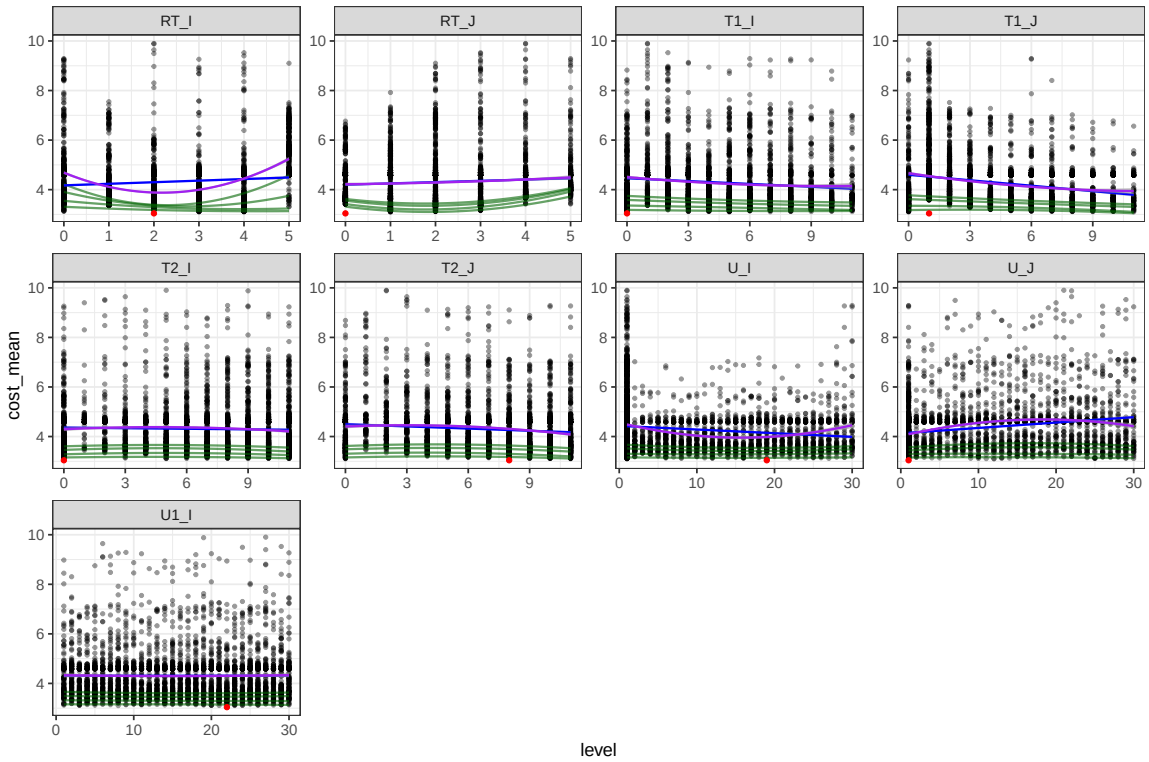

Figure 40: Fitting `lm` linear and cubic models to separate factors for the RS runs

6.4 Predicted Performance at Each Step

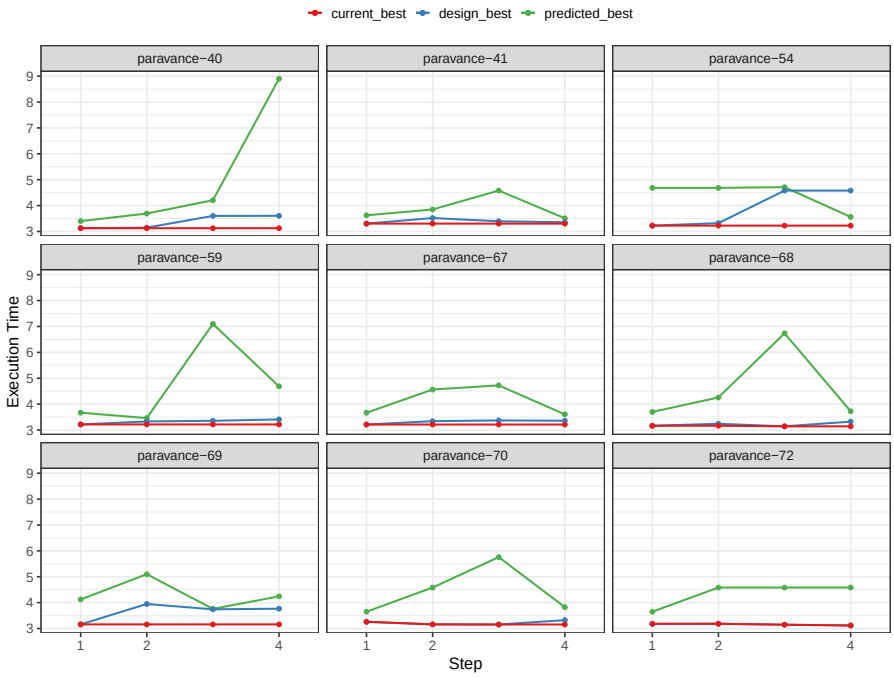

Figure 41: Performance of best points, step-by-step, by experiment

6.5 D-Optimality

6.5.1 D-Criterion

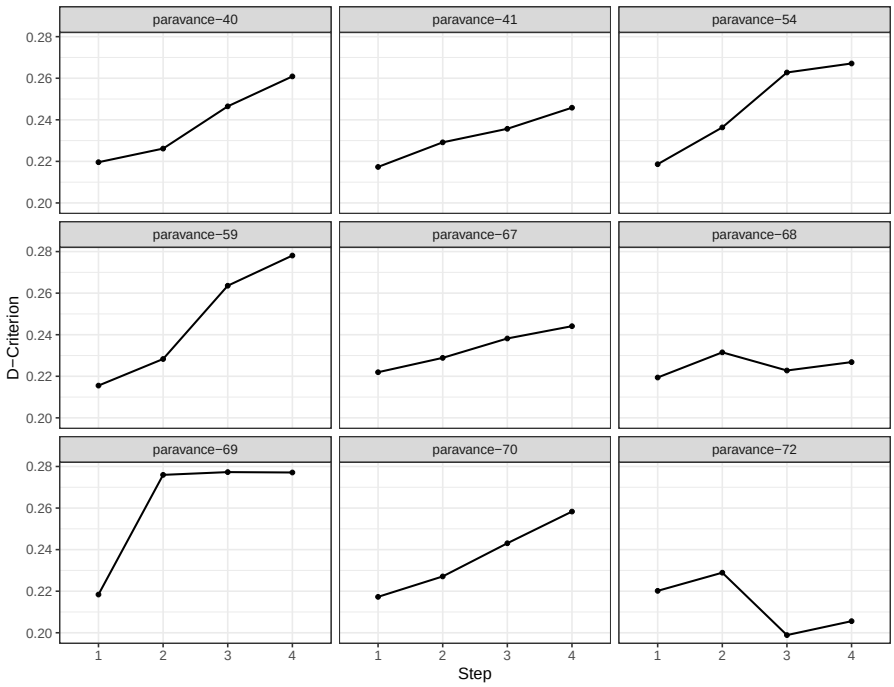

Figure 42: D-Criterion, step-by-step, by experiment

7 Updated DLMT with Quantile Regression ($\tau = 0.10$)

7.1 Iterations that found the Best Configuration

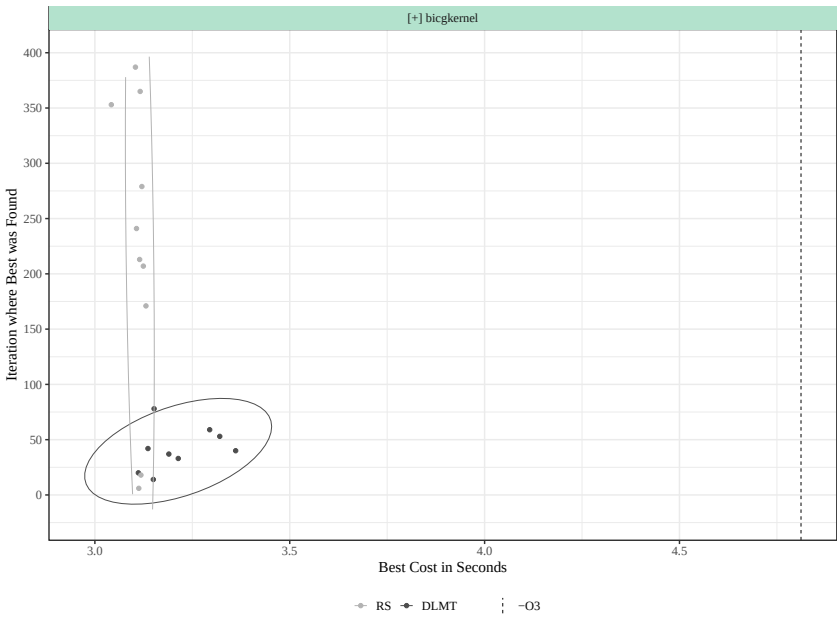

Figure 43: Iteration where best was found

Table 6:

experiment_id	technique	step	cost_mean	T1_I	T1_J	U_J	U_I	T2_I	T2_J	U1_I	RT_I	RT_J
paravance-16	DLMT	4	3.151937	32	1	24	1	128	64	14	8	2
paravance-18	DLMT	1	3.111585	32	2048	1	3	2048	1	30	16	2
paravance-20	DLMT	1	3.149921	2048	2048	1	30	1	1	22	8	2
paravance-24	DLMT	2	3.361233	4	1	1	2	1	2048	2	1	8
paravance-49	DLMT	2	3.189757	8	64	1	30	64	128	10	8	2
paravance-5	DLMT	2	3.213894	2048	64	16	1	2048	1	30	16	4
paravance-7	DLMT	2	3.136232	1	128	1	16	32	128	17	1	4
paravance-71	DLMT	3	3.320441	1	1	1	15	128	1	11	1	2
paravance-8	DLMT	4	3.294567	2048	1	1	23	2048	2048	11	4	4
paravance-16	RS	1	3.118132	1	256	25	1	32	512	20	16	4
paravance-18	RS	1	3.042194	1	2	1	19	1	256	22	4	1
paravance-2	RS	1	3.131061	1	1024	1	6	256	2048	20	16	2
paravance-20	RS	1	3.120612	256	1024	13	1	1	1024	26	16	2
paravance-21	RS	1	3.112708	32	1	30	1	64	512	20	16	2
paravance-22	RS	1	3.124333	256	1	1	12	256	1024	28	8	2
paravance-23	RS	1	3.11485	512	512	25	1	2048	2048	3	16	4
paravance-24	RS	1	3.106786	128	1	12	1	512	1	23	8	2
paravance-30	RS	1	3.104083	32	2048	12	1	1	2048	8	16	2
paravance-31	RS	1	3.116196	2048	256	1	19	2048	1	24	8	2

7.2 Eliminated Factors and Fixed Values

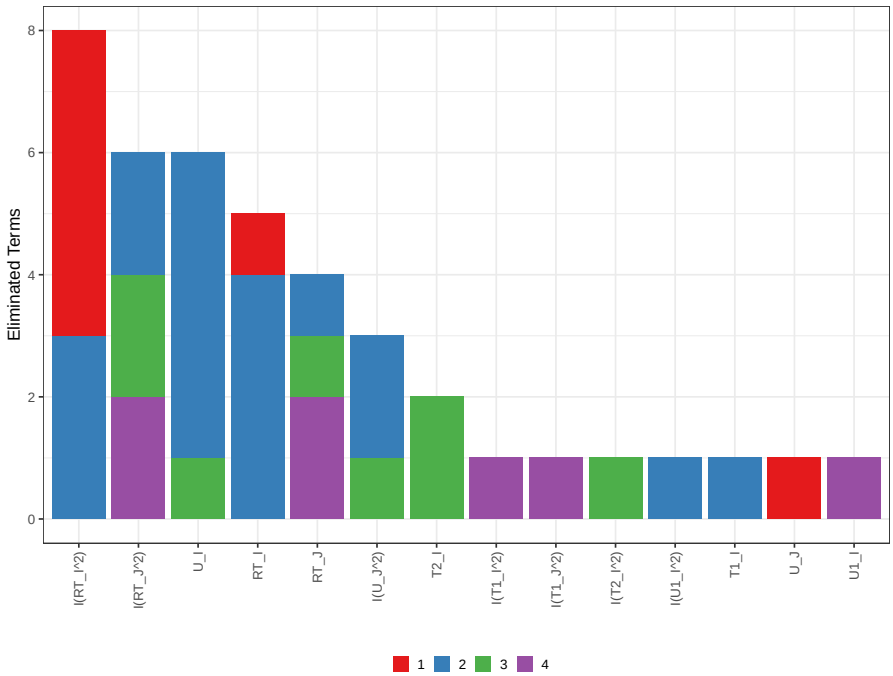

Figure 44: Eliminated terms

7.3 Fixed Factors & Values in the Explored Spaces

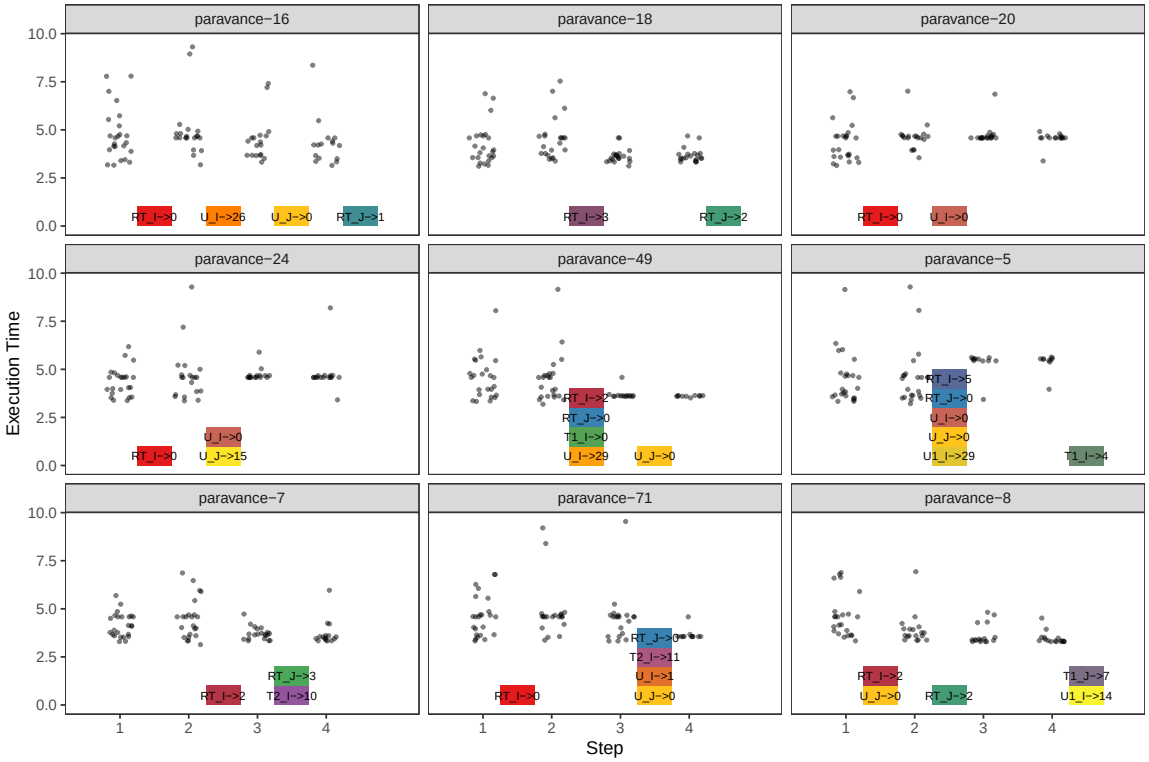

Figure 45: Execution time of Searched Space, step-by-step, by experiment

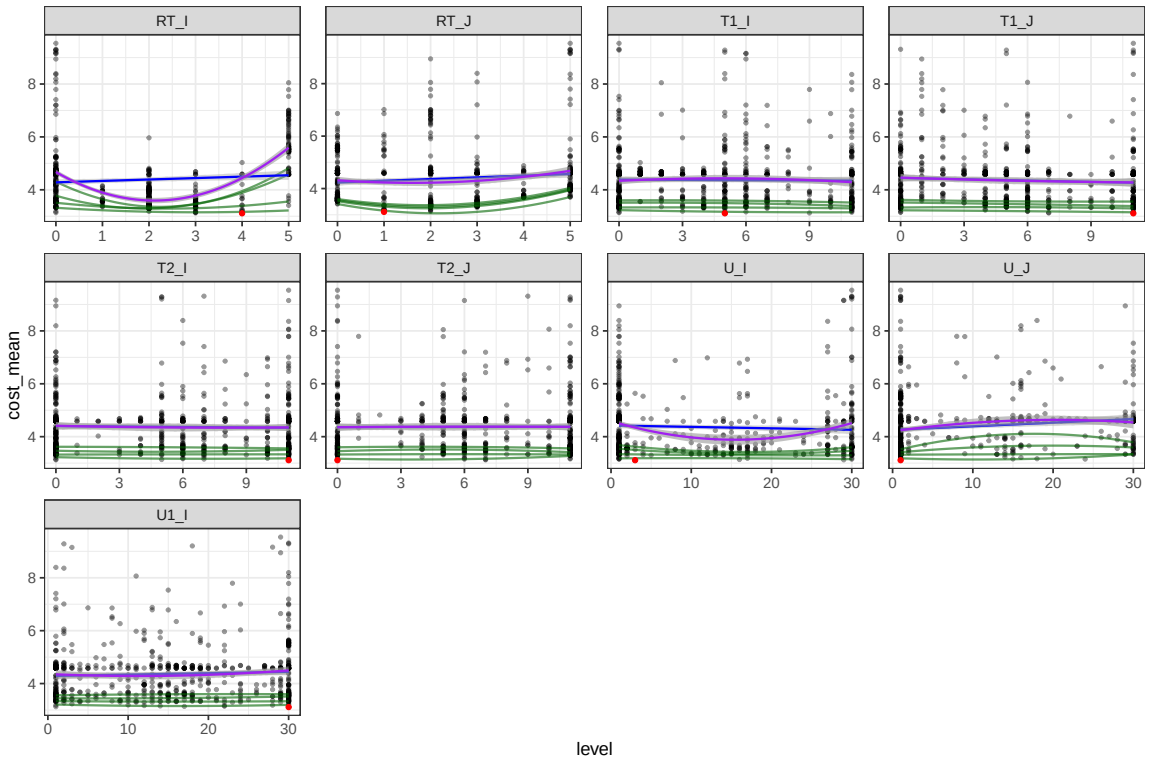

Figure 46: Fitting `lm` linear and cubic models to separate factors for the DLMT runs

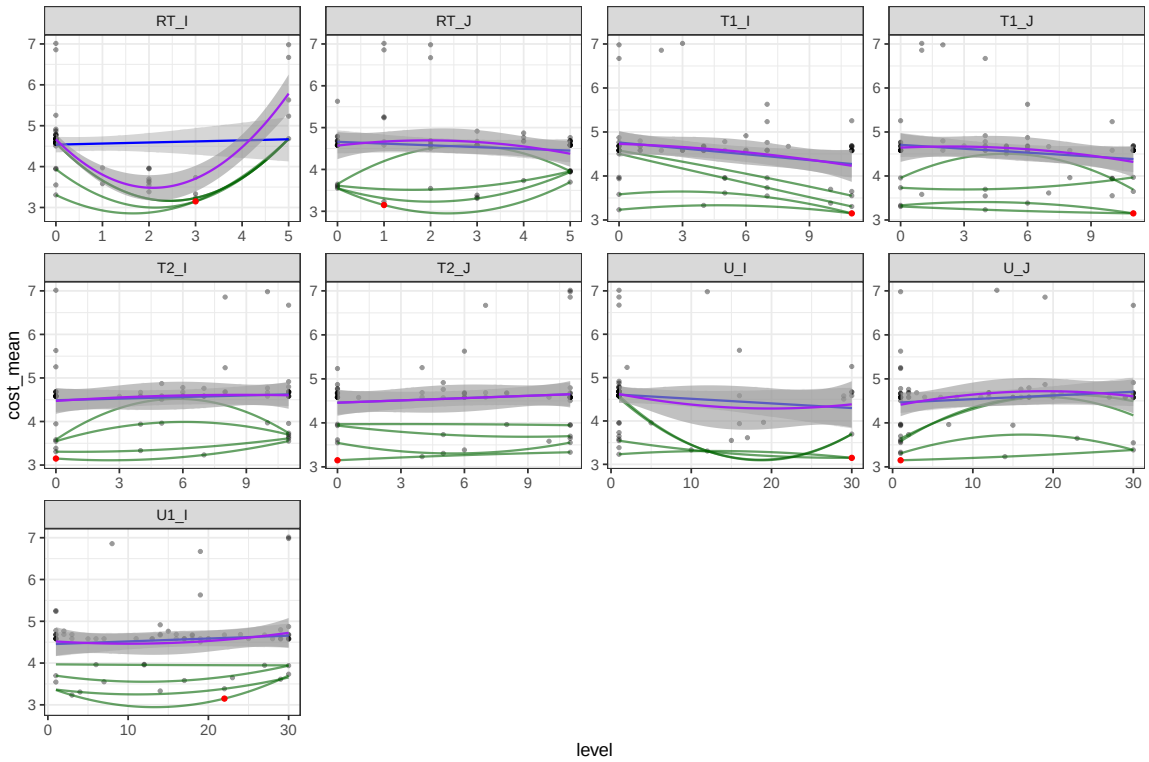

Figure 47: Fitting `lm` linear and cubic models to separate factors for the DLMT runs, for `paravance-20`

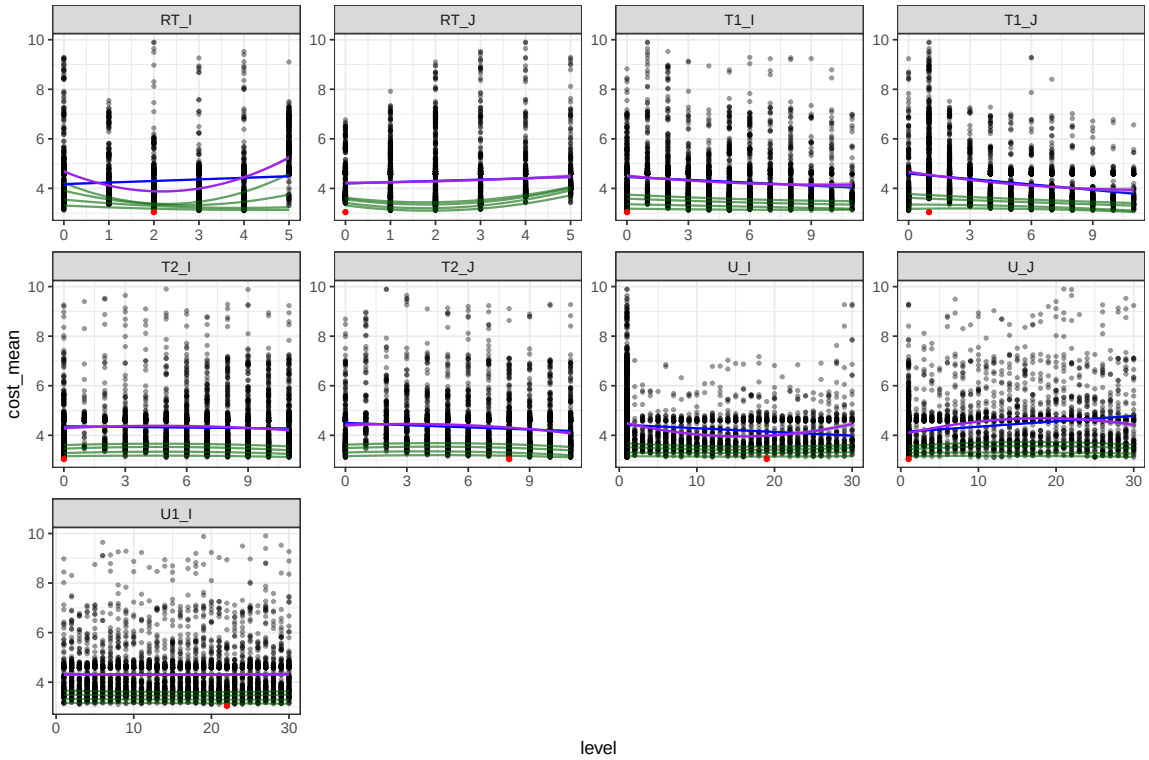

Figure 48: Fitting `lm` linear and cubic models to separate factors for the RS runs

7.4 Predicted Performance at Each Step

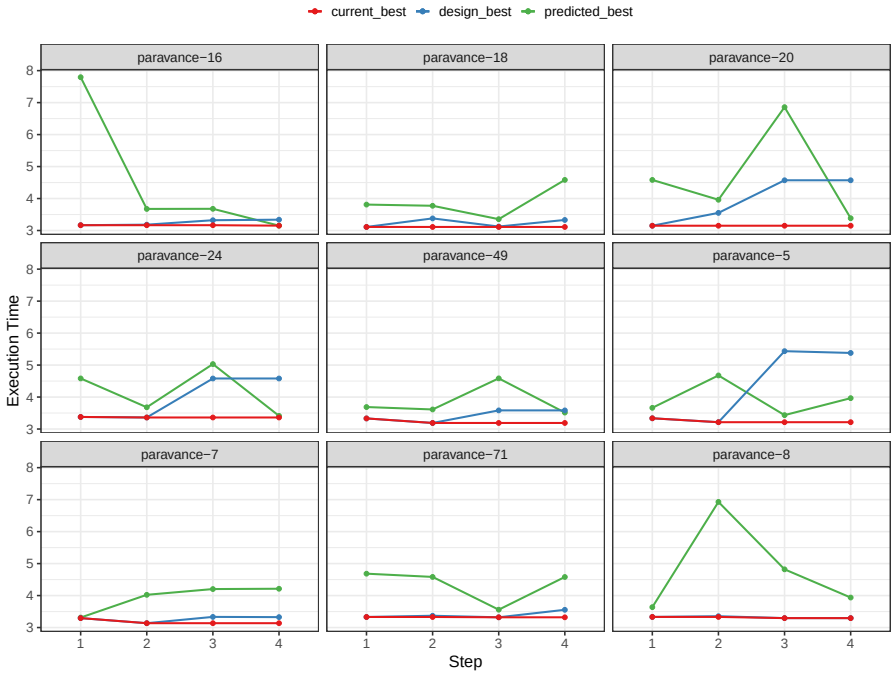

Figure 49: Performance of best points, step-by-step, by experiment

7.5 D-Optimality

7.5.1 D-Criterion

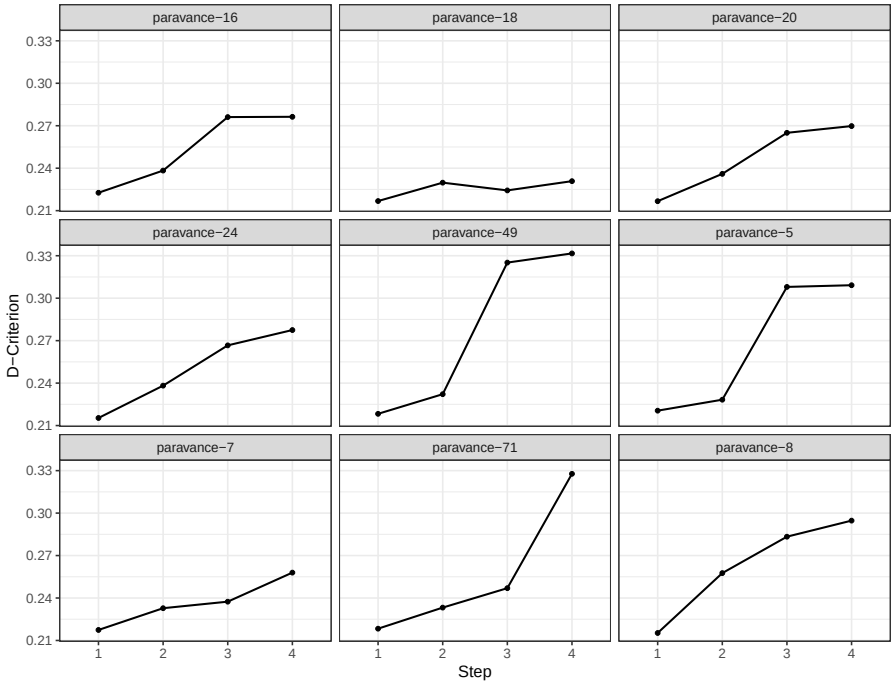

Figure 50: D-Criterion, step-by-step, by experiment

8 Updated DLMT with Quantile Regression ($\tau = 0.05$, "Carry Terms")

8.1 Iterations that found the Best Configuration

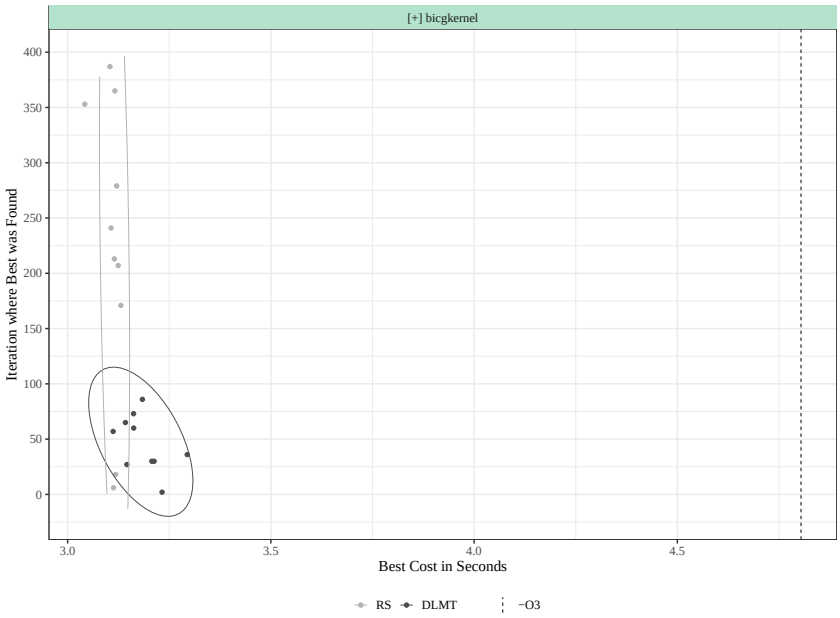

Figure 51: Iteration where best was found

Table 7:

experiment_id	technique	step	cost_mean	T1_I	T1_J	U_J	U_I	T2_I	T2_J	U1_I	RT_I	RT_J
paravance-19	DLMT	4	3.183816	64	256	1	11	1024	1024	9	16	8
paravance-25	DLMT	3	3.111797	2048	1	21	1	1	2048	1	8	2
paravance-26	DLMT	3	3.142036	2048	512	15	1	1	2048	10	16	2
paravance-27	DLMT	2	3.294139	1	2048	1	30	128	1	28	4	4
paravance-28	DLMT	3	3.162506	1	1	30	1	256	1	5	4	2
paravance-29	DLMT	1	3.232323	2048	32	1	17	2048	2048	11	4	2
paravance-3	DLMT	2	3.20722	2048	64	1	14	1	1024	16	1	4
paravance-30	DLMT	4	3.162193	8	1	30	1	64	1	8	4	2
paravance-31	DLMT	2	3.145759	1	1	1	16	32	64	1	1	4
paravance-32	DLMT	2	3.212736	512	16	1	30	1	1	30	8	2
paravance-16	RS	1	3.118132	1	256	25	1	32	512	20	16	4
paravance-18	RS	1	3.042194	1	2	1	19	1	256	22	4	1
paravance-2	RS	1	3.131061	1	1024	1	6	256	2048	20	16	2
paravance-20	RS	1	3.120612	256	1024	13	1	1	1024	26	16	2
paravance-21	RS	1	3.112708	32	1	30	1	64	512	20	16	2
paravance-22	RS	1	3.124333	256	1	1	12	256	1024	28	8	2
paravance-23	RS	1	3.11485	512	512	25	1	2048	2048	3	16	4
paravance-24	RS	1	3.106786	128	1	12	1	512	1	23	8	2
paravance-30	RS	1	3.104083	32	2048	12	1	1	2048	8	16	2
paravance-31	RS	1	3.116196	2048	256	1	19	2048	1	24	8	2

8.2 Eliminated Factors and Fixed Values

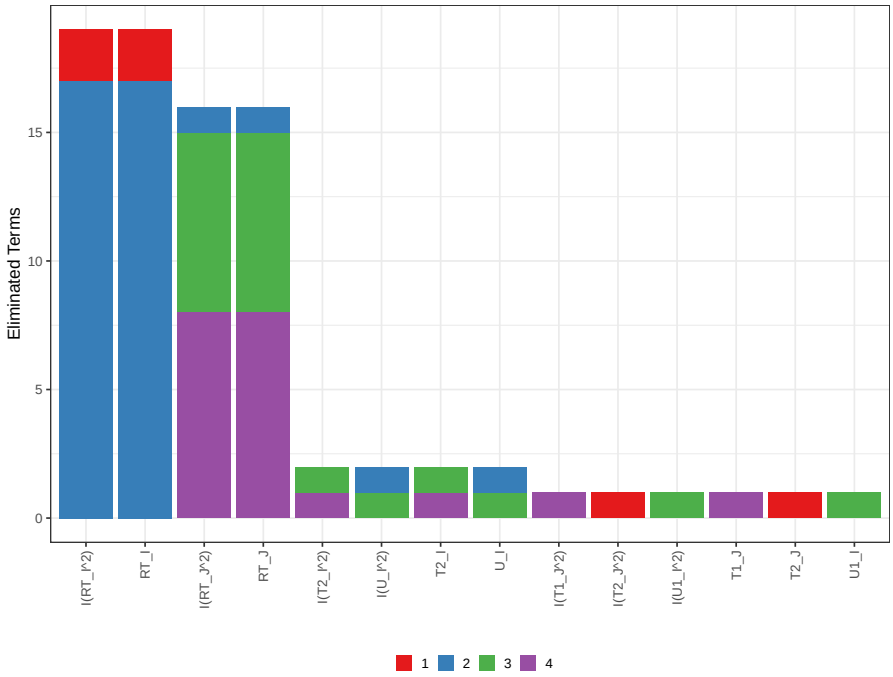

Figure 52: Eliminated terms

8.3 Fixed Factors & Values in the Explored Spaces

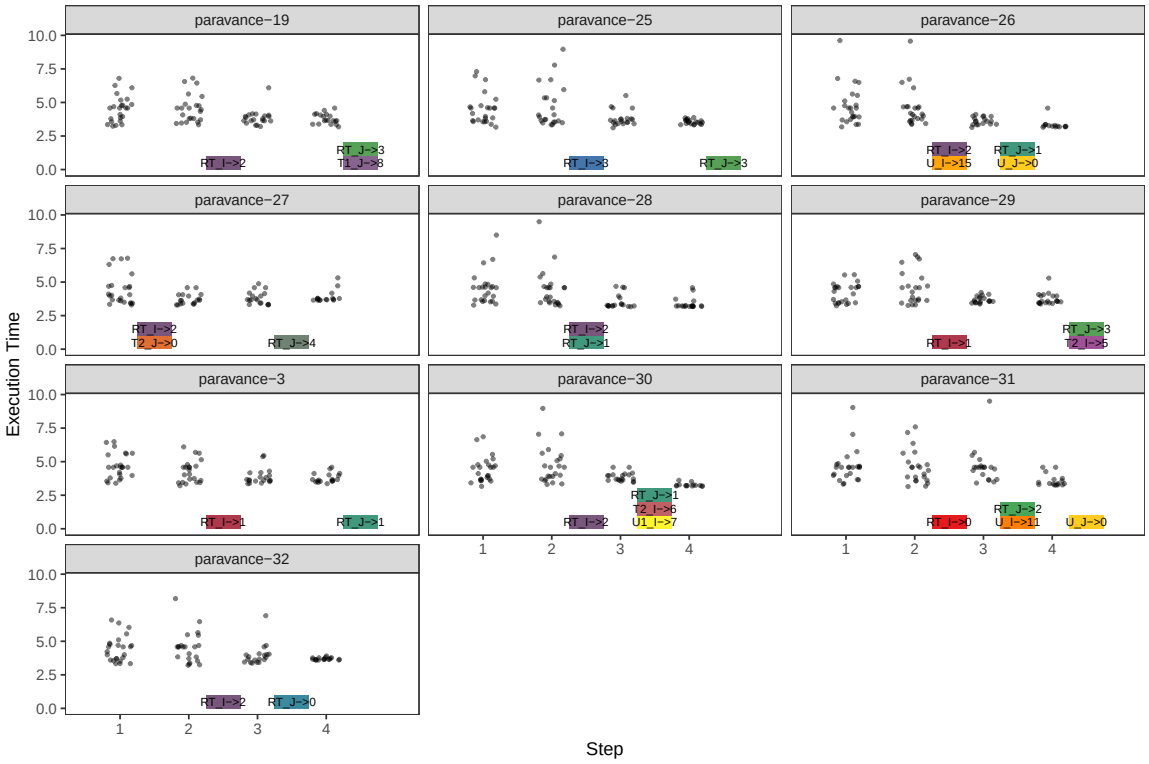

Figure 53: Execution time of Searched Space, step-by-step, by experiment

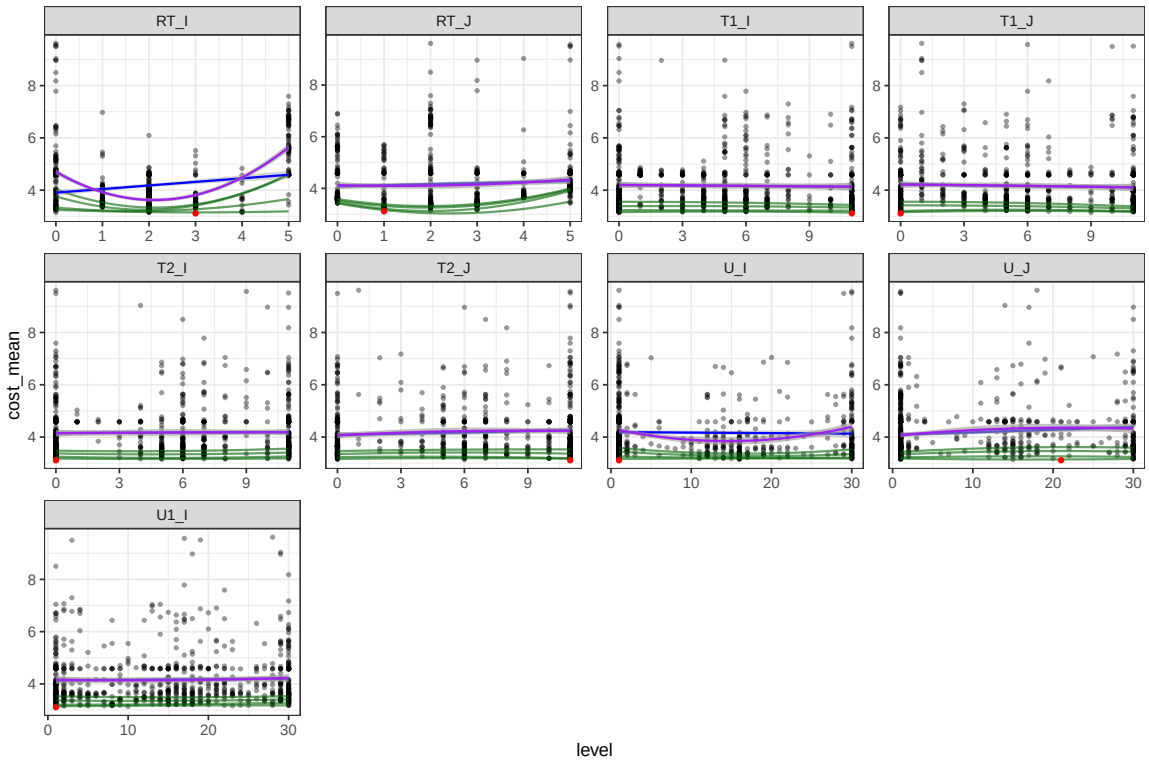

Figure 54: Fitting `lm` linear and cubic models to separate factors for the DLMT runs

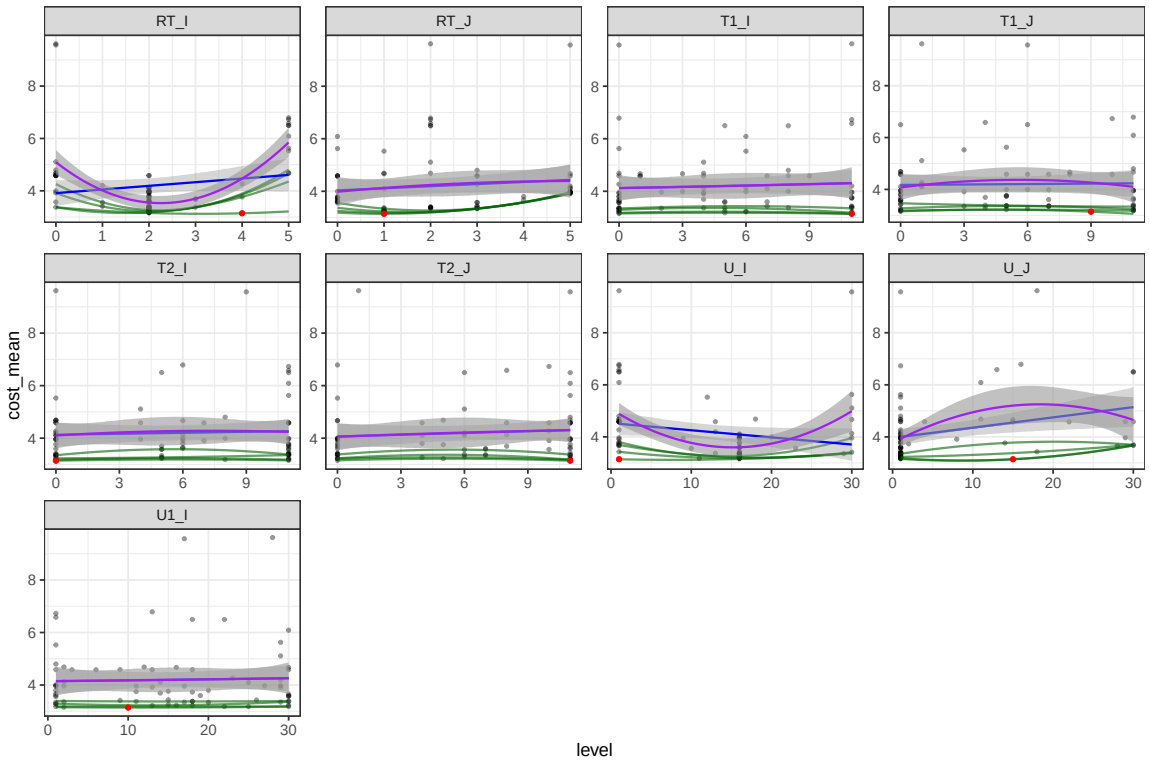

Figure 55: Fitting `lm` linear and cubic models to separate factors for the DLMT runs, for `paravance-26`

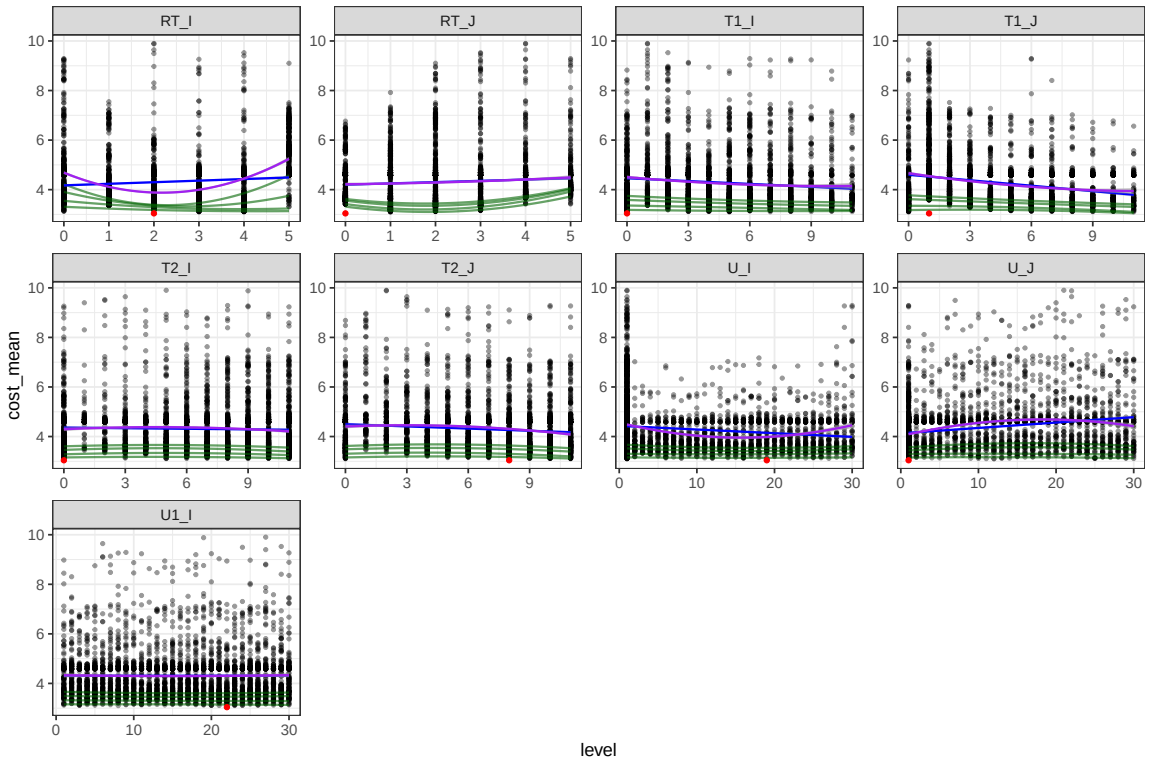

Figure 56: Fitting `lm` linear and cubic models to separate factors for the RS runs

8.4 Predicted Performance at Each Step

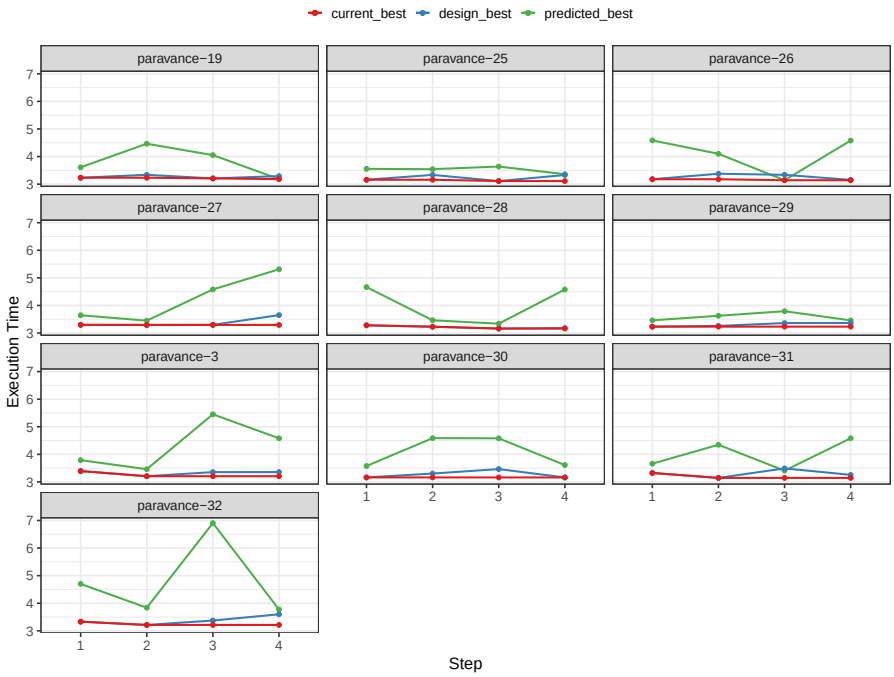

Figure 57: Performance of best points, step-by-step, by experiment

8.5 D-Optimality

8.5.1 D-Criterion

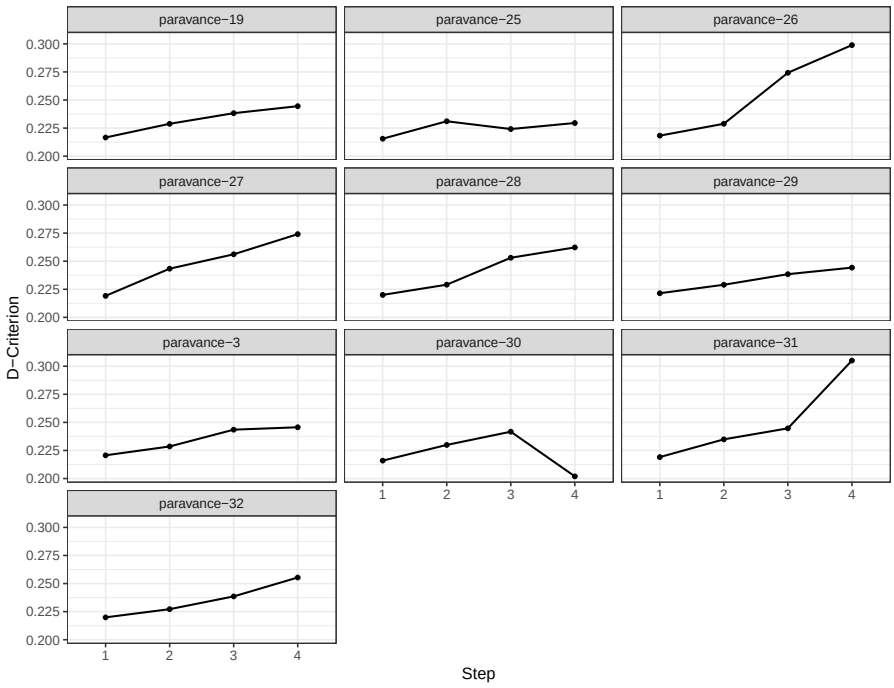

Figure 58: D-Criterion, step-by-step, by experiment

9 Updated DLMT with Quantile Regression ($\tau = 0.15$, "Carry Terms")

9.1 Iterations that found the Best Configuration

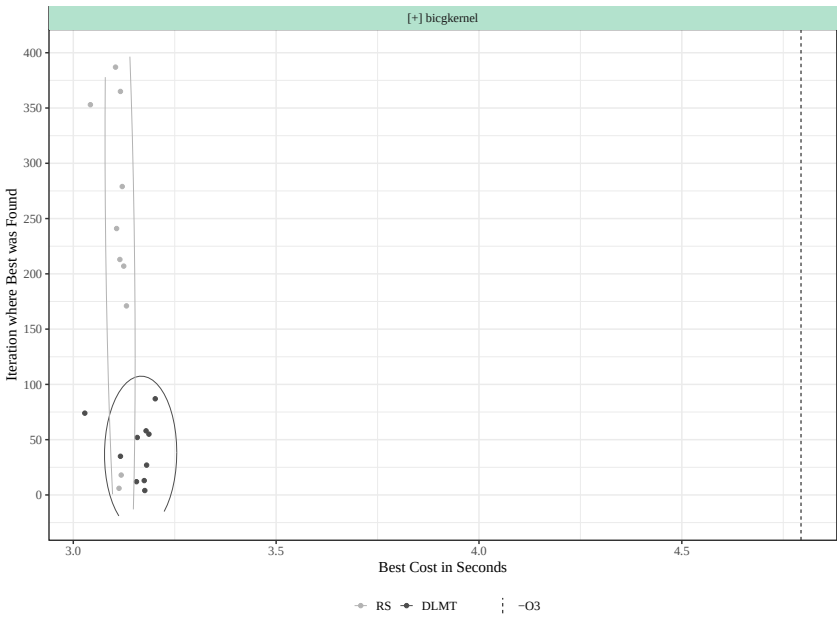

Figure 59: Iteration where best was found

Table 8:

experiment_id	technique	step	cost_mean	T1_I	T1_J	U_J	U_I	T2_I	T2_J	U1_I	RT_I	RT_J
paravance-40	DLMT	1	3.15577	1	2048	1	19	1	2048	27	1	8
paravance-44	DLMT	3	3.186451	2048	2048	30	1	1	2048	17	4	2
paravance-47	DLMT	3	3.179384	32	2048	1	1	1	2048	1	4	2
paravance-63	DLMT	3	3.157642	2048	1	1	14	1	1	1	4	2
paravance-64	DLMT	4	3.028382	1	2	1	30	1	2048	22	4	1
paravance-65	DLMT	2	3.180707	2048	2048	16	1	1	2048	8	4	2
paravance-66	DLMT	1	3.176287	1	2048	1	14	1	1	6	16	8
paravance-67	DLMT	1	3.174842	2048	2048	30	1	1	1	8	16	4
paravance-68	DLMT	4	3.202011	8	64	1	7	256	64	28	8	2
paravance-70	DLMT	2	3.116248	2048	1	29	1	2048	1	30	8	2
paravance-16	RS	1	3.118132	1	256	25	1	32	512	20	16	4
paravance-18	RS	1	3.042194	1	2	1	19	1	256	22	4	1
paravance-2	RS	1	3.131061	1	1024	1	6	256	2048	20	16	2
paravance-20	RS	1	3.120612	256	1024	13	1	1	1024	26	16	2
paravance-21	RS	1	3.112708	32	1	30	1	64	512	20	16	2
paravance-22	RS	1	3.124333	256	1	1	12	256	1024	28	8	2
paravance-23	RS	1	3.11485	512	512	25	1	2048	2048	3	16	4
paravance-24	RS	1	3.106786	128	1	12	1	512	1	23	8	2
paravance-30	RS	1	3.104083	32	2048	12	1	1	2048	8	16	2
paravance-31	RS	1	3.116196	2048	256	1	19	2048	1	24	8	2

9.2 Eliminated Factors and Fixed Values

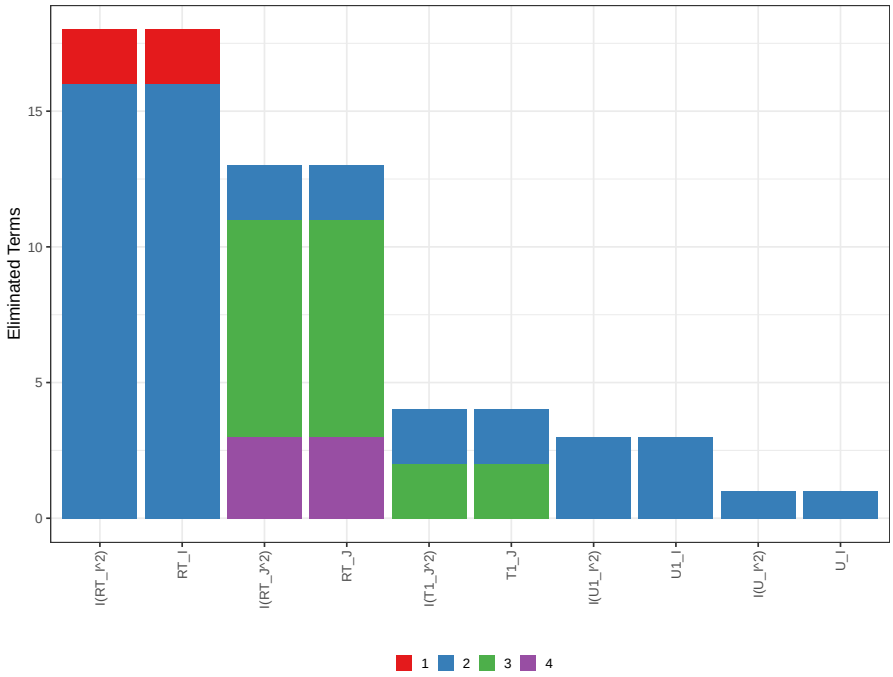

Figure 60: Eliminated terms

9.3 Fixed Factors & Values in the Explored Spaces

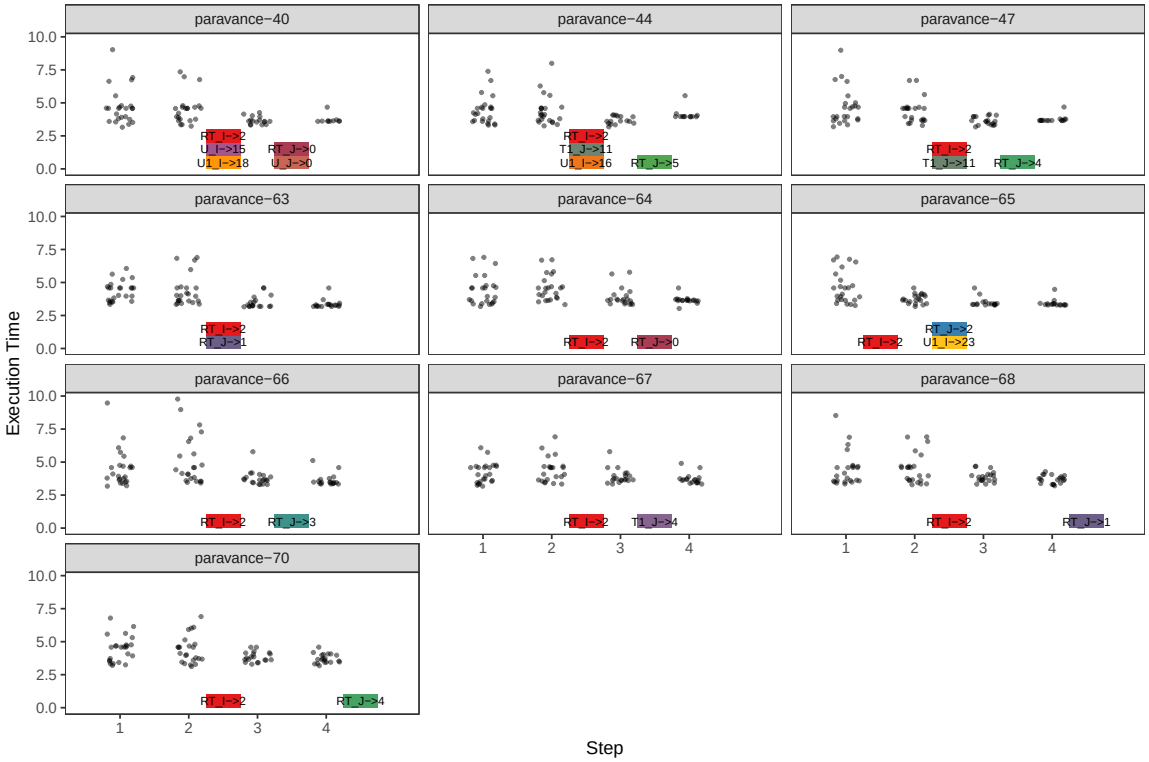

Figure 61: Execution time of Searched Space, step-by-step, by experiment

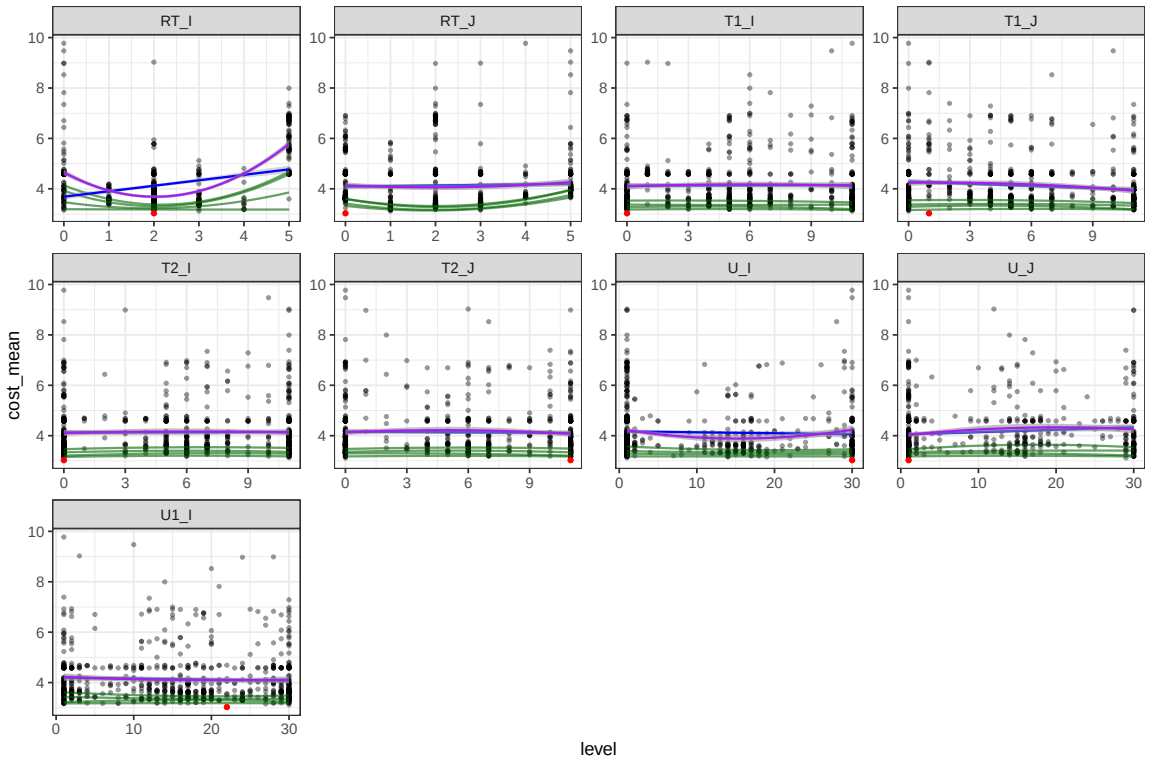

Figure 62: Fitting `lm` linear and cubic models to separate factors for the DLMT runs

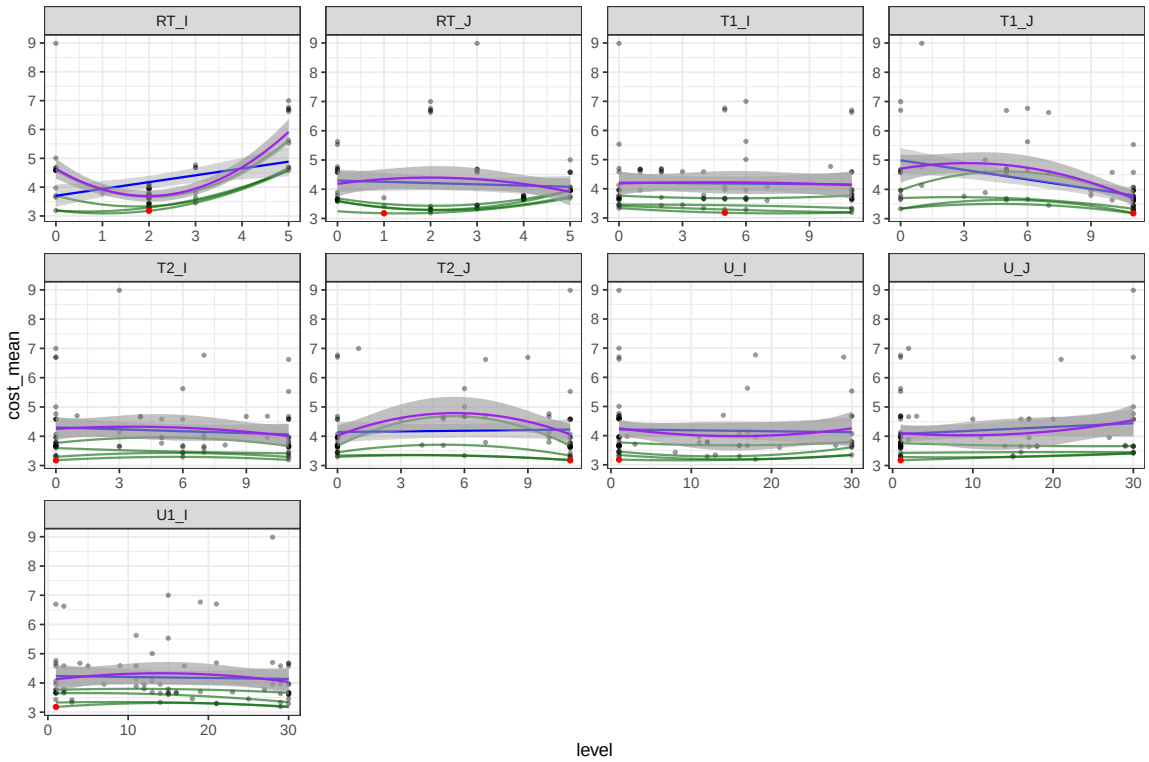

Figure 63: Fitting `lm` linear and cubic models to separate factors for the DLMT runs, for `paravance-47`

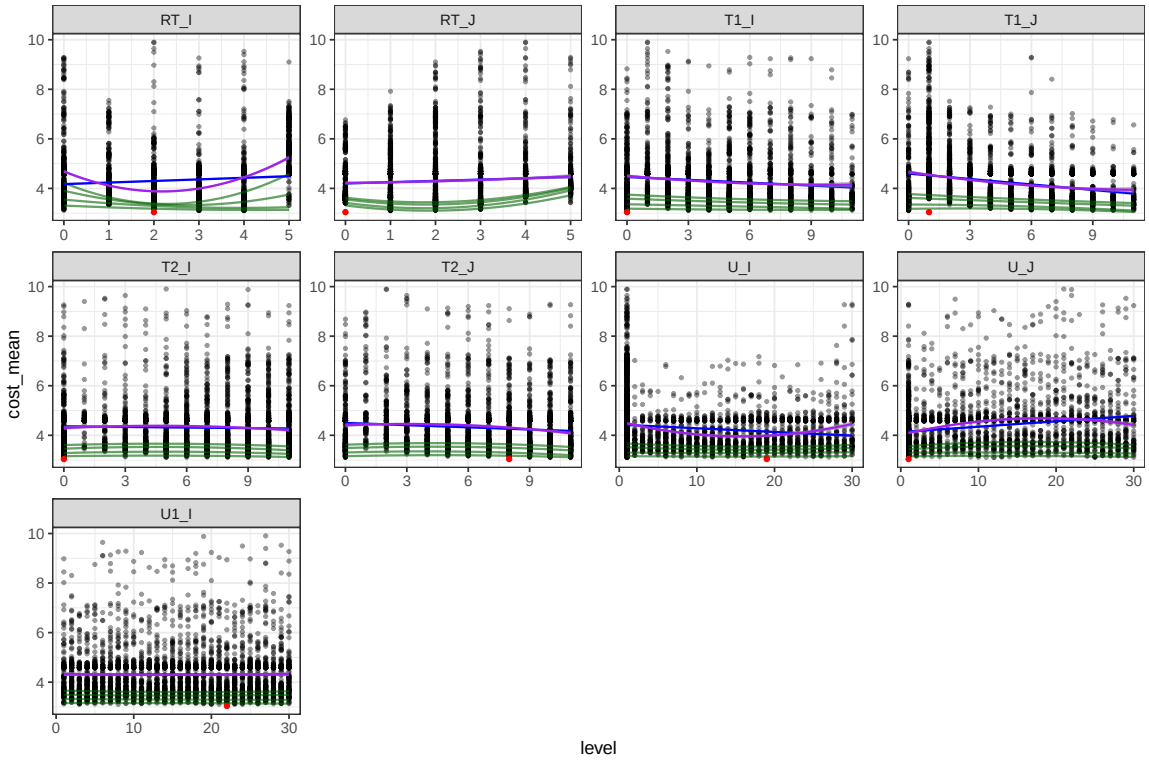

Figure 64: Fitting `lm` linear and cubic models to separate factors for the RS runs

9.4 Predicted Performance at Each Step

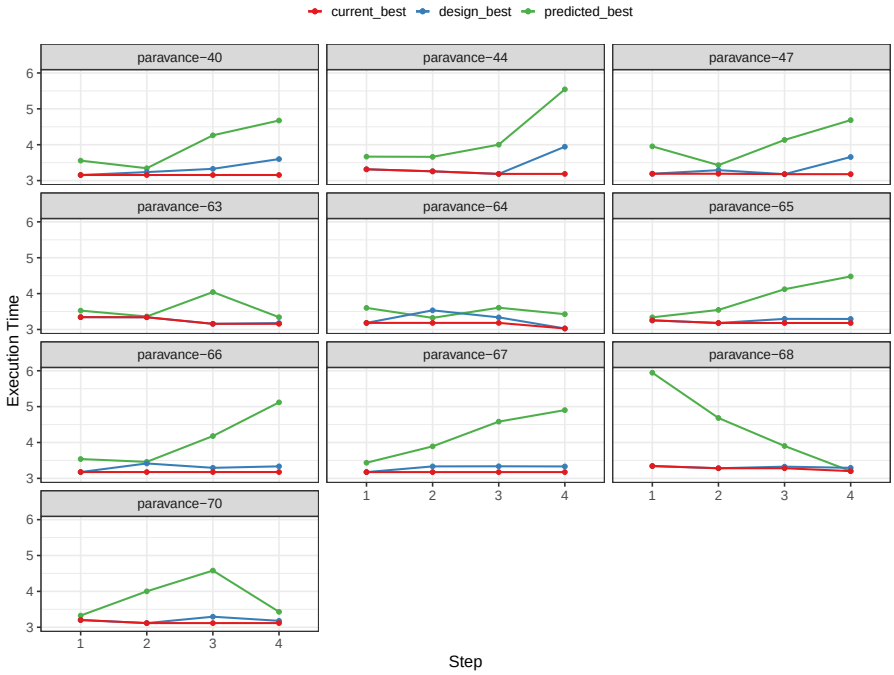

Figure 65: Performance of best points, step-by-step, by experiment

9.5 D-Optimality

9.5.1 D-Criterion

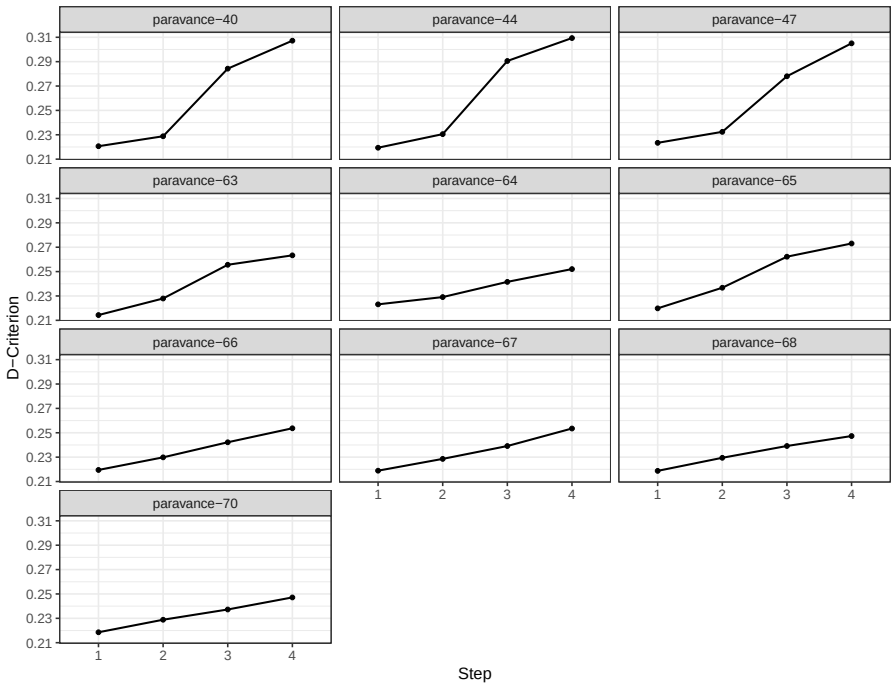

Figure 66: D-Criterion, step-by-step, by experiment