# CS 376 Hybrid Systems

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November 2014

## 1 Hybrid Timed Automaton

Construct a timed automaton that produces *tick* events in a periodic pattern.

$$1, 2, 3, 5, 6, 7, 8, 10, 11, \ldots \tag{1}$$

... or the times between events ...

$$1, 1, 1, 2, 1, 1, 1, 2, 1, 1, \ldots \tag{2}$$

That is three *tick* events one second apart and one *tick* event two seconds later repeated.

The formal model is that of a hybrid automaton. There are also two constants $T_1 = 1$ and $T_2 = 2$.

$$H = (Q, X, Init, f, Inv, E, G, R) \tag{3}$$

The set of discrete variables.

$$Q = \{1st, 2nd, 3rd, 4th\} \tag{4}$$

The set of continuous variables. $X = \mathbb{R}$

$$X = \{s(t)\} \tag{5}$$

The set of initial conditions. $Init \subseteq Q \times X$

$$Init = \{(1st, s(t) := 0)\} \tag{6}$$

The vector field. $f : Q \times X$

$$f = \{(1st, s(\dot{t})), 1) \tag{7}$$
$$(2nd, s(\dot{t})), 1) \tag{8}$$
$$(3rd, s(\dot{t})), 1) \tag{9}$$
$$(4th, s(\dot{t})), 1)\} \tag{10}$$

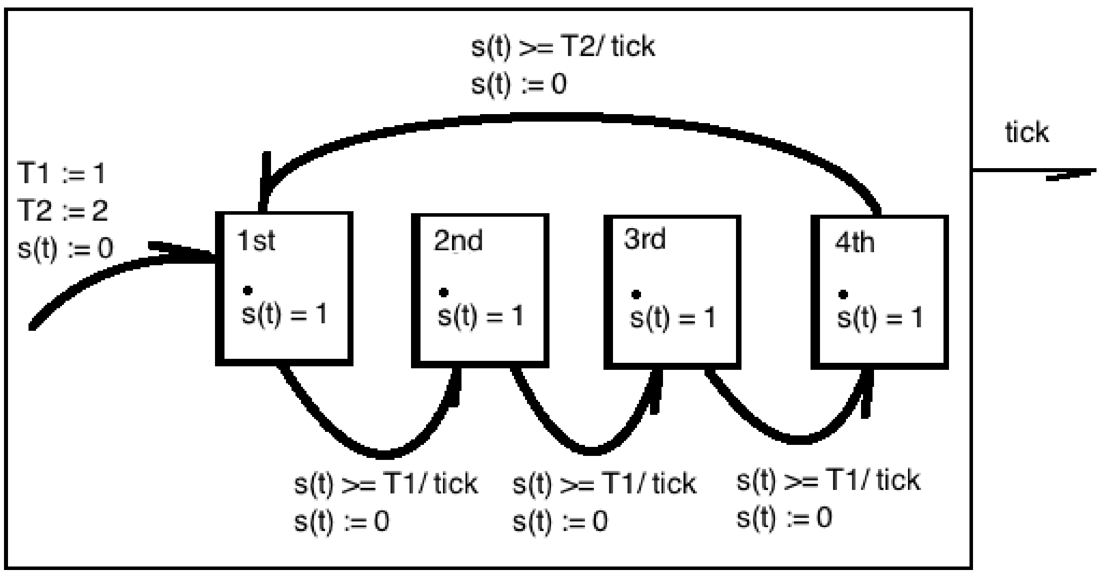


Figure 1: Hybrid timed automata

The invariant set. (all empty) $Q \mapsto 2^X$

$$Inv = \{(1st, \emptyset) \tag{11}$$
$$(2nd, \emptyset) \tag{12}$$
$$(3rd, \emptyset) \tag{13}$$
$$(4th, \emptyset)\} \tag{14}$$

The collection of discrete transitions. $E \subset Q \times Q$

$$E = \{(1st, 2nd) \tag{15}$$
$$(2nd, 3rd) \tag{16}$$
$$(3rd, 4th) \tag{17}$$
$$(4th, 1st)\} \tag{18}$$

The guards on the transitions. $G : E \mapsto 2^X$

$$G = \{((1st, 2nd), s(t) >= T_1/tick) \tag{19}$$
$$((2nd, 3rd), s(t) >= T_1/tick \tag{20}$$
$$((3rd, 4th), s(t) >= T_1/tick \tag{21}$$
$$((4th, 1st), s(t) >= T_2/tick\} \tag{22}$$

The reset relation on the transitions. $R : E \times X \mapsto 2^X$

$$R = \{((1st, 2nd, s(t)), 0) \tag{23}$$
$$((2nd, 3rd, s(t)), 0 \tag{24}$$
$$((3rd, 4th, s(t)), 0 \tag{25}$$
$$((4th, 1st, s(t)), 0\} \tag{26}$$

# 2 Automobile Features

## 2.1 Dome Light

The dome light is turned on as soon as any door is opened. It stays on for 30 seconds after all doors are shut. A sensor which can detect when a door's position, $\{opened, closed\}$ is needed for each door.

The formal model is that of a hybrid automaton.

$$H = (Q, X, Init, f, Inv, E, G, R) \tag{27}$$

The set of discrete variables.

$$Q = \{light_{off}, light_{on}, door_1, door_2, \ldots, door_n\} \tag{28}$$

The set of continuous variables. $X = \mathbb{R}$

$$X = \{s(t), l(t)\} \tag{29}$$

The set of initial conditions. $Init \subseteq Q \times X$

$$Init = \{(light_{off}, l(t) := 0, s(t) := 0)\} \tag{30}$$

The vector field. $f : Q \times X$

$$f = \{(light_{off}, \dot{s(t)}), 1) \tag{31}$$
$$(light_{on}, \dot{s(t)}), 1) \tag{32}$$
$$(door_1, \dot{s(t)}), 1) \tag{33}$$
$$\ldots \tag{34}$$
$$(door_n, \dot{s(t)}), 1)\} \tag{35}$$

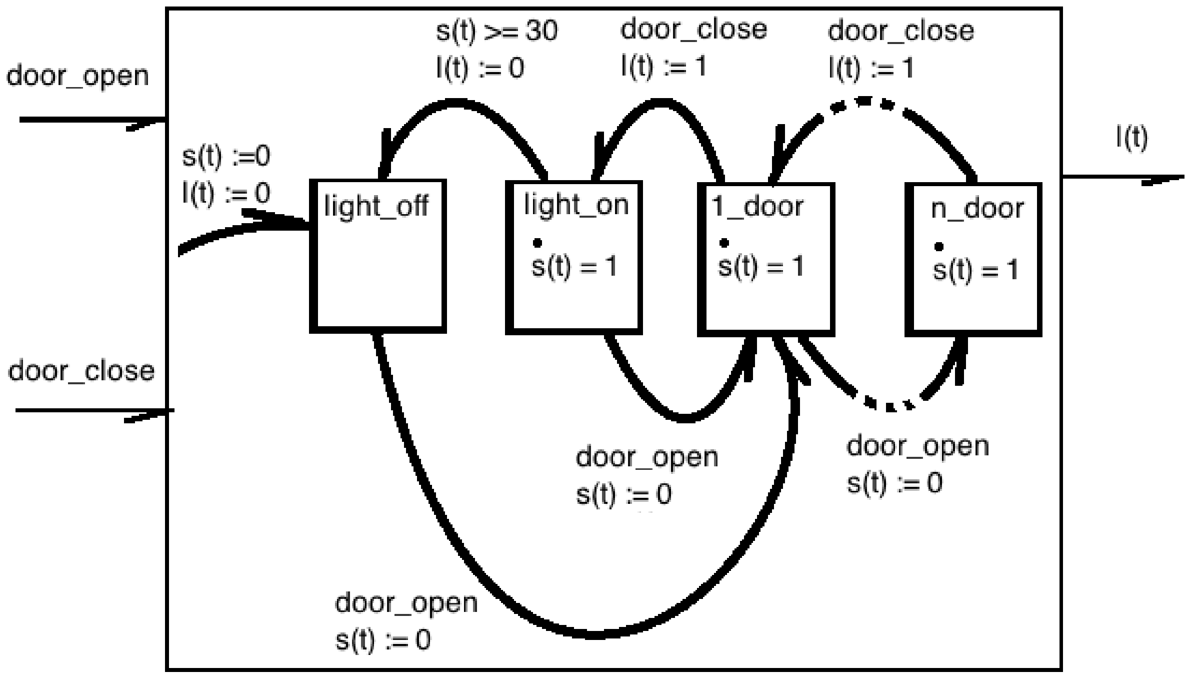


Figure 2: A dome light

The invariant set. (all empty) $Q \mapsto 2^X$

$$
\begin{aligned}
Inv = \{ & (light_{off}, \emptyset) && (36) \\
& (light_{on}, \emptyset) && (37) \\
& (door_1, \emptyset) && (38) \\
& \ldots && (39) \\
& (door_n, \emptyset)\} && (40)
\end{aligned}
$$

The collection of discrete transitions. $E \subset Q \times Q$

$$\begin{aligned} E = \{(light_{off}, door_1) && (41)\\ (light_{on}, door_1) && (42)\\ (door_1, door_2) && (43)\\ \ldots && (44)\\ (door_{n-1}, door_n) && (45)\\ (door_n, door_{n-1}) && (46)\\ \ldots && (47)\\ (door_2, door_1) && (48)\\ (door_1, light_{on}) && (49)\\ (light_{on}, light_{off})\} && (50) \end{aligned}$$

The guards on the transitions. $G : E \mapsto 2^X$

$$\begin{aligned} G = \{(light_{off}, door_1) \mapsto door_{open} && (51)\\ (light_{on}, door_1) \mapsto door_{open} && (52)\\ (door_1, door_2) \mapsto door_{open} && (53)\\ \ldots && (54)\\ (door_{n-1}, door_n) \mapsto door_{open} && (55)\\ (door_n, door_{n-1}) \mapsto door_{close} && (56)\\ \ldots && (57)\\ (door_2, door_1) \mapsto door_{close} && (58)\\ (door_1, light_{on}) \mapsto door_{close} && (59)\\ (light_{on}, light_{off} \mapsto s(t) \geq 30)\} && (60) \end{aligned}$$

The reset relation on the transitions. $R : E \times X \mapsto 2^X$

$$\begin{aligned} R = \{(light_{off}, door_1) \mapsto s(t) := 0 && (61)\\ (light_{on}, door_1) \mapsto s(t) := 0 && (62)\\ (door_1, door_2) \mapsto s(t) := 0 && (63)\\ \ldots && (64)\\ (door_{n-1}, door_n) \mapsto s(t) := 0 && (65)\\ (door_n, door_{n-1}) \mapsto l(t) := 1 && (66)\\ \ldots && (67)\\ (door_2, door_1) \mapsto l(t) := 1 && (68)\\ (door_1, light_{on}) \mapsto l(t) := 1 && (69)\\ (light_{on}, light_{off} \mapsto l(t) := 0)\} && (70) \end{aligned}$$

## 2.2 Safety Belt Alarm

Once the engine is runing, a beeper is sounded and a red light warning is indicated if thre are passengers that have not buckled their seat belt. The beeper stops sounding after 30 seconds, or as soon as the seat belts are buckled, whichever is sooner. The warning light remains on so long as the seatbelt on an occupied seat is not buckled.

The problem is made more tractable by replacing the engine running condition with the ignition on condition (for gasoline engines this is a reasonable assumption).

Each seat has two sensors, one indicating that the $seat \in \{occupiedempty\}$ and one indicating that the $seatbelt \in \{buckledunbuckled\}$. Another sensor indicates $ignition \in \{onoff\}$ In the diagram only the driver and one passenger seat witll be shown. The formal model will be extended to $n$ seats.

Actuators $light_{warning} \in \{onoff\}$ and $beeper_{warning} \in \{onoff\}$ are present.

*warn* an event generated when the *ignition* is turned *on* and any *seat* is *occupied* or when the *ignition* is already *on* and a *seat* becomes occupied.

The formal model is that of a multi-agent hybrid automaton.

$$H = (Q, X, Init, f, Inv, E, G, R) \tag{71}$$

The set of discrete variables.

$$Q_{total} = \{clear, beep, light\} \tag{72}$$

$$Q_{seat} = \{\} \tag{73}$$

$$Q_{aggregate} = \{\} \tag{74}$$

$$Q = Q_{total} \cup Q_{seat}^n \cup Q_{aggregate} \tag{75}$$

First the output variables to the actuators.

$$light_{warn} : l(t) = \begin{cases} 0, & \text{warning light off} \\ 1, & \text{warning light on} \end{cases} \tag{76}$$

$$beep_{warn} : b(t) = \begin{cases} 0, & \text{warning beeper off} \\ 1, & \text{warning beeper on} \end{cases} \tag{77}$$

And, the continuous variables from input sensors.

$$ignition : e(t) = \begin{cases} 0, & \text{ignition off} \\ 1, & \text{ignition on} \end{cases} belt : f(t,i) \quad = \begin{cases} 0, & \text{unbuckled} \\ 1, & \text{buckled} \end{cases} seat : s(t,i) = \begin{cases} 0, & \text{empty} \\ 1, & \text{occupied} \end{cases} \tag{78}$$

Giving us the set of continuous variables.

The set of continuous variables. $X = \mathbb{R}$ Notice that $n$ seats/seatbelts are accomdated and indexed by $i$. The system state comprises the following elements.

$$X = (e(t), r(t), (f(t,i), s(t,i))^n) \tag{79}$$

The model consists of three types of connected agent models.

*noWarn* is an event generated when a *seatbelt* is *buckled* a *seat* becomes *empty* or the *ignition* is turned *off*.

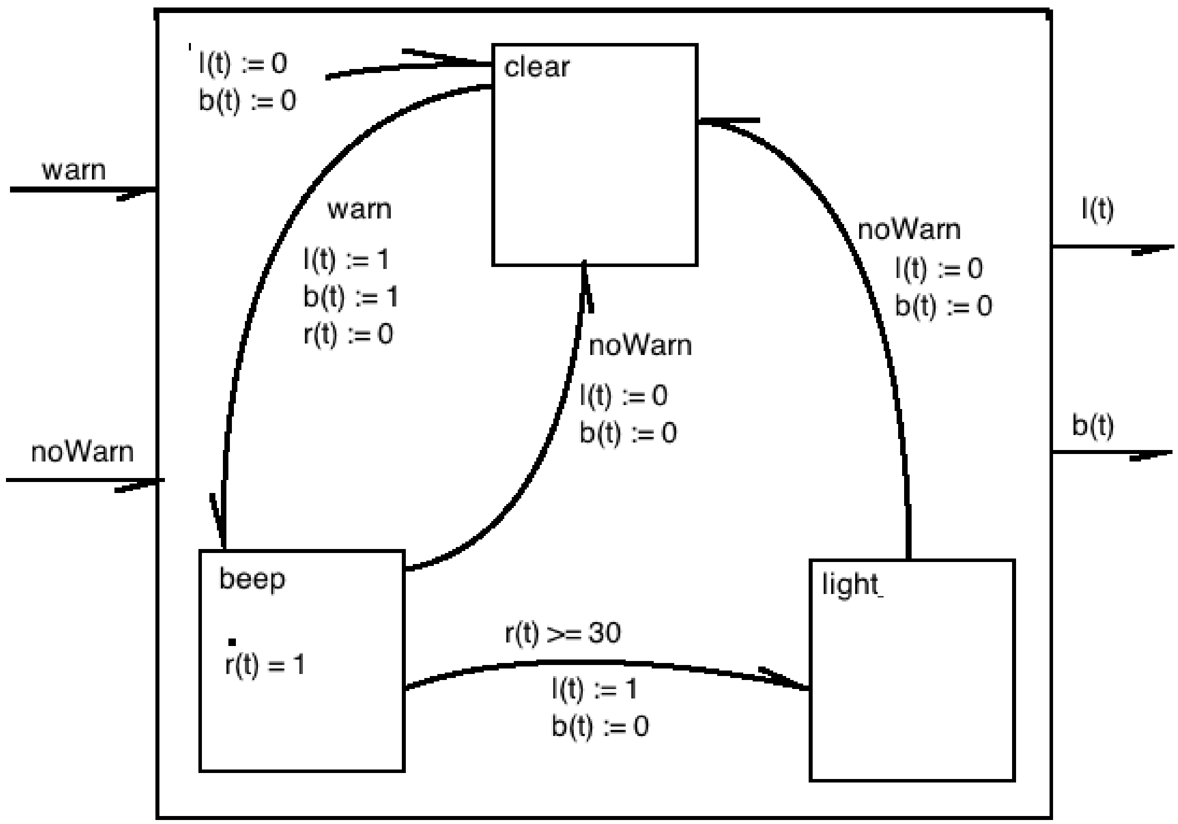


Figure 3: The final actor

The *warn* and *noWarn* inputs to the total model are produced by an aggregation agent with combines the events for the individual $seats_i$.

Feeding into the aggregator agent are $n$ seats.

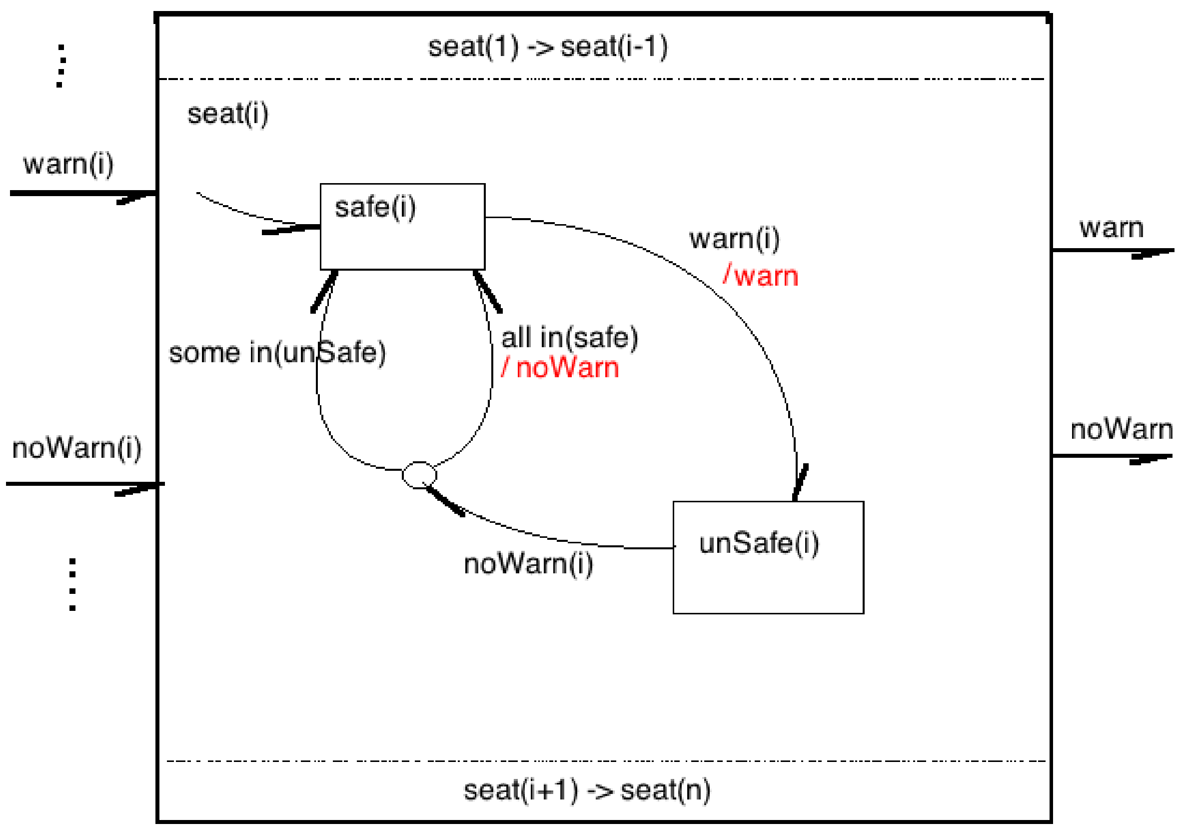


Figure 4: The aggregate actor

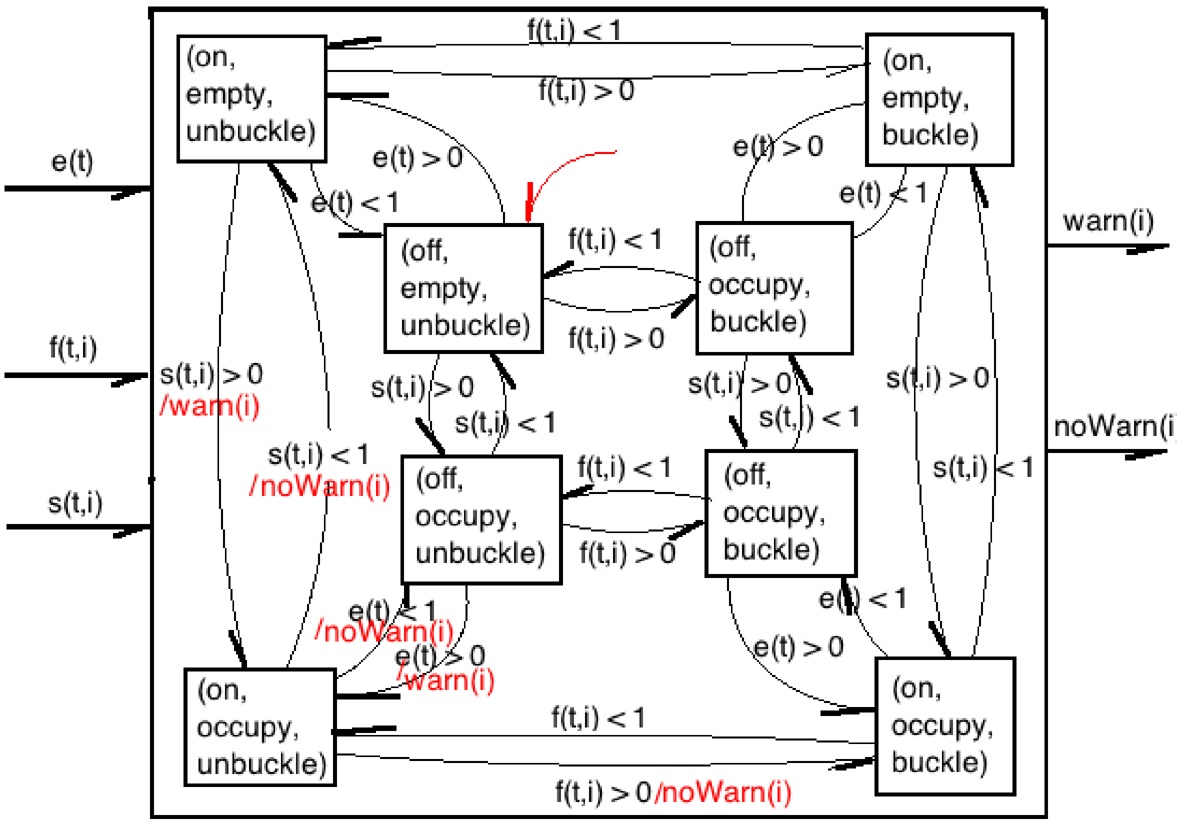


Figure 5: A seat actor