CS 376 : Assignment 6
Modeling and Simulation of Petri Nets

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1 Introduction

A tourist agency is setting up boat sightseeing tour where the boats are autonomous vehicles. You have been employed by the agency to design a control system that drives the boat along the channels as described in the following map.

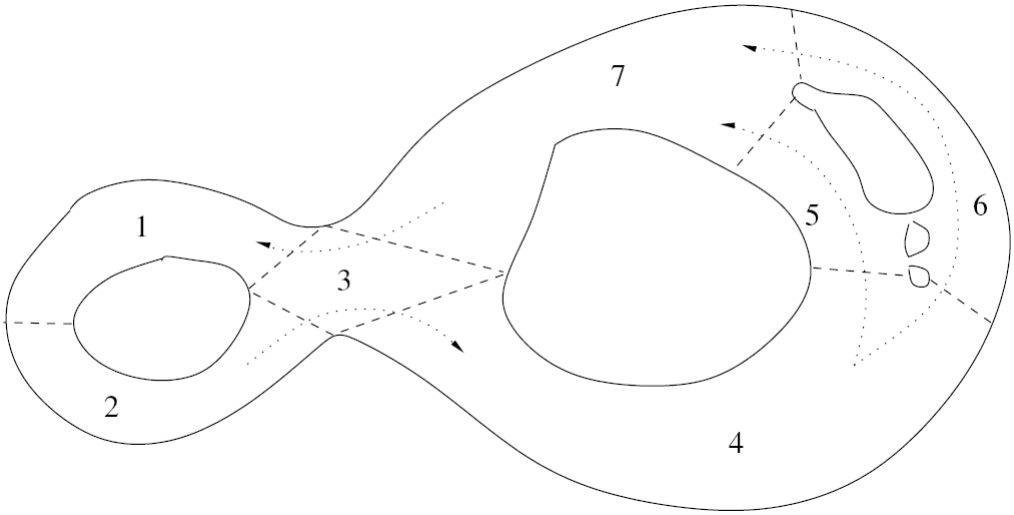

Figure 1: Boat Tour

T he channel, Figure 1, has been divided up into regions. In the Petri-nets, Figures 2 3 and 4, the places with names following the pattern P represent these regions. The exception to this rule is the $P3A$ and $P3B$ places which represent region 3. In that case the place represents the path through the region with

$P3A$ representing the lower path and $P3B$ representing the upper path. The transitions are named based on the boundaries between the regions T where the first digit is the start region and the second digit the destination. A token in any of these places represents a boat. Tokens in other places represent permision (not boats).

2 Single Boat Indeterminate Path

Model the problem of a single boat with a Petri net and describe in detail what each place and transition represents

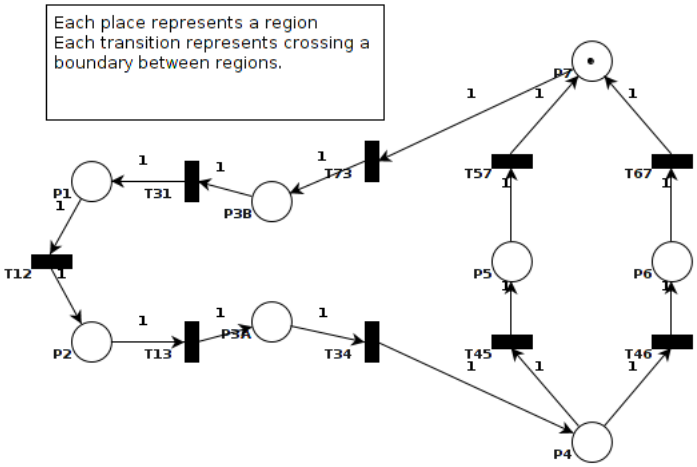

Figure 2: Single Boat with Indeterminate Path

T he formal Petri net is a 5-tuple,

$$PN = (P, T, F, W, M_0) \tag{1}$$

. . . alternatively . . .

$$N = (P, T, F, W) \tag{2}$$

$$PN = (N, M_0) \tag{3}$$

Where for this model . . .

The set of places representing the regions in the canal.

$$P_{canal} = \{P1, P2, P3A, P3B, P4, P5, P6, P7\} \tag{4}$$

$$P = P_{canal} \tag{5}$$

The set of transitions representing the region boundaries.

$$T_{canal} = \{T12, T23, T34, T45, T46, T57, T67, T73, T31\} \tag{6}$$

$$T = T_{canal} \tag{7}$$

The set of arcs representing the direction of movement across the region boundaries. As all weights are 1 it is not explicitly written in the following arc tuples rather it is implicit.

$$PT_{canal} = \{(P1, T12), (P2, T23), (P3A, T34), (P4, T45), (P4, T46), (P5, T57), (P6, T67), (P7, T73), (P3B, T31 \tag{8}$$

$$TP_{canal} = \{(T12, P2), (T23, P3A), (T34, P4), (T45, P5), (T46, P6), (T57, P7), (T67, P7), (T73, P3B), (T31, P1 \tag{9}$$

$$F = PT_{canal} \cup TP_{canal} \tag{10}$$

This completes the structural model for this problem there are a number of valid initial markings. An acceptable marking would be a single token in any of the P_{canal} places.

T his produces an indeterminate path as when there is a token in $P4$ both $T45$ and $T46$ are enabled.

3 Single Boat Determinate Path

The tourist agency wants the boat to pass alternatively through region 5 and 6. The Petri net in Section 2 is modified so that this constraint is satisfied.

This is done by introducing a pair of places, $P- > 5$, $P- > 6$ and a token. The token acts as a permit for the boat to enter either $P5$ or $P6$ depending on whether it is in place $P- > 5$ or $P- > 6$.

The set of places representing the next alternative are added.

$$P_{alternate} = \{P- > 5, P- > 6\} \tag{11}$$

$$P = P_{canal} \cup P_{alternate} \tag{12}$$

No new transitions are required. However new arcs are required (again all weights are 1).

$$PT_{alternate} = \{(P5- > 5, T45), (P- > 6, T46)\} \tag{13}$$

$$TP_{alternate} = \{(T45, P- > 6), (T46, P- > 5)\} \tag{14}$$

$$F = PT_{canal} \cup TP_{canal} \cup PT_{alternate} \cup TP_{alternate} \tag{15}$$

The initial marking now needs a new token in either $P- > 5$ or $P- > 6$ but not both.

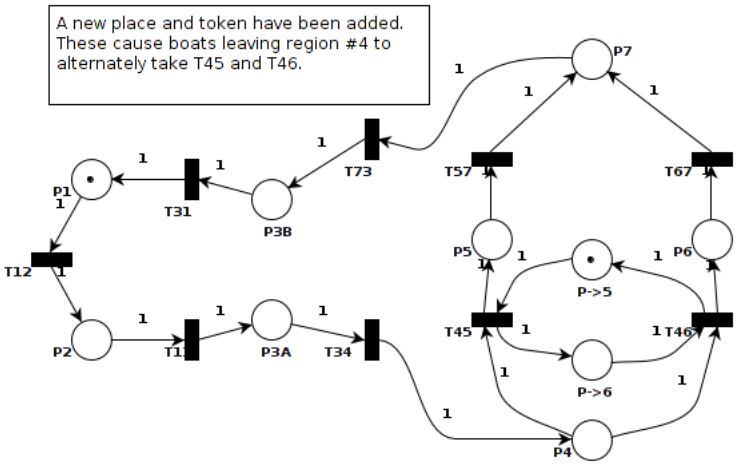

Figure 3: Single Boat with Determinate Path

4 Two Boats Place Exclusion

There are two boats. This Petri-net models a system like that of Section 3 but it additionally only allows one boat to access region 3 at a time.

T his is done by introducing a place, $PX3$ and a token. The token acts as a permit for the boat to enter $P3A$ or $P3B$. Recall that these two places represent one region. For this reason the two places compete for the net token.

The place holding the exclusion token is added.

$$P_{exclusion} = \{PX3\} \tag{16}$$

$$P = P_{canal} \cup P_{alternate} \cup P_{exclusion} \tag{17}$$

No new transitions are required. However new arcs are required (again all weights are 1).

$$PT_{exclusion} = \{(PX3, T73), (PX3, T23)\} \tag{18}$$

$$TP_{exclusion} = \{(T31, PX3), (T34, PX3)\} \tag{19}$$

$$F = PT_{canal} \cup TP_{canal} \cup PT_{alternate} \cup TP_{alternate} \cup PT_{exclusion} \cup TP_{exclusion} \tag{20}$$

The initial marking now needs a new token in $PX3$ unless there is a token in either $P3A$ or $P3B$. In addition the P_{canal} places may contain more than

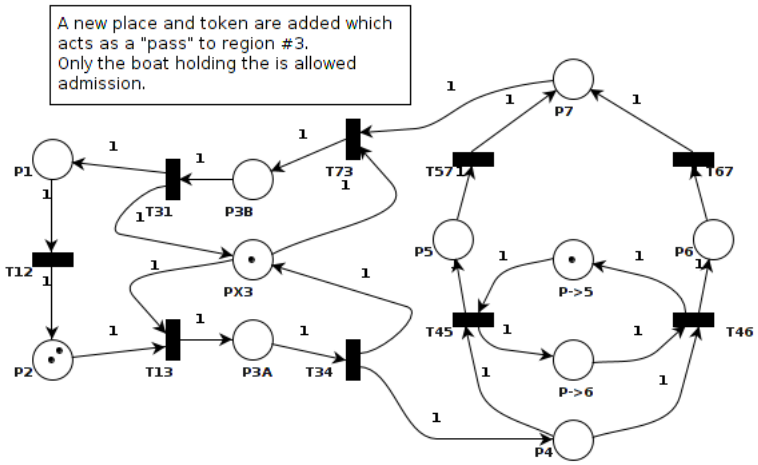

Figure 4: Two Boat with Exclusion Zone and Determinate Path

one token (the problem calls for exactly 2). However, there may be at most one token in $P3A \cup P3B$.