

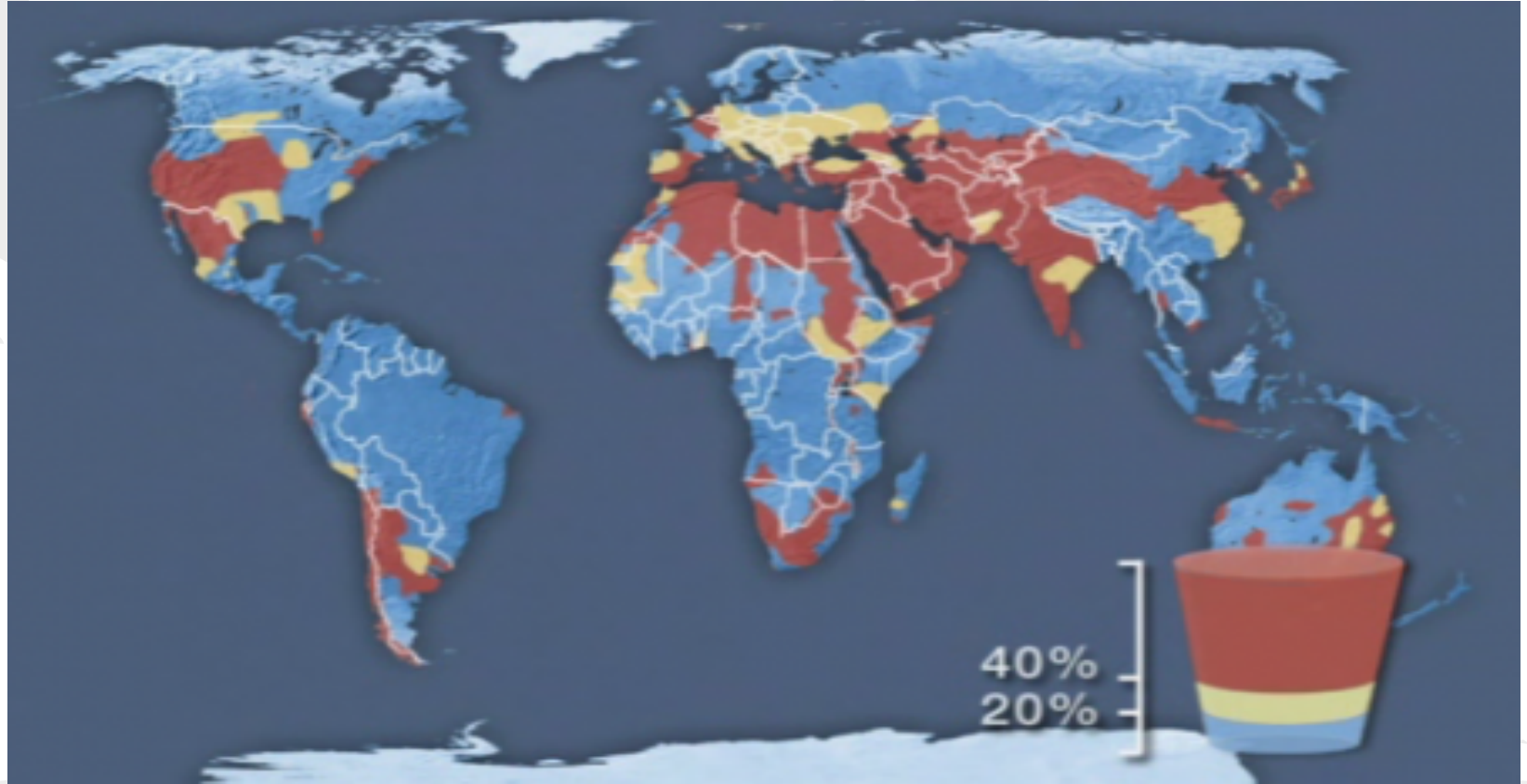
# Review: A Hybrid System Model of Seasonal Snowpack Water Balance

*Application of hybrid modeling to natural systems*

# Personal Motivation



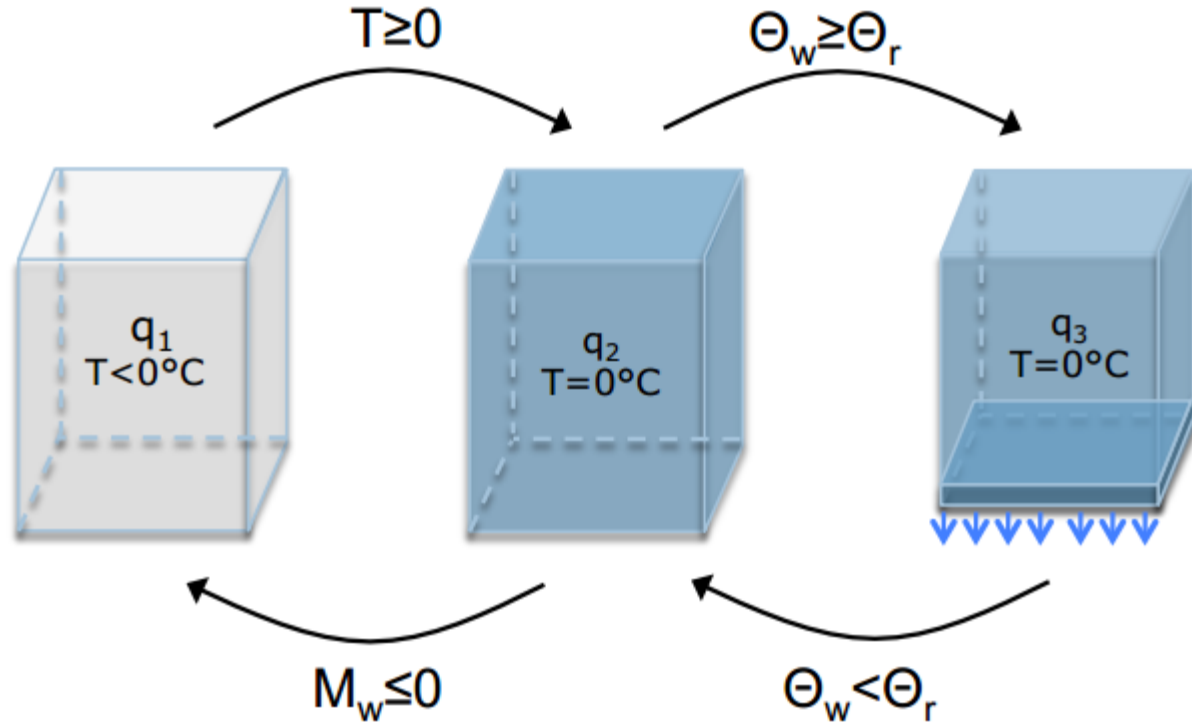
# Motivation



# The Discrete Model



$M = 0$



# The Basics

$$\frac{dT}{dt} = \frac{u(t)}{M_{snow}C_{snow}}$$

$$\begin{aligned}\frac{dM_{water}}{dt} &= -\frac{dM_{ice}}{dt} \\ &= \frac{u(t)}{L_f}\end{aligned}$$

# The Settling

$$\rho_{snow}(t) = \frac{A}{1 + B/t}$$

$$\frac{d\rho_{snow}(t)}{dt} = \frac{AB}{(B + t)^2}$$

$$t = \frac{\rho_{snow} B}{A - \rho_{snow}}$$

$$B = 20 \text{ days}$$

$$A = 450 \text{ kg/m}^3$$

$$\frac{d\rho_{snow}(t)}{dt} = \frac{A}{B(1 + \frac{\rho_{snow}(t)}{A - \rho_{snow}(t)})^2}$$

# The Melting Sponge

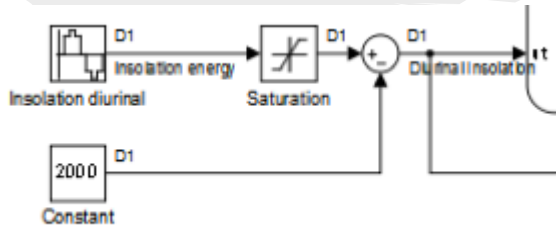
$$\begin{aligned}\theta_{snow} &= \frac{V_{water}}{V_{total}} \\ &= \frac{M_{water} / \rho_{water}}{M_{snow} / \rho_{snow}}\end{aligned}$$

$$M_{water} = \theta_r \rho_{water} \frac{M_{ice}}{\rho_{snow} - \theta_r \rho_{water}}$$

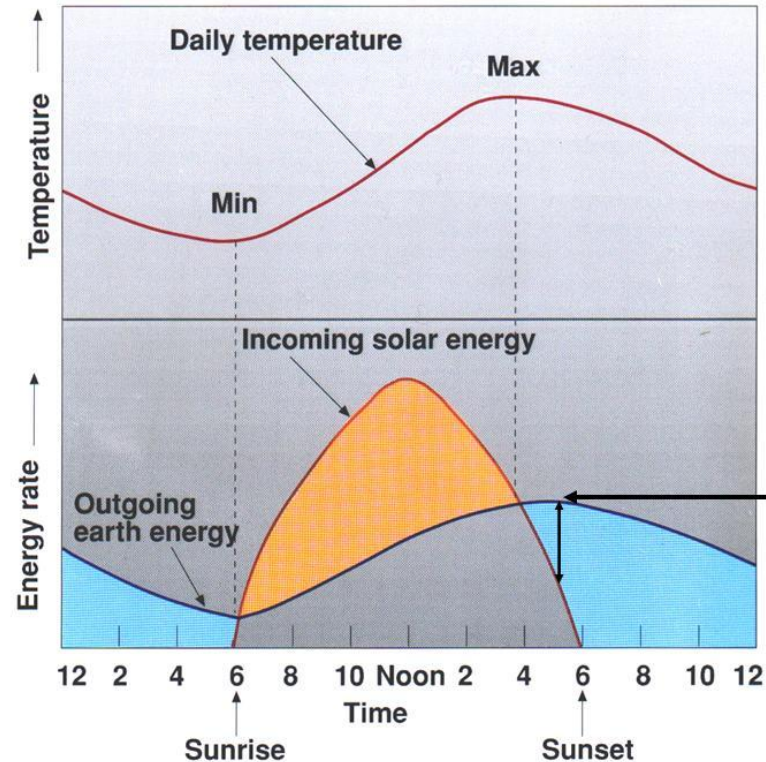
$$\frac{dM_{water}}{dt} = \theta_r \rho_{water} \left[ \frac{\frac{dM_{water}}{dt} (\rho_{snow} - \theta_r \rho_{water}) - M_{ice} \frac{d\rho_{snow}}{dt}}{(\rho_{snow} - \theta_r \rho_{water})^2} \right]$$

# Energy Input / Calibration

$$u(t)_{solar} = a \sin 2\pi t$$

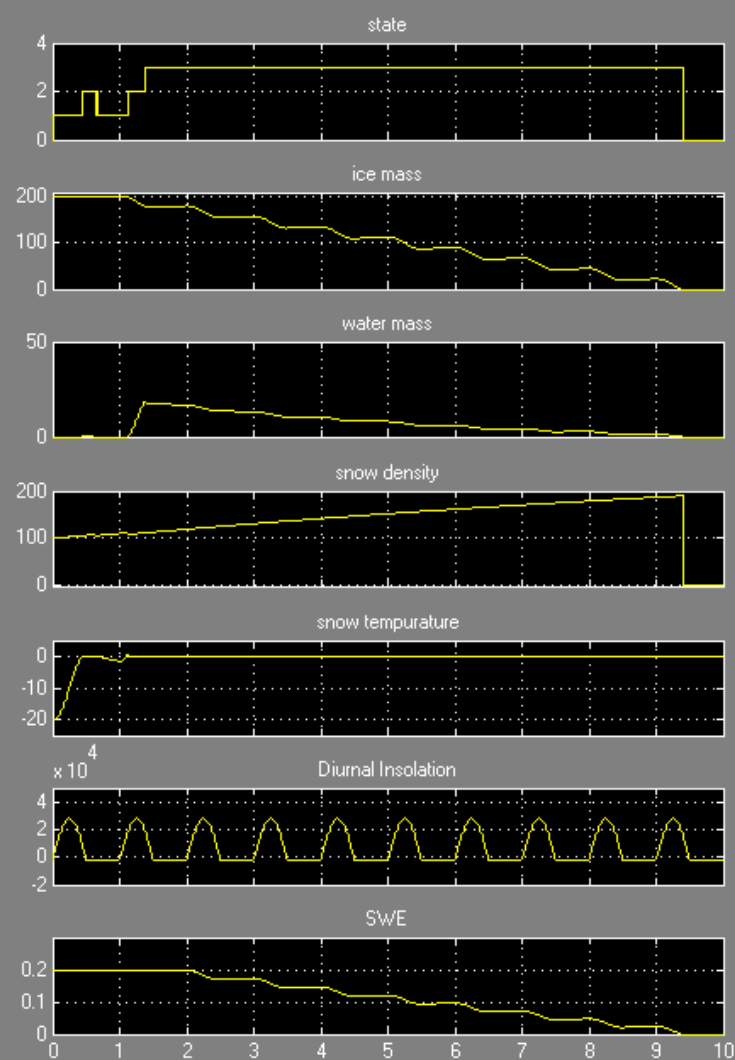
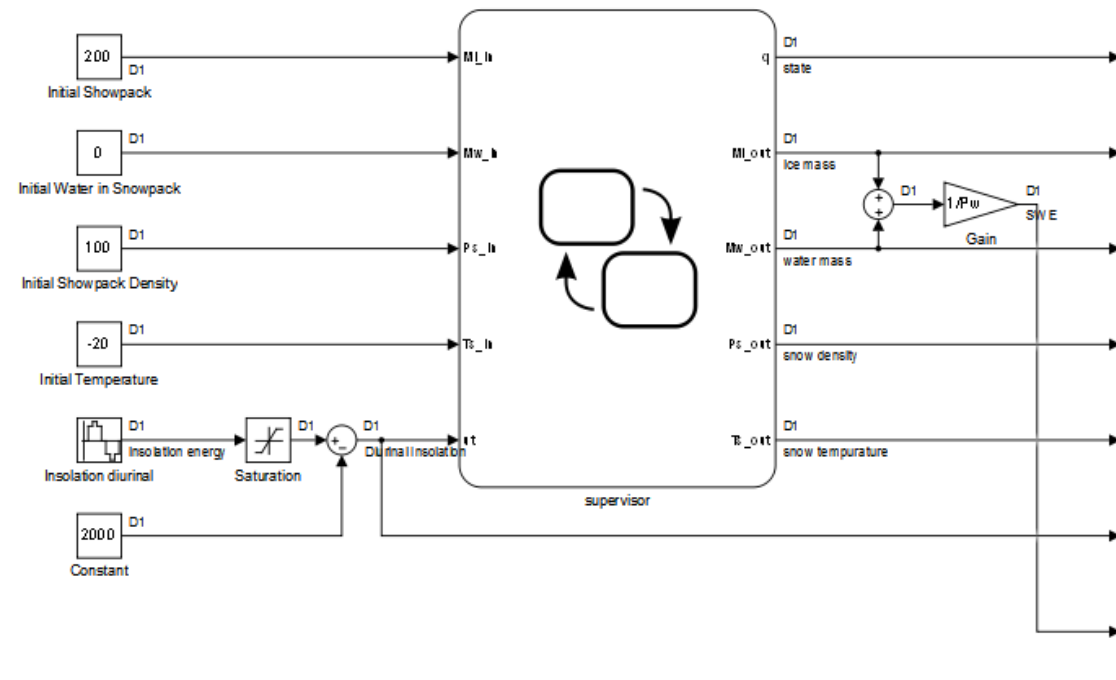


**kJ / m / day (not kW)**

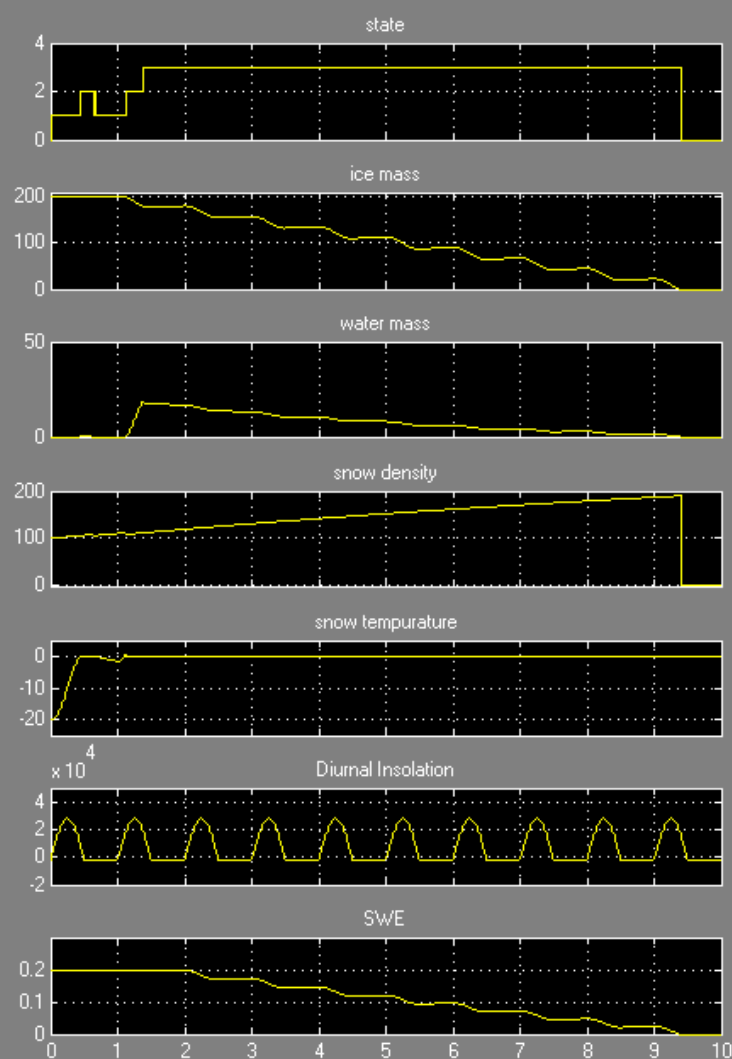
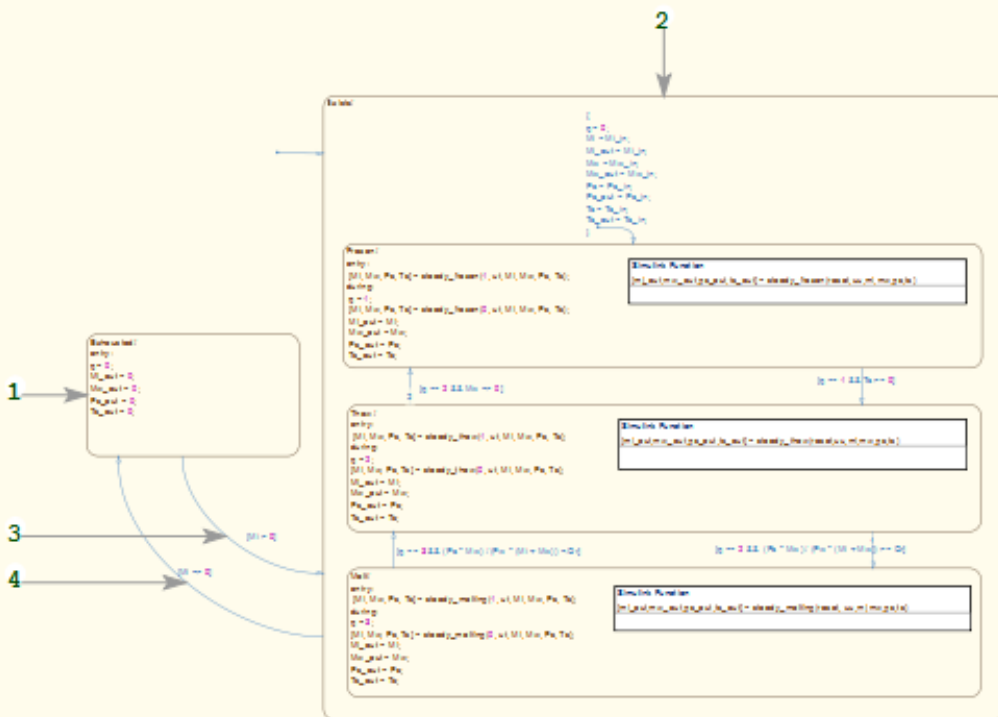




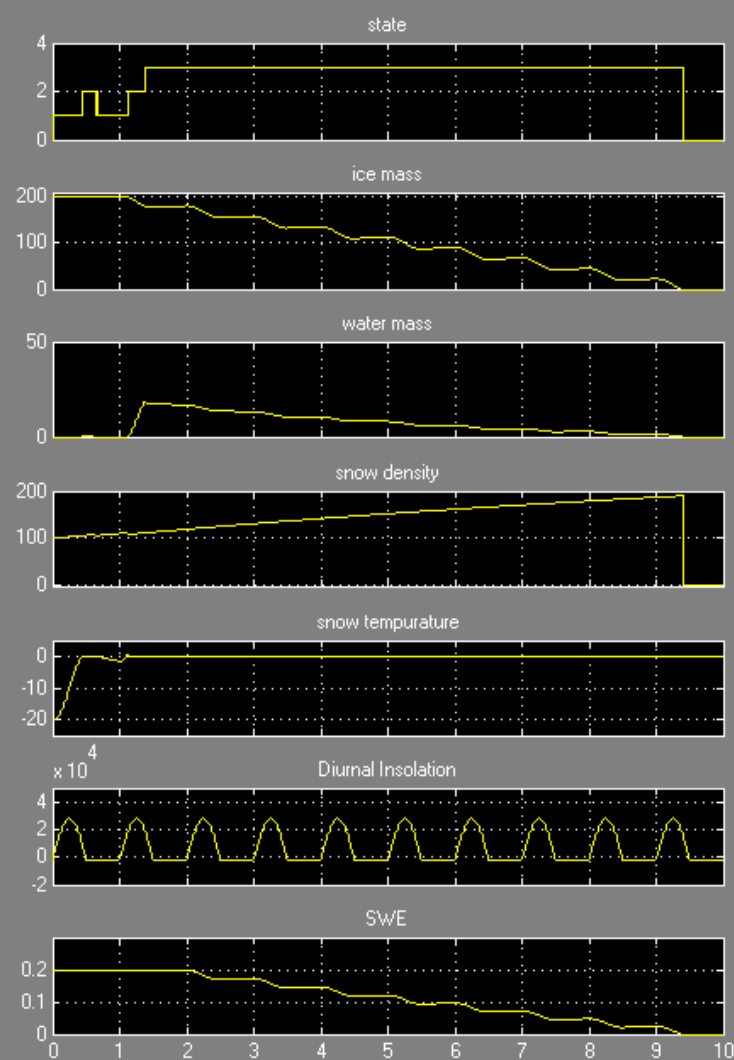
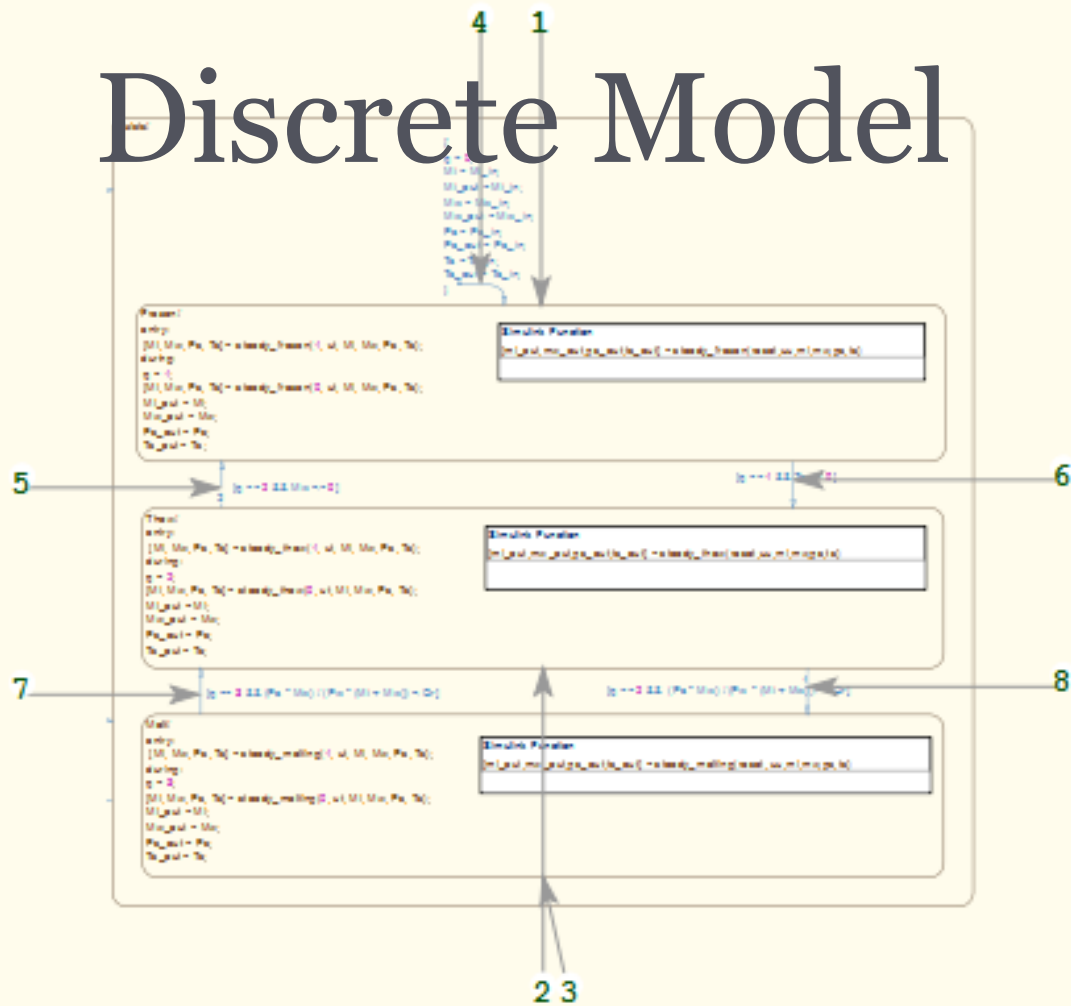
# Main Model



# Discrete Model



# Discrete Model

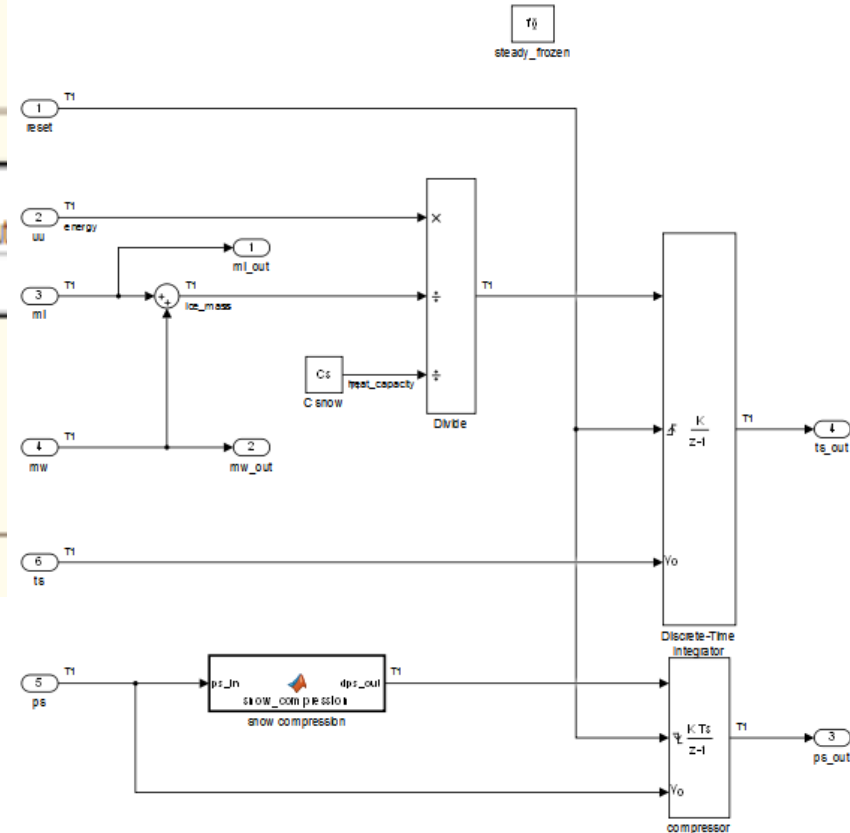


# Dynamic Model / Frozen

```

Frozen/
entry:
[Ml, Mw, Ps, Ts] = steady_frozen(1, ut, Ml, Mw, Ps, Ts);
during:
q = 1;
[Ml, Mw, Ps, Ts] = steady_frozen(0, ut, Ml, Mw, Ps, Ts);
Ml_out = Ml;
Mw_out = Mw;
Ps_out = Ps;
Ts_out = Ts;
    
```

Simulink Function  
[ml\_out,mw\_out,ps\_out,ts\_out



$$\frac{d\rho_{snow}(t)}{dt} = \frac{A}{B(1 + \frac{\rho_{snow}(t)}{A - \rho_{snow}(t)})^2}$$



*Questions*