CS 376

Foundations of Hybrid and Embedded Systems

Assignment 8

Hybrid Simulation of a Tank System Example

Consider the two-tank system shown in Figure 1. The system consists of two identical cylindrical tanks that are connected by a pipe at level h. We denote by h_1 and h_2 the water levels in tanks 1 and 2 respectively. The input flow Q_{in} is provided by a pump and it is described by

$$Q_{in} = V_{in} k_{in} u(t),$$

where $V_{in} \in \{0, 1\}$ represents a valve that can be used to turn on or off the pump, k_{in} is a linear gain, and $u(t)$ is the input signal representing the flow at the pump. The flow Q_a between the two tanks is controlled by a valve V_a. An outlet valve V_{out} located at the bottom of tank 2 is used to empty the tank. Tank 2 is equipped with a sensor that measures the output flow which is described by

$$Q_{out} = V_{out} k_{out} \sqrt{\rho g h_2} \tag{1}$$

where $V_{out} \in \{0, 1\}$ represents the outlet valve, k_{out} is a linear gain, ρ is the density of the water, and g is the gravity constant.

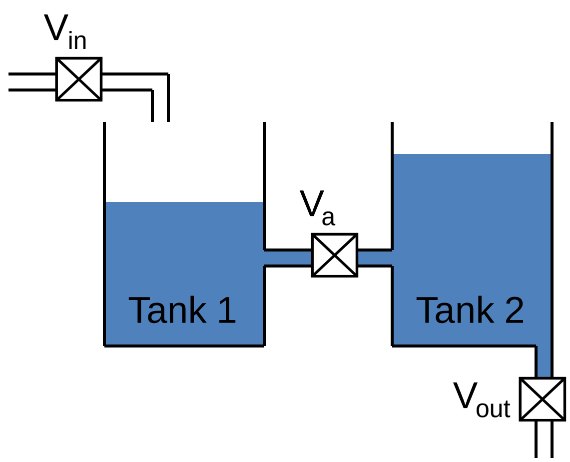

Figure 1: Two-tank system

The dynamic evolution of the system is described by

$$\begin{aligned} \dot{h}_1 &= \frac{1}{A}(Q_{in} - Q_a) \\ \dot{h}_2 &= \frac{1}{A}(Q_a - Q_{out}) \end{aligned}$$

where A is the section of each cylindrical tank. According to Toricelli's law, the flow Q_a depends on the water levels h_1 and h_2 as follows:

$$Q_a = \begin{cases} 0, & \text{if } h_1 < h \text{ and } h_2 < h \\ V_a k_a \sqrt{\rho g (h_1 - h)}, & \text{if } h_1 > h \text{ and } h_2 < h \\ -V_a k_a \sqrt{\rho g (h_2 - h)}, & \text{if } h_1 < h \text{ and } h_2 > h \\ \text{sign}(h_1 - h_2) V_a k_a \sqrt{\rho g |h_1 - h_2|}, & \text{if } h_1 > h \text{ and } h_2 > h \end{cases} \tag{2}$$

where $V_a \in \{0, 1\}$ and k_a is a linear gain. The evolution of the continuous state $x = [h_1, h_2]^T$ can be described by

$$\dot{x} = f_q(x(t), u(t)) \tag{3}$$

where q is the discrete mode of the system. The mode transitions are based on guard conditions on $x = [h_1, h_2]^T$ based on the equation (2) (you can use equality in the strict inequalities as you feel appropriate).

Assume the following values for the parameters of the system. All valves are open $V_{in} = V_a = V_{out} = 1$, the input $u(t)$ is a pulse with amplitude 1 and frequency 1Hz, $h = .3$m, $k_{in} = .06$, $k_a = .001$, $k_{out} = .001$, $g = 9.81$m/sec^2, $\rho = 1000$, and $A = 0.0154$m^2.

1. Develop a hybrid automaton model for the two-tank system.
2. Simulate your model for the following initial conditions: $x_0 = [.2, .75]^T, [.5, .2]^T, [.5, .5]^T$ and $[.2, .2]^T$. Plot the continuous state, discrete state, and output of the system.