CS 376 : HW 4 - Continuous Systems

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October 2014

This problem set is taken from [1] chapter 2 : Problem 6.

This system makes use of a helicopter model Figure 1. In this model the value of $a = 5$ and $T_top = 4$. This Simulink model corresponds to the mathematical model. For the Scale we have ...

$$\forall t \in \mathbb{R}, \qquad y_1(t) = ax_1(t), \qquad a = \frac{1}{I_{yy}} = 4 \quad units \tag{1}$$

... and for the Integrator ...

$$\forall t \in \mathbb{R}, \qquad y_2(t) = i + \int_0^t x_2(\tau)\,\mathrm{d}\tau, \qquad i = \dot{\theta}(0) = 0 \tag{2}$$

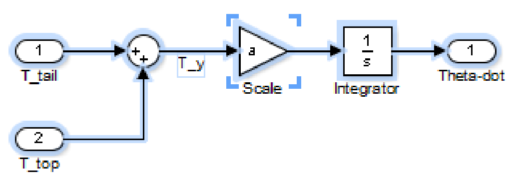

Figure 1: the helicopter plant model (SimuLink)

Using Simulink and its continuous-time modeling component I have built a model of the helicopter control system shown in Figure 2. This model has an upper section that models a system with a proportional (P) controller and a lower section that models a proportional-integrator (PI) controller. The P-controller subsystem illustrates problem a) with the lower subsystem modeling problem b). These are placed together so that the inputs and outputs may be shared.

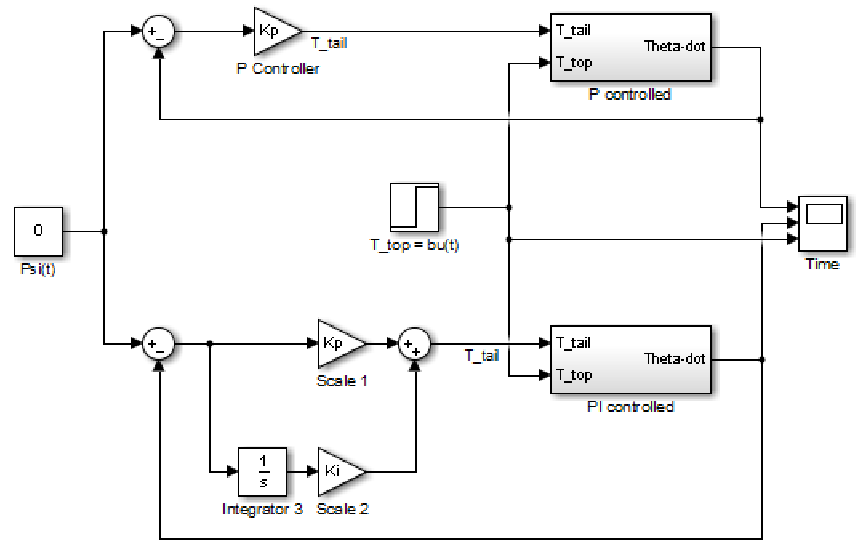

Figure 2: the control systems models (SimuLink)

1 P-controller system

Given some reasonable input parameters the actual angular velocity, $\dot{\theta}$, is shown as a function of time. The initial and operating conditions specify that the desired angular velocity is zero, $\phi(t) = 0$, and that the top-rotor torque is non-zero, $T_{top}(t) = bu(t)$ moving to that value as a step function. Given are plots for several values of K_p.

Once the T_{top} has changed a new stable but typically non-zero $\dot{\theta}$ is approached asymtotically. Larger values of K_p cause the convergence to happen more quickly and decrease the value of the stable $\dot{\theta}$.

2 PI-controller system

The lower portion of Figure 2 replaces the proportional (P) controller of the upper system with a propotional-integrator (PI) controller. This alternative controller has two parameter K_p and K_i. Experiment with the values of these parameters, give some plots of the behavior with the same inputs as in part (a), and discuss the behavior of this controller in contrast to the one of part (a).

Once the T_{top} has changed a new (typically non-zero) $\dot{\theta}$ is approached controlled-variable, $\Psi = 0$, is approached asymtotically by $\dot{\theta}$ after T_{top} changes. Larger values of K_p cause the convergence to happen more quickly.

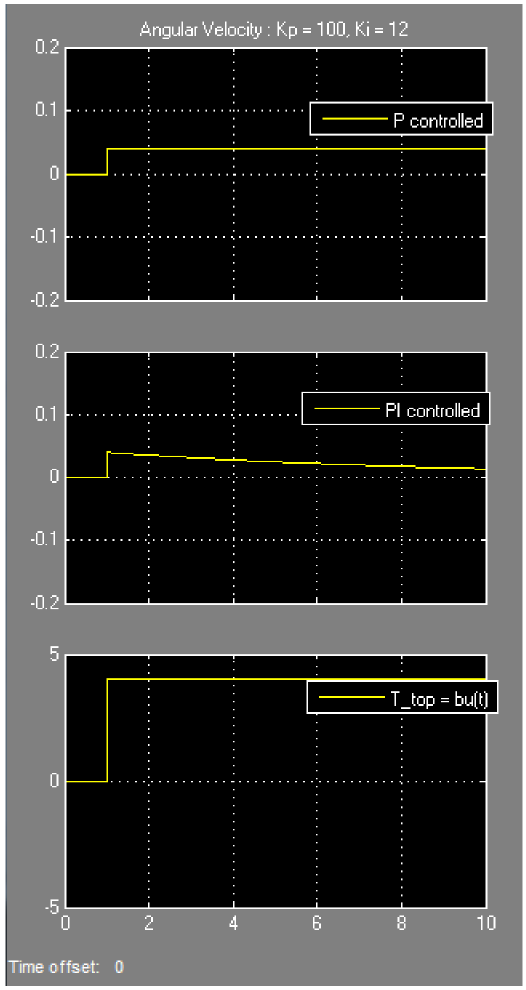

Figure 3: the control systems models (SimuLink)

References

[1] Edward Lee. *Introduction to Embedded Systems : A Cyber-Physical Systems Approach.* 2011-2012.

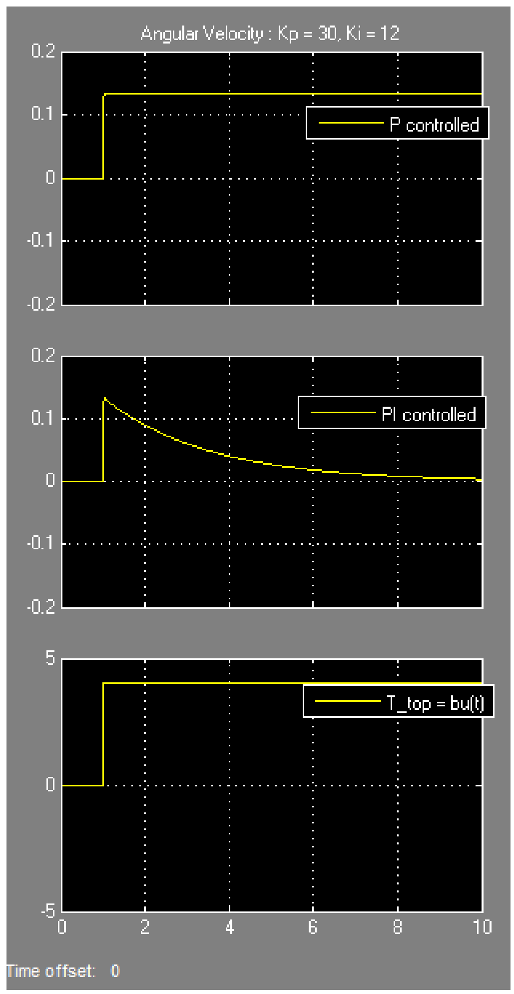

Figure 4: the control systems models (SimuLink)

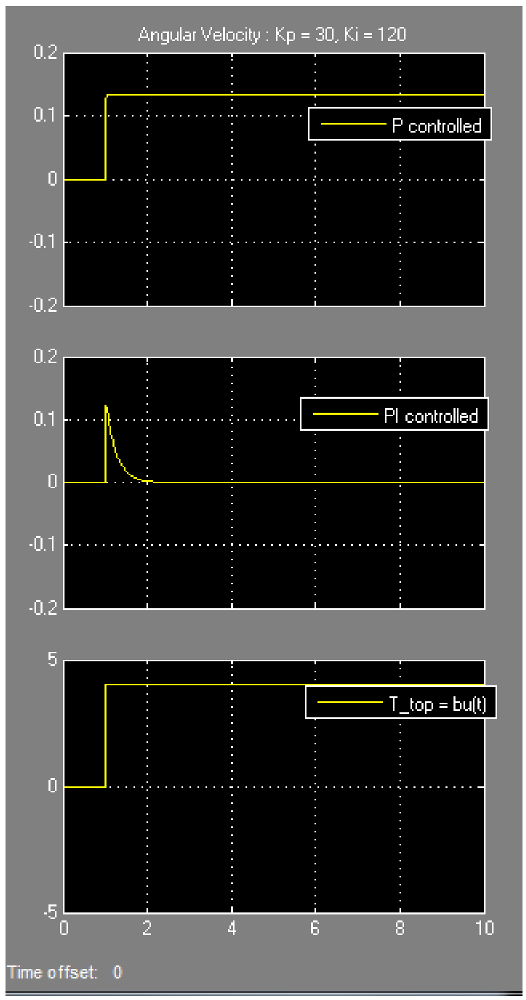

Figure 5: the control systems models (SimuLink)

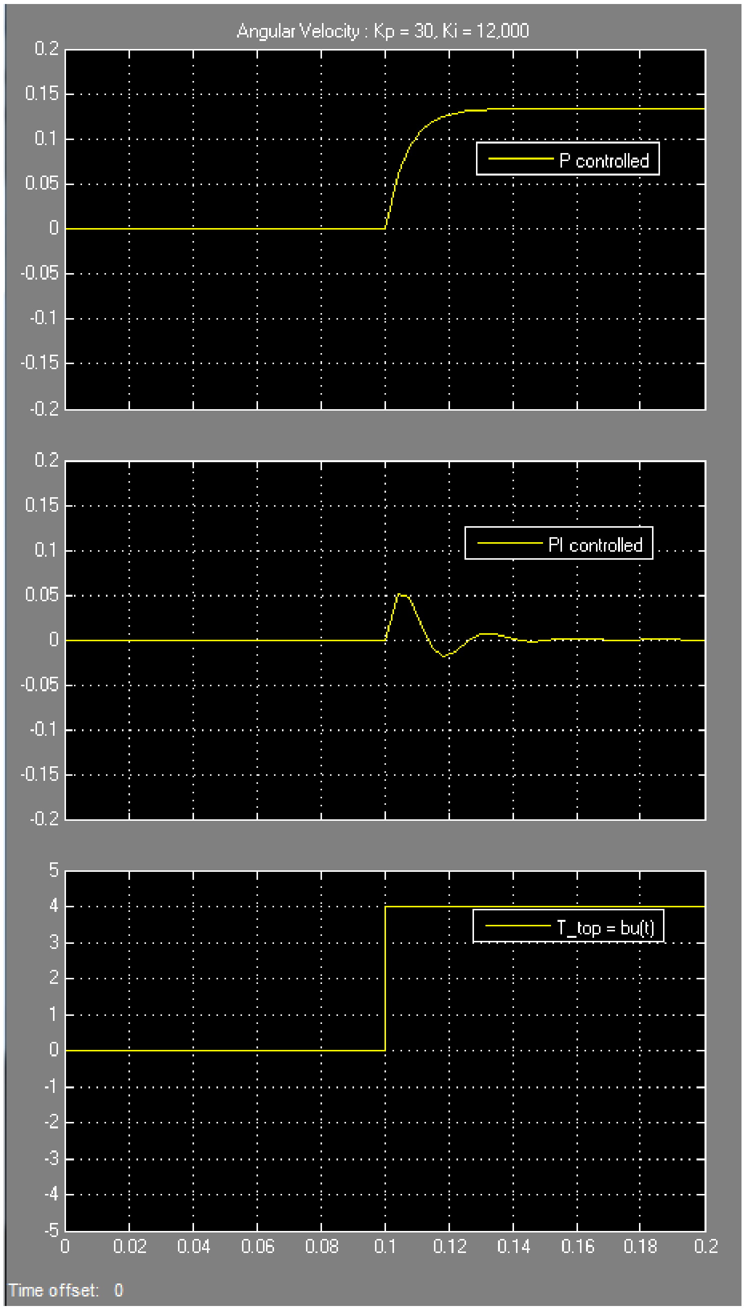

Figure 6: the control systems models (SimuLink)

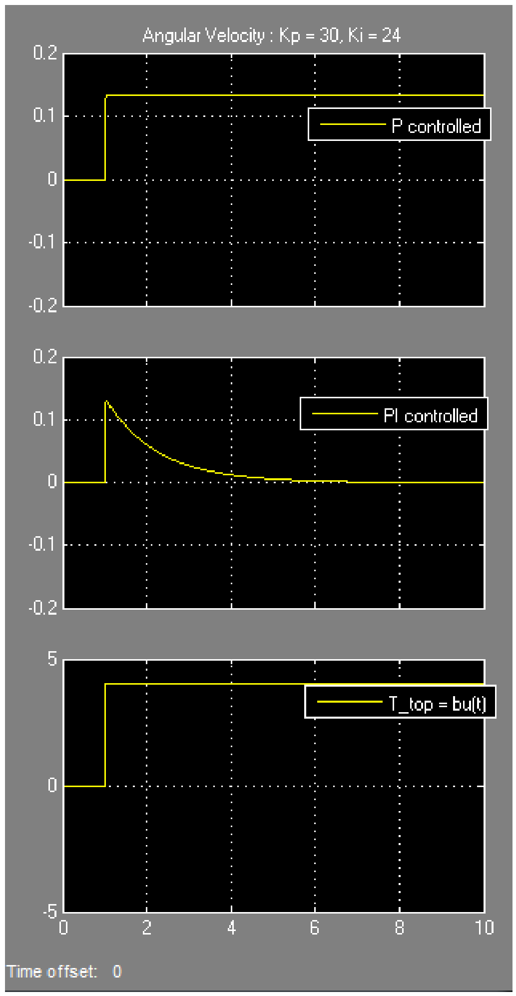

Figure 7: the control systems models (SimuLink)

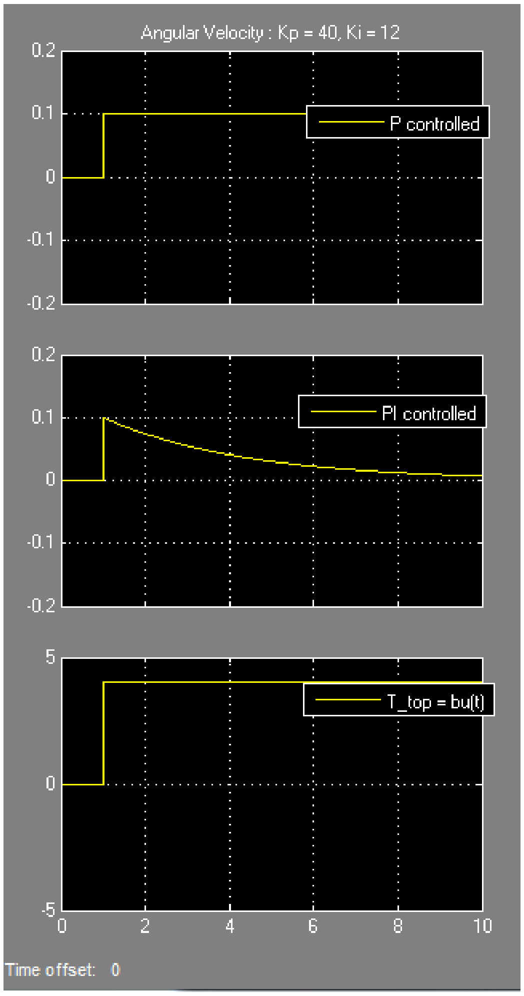

Figure 8: the control systems models (SimuLink)