CS 376 : HW 4 - Continuous Systems

Fred Eisele

15 October 2014

This problem set is taken from [1] chapter 2 : Problem 6. This system makes use of a helicopter model, Figure 1.

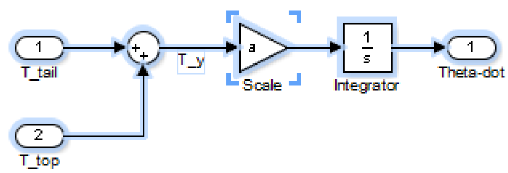

Figure 1: the helicopter plant model (SimuLink)

This Simulink model corresponds to the following mathematical model.

$$\forall t \in \mathbb{R}, \qquad y_1(t) = a x_1(t) \tag{1}$$

...and for the Integrator ...

$$\forall t \in \mathbb{R}, \qquad y_2(t) = i + \int_0^t x_2(\tau)\,\mathrm{d}\tau \tag{2}$$

The following expresses the mapping to the helicopter model. Notice that the memory carrying element in the models is the integrator in all cases.

$$a = \frac{1}{I_{yy}} \qquad \text{characteristic of the helicopter} \in \mathbb{R}_+ \tag{3}$$

$$i = \dot{\theta}(0) = 0 \qquad \text{initial } \textit{resting} \text{ angular velocity} \tag{4}$$

$$x_i = T_y \qquad \text{total y torque} \in \mathbb{R} \tag{5}$$

$$y_2 = \dot{\theta} \qquad \text{output angular velocity} \tag{6}$$

$$y_1 = x_2 \qquad \text{cascade composition} \tag{7}$$

$$T_{tail} \text{ follows } T_{top} \tag{8}$$

The tail-rotor torque compensates for the top-rotor torques.

Using Simulink and its continuous-time modeling component I have built a model of the helicopter control system shown in Figure 2. This model has an upper section that models a system with a proportional (P)-controller and a lower section that models a proportional-integrator (PI)-controller. The P-controller subsystem illustrates problem a) with the lower PI-controller subsystem illustrates problem b). These are placed together so that the inputs and outputs may be shared and displayed together. In this model the value of $a = 5$ and $T_top = 4$ are arbitrarily choosen.

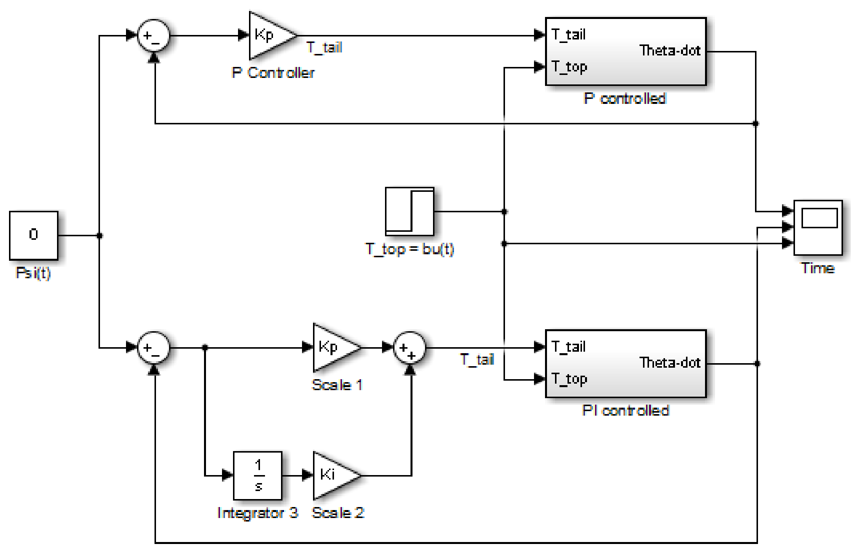

Figure 2: the control systems models (SimuLink)

1 P-controller system

Given some reasonable input parameters the actual angular velocity, $\dot{\theta}$, is shown as a function of time. The initial and operating conditions specify that the desired angular velocity is zero, $\phi(t) = 0$, and that the top-rotor torque is non-zero, $T_{top}(t) = bu(t) = 4$ moving to that value as a step function at time $t = 1$. Given are plots for several values of K_p.

Once the T_{top} has changed a new stable but typically non-zero $\dot{\theta}$ is approached asymtotically (from the text the value approached is b/K_p). The value of the new follows K_p as can be observed from the progression Figure 3 $\rightarrow Figure4 \rightarrow Figure5$.

The Figure 6 illustrates the convergence.

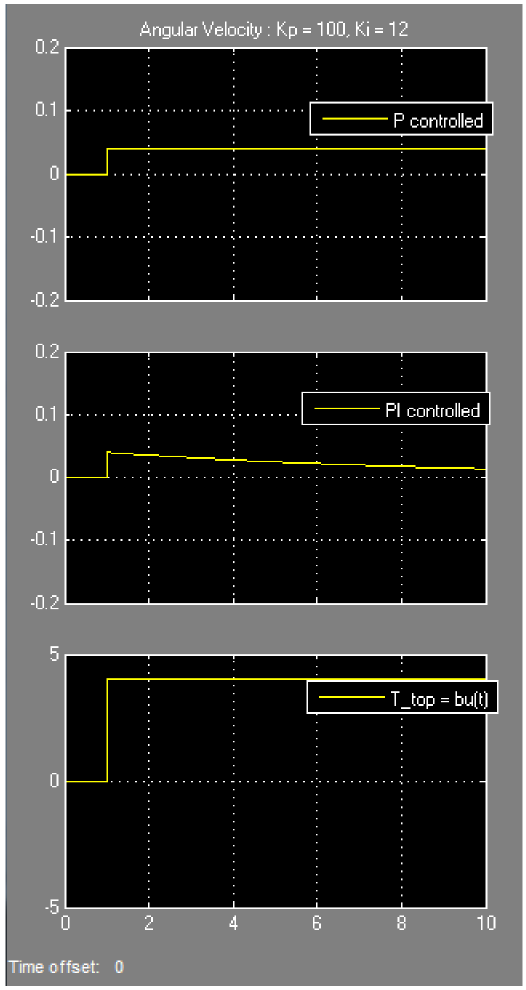

Figure 3: $K_p = 100, K_i = 12$

2 PI-controller system

The lower portion of Figure 2 replaces the proportional (P) controller of the upper system with a propotional-integrator (PI) controller. This alternative controller has an additional parameter associated with the intgrator, K_i. Experiment with this new value shows that the error is corrected for over time, recalling that the integrator has memory.

A larger value of K_i can be observed to cause a more rapid asymptotic convergence of $\dot{\theta}$ to the control-value, $\Psi = 0$ after T_{top} changes. This behavior can be seen by comparing Figure 7 to Figure 8 where K_p is held constant and K_i is changed.

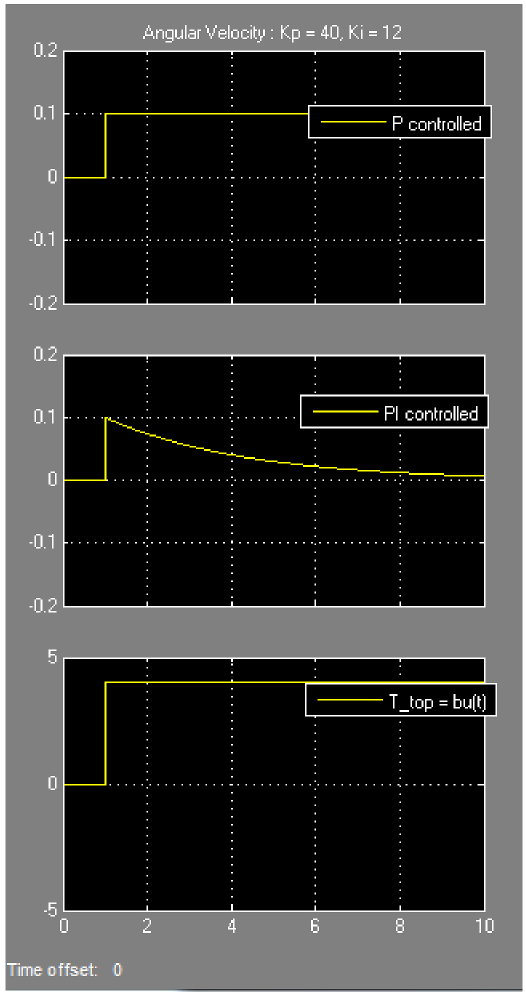

Figure 4: $K_p = 40, K_i = 12$

References

[1] Edward Lee. *Introduction to Embedded Systems : A Cyber-Physical Systems Approach.* 2011-2012.

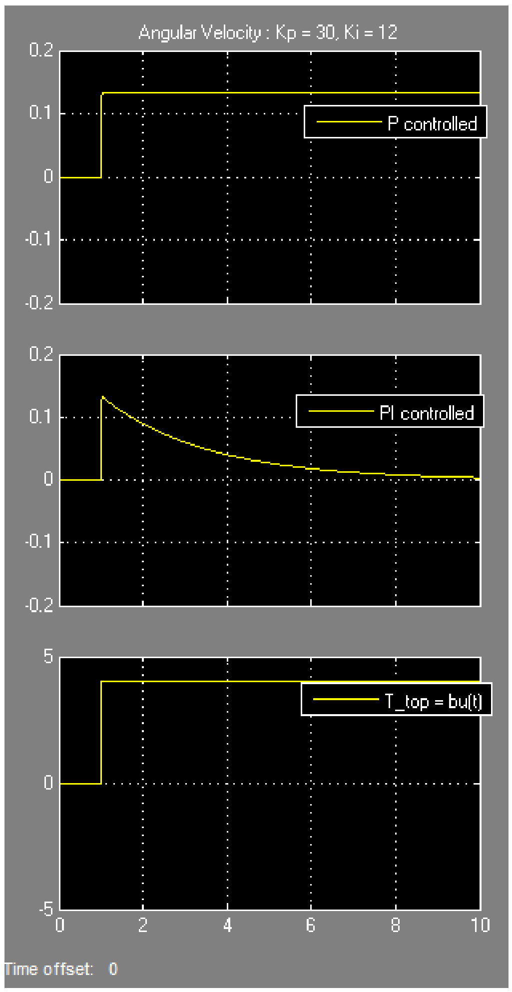

Figure 5: $K_p = 30, K_i = 12$

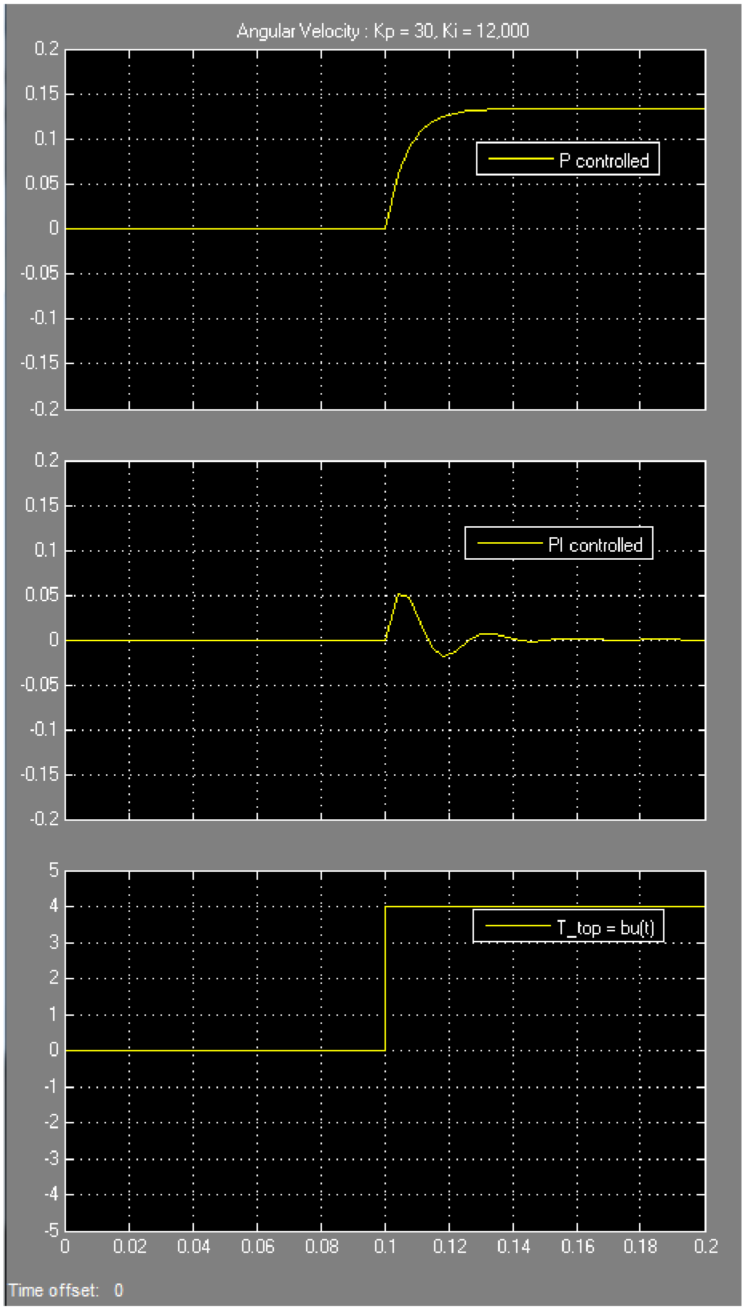

Figure 6: $K_p = 30, K_i = 12$

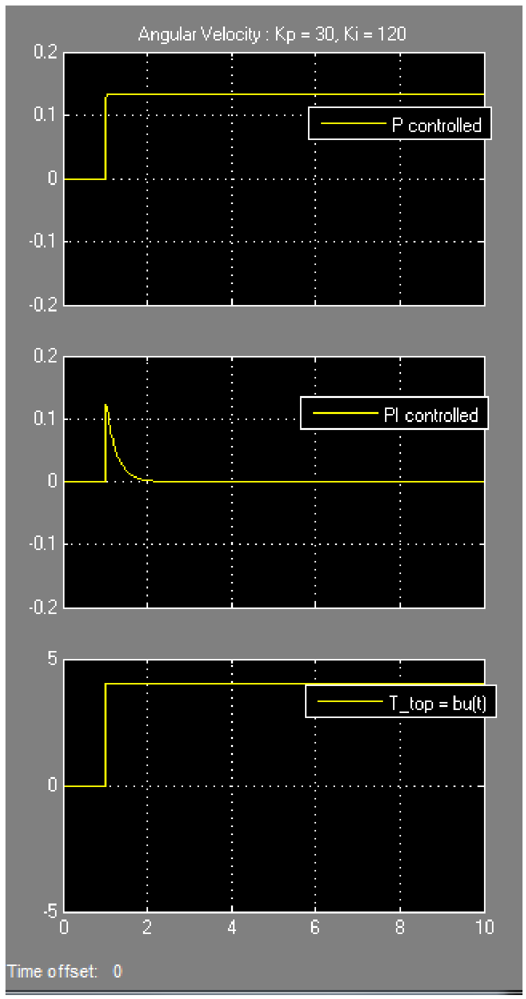

Figure 7: $K_p = 30, K_i = 120$

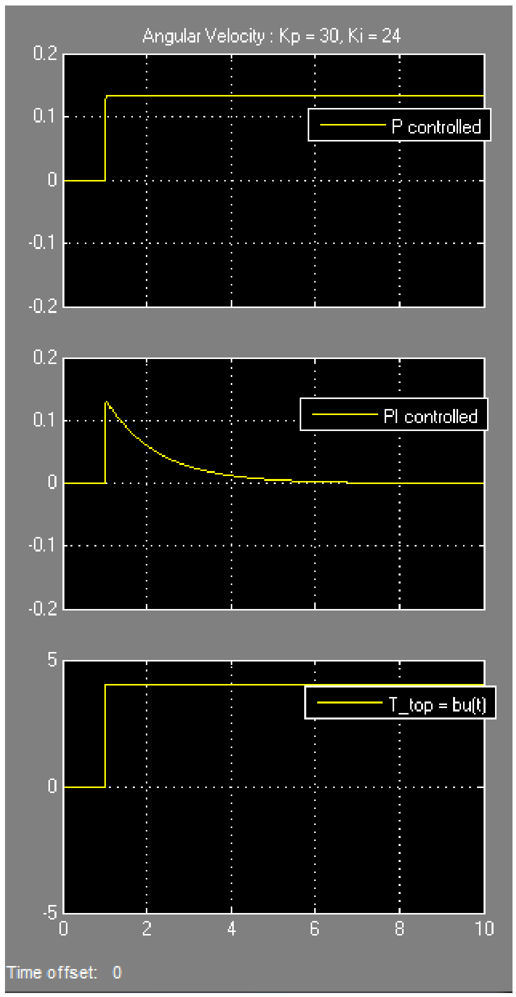

Figure 8: $K_p = 30, K_i = 24$