# Superconductivity and Electron Tunneling

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**Abstract**

We repeat the 1960 experiment by Giaver, in which the tunneling current from a superconductor through an insulating film is measured to obtain the width of its gap. Our results and the discrepancies to the BCS Theory are discussed. We measure the lead energy gap to be $(1.4 \pm 0.1) meV$.

## 1 Introduction

The theory of superconductivity presented by Bardeen, Cooper and Schrieffer in 1957 predicts a gap in the allowed energies for electrons. Even though this gap had been measured indirectly by several methods, it had not been measured directly until 1960, when Giaver conducted his electron tunneling experiment. Here we report our attempt to reproduce this seminal work.
In 1973 Giaver was awarded the Nobel Prize for his achievement. This is a feat we are not attempting to reproduce here; a 10 will be sufficient...

## 2 Theoretical Approach

The transition rate $W$ for an electron from an occupied state in the layer 1 with momentum $\mathbf{p_1}$ to a free state in the layer 2 with momentum $\mathbf{p_2}$ can be calculated by means of the Fermi Golden Rule like follows:

$$W^{1e}_{1\to 2} = \frac{2\pi}{\hbar}|T_{21}|^2 f(\epsilon_{\mathbf{p_1}})[1-f(\epsilon_{\mathbf{p_2}})]\delta(\epsilon_{\mathbf{p_1}}-\epsilon_{\mathbf{p_2}})\delta_{s_1 s_2}, \quad (1)$$

where $T_{21}$ is the transition amplitud, $f(\epsilon_{\mathbf{p_1}})$ the probability that the state $\mathbf{p_1}$ is occupied and $[1 - f(\epsilon_{\mathbf{p_2}})]$ the probability that the state $\mathbf{p_f}$ is empty, and the conservation of the energy and spin in the transition are assumed.

The total transition rate from the electrode 1 to the electrode 2 will be

$$W_{1\to 2} = \frac{2\pi}{\hbar}\sum_{\substack{s_1,s_2\\ \mathbf{p_1},\mathbf{p_2}}} |T_{21}|^2 f(\epsilon_{\mathbf{p_1}})[1-f(\epsilon_{\mathbf{p_2}})]\delta(\epsilon_{\mathbf{p_1}}-\epsilon_{\mathbf{p_2}})\delta_{s_1 s_2} \quad (2)$$

If the transition hamiltonian does not depend on the spin and couples weakly the electrodes when the applied voltage is small (a valid approximation for the voltages used in this experiment), we can simplify the expression (2):

$$W_{1\to 2} = \frac{4\pi}{\hbar}|T|^2 \sum_{\mathbf{p_1},\mathbf{p_2}} f(\epsilon_{\mathbf{p_1}})[1-f(\epsilon_{\mathbf{p_2}})]\delta(\epsilon_{\mathbf{p_1}}-\epsilon_{\mathbf{p_2}}). \quad (3)$$

The transition rate in the opposite way is analogous.

Now we can write explicitly the expression for the current in the $1 \to 2$ direction:

$$I = e\ (W_{1\to 2} - W_{2\to 1}), \quad (4)$$

that is

$$I = \frac{4\pi e}{\hbar}|T|^2 \sum_{\mathbf{p_1},\mathbf{p_2}} [f(\epsilon_{\mathbf{p_1}}) - f(\epsilon_{\mathbf{p_2}})]\delta(\epsilon_{\mathbf{p_1}} - \epsilon_{\mathbf{p_2}}).. \quad (5)$$

If we replace the summatories by integrals, considering that the momentums configure a quasicontinuum, and assuming a voltage difference $V$ between the electrodes that makes $\mu_2 - \mu_1 = eV$, we get

$$\begin{aligned} I &= \frac{4\pi e}{\hbar}|T|^2 \times \\ &\times \int_{-\infty}^{\infty} d\epsilon\ N_1(\epsilon - eV)\ N_2(\epsilon)[f(\epsilon - eV) - f(\epsilon)], \end{aligned} \quad (6)$$

where $N(E)$ is the density of states, needed to perform the change from summatories to the integral.

Now three cases can be distinguished, that is, when both electrodes are metals in the normal state, when only one of them is in the superconducting state and when both are in the superconducting state. The only difference between these situations is the form of the density of states $N(E)$, so it must be replaced by the adequate expression.

### 2.1 Normal-Normal junction

For a sufficient small voltage $V$, the state densities can be considered nearly constant, and equation (6) reads

$$I^{NN} = \frac{4\pi e}{\hbar}|T|^2 N_1(\mu)N_2(\mu)eV, \tag{7}$$

where we have used the fact that

$$\int_{-\infty}^{\infty} d\epsilon[f(\epsilon - eV) - f(\epsilon)] \simeq eV.$$

From this equation can be derived easily the normal conductance of the junction

$$C^{NN} = \frac{1}{R^{NN}} = \frac{dI^{NN}}{dV} = \frac{4\pi e^2}{\hbar}|T|^2 N_1(\mu)N_2(\mu). \tag{8}$$

### 2.2 Normal-Superconductor junction

The state density for a superconductor can be derived from considering a continuum spectrum of energy levels, and hence

$$N_N(\epsilon)d\epsilon = N_S(E)dE. \tag{9}$$

The relation between $\epsilon$ and $E$ in the range of the BCS Theory of Superconductivity is $E_{\mathbf{p}} = \sqrt{\epsilon_{\mathbf{p}}^2 + \Delta^2}$, with $\Delta$ the gap of the superconductor. So we can get

$$\begin{aligned} N_S(E) &= N_N(\epsilon)\left|\frac{d\epsilon}{dE}\right| = \\ &= \begin{cases} N_N(\epsilon)\frac{|E|}{\sqrt{E^2-\Delta^2}}, & |E| > \Delta \\ 0, & |E| \geq \Delta \end{cases} . \end{aligned} \tag{10}$$

If we replace this state density in (6) we get the $I^{NS}$, but only up to $|T|^2$ order. There are high order effects by means of Cooper pair transmission to the superconducting electrode. Neglecting these issues, the current for small voltages is

$$\begin{aligned} I^{NS} &= \frac{4\pi e}{\hbar}|T|^2 N_{1N}(\mu) \times \\ &\times \int_{-\infty}^{\infty} dE\ N_{2S}(E)[f(E-eV) - f(E)] = \\ &= \frac{4\pi e}{\hbar}|T|^2 N_{1N}(\mu)N_{2N}(\mu) \times \\ &\times \int_{-\infty}^{\infty} dE\ \frac{|E|}{\sqrt{E^2-\Delta^2}}[f(E-eV) - f(E)]. \end{aligned} \tag{11}$$

It can be expressed in terms of the $C^{NN}$ like follows:

$$I^{NS} = \frac{C^{NN}}{e}\int_{-\infty}^{\infty} dE\ \frac{|E|}{\sqrt{E^2-\Delta^2}}[f(E-eV) - f(E)]. \tag{12}$$

Finally, introducing $x = E - \Delta$ and noting that Fermi functions are even, we get the expression that is used for numerical analysis:

$$\begin{aligned} I^{NS} &= \frac{C^{NN}}{e}\int_{0}^{\infty} dx\ \frac{x+\Delta}{\sqrt{x(x+2\Delta)}} \times \\ &\times\ [f(x+\Delta-eV) - f(x+\Delta+eV)]. \end{aligned} \tag{13}$$

From eqrefins, the conductance will be the following

$$\begin{aligned} C^{NS} &= \frac{1}{R^{NS}} = \frac{dI^{NS}}{dV} = \\ &= \frac{C^{NN}}{e}\int_{-\infty}^{\infty} dE\ \frac{|E|}{\sqrt{E^2-\Delta^2}}\frac{\partial f(E-eV)}{\partial V} \end{aligned} \tag{14}$$

### 2.3 Superconductor-Superconductor junction

By analogy with the previous section, we write directly the expression of the current for this situation:

$$I^{SS} = \frac{C^{NN}}{e}\int_{-\infty}^{\infty} dE\ \frac{E^2\ [f(E-eV) - f(E)]}{\sqrt{(E^2-\Delta_1^2)}\sqrt{(E^2-\Delta_2^2)}}. \tag{15}$$

## 3 Experimental Method

Our aim in this experiment was to measure superconductor to normal tunneling as described above. To achieve this we choose Aluminum as the superconductor and Lead as the normal conductor. This is convenient because *Al* and *Pb* have a transition

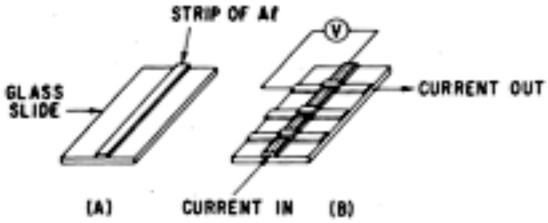


Figure 1: Hola

temperature of $1.140K$ and $7.193K$ respectively. Therefore the range in between the two allows the observation of the desired effect. Another benefit is that Aluminum readily oxidizes to $Al_2O_3$ at room temperature, providing an insulating layer.

### 3.1 Sample Preparation

Sample preparation was done in a manner similar to the one described in **??**. First a thin layer of Aluminum was vapor-deposited by means of a heated spiral filament onto a microscope slide in a vacuum chamber. Air was then let into the chamber, allowing a thin insulating film of $Al_2O_3$ to form. Finally the chamber was evacuated again, and the last layer, consisting of $Pb$, was deposited.

The layers were deposited in the shape of strips, a long strip of $Al$ crossed by 5 shorter strips of $Pb$, providing up to 5 tunnel junctions per slide. The shapes of the layers were determined by placing a mask between the heated filament and the slide.

LAYER THICKNESS

There are 2 main reasons for preparing the samples in a vacuum. Firstly it is necessary to keep the layers as free from contamination as possible, because due to their thinness even small amounts of foreign substances might cause a change in behavior. Secondly the vacuum significantly reduces scattering effects, making the thickness of the deposited layers as homogenous as possible. This is necessary so that the current is distributed evenly across the junction.

We used an oil diffusion pump to attain a high vacuum of about *??* bar.

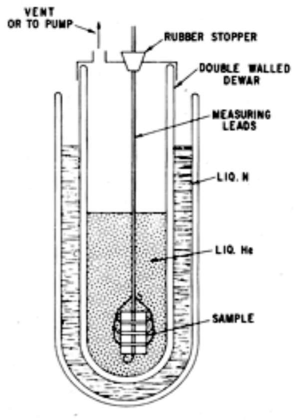


Figure 2: Hola

### 3.2 Measurement

The sample was immersed in a cryostat containing liquid $He$ surrounded by a layer of liquid $N$. A rotary pump was connected to the cryostat to reduce pressure inside. Since the vapor pressure is linked to the Temperature of the Helium this allowed us to perform measurements at several temperatures. A mechanical manometer was also connected to the chamber.

The tunnel junction was connected in a 4 terminal configuration, i.e. the voltage and current were measured in two separate circuits, which only joined at the tunneling junction. This setup has the advantage that only the potential difference at the junction itself is measured, and factors such as the resistivity of cables leading up to the sample, and the change with temperature thereof, do not have to be taken into account.

In our setup the Ampere-meter also functioned as a current source. Both units were connected to a computer via an IEEE interface, which was used to set the current and record the readings. In each measurement run the voltage across the junction was measured for a range of equally spaced current intensities, as produced by said current source. We measured ranges from *??* to *??* using divisions between 100 and 2000 steps. A time of $1s$ was left between measurements to allow for the readings to stabilize.

The resolution of voltage measurements depended greatly on the range in which the measurement was taken. The Volt-meter automatically changes the resolution, therefor we tried to avoid such changes of resolution by setting corresponding limits on the current. When the data contained a change of scale we omitted the data points beyond the change. The effects of this change of resolution are most severe in the numerical derivation, since this is done by dividing the differences.

CRYOSTAT GRAPHIC??

# 4 Results and Analysis

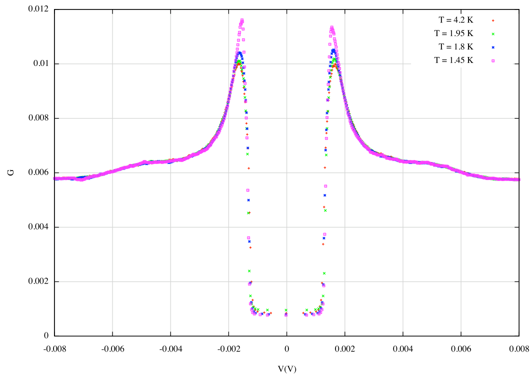


Figure 3: Hola

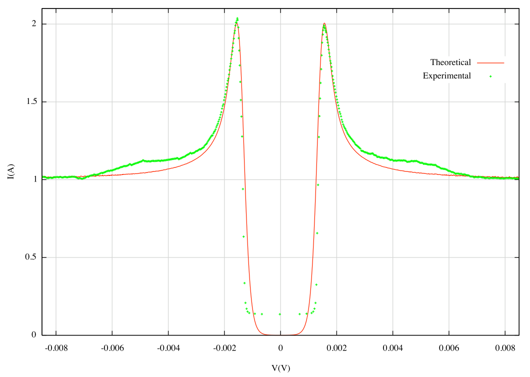


Figure 4: Hola

## 4.1 Energy gap

We have measured the lead's energy gap as $(1.4 \pm 0.1)meV$ below $4.2K$. The BCS theory predicts a temperature dependance, nevertheless in the range we have measured it, i.e. below the helium vaporization and above the Aluminum critical temperatures, this variation is less than the experimental error. This can be seen in fig.9.

The data almost agree with BCS theory. In fig.4 can be seen the I-V curve, which is modified near

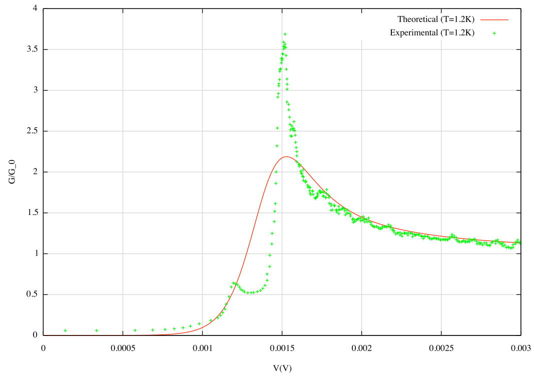


Figure 5: Hola

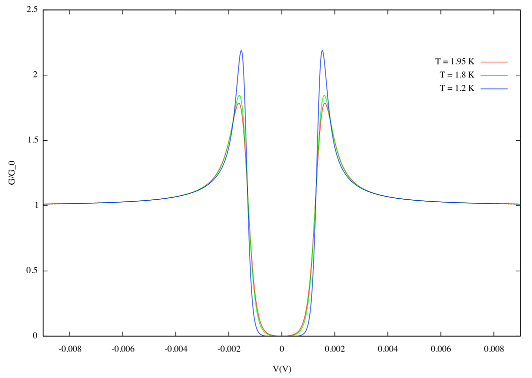


Figure 6: Hola

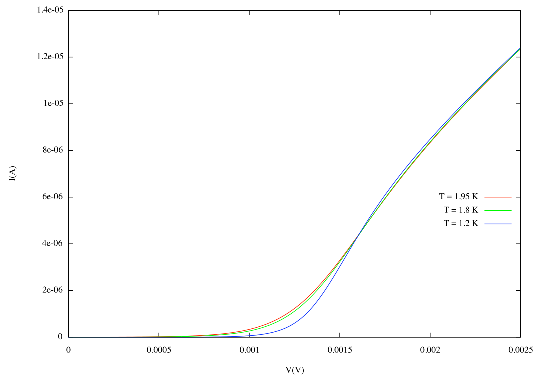


Figure 7: Hola

the gap when the temperature is below the $T_c$. The experimental and theoretical data are difficult to distinguish.

However,

1) Symmetric?

2) Al a little bit superconducting

3) Phonons

4) How have been made the graphs... smoothing... effects below the smoothing cannot be con-

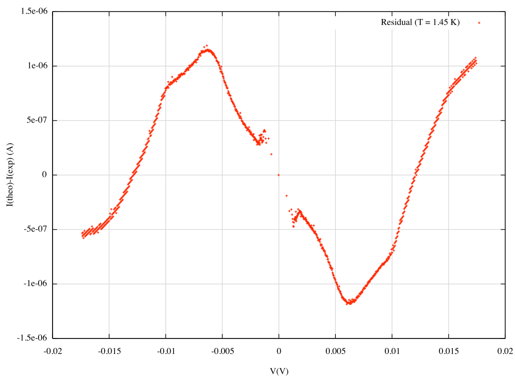


Figure 8: Hola

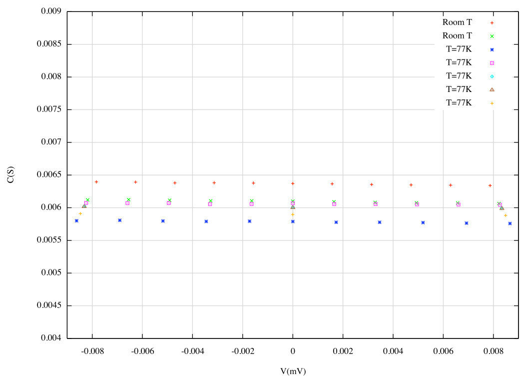


Figure 10: Conductance curves .

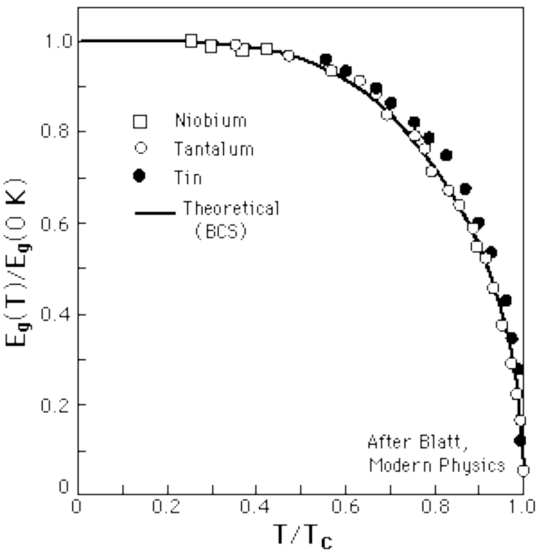


Figure 9: Reduced values of the observed energy gap as a function of the reduced temperature, after Towsend and Sutton. The solid curve is drawn for the BCS theory.

sidered.

5) Tc for films is greater than the bulk Tc's... So at 1.2K is possible that some parts of the Al film are superconductors. In this case, we have normal-super and super-super junctions added in parallel.

# References

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