Problem 1

1. 01 → ,10 → |
   01 → X, 10 → X, X# → #, X1 → 1, 1X → 1, X0 → 0, 0X → 0
2. There may be multiple options for applying rewrite rules, but only one rule can be applied at a time. For example there are 2 options in rewriting $0110#. $0110 → $10# (using rule 01 →) or $0110# (using rule 10 →).
3. $0110# use rule 01 → ϵ
   $10# use rule 10 → ϵ
   $# STOP
   OR
   $0110# use rule 10 → ϵ
   $01# use rule 01 → ϵ
   $# STOP

   $00010# use rule 10 → ϵ
   $000# STOP
   OR
   $00010# use rule 01 → ϵ
   $000# STOP

Problem 2

1. #fand#f → #f, #for#f → #f
   #fand#t → #f, #for#t → #t
   #tand#f → #f, #tor#f → #t
   #tand#t → #t, #tor#t → #t
   #fand#? → #f, #for#? → #?
   #?and#f → #f, #?or#f → #?
   #tand#? → #?, #tor#? → #t
   #?and#t → #?, #?or#t → #t
   #?and#? → #?, #?or#? → #?

2. ((#tor#f)and#?) Use rule #tor#f → #t
   ((#t)and#?) Use rule #tand#? → #?
   #? STOP

   (#tor(#?and#?)) Use rule #?and#?
   (#tor(#?)) Use rule #tor#?
   #t STOP

Problem 3

(+ | - | ϵ) (0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9) $^+$
(ϵ | . (0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9) $^+$
(ϵ | E (ϵ | -) (0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9) $^+$))
::= (+ | 0 | ϵ) digit$^+$ (ϵ | . digit$^+$ (ϵ | E (ϵ | -) digit$^+$))

Problem 4

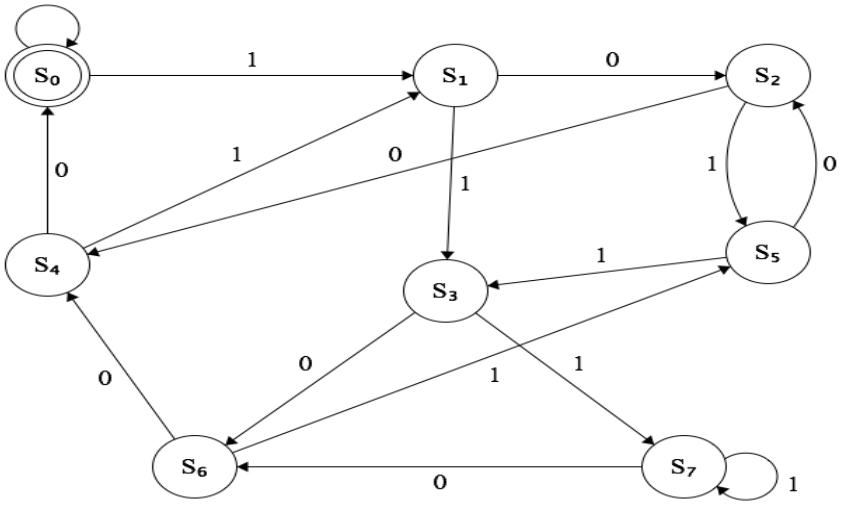
1. All binary strings including the empty string.
2. All binary string of length greater than or equal to 4 with 0 as the fourth to last digit and 0 as the last digit.

Problem 5

1. No a's directly following any b's: $(bc^+ \mid c \mid a)^*b^*$ or $a^*(c^+ a^* \mid b)^*$
   No a's following any b's: $(c \mid a)^*(c \mid b)^*$
2. Does not contain more than 2 b 2 a
   $c^*(a \mid \epsilon)c^*(a \mid \epsilon)c^*(b \mid \epsilon)c^*(b \mid \epsilon)c^*)$

Problem 6

1. <S, s, F, T>
   S = {S0, S1, S2,S3,S4,S5,S6,S7}
   s = S0
   F = {S0}
   T = {S0 x 0 → S0, S0 x 1 → S1,
   S1 x 0 → S2, S1 x 1 → S3,
   S2 x 0 → S4, S2 x 1 → S5,
   S3 x 0 → S6, S3 x 1 → S7,
   S4 x 0 → S0, S4 x 1 → S1,
   S5 x 0 → S2, S5 x 1 → S3,
   S6 x 0 → S4, S6 x 1 → S5,
   S7 x 0 → S6, S7 x 1 → S7}



S0 represents mod8 = 0, S1 is mod8 = 1, S2 is mod8 = 2, and S3 is mod8=3, S4 is mod8=4, S5 is mod8=5, S6 is mod8=6, S7 is mod8=7

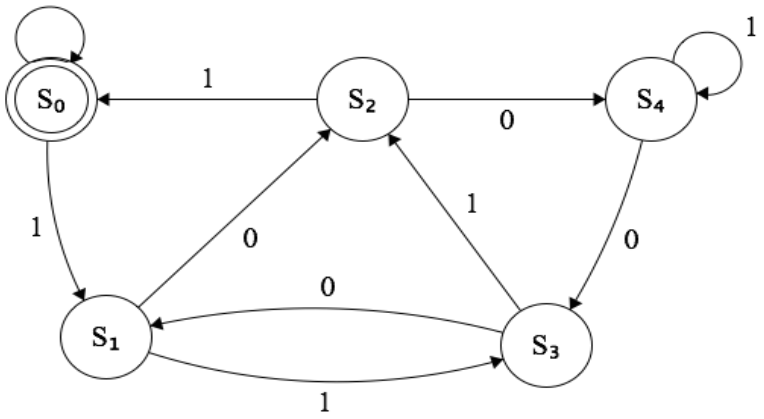
2. <S, s, F, T>

S = {S0, S1, S2,S3,S4}

s = S0

F = {S0}

T = { S0 x 0 → S0, S0 x 1 → S1,
S1 x 0 → S2, S1 x 1 → S3,
S2 x 0 → S4, S2 x 1 → S0,
S3 x 0 → S1, S3 x 1 → S2,
S4 x 0 → S0, S4 x 1 → S4}



S0 represents mod5 = 0, S1 is mod5=1, S2 is mod5 = 2, S3 is mod5 = 3, S4 is mod5=4