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On Robust Distance-Based Intensity Estimators

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SUMMARY

Several 'compound' estimators of the average intensity of a point process in some region of interest are introduced. Simulation is used to study the robustness of these and other estimators to changes in the underlying process, and to examine the effect of semi-systematic sampling in place of random sampling. Both homogeneous and heterogeneous patterns are considered. The estimators are based on distances from selected positions to the rth nearest point ($r = 1, 2, 3$). A 'compound' estimator formed from 'T-square' distances to only the nearest points is recommended for all patterns except those which are visually highly clustered. In these cases, one of more usual estimators should be used, preferably in conjunction with a bias-correction factor. Semi-systematic sampling is recommended.

1. Introduction

A reliable estimate of the number of trees in a forest is often required for ecological or economic reasons. The pattern formed by the tree centres may be thought of as an outcome of a spatial point process. If $\lambda(\mathbf{x})$ is the intensity of this process at the point $\mathbf{x}$, then the problem is to find a suitable estimator of $\bar{\lambda}$, where

$$\bar{\lambda} = \int_S \lambda(\mathbf{x})\, d\mathbf{x} \Big/ \int_S d\mathbf{x}$$

and is simply the 'average intensity' in the study region S.

Counts in randomly-located quadrats can always be used to give an unbiased estimate of $\bar{\lambda}$, irrespective of the form of the underlying process. This is not true of 'distance' methods. Nevertheless the latter are often preferred since they can be easier to use in the field. The aim of this paper is to establish which distance-based estimators of $\bar{\lambda}$ are robust, in a sense to be described, to changes in the distribution of the underlying process. Although the discussion is in terms of trees and forests, the methods to be considered have obvious wider applications.

Persson (1971) reviewed the distance-based estimators of $\bar{\lambda}$ then available, and calculated their bias for regular lattice processes and for randomly-distributed point cluster processes. Diggle (1975, 1977) examined estimates of $1/\bar{\lambda}$ for some regular and clustered processes. He used analytical arguments or simulation to show that his new 'compound' estimators are more reliable than the usual distance-based estimators from which they are constructed.

Analogues of Diggle's 'compound' estimators are presented in this paper. These new functions estimate $\bar{\lambda}$ rather than $1/\bar{\lambda}$. They include estimators based on distances from

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selected points or trees to the second and third nearest trees, as well as those which, like Diggle's, use only the distances to the nearest tree. The estimators are formed either from the squares of the mean or median distances or, as for Diggle's estimators, from the mean of the squared distances. Simulation is used to study the bias and variance of both these new estimators and others already described, including those introduced by Morisita (1957) and Batcheler and Hodder (1975). To date, nearly all comparisons of distance-based estimators have considered only the homogeneous case $\lambda(\mathbf{x}) \equiv \bar{\lambda}$, a constant. In this paper, $\lambda(\mathbf{x})$ is allowed to vary over the study region. Results are compared for completely random (i.e. Poisson) processes and for clustered processes, under both random and semi-systematic sampling schemes.

One of the new 'compound' estimators based on point-to-nearest-tree and '*T*-square' tree-to-nearest-tree measurements performs well in all situations except those which are visually extremely clustered. In such cases it is recommended that one of the current estimators be used in conjunction with a bias-correction factor. Semi-systematic sampling is recommended.

Patil, Burnham and Kovner (1979) introduced a nonparametric intensity estimator with certain desirable properties. It is not included in the present study for several reasons. Firstly it is based only on point–tree distances. Distance measurements are usually put to a dual purpose: intensity estimation and testing for spatial randomness. Byth and Ripley (1980) showed that tests of randomness based on just point–tree measurements are considerably less powerful against clustered and regular alternatives than those based on both point–tree and tree–tree measurements. Given that a certain number of distances are to be measured, the 'compound' estimators seemed preferable.

Secondly, the estimator of Patil *et al.* is consistent and asymptotically normal only for the homogeneous case. This includes some, but by no means all, of the clustered processes examined here. Thirdly, as noted by Patil *et al.*, there are problems concerning their estimator $\hat{f}_n(0)$: in particular, it is not very efficient unless the number of distance measurements sampled is considerably larger than that envisaged in the present study.

2. The Estimators

Suppose that we are interested in some region of forest, R. To minimize edge effects it is necessary to measure distances from points or trees selected from a study region S lying well within R. Under either the usual random or semi-systematic sampling as defined in Byth and Ripley (1980), let

(i) x_r be the distance from the selected point P to the rth nearest tree Q_r;
(ii) y_r be the distance from the selected tree to the rth nearest tree;
(iii) z_r be the distance from Q_1 to the rth nearest tree in a direction away from P, that is the '*T*-square' distance to the rth nearest tree from Q_1;
(iv) w be the distance from Q_1 to the nearest tree.

Denote samples of size m in the obvious way by

$$\mathbf{x}_1, \ldots, \mathbf{x}_r, \quad \mathbf{y}_1, \ldots, \mathbf{y}_r, \quad \mathbf{z}_1, \ldots, \mathbf{z}_r, \quad \mathbf{w}$$

In this paper, distances corresponding to $r \leqslant 3$ are of interest and sampling is assumed to be sufficiently sparse to ensure the approximate independence of the samples (see Byth and Ripley, 1980).

Estimators of $\bar{\lambda}$ based on the measurements $\mathbf{y}_r$ are practicable only under semi-systematic sampling. Random sampling of $\mathbf{y}_r$ necessitates complete enumeration of the N

trees in the study region S and $N/\int_S d\mathbf{x}$ would then be an excellent estimator of $\bar{\lambda}$. A modest amount of enumeration is required to obtain $\mathbf{y}_r$ under semi-systematic sampling. Before estimators which use $\mathbf{y}_r$ are recommended, they must therefore be more robust than others which do not. Although impracticable, estimators which use $\mathbf{y}_r$ are considered under random sampling in the simulation study. The inclusion of these estimators allows the effect of semi-systematic sampling to be assessed for a wider variety of estimators.

The case which is analytically most tractable is the homogeneous Poisson process for which $\lambda(\mathbf{x}) \equiv \bar{\lambda} \equiv \lambda$, say. It is well-known that, for such a process under the assumption of sparse sampling, $\lambda\pi x_{ri}^2$, $\lambda\pi y_{ri}^2$ and $\lambda\pi z_{ri}^2$ $(i = 1, \ldots, m)$ are approximately jointly independent and distributed as gamma variates with index r. Following Persson (1964), it is therefore easy to show that if $d_{ri} = x_{ri}$ or y_{ri} or $z_{ri}/\sqrt{2}$ $(r = 1, 2, 3; i = 1, \ldots, m)$, and if $\overline{d_r^2} = \sum_i d_{ri}^2/m$, $\bar{d}_r^2 = (\sum_i d_{ri}/m)^2$, $d_r^* =$ median of $d_{r1}, \ldots, d_{rm}$, and $\overline{1/d_r^2} = (\sum_i 1/d_{ri}^2)/m$, then, at least asymptotically as $m \to \infty$, the method of moments estimators of λ given in Table 1 are unbiased and have the given asymptotic variances. The estimator based on the function $\overline{1/d_3^2}$ is due to Morisita (1957) who claimed that it should be robust if the pattern consists of areas of different intensity within each of which the process is Poisson. Estimators based on $\overline{1/d_r^2}$ for $r = 1, 2$ have not been included in Table 1, because the expectation of $1/d_{1i}^2$ and the variance of $1/d_{ri}^2$ $(r = 1, 2)$ are infinite for a homogeneous Poisson process. For such a process the asymptotic variance of the estimator based on $\overline{d_r^2}$ is as small as that based on any other function of $\mathbf{d}_r$, since this estimator is maximum likelihood. Of course for other processes this need not be so.

The usual estimators of $\bar{\lambda}$ based on point–tree distances and those based on tree–tree distances are biased in opposite directions for clustered processes. The former tend to underestimate, and the latter to overestimate $\bar{\lambda}$. The converse is true for regular processes. The new 'compound' estimators of $\bar{\lambda}$ are arithmetic and geometric means of the usual estimators based on point–tree and on tree–tree measurements. They are analogous to $\tilde{\gamma}$, $\tilde{\gamma}_T$, γ^* and γ_T^*, 'compound' estimators of $1/\bar{\lambda}$ introduced by Diggle (1975). It is hoped that

Table 1

The function $f(\cdot)$ of the distance measurement $\mathbf{d}_r$ $(r = 1, 2, 3)$ used for the method of moments estimator, $\hat{\lambda}$, of the intensity, λ, of a homogeneous Poisson process. The asymptotic variance, var$(\hat{\lambda})$, is also shown

$f(\mathbf{d}_r)$	$\hat{\lambda}$	var$(\hat{\lambda})$
$\overline{d_1^2}$	$1/(\pi\overline{d_1^2})$	λ^2/m
$\bar{d}_1^2$	$1/(4\bar{d}_1^2)$	$1.09296\lambda^2/m$
d_1^{*2}	$0.069315/(\pi d_1^{*2})$	$2.08138\lambda^2/m$
$\overline{d_2^2}$	$2/(\pi\overline{d_2^2})$	$0.5\lambda^2/m$
$\bar{d}_2^2$	$9/(16\bar{d}_2^2)$	$0.52707\lambda^2/m$
d_2^{*2}	$1.67835/(\pi d_2^{*2})$	$0.90408\lambda^2/m$
$\overline{d_3^2}$	$3/(\pi\overline{d_3^2})$	$0.3\lambda^2/m$
$\bar{d}_3^2$	$225/(256\bar{d}_3^2)$	$0.34599\lambda^2/m$
d_3^{*2}	$2.67406/(\pi d_3^{*2})$	$0.57495\lambda^2/m$
$\overline{1/d_3^2}$	$2(\overline{1/d_3^2})/\pi$	λ^2/m

the new 'compound' estimators are more robust than the usual estimators used in their formation: the opposing effects on the bias of the usual estimators should at least partially cancel each other out in the 'compound' estimators.

For each function $\overline{d_r^2}$, $\bar{d}_r^2$, d^{*2} and $\overline{1/d_r^2}$ of the distance measurements to the rth nearest tree there are four corresponding new 'compound' estimators. For example, the estimators based on $\overline{d_1^2}$ are

$$\tfrac{1}{2}m\left\{1\Big/\left(\pi\sum x_{1i}^2\right)+1\Big/\left(\pi\sum y_{1i}^2\right)\right\},$$

$$\tfrac{1}{2}m\left\{1\Big/\left(\pi\sum x_{1i}^2\right)+2\Big/\left(\pi\sum z_{1i}^2\right)\right\},$$

$$m\left[\left\{1\Big/\left(\pi\sum x_{1i}^2\right)\right\}\left\{1\Big/\left(\pi\sum y_{1i}^2\right)\right\}\right]^{\frac{1}{2}}$$

and

$$m\left[\left\{1\Big/\left(\pi\sum x_{1i}^2\right)\right\}\left\{2\Big/\left(\pi\sum z_{1i}^2\right)\right\}\right]^{\frac{1}{2}}, \tag{1}$$

where the summation is in each case over $i=1,\ldots,m$. The other new estimators are defined in the obvious way, using the remaining nine estimators given in Table 1. Thus there are in all forty 'compound' estimators to be compared in addition to the usual estimators formed by using either the point–tree or the tree–tree distance measurements in the estimators of Table 1.

The estimator suggested on empirical grounds by Batcheler and Hodder (1975) is also considered. This estimator is

$$m\Big/\left(\pi\sum x_{1i}^2\times 3.473\times 3.717^{-1.913X}\right),$$

where

$$X=\left[\left\{m\sum x_{1i}^2-\left(\sum x_{1i}\right)^2\right\}\Big/\left(\sum x_{1i}\sum w_i\right)\right]^{\frac{1}{2}}.$$

It is possible to construct 'compound' estimators based on distance measurements from selected points to the rth nearest tree and from selected trees to the sth nearest tree when $r\neq s$. These estimators have not been included here since there is no reason to expect that they are more robust than the 'compound' estimators already described.

The 'standardized bias' B, and the 'standardized variance' V, of an estimator $\hat{\lambda}$ of the average intensity $\bar{\lambda}$ in the study region S are defined by $B=\mathrm{E}\{\hat{\lambda}/\bar{\lambda})-1\}$ and $V=\mathrm{var}(\hat{\lambda}/\bar{\lambda})$. With the exception of the Batcheler and Hodder estimator it is clear that, for a homogeneous Poisson process, $B\to 0$ for each estimator under investigation, at least asymptotically as $m\to\infty$. In §3 simulation is used in a study of the robustness of the estimators to changes in the form of the underlying process. An estimator is said to be 'robust' if B is close to zero and if V is not excessively large, for a wide variety of processes. This restriction on the size of V ensures the efficiency of the estimator.

3. Simulation Study

The analytical result of Matérn (1960, p. 81), concerning the variance of an estimator based on systematic sampling, suggests that semi-systematic sampling should yield more

efficient intensity estimators than random sampling when there is strong local positive correlation in the pattern. Many authors have been concerned about the possible bias introduced under systematic sampling in the presence of periodic variation (e.g., see Finney, 1947, 1948, 1950). Milne (1959) points out that five conditions must be satisfied before 'centric' systematic sampling gives rise to a severely biased estimate. The probability of the simultaneous occurrence of these five conditions is so small that he contends that 'the risk of periodic variation defeating the 'centric' systematic sample . . . . . can justifiably be ignored'.

We now use simulation to examine the performance of the intensity estimators in the presence of trend. Since so many estimators are being compared the full set of tables of results is not given. Instead, the main features of these tables are described and the actual results are given in each case for the most robust estimator. Details are available from the author on request.

In all cases, 250 realizations of each process were used to provide estimates of B and V for the various estimators under consideration. It is easier to present the results for the percentage standardized bias and variance, respectively $B' = 100B$ and $V' = 100V$, rather than those for B and V. The estimated standard error associated with B' is therefore $(100V'/250)^{\frac{1}{2}} \simeq 0.6(V')^{\frac{1}{2}}$. For the 'compound' estimators $m = 9$, whilst for the usual estimators $m = 18$. Thus, the results concerning either type of estimator are based on the same total number of distance measurements. This fact ensures that the effort required to collect the data used in a particular estimate is comparable with that needed to form any other estimate. The simulated patterns were all restricted to have 500 'trees' in the unit square R. The points or 'trees' from which distance measurements were made were selected according to the random and semi-systematic schemes used in Byth and Ripley (1980). They were chosen from within the square study region $S = [0.1, 0.9]^2$ so as to minimize edge effects. The results of Byth and Ripley suggest that the resulting sampling intensity is sufficiently small to ensure the approximate validity of the assumptions made under sparse sampling.

Realizations of Poisson and clustered processes were simulated. The results for the former are discussed first. The intensity function for these processes was chosen to be

$$\lambda(\mathbf{x}) \propto \alpha(x_1 - \tfrac{1}{2})^2 + \tfrac{1}{2} \tag{2}$$

for $\mathbf{x} = (x_1, x_2) \in R$. Here α is a real-valued parameter which may be varied. The cases with $\alpha = 2, 4$ and 6 were considered, $\alpha = 0$ corresponding to a homogeneous Poisson process. A realization of a Poisson process is illustrated in Fig. 1(a). The x-coordinates of the 'trees' in each pattern were generated by acceptance sampling from the distribution with density proportional to $\alpha(x - \frac{1}{2})^2 + \frac{1}{2}$, $x \in [0, 1]$, whilst the y-coordinates were random numbers generated from a uniform distribution on $[0, 1]$.

The estimators which performed best were those based on the distances from selected points or trees to the nearest tree. This result is probably due to the fact that the larger distances to other trees are more affected by the underlying trend. On the whole the 'compound' estimators seem to be more reliable than the usual estimators. In nearly all cases considered, the use of semi-systematic sampling in place of random sampling led to a decrease in both the estimated absolute bias and the variance of each estimator. The estimator $E^* = m^2/\{(2\sum x_{1i})(\sqrt{2}\sum z_{1i})\}$ is the most robust. The results of the study for E^* are presented in Table 2.

The clustered processes which were examined are extensions of the modified Matérn cluster processes described by Byth and Ripley (1980). The present processes are simulated in the same way as these earlier processes except that the cluster centres are

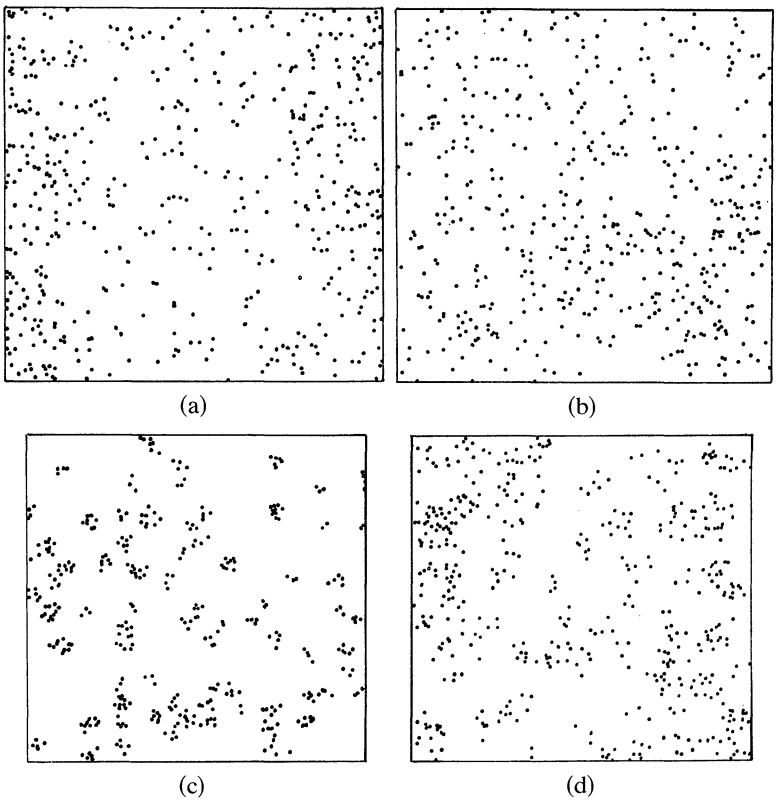

Figure 1. Realizations on the unit square R of (a) Poisson process with $\lambda(\mathbf{x}) \propto 4(x_1 - \frac{1}{2})^2 + \frac{1}{2}$, $\mathbf{x} = (x_1, x_2) \in R$, and of modified Matérn cluster processes with (b) $\mu = 2$, $D = 0.2$, $\alpha = 0$, (c) $\mu = 4$, $D = 0.05$, $\alpha = 0$, and (d) $\mu = 4$, $D = 0.1$, $\alpha = 4$. Each realization consists of 500 points.

now a realization of a Poisson process with intensity $\lambda(\mathbf{x})$ given by (2). Thus, there are three parameters needed to describe such processes: the trend parameter α, the mean number of 'trees' per cluster μ, and the cluster diameter D. In the simulated patterns, $\mu = 2$, 4 or 5, and $D = .05$, .1 or .2. Increasing μ or decreasing D produces more obvious visual clustering. When $D = .05$ the simulated patterns are highly clustered, whilst the patterns for $\mu = 2$ and $D = .2$ are visually similar to Poisson patterns. The cases with $\alpha = 0$, 2 and 4 were considered. These represent no trend, slight and moderate trend, respectively. As Bartlett (1963) points out, it is possible to consider the homogeneous

Table 2
Simulation results for E^* for Poisson processes with intensity $\lambda(\mathbf{x}) \propto \alpha(x_1 - \frac{1}{2})^2 + \frac{1}{2}$ for $\mathbf{x} = (x_1, x_2) \in [0, 1]^2$. Both random ($R$) and semi-systematic ($S$) sampling are considered

Percentage standardized values for	$\alpha = 2$		$\alpha = 4$		$\alpha = 6$	
	R	S	R	S	R	S
(i) bias	5	3	3	2	10	4
(ii) variance	5	3	6	5	7	5

cluster process corresponding to $\alpha = 0$ as a heterogeneous process, but such an interpretation is not used here. Realizations of clustered processes are illustrated in Fig. 1(b)–(d).

Results of the simulation study for clustered processes suggest that the 'compound' estimators are more robust than the usual estimators in all except the highly-clustered situations ($D = .05$). For moderately-clustered patterns there seems to be no advantage in using distances from the selected point or tree to the second or third nearest tree. Nor is there any apparent advantage in choosing estimators which make use of $\mathbf{y}_r$ and so require partial enumeration. In the situations considered, the performance of such estimators is at best comparable with that of the analogous estimators which use $\mathbf{z}_r$. Semi-systematic sampling is always a precaution against oversampling (Byth and Ripley, 1980) and, as expected, in the heterogeneous case it tends to decrease the variance of all the estimators without causing large increases in their absolute biases. Indeed the absolute bias is often reduced. The estimator E^* is again recommended for all except highly-clustered patterns. The results for this estimator are given in Table 3.

In highly-clustered situations all the estimators have considerable bias. The estimator $3m/(\pi \sum x_{3i}^2)$, based on the distances $\mathbf{x}_3$ under semi-systematic sampling, generally appears to have bias comparable with, and variance less than, the best of the other estimators in

Table 3

Results of the simulation study for E^* under both random (R) and semi-systematic (S) sampling in moderately-clustered situations, and for $3m/(\pi \sum x_{3i}^2)$ under only S sampling in extremely-clustered situations. The estimated percentage standardized bias is listed with the estimated percentage standardized variance beneath in parentheses

Mean cluster size, μ	Cluster diameter, D	Trend parameter, α					
		0		2		4	
Moderate clustering: results for E^*		R	S	R	S	R	S
2	.2	6	7	7	8	3	3
		(8)	(7)	(11)	(3)	(13)	(6)
	.1	−2	−1	−1	1	6	−3
		(7)	(9)	(9)	(8)	(4)	(3)
4	.2	4	2	−4	−1	−3	0
		(6)	(4)	(6)	(4)	(14)	(10)
	.1	−1	3	7	9	−2	−4
		(11)	(9)	(17)	(11)	(8)	(6)
5	.2	7	1	−3	1	12	−7
		(17)	(7)	(12)	(7)	(13)	(5)
	.1	3	−2	−13	3	−9	−3
		(18)	(11)	(11)	(12)	(10)	(11)
Extreme clustering: results for $3m/(\pi \sum x_{3i}^2)$							
2	.05		−15		−14		−16
			(1)		(1)		(1)
4			−26		−25		−30
			(1)		(1)		(1)
5			−41		−38		−44
			(2)		(1)		(2)

such cases. There is a further advantage in using this estimator. If the field worker has some idea of the average number μ of trees in each cluster, not an unreasonable assumption in a highly-clustered situation, then the standardized bias of $rm/(\pi \sum x_{ri}^2)$ is about $(r/\mu)-1$ for $r \leqslant \mu$. In such situations it is therefore possible approximately to correct for bias. Even if the field worker can only give a probable range of values for μ, at least some idea of the possible size of the bias is available.

To see how the correction term arises, suppose that the centres are randomly-distributed point clusters each containing exactly μ 'trees'. Then for $r \leqslant \mu$, $\mathbf{x}_r$ are the distances from selected points to the nearest cluster, and the cluster intensity in the homogeneous case is λ/μ. Thus, in this case the expected squared distance from a selected point to the nearest cluster is $\mu/(\pi\lambda)$. Hence the estimate $rm/(\pi \sum x_{ri}^2) \doteq r\lambda/\mu$. In other words, for $\mu \geqslant 3$ the standardized bias of $3m/(\pi \sum x_{3i}^2)$ is about $(3/\mu)-1$. Results for $3m/(\pi \sum x_{3i}^2)$ under semi-systematic sampling in highly-clustered situations are given in Table 3. The estimated percentage standardized biases for the cases $\mu = 4$ and 5 should be compared with the approximate expected values of $100\{(3/\mu)-1\} = -25$ and -40, respectively.

4. Recommendations

The 'compound' estimator E^* should be used for estimation of the average intensities of processes underlying all but visually highly-clustered patterns. In each case considered, the performance of this estimator is comparable with or better than that of $1/\gamma_T^*$, the estimator given in (1). Here, γ_T^* is the estimator of $1/\bar{\lambda}$ recommended by Diggle (1975, 1977). This improvement in performance is possibly due to the fact that γ_T^* is a function of $\sum x_{1i}^2/m$ and $\sum z_{1i}^2/m$, whilst E^* is based on $(\sum x_{1i}/m)^2$ and $(\sum z_{1i}/m)^2$, the latter being less affected by an unusually large observation. Use of the estimator $3m/(\pi \sum x_{3i}^2)$ in conjunction with a bias-correction factor is recommended for visually highly-clustered patterns. Semi-systematic sampling is recommended as a precaution against oversampling and, in heterogeneous cases, as a variance-reduction technique. It should be noted that neither the estimator due to Morisita (1957), nor that suggested by Batcheler and Hodder (1975), is robust to the types of changes in the underlying distributions considered here.

Résumé

On introduit plusieurs estimateurs 'composés' de l'intensité moyenne d'un processus ponctuel dans une région donnée. On utilise la simulation pour étudier la robustesse d'estimateurs de ce type ou différents à des modifications du processus observés et pour examiner l'effet de l'échantillonnage semi-systématique à la place de l'échantillonnage aléatoire. On considère des dispersions homogènes et hétérogènes. Les estimateurs sont calculés à partir des distances de points choisis du support au rième plus proche point de la réalisation ($r = 1, 2, 3$). Un estimateur 'composé' calculé à partir des distances 'T-carré' au point le plus proche est recommandé pour toutes les dispersions exceptées celles qui sont très agrégées à vue d'œil. Dans ce cas, un des estimateurs les plus usuels doit être employé, si possible en même temps qu'un facteur de correction de biais. On recommande l'échantillonnage semi-systématique.

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