closed - form phase deriv of a LTI / OppSch p. 292

In[35]:=
```
ph = ArcTan[1 - r * Cos[omega - theta], r * Sin[omega - theta]]
```

Out[35]= ArcTan[1 - r Cos[omega - theta], r Sin[omega - theta]]

In[36]:=
```
grd = Simplify[D[ph, {omega, 1}]]
```

Out[36]= $$-\frac{r\,(r-\mathrm{Cos}[omega-theta])}{1+r^2-2\,r\,\mathrm{Cos}[omega-theta]}$$

In[37]:=
```
grd' = Simplify[D[grd, {omega, 1}]]
```

Out[37]= $$\frac{r\,\left(-1+r^2\right)\mathrm{Sin}[omega-theta]}{\left(1+r^2-2\,r\,\mathrm{Cos}[omega-theta]\right)^2}$$

bilinear LP / OppSch p .504, 388

In[38]:=
```
s = c * (1 - z^(-1)) / (1 + z^(-1))
```

Out[38]= $$\frac{c\left(1-\frac{1}{z}\right)}{1+\frac{1}{z}}$$

In[39]:=
```
s = c * (1 - y) / (1 + y)
```

Out[39]= $$\frac{c\,(1-y)}{1+y}$$

In[40]:=
```
Collect[Expand[s^2], y]
```

Out[40]= $$\frac{c^2}{(1+y)^2}-\frac{2\,c^2\,y}{(1+y)^2}+\frac{c^2\,y^2}{(1+y)^2}$$

In[41]:=
```
? Conjugate
```

Conjugate[*z*] or *z** gives the complex conjugate of the complex number *z*. ≫

In[42]:=
```
lpdenom2 = FullSimplify[s^2 - 2 * Real[p0] * k * s + (k^2) * ((Real[p0])^2 + (Imag[p0])^2)]
```

Out[42]= $$\frac{k^2\,(1+y)^2\,\mathrm{Imag}[p0]^2+(c\,(-1+y)+k\,(1+y)\,\mathrm{Real}[p0])^2}{(1+y)^2}$$

In[43]:=
```
lpnumh2 = Collect[Expand[Denominator[lpdenom2]], y]
```

Out[43]= $1+2\,y+y^2$

In[44]:=
```
lpdenomh2 = Collect[Expand[Numerator[lpdenom2]], y]
```

Out[44]= $$c^2+k^2\,\mathrm{Imag}[p0]^2-2\,c\,k\,\mathrm{Real}[p0]+k^2\,\mathrm{Real}[p0]^2+ y^2\left(c^2+k^2\,\mathrm{Imag}[p0]^2+2\,c\,k\,\mathrm{Real}[p0]+k^2\,\mathrm{Real}[p0]^2\right)+y\left(-2\,c^2+2\,k^2\,\mathrm{Imag}[p0]^2+2\,k^2\,\mathrm{Real}[p0]^2\right)$$

In[45]:=
```
hpdenom2 =
 FullSimplify[(s^2) * (kw^2) * ((Real[p0])^2 + (Imag[p0])^2) - 2 * Real[p0] * ks * kw * s + ks^2]
```

Out[45]= $$\frac{c^2\,kw^2\,(-1+y)^2\,\mathrm{Imag}[p0]^2+(ks\,(1+y)+c\,kw\,(-1+y)\,\mathrm{Real}[p0])^2}{(1+y)^2}$$

In[46]:=
```
hpnum2 = FullSimplify[kn^2 * s^2]
```

Out[46]=
$$\frac{c^2\ kn^2\ (-1+y)^2}{(1+y)^2}$$

In[47]:=
```
hpnumh2 = Collect[Expand[Numerator[hpnum2]], y]
```

Out[47]=
$$c^2\ kn^2 - 2\ c^2\ kn^2\ y + c^2\ kn^2\ y^2$$

In[48]:=
```
hpdenomh2 = Collect[Expand[Numerator[hpdenom2]], y]
```

Out[48]=
$$ks^2 + c^2\ kw^2\ \mathrm{Imag}[p0]^2 - 2\ c\ ks\ kw\ \mathrm{Real}[p0] + c^2\ kw^2\ \mathrm{Real}[p0]^2 + y\left(2\ ks^2 - 2\ c^2\ kw^2\ \mathrm{Imag}[p0]^2 - 2\ c^2\ kw^2\ \mathrm{Real}[p0]^2\right) + y^2\left(ks^2 + c^2\ kw^2\ \mathrm{Imag}[p0]^2 + 2\ c\ ks\ kw\ \mathrm{Real}[p0] + c^2\ kw^2\ \mathrm{Real}[p0]^2\right)$$