Self consistency calculation for Pseudo Bogoliubov Landau levels*

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Abstract

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I. INTRODUCTION

II. PSEUDO-LANDAU LEVELS

It is shown that if a Weyl semi-metal becomes superconducting, the magnitude of the superconducting order parameter controls the separation of the Weyl nodes. Thus if our system have a superconductivity with spatially modulating order parameter, we expect to have spatially modulating separation length of the Weyl nodes. In this section we quantitatively show our arguments, and in the following section we will calculate the superconducting order parameter self-consistently. We start adding s-wave superconductivity to a Weyl semi-metal.

$$H = \sum_k c^\dagger_{\mathbf{k}}[\mathbf{d}(\mathbf{k}) \cdot \sigma]c_{\mathbf{k}} + (\Delta_0 c^\dagger_{\uparrow k} c^\dagger_{\downarrow -k} + h.c.), \tag{1}$$

where $c_k = (c_{\uparrow k}, c_{\downarrow k})^{\mathrm{T}}$, $\mathbf{d}(\mathbf{k}) = -(\sum_i \cos(k_i) - \cos(k_0) - m_g, \sin(k_2), \sin(k_3))$, m_g controls the size of the gap and we take it always positive k_0 controls the position of the Weyl nodes. We will denote the first term in eq(1) as H_0. H_0 hosts Weyl nodes at $k = (\pm k_0, 0, 0)$ for $m_g = 0$. If we let m_g, k_0 and Δ_0 to be zero we would get a quadratic band touching at $\mathbf{k} = 0$ and then if we let $\Delta_0 > 0$ this Dirac node would be separated into two Weyl nodes. It is known that the Dirac node can be transformed into a Weyl nodes either by breaking time reversal symmetry or inversion symmetry. Time reversal symmetry can be broken by applying external magnetic field to the system with Dirac node. In our system we break time reversal symmetry by adding superconductivity thus, superconducting order parameter behaves like a pseudo-electromagnetic potential.[Check this!] If the superconducting order parameter spatially inhomogeneous then we would observe finite pseudo magnetic field.

First we write 1 as a Bogoliubov Hamiltonian such that

$$H_{\mathbf{bog}} = \frac{1}{2}\sum_k \gamma^\dagger_k[\tau_3 \otimes d(k_\parallel) \cdot \sigma_\parallel + \tau_0 \otimes \sigma_3 d_3(k_3) + [\tau^+\Delta_0 - \tau^-\Delta_0^*] \otimes \mathbf{i}\sigma_2]\gamma_k + \text{const.}, \tag{2}$$

where σ_i are spin degree of freedom, τ_i are particle-hole degree of freedom, $\gamma_k = (c_{k\uparrow}, c_{k\downarrow}, c^\dagger_{-k\uparrow}, c^\dagger_{-k\downarrow})^{\mathrm{T}}$, $k_\parallel = (k_1, k_2)$ and $\sigma_\parallel = (\sigma_1, \sigma_2)$. In order to diagonalize eq.(2) in particle hole space we first make a canonical transformation in order to decouple σ and τ degree of freedom ,

$$(c_{k\uparrow}, c_{k\downarrow}, c^\dagger_{-k\uparrow}, c^\dagger_{-k\downarrow})^{\mathrm{T}} \to (c_{k\uparrow}, c_{k\downarrow}, c^\dagger_{-k\uparrow}, -c^\dagger_{-k\downarrow})^{\mathrm{T}} \tag{3}$$

then after diagonalizing in τ degree of freedom we get

$$H_{\mathbf{bog}} = \frac{1}{2}\sum_k \phi_k^\dagger \begin{bmatrix} d(k)\cdot\sigma + |\Delta_0|\sigma_1 & \mathbf{0} \\ \mathbf{0} & d(k)\cdot\sigma - |\Delta_0|\sigma_1 \end{bmatrix} \phi_k + \text{const.} \tag{4}$$

We denote the upper(lower) block of eq(4) as $H_{+(-)}$. $H_\pm$ hosts two Weyl points if $1 < (2+\cos(k_0)+m_g \pm \Delta_0) < 3$ and it is an insulator for $(2+\cos(k_0)+m_g\pm\Delta_0) > 3$. We will let $k_0 = 0$ so only H_- will host Weyl points and H_+ will be an insulator, and our system in total will host only two Weyl nodes. We add mass term to make on of the block Hamiltonian always an insulator, and in order to have a smooth transition from insulating phase to Weyl phase in the boundary of the finite slab. The separation between two Weyl nodes is

$$b = 2\cos^{-1}(\cos(k_0) - |\Delta_0|) \tag{5}$$

For the next step we will write the Bogoliubov Hamiltonian with spatially inhomogeneous order parameter. We will assume that we have a slab in direction $\hat{x}_2$ with N sites and order parameter will be spatially inhomogeneous in in direction $\hat{x}_2$. Thus the separation of Weyl nodes becomes $b(x_2) = 2\cos^{-1}(\cos(k_0) - |\Delta_0(x_2)|$, thus we have a pseudo magnetic field in $\hat{x}_3$ direction. In order to do so we first start with an Hamiltonian with quadratic interaction and then we will apply mean field theory. The Hamiltonian is given as,

$$H = H_0 + \sum_{k_\perp ij} \frac{g_{ij}}{V} c^\dagger_{k_\perp i\uparrow} c^\dagger_{-k_\perp i\downarrow} c_{-k_\perp j\downarrow} c_{k_\perp j\uparrow} \tag{6}$$

where $k_\perp = (k_1, k_3)$ and V is the volume of the system. We can assume contact interaction by letting $g_{ij} = -g_0\delta_{ij}$. We can apply a mean-field theory by defining

$$\Delta_j = -\frac{g_0}{V}\sum_{k_\perp} \langle c_{-k_\perp j\downarrow} c_{k_\perp j\uparrow}\rangle_{\mathbf{th}}, \tag{7}$$

then we get,

$$H = H_0 + \sum_{k_\perp i} [\Delta_i c^\dagger_{k_\perp i\uparrow} c^\dagger_{-k_\perp i\downarrow} + \text{h.c.}] + \text{const.}. \tag{8}$$

We can write eq.(8) in Bogoliubov Hamiltonian form,

$$H_{\mathbf{bog}} = \frac{1}{2}\sum_{k_\perp} \psi^\dagger_{k_\perp} \begin{bmatrix} H(k_\perp) & \Delta \\ \Delta^\dagger & -H(-k_\perp)^{\mathrm{T}} \end{bmatrix} \psi_{k_\perp} + \text{const.} \tag{9}$$

where Δ is $2N \times 2N$ block-diagonal matrix with blocks of $\mathbf{i}\sigma_2\Delta_i$, $H(k_\perp)$ is position space representation of H_0, $\psi_{i\sigma k_\perp} = c_{i\sigma k_\perp}$ and $\psi_{i+2N\sigma k_\perp} = c^\dagger_{i\sigma -k_\perp}$. If we solve eq(9) with appropriate Δ_i we observe Landau levels.

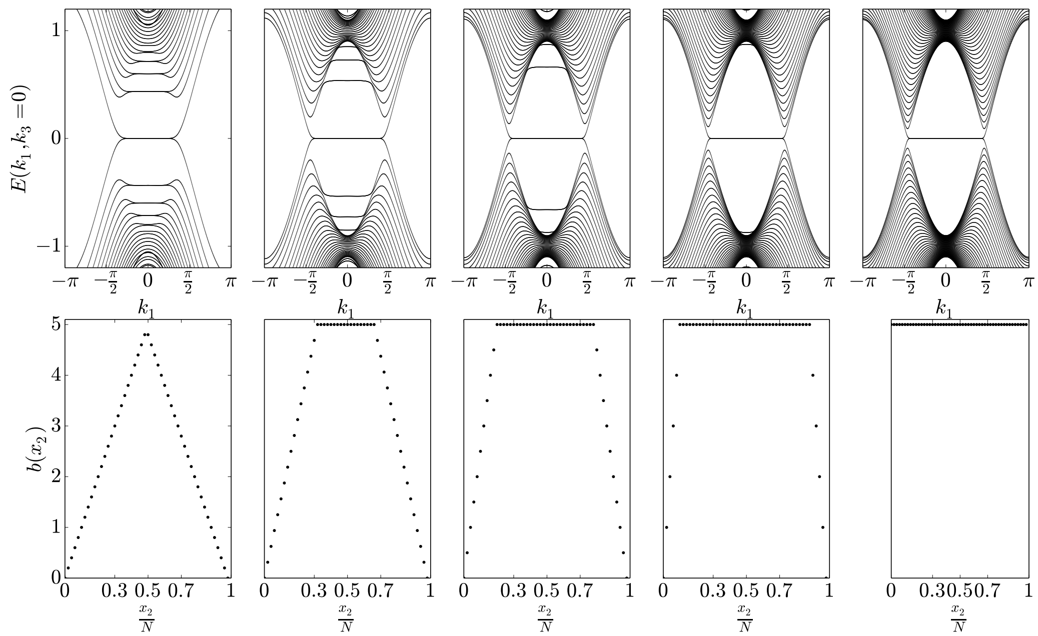

FIG. 1. I will add comment

III. SELF CONSISTENCY EQUATION

The spatial inhomogeneity of the superconducting order parameter can be achieved may spatially modulating strain, temperature or chemical potential. We will consider the latter case. We assume periodic boundary conditions, i.e. $\mu(i+n) = \mu(i)$ where n is the number of sites in super-cell. The superconducting order parameter id

$$\Delta_{\alpha_i} = \frac{1}{V} \sum_k g(k) \langle c_{-k\alpha_i\downarrow} c_{k\alpha_i\uparrow} \rangle_{\mathrm{th}}, \tag{10}$$

where α_i is the index of the lattice site in the super-cell and runs from 0 to n. The particle and hole eigenstates states of the Bogoliubov Hamiltonian are

$$\begin{bmatrix} H(k) & \Delta \\ \Delta^\dagger & -H(-k)^{\mathrm{T}} \end{bmatrix} \begin{bmatrix} u_m(k) \\ v_m(k) \end{bmatrix} = E_m(k) \begin{bmatrix} u_m(k) \\ v_m(k) \end{bmatrix} \tag{11}$$

$$\begin{bmatrix} H(k) & \Delta \\ \Delta^\dagger & -H(-k)^{\mathrm{T}} \end{bmatrix} \begin{bmatrix} v_m^*(-k) \\ u_m^*(-k) \end{bmatrix} = -E_m(-k) \begin{bmatrix} v_m^*(-k) \\ u_m^*(-k) \end{bmatrix} \tag{12}$$

where m is the superconducting band index, u and v are $2n \times 1$ sized column vectors. If we diagonalize the Bogoliubov Hamiltonian by using eq(11) and eq(12), we have

$$c_{k\alpha_i\uparrow} = \sum_m u_{m\uparrow\alpha_i}(k)\gamma_{km} + v^*_{\alpha_i m\uparrow}(-k)\gamma^\dagger_{-km} \tag{13}$$

$$c_{-k\alpha_i\downarrow} = \sum_m u_{m\downarrow\alpha_i}(-k)\gamma_{-km} + v^*_{\alpha_i m\downarrow}(k)\gamma^\dagger_{km} \tag{14}$$

Thus the self consistency condition at absolute zero is

$$\Delta_{\alpha_i} = \frac{1}{V}\sum_{km} g(k) u_{m\downarrow\alpha_i}(k) v^*_{\alpha_i m\uparrow}(k) \tag{15}$$

with

$$g(k) = \begin{cases} -g_0, & \text{for } |\xi_k| \leq \omega_d \\ 0, & \text{otherwise} \end{cases} \tag{16}$$

where ξ_k [there are 2n ξ_k for which one we should check this condition!]are eigenvalues of $H(k)$ and ω_d is the Deby frequency. The self consistency equation can be solved analytically for small n and numerically for large n. If solve for constant μ eq(15) numerically we get exponential dependency of Δ to μ. We can add an analytic calculation of dos and Δ for $n = 2$ here.

For the next step we will solve the self consistency equation for spatially inhomogeneous μ.

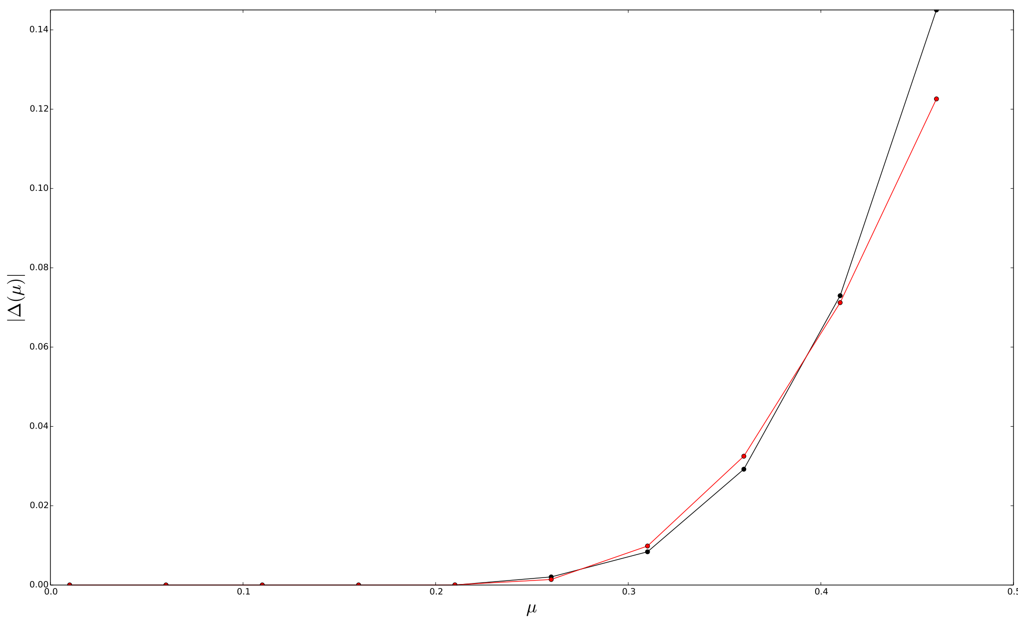

FIG. 2. This is calculated with $n = 2$, $\omega_d = 0.2$, $g_0 = 100$, $k_0 = \frac{\pi}{3}$, $\delta\mu = 0.05$ and $\delta k = 0.2$. Red is analytic and black is numeric. The g_0 is unphysically high because otherwise both of them(numeric and analytic solution) would be zero in the regime of the approximation of the analytic calculation holds.

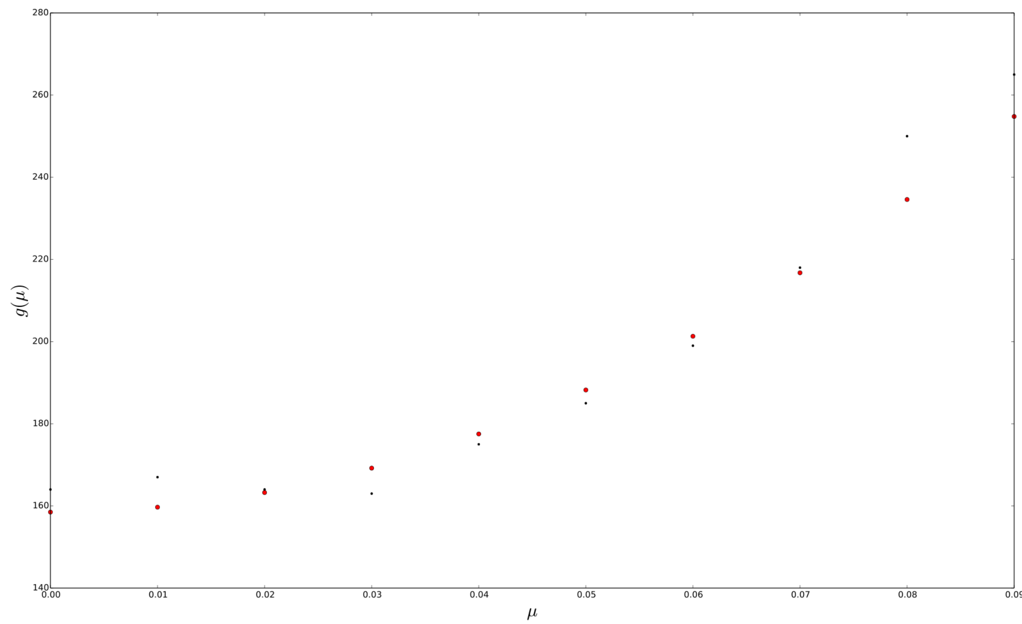

FIG. 3. Density of states

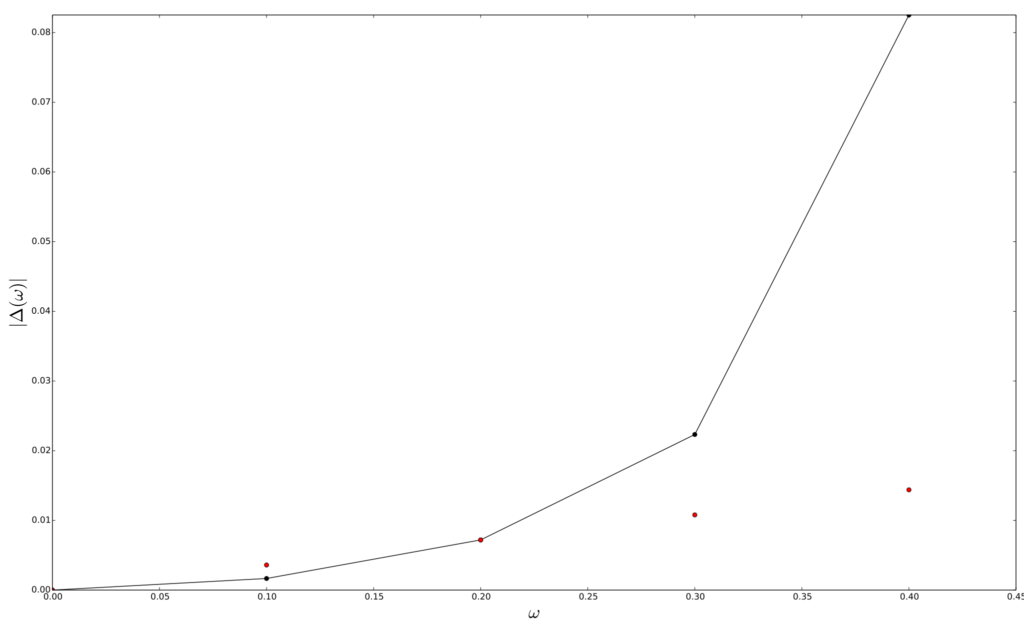

FIG. 4. I think this occurs because of the finite size of k here $\mu = 0.4$