Lab 03 - Conditional Probability

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1. Conditional probability

The probability of an event may depend on many different factors and conditions, such as randomly choosing two students to join a game, the probability of choosing a second female student depends on the first selection. The concept of conditional probability shows that a probability of a certain event A depends on a certain condition B. Condition B is also an event, in other words, the event of B.

In any probability space, condition B (or event B) has the probability $P(B) > 0$, and **the probability of event A under condition B**, the symbol $P(A|B)$ is defined as:

$$P(A|B) = \frac{P(A \cap B)}{P(B)} \tag{1}$$

A direct consequence of the conditional probability formula (1) is the following formula:

$$P(A \cap B) = P(A|B).P(B) \tag{2}$$

1.1. Problem 1

Mr. Smith has two children, including at least one son. What is the probability of both children is boys?

The above problem can be expressed through probability theory with the following conditions: Let $\Omega = \{\text{'BB'}, \text{'BG'}, \text{'GB'}, \text{'GG'}\}$ is the probability space of gender of Mr. Smith's two children, in which 'B' is male and 'G' is female. What is the probability of Mr. Smith's two children being boys under the condition that at least one child is a son?

To solve the above problem, we call:

- A is the event that Mr. Smith's two children to be boys.
- B is the event that at least one of Mr. Smith's children is a boy.

So we need to calculate $P(A|B)$. According to (1) we need to specify $P(A \cap B)$ and $P(B)$.

```python
from fractions import Fraction

def P(event, space):
    '''The probability of an event, given a sample space of
    equiprobable outcomes.'''
    return Fraction(len(event & space), len(space))

S = {'BB', 'BG', 'GB', 'GG'}

B = {s for s in S if 'B' in s}
A_B = {s for s in B if s.count('B')==2}

P_B = P(B, S)
P_A_B = P(A_B, S)

P_A_with_B = P_A_B/P_B
print(P_A_with_B)
```

1.2. Problem 2

One class has 17 students, including 10 female and 7 male. There are 3 friends named Thanh, including 1 female and 2 male. The teacher randomly calls a student to the board. The probability that this student' s name is Thanh is 3/17. But with the condition that "this is a female", the probability of that student' s name is Thanh is 1/10.

In this problem, the event A = "student' s name is Thanh", and condition B = "is a female". The probability space Ω has 17 elements with the uniform distribution. A has 3 elements, B has 10 elements and $A \cap B$ has 1 element. Therefore:

$$P(A) = \frac{\#A}{\#\Omega} = \frac{3}{17}$$

$$P(B) = \frac{\#B}{\#\Omega} = \frac{10}{17}$$

$$P(A \cap B) = \frac{\#A \cap B}{\#\Omega} = \frac{1}{17}$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{1}{17}}{\frac{10}{17}} = \frac{1}{10}$$

2. Bài tập

1. Know that a house has 3 cats.

   (a) Performance probability space of the problem through the variable S in the form ['MMM', 'MMF', 'MFM', ...] where M stands for "Male" and F stands for "Female".

(b) Determine the number of elements in the probability space S.

(c) Let B is the event that there is at least one female cat. List the elements of B and save to variable B.

(d) Let A is the event that all three cats are female. List elements of event A and save to variable A.

(e) Calculate the probability that all three cats are female under the condition that at least one cat is a female.

2. Give the probability space Ω of Problem 2 (section 1.2) as follows:

```
S = [('Thanh', 'Nữ'), ('Hồng', 'Nữ'), ('Thương', 'Nữ'), ('
    Đào', 'Nữ'), ('My', 'Nữ'), ('Yến', 'Nữ'), ('Hạnh', 'Nữ'),
     ('My', 'Nữ'), ('Vy', 'Nữ'), ('Tiên', 'Nữ'), ('Thanh', '
    Nam'), ('Thanh', 'Nam'), ('Bình', 'Nam'), ('Nhật', 'Nam')
    , ('Hào', 'Nam'), ('Đạt', 'Nam'), ('Minh', 'Nam')]
```

Write a program to:

(a) List elements of event A and store in variable A.

(b) List the elements of event B and save to variable B.

(c) List the elements of the event $A \cap B$ and store it in the variable A_B.

(d) Calculate the probability of the three above events and stored in three variables P_A, P_B, P_A_B respectively.

(e) Calculate the probability that the called student' s name is "Thanh" under the condition that "this is a female".

3. A standard deck of playing cards consists of 52 cards. All cards are divided into 4 suits including spades (♠), clubs (♣), diamonds (♦) and hearts (♥). In each suit there are 13 cards including a 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, J (jack), Q (queen), K (king). Write a program to:

(a) Create a set Cards to store cards in the form of '10♠', '10♡', '10♢', '10♣', '1♠', '1♡', '1♢', ...

(b) Randomly collect 3 cards, save the elements in variable B.

(c) $A1$ is the event that 3 cards include 1 or 2 K. Save the elements of event $A1$ to variable A1.

(d) $A2$ is the event that 3 cards include at least 1 K. Save the elements of event $A2$ into variable A2.

(e) Calculate the probability of two events $A1$ and $A2$. (Hint: P(A1) = 0.2172; P(A2) = 0.2174)