2-Dim case

$$f(x,y) = \frac{1}{\sqrt{(2\pi)^k |\Sigma|}} \exp\left[-\frac{1}{2}(\bar{x}-\bar{\mu})^T \cdot \Sigma^{-1}(\bar{x}-\bar{\mu})\right] \longrightarrow (3)$$

Determinant of covariance matrix:

① $$|\Sigma| = \begin{vmatrix} \sigma_x^2 & \rho\,\sigma_x\sigma_y \\ \rho\sigma_y\sigma_x & \sigma_y^2 \end{vmatrix} = \sigma_x^2\sigma_y^2 - \rho^2\sigma_x^2\sigma_y^2$$

$$= \sigma_x^2\sigma_y^2(1-\rho^2)$$

Inverse of the co. var. matrix:

② $$\Sigma^{-1} = \frac{1}{Det(\Sigma)} \qquad \Sigma^{-1} = \frac{1}{\sigma_x^2\sigma_y^2(1-\rho^2)} \begin{bmatrix} \sigma_y^2 & -\rho\sigma_x\sigma_y \\ -\rho\sigma_y\sigma_x & \sigma_x^2 \end{bmatrix}$$

Calculating exp. part only:

③ $$-\frac{1}{2}\begin{bmatrix} x-\mu_x \\ y-\mu_y \end{bmatrix}^T \cdot \Sigma^{-1} \begin{bmatrix} x-\mu_x \\ y-\mu_y \end{bmatrix}$$

$$-\frac{1}{2}\begin{bmatrix} x-\mu_x \\ y-\mu_y \end{bmatrix}^T \cdot \underbrace{\frac{1}{\sigma_x^2\sigma_y^2(1-\rho^2)} \begin{bmatrix} \sigma_y^2 & -\rho\sigma_x\sigma_y \\ -\rho\sigma_y\sigma_x & \sigma_x^2 \end{bmatrix}}_{\Sigma^{-1}} \begin{bmatrix} x-\mu_x \\ y-\mu_y \end{bmatrix}$$

$$-\frac{1}{2}\left[\begin{array}{cc} X-\mathcal{M}_x & y-\mathcal{M}_y \end{array}\right] \cdot \frac{1}{\sigma_x^2 \sigma_y^2(1-\rho^2)}\left[\begin{array}{c} \sigma_y^2(x-\mathcal{M}_x)-(\rho\sigma_x\sigma_y)(y-\mathcal{M}_y) \\ \rho\sigma_y\sigma_x(x-\mathcal{M}_x)+\sigma_x^2(y-\mathcal{M}_y) \end{array}\right]$$

multiply

$$-\frac{1}{2\sigma_x^2\sigma_y^2(1-\rho^2)}\left[\sigma_y^2(x-\mathcal{M}_x)^2-(\rho\sigma_x\sigma_y)(y-\mathcal{M}_y)(x-\mathcal{M}_x)+\rho\sigma_y\sigma_x(x-\mathcal{M}_x)(y-\mathcal{M}_y)\right.$$
$$\left.+\sigma_x^2(y-\mathcal{M}_y)^2\right]$$

minus

$$=\frac{1}{2\sigma_x^2\sigma_y^2(1-\rho^2)}\left[\sigma_y^2(x-\mathcal{M}_x)^2+\sigma_x^2(y-\mathcal{M}_y)^2\right]-2\rho\sigma_x\sigma_y(x-\mathcal{M}_x)(y-\mathcal{M}_y)$$

$$=\frac{1}{2(1-\rho^2)}\left[\frac{\sigma_y^2(x-\mathcal{M}_x)^2}{\sigma_x^2\sigma_y^2}+\frac{\sigma_x^2(y-\mathcal{M}_y)^2}{\sigma_y^2\sigma_x^2}\right]-\frac{2\rho\sigma_x\sigma_y(x-\mathcal{M}_x)(y-\mathcal{M}_y)}{\sigma_x^2\sigma_y^2}$$

③
$$=\frac{1}{2(1-\rho^2)}\left[\frac{(x-\mathcal{M}_x)^2}{\sigma_x^2}+\frac{(y-\mathcal{M}_y)^2}{\sigma_y^2}-\frac{2\rho(x-\mathcal{M}_x)(y-\mathcal{M}_y)}{\sigma_x\sigma_y}\right]$$

$$f(x,y)=\frac{1}{2\pi\sigma_x\sigma_y\sqrt{1-\rho^2}}\exp\left[\frac{1}{2(1-\rho^2)}\left(\frac{(x-\mathcal{M}_x)^2}{\sigma_x^2}+\frac{y-\mathcal{M}_y^2}{\sigma_y^2}-\frac{2\rho(x-\mathcal{M}_x)(y-\mathcal{M}_y)}{\sigma_x\sigma_y}\right)\right.$$

$$f(x,y) = \frac{1}{2\pi \sigma_x \sigma_y \sqrt{(1-\rho^2)}} \exp\left[\frac{1}{2(1-\rho^2)}\left(\frac{x-\mu_x^2}{\sigma_x^2} + \frac{y-\mu_y^2}{\sigma_y^2} - \frac{2\rho(x-\mu_x)(y-\mu_y)}{\sigma_x \sigma_y}\right)\right]$$