Heuristic search

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Outline

Generate-and-Test

Algorithm

1. Generate a possible solution
2. Test to see if this is actually a solution
3. Quit if a solution has been found. Otherwise, return to step 1

Generate-and-Test

- Acceptable for simple problem
- Inefficient for problems with large space

Generate-and-Test

- Exhaustive generate-and-test.
- Heuristic generate-and-test: not consider paths that seem unlikely to lead to a solution.
- Plan generate-test:
  - Create a list of candidates.
  - Apply generate-and-test to that list.

Example (colored blocks)

Arrange four 6-sided cubes in a row, with each side of each cube painted one of four colors, such that on all four sides of the row one block face of each color is showing.

Generate-and-Test

Example (heuristic)

if there are more red faces than other colors then, when placing a block with several red faces, use few of them as possible as outside faces.

Hill Climbing

Searching for a goal state = Climbing to the top of a hill

Hill Climbing

- Generate-and-Test + direction to move.
- Heuristic function: estimate how close a given state to a goal state

Simple Hill Climbing

Algorithm

1. Evaluate the initial state
2. Loop until a solution is found or there are no new operators left to be applied:
   - Select and apply a operator
   - Evaluate the new state:

     goal $\rightarrow$ quit

     better than current state $\rightarrow$ new current state

Simple Hill Climbing

Algorithm

1. Evaluate the initial state
2. Loop until a solution is found or there are no new operators left to be applied:
   - Select and apply a operator
   - Evaluate the new state:

     goal $\rightarrow$ quit

     better than current state $\rightarrow$ new current state

not try all possible new states!

Simple Hill Climbing

Example (colored blocks)

Heuristic function: the sum of the number of different colors on each of the four sides (solution $= 16$).

Heuristic function

Heuristic function as a way to inject task-specific knowledge into the control process.

Steepest-Ascent Hill Climbing (Gradient Search)

- Considers all the moves from the current state.
- Selects the best one as the next state.

Steepest-Ascent Hill Climbing (Gradient Search)

Algorithm

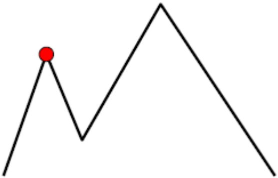
1. Evaluate the initial state
2. Loop until a solution is found or there are no new operators left to be applied:
   - Apply all the possible operators
   - Evaluate the best new state:

     goal $\rightarrow$ quit

     better than current state $\rightarrow$ new current state

Hill Climbing: Disadvantages

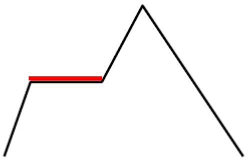
Local maximum
A state that is better than all of its neighbors, but not better than some other states far away.



Hill Climbing: Disadvantages

Plateau

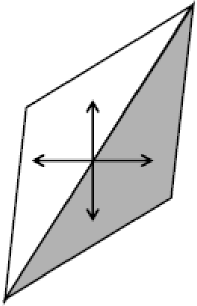
A flat area of the search space in which all neighboring states have the same value.



Hill Climbing: Disadvantages

Ridge

The orientation of the high region, compared to the set of available moves, makes it impossible to climb up. However, two moves executed serially may increase the height.



Hill Climbing: Disadvantages

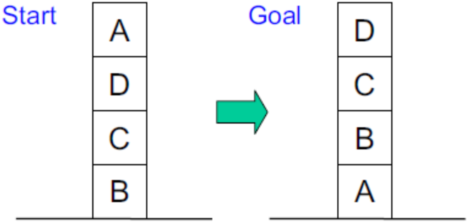
Ways Out

- Backtrack to some earlier node and try going in a different direction.
- Make a big jump to try to get in a new section.
- Moving in several directions at once.

Hill Climbing: Disadvantages

- Hill climbing is a local method: Decides what to do next by looking only at the "immediate" consequences of its choices.
- Global information might be encoded in heuristic functions.

Hill Climbing: Block World

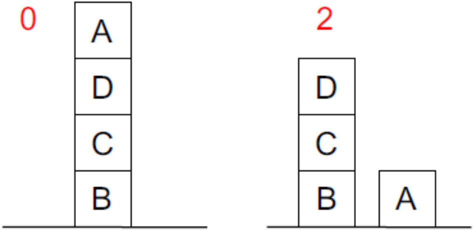


Hill Climbing: Block World

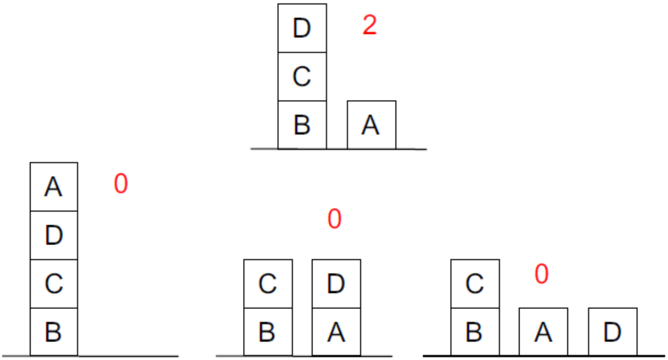
Local heuristic:

- $+1$ for each block that is resting on the thing it is supposed to be resting on.
- -1 for each block that is resting on a wrong thing.

Hill Climbing: Block World



Hill Climbing: Block World

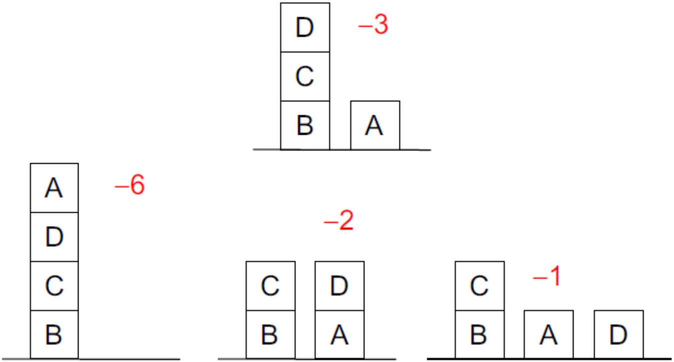


Hill Climbing: Block World

Global heuristic:

- For each block that has the correct support structure: $+1$ to every block in the support structure.
- For each block that has a wrong support structure: -1 to every block in the support structure.

Hill Climbing: Block World



Hill Climbing: Conclusion

- Can be very inefficient in a large, rough problem space.
- Global heuristic may have to pay for computational complexity.
- Often useful when combined with other methods, getting it started right in the right general neighborhood.

Simulated Annealing

A variation of hill climbing in which, at the beginning of the process, some downhill moves may be made.

Simulated Annealing

- To do enough exploration of the whole space early on, so that the final solution is relatively insensitive to the starting state.
- Lowering the chances of getting caught at a local maximum, or plateau, or a ridge.

Physical Annealing

- Physical substances are melted and then gradually cooled until some solid state is reached.
- The goal is to produce a minimal-energy state.
- Annealing schedule: if the temperature is lowered sufficiently slowly, then the goal will be attained.
- Nevertheless, there is some probability for a transition to a higher energy state: $e^{-\Delta E/kT}$

Simulated Annealing

Algorithm

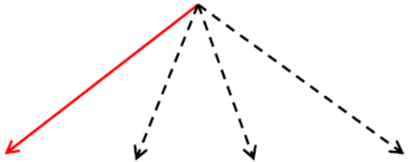
1. Evaluate the initial state
2. Loop until a solution is found or there are no new operators left to be applied:
   - Set T according to an annealing schedule.
   - Selects and applies a new operator
   - Evaluate the new state:

     goal $\rightarrow$ quit
     $\Delta E =$ Val(current state) – Val(new state)
     $\Delta E < 0 \rightarrow$ new current state
     else $\rightarrow$ new current state with probability $e^{-\Delta E/kT}$

Best-First Search

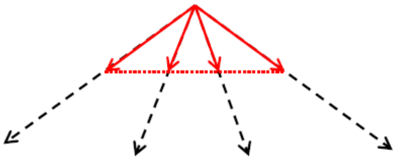
Depth-first search

- Pros: not having to expand all competing branches
- Cons: getting trapped on dead-end paths



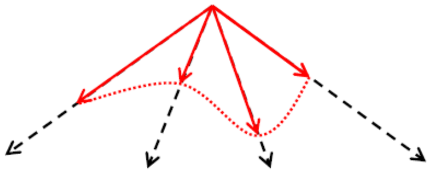
Breadth-first search

- Pros: not getting trapped on dead-end paths
- Cons: having to expand all competing branches

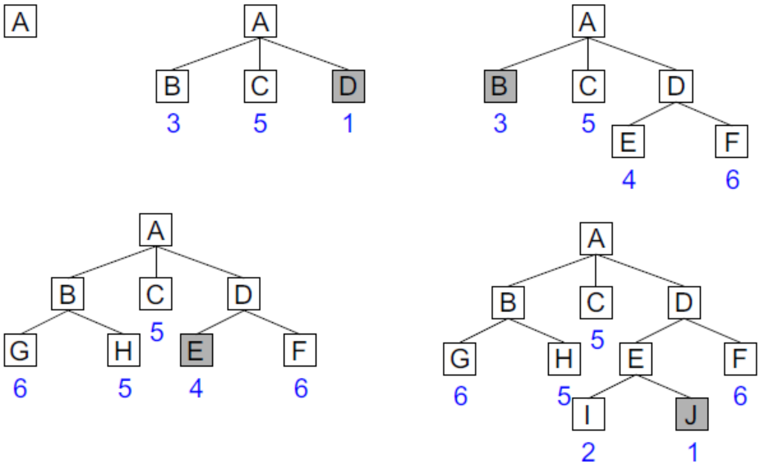


Best-First Search

⇒ Combining the two is to follow a single path at a time, but switch paths whenever some competing path looks more promising than the current one.



Best-First Search



Best-First Search

- OPEN: nodes that have been generated, but have not examined.

  This is organized as a priority queue.

- CLOSED: nodes that have already been examined.

  Whenever a new node is generated, check whether it has been generated before.

Best-First Search

Algorithm

1. OPEN = {initial state}.
2. Loop until a goal is found or there are no nodes left in OPEN:
   - Pick the best node in OPEN
   - Generate its successors
   - For each successor:

     new $\rightarrow$ evaluate it, add it to OPEN, record its parent generated before

     $\rightarrow$ change parent, update successors

- Greedy search:

  $h(n)$ = cost of the cheapest path from node n to a goal state.

Best-First Search

- Greedy search:
  $h(n)$ = cost of the cheapest path from node n to a goal state.

- Uniform-cost search:
  $g(n)$ = cost of the cheapest path from the initial state to node n.

Best-First Search

- Greedy search:

  $h(n)$ = cost of the cheapest path from node n to a goal state.

  Neither optimal nor complete

- Uniform-cost search:

  $g(n)$ = cost of the cheapest path from the initial state to node n.

Best-First Search

- Greedy search:

  $h(n)$ = cost of the cheapest path from node n to a goal state.

  Neither optimal nor complete

- Uniform-cost search:

  $g(n)$ = cost of the cheapest path from the initial state to node n.

  Optimal and complete, but very inefficient

Best-First Search

Algorithm: A^* (Hart et al., 1968)

$$f(n) = g(n) + h(n)$$

$h(n)$ = cost of the cheapest path from node n to a goal state.

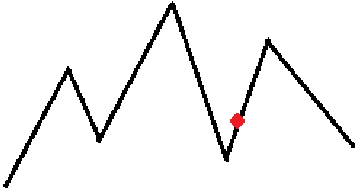
$g(n)$ = cost of the cheapest path from the initial state to node n.

Algorithm: A^* (Hart et al., 1968)

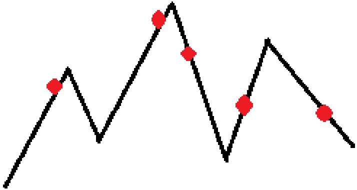
$$f^*(n) = g^*(n) + h^*(n)$$

$h^*(n)$ (heuristic factor) = estimate of $h(n)$.

$g^*(n)$ (depth factor) = approximation of $g(n)$ found by A^* so far.



Algorithm: Gradient search



Algorithm: Genetic algorithm

Genetic Algorithm

- A genetic algorithm is a heuristic search that mimics the process of natural evolution.
- There are five phases:
  - Initial Population
  - Fitness function
  - Selection
  - Crossover
  - Mutation

Genetic Algorithm: Initial Population

- Identify the information already presents in the problem as well as information that needs to be computed or derived
- Encode the solution in a form that is analogous to biological's chromosomes or sequence of DNA
- Generate randomly states that satisfy the problem

Genetic Algorithm: Initial Population

Example (8-queens)

The eight queens puzzle is the problem of placing eight chess queens on an 8x8 chessboard so that no two queens threaten each other

Representation

- Each state much have 8 queens
- One queen in each column
- Usually represented by a bit-string

  EX: [1,2,3,4,5,6,7,8]

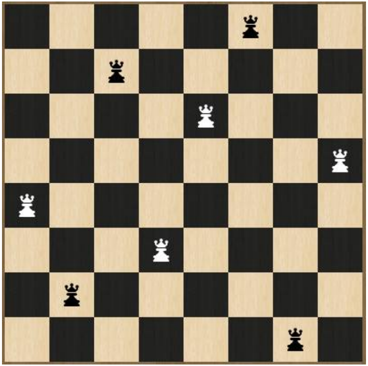
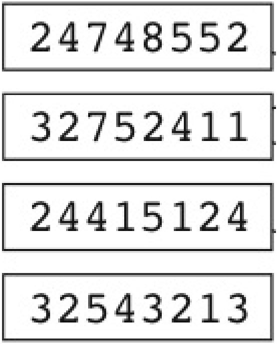
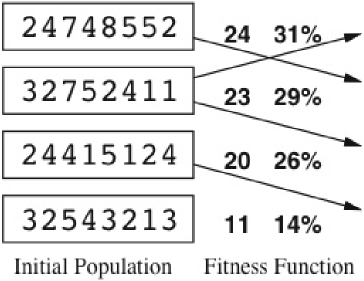


Figure: [4,2,7,3,6,8,1,5]



Initial Population

- Evaluate the "goodness" of each candidate solution
- The probability of being chosen for reproduction is based on your fitness score
- EX: 8-queens → fitness function: the number of nonattacking pairs of queens



Genetic Algorithm: Selection

- Fitness Proportionate Selection

  based on probability of selection: $p_i = f_i / \Sigma f_i$

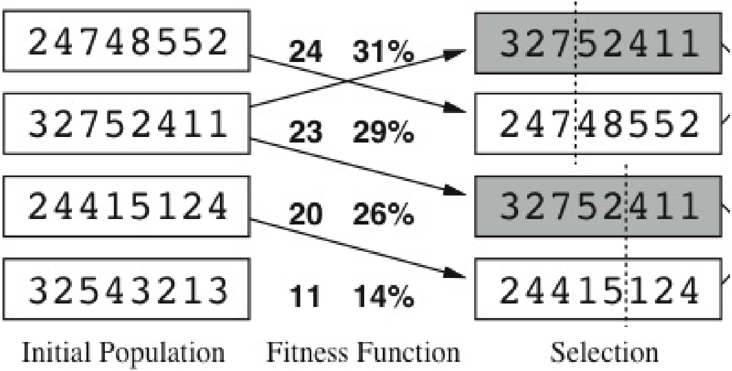
- Tournament Selection

  based on pre-determine number T (tournament size)

- Rank Selection

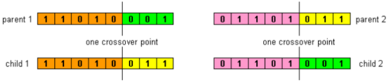
  rank the individuals in the population in ascending order according to their fitness values. Then, select similar to "Fitness Proportionate Selection"

Genetic Algorithm: Selection

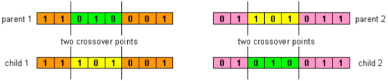


Genetic Algorithm: Crossover

Single-point Crossover

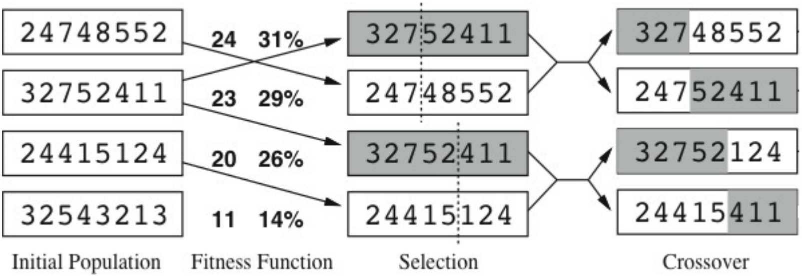


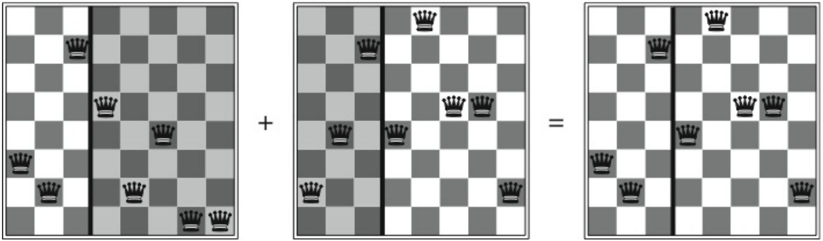
Multi-point Crossover



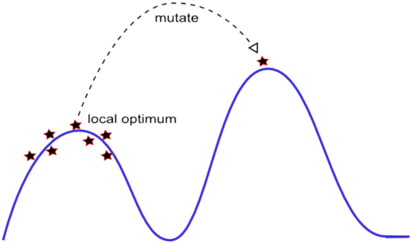
- Not all chosen chromosome pairs will undergo crossover
- p_c: probability of crossover or crossover rate

Genetic Algorithm: Crossover





- Mutation replaces the values of some randomly chosen genes of a chromosome by some arbitrary new values
- Bit inversion
- p_m: probability of mutation or mutation rate



Genetic Algorithm: Mutation

Exercises

- 8-puzzle
- Graph-coloring
- 8-queens
- Sudoku