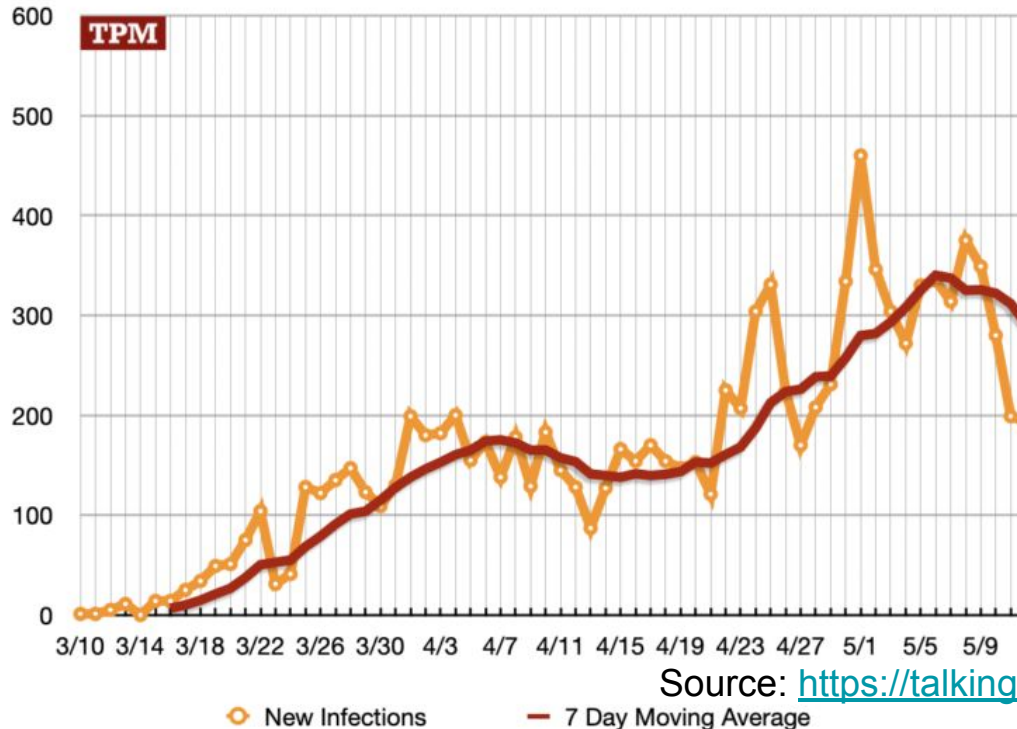


# CSC 369

Introduction to Distributed Computing

# Computing Moving Average

Daily New COVID19 Infections - Wisconsin (Data per @COVID19Tracking)



Smooth, captures trends

Precise, obscures trends

Source: <https://talkingpointsmemo.com/edblog/looking-at-the>

# Moving Average

**Problem: given COVID-19 data, report  $k$ -day moving averages for new infections for each state.**

In our examples  $k=3$  for simplicity

**Data Source:**

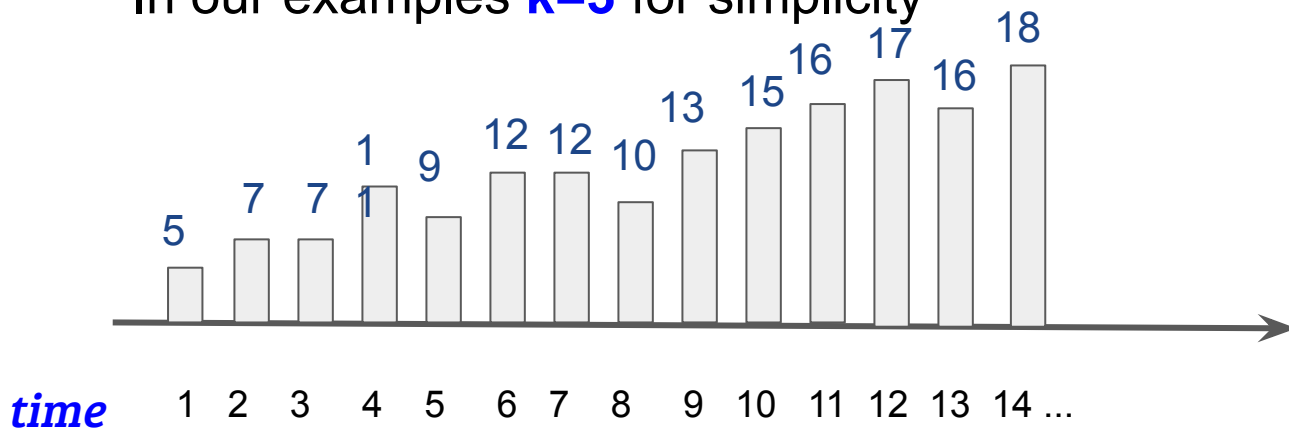
<https://api.covidtracking.com/v1/states/wi/daily.csv>

The "positiveIncrease" field represents new cases.

# Moving Average

**Problem:** given COVID-19 data, report **k**-day moving averages for new infections for each state.

In our examples **k=3** for simplicity

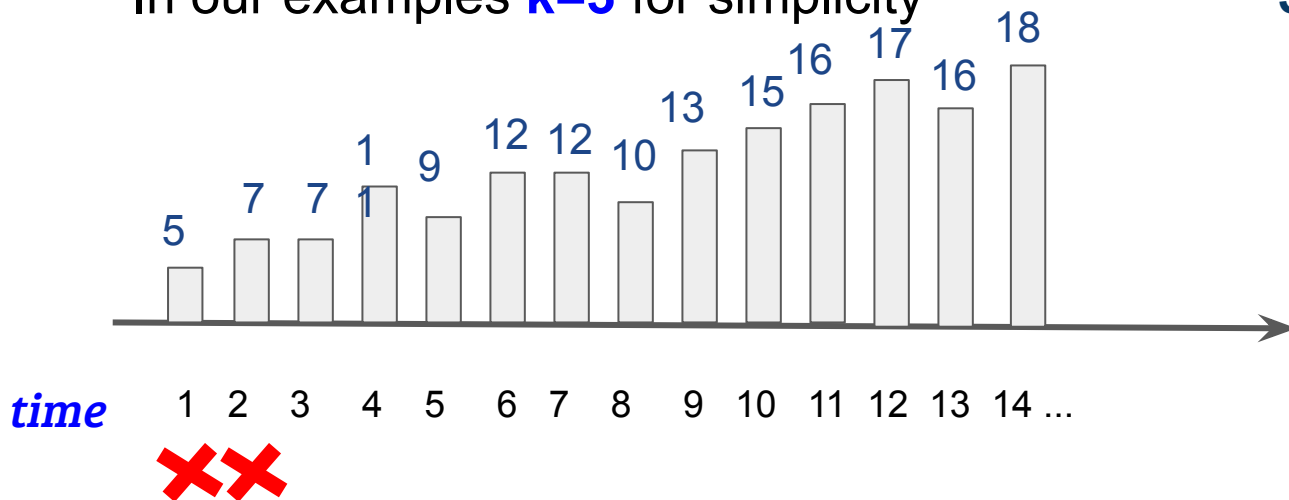


# Moving Average

**Problem:** given COVID-19 data, report **k**-day moving averages for new infections for each state.

In our examples **k=3** for simplicity

**Skip days 1, and 2**



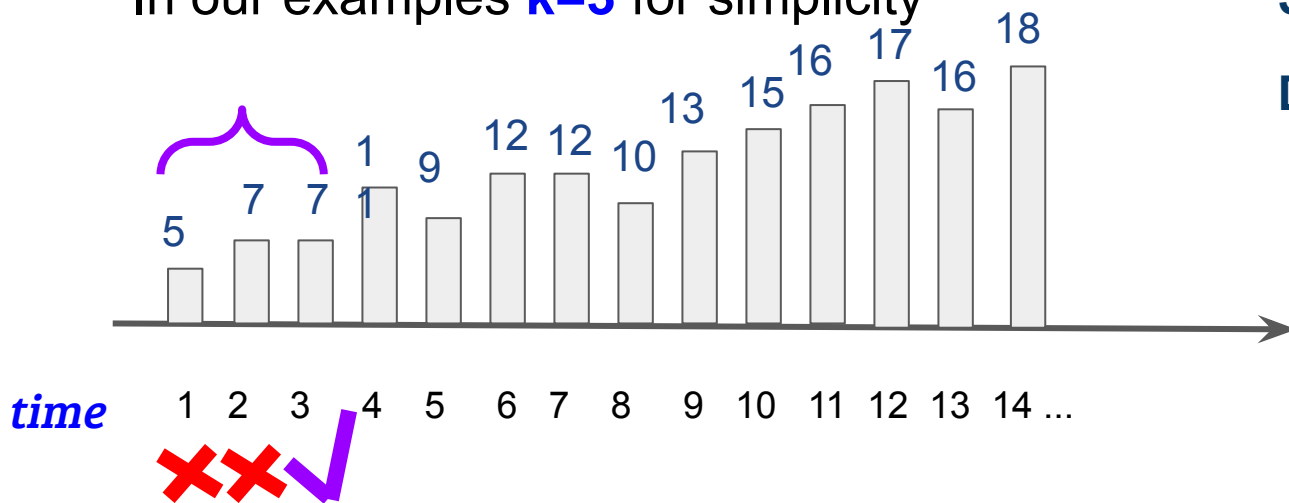
# Moving Average

**Problem:** given COVID-19 data, report **k**-day moving averages for new infections for each state.

In our examples **k=3** for simplicity

**Skip days 1, and 2**

**Day 3: include 1,2,3**



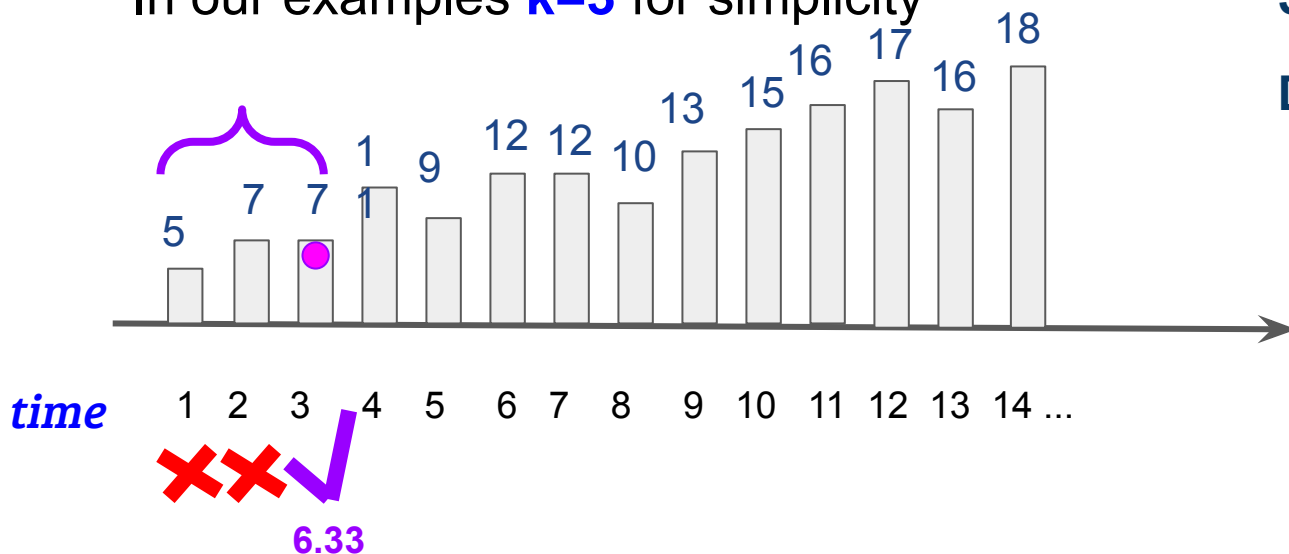
# Moving Average

**Problem:** given COVID-19 data, report **k**-day moving averages for new infections for each state.

In our examples **k=3** for simplicity

**Skip days 1, and 2**

**Day 3: include 1,2,3**



# Moving Average

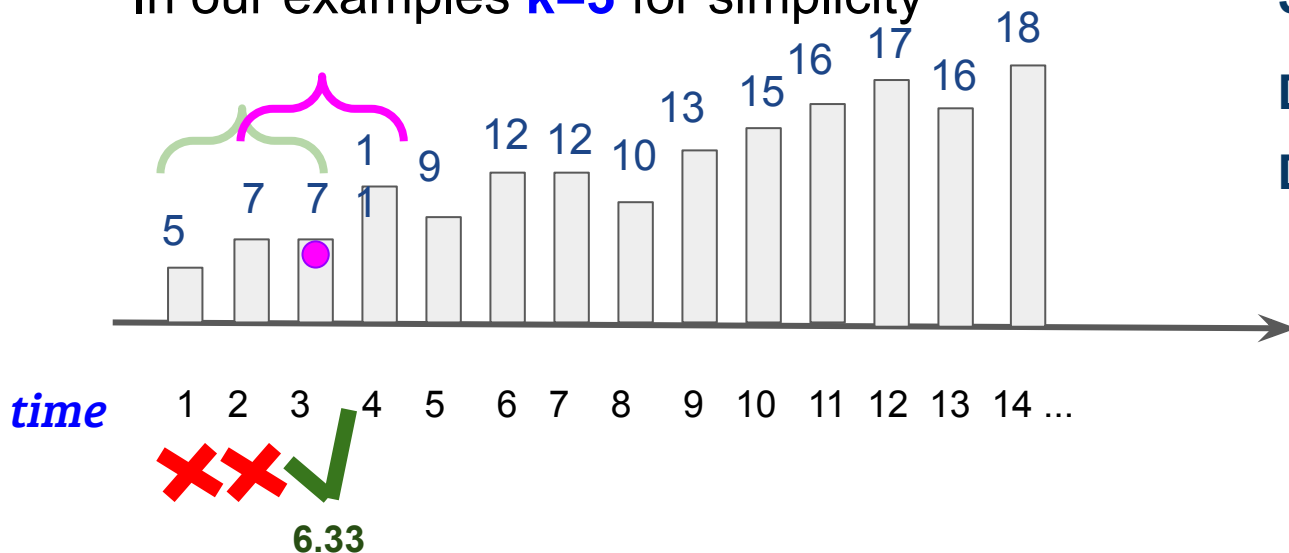
**Problem:** given COVID-19 data, report **k**-day moving averages for new infections for each state.

In our examples **k=3** for simplicity

**Skip days 1, and 2**

**Day 3: include 1,2,3**

**Day 4: include 2,3, 4**





# Moving Average

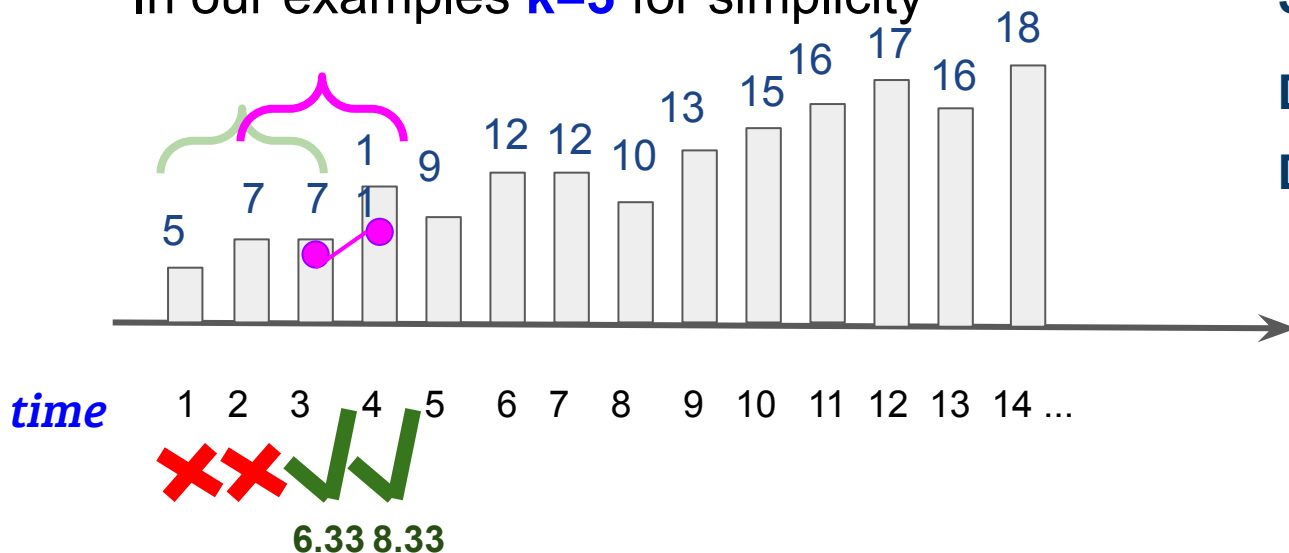
**Problem:** given COVID-19 data, report **k**-day moving averages for new infections for each state.

In our examples **k=3** for simplicity

Skip days 1, and 2

Day 3: include 1,2,3

Day 4: include 2,3, 4



# Moving Average

**Problem:** given COVID-19 data, report **k**-day moving averages for new infections for each state.

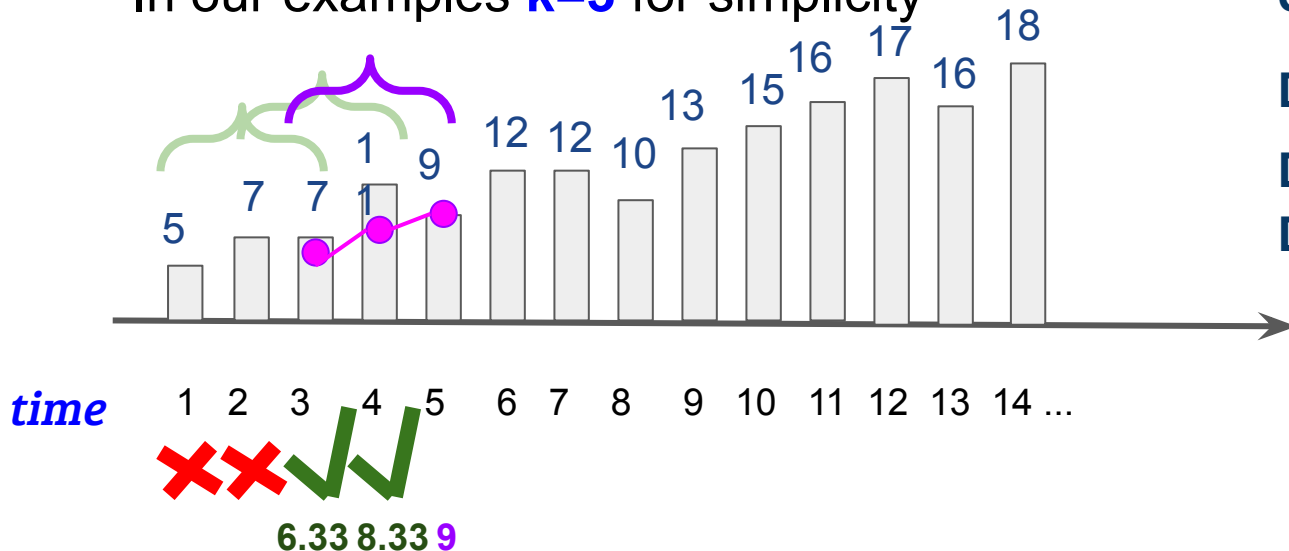
In our examples **k=3** for simplicity

**Skip days 1, and 2**

**Day 3: include 1,2,3**

**Day 4: include 2,3, 4**

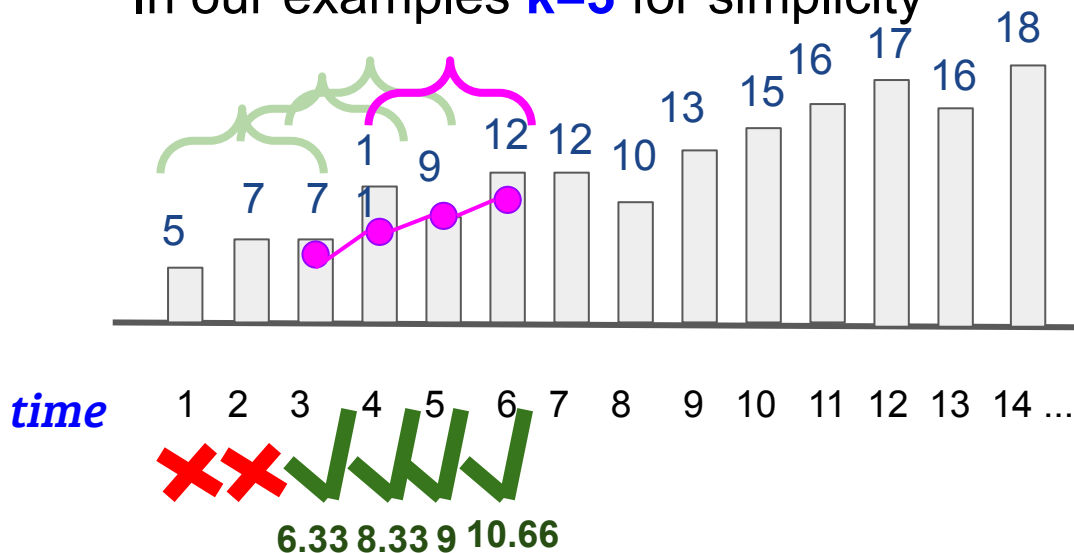
**Day 5: include 3, 4, 5**



# Moving Average

**Problem:** given COVID-19 data, report **k**-day moving averages for new infections for each state.

In our examples **k=3** for simplicity



Skip days 1, and 2

Day 3: include 1,2,3

Day 4: include 2,3, 4

Day 5: include 3, 4, 5

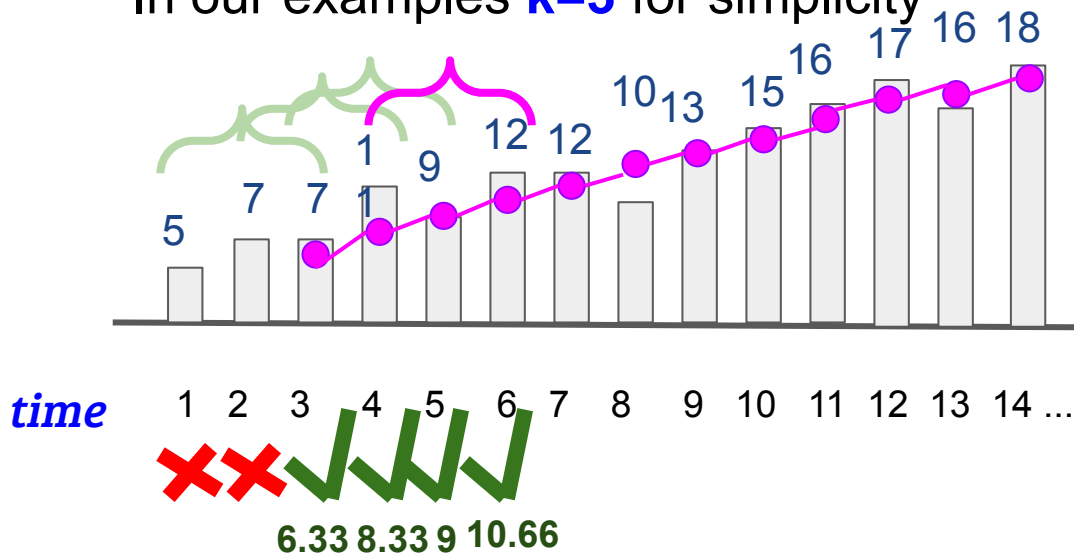
Day 6: include 4, 5, 6

... and so on

# Moving Average

**Problem:** given COVID-19 data, report **k**-day moving averages for new infections for each state.

In our examples **k=3** for simplicity



Skip days 1, and 2

Day 3: include 1,2,3

Day 4: include 2,3, 4

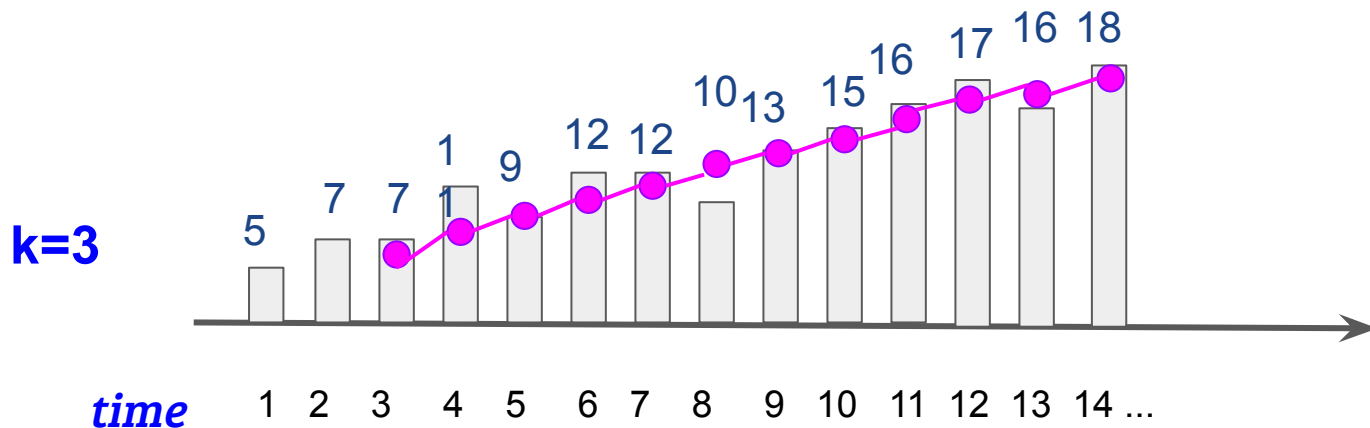
Day 5: include 3, 4, 5

Day 6: include 4, 5, 6

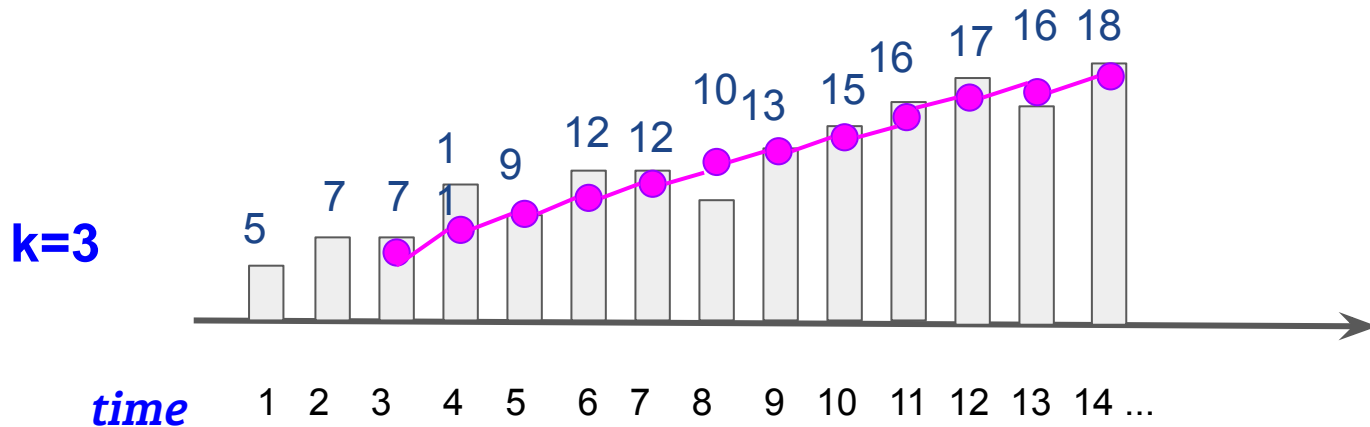
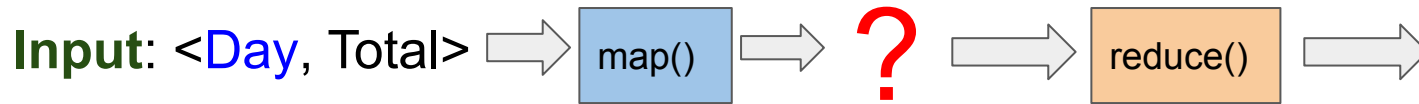
... and so on

# Moving Average via MapReduce

**Input:** <Day, Total>

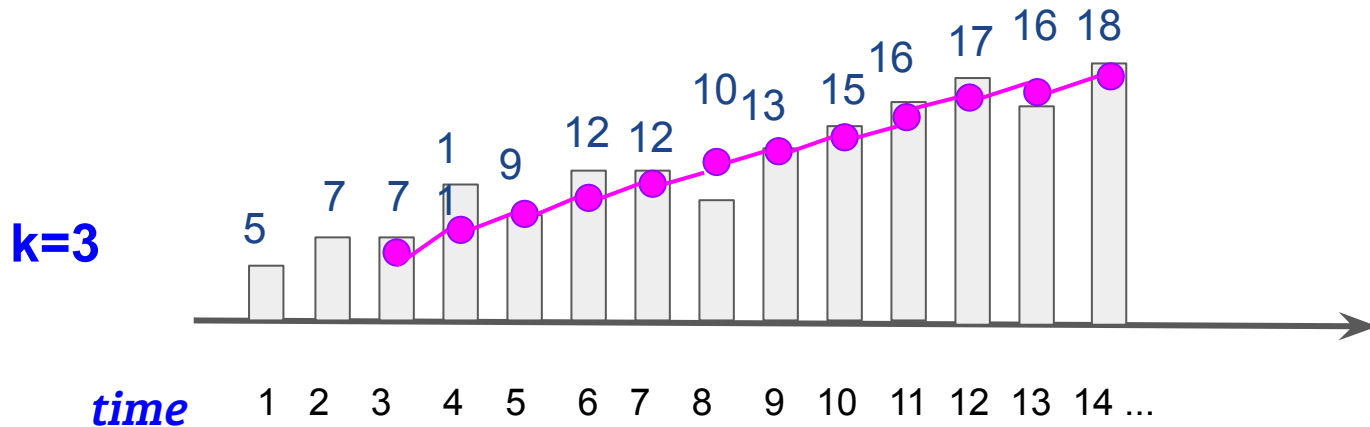


# Moving Average via MapReduce

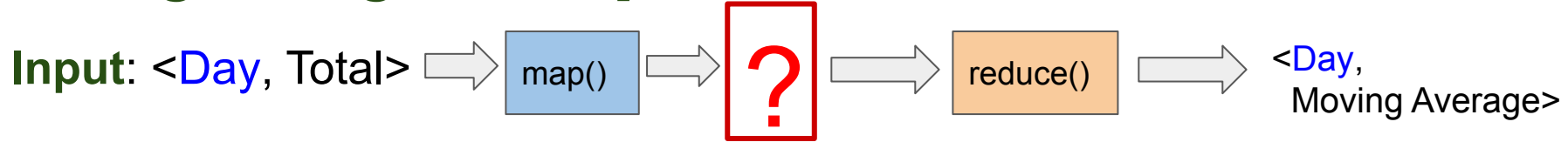


# Moving Average via MapReduce

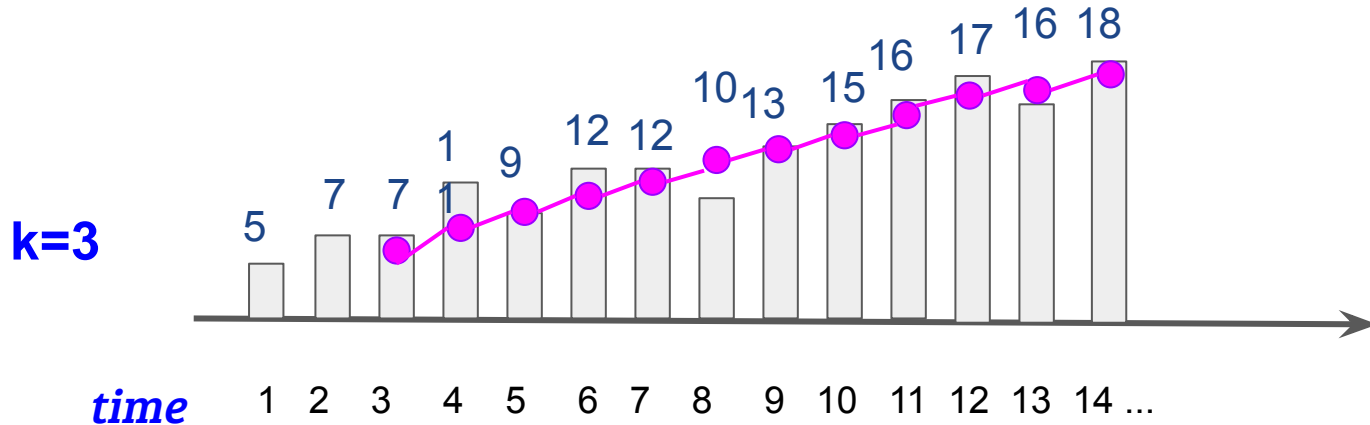
**Input:** <Day, Total>  $\Rightarrow$  `map()`  $\Rightarrow$  ?  $\Rightarrow$  `reduce()`  $\Rightarrow$  <Day, Moving Average>



# Moving Average via MapReduce

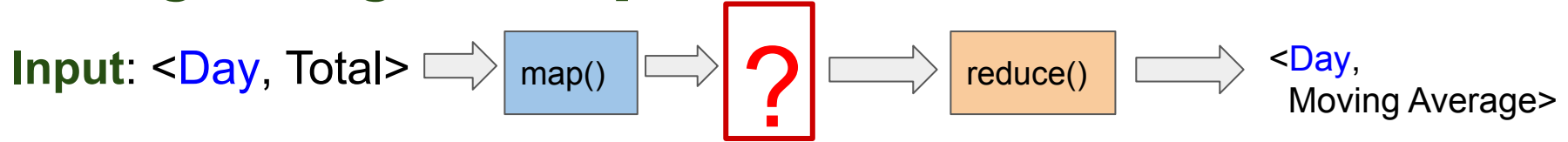


What should `reduce()` see?

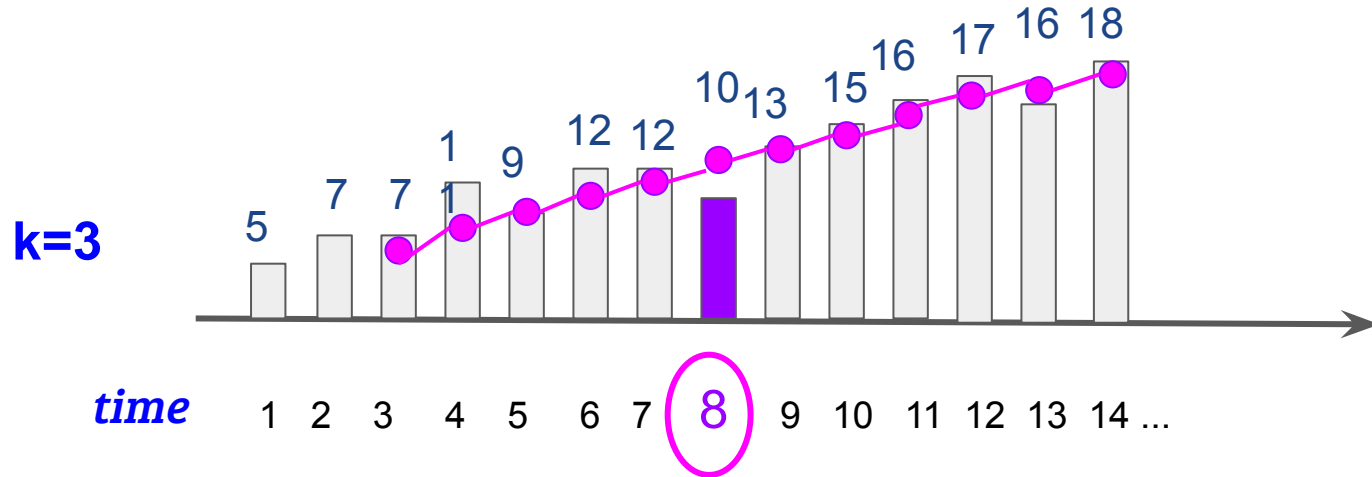




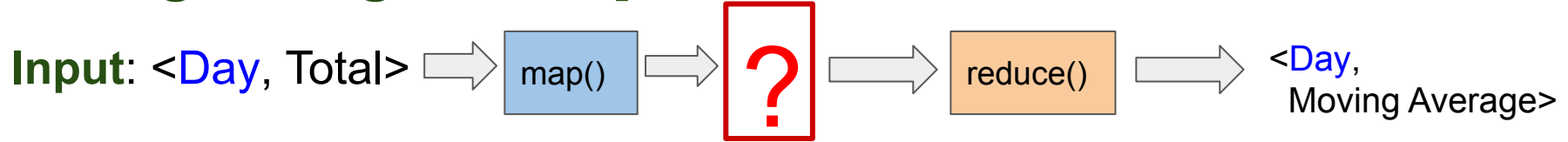
# Moving Average via MapReduce



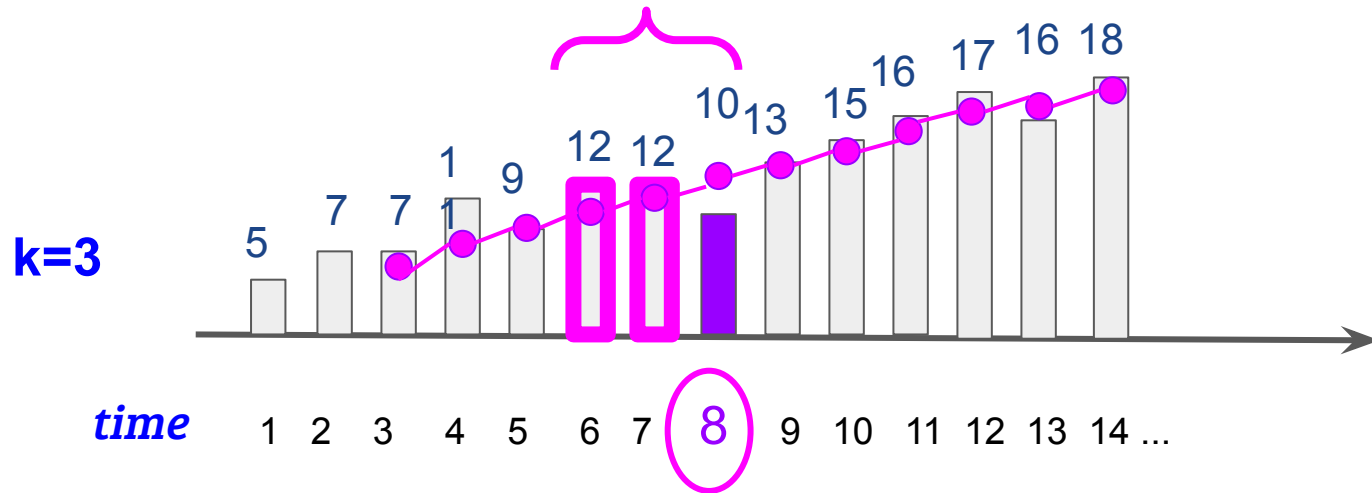
<Day: 8, Values: [ ???]>



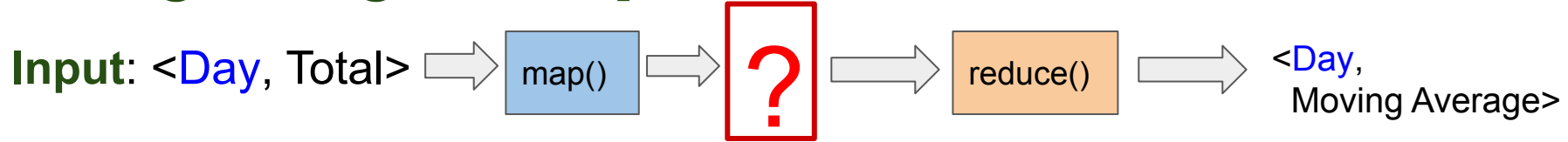
# Moving Average via MapReduce



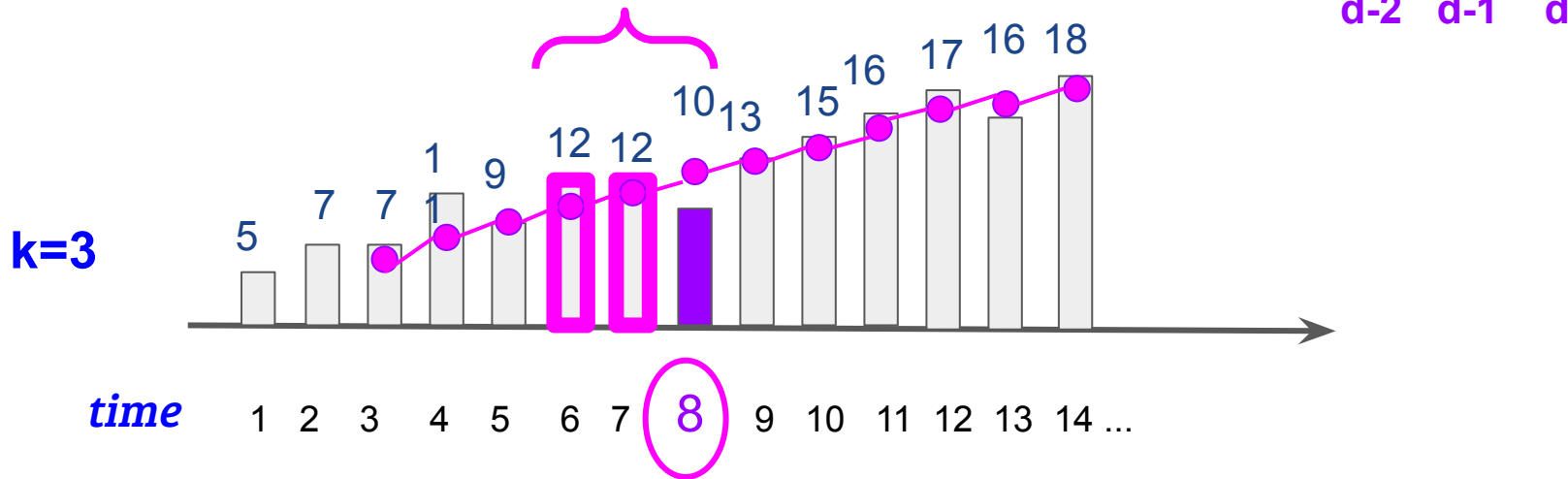
<Day: 8, Values: [ ???]>



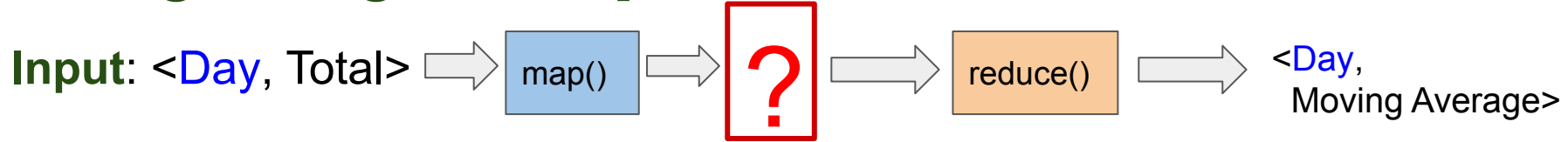
# Moving Average via MapReduce



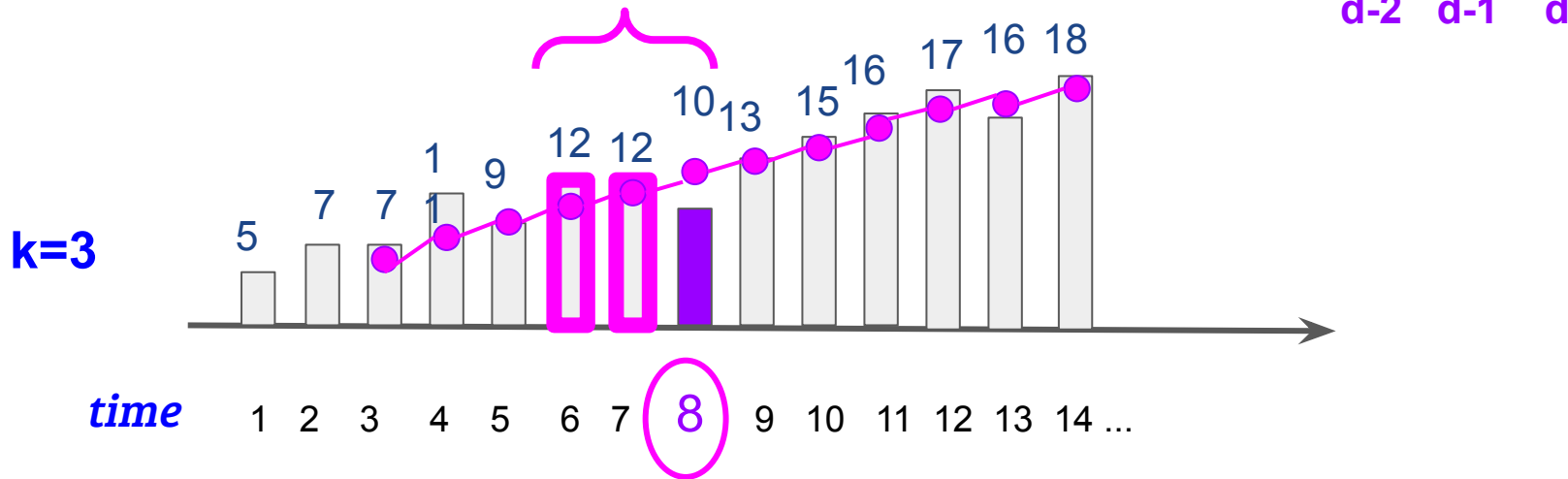
$\langle \text{Day: 8, Values: [12, 12, 10]} \rangle$



# Moving Average via MapReduce



$\langle \text{Day: 8, Values: [12, 12, 10]} \rangle$



reduce() writes itself

```
reduce( key, Iterable values):  
    sum = 0  
    count = 0  
    for v in values do  
        sum = sum + v  
        count = count + 1  
    end for  
    movingAverage = sum/count  
    emit (key, movingAverage)
```

# What about map()?

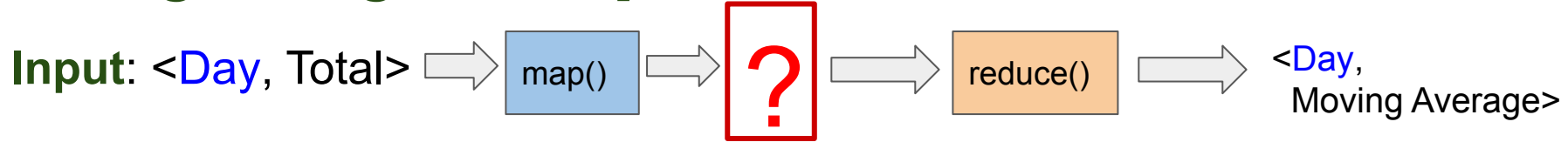
```
map(key, value):    // key: date;  value: total
```

# What about map()?

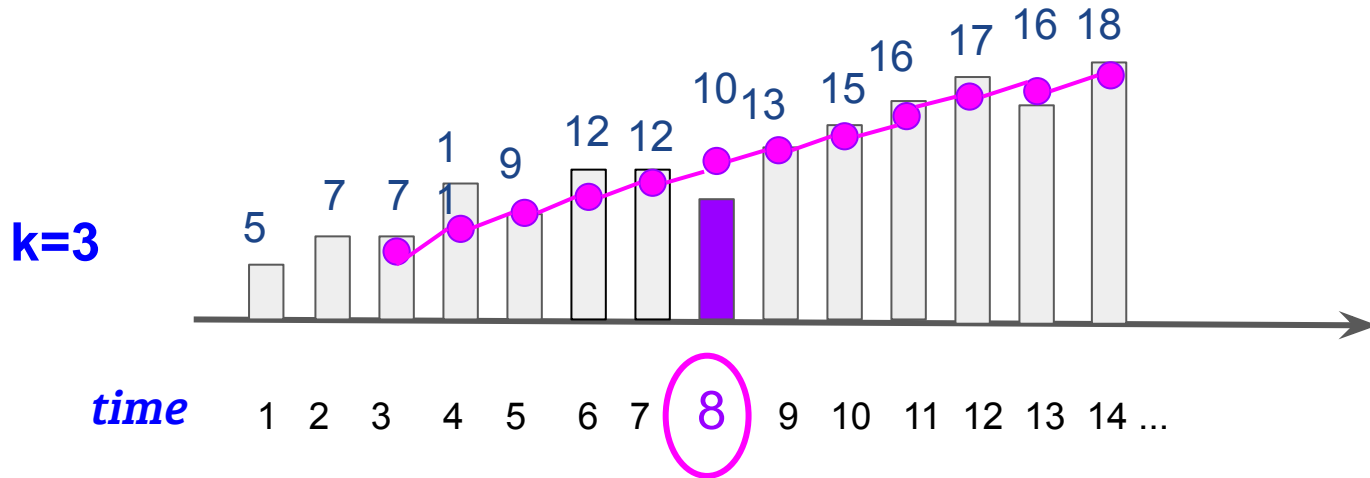
```
map(key, value):    // key: date;  value: total
```

```
// what computations will need our value???
```

# Moving Average via MapReduce

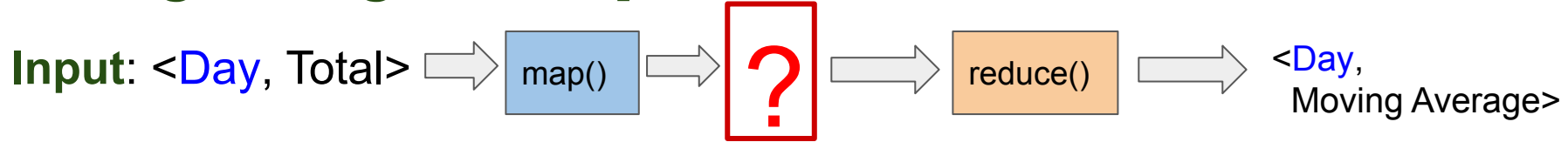


$\langle \text{Day:8, Total:10} \rangle$



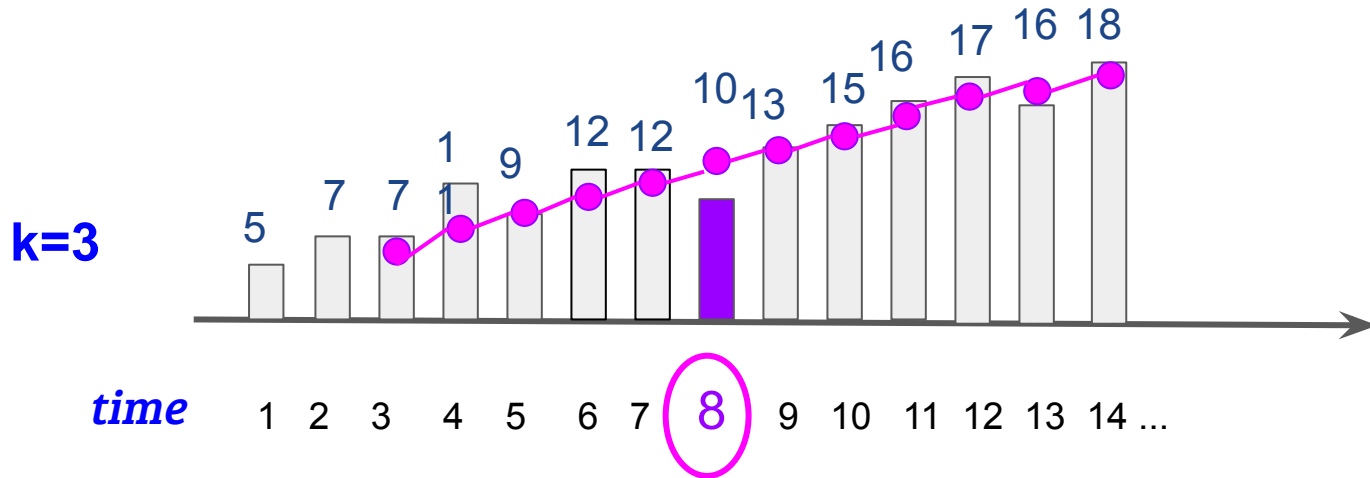


# Moving Average via MapReduce

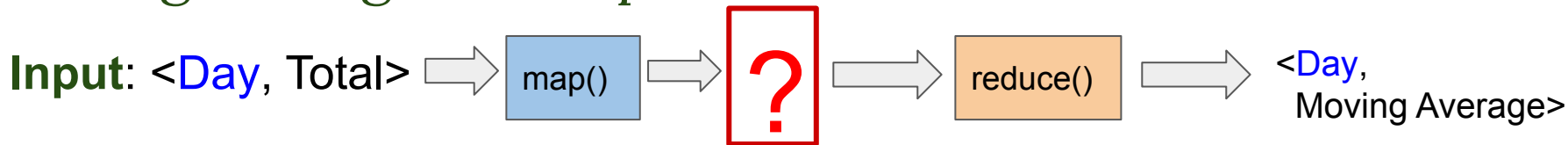


$\langle \text{Day:8}, \text{Total:10} \rangle$

Which Reduce() computations will Day 8 participate in?

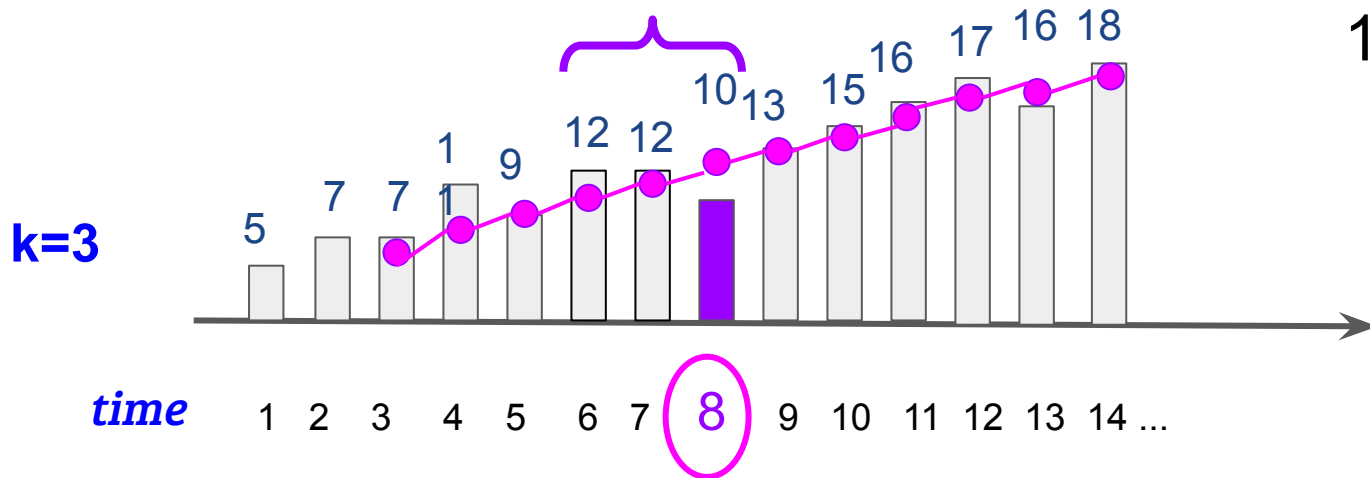


# Moving Average via MapReduce



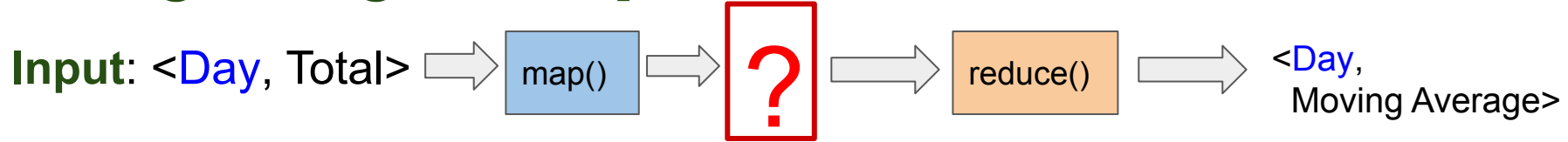
$\langle \text{Day:8}, \text{Total:10} \rangle$

Which Reduce() computations will Day 8 participate in?



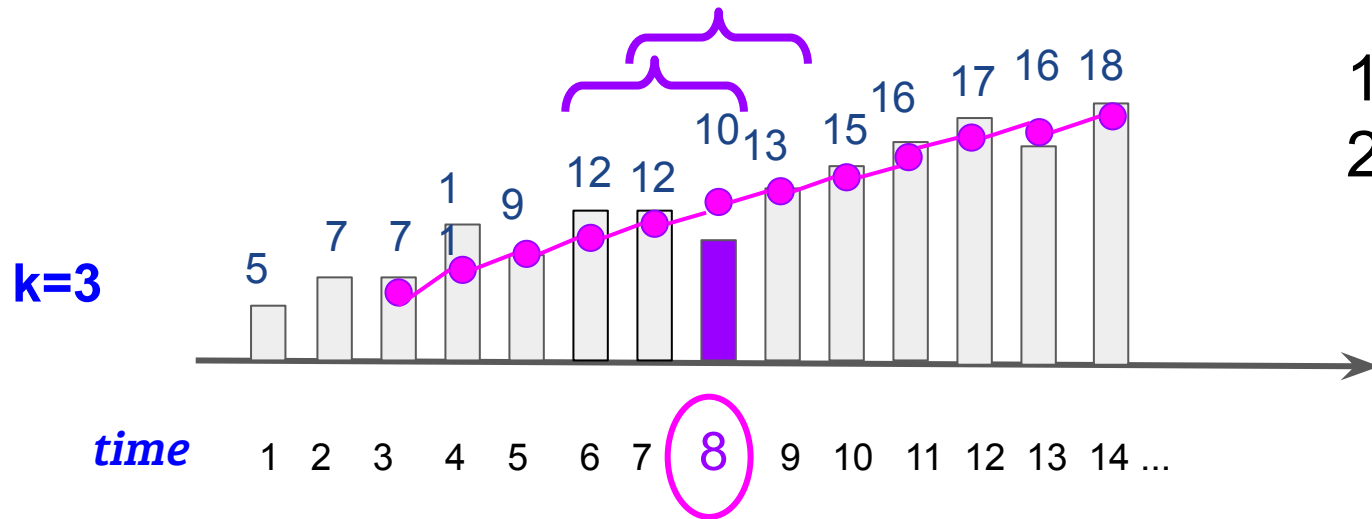
1. Day 8

# Moving Average via MapReduce



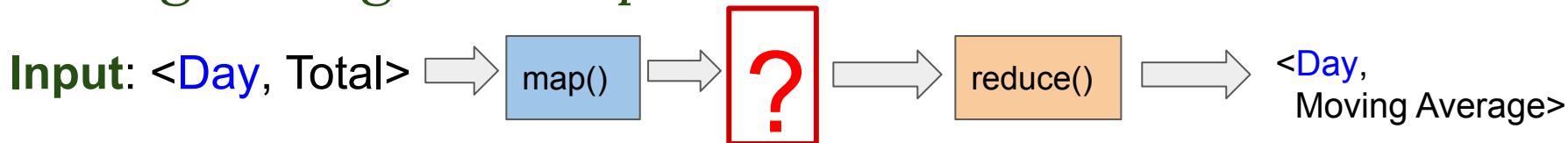
**$\langle \text{Day:8}, \text{Total:10} \rangle$**

Which Reduce() computations will Day 8 participate in?



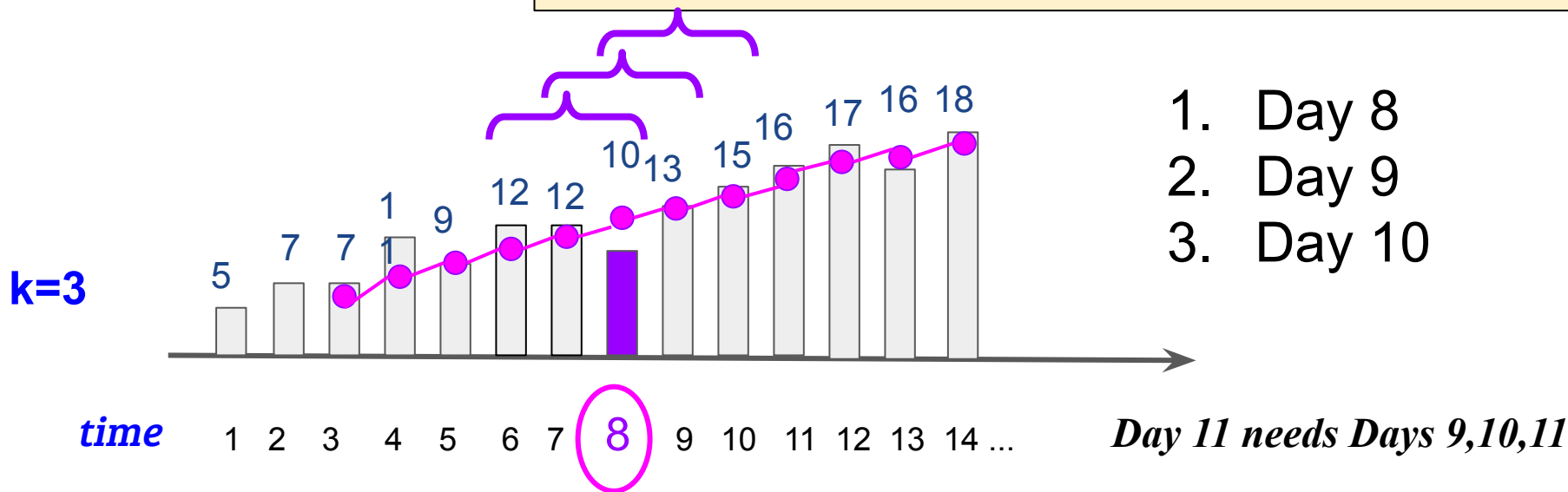
1. Day 8
2. Day 9

# Moving Average via MapReduce



$\langle \text{Day:8, Total:10} \rangle$

Which Reduce() computations will Day 8 participate in?



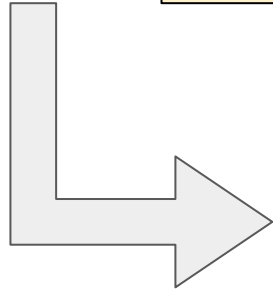
**<Day:8, Total:10>**

Which Reduce() computations will Day 8 participate in?

1. Day 8
2. Day 9
3. Day 10

**<Day:8, Total:10>**

Which Reduce() computations will Day 8 participate in?



map()



1. Day 8
2. Day 9
3. Day 10

And now, map() writes itself too

<Day:8, Total:10>

1. Day 8
2. Day 9
3. Day 10

```
map(key, value):    // key: date;  value: total

movingWindow = 3 // set moving window

for i = 0 to movingWindow-1 do
    newKey = key+i
    emit(key,value)
end for
```

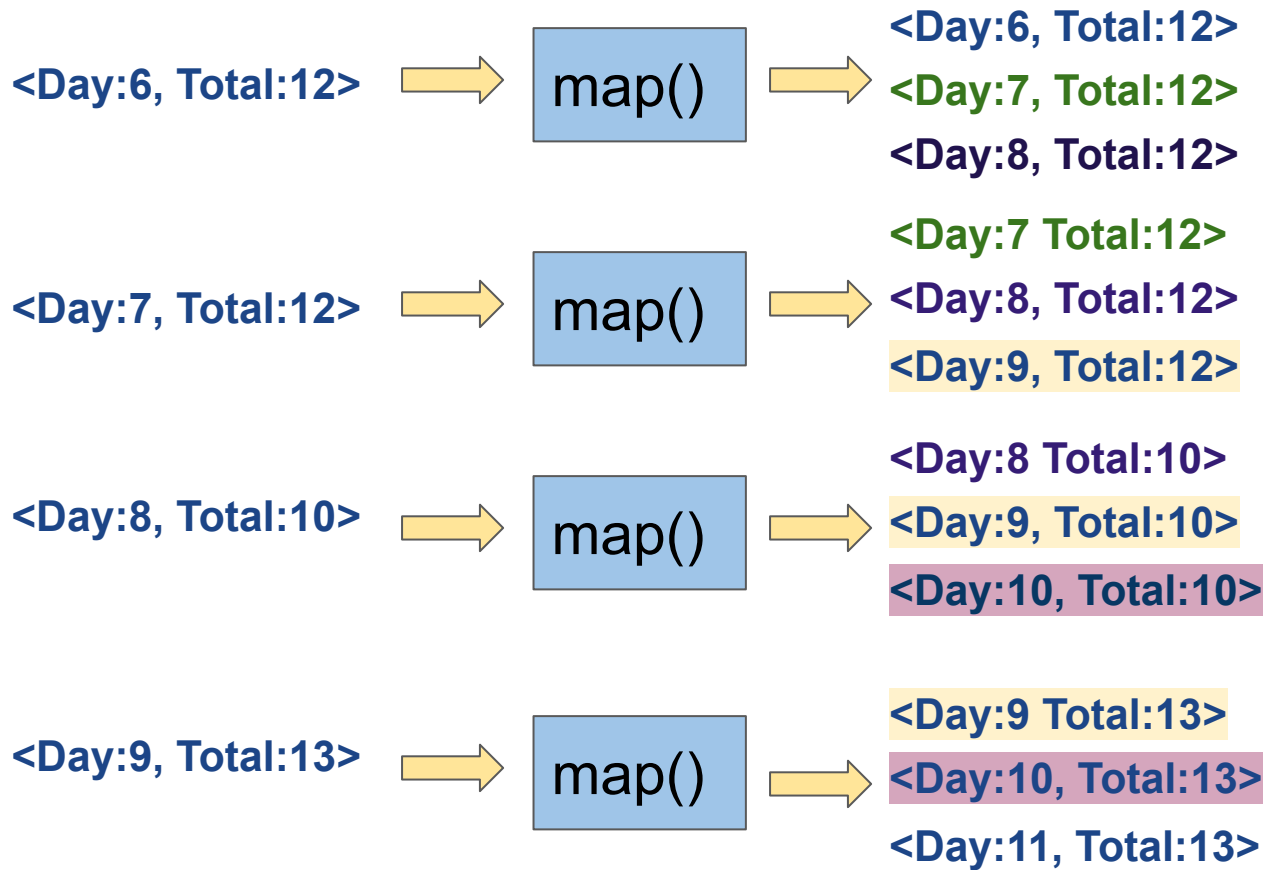
<Day:6, Total:12> → map() →  
<Day:6, Total:12>  
<Day:7, Total:12>  
<Day:8, Total:12>

<Day:7, Total:12> → map() →  
<Day:7 Total:12>  
<Day:8, Total:12>  
<Day:9, Total:12>

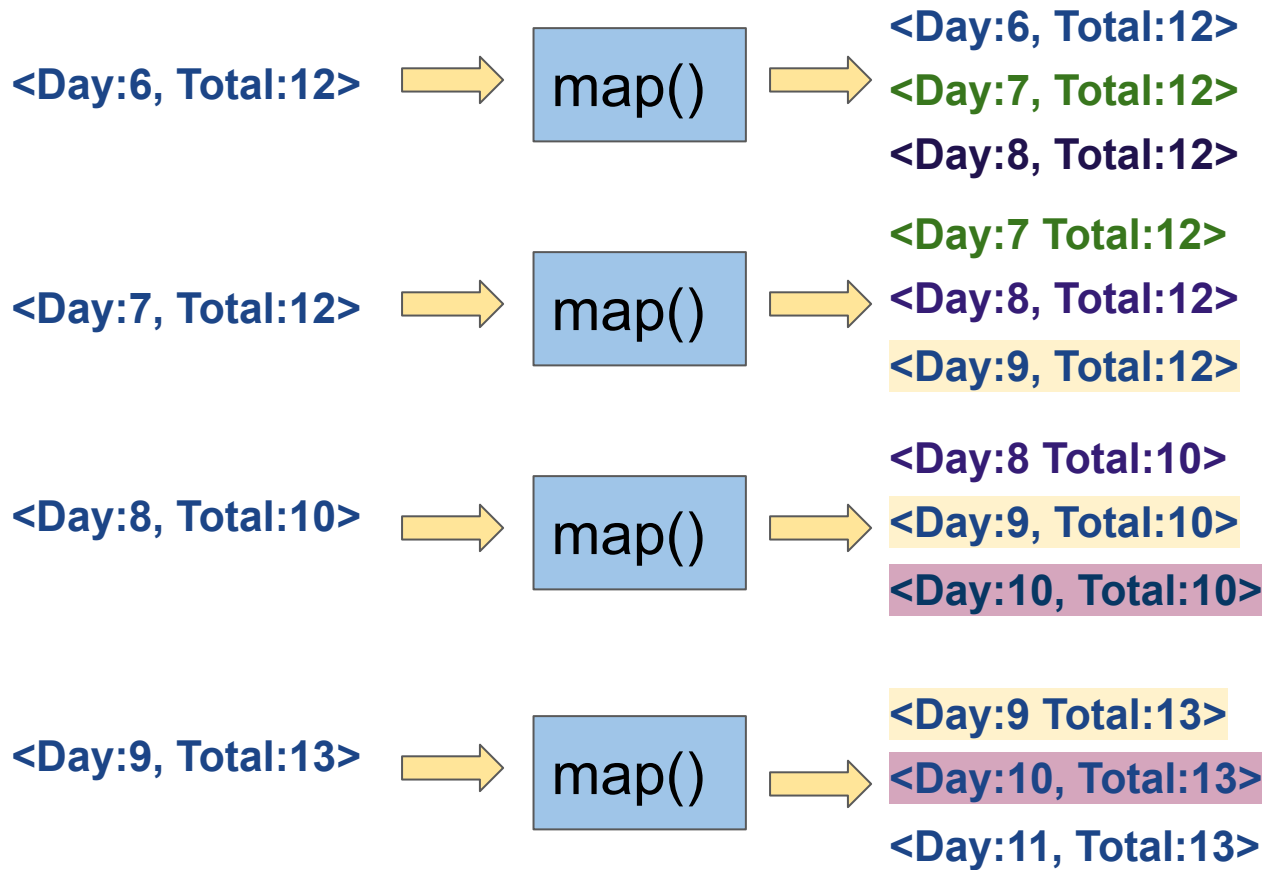
<Day:8, Total:10> → map() →  
<Day:8 Total:10>  
<Day:9, Total:10>  
<Day:10, Total:10>

<Day:9, Total:13> → map() →  
<Day:9 Total:13>  
<Day:10, Total:13>  
<Day:11, Total:13>





Let shuffle do its job



Day 7

....

Total: 12

Total: 12

Day 8

Total: 12

Total: 12

Total: 10

Day 9

Total: 12

Total: 10

Total: 13

Day 10

Total: 10

Total: 13

...

Let shuffle do its job

# Moving Average

**Solution idea for map():** we know what keys will need “**our**” value

# Moving Average

**Solution idea for map():** we know what keys will need “**our**” value

**Transfers over to other problems**

**Matrix Multiplication**

# Why Matrix Multiplication?

**Lies at the heart of many Machine Learning methods**

- Linear regression
- Support Vector Machines
- Neural Networks

**Used in many graph algorithms**

- Multiplying adjacency matrices by themselves

# Multiplying matrices

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

3 x 3

**X**

$$\begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \\ b_{31} & b_{32} \end{pmatrix}$$

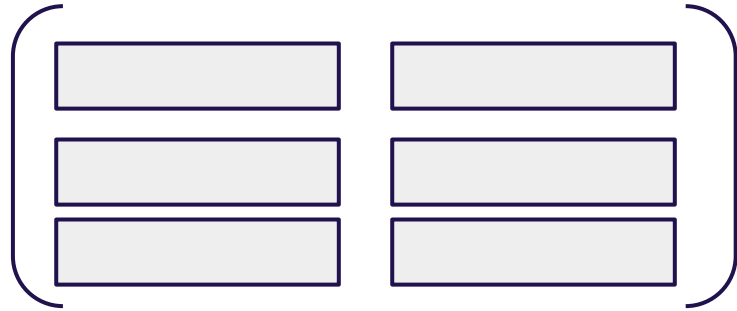
3 x 2

# Multiplying matrices

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \times \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \\ b_{31} & b_{32} \end{pmatrix}$$

3 x 3

3 x 2



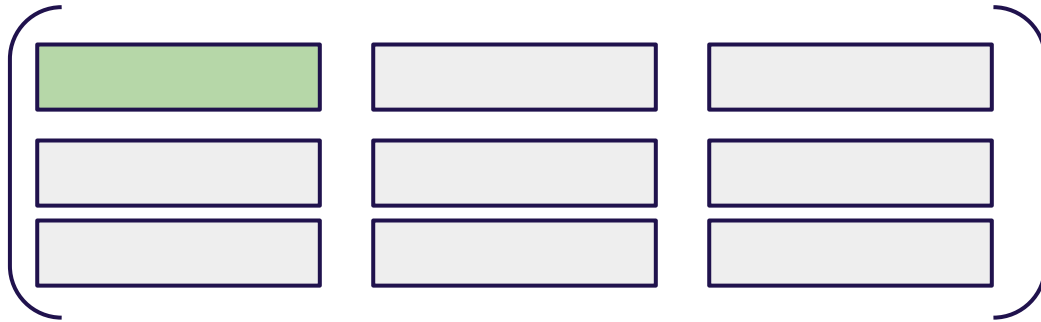
3 x 2

# Multiplying matrices

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \times \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \\ b_{31} & b_{32} \end{pmatrix}$$

3 x 3

3 x 2



3 x 2



# Multiplying matrices

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \times \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \\ b_{31} & b_{32} \end{pmatrix}$$

3 x 3

3 x 2

$$\begin{pmatrix} a_{11} \cdot b_{11} + a_{12} \cdot b_{21} + a_{13} \cdot b_{31} & \phantom{a_{11} \cdot b_{11} + a_{12} \cdot b_{21} + a_{13} \cdot b_{31}} \\ \phantom{a_{11} \cdot b_{11} + a_{12} \cdot b_{21} + a_{13} \cdot b_{31}} & \phantom{a_{11} \cdot b_{11} + a_{12} \cdot b_{21} + a_{13} \cdot b_{31}} \\ \phantom{a_{11} \cdot b_{11} + a_{12} \cdot b_{21} + a_{13} \cdot b_{31}} & \phantom{a_{11} \cdot b_{11} + a_{12} \cdot b_{21} + a_{13} \cdot b_{31}} \end{pmatrix}$$

3 x 2

# Multiplying matrices

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \times \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \\ b_{31} & b_{32} \end{pmatrix}$$

3 x 3

3 x 2

$$\begin{pmatrix} a_{11} \cdot b_{11} + a_{12} \cdot b_{21} + a_{13} \cdot b_{31} & \phantom{a_{11} \cdot b_{11} + a_{12} \cdot b_{21} + a_{13} \cdot b_{31}} \\ a_{21} \cdot b_{11} + a_{22} \cdot b_{21} + a_{23} \cdot b_{31} & \phantom{a_{21} \cdot b_{11} + a_{22} \cdot b_{21} + a_{23} \cdot b_{31}} \\ \phantom{a_{21} \cdot b_{11} + a_{22} \cdot b_{21} + a_{23} \cdot b_{31}} & \phantom{a_{21} \cdot b_{11} + a_{22} \cdot b_{21} + a_{23} \cdot b_{31}} \end{pmatrix}$$

3 x 2

# Multiplying matrices

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \times \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \\ b_{31} & b_{32} \end{pmatrix}$$

3 x 3

3 x 2

$$\begin{pmatrix} a_{11} \cdot b_{11} + a_{12} \cdot b_{21} + a_{13} \cdot b_{31} & \phantom{a_{11} \cdot b_{11} + a_{12} \cdot b_{21} + a_{13} \cdot b_{31}} \\ a_{21} \cdot b_{11} + a_{22} \cdot b_{21} + a_{23} \cdot b_{31} & \phantom{a_{21} \cdot b_{11} + a_{22} \cdot b_{21} + a_{23} \cdot b_{31}} \\ a_{31} \cdot b_{11} + a_{32} \cdot b_{21} + a_{33} \cdot b_{31} & \phantom{a_{31} \cdot b_{11} + a_{32} \cdot b_{21} + a_{33} \cdot b_{31}} \end{pmatrix}$$

3 x 2

# Multiplying matrices

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

3 x 3

X

$$\begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \\ b_{31} & b_{32} \end{pmatrix}$$

3 x 2

$$\begin{pmatrix} a_{11} \cdot b_{11} + a_{12} \cdot b_{21} + a_{13} \cdot b_{31} & a_{11} \cdot b_{12} + a_{12} \cdot b_{22} + a_{13} \cdot b_{23} \\ a_{21} \cdot b_{11} + a_{22} \cdot b_{21} + a_{23} \cdot b_{31} & a_{21} \cdot b_{12} + a_{22} \cdot b_{22} + a_{23} \cdot b_{23} \\ a_{31} \cdot b_{11} + a_{32} \cdot b_{21} + a_{33} \cdot b_{31} & a_{31} \cdot b_{12} + a_{32} \cdot b_{22} + a_{33} \cdot b_{23} \end{pmatrix}$$

3 x 2

# Matrix Multiplication via MapReduce

## Input

Two files

Row-wise format

Includes row number

# Input Version 1

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{23} & a_{33} \end{pmatrix}$$

**X**

$$\begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \\ b_{31} & b_{32} \end{pmatrix}$$

**A.csv**

```
1, a11, a12, a13  
2, a21, a22, a23  
3, a31, a32, a33
```

**B.csv**

```
1, b11, b12  
2, b21, b22  
3, b31, b32
```

# Input Version 1

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{23} & a_{33} \end{pmatrix}$$

**X**

$$\begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \\ b_{31} & b_{32} \end{pmatrix}$$

A.csv

```
1, a11, a12, a13
2, a21, a22, a23
3, a31, a32, a33
```

B.csv

```
1, b11, b12
2, b21, b22
3, b31, b32
```

# Matrix Multiplication via MapReduce: Mappers

A.csv

```
1, a11, a12, a13  
2, a21, a22, a23  
3, a31, a32, a33
```

map(**key**, **value**)



# Matrix Multiplication via MapReduce: Mappers

A.csv

```
1, a11, a12, a13  
2, a21, a22, a23  
3, a31, a32, a33
```

Let's use THE SAME idea as for moving average

```
map(key, value)
```

# Matrix Multiplication via MapReduce: Reduce!

A.csv

```
1, a11, a12, a13  
2, a21, a22, a23  
3, a31, a32, a33
```

Let's use THE SAME idea as for moving average

**reduce**(key, Iterable values)

# Matrix Multiplication via MapReduce: Reduce!

A.csv

1,	a11,	a12,	a13
2,	a21,	a22,	a23
3,	a31,	a32,	a33

B.csv

1,	b11,	b12
2,	b21,	b22
3,	b31,	b32

**reduce**(key, Iterable values)

key    **(rowID, columnID)**

values

[ (1,A, a11), (2,A, a12), (3,A,a13), (1,B, b11), (2,B, b21), (3,B,b31) ]

**Position, Matrix, Value**

# Matrix Multiplication via MapReduce: Reduce!

A.csv

```
1, a11, a12, a13  
2, a21, a22, a23  
3, a31, a32, a33
```

B.csv

```
1, b11, b12  
2, b21, b22  
3, b31, b32
```

**reduce**(key, Iterable values)

a11 a12 a13

x x x

b11 b21 b31

# Matrix Multiplication via MapReduce: Reduce!

A.csv

```
1, a11, a12, a13  
2, a21, a22, a23  
3, a31, a32, a33
```

B.csv

```
1, b11, b12  
2, b21, b22  
3, b31, b32
```

**reduce**(key, Iterable values)

+

a11 a12 a13

x x x

b11 b21 b31

# Matrix Multiplication via MapReduce: Reduce!

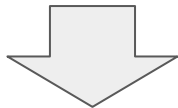
A.csv

1,	a11,	a12,	a13
2,	a21,	a22,	a23
3,	a31,	a32,	a33

B.csv

1,	b11,	b12
2,	b21,	b22
3,	b31,	b32

**reduce**(key, Iterable values)



[(1,A, a11), (2,A, a12), (3,A,a13), (1,B, b11), (2,B, b21), (3,B,b31)]

Identify Source Matrix

Identify Position for dot-product computation

Identify Value

# Matrix Multiplication via MapReduce: Reduce!

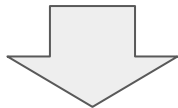
A.csv

```
1, a11, a12, a13  
2, a21, a22, a23  
3, a31, a32, a33
```

B.csv

```
1, b11, b12  
2, b21, b22  
3, b31, b32
```

**reduce**(key, Iterable values)



[ (1,1,A, a11), (1,2,A, a12), (1,3,A,a13), (1,1,B, b11), (1,2,B, b21), (1,3,B,b31) ]

Identify Source Matrix  
Identify Position for dot-product computation  
Identify Value



map()

# Matrix Multiplication via MapReduce: Mappers

A.csv

```
1, a11, a12, a13  
2, a21, a22, a23  
3, a31, a32, a33
```

a11 is needed for computing cells (1,1), (1,2)

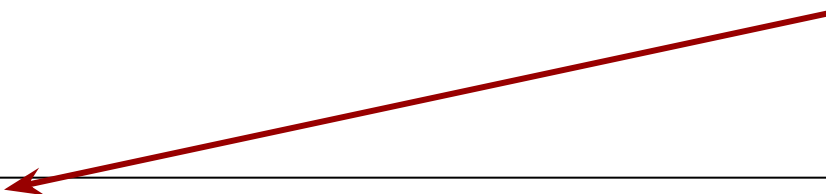
```
mapA(key, value)  
// output the values for each computation they are in  
// key = rowId  
// reduce key is <rowId, colId>  
for each pos = 1 to rowSize do  
  cell = value[pos]  
  for column = 1 to maxColumn do  
    emit((key,column), ("A",pos, cell))  
  end for  
end for
```



# Matrix Multiplication via MapReduce: Mappers

B.csv

1, b11, b12
2, b21, b22
3, b31, b32



```
mapB(key, value)
// output the values for each computation they are in
// key = rowId
// reduce key is <rowId, colId>
for each pos = 1 to rowSize do
  cell = value[pos]
  for column = 1 to maxColumn do
    emit((pos,column), ("B",row, cell))
  end for
end for
```

# Matrix Multiplication via MapReduce: Mappers

b1<sup>1</sup> needs to go to reduce keys (1,<sup>1</sup>), (2,<sup>1</sup>), (3,<sup>1</sup>)

b1<sup>2</sup> needs to go to reduce keys (1,<sup>2</sup>), (2,<sup>2</sup>), (3,<sup>2</sup>)

B.csv

1, b11, b12

2, b21, b22

3, b31, b32

```
mapB(key, value)
```

```
// output the values for each computation they are in
```

```
// key = rowId
```

```
// reduce key is <rowId, colId>
```

```
for each pos = 1 to rowSize do //rowSize is for B
```

```
  cell = value[pos]
```

```
  for row = 1 to maxRow do //maxRow is for A
```

```
    emit((row,pos), ("B",key, cell))
```

```
  end for
```

```
end for
```

# Distributed Batch Processing - MapReduce

- Partitioning

- Splitting data to make high-volume storage possible and to allow efficient distributed processing: (a) input data partitioning, and (b) repartitioning for processing by multiple reducers

- Fault Tolerance

- MapReduce writes to disk ("materializes" data) frequently. This simplifies recovery from task failure (hardware, software, network, human) at the cost of slower overall processing