1 Forces

Three forces act on the driver of a speaker. We want the driver to **only** be affected by Force #1, the Electromagnetic Force, but this is impossible. In essence, the driver is affected by two other forces that distort the sound: the Damping Force and Spring/Restorative Force.

These extra forces make the real-world driver sound bad. As a goal, we want to eliminate these extra forces, to make the acceleration of the real-world driver mimic the voltage of the audio signal.

1.1 Electromagnetic Force

$$F_E = k_E V = m a_E \tag{1}$$

$$V = \frac{m a_E}{k_E} \tag{2}$$

1. F_E is the electromagnetic force in Newtons
2. k_E is a constant, called the **Electromagnetic Force Constant** in units Newtons/Volt
3. V is the voltage that is encoded in the audio signal in Volts

1.2 Damping Force

$$F_f = -k_f v \tag{3}$$

1. $F_f =$ is the damping/frictional force in Newtons
2. k_f is a constant, called the **Damping Force Constant**
3. v is the velocity of the driver

1.3 Spring/Restorative Force

$$F_s = -k_s x \tag{4}$$

1. F_s is the spring force
2. k_s is the a constant, called the **Spring Constant**
3. x is the displacement of the driver from its resting position

2 Adjusting for These Extra Forces

The electromagnet in the speaker is given a voltage and produces a force which accelerates the cone which produces sound. By Newton's Law

$$F_{\text{net}} = ma \tag{5}$$
$$F_{\text{net}} = F_E + F_f + F_s \tag{6}$$

1. F_{net} is the net force acted on the cone
2. m is the total summed moving mass
3. a is the acceleration that makes the cone move

Thus, the net force of the driver should be proportional to the voltage of the original signal.

Let's let V_i be the voltage of the input signal. We want V_i to be proportional to the acceleration of the driver. The acceleration of the driver, as shown above, is proportional to the Net Force that is acted upon the driver.

Let V_o be the output voltage for the electromagnetic force, as seen in Section 1.1 Equations 1 and 2.

$$a = \text{acceleration} \tag{7}$$
$$v = \text{velocity} = \int a\,dt \tag{8}$$
$$x = \text{position} = \int v\,dt = \iint a\,dt \tag{9}$$

$$V_i = c(F_{\text{net}}) \tag{10}$$
$$F_{\text{net}} = F_E + F_f + F_s \tag{11}$$
$$= k_E V_o - k_f v - k_s x \tag{12}$$

2.1 Compensating for the Damping Force With an Equal and Opposite Force

Let's solve a different equation first. Let V_f be the voltage required to generate an electromagnetic force equal and opposite to the damping force.

$$k_f v = k_E V_f \tag{13}$$
$$V_f = \frac{k_f v}{k_E} = \frac{k_f}{k_E} v \tag{14}$$

By equation 1, $F = K_e V_i$ so

$$a = \frac{F}{m} = \frac{k_e V_i}{m} \tag{15}$$

$$v = \int \frac{k_e V_i}{m}\, dt \tag{16}$$

$$= \frac{k_E}{m} \int V_i\, dt \tag{17}$$

Thus, substituting this in to Equation #14 yields

$$V_f = \frac{k_f}{k_E} \times \frac{k_E}{m} \int V_i\, dt \tag{18}$$

$$= \frac{k_f}{m} \int V_i\, dt \tag{19}$$

2.2 Compensating for the Spring Force With an Equal and Opposite Force

Now let V_s be the voltage required to generate an electromagnetic force equal and opposite to the spring force.

Using Equation #17 where $v = \frac{k_E}{m} \int V_i\, dt$,

$$x = \int v\, dt = \frac{k_E}{m} \iint V_i\, dt \tag{20}$$

$$k_s x = k_E V_s \tag{21}$$

$$V_s = \frac{k_s}{k_E} x \tag{22}$$

$$= \frac{k_s}{k_E} \times \frac{k_E}{m} \iint V_i\, dt \tag{23}$$

$$= \frac{k_s}{m} \iint V_i\, dt \tag{24}$$

3 Final Compensated Output Voltage

Because $F_{\text{net}} = k_E V_o - k_f v - k_s x$ and $k_f v = k_E V_f$ and $k_s x = k_E V_s$,

$$F_{\text{net}} = k_E V_i = k_e V_o - k_E V_f - k_E V_s \tag{25}$$

$$V_o = V_i - V_f - V_s \tag{26}$$

Substituting our earlier equations, we get our final compensated output voltage V_o:

$$V_o = V_i - \frac{k_f}{m} \int V_i \, dt - \frac{k_s}{m} \iint V_i \, dt \tag{27}$$

As a reminder, our variables are

1. V_o is the output voltage, basically what we're going to send to the speaker
2. V_i is what the audio file or audio source has encoded
3. k_f is the damping force constant, which depends on the driver
4. m is the moving mass (Mms)
5. k_s is the spring constant, which depends on the driver
6. t is time
    - Time will be from the beginning of the listening session in order to get the correct velocity and position

3.1 Finding the Constants

3.1.1 The Spring Constant k_s

$$k_s = m(2\pi f_{sc})^2 \tag{28}$$

A couple variables are used here:

1. f_{sc} is the Resonant Frequency of the driver
2. m is the Thiele-Small parameter (Mms)

3.1.2 The Damping Force Constant k_f

$$\zeta = \frac{1}{2Q} \tag{29}$$

$$\zeta = \frac{k_f}{c_c} \tag{30}$$

$$c_c = 2\sqrt{k_s m} \tag{31}$$

$$k_f = \frac{\sqrt{k_s m}}{Q} \tag{32}$$

1. ζ is an intermediary variable, which represents the damping ratio
2. k_f is the actual damping coefficient
3. c_c is the critical damping coefficient
4. k_s is the spring coefficient
5. Q is the quality factor of the uncompensated system (Qtc)
6. m is the Thiele-Small parameter (Mms)

3.2 Incorporating It

We have our input voltage V_i and output voltage V_o. V_o is the compensated voltage. In other words, V_o is what we're going to send to the speaker, whose spring and damping forces will be cancelled by the adjustments made. This means the actual output of the speaker (in units of acceleration) will be proportional to the voltage of the original, input signal.

We can't really represent the input voltage as a function, so taking the integral won't be like in calculus class. We'll just use tools to get points of the input at very frequent rates. Let the sampling rate s be the number of samples in per second, thus s has units Hz. For example, a sampling rate of $s = 44.1\,\text{kHz}$ is used in a .wav file of a 100 Hz tone. The sample rate matters for when we calculate the integral.

3.2.1 Required Parameters

1. Audio input signal
2. Q, the uncompensated quality factor (Qtc)
3. m, the moving mass and air load (Mms)
4. f_{sc}, the resonant frequency of the driver

3.2.2 Calculating Integrals

Using the Riemann sum, we can get a very good estimation of the integral.

3.3 Fighting DC Offset

To filter out the DC Offset, we must use a high order butterworth highpass at 20 Hz. This also protects the drivers. If the subwoofer wants to go under 20 Hz, we might adjust this value.